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Over-aging - Are present human populations too old?*

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Abstract

This paper investigates the problem of an “optimum population” with respect to the age structure. Within a 3-period OLG model, with endogenous fertility and longevity, the optimal age structure, identified by number-dampened total utilitarianism, is generally failed in the laissez-faire economy. The individual decisions on the number of offspring as well as on health expenditures are biased. Tendencies concerning the distortions of the age structures are identified by decentralizing the first-best solution. A calibration of the model for 84 countries emphasizes that mean age in “Golden Age” always exceeds the observed, especially due to a very low fertility. Introducing a preference for the population stock increases the number of children. As optimal mean age shrinks, an over-aging of the laissez-faire economy becomes likely. To decentralize the optimal age structure, children are either taxed or subsidized, whereas health expenditures are taxed.

Keywords: endogenous fertility; adult mortality; optimal age structure; over-aging; optimal taxation

JEL classification: H20; I10; J18

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1 Introduction

The demographic transition, most countries underwent during the last 150 years, leads to a variety of age structures, illustrated in figure 1. Developed Countries, like in Europe, are plotted on the upper left in the left picture. The high fraction of old individuals arouse fear of an over-aged population. On the contrary, developed countries on the lower right, featured by many children and a low survival probability, are seen to be too young. Since both suggest sub-optimal situations, this papers aims to highlight the direction and extent of the differences between observed and optimal age structures. A discussion of optimal age structures and their divergence from observed situations visualized in fig. 1 is the main contribution of the paper.

Varying age structures caused by different timings of the onset of the demographic transition go along with specific social and political challenges. Developed economies are illustrated on the RHS of figure 1 by the European countries. Low fertility and mortality imply relatively high dependency ratios and mean ages. Due to intergenerational transfers, demographic aging involves strong economic effects (Weil, 2006). The high dependency ratios have consequences on old age security, health care and labor markets and are seen as a risk on social and individual welfare quite often. On the contrary, the transition started more recently in developing economies. Mean ages are still low, but dependency ratios likewise high. The large number of births combined with a low life expectancy is as challenging to societies and especially policy makers as over-aged populations. However, each policy implemented to avoid a population that is either over-aged or too young requires a point of reference; an optimal age structure.

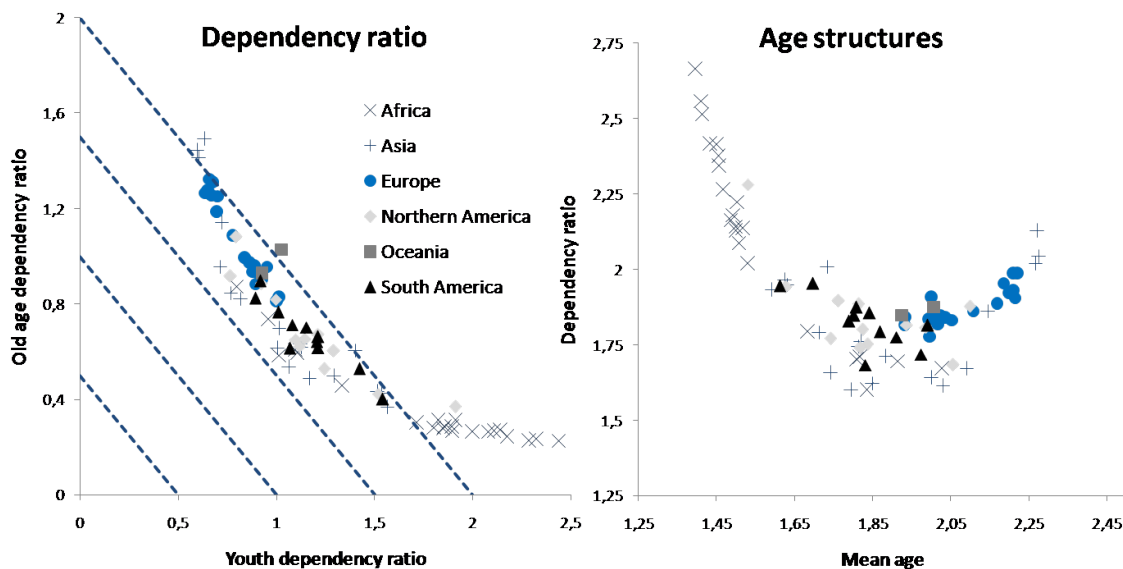


Figure 1: Steady state age structures across countries

The figure presents the stable populations arising from NRR (Net Reproduction Rate) and female life expectancies at age 60, according to section 3.1. The LHS illustrates dependency ratios across countries distinguished by youth and old age dependency ratio. Their overall value increases with distance to the point of origin, see Weil (2006). The U-shaped relation between mean ages and overall dependency ratios is plotted on the RHS. (Data source: United Nations (2011))

An Overlapping-generations model is chosen to investigate the optimality of age structures. Comparing a laissez-faire (LF) and first-best (FB) economy enables to evaluate the discrepancy

between the observed and optimal situation. The laissez-faire solution reflects the age structure endogenously determined by individual decisions. Selfish representative individuals choose the number of children and health expenditures to influence longevity.¹ A flexible approach is used in the first-best solution. The social welfare function (SWF) follows number-dampened total utilitarianism (Ng, 1986). Hence, maximizing social welfare is defined as the objective for optimality. Comparing both economies points out, that the optimal solution is generally failed by individuals, due to distorted decisions on fertility and mortality as well as different time horizons of the social planner and individuals. A decentralization of the first-best solution by means of health subsidies, child allowances and lump-sum transfers already enables to identify tendencies considering a potential over-aging. Afterwards the laissez-faire economy is calibrated to reproduce the age structures of 84 countries presented in figure 1. The resulting parameter set permits to compute the optimal age structures in “Golden Age” - the optimal solutions with the highest feasible individual life-cycle utility among all first best - as point of reference. Mean ages in all countries are considerably above the observed in “Golden Age”, whereas the findings on the dependency ratio depend on the observed level of fertility. Even if “Golden Age” implies the highest expected life-cycle utility, the results are striking from a demographic point of view. Fertility is substantially below the replacement level. Introducing a preference for the population stock and/or a social discount factor (SDF) in the SWF increases fertility and alters the age structure. If the weight of the population in the SWF is sufficiently high, outcome changes and populations are over-aged. However, both the weight on the population size and the SDF are fixed arbitrary. Finally, the findings lead to the following dilemma: The “Golden Age” is favorable from an ethical and economic point of view but problematic from a demographic perspective. On the contrary, fixing the weight of the population size and the SDF arbitrarily facilitates plausible demographic results but is problematic from an ethical point of view.

The outline of the paper is as follows. Before introducing the theoretical framework in section 3, the related literature is reviewed. The first subsection (3.1) presents the demographic structure of the OLG model. Afterwards the solution chosen by individuals in a laissez-faire economy (3.2) and the optimal decisions of a social planner (3.3) are evolved. A decentralization of the first-best solution in subsection (3.4) enables to discuss first tendency concerning an over-aged population. Section 4 presents the calibration exercise. After describing the data set (4.1), the laissez-faire model is calibrated in subsection 4.2. Subsection 4.3 compares the observed and optimal age structures and discusses the outcomes. Section 5 concludes.

2 Related literature

The idea of an optimal age structure contributes to the literature on “optimum population” traditionally dealing with population size by applying the “economic principle” on the population (Wolf, 1908). In other words: “An optimum population is the one that achieves a given aim in the most satisfactory way” (Sauvy, 1969). Rawls’ Maximin principle, maximization of output, military power or social welfare, are possible objectives for the optimization (see e.g. Sauvy (1969)). A common approach in (at least more recent) theories on optimal populations is the maximization of a social welfare function, whereby different types exist. A Millian or Average SWF optimizes utility of an average or representative individuals. The population size does not matter, but is fully internalized in Benthamite or Classical utilitarianism. The sum over welfare or utility of all individuals is maximized. However, both types of welfare functions may lead to undesirable out-

¹An alternative is to focus on the relation between child mortality and fertility, see e.g. Becker and Barro (1988), Soares (2005) and Baudin (2012).

comes: a very small number of individuals characterized by a high utility for Average SWF or the emergence of the repugnant conclusion in total utilitarianism (Parfit [1976, 1982, 1984]). A very large number of individuals, featured by a very low individual utility or welfare level is optimal. Modifying the SWF prevents this problematic outcomes. Blackorby and Donaldson (1984) propose a critical-level generalized utilitarianism to avoid a population that is too large. Individual utility only contributes positively to the SWF if it exceeds a critical level. We choose another flexible approach, that follows the idea of Ng (1986). The so called number-dampened total utilitarianism allows different weights of population size in the SWF; between the two extreme cases of no weight at all for Millian and a linear weight for Benthamite utilitarianism.

Investigating the growth rate adds the dynamic perspective to theories of optimum populations. For an OLG-framework with two generations the famous “serendipity theorem” (Samuelson, 1975) claims that in the case of a unique and stable steady state the laissez-faire economy converges to the most advantageous golden rule steady state, the “Golden Age”, if fertility is fixed optimally. Unfortunately, supposing a Cobb-Douglas utility and production function leads to a global minimum (Deardorff, 1976). The exact general conditions for an interior optimal growth rate were added by Jäger and Kuhle (2009). Unfortunately, this model in general tends to an overestimation of the optimal population growth rate since child rearing is free of cost. Only the negative capital widening and positive intergenerational transfer effect are balanced (Arthur and McNicoll, 1978). The “serendipity theorem” still holds if mortality, and hence an uncertain life time for the second period, is added (de la Croix *et al.*, 2012). However, the survival probability and child-rearing still cause no costs.

Based on the OLG-framework, the idea of an optimal age structure introduces a third aspect to theories of “optimum population”. Considering an age structure, individuals not only evolve along the dimension time. Individual aging adds another dimension (Arthur and McNicoll, 1977) generally neglected in theories of optimal size. Indeed, population size, growth rate as well as age structure are jointly determined by the interaction of fertility and mortality. As theories on optimal population should consider these demographic processes, the paper is linked to frameworks on an optimal fertility and longevity.

Different approaches exist to model endogenously the number of birth. Like Razin and Ben-Zion (1975), Becker and Barro (1988) and Baudin (2011), the framework of Nerlove *et al.* (1986) uses altruistic parents to compare individual and optimal fertility. The utility of the offspring is one argument in their parents’ utility function. This kind of perfect altruism leads to dynastic preferences such that the time horizon of the parents is infinite. The socially optimal level of fertility is strongly related to the choice of the SWF. In general, fertility and hence the population size of a Benthamite SWF exceeds those of a Millian SWF. However, the number of children in the LF economy is either above or below the optimal level in both kinds of SWF (Nerlove *et al.*, 1985). Optimal family policies to decentralize the FB fertility in an altruistic framework, considering a quality quantity trade off, has to account for the interaction between the instruments on education and fertility, due to the non-linear budget constraint (Baudin, 2011). Totally selfish parents are a second possibility, used in the framework of Eckstein and Wolpin (1985). In contrast to altruistic individuals, time horizon is limited by death and therefore finite.² Related to the idea of an optimal fertility, missing property rights, causing an inefficiently low number of offspring, are another possible explanation for the low fertility in developed countries (Schoonbroodt and Tertilt, 2013).

²Different frameworks of selfish parents are detectable in the literature. Children either appear in the utility function as a kind of consumption good, like in Eckstein and Wolpin (1985), Galor and Weil (1996) or van Groezen and Mejidam (2008), or they are considered as an investment good, as presented in Neher (1989) or Bental (1989). A third possibility are “warm glove” preferences or ad hoc altruism. Parents achieve utility not only by the number of children, but also by an argument of their children’s utility function. For an example see Benabou (1996).

Mortality is a very complex phenomenon, influenced by many variables like nutrition, medical care, education and so on (Cutler *et al.*, 2006). Endogenous health expenditures are one possibility to model an endogenous length of life. Individuals invest in their health capital stock, simultaneously depreciated age specifically in each period. They die, if the health capital stock is zero (Grossman (1972), Ehrlich and Chuma (1990), ?). However, the optimal survival probability might be failed as several effects, like externalities in preventing contagious diseases or the negative impact of health expenditures on the life-cycle income, are not taken into account (Davies and Kuhn (1992), Philipson and Becker (1998)).

3 The model

3.1 The demographic structure

The model is an extension of the overlapping generations model introduced in the literature by Allais (1947), Diamond (1965) and Samuelson (1958) in discrete time t from 0 to infinity. For simplicity, a single sex population in a closed economy is assumed. The age structure is pooled in three age groups, children, working population and retirees. Hence, at any point in time three generations coexist. The cohort born in period t constitutes the children N_t^t .³ Childhood as first period of life is passive. Simultaneously the cohort N_t^{t-1} born in $t - 1$ is in middle age as working adults. During this active period of life, individuals solve their maximization problem endowed with one unit of time, allocated on child-rearing and labor. As the old generation N_t^{t-2} is retired and does not participate in labor market, total labor supply is:

$$L_t = (1 - \tau n_t) N_t^{t-1}, \quad (1)$$

where τ represents the time required to bring up a child. The size of a cohort born in t is determined by the number of children in t , hence $N_t^t = n_t N_t^{t-1}$. Whereas all individuals survive childhood and undergo middle age, only a fraction q_t becomes old: $N_t^{t-2} = q_t N_{t-1}^{t-2}$.⁴ Thus, the age structure is as simple as possible and composed of the three at least necessary age groups to describe a human life-cycle: the working period including parenthood surrounded by the two periods in dependency, childhood and retirement (Bommier and Lee, 2003).

The dependency ratio (DR_t) and the mean age (\bar{x}_t) are used to describe age structures. The former indicator links economic and demographic aspects. Individuals participating in the labor market are compared to non-working and thus dependent people. Due to interaction between the generation, e.g. intergenerational transfers, this indicator is of particular interest for policy makers. Applied to the population of the model, the dependency ratio is:

$$DR_t = \frac{N_{t,t-2} + N_{t,t}}{N_{t,t-1}}.$$

Rearranging allows to express the dependency ratio in terms of the survival probability and the number of children:

$$DR_t = \frac{q_t + n_{t-1} n_t}{n_{t-1}}. \quad (2)$$

It has to be mentioned that the term “dependent” is deceptive in the particular case of the presented framework. Indeed, the old generation does not participate in the labor market. However, retirees

³The subscript indicates the period and the superscript the birth cohort.

⁴Likewise $N_t^{t-2} = q_t N_{t-1}^{t-2}$ holds, as the survival probability between childhood and middle age is 1.

are not supported by the working generation. Nevertheless, it is not possible to conclude on the age of a population by means of the dependency ratio. Both children and retirees are elements of the numerator. Thus a high value can be explained either by a very young or old population, as emphasized on the RHS of figure 1. In other words, an age structure with the shape of a pyramid as well as an inverted pyramid can have the same dependency ratio. Moreover, this demographic indicator is inappropriate to describe the process of aging. Therefore the mean age is additionally considered.⁵ Applied to the model the mean age is calculated as:

$$\bar{x}_t = \frac{\sum_{j=1,2,3} j N_{t,t-j+1}}{\sum_{j=1,2,3} N_{t,t-j+1}},$$

whereby young individuals are defined to be at age $j = 1$. Accordingly, middle age is 2 and old age 3. Rearranging leads to:

$$\bar{x}_t = \frac{3q_t + 2n_{t-1} + n_{t-1}n_t}{q_t + n_{t-1} + n_{t-1}n_t}.$$

Hence, mean age and dependency ratio are expressed in terms of the survival probability and the individual number of children. As both are endogenous variables, the age structure is endogenous as well.

3.2 The laissez-faire economy

3.2.1 Production and capital market

Production follows the common neoclassical OLG framework (see e.g. de la Croix and Michel (2002)). At each point in time homogeneous firms produce a single commodity Y_t^{LF} with capital K_t^{LF} and labor L_t^{LF} by means of a neoclassical production function with constant returns to scale. Since capital is fully depreciated every period, $Y_t^{\text{LF}} = F(K_t^{\text{LF}}, L_t^{\text{LF}})$ is the net production. Dividing by labor yields production in intensive terms $f(k_t^{\text{LF}}) = F\left(\frac{K_t^{\text{LF}}}{L_t^{\text{LF}}}, 1\right)$, where k_t^{LF} is the capital-labor ratio. The representative firm maximizes its profits in fully competitive markets. Labor is provided by the generation in middle age and remunerated by the wage rate w_t^{LF} . The interest factor R_t^{LF} is paid on capital. The maximization problem of firms in intensive terms is:

$$\max_{k_t^{\text{LF}}} = f(k_t^{\text{LF}}) - R_t^{\text{LF}} k_t^{\text{LF}} - w_t^{\text{LF}}.$$

Production factors are compensated by their marginal products:

$$R_t^{\text{LF}} = f'(k_t^{\text{LF}}), \tag{3}$$

$$w_t^{\text{LF}} = f(k_t^{\text{LF}}) - k_t^{\text{LF}} f'(k_t^{\text{LF}}). \tag{4}$$

⁵Alternatively, a decomposition in young and old age dependency ratio solves, as illustrated in figure 1, allows to conclude on the age structure (see e.g. Weil (2006)). Nevertheless, the mean age is preferred, as this single indicator measures the full age structure.

Savings of the working generation are invested and constitute productive capital in the next period $S_t^{\text{LF}} = K_{t+1}^{\text{LF}}$. Therefore, capital of firms is owned by the current old generation. Since labor is determined by equation 1, the capital-labor ratio is:

$$k_t^{\text{LF}} = \frac{K_t^{\text{LF}}}{L_t^{\text{LF}}} = \frac{K_t^{\text{LF}}}{(1 - \tau n_t^{\text{LF}}) N_t^{t-1}}.$$

Hence, the labor decreasing effect, caused by the time needed to rear a child, is considered. The equilibrium of the capital market in terms of individuals in working age is:

$$s_t^{\text{LF}} = (1 - \tau n_{t+1}^{\text{LF}}) n_t^{\text{LF}} k_{t+1}^{\text{LF}}. \quad (5)$$

Savings per capita of a middle-aged individual in t are equal to capital per individual in working age in $t + 1$.

3.2.2 Individuals

The representative individual lives at most three periods, childhood, working age including parenthood and retirement. She maximizes her expected life-cycle utility at the beginning of the working period and obtains utility from consumption in middle c_t^{LF} and old age d_{t+1}^{LF} . Furthermore, parents receive a “consumption” utility from rearing their children n_t^{LF} , weighted in relation to those of material consumption $u(c_t^{\text{LF}})$ by $\gamma > 0$. For simplicity, only the pure number of own children generates utility. Hence, no quality quantity trade off is considered.

With all individuals undergoing child- and parenthood, to experience retirement is uncertain. If she is alive, she consumes d_{t+1}^{LF} and receives a utility discounted by $0 < \beta \leq 1$. Otherwise if she dies, her utility is implicitly assumed to be zero. The survival probability q is a strictly increasing function of health expenditures (h_t^{LF}) during middle age and upper bounded by one:

$$q'(h) > 0, \quad q(h \rightarrow \infty) = 1.$$

Like in Leung and Wang (2010) individuals themselves choose health expenditures, which are a pure private good. As she is risk neutral with respect to longevity her expected life-cycle utility function is additive and time separable:⁶

$$EU_t = u(c_t^{\text{LF}}) + \beta q(h_t^{\text{LF}}) u(d_{t+1}^{\text{LF}}) + \gamma v(n_t^{\text{LF}}) \quad (6)$$

and is maximized with respect to the budget constraints in working age and retirement:

$$(1 - \tau n_t^{\text{LF}}) w_t^{\text{LF}} = c_t^{\text{LF}} + a n_t^{\text{LF}} + s_t^{\text{LF}} + h_t^{\text{LF}}, \quad (7)$$

$$d_{t+1}^{\text{LF}} = \tilde{R}_{t+1}^{\text{LF}} s_t^{\text{LF}}. \quad (8)$$

In midlife individuals are endowed with one unit of time used either to work or to bring up children. Child rearing causes two kinds of costs, exogenous goods cost a , like in van Groezen and Mejidam (2008), and time cost τ , see for example de la Croix and Doepke (2003). For common utility functions the number of children depends positively on labor income for the former case and negatively

⁶Risk neutral individuals with respect to longevity are a common assumption in literature. Supposing risk neutrality facilitates a time separable and additive life-cycle utility function in contrast to risk averse individuals (Bommier, 2006). In the more general case of risk aversion the shift of certain utilities is concave. Kuhn *et al.* (2010) show that in the case of constant health expenditures the optimal savings are lower in the case of risk aversion. The effect on health expenditures is unambiguous, if savings are assumed to be constant.

for the latter. Furthermore, time cost fixes an upper bound on the individual number of children $n_t^{LF} < \frac{1}{\tau}$. Labor income is spent on consumption in middle age c_t^{LF} , direct child cost in terms of goods an_t^{LF} , health expenditures h_t^{LF} and savings for old age consumption s_t^{LF} . Supposing perfect annuity markets with an annuity for life (Yaari, 1965), the return factor on savings \tilde{R}_t^{LF} depends on the average survival probability of individuals:

$$\tilde{R}_t^{LF} = \frac{R_t^{LF}}{q(h_{t-1}^{LF})}. \quad (9)$$

Taking prices as given, maximizing expected life-cycle utility with respect to health expenditures, the number of children, consumption in working age and retirement yields to the following set of first-order conditions (FOC):

$$u'(c_t^{LF}) = \beta \tilde{R}_{t+1}^{LF} q(h_t^{LF}) u'(d_{t+1}^{LF}), \quad (10)$$

$$u'(c_t^{LF}) = \frac{\gamma}{a + \tau w_t^{LF}} v'(n_t^{LF}), \quad (11)$$

$$u'(c_t^{LF}) = \beta q'(h_t^{LF}) u(d_{t+1}^{LF}). \quad (12)$$

The strictly positive set of variables $\{c_t^{LF}, d_t^{LF}, h_t^{LF}, k_t^{LF}, n_t^{LF}, s_t^{LF}, w_t^{LF}, R_t^{LF}\}_{t=0}^{t=+\infty}$, solving the system of FOCs (eq. 10-12) as well as the equations of wage (eq. 4), return factor on savings (eq. 9), capital market equilibrium (eq. 5) and budget constraints (eq. 7, 8) define the interior solution of the laissez-faire economy.⁷ The first FOC (eq. 10) is the familiar Euler equation describing the distribution of consumption between midlife and old age. The individual decision on the number of children is illustrated by the second FOC (eq. 11). The marginal costs of children, goods ($au'(c_t^{LF})$) as well as time cost ($\tau w_t^{LF} u'(c_t^{LF})$) balance the marginal utility of a child ($\gamma v'(n_t^{LF})$). The decision on health expenditures and hence on the survival probability is presented by the third FOC (eq. 12). Individuals compare the additional utility, due to a higher survival probability in old age, with the loss of consumption possibilities in middle age. The gain achieved from the last marginal unit of health expenditures ($\beta q'(h_t^{LF}) u(d_{t+1}^{LF})$) equalizes its opportunity cost ($u'(c_t^{LF})$).⁸ Thus, individuals do not take into account the influence of the changing survival probability on life-cycle income. The rising survival probability shrinks returns on savings \tilde{R}^{LF} . This reduces consumption possibilities during retirement and thus life-cycle income. Hence, individuals do not internalize the so-called Philipson-Becker-effect (Philipson and Becker, 1998).

3.3 The first-best solution

A point of reference is required to evaluate the age structures reflecting individual decisions. This optimal or first-best age structure results from a SWF following the idea of number-dampened total utilitarianism (Ng, 1986). Expected life-cycle utility of the current and all future generations, weighted by the size of the birth cohorts and the social discount factor ρ , is maximized:

$$W = \sum_{t=0}^{\infty} \rho^t (N_t^{t-1})^x EU(c_t^{FB}, d_t^{FB}, n_t^{FB}, h_t^{FB}), \quad \text{with} \quad 0 \leq x \leq 1, 0 < \rho \leq 1$$

⁷The Hessian-matrix, required to control for the second-order conditions (SOC), is presented appendix A.

⁸As $u'(c_t) > 0$, a positive level of utility in old age $u(d_{t+1}) > 0$ is required. Intuitively, rational individuals only invest in health, if they enjoy old age.

The characteristic feature of number-dampened total utilitarianism, a concave weight of population size in the welfare function, holds for $0 < x < 1$. Furthermore, the two extreme cases of Average and Total utilitarianism are included. Population size doesn't matter at all, if the planner is Millian ($x = 0$). Only the discounted life-cycle utility of a representative individual of each birth cohort is taken into account. On the contrary, a Benthamite Planner ($x = 1$) considers the discounted life-cycle utility of each born individual. Thus number-dampened total utilitarianism has the advantage to imply the two extreme and the whole spectrum of intermediate cases.

As the time horizon of the planner's dynamic maximization problem is infinite, an upper bound requires $\rho < n^{-x}$. Certainly, a discounting of future generation's utility $\rho < 1$ is problematic from an ethical point of view. Blackorby *et al.* (2005) e.g. claim that "for the purpose of social evaluation, the well-being of future generations should not be discounted." A different weight of present and (unborn) future individuals in the SWF is hardly defensible. Additionally, the choice of the social discount factor (SDF) is arbitrary. One way to avoid any kind of social discounting is to convert the dynamic maximization problem of the Millian planner into a static. This "Golden Age" situation is treated as the point of reference.⁹

The planner includes the preferences of the selfish individuals of each generation, implying that both, individuals and the planner, have different time horizons. An individual only achieves utility in periods, she is alive. In contrast, the planner has a dynastic function. Due to the varying preferences, solutions differ, even if there is no externality or imperfection in the economy. Considering the preferences of the representative individual and rearranging from a longitudinal in a cross-section point of view results in:

$$W = \sum_{t=0}^{\infty} \rho^t \left[(N_t^{t-1})^x [u(c_t^{\text{FB}}) + \gamma v(n_t^{\text{FB}})] + (N_{t-1}^{t-2})^x \frac{\beta}{\rho} q(h_{t-1}^{\text{FB}}) u(d_t^{\text{FB}}) \right],$$

where consumption in working age of the first old generation (c_{-1}^{FB}) is not taken into account and the initial health expenditures (h_{-1}^{FB}) are preexisting. The social planner's resource constraint at time t is:

$$Y_t = N_t^{t-1} (c_t^{\text{FB}} + an_t^{\text{FB}} + h_t^{\text{FB}}) + K_{t+1}^{\text{FB}} + N_t^{t-2} d_t^{\text{FB}}.$$

The initial capital stock K_0 and the two cohorts N_0^{-1}, N_0^{-2} are as well historically given. Aggregated production Y_t is allocated on consumption c_t^{FB} , health expenditures h_t^{FB} and exogenous goods cost to bring up children an_t^{FB} for all adults N_t^{t-1} as well as for consumption of the retirees alive $d_t^{\text{FB}} N_{t,t-2}$. Additionally, output is used to install capital for production in the next period K_{t+1} . Dividing by the size of the cohort N_t^{t-1} leads to the resource constraint in terms of individuals in working age. In doing so, time cost to raise a child has to be taken into account. Hence, output in terms of working individuals is $y_t^{\text{FB}} = (1 - \tau n_{t+1}^{\text{FB}}) f(k_t^{\text{FB}})$ and the resource constraint at t :

$$y_t^{\text{FB}} = c_t^{\text{FB}} + an_t^{\text{FB}} + (1 - \tau n_{t+1}^{\text{FB}}) n_t^{\text{FB}} k_{t+1}^{\text{FB}} + q(h_{t-1}^{\text{FB}}) \frac{d_t^{\text{FB}}}{n_{t-1}^{\text{FB}}} + h_t^{\text{FB}}. \quad (13)$$

In order to maximize the social welfare function, the benevolent planner chooses consumption in working and retirement age, health expenditures, the number of children and the capital-labor ratio

⁹The "Golden Age" situation additionally enables to relate the results to the literature on optimal population growth, e.g. Samuelson (1975), Deardorff (1976) and de la Croix *et al.* (2012). Different solutions to ensure an upper bound of the problem, required to determine the optimal path, are discussed in the literature. Already Ramsey (1928) discussed this problem in his paper "A Mathematical Theory of Saving". For an overview on social discounting see e.g. Heal (2005).

for the next point in time. The optimal decisions are given by the following system of FOCs:

$$u'(c_t^{\text{FB}}) = \frac{\beta}{\rho} (n_{t-1}^{\text{FB}})^{1-x} u'(d_t^{\text{FB}}), \quad (14)$$

$$u'(c_t^{\text{FB}}) = \frac{\rho}{(n_{t-1}^{\text{FB}})^{1-x}} f'(k_{t+1}^{\text{FB}}) u'(c_{t+1}^{\text{FB}}), \quad (15)$$

$$u'(c_t^{\text{FB}}) = \frac{\gamma v'(n_t^{\text{FB}}) + \rho x (n_t^{\text{FB}})^{x-1} (u(c_{t+1}^{\text{FB}}) + \gamma v(n_{t+1}^{\text{FB}}) + \beta q(h_{t+1}^{\text{FB}}) u(d_{t+2}^{\text{FB}}))}{\tau \omega + a + (1 - \tau n_{t+1}^{\text{FB}}) k_{t+1}^{\text{FB}} - \frac{q(h_t^{\text{FB}}) d_{t+1}^{\text{FB}}}{n_t^{\text{FB}} f'(k_{t+1}^{\text{FB}})}}, \quad (16)$$

$$u'(c_t^{\text{FB}}) = \beta q'(h_t^{\text{FB}}) (u(d_{t+1}^{\text{FB}}) - u'(d_{t+1}^{\text{FB}}) d_{t+1}^{\text{FB}}), \quad (17)$$

with $\omega = f(k_t^{\text{FB}}) - f'(k_t^{\text{FB}}) k_t^{\text{FB}}$ the marginal product of labor. The interior first-best solution is given by the set of variables $\{c_t^{\text{FB}}, d_t^{\text{FB}}, h_t^{\text{FB}}, k_t^{\text{FB}}, n_t^{\text{FB}}\}_{t=0}^{t=+\infty}$ satisfying the 4 FOCs (eq. 14-17) and the resource constraint (eq. 13).¹⁰ Consumption is allocated optimally between the working and retired generation according to eq. 14. By means of eq. 15 the FOC is converted in the familiar Euler equation already known from laissez-faire economy.

$$u'(c_t^{\text{FB}}) = \beta f'(k_{t+1}^{\text{FB}}) u'(d_{t+1}^{\text{FB}}) \quad (18)$$

The modified golden rule of capital accumulation¹¹ (eq. 15) describes the optimal investment in the capital stock. In steady state, a capital-labor ratio is installed that balances its marginal product and the inverse of the SDF in the extreme case of total utilitarianism ($x = 1$). Additionally the number of children and thus implicitly population growth is considered if the weight of population size is concave or even absent in the social welfare function ($0 \leq x < 1$).

Considering all effects of a newborn, fertility is optimal if marginal utility is equal to marginal costs. According to FOC 16 both consist of three elements. First of all, parents enjoy marginal utility ($\gamma v'(n_t^{\text{FB}})$) from rearing an additional child. Furthermore, the intergenerational or (old age) dependency ratio effect $\left(\frac{q(h_t^{\text{FB}}) d_{t+1}^{\text{FB}}}{n_t^{\text{FB}} f'(k_{t+1}^{\text{FB}})} u'(c_t^{\text{FB}}) \right)$, see e.g. van Groezen and Mejidam (2008), captures the effect of an additional child on the future relation between working and retired generation. Children increase population growth and hence the number of working (and consuming) individuals related to the old, only consuming. A third effect arise if the planner is characterized by a preference for the population stock, see e.g. Baudin (2011). The discounted marginal life-cycle utility of an additional child $\left(\rho x n_t^{\text{FB}x-1} EU_{t+1}^{\text{FB}} \right)$ is considered in the SWF and boosts fertility if $EU_{t+1} > 0$. On the contrary, fertility is reduced if output does not ensure a positive expected life-cycle utility. A sufficient output level $y_{t+1} > \bar{y}_{t+1}$ s.t. $EU_{t+1} > 0$ is assumed below. Obviously this effect vanishes if the planner is Millian ($x = 0$). The first two costs in eq. 16 are the marginal goods ($au'(c_t^{\text{FB}})$) and time costs $\tau f'(k_t^{\text{FB}}) u'(c_t^{\text{FB}})$ to bring up children. Due to the time cost for child rearing, the capital widening effect of children, as third costly aspect, is composed of two elements. Current fertility increases the present capital-labor ratio $\tau f'(k_t^{\text{FB}}) k_t^{\text{FB}} u'(c_t^{\text{FB}})$, as time cost lowers labor supply. Nevertheless, the capital-labor ratio in the next period decreases, because labor supply

¹⁰The Hessian-matrix to control for the SOC in “Golden Age” as well as in the general FB solution are in appendices B and C.

¹¹The term “golden rule” was introduced by Phelps (1961).

tomorrow is higher $(1 - \tau n_{t+1}^{\text{FB}}) k_{t+1}^{\text{FB}} u'(c_t^{\text{FB}})$. Consequently, the time cost of children slows down the capital widening effect.

Hence, two effects influencing fertility in opposite directions are not internalized by the individuals if the optimal solution follows the idea of Millian utilitarianism. Taking into account the dependency ratio effect increases the number of children, whereas a consideration of the capital dilution effect leads to a reduction. An existing preference for the population stock ($x > 0$) leads to an additional effect boosting fertility.

Optimally fixed health expenditures follow the last FOC (eq. 17). Compared to de la Croix *et al.* (2012), a survival probability determined by health investments helps to obtain an interior solution. Rearranging eq. 17 emphasizes that

$$\varepsilon_{u(d_t^{\text{FB}}, d_t^{\text{FB}})} = \frac{u'(d_t^{\text{FB}})}{u(d_t^{\text{FB}})} d_t^{\text{FB}} < 1$$

is a necessary condition for an interior solution of the social planner problem. The instantaneous utility function $u(d_t^{\text{FB}})$ has to be inelastic with respect to changes in old age consumption d_t^{FB} ; implying a sufficient consumption level. In other words, the absolute level of consumption of retirees matters. An interior solution is only feasible, if consumption and hence utility during retirement exceeds a critical level. The gain achieved from an additional unit of health expenditures ($\beta q'(h_t^{\text{FB}}) u(d_{t+1}^{\text{FB}})$) has to be as high as the arising costs. These opportunity costs consist of two elements, less consumption in middle age ($u'(c_t^{\text{FB}})$) and the decreasing consumption possibilities in old age ($\beta q'(h_t^{\text{FB}}) u'(d_{t+1}^{\text{FB}}) d_{t+1}^{\text{FB}}$). Hence, the Philipson-Becker effect is internalized.

All effects are taken into account by the benevolent planner. Both fertility and mortality are chosen in an optimal way and hence implicitly the age structure. Thus, first-best age structures are considered as optimal and acts as point of reference.

3.4 Decentralization of the optimal age structure

The interior first-best and laissez-faire solution imply the optimal as well as the age structure, reflecting individual decisions. As well known, nothing guarantees that the “modified golden rule of capital accumulation” holds in laissez-faire economies. In contrast, in general FB and LF solutions differ, because of the double infinity in goods and agents (Shell, 1971). Fixing fertility optimally is one possibility to achieve the golden rule level in the standard OLG framework, claimed by the “serendipity theorem”. Anyway, this intervention is excluded, since individuals decide themselves on the number of offspring.

As individuals don’t consider all effects, fertility and mortality are biased and would deviate c.p. from their optimal values. However, a conclusion on the relation between LF and FB age structure isn’t possible. Due to the double infinity in goods and agents, LF and FB capital-labor ratio vary. Thus, beside the biased decisions on demographic variables, the production levels differ. Only a discussion of distortions is possible either by comparing the FOCs or by decentralizing the optimal age structures. The latter is preferred, because of the advantage to display the required policy instruments.

Four instruments are required to implement the optimal solution in a decentralized economy. Two pigouvian taxes ensure, that individuals internalize all effects in fertility as well as mortality. A child allowance or tax η_t corrects the desired number of children and a health subsidy or tax ϕ_t the health expenditures. In addition lump-sum transfers in working age z_t^m and retirement age z_t^o are implemented. The former guarantees that the capital-labor ratio in the laissez-faire economy satisfies the modified golden rule level. The latter balances the budget constraint of the government

in each point in time, since debt is excluded:

$$(\eta_t n_t + \phi_t h_t + z_t^m) N_t^{t-1} + z_t^o N_t^{t-2} = 0,$$

or, in terms per individual in middle age:

$$\eta_t n_t + \phi_t h_t + z_t^m + \frac{q(h_{t-1})z_t^o}{n_{t-1}} = 0. \quad (19)$$

Individuals maximize their expected life-cycle utility function (eq. 6) with respect to the adjusted budget constraints, due to the interventions of the government:

$$c_t = (1 - \tau n_t)w_t - (a - \eta_t)n_t - s_t - (1 - \phi_t)h_t + z_t^m, \quad (20)$$

$$d_{t+1} = \tilde{R}_{t+1}s_t + z_{t+1}^o.$$

The corresponding system of FOCs is:

$$u'(c_t) = \tilde{R}_{t+1}\beta q(h_t)u'(d_{t+1}), \quad (21)$$

$$u'(c_t) = \frac{\gamma v'(n_t)}{\tau w_t + a - \eta_t}, \quad (22)$$

$$u'(c_t) = \frac{\beta q'(h_t)u(d_{t+1})}{(1 - \phi_t)}. \quad (23)$$

By means of FOCs eq. 16, 17, 22 and 23, the middle age budget and resource constraint (eq. 13 and 20), as well as the constraint of the government (eq. 19), the necessary levels for the four policy measures are determined such that $\{c_t^{\text{FB}}, d_t^{\text{FB}}, h_t^{\text{FB}}, k_t^{\text{FB}}, n_t^{\text{FB}}\}_{t=0}^{t=+\infty} = \{c_t, d_t, h_t, k_t, n_t\}_{t=0}^{t=+\infty}$. The lump-sum transfer in working age corrects the capital accumulation in the laissez-faire economy (eq. 24) and that in retirement age (eq. 25) balances the budget constrain:

$$z_t^m = (1 - \tau n_t) f'(k_t)k_t - \eta_t n_t - \phi_t h_t - \frac{q(h_{t-1})d_t}{n_{t-1}}, \quad (24)$$

$$z_t^o = -\frac{n_{t-1}(\eta_t n_t + \phi_t h_t + z_t^m)}{q(h_{t-1})}. \quad (25)$$

Proposition 1 *Taxes on health expenditures*

Individual health expenditures are always taxed according to

$$\phi_t = \frac{d_{t+1}u'(d_{t+1})}{d_{t+1}u'(d_{t+1}) - u(d_{t+1})}, \quad (26)$$

in order to implement the Philipson-Becker-Effect.

Proof At each point in time the optimal policy on health expenditures is given by 26, resulting from equalizing 17 to 23 and rearranging. Due to the non-negativity in consumption and $u'(\cdot) > 0$, the numerator is always positive. In contrast, the denominator must be negative to allow a solution of the FOC (eq. 17). The negative result for ϕ_t confirms that health expenditures are taxed. ■

The tax on health expenditures indicates that the price of health investments is too low and individuals tend to over invest in their longevity. In contrast to the unambiguous findings for mortality, results concerning fertility are less obvious. The effects, not considered by individuals, operate in different directions.

Proposition 2 *Child allowances*

The optimal path on child policies to decentralize the first-best fertility follows:

$$\eta_t = \frac{\gamma v'(n_t) \left[\frac{q(h_t)d_{t+1}}{n_t f'(k_t)} - (1 - \tau n_{t+1}) k_{t+1} \right] + \rho x n_t^{x-1} EU_{t+1} (a + \tau w_t)}{\gamma v'(n_t) + \rho x n_t^{x-1} EU_{t+1}}. \quad (27)$$

1. *Average utilitarianism*

The child allowance is positive if the dependency ratio effect $\left(\frac{q(h_t)d_{t+1}}{n_t f'(k_t)} \right)$ dominates the capital dilution effect $((1 - \tau n_{t+1}) k_{t+1})$ and negative in a vice versa situation.

2. *Number-dampened total and total utilitarianism*

The child allowance is positive if the dependency ratio effect $\left(\frac{q(h_t)d_{t+1}}{n_t f'(k_t)} \right)$ and the preference for the population stock weighted by the ratio of marginal time and good costs to the marginal consumption utility of child rearing $\left(\rho x n_t^{x-1} EU_{t+1} \frac{a + \tau w_t}{\gamma v'(n_t)} \right)$ dominate the capital dilution effect $((1 - \tau n_{t+1}) k_{t+1})$. Children are taxed in a vice versa situation.

Proof Equalizing the FOC on fertility (eq. 16 and 22) and transposing for η_t leads to 27. If $x = 0$, the extreme case of Millian utilitarianism, the optimal policy on fertility simplifies to:

$$\eta_t|_{x=0} = \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} - (1 - \tau n_{t+1}) k_{t+1} \quad \text{with} \quad \begin{cases} \eta < 0 \text{ if } LHS < RHS \\ \eta = 0 \text{ if } LHS = RHS \\ \eta > 0 \text{ if } LHS > RHS \end{cases}$$

In the case of number-dampened total or total utilitarianism $0 < x \leq 1$ the sign of the policy is determined by the numerator of eq. 27. Rearranging leads to:

$$\eta_t|_{0 < x \leq 1} \begin{cases} < 0 \text{ if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} EU_{t+1} \frac{a + \tau w_t}{\gamma v'(n_t)} < (1 - \tau n_{t+1}) k_{t+1} \\ = 0 \text{ if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} EU_{t+1} \frac{a + \tau w_t}{\gamma v'(n_t)} = (1 - \tau n_{t+1}) k_{t+1} \\ > 0 \text{ if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} EU_{t+1} \frac{a + \tau w_t}{\gamma v'(n_t)} > (1 - \tau n_{t+1}) k_{t+1} \end{cases}$$

■

Hence, the dominating effect (or effects) determine(s) the sign of the child allowances. A positive value indicates costs of offspring in the laissez-faire economy that are too high. A situation more likely if the planner takes into account the population size in the SWF, as this adds an additional positive effect not considered by the individuals. The allowance reduces costs and fertility increases, since children are a normal good.¹² The opposite is true, if the capital dilution effect dominates. The costs of a child are too low to consider the dominating negative influence of additional children on the capital-labor ratio. The tax increases costs and lowers fertility.

Supposed both children and health expenditures are normal goods, taxing health expenditures

¹²If the quantity of children is a Giffen good, fertility decreases in child allowance. This has to be checked if a quantity quality trade off is considered.

implies an over-investment in health and a upwards biased survival probability. The lower mortality raises the mean age. This tendency to an over-aged population is strengthened by fertility, if the dependency ratio effect, supported by the preference for the population stock, dominates. The costs for children are too high in the LF economy. Hence, the number of children is below its optimal level and thus upwards biases the mean age, too. On the contrary, a dominating capital dilution effect goes along with a LF fertility that tends to exceed the FB level, reducing the LF mean age below the optimal.

The case of the dependency ratio is similar and depends positively on the survival probability, too. As the Philipson-Becker effect is not internalized by individuals, the fraction of the non-working population is biased upwards. The effect of fertility is less obvious, based on different reasons. First of all, according to the relationship between capital dilution and the sum of dependency ratio effect and the preference for the population stock, costs of children are either too high or low in the laissez-faire solution. Additionally, fertility either rises or lowers the dependency ratio. A higher fertility boosts the dependency ratio in t but reduces its value in $t + 1$. According to

$$\frac{\partial DR}{\partial n} = \begin{cases} > 0 & \text{if } n^2 > q \\ < 0 & \text{if } n^2 < q \end{cases}$$

the steady state effect is ambiguous. Developing countries, commonly characterized by a higher fertility, are featured by a positive steady state relation between the number of children and the dependency ratio. India with a Net reproduction rate¹³ (NRR) of 1.69 and a survival probability of 0.57, used in the calibration exercise, belongs to this group. On the contrary, developed countries with a small number of offspring per women, like Germany with a NRR of 0.654 and high survival probability (0.83), are featured by a shrinking dependency ratio if fertility rises. Hence, four steady state scenarios are possible. Developing countries with high youth dependency ratios have a tendency to a downward distortion if the intergenerational effect and the preference for the population stock dominate. Developed countries are featured by a tendency to exceed the optimal level. If the capital dilution effect dominates, the situation is vice versa. However, as the effect of the capital-labor ratios is not considered, only a discussion on tendencies is possible. To get a clear picture, the model is calibrated in the next section.

4 Calibration exercise

4.1 Data

Laissez-faire age structures always differ from the optimal. In order to analyze the deviation, the LF economy is calibrated to fit the data of 84 countries ($i = 1, 2, \dots, 84$). The resulting parameters enable to replicate the age structures illustrated in figure 1. Afterwards, a calculation of the FB solutions is possible and allows a discussion on the direction and extent of the difference between the observed and optimal age structures.

A period and implicitly each age, childhood, parenthood and retirement, is fixed to 30 years. The steady state values of output Y_i are approximated by a ten-year average of GDP between 2000 and 2009 times the length of a period. Like all economic variables, GDP is expressed in PPP (constant 2005 international \$). As well as for labor, health expenditures and gross fixed capital formation ratio in percentage of GDP, the world bank database is the source. The estimation of the gross

¹³The NRR is “the average number of daughters a hypothetical cohort of women would have at the end of their reproductive period if they were subject during their whole lives to the fertility and mortality rates of a given period (United Nations, 2011).”

domestic fixed capital stock for 93 countries until 1990, offered by Nehru and Dhareshwar (1993), is used to approximate the real capital stocks. Converting their values in PPP and adjusted to the future according to:

$$K_{i,t+1} = (1 - \Delta) K_{i,t} + I_{i,t} \times GDP_{i,t},$$

leads to the capital stock $K_{i,t}$. Yearly investments are computed by means of the gross fixed capital formation ratio in percentages of GDP ($I_{i,t}$) times GDP in PPP. The capital depreciation rate Δ is fixed to 5% p.a. Furthermore, health expenditures between age 30 and 59 are required. As health expenditures per capita are fundamentally driven by age structures and no adequate data by age group is available, they are approximated by controlling for the effects of age structures. By means of total health expenditures in percent of GDP, the average health expenditures per capita in PPP are derived. In a second step the values are corrected by supposing the distribution of health expenditures over age groups and sexes in Germany, which leads to the health expenditures between age 30 and 59 $h_{i,t}$.¹⁴ Summary statistics of the economic as well as the demographic variables are offered in table 1.

Table 1:
SUMMARY STATISTICS OF ECONOMIC AND DEMOGRAPHIC VARIABLES

Variable	Mean	Variance	Min	Max
GDP in bn PPP	17748	46464	59.431	366698
Health exp. in K PPP	37.381	41.163	1.452	183.492
Total labor force in K	30116	96880	165	761269
Capital stock in bn PPP	1573.3	3992	6.8659	30618
Survival prob.	0.718	0.122	0.510	0.950
NRR	1.207	0.477	0.600	2.437

In addition to the economic, the demographic variables are required to calibrate the model. Ten-year averages of total labor forces are used for the population in working age $N_{i,t}^{t-1}$. Both fertility as well as mortality emanate from the United Nations (2011) world prospects 2010 and focus on female populations. The survival probability q_i is calculated from female life tables in 2009. The average life-expectancy at age 60 is related to the length of a period. The NRR 2005-2010 reflects the variable $n_{i,t}$ and thus implicitly captures mortality within childhood and reproductive age.¹⁵

4.2 Calibration of the laissez-faire economy

Specific functional forms are required To calibrate the model, logarithmic instantaneous utility functions are supposed for consumption in middle and old age, as well as for the utility generated by children $u(\cdot) = v(\cdot) = \ln(\cdot)$. Production technology follows a common Cobb-Douglas function with A_i as total factor productivity (TFP):

$$Y_{i,t} = F(K_{i,t}, L_{i,t}) = A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha}.$$

¹⁴In general the distribution of health expenditures over age groups is similar across countries (Dalgaard and Strulik, 2014). Details are presented in appendix D.

¹⁵The implied age structures are illustrated in figure 1.

Its partial production elasticity of capital (α) is fixed to 0.3. The survival probability function is a monotonically increasing function of health expenditures $q' > 0$:

$$q(h_{i,t-1}) = \frac{1}{1 + \exp^{h_{0,i} - h_{i,t-1}^{\delta}}} \quad (28)$$

upper bounded by 1. Determined by the parameters $h_{0,i}, \delta_i$ the function is either s-shaped or concave. The country specific parameter $h_{0,i}$ reflects general climate, medical and hygienic environmental situations of the economies and is independent from individual health expenditures. Their influence is captured by the country specific parameter $\delta_i > 0$.¹⁶

In a first step of the calibration, the country non-specific time to bring up a child and her utility weight are fixed. Within the model economies parents allocate their time between child rearing and paid work. Relating the time used for care to the sum of time for care and paid work leads to an average of 15.3% for both men and women over age 15 in 23 OECD countries. However, this value is sensitive to age structures and the average number of children within the countries. Focusing alternatively on the time used for childcare as main or second activity compared to the total time for both activities in all families with at least one child in preschool age results in a much higher average value of 30.5% in 14 OECD countries. As the time to bring up a child varies over age and potentially more than one is reared in the household, this value is likely to be an overestimation. Following de la Croix and Doepke (2003), assuming that only half of the time is necessary over a period of 30 years, results once again in a value around 15%; supposed for all countries in the calibration (OECD, 2012). Implicitly this fixes the average maximal fertility to around 13 children per women. This outcome seems to be rather realistic as Gagnon *et al.* (2008) state: “fertility was particularly high among the settlers of New France ... an average of 9.97 children for each fertile woman who survived married to the age of 50.” Thus, even this very high fertility is below the fixed natural limit.¹⁷ As part of the preferences, the country non-specific weight of utility generated by child rearing is fixed to fulfill the FOC on the number of children (eq. 11) in the US. The required good costs are 207.43 K and defined as total expenditures of a child for housing, food, health care etc. converted in constant international PPP \$.¹⁸

In a second step, the eight remaining parameters are fixed, supposing utility generated by child rearing and the share of time required to do so are constant across countries. As the laissez-faire solution comprises of eight equations, the system is identified and the model perfectly replicates the observed data. Following the definition of labor in eq. 1, L_i is calculated considering the NRR, the total labor force and the time to bring up children. Afterwards, the TFP remains as residual in the Cobb-Douglas function and is ascertained. Additionally, the interest factors, wages and savings are fixed according to eq. 3, 4 and the capital market equilibrium (eq. 5). By means of the savings, required to fit the capital-labor ratio, the discount factor β_i is chosen corresponding to the Euler

¹⁶A common assumption in the literature, see e.g. Chakraborty (2004), Leung and Wang (2010) and Leroux *et al.* (2011), is a concave survival probability function with: $q'(h) > 0, q''(h) < 0, q(h \rightarrow \infty) = 1$ and $0 \leq q \leq 1$. The specification $q(h_{i,t-1}) = \frac{h_{0,i} + h_{i,t-1}^{\delta_i - 1}}{1 + h_{0,i} + h_{i,t-1}^{\delta_i}}$ satisfies an upper bound of 1 and concavity ($q' > 0, q'' < 0$) for the set of parameters $(\delta_i, h_{0,i})$ fulfilling $h_{i,t-1}^{\delta_i} + h_{0,1} > 0$ and $h_{i,t-1}^{\delta_i} + \frac{(1+h_{0,i})(1-\delta)}{1+\delta} > 0$ for steady state solutions. As this specification inserts additional restrictions and qualitative results are not altered, the more flexible functional form in equ. 28 is preferred.

¹⁷Fixing the time to bring up a child to 0.15 % goes along with the values detectable in OLG models with endogenous fertility. For a period of 25 years Doepke (2004) chooses a time costs of 0.155. Half of the time cost of 0.15 %, estimated by Knowles (1999), is used in the calibration of de la Croix and Doepke (2003).

¹⁸The good costs are the sum over total expenditures at ages 0 to 17 for the younger child in middle-income, husband-wife households with two children. Child care and education expenses are only considered for families with expense (Lino, 2012).

equation (eq. 10). The good cost in each country is fixed to fit the individual choice on the number of children (eq. 11). Finally, the two parameters of the survival probability function are determined by the decision on health expenditures (eq. 12) and the survival probability itself (eq. 28).

Table 2:

SUMMARY STATISTICS OF THE PARAMETERS				
Parameter	Mean	Std. dev.	Min	Max
α	0.30			
τ	0.15			
γ	0.46			
A_i (in K)	31.55	21.27	4.59	107.92
a_i (in K)	118.12	149.91	-4.27	735.68
$h_{0,i}$	1.10	0.43	0.33	3.02
β	0.30	0.15	0.07	0.85
δ	0.07	0.03	0.02	0.16

The country non-specific and specific parameters, determined according to the defined calibration strategy, are summarized in table 2. The elasticity of production and the time required to bring up a child are fixed by means of the literature. The utility generated by child rearing is weighted slightly below half of those originated by consumption during working age.

On average the TFP (A_i) is 31,546, with a range between 4,592 in Mozambique and 107,924 in Luxembourg. The latter is in addition characterized by the highest total good costs per child of 735.68 K PPP. In contrast, in many very poor, especially African countries, children still have to contribute to household income. To fit the data, their contribution is highest in Nigeria (-4.27 K PPP). On average the expenditures to bring up a child are 118.12 K PPP. The average individual discount factor corresponds to the text book value of 0.3 for 30 years, see e.g. de la Croix and Michel (2002). Commonly, values are below the mean in developed countries, e.g. Germany, UK or the US, whereas developing countries like Malawi, Nigeria or Zambia have rather high values, implying a low discounting. Pakistan, as an outlier, has a rather considerable discounting ($\beta = 0.07$) that goes along with a high interest rate of around 10 % per annum. The two parameters in the survival probability function have an average of $\delta = 0.07$ and $h_0 = 1.1$. To fit both, mortality and health expenditures, Japan has the lowest survival probability in absence of health investments, due to $h_0 = 3.02$. However, health expenditures have c.p. the highest influence ($\delta = 0.16$). On the contrary, the situation is vice versa in Israel with the highest survival probability in a hypothetical situation without health investments ($h_0 = 0.33$ and $\delta = 0.07$). Nigeria is featured by the lowest impact of health expenditures on longevity ($\delta = 0.02$).

4.3 Numerical results

The set of parameters perfectly replicates the observed age structures in the laissez-faire scenario (LF) illustrated in figure 1. Additionally, it enables to compute the corresponding optimal solutions in the first-best economies. The demographic variables of three first-best as well as the LF scenario are presented in table 3.¹⁹

¹⁹The values are non-weighted averages across continents. Parameters, variables and the SOC for selected countries are presented in appendix E.

Table 3:

AVERAGE OBSERVED AND OPTIMAL DEMOGRAPHIC VARIABLES								
Area	Fertility				Survival probability			
	LF	GA	FB1	FB2	LF	GA	FB1	FB2
Africa	1.76	0.75	1.49	3.60	0.57	0.58	0.58	0.57
Asia	1.04	0.39	0.87	1.98	0.71	0.73	0.72	0.71
Europe	0.81	0.31	0.70	1.60	0.84	0.88	0.86	0.84
Northern America	1.17	0.45	0.97	2.23	0.75	0.77	0.76	0.74
Ozeania	0.98	0.40	0.84	1.89	0.86	0.89	0.87	0.85
South America	1.16	0.47	0.97	2.17	0.75	0.78	0.76	0.75
World	1.21	0.49	1.02	2.38	0.72	0.74	0.73	0.71

The “Golden Age” scenario (GA) constitutes the point of reference, because of two reasons. First, this optimal solution for a Millian SWF with a SDF of $\rho = 1$ avoids the arbitrary and hardly defensible discounting of future generations. Furthermore, the “Golden Age” is characterized as the most favorable solution among the optimal from an individual point of view, as it goes along with the highest expected life-cycle utility. However, the neglected population size in the SWF and the dominating capital dilution effect lead to a very low “Golden Age” fertility. Optimality of such a very fast shrinking population is highly questionable from a demographic point of view. On average populations halve every generation and reduces actually to 30 % in the European area. Africa, the continent with the largest observable number of offspring, coincidences with the highest “Golden Age” fertility. Still, on average the NRR of 0.75 is below replacement level. Only in Kenya, Malawi, Nigeria, Senegal and Zambia optimal populations grow. Thus, adding children to the SWF and considering their goods and time cost avoids the overestimation of fertility, detectable in theories on optimal growth rates (Arthur and McNicoll, 1978). Nevertheless, the low “Golden Age” fertility is rather intuitive. The planner only cares about utility of the representative individual, not about population size. Furthermore, accounting for the “modified golden rule” (eq. 15) a low fertility goes along with a lower marginal product of k^{FB} and hence c.p. a higher output per work-force. Finally, this implies more consumption possibilities and thereby utility for the representative individual. In addition, the higher output enables to invest more in health. Thus, even if taking into account the Philipson-Becker effect would suggest a lower survival probability than in the LF economy, the “Golden Age” exceeds the observed survival probability in all 84 countries.

Two additional first-best scenarios are presented, as the very low “Golden Age” fertility is striking from a demographic point of view. Introducing a SDF and considering the population size in the welfare function ($x > 0$) changes the optimal number of children. The SDF is fixed to 0.5 % p.a in both scenarios FB1 and FB2. To achieve an optimal average fertility across countries close to the replacement level, the weighting of population size in the SWF (x) is fixed to 0.75 % in scenario FB1. In general this scenario implies an optimal NRR slightly smaller than the observed. The number of offspring in less developed countries exceeds the replacement level but still remains below in developed economies. As output persists above the observed level, most populations in FB1 have similar or even higher survival probabilities than observed.

Optimum populations featured by a high fertility are represented by scenario FB2. Supposing a weight of population size of $x = 7.5\%$ still guarantees a upper bounded maximization ($\rho n^x < 1$) in all economies. On average the cohort size more than doubles every period and fertility exceeds the replacement level everywhere. Due to the lower output per capita, the survival probability further decreases compared to “Golden Age” and FB1. With exception of Korea, Pakistan, Singapore and Thailand all economies are now featured by a survival probability below the observed.

Table 4:
AVERAGE OBSERVED AND OPTIMAL AGE STRUCTURES

Area	Mean age				Dependency ratio			
	LF	GA	FB1	FB2	LF	GA	FB1	FB2
Africa	1.57	2.05	1.66	1.32	2.13	1.69	1.94	3.79
Asia	1.91	2.46	2.01	1.55	1.82	2.60	1.79	2.38
Europe	2.09	2.61	2.18	1.67	1.88	3.21	1.95	2.13
Northern America	1.84	2.41	1.95	1.49	1.85	2.39	1.81	2.58
Oceania	1.97	2.50	2.07	1.57	1.86	2.61	1.88	2.35
South America	1.83	2.38	1.94	1.49	1.83	2.24	1.78	2.53
World	1.84	2.37	1.94	1.50	1.92	2.42	1.87	2.74

The optimal age structures, involved by fertility and mortality, are presented in table 4 as non-weighted averages across continents. Additionally, the optimal dependency ratios and mean ages are added in figure 2 to the observed on the RHS of figure 1. Figure 3 supplements the LHS of figure 1 by the optimal dependency ratios distinguished by youth and old age dependency ratio.

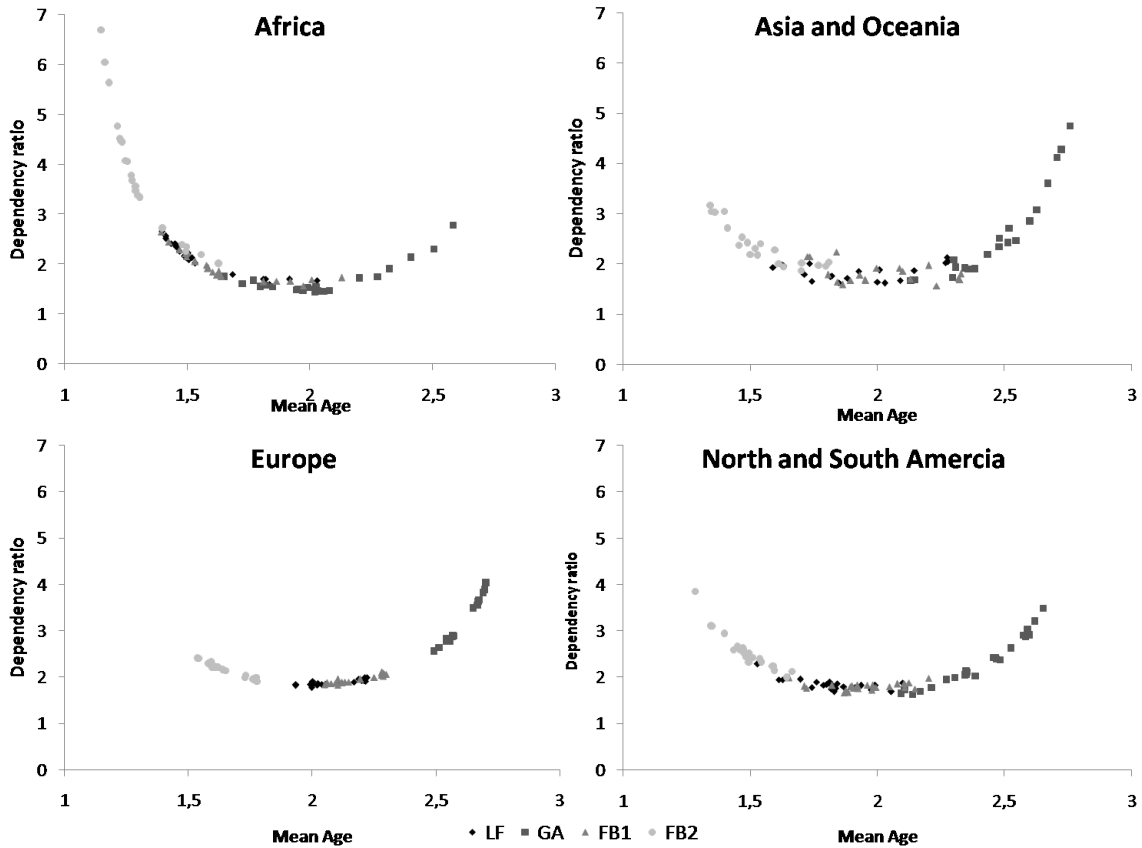


Figure 2: Laissez-faire and first-best age structures across countries

Both, the very low NRR and high survival probability, lead to a “Golden Age” mean age above the observed, plotted to the right of the LF scenario in figure 2. Hence, observed populations

are far too young compared to the “Golden Age” situation, as point of reference. The increasing optimal number of children combined with a shrinking survival probability decreases the mean age, if future generation’s utility is discounted and/or the population size is considered in the SWF. As soon as the optimal solution is plotted to the left of the LF economy, the population is over-aged, like illustrated by scenario FB2.

Whether the “Golden Age” exceeds the laissez-faire share of non-working individuals or not, in particular depends on the observed number of children. All countries feature by a higher old-age and a lower youth dependency ratio than observed, illustrated in figure 3. Even if the lower “Golden Age” fertility reduces the youth dependency ratio, the higher old age dependency ratio dominates in most countries. These economies are featured by a “Golden age” dependency ratio that exceeds the observed level. Only economies with high observed NRR and low survival probabilities plotted on the lower left in fig. 1, like Cameroon, Egypt, Kenya, Malawi, Nigeria and Zambia, are featured by a lower distance to the point of origin and thus smaller “Golden Age” dependency ratio.

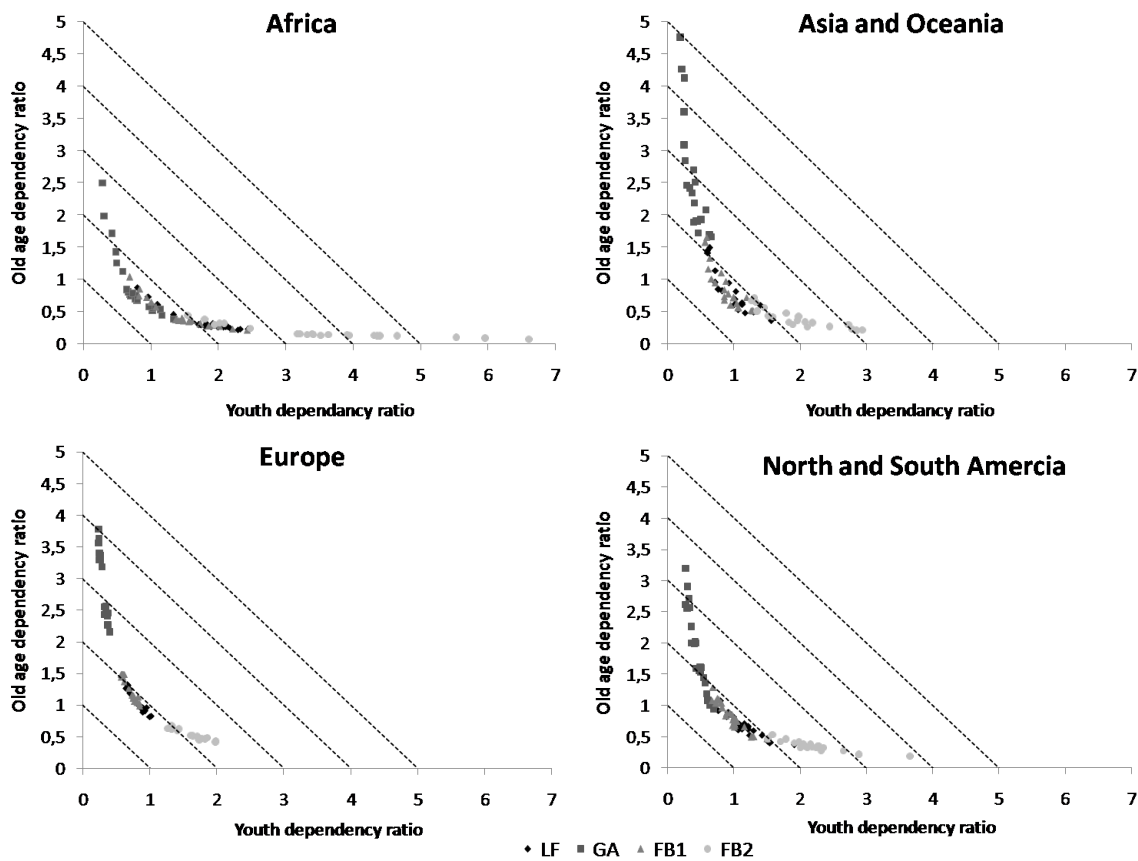


Figure 3: Laissez-faire and first-best dependency ratios across countries

Introducing a SDF and a preference for the population stock ($x > 0$) increase the optimal fertility and reduces the survival probability. The optimal mean age and the old age dependency shrink, whereas the youth dependency ratio increases. The first-best age structure moves to the lower right following a convex curve, accompanied by an initially shrinking share of non-working individuals, visualized in figure 3. The decreasing old-age dependency ratio dominates the growing youth dependency ratio. As soon as they balance each other, the minimal overall dependency ratio

is obtained. A further increasing SDF and/or weight of population size in the SWF leads to a growing overall share of non active individuals. Thus the optimal dependency ratio follows the same U-shaped pattern in the decreasing mean age detectable in the observed data of figure 1.

The U-shaped relation between the optimal dependency ratio and the mean age, if the SDF and/or weight of the population size is increased, is as well illustrated by the three FB scenarios in figure 2. Starting from the reference point of the “Golden Age”, the share of non-working individuals in most economies shrinks and converges to the observed in scenario FB1. Populations featured by a lower “Golden Age” than observed dependency ratio are still an exception. In general their “Golden Age” is below their FB1 dependency ratio. Stronger preferences for the population stock (FB2) imply younger optimal populations (a low mean age) and higher dependency ratios. The larger number of fertility further boost the youth dependency ratio, now dominating the shrinking old age dependency ratio.

All optimal populations plotted in figure 2 and 3 differ from the observed situations in figure 1. Whereby the SWF function determines whether optimal or observed age structures are characterized by a higher or lower mean age and share of non-working individuals. Compared to the “Golden Age” all economies are featured by too young populations and most developed economies have a higher share of working individuals than optimal. In scenario FB1 the dependency ratio is similar to the observed. However, except of Kenja, all populations are still too young. On the contrary the strong preference for the population stock leads to an over-aging in all countries, accompanied by an overshooting of the working individuals in most laissez-faire economy.²⁰ The varying relation between optimal and observed age structures are summarized on average across continents in tab. 4. The policy instruments to decentralize the different optimal solutions are presented in tab. 5.

Table 5:
AVERAGE POLICY MEASURES TO IMPLEMENT THE “GOLDEN AGE”

Area	Health Tax			Child allowance in K			Lump-sum transfer					
							middle age in K			old age in K		
	GA	FB1	FB2	GA	FB1	FB2	GA	FB1	FB2	GA	FB1	FB2
Africa	-0.10	-0.10	-0.10	-265	-36	27	222	94	-25	-87	-98	-123
Asia	-0.10	-0.09	-0.09	-2030	-274	145	1181	527	-27	-251	-330	-506
Europe	-0.09	-0.09	-0.09	-4118	-576	340	2461	1113	-40	-425	-574	-943
N. America	-0.10	-0.09	-0.09	-1461	-192	116	985	431	-38	-216	-285	-448
Ozeania	-0.09	-0.09	-0.09	-1923	-281	230	1554	693	105	-347	-429	-705
S. America	-0.10	-0.09	-0.09	-651	-92	64	522	230	-31	-136	-169	-265
World	-0.10	-0.09	-0.09	-1776	-245	146	1114	498	-34	-230	-299	-469

The “Golden Age” goes along with the highest feasible expected life-cycle utility for the represented individuals in all points in time. In order to achieve this most favorable optimal age structure from an individual point of view, which is beyond doubt problematic from a demographic perspective, extensive policy measures are required. Their signs are identical across countries, only the magnitudes differ. As expected, health expenditures are taxed by around 10 %, to correct the Philipson-Becker effect. The children are taxed too, due to a capital dilution effect that dominates the intergenerational transfer effect in all countries. Additional to the good cost a , a child costs 1,776 K PPP on average, certainly with a large variation between 7.97 K PPP in Zambia and 13.45 million PPP in Luxembourg. The working generation obtains a lump-sum transfer to increase the

²⁰It has to be mentioned, that the expected first-best life-cycle utility is below laissez-faire in certain circumstances.

capital-labor ratio and the retired generations pays a transfer to close the budget constraint in all 84 countries. The signs are unchanged in scenario FB1, but the extents of the instruments decrease. The higher preference for the population stocks in FB2 leads to a dominated capital dilution effect and a child allowance across countries, between 2.5 K PPP in Malawi and 838 K PPP in Luxembourg. The sign of health expenditures isn't altered. The lump-sum transfers are either both negative, e.g. India, UK or the US, only positive in working age to increase the capital-labor ratio, e.g. China, Germany or Japan, or during retirement age as in Malawi, Uganda, Zambia to implement the optimal capital-labor ratio and close the budget constraint. Hence, beside optimal age structure, the assumptions concerning the SWF are important for the direction and magnitudes of the policy instruments.

5 Conclusion

Observed age structures, illustrated in figure 1, are replicated by a simple OLG-model with endogenous fertility and mortality to evaluate a possible over-aging. The objective for an optimal age structure is defined by a number-dampened SWF. Plotting the optimal age structure in addition to the observed in figure 2 and 3 enables to discuss the differences. Three first-best scenarios point out the sensitivity of the results concerning the definition of the SWF.

“Golden Age” populations are always older than the observed and featured by a shrinking number of very happy individuals in most economies. A successive increase in the preference for the population stock rises fertility and lowers the survival probability as well as the expected life-cycle utility. Optimal mean age and dependency ratio trace the U-shape pattern detectable in observed age structures. A sufficient weighting of the population size in SWF, as in scenario FB2, leads to an over-aging of observed populations. To implement the varying optimal age structures, four policies are required. Health expenditures are always taxed, whereas the sign of the child allowances and the lump-sum transfers in working as well as retirement age are sensitive to the SDF and the preference for the population stock. A positive child allowance is more likely the higher the latter. However, the discounting of future generation's utility is problematic from an ethical point of view. Furthermore, both parameters, the SDF (ρ) and the magnitude of the preference for the population size (x), are fixed arbitrary and enable first-best solutions with lower life-cycle utilities than in the laissez-faire economy. Finally, the following dilemma arises:

At least one aspect of the different optimal solutions is problematic. The shrinking, very old populations in the “Golden Age” are striking from a demographic point of view but favorable from an ethical and economic perspective. In contrast, first-best solutions with younger age structures are demographically less problematic, but imply ethical problems and are less advantageous for the individuals, due to an arbitrary discounting of future generation's utility and preference for the population stock.

However, due to assumptions and simplifications, findings should be interpreted carefully. First of all, age structures are aggregated to three generations. The child survival probability and age-dependent heterogeneity within age groups are not considered, e.g. fast growing health expenditures in old age. Furthermore, the calibration focuses on steady state and hence stable populations. Aging as a dynamic phenomenon is not investigated. Indeed, non of the 84 calibrated economies is in a steady state situation. Beside the demographic aspects, economic assumptions limit results. Estimated parameters might be biased, in consequence of country-specific policies and social security systems neglected in the calibration. However, the diversity of social security systems across countries would require totally different models to considered them adequately.

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A Hessian-matrix of the laissez-faire economy

The FOCs describe a maximum if the following 3x3 Hessian-matrix is negative definite:

$$H \equiv \begin{pmatrix} \frac{\partial^2 EU_t}{\partial s_t^2} & \frac{\partial^2 EU_t}{\partial s_t \partial n_t} & \frac{\partial^2 EU_t}{\partial s_t \partial h_t} \\ \frac{\partial^2 EU_t}{\partial n_t \partial s_t} & \frac{\partial^2 EU_t}{\partial n_t^2} & \frac{\partial^2 EU_t}{\partial n_t \partial h_t} \\ \frac{\partial^2 EU_t}{\partial h_t \partial s_t} & \frac{\partial^2 EU_t}{\partial h_t \partial n_t} & \frac{\partial^2 EU_t}{\partial h_t^2} \end{pmatrix}$$

Following Young's theorem, the Hessian is symmetric. The 6 necessary partial derivations of the matrix H are:

$$\frac{\partial^2 EU_t}{\partial s_t^2} = u''(c_t) + \beta \tilde{R}_{t+1}^2 q(h_t) u''(d_{t+1})$$

$$\frac{\partial^2 EU_t}{\partial s_t \partial n_t} = (\tau w_t + a) u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial s_t \partial h_t} = u''(c_t) + \beta q'(h_t) \tilde{R}_{t+1} u'(d_{t+1})$$

$$\frac{\partial^2 EU_t}{\partial n_t^2} = \gamma v''(n_t) + (\tau w_t + a)^2 u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial n_t \partial h_t} = (a + \tau w_t) u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial h_t^2} = u''(c_t) + \beta q''(h_t) u(d_{t+1})$$

To determine the SOC the Eigenvalues and Sylvester's Criterion are computed by the matrix H in the calibration exercise.

B Hessian-matrix of the “Golden Age” solution

To verify that the optimality conditions describe a maximum in the Golden age situation, the following Hessian-matrix is computed for the static case of $\rho = 1$:

$$H \equiv \begin{pmatrix} \frac{\partial^2 W}{\partial d^2} & \frac{\partial^2 W}{\partial d \partial k} & \frac{\partial^2 W}{\partial d \partial n} & \frac{\partial^2 W}{\partial d \partial h} \\ \frac{\partial^2 W}{\partial k \partial d} & \frac{\partial^2 W}{\partial k^2} & \frac{\partial^2 W}{\partial k \partial n} & \frac{\partial^2 W}{\partial k \partial h} \\ \frac{\partial^2 W}{\partial n \partial d} & \frac{\partial^2 W}{\partial n \partial k} & \frac{\partial^2 W}{\partial n^2} & \frac{\partial^2 W}{\partial n \partial h} \\ \frac{\partial^2 W}{\partial h \partial d} & \frac{\partial^2 W}{\partial h \partial k} & \frac{\partial^2 W}{\partial h \partial n} & \frac{\partial^2 W}{\partial h^2} \end{pmatrix}$$

The partial derivations of the matrix H are:

$$\frac{\partial^2 W}{\partial d^2} = \left(\frac{q(h)}{n} \right)^2 u''(c) + \beta q(h) u''(d)$$

$$\frac{\partial^2 W}{\partial d \partial k} = -(f'(k) - n) \frac{q(h)}{n} (1 - \tau n) u''(c) = 0$$

$$\frac{\partial^2 W}{\partial d \partial n} = \frac{q(h)}{n} \left(\tau (f(k) - nk) + a + (1 - \tau n) k - \frac{q(h)d}{n^2} \right) u''(c_t) + \frac{q(h)}{n^2} u''(c)$$

$$\frac{\partial^2 W}{\partial d \partial h} = \frac{q(h)}{n} \left(1 + \frac{q'(h)d}{n} \right) u''(c)$$

$$\frac{\partial^2 W}{\partial k^2} = (1 - \tau n) f''(k) u'(c)$$

$$\frac{\partial^2 W}{\partial k \partial n} = -(1 - \tau n) u'(c)$$

$$\frac{\partial^2 W}{\partial k \partial h} = (1 - \tau n) (n - f'(k)) \left(1 + \frac{q'(h)}{n} \right) u''(c) = 0$$

$$\frac{\partial^2 W}{\partial n^2} = \left(a + \tau (f(k) - nk) + (1 - \tau n) k - \frac{q(h)d}{n^2} \right)^2 u''(c) + \gamma u''(n) + 2 \left(\tau k - \frac{q(h)d}{n^3} \right)^2 u'(c)$$

$$\frac{\partial^2 W}{\partial n \partial h} = \frac{q'(h)d}{n^2} u'(c) + \left(a + \tau (f(k) - nk) + (1 - \tau n) k - \frac{q(h)d}{n^2} \right) \left(1 + \frac{q'(h)d}{n} \right) u''(c_t)$$

$$\frac{\partial^2 W}{\partial h^2} = q''(h) \left(\beta u(d) - \frac{d}{n} u'(c) \right) + \left(1 + \frac{q'(h)d}{n} \right)^2 u''(c)$$

C Hessian-matrix of the first-best solution

The Hessian matrix, to verify that the optimality conditions describe a maximum, is:

$$H \equiv \begin{pmatrix} \frac{\partial^2 W}{\partial d_t^2} & \frac{\partial^2 W}{\partial d_t \partial k_{t+1}} & \frac{\partial^2 W}{\partial d_t \partial n_t} & \frac{\partial^2 W}{\partial d_t \partial h_t} \\ \frac{\partial^2 W}{\partial k_{t+1} \partial d_t} & \frac{\partial^2 W}{\partial k_{t+1}^2} & \frac{\partial^2 W}{\partial k_{t+1} \partial n_t} & \frac{\partial^2 W}{\partial k_{t+1} \partial h_t} \\ \frac{\partial^2 W}{\partial n_t \partial d_t} & \frac{\partial^2 W}{\partial n_t \partial k_t} & \frac{\partial^2 W}{\partial n_t^2} & \frac{\partial^2 W}{\partial n_t \partial h_t} \\ \frac{\partial^2 W}{\partial h_t \partial d_t} & \frac{\partial^2 W}{\partial h_t \partial k_t} & \frac{\partial^2 W}{\partial h_t \partial n_t} & \frac{\partial^2 W}{\partial h_t^2} \end{pmatrix}$$

The partial derivations of the matrix H are:

$$\frac{\partial^2 W}{\partial d_t^2} = \left(\frac{q(h_t)}{n_{t-1}} \right)^2 u''(c_t) + \frac{\beta}{\rho n_{t-1}^x} q(h_t) u''(d_t)$$

$$\frac{\partial^2 W}{\partial d_t \partial k_{t+1}} = \frac{q(h_t)}{n_{t-1}} ((1 - \tau n_{t+1}) n_t) u''(c)$$

$$\frac{\partial^2 W}{\partial d_t \partial n_t} = \frac{q(h_{t-1})}{n_{t-1}} (\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1}) u''(c_t)$$

$$\frac{\partial^2 W}{\partial d_t \partial h_t} = \frac{q(h_{t-1})}{n_{t-1}} u''(c_t)$$

$$\frac{\partial^2 W}{\partial k_{t+1}^2} = (1 - \tau n_{t+1}) \left(\rho n_t^x f''(k_{t+1}) u'(c_{t+1}) + (1 - \tau n_{t+1}) \left(f'(k_{t+1})^2 u''(c_{t+1}) \rho n_t^x + n_t^2 u''(c_t) \right) \right)$$

$$\begin{aligned} \frac{\partial^2 W}{\partial k_{t+1} \partial n_t} &= (1 - \tau n_{t+1}) \left(\rho n_t^x f'(k_{t+1}) \left(\frac{1}{n_t} u'(c_{t+1}) + \frac{q(h_t) d_{t+1}}{n_t^2} u''(c_{t+1}) \right) - u'(c_t) \right. \\ &\quad \left. + (\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1}) n_t u''(c_t) \right) \end{aligned}$$

$$\frac{\partial^2 W}{\partial k_{t+1} \partial h_t} = (1 - \tau n_{t+1}) \left(n_t u''(c_t) - \rho \frac{q'(h_t) d_{t+1}}{n_t} f'(k_{t+1}) \right) u''(c_{t+1})$$

$$\begin{aligned} \frac{\partial^2 W}{\partial n_t^2} &= (\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1})^2 u''(c_t) + \gamma u''(n_t) + \rho x(x-1) n_t^{x-2} E U_t \\ &\quad + \rho x n_t^{x-1} \frac{q(h_t) d_{t+1}}{n_t^3} u'(c_{t+1}) - \rho(2-x) \frac{q(h_t) d_{t+1}}{n_t^{3-x}} u'(c_{t+1}) + \\ &\quad \rho \left(\frac{q(h_t) d_{t+1}}{n_t^3} \right)^2 n_t^x u''(c_{t+1}) + \frac{1}{\rho} \tau^2 k_t^2 n_t^{2-x} u''(c_{t-1}) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 W}{\partial n_t \partial h_t} &= \rho(1-x) \frac{q'(h_t) d_{t+1}}{n_t^{2-x}} u'(c_{t+1}) - \rho \frac{q'(h_t) q(h_t) d_{t+1}^2}{n_t^{3-x}} u''(c_{t+1}) \\ &\quad + (\tau (f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1})) u''(c_t) \end{aligned}$$

$$\frac{\partial^2 W}{\partial h_t^2} = \beta q''(h_t) u(d_{t+1}) - \rho \frac{d_{t+1}}{n_t^{1-x}} \left(q''(h_t) u'(c_{t+1}) - \frac{q'(h_t)^2 d_{t+1}}{n_t} u''(c_{t+1}) \right) + u''(c_t)$$

D Calculation of the Health expenditures

The total health expenditures per capita in PPP are the weighted sum over sexes and age groups:

$$H_\Sigma = \sum_{\text{age group}} \sum_{\text{sex}} x_{\text{age group, sex}} h_{\text{age group, sex}}$$

with $h_{\text{age group, sex}}$, the age and sex specific health expenditures and $x_{\text{age group, sex}}$, the share on total population. Supposed $\frac{h_{\text{Germany, age group}+j}}{h_{\text{Germany, 0-14, female}}}$, the relation of health expenditures between age groups and sexes in Germany, is fixed across countries, rearranging allows to express the female health expenditure in the youngest age group for each country:

$$h_{i,0-14,\text{female}} = \frac{H_\Sigma}{\sum_{j=0}^{J=12} x_{\text{age group, sex}} \frac{h_{\text{Germany, age group}+j}}{h_{\text{Germany, 0-14, female}}}}$$

with J as the age-groups of both sexes. By means of $H_{j,\text{age group},\text{female}}$ and the age structures given by the (United Nations, 2011), it is possible to approximate the total health expenditures of females between age 30 and 59 for the 84 countries:

$$h_i = 15 * h_{i,0-14} * \left(\frac{h_{30-44}^{Germany}}{h_{0-14}^{Germany}} + \frac{h_{30-64}^{Germany}}{h_{0-14}^{Germany}} \right).$$

E Results for selected countries

Table 6:

EXPECTED LC-UTILITIES IN SELECTED COUNTRIES

Country	Life-cycle utility			
	<i>LF</i>	<i>GA</i>	<i>FB1</i>	<i>FB2</i>
Argentina	15.26	15.86	15.78	15.35
Australia	16.47	17.03	16.95	16.51
Brazil	14.33	15.06	14.98	14.55
Cameroon	14.36	14.74	14.68	14.26
Canada	15.56	16.34	16.25	15.80
Chile	14.86	15.59	15.51	15.08
China	12.85	13.68	13.60	13.22
Egypt	14.71	15.28	15.20	14.78
Finland	16.04	16.66	16.59	16.15
France	16.25	16.86	16.79	16.34
Germany	15.44	16.23	16.14	15.71
Guatemala	15.35	15.73	15.66	15.19
India	13.54	14.10	14.04	13.66
Israel	17.01	17.45	17.38	16.90
Japan	15.00	15.85	15.77	15.35
Kenya	15.91	16.03	15.97	15.53
Luxembourg	16.21	17.08	16.99	16.52
Malawi	14.62	14.75	14.68	14.17
Mexico	15.92	16.41	16.34	15.90
Mozambique	13.18	13.50	13.44	13.05
Nigeria	15.91	16.07	16.01	15.55
Pakistan	12.39	13.66	13.56	13.12
Philippines	14.73	15.12	15.06	14.65
South Africa	14.53	15.32	15.23	14.79
Switzerland	15.34	16.18	16.92	16.47
Sweden	16.43	16.99	16.10	15.65
Uganda	14.35	14.57	14.50	13.99
United Kingdom	15.92	16.60	16.52	16.07
United States	16.41	17.11	17.02	16.53
Zambia	16.14	16.21	16.14	15.57

Table 7:

OVERVIEW ON PARAMETERS, SAVINGS, CAPITAL-LABOR RATIO AND HEALTH EXPENDITURES IN SELECTED COUNTRIES

Country	Parameters				Capital-labor ratio				Health expenditures					
	A_i	a_i	β	h_0i	δ	sLF	LF	GA	$FB1$	$FB2$	LF	GA	$FB1$	$FB2$
Argentina	30420	57870	0.25	0.08	1.01	68512	33770	88871	430213	142042	75644	1668690	57186	27263
Australia	53604	199684	0.24	0.09	0.92	184138	80868	183480	1110560	378697	230720	3919230	125320	63881
Brazil	23041	60926	0.22	0.09	1.44	39273	27480	80435	381585	122134	50781	1662710	50973	24791
Cameroon	11046	367	0.46	0.03	1.23	18749	4355	9816	54890	18796	14702	183223	6369	2917
Canada	56072	278362	0.18	0.11	1.81	131170	98373	255972	1497250	475533	187333	6644440	172066	90236
Chile	35155	108573	0.18	0.09	0.95	66202	32186	91471	644422	210431	83554	2689660	59090	28818
China	12091	39532	0.20	0.08	1.29	13463	7830	25068	198663	65352	21118	846265	15907	7827
Egypt, Arab Rep.	23304	21169	0.29	0.04	0.94	40710	9991	27577	225899	75851	38087	849669	17411	7897
Finland	52990	216677	0.22	0.09	0.84	155287	67181	169379	1164210	390773	201362	4372660	112348	55441
France	56680	199172	0.21	0.11	1.36	173491	87176	192174	1179600	394477	212783	4401450	134831	72480
Germany	52381	382063	0.18	0.10	1.55	129877	89521	244249	1716810	552840	220190	7347990	161594	82719
Guatemala	20867	-4145	0.33	0.05	0.66	43609	11268	25926	119316	38461	31990	431028	16395	7117
India	11037	11648	0.32	0.04	1.08	15409	3940	10440	92323	32261	15984	322867	6794	3255
Israel	59068	58097	0.27	0.07	0.33	206188	57888	132288	741255	250207	186037	2596350	86217	39544
Japan	50670	377244	0.13	0.16	3.02	101315	62258	121965	1623780	533889	175854	6742380	94305	60999
Kenya	8231	-2577	0.68	0.02	0.79	22519	2687	3882	22382	8233	16514	58166	2675	1239
Luxembourg	107924	735677	0.15	0.10	1.54	267441	183492	554289	3798960	1182030	389606	18696200	352660	172386
Malawi	5036	-2200	0.72	0.03	1.23	9423	2205	3450	9119	2652	6268	28041	2244	895
Mexico	33396	58843	0.30	0.05	0.61	95246	27328	64744	430119	150082	100364	1441190	42497	19992
Mozambique	4592	-511	0.51	0.04	1.24	6006	1629	3349	14056	4876	4480	44676	2212	1045
Nigeria	11719	-4265	0.66	0.02	1.08	30624	4315	7140	38762	13448	21868	111241	4769	2115
Pakistan	21128	7766	0.07	0.06	1.30	4957	2418	15963	197760	54665	4139	1628820	8163	3285
Philippines	13994	4869	0.37	0.03	0.65	27924	4680	10233	87091	31080	23830	273841	6792	3195
South Africa	31394	69577	0.23	0.06	1.52	47848	29421	102303	506967	158071	55608	2452110	60556	27052
Switzerland	55464	337562	0.16	0.13	2.77	120764	120147	285207	1096800	374014	192034	7949710	120960	59605
Sweden	52434	201272	0.25	0.08	0.82	179172	76176	181068	1719480	538581	227176	3883800	201792	115571
Uganda	7372	-3531	0.52	0.04	1.14	12043	3144	6048	17855	5056	7789	62781	3798	1497
United Kingdom	56035	233943	0.21	0.08	1.01	152262	76245	205215	1289980	423433	198763	5182690	133248	64644
United States	69070	207432	0.22	0.10	2.03	200047	183491	484919	1582340	480954	235933	7495330	312622	155410
Zambia	7009	-3648	0.85	0.03	1.14	18778	3492	4670	12790	3709	12817	36234	3085	1185

Table 8:

OVERVIEW ON DEMOGRAPHIC VARIABLES AND AGE STRUCTURES IN SELECTED COUNTRIES

Country	Net Reproduction Rate			Survival probability			Mean age			Dependency ratio		
	LF	GA	FB1	FB2	LF	GA	FB1	FB2	LF	GA	FB1	FB2
Argentina	1.081	0.402	0.893	2.050	0.770	0.799	0.786	0.764	1.868	2.467	1.995	1.510
Australia	0.927	0.390	0.810	1.800	0.880	0.903	0.892	0.873	2.008	2.519	2.100	1.600
Brazil	0.893	0.306	0.735	1.702	0.737	0.782	0.763	0.732	1.975	2.583	2.109	1.594
Cameroon	1.718	0.686	1.375	3.167	0.517	0.525	0.521	0.513	1.531	2.032	1.638	1.306
Canada	0.795	0.282	0.686	1.591	0.860	0.903	0.886	0.856	2.100	2.651	2.203	1.663
Chile	0.919	0.333	0.777	1.780	0.823	0.855	0.842	0.820	1.992	2.573	2.107	1.593
China	0.714	0.257	0.608	1.360	0.683	0.725	0.709	0.683	2.091	2.629	2.201	1.700
Egypt, Arab Rep.	1.337	0.494	1.076	2.470	0.610	0.622	0.617	0.607	1.685	2.278	1.810	1.401
Finland	0.890	0.357	0.774	1.736	0.853	0.878	0.867	0.848	2.024	2.551	2.119	1.613
France	0.951	0.380	0.821	1.854	0.910	0.935	0.924	0.904	2.002	2.541	2.103	1.591
Germany	0.654	0.245	0.582	1.319	0.833	0.875	0.859	0.830	2.212	2.690	2.292	1.766
Guatemala	1.911	0.712	1.510	3.665	0.707	0.719	0.712	0.700	1.530	2.109	1.652	1.285
India	1.169	0.461	0.952	2.103	0.570	0.582	0.577	0.568	1.744	2.295	1.865	1.457
Israel	1.404	0.573	1.188	2.736	0.850	0.864	0.857	0.843	1.735	2.303	1.840	1.400
Japan	0.637	0.252	0.585	1.306	0.950	0.974	0.966	0.949	2.273	2.706	2.329	1.809
Kenya	1.912	1.141	1.925	4.304	0.597	0.599	0.597	0.592	1.504	1.769	1.501	1.234
Luxembourg	0.777	0.263	0.689	1.621	0.843	0.886	0.869	0.841	2.108	2.670	2.194	1.649
Malawi	2.290	1.163	2.211	5.965	0.517	0.522	0.517	0.507	1.413	1.726	1.426	1.166
Mexico	1.146	0.489	0.982	2.175	0.750	0.764	0.757	0.745	1.824	2.351	1.924	1.479
Mozambique	1.859	0.766	1.485	3.405	0.527	0.536	0.531	0.521	1.499	1.974	1.603	1.286
Nigeria	2.001	1.032	1.866	4.350	0.527	0.530	0.527	0.522	1.468	1.796	1.497	1.227
Pakistan	1.565	0.284	1.071	2.847	0.573	0.619	0.602	0.581	1.591	2.547	1.807	1.348
Philippines	1.517	0.656	1.260	2.796	0.657	0.664	0.660	0.653	1.632	2.134	1.735	1.364
South Africa	1.015	0.317	0.821	1.956	0.593	0.630	0.614	0.591	1.834	2.505	1.971	1.492
Switzerland	0.703	0.246	0.799	1.775	0.880	0.929	0.856	0.837	2.186	2.703	2.095	1.599
Sweden	0.914	0.384	0.616	1.431	0.843	0.866	0.912	0.877	2.003	2.514	2.279	1.731
Uganda	2.437	0.969	2.020	5.526	0.553	0.562	0.556	0.544	1.397	1.847	1.470	1.181
United Kingdom	0.883	0.335	0.762	1.736	0.827	0.856	0.844	0.822	2.019	2.571	2.120	1.607
United States	0.997	0.319	0.815	1.976	0.817	0.866	0.845	0.807	1.937	2.594	2.078	1.537
Zambia	2.174	1.353	2.430	6.615	0.527	0.529	0.526	0.518	1.435	1.649	1.393	1.150

Table 9:
OVERVIEW ON POLICY INSTRUMENTS IN SELECTED COUNTRIES

Country	Tax on health expenditures			Child allowance			Lump-sum transfer middle age			Lump-sum transfer old age		
	GA	FB1	FB2	GA	FB1	FB2	GA	FB1	FB2	GA	FB1	FB2
Argentina	-0.094	-0.090	-0.090	-1002920	-141352	97275	815415	360031	-43377	-203279	-259916	-412120
Australia	-0.088	-0.085	-0.085	-2263860	-336817	273868	1781970	802972	-110277	-381493	-471537	-778193
Brazil	-0.100	-0.095	-0.095	-1085550	-150849	75114	671368	303306	2245	-129636	-180743	-296729
Cameroon	-0.099	-0.096	-0.096	-92251	-12108	13799	127515	50744	-25403	-82660	-88469	-111367
Canada	-0.091	-0.087	-0.087	-4494960	-606298	316572	2557800	1153310	-214	-395860	-559564	-921435
Chile	-0.095	-0.091	-0.091	-1797490	-255131	138869	1205510	546144	-13930	-232891	-316032	-500738
China	-0.112	-0.106	-0.105	-594093	-91680	35070	308464	146812	15569	-54205	-76672	-124314
Egypt, Arab Rep.	-0.095	-0.091	-0.091	-517419	-73119	53471	514233	223084	-43285	-203257	-249199	-358319
Finland	-0.089	-0.086	-0.086	-2703320	-398884	276518	1945760	883922	-74458	-392539	-504783	-820958
France	-0.088	-0.085	-0.085	-2665110	-386600	281925	2043600	918533	-103057	-412223	-523852	-848577
Germany	-0.092	-0.088	-0.088	-4997270	-715994	360466	2475700	1147770	43868	-344123	-485967	-813702
Guatemala	-0.094	-0.090	-0.090	-219023	-24253	30937	314287	116409	-78644	-154415	-166028	-178528
India	-0.105	-0.100	-0.100	-194471	-31498	20275	180371	82741	-7424	-70934	-85939	-129230
Israel	-0.084	-0.082	-0.082	-1379570	-170972	201813	1593130	643170	-281113	-524766	-600277	-869035
Japan	-0.094	-0.090	-0.090	-4842420	-731484	342852	2453810	1161600	68305	-316116	-439103	-702924
Kenya	-0.098	-0.096	-0.096	-15861	258	7263	36578	6500	-30156	-34478	-21749	-7169
Luxembourg	-0.085	-0.081	-0.081	-13447900	-1633980	838369	7121570	3120040	-37084	-1050530	-1557750	-2522280
Malawi	-0.105	-0.101	-0.101	-8594	-22	2515	20358	3491	-17609	-22299	-13751	31740
Mexico	-0.090	-0.087	-0.087	-789268	-116091	109993	778482	340709	-82171	-247352	-289073	-453358
Mozambique	-0.112	-0.107	-0.107	-20440	-2644	3409	31680	12158	-7588	-22346	-22362	-25558
Nigeria	-0.094	-0.091	-0.092	-38136	-1761	11850	79372	20553	-47716	-76644	-59556	-30325
Pakistan	-0.112	-0.103	-0.103	-1404430	-108220	34616	800141	299071	-11331	-183241	-324052	-425996
Philippines	-0.098	-0.095	-0.095	-137854	-19608	22716	181832	75385	-32271	-89314	-95521	-132412
South Africa	-0.095	-0.090	-0.090	-1688460	-209406	102701	1080760	469110	-12878	-269747	-390026	-614083
Switzerland	-0.092	-0.085	-0.085	-5537470	-334843	270095	2750460	786061	-104018	-360538	-474667	-785223
Sweden	-0.088	-0.088	-0.088	-2248930	-741702	343255	1742760	1251540	55890	-382774	-524881	-875692
Uganda	-0.103	-0.099	-0.099	-25816	-1735	4561	50638	14043	-26833	-43131	-36948	18021
United Kingdom	-0.089	-0.085	-0.085	-3316120	-468110	296540	2241800	1009720	-63797	-435213	-579521	-941091
United States	-0.086	-0.083	-0.082	-4707950	-552255	336497	3047440	1291610	-118308	-553771	-785942	-1306430
Zambia	-0.098	-0.095	-0.096	-7971	1145	3885	22032	-741	-32568	-27588	-8080	89202

Table 10:
OVERVIEW ON EIGENVALUES (EV) IN SELECTED COUNTRIES

Country	Laissez-faire economy			"Golden Age"			FB1			FB2						
	EV1	EV2	EV3	EV1	EV2	EV3	EV1	EV2	EV3	EV1	EV2	EV3	EV4			
Argentina	-0.6	-4.7E-11	-3.2E-11	-6.2	-1.8E-11	-3.8E-12	-2.0E-14	-2.2	-9.8E-12	-7.8E-12	-1.4E-12	-1.4E-12	-1.4	-4.0E-11	-3.0E-11	-6.5E-12
Australia	-0.8	-7.5E-12	-6.6E-12	-6.8	-4.5E-12	-1.1E-12	-4.4E-15	-2.8	-2.5E-12	-2.1E-12	-2.2E-13	-2.2E-13	-1.9	-9.2E-12	-4.6E-12	-1.9E-12
Brazil	-0.8	-1.2E-10	-4.4E-11	-10.4	-5.6E-11	-4.1E-12	-1.4E-14	-2.9	-2.3E-11	-1.1E-11	-1.7E-12	-1.7E-12	-1.8	-4.4E-11	-3.9E-11	-1.7E-11
Cameroon	-0.2	-1.1E-09	-7.8E-10	-2.2	-1.9E-10	-6.5E-11	-2.0E-12	-1.1	-4.3E-10	-8.8E-11	-3.6E-11	-3.6E-11	-0.7	-1.9E-09	-1.3E-09	-3.0E-11
Canada	-1.1	-9.7E-12	-3.3E-12	-11.8	-6.6E-12	-3.6E-13	-9.2E-16	-3.2	-2.7E-12	-8.5E-13	-1.1E-13	-1.1E-13	-2.1	-3.2E-12	-2.8E-12	-2.1E-12
Chile	-0.8	-3.8E-11	-2.9E-11	-8.4	-1.8E-11	-2.9E-12	-6.9E-15	-2.5	-8.1E-12	-7.0E-12	-5.9E-13	-5.9E-13	-1.7	-3.0E-11	-1.3E-11	-6.6E-12
China	-1.3	-8.1E-10	-3.8E-10	-13.8	-4.0E-10	-3.0E-11	-5.7E-14	-3.8	-1.6E-10	-7.6E-11	-6.0E-12	-6.0E-12	-2.3	-3.0E-10	-1.3E-10	-1.2E-10
Egypt	-0.4	-1.8E-10	-1.2E-10	-3.9	-2.2E-11	-1.6E-11	-8.1E-14	-1.4	-5.1E-11	-8.7E-12	-4.8E-12	-4.8E-12	-1.0	-2.4E-10	-9.3E-11	-7.6E-12
Finland	-0.8	-8.9E-12	-8.0E-12	-7.8	-5.1E-12	-1.1E-12	-3.2E-15	-2.8	-2.6E-12	-2.3E-12	-1.9E-13	-1.9E-13	-1.9	-1.0E-11	-4.2E-12	-2.0E-12
France	-0.7	-7.3E-12	-5.2E-12	-6.9	-4.4E-12	-8.4E-13	-3.2E-15	-2.5	-2.1E-12	-1.8E-12	-1.9E-13	-1.9E-13	-1.8	-6.3E-12	-4.2E-12	-1.8E-12
Germany	-1.6	-9.9E-12	-3.8E-12	-15.8	-8.8E-12	-4.0E-13	-8.0E-16	-4.4	-3.7E-12	-9.7E-13	-8.6E-14	-8.6E-14	-2.9	-3.8E-12	-2.8E-12	-2.1E-12
Guatemala	-0.2	-2.4E-10	-1.4E-10	-2.0	-4.2E-11	-1.9E-11	-3.5E-13	-0.9	-1.0E-10	-1.8E-11	-1.0E-11	-1.0E-11	-0.6	-5.2E-10	-2.9E-10	-8.7E-12
India	-0.5	-1.1E-09	-8.7E-10	-4.6	-1.7E-10	-1.2E-10	-5.9E-13	-1.8	-3.3E-10	-7.1E-11	-2.9E-11	-2.9E-11	-1.2	-1.4E-09	-5.2E-10	-6.0E-11
Israel	-0.3	-1.4E-11	-6.2E-12	-3.1	-2.5E-12	-1.8E-12	-1.1E-14	-1.4	-5.6E-12	-9.7E-13	-4.9E-13	-4.9E-13	-1.0	-2.6E-11	-9.3E-12	-8.9E-13
Japan	-1.7	-1.3E-11	-5.5E-12	-14.0	-1.2E-11	-1.0E-12	-1.1E-15	-4.1	-5.4E-12	-1.8E-12	-9.0E-14	-9.0E-14	-2.8	-5.3E-12	-4.1E-12	-2.2E-12
Kenya	-0.2	-3.4E-09	-1.0E-09	-1.0	-1.5E-09	-1.1E-10	-2.4E-11	-0.8	-3.0E-09	-6.2E-10	-6.8E-11	-6.8E-11	-0.6	-1.4E-08	-5.8E-09	-6.0E-11
Luxembourg	-1.1	-1.9E-12	-7.3E-13	-12.9	-1.2E-12	-6.2E-14	-1.1E-16	-3.0	-4.4E-13	-1.6E-13	-1.7E-14	-1.7E-14	-2.1	-6.7E-13	-4.6E-13	-3.7E-13
Malawi	-0.1	-7.0E-09	-5.0E-09	-0.9	-2.6E-09	-3.7E-10	-6.4E-11	-0.6	-5.7E-09	-3.5E-09	-1.9E-10	-1.9E-10	-0.4	-3.3E-08	-1.6E-08	-1.5E-10
Mexico	-0.5	-4.6E-11	-2.8E-11	-4.4	-1.0E-11	-6.5E-12	-3.5E-14	-1.9	-1.7E-11	-4.7E-12	-1.5E-12	-1.5E-12	-1.4	-7.3E-11	-2.7E-11	-4.2E-12
Mozambique	-0.2	-1.0E-08	-8.6E-09	-1.8	-2.2E-09	-7.1E-10	-3.3E-11	-1.0	-4.7E-09	-1.4E-09	-4.0E-10	-4.0E-10	-0.6	-2.0E-08	-1.8E-08	-3.2E-10
Nigeria	-0.2	-1.1E-09	-4.5E-10	-1.1	-3.5E-10	-3.6E-11	-6.0E-12	-0.8	-7.7E-10	-1.9E-10	-2.0E-11	-2.0E-11	-0.5	-3.8E-09	-2.1E-09	-1.8E-11
Pakistan	-0.3	-1.8E-09	-9.7E-10	-8.9	-1.1E-10	-1.7E-11	-8.4E-15	-1.0	-7.0E-11	-2.9E-11	-4.8E-12	-4.8E-12	-0.6	-4.2E-10	-1.4E-10	-2.4E-11
Philippines	-0.3	-9.9E-10	-3.6E-10	-2.4	-1.8E-10	-5.7E-11	-1.0E-12	-1.2	-4.1E-10	-3.5E-11	-3.1E-11	-3.1E-11	-0.9	-1.8E-09	-5.4E-10	-2.8E-11
South Africa	-0.7	-6.6E-11	-2.6E-11	-9.1	-1.5E-11	-1.8E-12	-5.5E-15	-2.2	-6.0E-12	-5.1E-12	-9.4E-13	-9.4E-13	-1.4	-2.5E-11	-2.2E-11	-4.6E-12
Switzerland	-1.4	-1.1E-11	-2.2E-12	-15.3	-8.8E-12	-2.5E-13	-5.0E-16	-2.8	-2.6E-12	-2.1E-12	-2.3E-13	-2.3E-13	-2.0	-1.0E-11	-4.7E-12	-1.9E-12
Sweden	-0.8	-8.0E-12	-7.0E-12	-7.1	-4.5E-12	-1.1E-12	-4.5E-15	-3.9	-3.5E-12	-5.6E-13	-8.4E-14	-8.4E-14	-2.5	-3.0E-12	-2.3E-12	-1.5E-12
Uganda	-0.1	-3.0E-09	-2.3E-09	-1.1	-7.3E-10	-1.6E-10	-1.4E-11	-0.6	-1.7E-09	-8.4E-10	-8.3E-11	-8.3E-11	-0.4	-1.0E-08	-6.6E-09	-6.7E-11
UK	-0.9	-8.3E-12	-5.8E-12	-8.6	-4.6E-12	-6.5E-13	-2.0E-15	-2.8	-2.1E-12	-1.6E-12	-1.5E-13	-1.5E-13	-1.9	-6.7E-12	-3.6E-12	-1.7E-12
US	-0.7	-5.2E-12	-1.3E-12	-9.8	-2.8E-12	-1.4E-13	-5.3E-16	-2.6	-1.1E-12	-3.6E-13	-1.1E-13	-1.1E-13	-1.6	-2.9E-12	-1.4E-12	-7.3E-13
Zambia	-0.1	-2.81E-09	-1.6E-09	-0.7	-1.5E-09	-1.6E-10	-2.7E-11	-0.6	-3.1E-09	-1.9E-09	-6.7E-11	-6.7E-11	-0.4	-2.0E-08	-6.0E-10	-5.7E-11