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# Smooth, strategic communication

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## Abstract

We study strategic communication between a Sender and Receiver who are both uncertain about their preferred actions. The Sender observes noisy signals about both players' ideal policies and then communicates with the Receiver. Even though Sender and Receiver disagree about ideal policies as a function of the Sender's information, we can show that: i) there are information structures such that in equilibrium the Sender credibly communicates his ideal policy and the Receiver correctly takes the Sender's advice at face value and ii) the unique outcome of Nash-bargaining over information structures induces precisely a situation where communication about ideal policies is credible. The resulting equilibrium features message strategies that are smooth in a subspace of the Sender's information. Smooth communication equilibria are extremely tractable. Senders with better aligned preferences are endogenously endowed with better information and therefore give more accurate advice.

**JEL:** D82

**Keywords:** strategic information transmission, monotone strategies, elliptical distributions, endogenous information, bargaining

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# 1 Introduction

In a seminal paper, (Crawford & Sobel (1982)), henceforth CS, study strategic communication in a Sender-Receiver-game. The Sender is perfectly informed about some state of nature, the Receiver has no information. Even though the Sender's information is useful in reaching a good decision, a Sender whose interests differ from those of the Receiver garbles his information in any equilibrium implying that only a discrete number of different meanings can be conveyed.

Almost any communication in practice is strategic. Thus, the literature has applied the CS model to various contexts to explain e.g., communication between central bankers and the public (Stein (1989), Moscarini (2007)), or communication between financial analysts and investors (Morgan & Stocken (2003), Ottaviani & Sørensen (2006*a*), Ottaviani & Sørensen (2006*b*)), and to analyze trade-offs between various forms of organizations (Dessein (2002), Alonso et al. (2008), and Goltsman et al. (2009)). The CS framework has been extended in various important directions, which include multiple senders (Battaglini (2002)), the selection of equilibria (Chen et al. (2008)), higher dimensional preferences and choices (Chakraborty & Harbaugh (2007)), (Levy & Razin (2007)), noisy talk (Blume et al. (2007)), lying costs (Kartik (2009), Kartik et al. (2007)), and many others.<sup>1</sup>

Even though many questions have been answered, there remain interesting and important problems that are hard to tackle analytically. In the CS model, the Sender's information structure is part of the model's primitives. Yet, in many situations in practice, it is the prospect of communication itself that provides the incentives to be informed in the first place. How does the advice and the quality of decision-making depend on the quality of information? What are the trade-offs involved in improving the Sender's information? Which Sender types would we expect to have better information and to give more accurate advice?

The discreteness inherent in strategic communication equilibria makes these questions extremely hard to answer. Changing the information structure involves complex trade-offs because the information structure affects the equilibrium meaning of messages and hence the equilibrium partition of the type space in non-obvious ways. We offer a solution to this problem.

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<sup>1</sup>This list is obviously short and therefore incomplete.

The model we study in this paper has three key ingredients. Firstly, Sender and Receiver are uncertain about their preferred actions *ex ante* and agreed upon the ideal action based on prior information. Secondly, the Sender obtains noisy signals about both players' ideal actions. Thirdly, Sender and Receiver jointly determine the optimal noise level in the Sender's information prior to the communication stage.

The first two components describe situations in which Sender and Receiver disagree on the optimal use of news; they are, however, agreed upon the use of information they already possess. This is a natural assumption in many advisor-advisee situations. It captures the idea that *ex ante* differences of opinions can be eliminated by carefully selecting the advisor. However, it may be more difficult, and not always possible, to eliminate all conflicts of interests that will arise in the future. E.g., a financial advisor's past action may provide a good signal about his current information; but past actions may not provide any information as to how future information is used. Likewise, an advisor who thinks of himself as being more competent than the advisee thinks he is will to act more aggressively on future information than the advisee would like him to act. Many more situations come to mind.

To capture such situations neatly, we assume that Sender and Receiver have symmetric preferences around some uncertain bliss points and that the priors on these bliss points are elliptical distributions. Moreover, news - that is, the Sender's signals - is modelled again by elliptically distributed signals about the underlying bliss points. As a result of symmetric preferences and elliptical distributions, ideal choices as a function of news correspond to conditional means, which are linear functions of the underlying news. The slopes of these conditional means measure how aggressively the players would like to respond to new information.

The third important ingredient in our model is that Sender and Receiver get together before the communication stage to determine cooperatively what signals will be available to the Sender. Specifically, Sender and Receiver engage in Nash-bargaining to determine the information structure that the Sender will face. As usual, the cooperative perspective should be taken as a metaphor for saying that players get together to structure the environment in ways that are most conducive to reaching their goals.

Though seemingly more complex than the standard one, our model is extremely tractable. Given symmetry of preferences and the available information structures, the Sender's ideal

choice simply corresponds to the conditional mean, and he compares any feasible choices just with respect to how far they are from this ideal choice. Hence, in any equilibrium he will communicate at most the conditional expected value of his bliss point given the information he observes, but not more. For arbitrary information structures, equilibria of the communication game therefore take the standard form of partitional equilibria in the space of conditional means. However, for some information structures, a smooth communication equilibrium exists. In a smooth communication equilibrium, the Sender truthfully announces what action he would like to take - the conditionally expected value of his bliss point, conditional on the signals he has observed - and the Receiver follows the Sender's advice. The Receiver is not naively credulous but wisely so: the Receiver takes the optimal action relative to his preferences given the information he has obtained - the conditional expected value of the Receiver's bliss point conditional on the recommendation made by the Sender. The construction is an equilibrium if the regression of the Receiver's bliss point on the Sender's recommendation has a slope of exactly one, which we show to be possible under very general conditions.

When the players get together *ex ante* to agree upon the information structure, they anticipate that the noise structure has two functions. Firstly, noise determines the overall surplus the players can reach through communication. Secondly, noise shifts surplus between the Sender and the Receiver. With two sources of noise, these functions can be performed separately, and we can think of the players as first maximizing the size of the cake and then bargain over its division. For a class of environments, we show that Nash-bargaining over noise levels results in an information structure that enables the players to communicate smoothly. More precisely, the players even coordinate on the most efficient information structure that allows them to communicate smoothly about conditional means.

The equilibrium has a number of intuitive properties. In particular, our model allows us to answer the questions that have proven to be out of reach so far. We show that a Sender whose interests are better aligned with those of the Receiver will be endowed with more precise information. As a result, the quality of equilibrium advice increases with the alignment of preferences. Moreover, choices become more variable, because the variance of the *ex ante* distribution of conditional means is increased.

In terms of practical implications, we advocate transparency of methods coupled with

communication of aggregated data. The president of the Federal Reserve Board is heavily concerned with controlling the market's reaction to the Fed's press releases. This is a typical situation, where there would be no disagreement only based on prior information, but the interpretation of news may differ substantially between the Fed and the market. According to our theory, we expect there to be transparency with respect to the key economic factors that the Fed investigates and transparency with respect to the conclusions that the Fed reaches, as long as the Fed can be silent as to how it reaches its conclusions. A means to achieve this goal is to present aggregated data. Similarly, financial institutions engaged in investment advice should put their research methods on the table - alongside with their own financial interests. This enables them to communicate informative advice - as long as they are not forced to say how they reach their conclusions. Other examples abound.

Our approach is connected to several others in the literature. Moscarini (2007) investigates how the quality of information relates to the quality of decision making. He shows that better information for a given bias enables better equilibrium communication - in the sense that the Sender is able to communicate finer partitions and that the maximum informative equilibrium can induce a greater number of actions. In contrast to his theory, the quality of information is endogenous here and the advisor is ultimately more competent because he is more reliable.

The statistical model we use has been exploited to great success in the finance literature on market making. While much of this literature studies the special case of normally distributed uncertainty, Noldeke & Tröger (2004) show that the existence of linear equilibria in a class of models extends to the case of elliptical distributions. To the best of our knowledge, the usefulness of finance market models for the analysis of strategic communication games has not been noted anywhere.

An important building block in our model is the idea that the information structure can be chosen before the information transmission game is played, which allows us to select the equilibrium we focus on. Ivanov (2010) studies informational control in the strategic communication game. However, the optimal information structure in Ivanov (2010) remains partitional so perfect communication is impossible. Ivanov (2013) studies a dynamic model where the Receiver can control what information the Sender obtains in each step. Our information structure is determined in one step only and our statistical environment is very

different from Ivanov's. Moreover, our approach allows us to derive novel predictions relating conflicts of interest to the quality of decision-making.

An alternative to bargaining over information structures is costly information acquisition by the Sender. Szalay (2005) studies the case of aligned ex post preferences with commitment to decision rules; Esó and Szalay (in preparation) study the same model without commitment to decision rules. Argenziano et al. (2013) look at endogenous information in the case of a sender who is uniformly biased in one direction; Di Pei (2013) studies a model where the Sender can choose the partitional information that he observes. With a biased sender, the equilibrium remains partitional.

The equilibrium of our model is so easily tractable because an uncountable infinity of actions is induced - communication is smooth in the appropriate space (of conditional means). We are not aware of any other model in the literature that shares this feature of "large action sets". However, in some models equilibria featuring a countable infinity of induced actions arise. Gordon (2010) has shown that, depending on how the ideal choice functions of Sender and Receiver intersect, the most informative equilibrium induces either a finite or a countably infinite number of distinct Receiver actions. Alonso et al. (2008) characterize the most informative equilibrium for the case of uniform distributions and quadratic utilities.

We confine ourselves to communication between one sender and one receiver in this paper; the multisender case is analyzed in companion work.

The paper is organized as follows. In section two, we present the model and the main assumptions. In section three, we define strategies and analyze equilibrium communication. In section four we prove our result on the existence of a perfectly informative equilibrium in conditional expectations. In section five we analyze the optimal noise structure. In section six, we focus on comparative statics. Lengthy proofs are gathered in the appendix.

## 2 The model

### 2.1 Timing

We consider a Sender-Receiver-game with two players and the following underlying decision problem. The Receiver needs to choose a decision  $x$  that affects the utility of both the Sender

and the Receiver. Preferences are given by

$$u^R(x - \omega)$$

and

$$u^S(x - \eta)$$

where  $u^i(x - \cdot)$  for  $i = R, S$  is a strictly concave, symmetric and differentiable function. The preferences only depend on the distance to the bliss points  $x^R(\omega) = \omega$  and  $x^S(\eta) = \eta$ , respectively.

At the outset, neither the Sender nor the Receiver know the realization of the random variables  $\omega$  and  $\eta$ . However, the Sender observes noisy signals of their realizations according to

$$s_\omega = \omega + \varepsilon_\omega$$

and

$$s_\eta = \eta + \varepsilon_\eta,$$

where  $\varepsilon_\omega$  and  $\varepsilon_\eta$  are independent noise terms with variances that are cooperatively determined at the start of the game, where the Sender and the Receiver get together to determine the optimal amount of noise in the Sender's information structure.

After observing the realizations of  $s_\omega$  and  $s_\eta$ , the Sender sends a message to the Receiver. The Receiver observes the message - but not the Sender's information - and then chooses  $x$ . The Receiver cannot commit ex ante how to use the information he receives from the Sender.

## 2.2 Feasible Information Structures

We assume that the random variables  $\omega, \eta, \varepsilon_\omega, \varepsilon_\eta$  are jointly elliptically distributed, with finite first and second moments. Each of the marginals has mean zero; let  $\sigma_\omega^2, \sigma_\eta^2, \sigma_{\varepsilon_\omega}^2, \sigma_{\varepsilon_\eta}^2$  denote the variances of the random variables. For convenience, we assume that the joint distribution has a density. More assumptions are imposed later when needed.

Elliptical distributions can be defined as follows (e.g. Owen & Rabinovitch (1983), p.746).



**Definition 1** Let  $\mu$  be a fixed  $n$ -component vector and  $\Sigma$  an  $(n \times n)$  positive definite symmetric matrix. Then, an  $n$ -component random vector  $X = (X_1, \dots, X_n)'$  is said to be distributed elliptically,  $X \sim \mathcal{E}_n(\mu, \Sigma)$ , if and only if for all nonzero  $n$ -component scalar vectors  $\alpha$ , all the univariate random variables  $\alpha'X$  such that  $\text{Var}(\alpha'X)$  is constant follow the same distribution. If  $X$  has a density, then the density function of  $X$ ,  $f_X(x)$ , can be expressed in the form  $f_X(x) = c_n |\Sigma|^{-1/2} g((x - \mu)' \Sigma^{-1} (x - \mu))$ , for some constant  $c_n$  and some function  $g$  that is independent of  $n$ .

Given that the first two moments exist,  $\mu$  is mean vector and  $\Sigma$  is proportional (that is, up to a constant factor equal to) the variance-covariance matrix.

Elliptical distributions have convenient properties. We summarize the properties we use in the following Lemma:

**Lemma 1** Let  $X \sim \mathcal{E}_n(\mu, \Sigma)$  be elliptically distributed. Further let

$$X = (X_1, X_2), \quad \mu = (\mu_1, \mu_2), \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

where the dimensions of  $X_1$ ,  $\mu_1$  and  $\Sigma_{11}$  are, respectively,  $m$ ,  $m$  and  $m \times m$ .

i) The elliptical distribution is symmetric about  $\mu$ , i.e.

$$f(\mu + \Delta) = f(\mu - \Delta) \quad \forall \Delta.$$

ii) Linear combinations of elliptically distributed random variables are again elliptical.

iii) The conditional distribution of  $X_1|X_2$  is elliptical, i.e.

$$X_1|X_2 \sim \mathcal{E}_m(\mu_{X_1} + \Sigma_{12}\Sigma_{22}^{-1}(X_2 - \mu_{X_2}), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}).$$

**Proof.** i) by definition, ii) see for example, Fang et al. (1990) Thm 2.16, iii) see for example, Cambanis et al. (1981) Cor.5.

Examples for elliptical distributions are the normal distribution, the Student-t distribution, the exponential power distribution, and the logistic distribution.

### 2.3 Conflicting interest with respect to news

Since  $\omega$  and  $\eta$  have the same prior mean, Sender and Receiver are agreed upon the optimal action if the Sender does not obtain any new information; formally, this is the limiting case when  $\sigma_{\varepsilon_\omega}^2$  and  $\sigma_{\varepsilon_\eta}^2$  both go to infinity. So, potential conflicts arise only with respect to how new information should be used. The following Lemma characterizes ideal choice functions for given information and identifies conflicts between Sender and Receiver. Let  $\rho_{\omega\eta} \equiv \frac{Cov(\omega,\eta)}{\sigma_\omega\sigma_\eta}$ .

**Lemma 2** *The ideal choice functions of Receiver and Sender are*

$$x^R(s_\omega, s_\eta) \equiv \arg \max_x \mathbb{E} [u^R(x - \omega) | s_\omega, s_\eta] = \mathbb{E} [\omega | s_\omega, s_\eta]$$

and

$$x^S(s_\omega, s_\eta) \equiv \arg \max_x \mathbb{E} [u^S(x - \eta) | s_\omega, s_\eta] = \mathbb{E} [\eta | s_\omega, s_\eta].$$

Moreover,

$$\mathbb{E} [\omega | s_\omega, s_\eta] = \mathbb{E} [\eta | s_\omega, s_\eta] \quad \text{for all } s_\omega, s_\eta$$

only if the noise variances  $\sigma_{\varepsilon_\omega}^2, \sigma_{\varepsilon_\eta}^2$  are bounded away from zero. For  $\sigma_{\varepsilon_\omega}^2, \sigma_{\varepsilon_\eta}^2$  strictly positive and finite the above equality holds if and only if  $\sigma_\omega = \sigma_\eta$  and  $\rho_{\omega\eta} = 1$ .

Furthermore,

$$\lim_{\sigma_{\varepsilon_i}^2 \rightarrow \infty} \mathbb{E} [\omega | s_\omega, s_\eta] = \lim_{\sigma_{\varepsilon_i}^2 \rightarrow \infty} \mathbb{E} [\eta | s_\omega, s_\eta]$$

for  $\sigma_{\varepsilon_j}^2$  positive and finite if  $\rho_{\omega\eta}\sigma_i = \sigma_j$ , with  $i, j = \eta, \omega$ .

Preferences are symmetric around their bliss point and all conditional distributions are elliptical and hence symmetric around their mean. Hence, expected losses are minimized, if the action is set equal to the expected value of the bliss point  $\omega$ , conditional on the available information. The formal proof is in the Appendix.

Sender and Receiver disagree on how to use new information, except in very special cases. Clearly, if perfect signals are available, then the Sender is interested in  $s_\eta$  while the Receiver is interested in  $s_\omega$ . To make them agree on how to use new information with positive and finite noise variances, they have to be equally uncertain ex ante and need to have perfectly aligned interests; this is virtually the case where their bliss points depend on one and the

same random variable only. Finally, unless the covariance of  $\omega$  and  $\eta$  happens to be identical to either one of the variances of  $\omega$  or  $\eta$ , Sender and Receiver also disagree on how to use a single signal if only one is available - formally, this is the limiting case where the variance of the other signal goes to infinity.

We are clearly interested in the cases where communication is plagued by differences of objectives. Therefore, we impose the following Assumption:

**Assumption 1:**  $\rho_{\omega\eta} \neq \min \left\{ \frac{\sigma_\eta}{\sigma_\omega}, \frac{\sigma_\omega}{\sigma_\eta} \right\}$ .

Assumption 1 implies conflict with respect to the use of the signals.

### 3 Strategic Communication

We now solve our game under asymmetric information. There are two stages of the game. In the first stage, Sender and Receiver get together and engage in Nash-bargaining over the information that the Sender can observe. The cooperative nature of this bargaining game captures the idea that Sender and Receiver wish to stack the deck in a way that enables them to reach the highest possible surplus in the ensuing communication game. The communication game is modelled non-cooperatively. We assume that the Receiver cannot commit to what action he will take as a function of the message he receives; moreover, information is soft, so the Sender cannot credibly commit to being honest either. The equilibrium concept we use is Perfect Bayes Nash Equilibrium. In the equilibria we study, there won't be any out of equilibrium messages, so Perfect Bayes Nash Equilibrium and Bayes Nash Equilibrium coincide.

We solve our game proceeding backwards from the communication phase to the bargaining phase.

#### 3.1 Strategies

Take the experiment - that is the noise variances  $\sigma_{\varepsilon_\omega}^2$  and  $\sigma_{\varepsilon_\eta}^2$  - as given. After the Sender has observed his signals, he communicates a message  $m \in \mathbb{M}$  to the Receiver. The message space

$\mathbb{M}$  is assumed to be rich. It will be enough to consider pure message strategies for the Sender.<sup>2</sup> A pure Sender strategy maps the Sender's information,  $I$ , into messages  $M : \mathbb{R}^2 \rightarrow \mathbb{M}$ ,  $I \mapsto m$ . A pure Receiver strategy maps messages into actions,  $X : \mathbb{M} \rightarrow \mathbb{R}$ ,  $m \mapsto x$ . Let  $f(\omega|m)$  denote the pdf of the Receiver's posterior over  $\omega$  given the message sent by the Sender.

The Receiver's optimal action given the Sender's message is

$$x^R(m) \in \arg \max_{x \in \mathbb{R}} \int u^R(x - \omega) f(\omega|m) d\omega.$$

Note that the Receiver solves a concave problem for arbitrary posteriors  $f(\omega|m)$ . Hence  $x^R(m)$  is in fact unique and the Receiver never wishes to use a mixed strategy.

The Sender's optimal message solves

$$m(I) \in \arg \max_{m \in \mathbb{M}} \int u^S(x^R(m) - \eta) f(\eta|I) d\eta.$$

The Receiver updates his information about  $\omega$  based on the message he receives, so the posterior distribution satisfies Bayes law. Let  $s(m) \equiv \{I = \{s_\omega, s_\eta\} : m(I) = m\}$ . Then,

$$f(\omega|m) = \frac{f(\omega, m)}{\iint_{s(m)} f(s_\omega, s_\eta) ds_\omega ds_\eta}.$$

The numerator on the right-hand side is the joint likelihood that the state is  $\omega$  and message  $m$  is received. The denominator is the likelihood of receiving message  $m$ ; it is obtained by integrating over the Sender types that send message  $m$ .

### 3.2 Non-existence of full communication equilibria

Under Assumption 1, there cannot be any equilibrium where the Sender communicates all his information truthfully. Formally, we have the following result:

**Lemma 3** *Under Assumption 1, there does not exist any equilibrium where  $m(I) = I$ .*

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<sup>2</sup>More specifically, the *best* equilibrium of our game will involve pure strategies. Therefore, we abstain from introducing the notational clutter to deal formally with mixed strategies.

**Proof.** Suppose there exists an equilibrium with  $m(I) = I$ . Then, the Receiver's posterior is  $f(\omega|I)$  resulting in an optimal choice  $x^R(I) = \mathbb{E}[\omega|I]$ . By Assumption 1,  $\mathbb{E}[\omega|I] \neq \mathbb{E}[\eta|I]$  for any  $I$  in the relevant set. Hence, regardless of the signal structure, as long as the informational environment satisfies Assumption 1, there cannot be a fully informative equilibrium. ■

Even if Sender and Receiver agree that both signals should be used to reach a good decision, they disagree with respect to the weight they attach to the individual sources of information. Therefore, the Sender is not willing to reveal his information truthfully to the Receiver if the latter cannot commit to use the information the way the Sender proposes.

### 3.3 Sender preferences and conditional expectations

Given symmetry of the Sender's payoff and symmetry of the distributions, there is a neat simplification available. The Sender cares only about the conditionally expected value of  $\eta$  given his information. To see this, suppose there are two messages  $m$  and  $m'$  and the Receiver's optimal actions are  $x^R(m)$  and  $x^R(m')$ , respectively, where wlog  $x^R(m') > x^R(m)$ . Then, a Sender of type  $I = \{s_\omega, s_\eta\}$  prefers to send message  $m$  rather message  $m'$  if

$$\int u^S(x(m) - \eta) f(\eta|s_\omega, s_\eta) d\eta \geq \int u^S(x(m') - \eta) f(\eta|s_\omega, s_\eta) d\eta.$$

Given that preferences have bliss point  $\mathbb{E}[\eta|s_\omega, s_\eta]$  and given the symmetry of the distribution, the Sender simply prefers to induce the action that comes closer to  $\mathbb{E}[\eta|s_\omega, s_\eta]$ . Conversely, the set of types that are indifferent between sending message  $m$  and message  $m'$  are those  $s_\omega, s_\eta$  for which

$$\mathbb{E}[\eta|s_\omega, s_\eta] = \frac{x(m') + x(m)}{2}. \tag{1}$$

Equation (1) describes a locus of points  $s_\omega, s_\eta$  that induce a constant conditional expectation; the constant depends on the induced actions.

Clearly, we cannot expect to have meaningful communication beyond what is of interest to the Sender. Hence, without loss of generality we can reduce the Sender's message space to the unidimensional space of conditionally expected means. Define

$$\theta \equiv \mathbb{E}[\eta|s_\omega, s_\eta].$$

Given our informational assumptions,  $\mathbb{E}[\eta | s_\omega, s_\eta]$  is a linear function of  $s_\omega$  and  $s_\eta$ ; formally, we have  $\mathbb{E}[\eta | s_\omega, s_\eta] = \alpha s_\omega + \beta s_\eta$ , where  $\alpha$  and  $\beta$  are constants and given by

$$\begin{aligned}\alpha &= \frac{\sigma_{\varepsilon_\eta}^2 \sigma_\eta \sigma_\omega \rho_{\omega\eta}}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2)(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - \rho_{\omega\eta}^2 \sigma_\omega^2 \sigma_\eta^2} \\ \beta &= \frac{\sigma_\eta^2 (\sigma_{\varepsilon_\omega}^2 + \sigma_\omega^2) - \sigma_\eta^2 \sigma_\omega^2 \rho_{\omega\eta}^2}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2)(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - \rho_{\omega\eta}^2 \sigma_\omega^2 \sigma_\eta^2}.\end{aligned}\tag{2}$$

From the perspective of the Receiver,  $s_\omega$  and  $s_\eta$  are unknown. Since linear functions of elliptically distributed random variables are again elliptically distributed,  $\theta$  follows an elliptical distribution  $\theta \sim \mathcal{E}(0, \sigma_\theta^2)$  and the joint distribution of  $\omega, \eta$ , and  $\theta$  is elliptical too.

### 3.4 Equilibria

Our game has the standard plethora of equilibria in the communication game. Since this is standard, we contend ourselves with a heuristic description. It is always an equilibrium that all Sender types  $\theta$  pool on the same message (or mix over all available messages) and the Receiver always chooses  $x = 0$ . This is the babbling equilibrium. Depending on the conflict of interest, it may also be possible to sustain partitional equilibria, where sets of Sender types within given intervals pool on the same message. Again depending on the degree of conflict between Sender and Receiver, the number of actions induced in such an equilibrium may go out of bounds; that is a countable infinity of actions is induced.

We can keep this discussion heuristic, because all these equilibria will not be played when Sender and Receiver agree on an optimal information structure and then play the best equilibrium for both of them. The most informative equilibrium for the optimal information structure induces an uncountable infinity of actions. The equilibrium is fully revealing in the space of conditional expectations.

Before we prove this statement, we establish a weaker result: there are information structures that enable perfect communication about conditional means in the most informative equilibrium.

## 4 A benchmark: perfect communication about conditional means

Suppose the Sender communicates his conditionally expected value of  $\eta$  given his information truthfully to the Receiver, that is

$$m(\theta) = \theta \text{ for all } \theta.$$

We know from Lemma 1 that the Receiver's posterior based on information  $\theta$  is

$$\omega | \theta \sim \mathcal{E} \left( \rho_{\omega\theta} \frac{\sigma_\omega}{\sigma_\theta} \theta, \sigma_\omega^2 (1 - \rho_{\omega\theta}^2) \right),$$

where  $\rho_{\omega\theta} = \frac{Cov(\omega, \theta)}{\sigma_\theta \sigma_\omega}$ . Hence the Receiver's ideal policy given information  $\theta$  is

$$\mathbb{E}[\omega | \theta] = \frac{Cov(\omega, \theta)}{Var(\theta)} \theta = \rho_{\omega\theta} \frac{\sigma_\omega}{\sigma_\theta} \theta.$$

Clearly, the Sender will only be happy to communicate  $\theta$  truthfully if the Receiver does exactly what the Sender would want to do, so

$$\mathbb{E}[\omega | \theta] = \theta \text{ for all } \theta,$$

which requires that

$$Cov(\omega, \theta) = Var(\theta). \tag{3}$$

The following theorem relates equation (3) to the underlying noise structure:

**Theorem 1** *For any conflict described by  $\rho_{\omega\eta} \in (0, 1]$  and  $\sigma_\omega^2, \sigma_\eta^2 \in (0, \infty)$  satisfying  $\frac{1}{\rho_{\omega\eta}} > \frac{\sigma_\omega}{\sigma_\eta} > \rho_{\omega\eta}$ , there is a continuum of information structures  $\sigma_{\varepsilon_\omega}^2, \sigma_{\varepsilon_\eta}^2$  such that a perfectly informative equilibrium in conditional expectations  $\theta$  exists if  $\sigma_{\varepsilon_\omega}^2, \sigma_{\varepsilon_\eta}^2$  satisfy*

$$\frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2} = \frac{\left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right)}{\rho_{\omega\eta} \left(\frac{\sigma_\omega}{\sigma_\eta} - \rho_{\omega\eta}\right)} \frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2} + \frac{\left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right)}{\rho_{\omega\eta} \left(\frac{\sigma_\omega}{\sigma_\eta} - \rho_{\omega\eta}\right)} (1 - \rho_{\omega\eta}^2). \tag{4}$$

Equation (4) arises from plugging  $\alpha$  and  $\beta$  from (2) into (3) and solving. The details can be found in the appendix. The conditions in the theorem ensure that the locus defined by equation (4) describes a line with positive intercept and slope.

Perfect communication about conditional expectations is possible provided that the Sender's and the Receiver's preferences are perfectly aligned in that subspace of the Sender's information. Interests are perfectly aligned if the regression of  $\omega$  on  $\theta$  has a slope of unity: the Receiver cannot extract more than  $\theta$  from what he is told. Notice that there is still bunching of Sender types; the Sender makes garbled statements about the underlying information. However, in contrast to partitional equilibria, the bunches are very easy to characterize.

For the obvious reasons, we impose henceforth

**Assumption 2:**  $\frac{1}{\rho_{\omega\eta}} > \frac{\sigma_{\omega}}{\sigma_{\eta}} > \rho_{\omega\eta} > 0$ .

Note that Assumption 2 implies that there is conflict between Sender and Receiver; formally, Assumption 2 implies Assumption 1. Notice also that for the case where Sender and Receiver are equally uncertain ex ante,  $\frac{\sigma_{\omega}}{\sigma_{\eta}} = 1$ , Assumption 2 simply requires that the correlation be positive but smaller than one.

The existence of a perfectly revealing equilibrium about  $\theta$  depends crucially on the details of the noise structure. For *given* noise variances, generically a fully revealing equilibrium does not exist. Rather, the noise structures have to be carefully adjusted to make sure they satisfy equation (4). We now show that Nash-bargaining about the experiment ex ante achieves precisely that the noise variances are set so as to enable perfect communication about  $\theta$  for a class of preferences.

## 5 Bargaining over information

Suppose any given equilibrium in the communication game is being played and Sender and Receiver both correctly anticipate that this equilibrium will be played. Suppose further that they both anticipate the ensuing equilibrium for any given information structure. By definition, for a given information structure, surplus is highest if players coordinate on an equilibrium that achieves the highest joint surplus. Assume this. We can then delineate a



bargaining frontier, that associates to each information structure a pair of utility gains for Sender and Receiver.

Describing the bargaining frontier requires solving for the best equilibrium for each given information structure, a complex task. Fortunately, we do not have to do this, to solve our game. Rather, we will provide an upper bound on what can be reached and show that Nash-bargaining achieves the upper bound. We split the following analysis into two parts. For the class of quadratic preferences, we can derive a complete Nash-bargaining solution. For more general preferences, we simply show that our equilibrium maximizes joint surplus and splits the difference between Sender and Receiver.

## 5.1 Quadratic loss functions

For the purpose of this section, we specialize our environment and assume that Sender and Receiver's preferences are described by quadratic loss functions around their bliss points  $\omega$  and  $\eta$ .

Suppose Sender and Receiver anticipate playing a partitional equilibrium with  $n$  (finite or infinite) induced actions. Upon receiving a message  $m_i$  sent by types  $\theta \in [\theta_{i-1}, \theta_i]$ , the Receiver takes action

$$x_i^* = \mathbb{E}[\omega | \theta \in [\theta_{i-1}, \theta_i]].$$

It is easy to show - see the appendix for details - that relative to making a decision based on prior information only, the utility gain to the Sender can be written as

$$\Delta U^S = - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} (x_i^* - \theta)^2 f(\theta) d\theta + Var(\theta) \quad (5)$$

and the utility gain to the Receiver can be written as

$$\Delta U^R = - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} (x_i^* - \mathbb{E}[\omega | \theta])^2 f(\theta) d\theta + \frac{Cov(\theta, \omega)^2}{Var(\theta)}. \quad (6)$$

$Var(\theta)$  is the utility gain to the Sender if his preferred policy  $x(\theta) = \theta$  for all  $\theta$  is implemented. The actual utility gain is the maximum utility gain net of the losses due to limited communication in a partitional equilibrium. Likewise,  $\frac{Cov(\theta, \omega)^2}{Var(\theta)}$  is the maximum utility gain to the Receiver if his preferred policy  $x(\theta) = \mathbb{E}[\omega | \theta]$  for all  $\theta$  is implemented. His actual

utility gain in any equilibrium is therefore equal to the maximum possible gain net of any losses due to limited communication.

It follows that, in any equilibrium, the Sender cannot expect to benefit more than if the Receiver commits to just blindly follow his advice and choose  $x(\theta) = \theta$  for all  $\theta$ , so

$$\Delta U^S \leq Var(\theta);$$

and the Receiver cannot expect to benefit more than if the Sender would always convey  $\theta$  truthfully and the Receiver is allowed to choose  $x(\theta) = \mathbb{E}[\omega|\theta]$  for all  $\theta$ , that is

$$\Delta U^R \leq \frac{Cov(\theta, \omega)^2}{Var(\theta)}.$$

Consider an ideal - and in general unrealistic - world, where both players indeed get their way, so that

$$\Delta U^S = Var(\theta)$$

and

$$\Delta U^R = \frac{Cov(\theta, \omega)^2}{Var(\theta)}.$$

Of course, this is unrealistic, except for the case where  $\mathbb{E}[\omega|\theta] = \theta$  for all  $\theta$ . However, suppose for a moment that Sender and Receiver bargain in paradise. In this game, we have

$$\Delta U^R = \frac{Cov(\theta, \omega)^2}{\Delta U^S}. \tag{7}$$

$Var(\theta)$  and  $Cov(\theta, \omega)$  can be varied independently of each other. Therefore, we can view (7) as a bargaining frontier of a paradise bargaining game, where  $Cov(\theta, \omega)$  varies the level of the surplus and  $Var(\theta)$  divides the surplus between the Sender and the Receiver. The Nash-bargaining problem on this frontier solves

$$\begin{aligned} \max \Delta U^S \Delta U^R \\ s.t. \Delta U^R = \frac{Cov(\theta, \omega)^2}{\Delta U^S}. \end{aligned} \tag{8}$$

Notice that the objective coincides with the constraint. Hence, *all* the allocations on the frontier solve the bargaining problem in the artificial game. The value of the objective is

equal to  $Cov(\theta, \omega)^2$  and is independent of  $Var(\theta)$ . As part of the proof of the theorem that follows, we show in the appendix that setting  $\sigma_{\varepsilon_\omega}^2 = 0$  maximizes  $Cov(\theta, \omega)$  and the objective takes value  $Cov(\eta, \omega)$  in that case. Moreover, for  $\sigma_{\varepsilon_\omega}^2 = 0$ ,  $Var(\theta)$  ranges from  $\rho_{\omega\eta}^2 \sigma_\eta^2$  (for  $\sigma_{\varepsilon_\eta}^2 = \infty$ ) to  $\sigma_\eta^2$  (for  $\sigma_{\varepsilon_\eta}^2 = 0$ ). So,  $\sigma_{\varepsilon_\omega}^2 = 0, \sigma_{\varepsilon_\eta}^2 \geq 0$  solves the bargaining game in the artificial game where both players get the upper bound of their utility gains. These arguments make the following theorem immediate:

**Theorem 2** *Consider a Nash-bargaining game where the utility possibility frontier is given by the payoffs arising from playing the most informative equilibrium for any given information structure. Under Assumption 2, the unique outcome of this bargaining game is*

$$\left(\sigma_{\varepsilon_\omega}^2, \sigma_{\varepsilon_\eta}^2\right) = \left(0, \frac{\left(1 - \frac{\sigma_\omega}{\sigma_\eta} \rho_{\omega\eta}\right)}{\rho_{\omega\eta} \left(\frac{\sigma_\omega}{\sigma_\eta} - \rho_{\omega\eta}\right)} (1 - \rho_{\omega\eta}^2) \sigma_\eta^2\right)$$

so that  $Var(\theta) = Cov(\theta, \omega) = Cov(\eta, \omega)$ .

The intuition is very simple. Note that the bargaining frontier cannot be characterized for all information structures. From our preliminary thoughts we know that the Sender cannot be made better off for any given information structure than when the Receiver is credulous; and the Receiver cannot be made better off than when the Sender non-strategically tells him the truth even if the Receiver responds strategically to such messages. For this artificial game, the bargaining frontier was easy to describe. Moreover, there is a unique situation in which the bargaining frontier of the artificial game and the true bargaining game coincide: when the Receiver is rightly credulous and the Sender is rightly non-strategic. This is precisely then the case when the Sender can announce  $\theta$  and the Receiver optimally responds by taking this advice at face value, which requires that  $Var(\theta) = Cov(\eta, \omega)$ . So, for this unique situation, the bargaining frontiers - and hence the objectives - coincide in the two problems. For all other information structures, the true bargaining frontier must be strictly below the bargaining frontier of the artificial problem.

As is well known, Nash-bargaining with equal weights results in splitting the difference. For  $Var(\theta) = Cov(\theta, \omega)$ , the utility gains (5) and (6) are equal for the equilibrium that fully reveals  $\theta$  and moreover, the utility gain to each of them is maximal if  $Cov(\theta, \omega) = Cov(\eta, \omega)$ .

## 5.2 General preferences

Nash-bargaining with general preferences is not straight-forward. However, we can establish a simpler result. The information structure described in Theorem 2 maximizes joint surplus. Moreover, if Sender and Receiver have the same utility functions and are ex ante equally uncertain about their bliss points (the prior variances are the same), then the information structure divides the gains from the experiment equally between the Sender and the Receiver.

If Sender and Receiver anticipate that  $x(\theta) = \theta$ , then their utility gains from the experiment can be written as

$$\Delta U^S = \mathbb{E}u^S(\theta - \eta) - \mathbb{E}u^S(0 - \eta)$$

and

$$\Delta U^R = \mathbb{E}u^R(\theta - \omega) - \mathbb{E}u^R(0 - \omega).$$

For equal prior variances and  $u^S(\cdot) = u^R(\cdot)$ , Sender and Receiver have the same utility if they had to choose  $x$  based on prior information only. Hence, splitting the gains equally requires that Sender and Receiver have the same utility from decision making when  $x(\theta) = \theta$  for all  $\theta$  is being chosen. Note that the random variables  $\theta - \eta$  and  $\theta - \omega$  both have mean zero and moreover arise from linear combinations of elliptically distributed random variables. Hence, by their defining property, the variances of these distributions uniquely define the distributions. Therefore, the utility gains are identical if and only if the distributions of both differences have identical variance. That is, if

$$\sigma_\eta^2 - \sigma_\theta^2 = \sigma_\theta^2 - 2Cov(\omega, \theta) + \sigma_\omega^2,$$

where we have used the fact that  $Cov(\eta, \theta) = \sigma_\theta^2$  - which follows from  $\mathbb{E}[\eta|\theta] = \theta$ . Obviously, for  $\sigma_\eta^2 = \sigma_\omega^2$  the equation holds if and only if  $\sigma_\theta^2 = Cov(\omega, \theta)$ . Moreover, the uncertainty is minimized if  $Cov(\omega, \theta)$  is maximal, that is if  $Cov(\omega, \theta) = Cov(\omega, \eta)$ , which is precisely achieved by the information structure in Theorem 2.

## 6 Comparative statics

How does the quality of advice and of decision-making depend on conflicts of interests? How does prior uncertainty impact on decision-making? Our model gives very clear and simple

answers to these questions. The equilibrium quality of advice is measured by  $\sigma_\theta^2$ . The higher the variance of induced choices, the better off are both Sender and Receiver. In the most informative equilibrium for the optimal information structure, we have

$$\sigma_\theta^2 = Cov(\omega, \eta) = \rho_{\omega\eta}\sigma_\omega\sigma_\eta.$$

If interests are better aligned, then both Receiver and Sender are better off. The reason is that a Sender with better aligned preferences can be given access to better information. Therefore, an advisor who is more trustworthy will appear more competent: his equilibrium information is of better quality. It is interesting to note that higher prior uncertainty has a positive impact on the quality of decision-making. The reason is that with a high level of prior uncertainty, the weight attached to new information tends to be higher. Since the variance of ideal choices based on prior information only is zero, placing more weight on new information unambiguously increases the variance.

## 7 Conclusion

We analyze a Sender-Receiver-Game of strategic communication in which the most informative equilibrium involves smooth communication and is therefore extremely tractable. The key ingredients are that Sender and Receiver are uncertain about but agreed upon their most preferred action ex ante; the Sender observes signals about the underlying states of nature and finally that the players get together before communicating to determine the level of noise in the Sender's information.

Since the Sender's information is of a higher dimension than his preferred policy, there is room for honest communication that is not completely revealing: the Sender can reveal what he would suggest to do - the conditionally expected value of his bliss point given the signal realizations - without revealing why he would do that - that is, without revealing the underlying signals. Thus, we obtain the standard partial pooling result of cheap talk equilibria; however, pooling is much easier to characterize in the most efficient equilibrium of our game, because only Sender types with the same conditional expectation pool and so we obtain communication that is perfectly revealing in that space.

The upshot of our theory is that transparency of methods is good and allows strategic

Senders to communicate credibly about statistics of the underlying information. The equilibrium is the more informative the better interests are aligned, with a slightly different twist than other theories predict. The Sender is endogenously endowed with better information when his interests are better aligned with those of the Receiver. Thus, more trustworthy Senders are better informed and therefore give more accurate advice.

## 8 Appendix

**Conditioning for elliptical distributions.** The expectation and covariance matrix are as follows

$$E[\omega, \eta, s_\omega, s_\eta] = (0, 0, 0, 0)'$$

and

$$Cov(\omega, \eta, s_\omega, s_\eta) = \begin{pmatrix} \sigma_\omega^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\omega^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta \\ \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\eta^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\eta^2 \\ \sigma_\omega^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta \\ \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\eta^2 & \rho_{\omega\eta}\sigma_\omega\sigma_\eta & \sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2 \end{pmatrix}.$$

By Lemma 1 the conditional variances can be calculated via

$$E[\mathbf{X} | \mathbf{Y}] = \mu_X + \Sigma_{12}\Sigma_{22}^{-1}(\mathbf{Y} - \mu_Y)$$

with

$$Cov(X, Y) = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}.$$

We obtain

$$\begin{aligned} \mathbb{E}[\omega | s_\omega] &= \frac{Cov(\omega, s_\omega)}{\sigma_{s_\omega}^2} s_\omega = \frac{\sigma_\omega^2}{\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2} s_\omega, \\ \mathbb{E}[\omega | s_\eta] &= \frac{\rho_{\omega\eta}\sigma_\omega\sigma_\eta}{\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2} s_\eta, \\ \mathbb{E}[\eta | s_\omega] &= \frac{\rho_{\omega\eta}\sigma_\omega\sigma_\eta}{\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2} s_\omega, \\ \mathbb{E}[\eta | s_\eta] &= \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2} s_\eta, \end{aligned}$$

and moreover

$$\begin{aligned}\mathbb{E}[\omega | s_\omega, s_\eta] &= s_\omega \left( \frac{\sigma_{\varepsilon_\eta}^2 + \sigma_\eta^2 - \sigma_\eta^2 \rho_{\eta\omega}^2}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2) (\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2} \right) \\ &\quad + s_\eta \left( \frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\eta \sigma_\omega \rho_{\omega\eta} (\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2) (\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2} \right)\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}[\eta | s_\omega, s_\eta] &= s_\omega \left( \frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta \sigma_\omega \rho_{\omega\eta} (\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2) (\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2} \right) \\ &\quad + s_\eta \left( \frac{\sigma_{\varepsilon_\omega}^2 + \sigma_\omega^2 - \sigma_\omega^2 \rho_{\omega\eta}^2}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2) (\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2} \right).\end{aligned}$$

■

**Proof of Lemma 2.** Part i) Let  $I$  denote the information available and let  $z = \omega, \eta$  and consider the problem

$$\max_x \int_{-\infty}^{\infty} u(x - z) f(z | I) dz$$

Since the utility depends only on the distance between  $x$  and  $z$  we have  $u'(x - z) > 0$  for  $z < x$ ,  $u'(x - z) = 0$  for  $x = z$ , and  $u'(x - z) < 0$  for  $z > x$ .

The first-order condition can be written as

$$\int_{-\infty}^{\infty} u'(x^* - z) f(z | I) dz = 0.$$

Consider the candidate solution

$$x^* = \mu \equiv E[z | I].$$

Substituting back, we can write

$$\int_{-\infty}^{\infty} u'(\mu - z) f(z | I) dz = 0.$$

Consider two points  $z_1 = \mu - \Delta$  and  $z_2 = \mu + \Delta$  for arbitrary  $\Delta > 0$ . By symmetry of  $u$  around its bliss point and symmetry of the distribution around  $\mu$ , we have

$$u'(\Delta) f(\mu - \Delta | I) = -u'(-\Delta) f(\mu + \Delta | I).$$

Since this holds pointwise for each  $\Delta$ , it also holds if we integrate over  $\Delta$ . Thus, the first-order condition holds at  $x^* = \mu$ . By concavity of  $u$  in  $x$ , only one value of  $x$  solves the first-order condition. Hence, the solution is the one stated in the Lemma.

Part ii): The conditional expectations are as follows:

$$\begin{aligned} \mathbb{E}[\eta | s_\omega, s_\eta] &= \sigma_{\varepsilon_\eta}^2 \frac{\rho_{\omega\eta} \sigma_\omega \sigma_\eta}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2)(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2} s_\omega \\ &\quad + \sigma_\eta^2 \frac{\sigma_{\varepsilon_\omega}^2 - \sigma_\omega^2 \rho_{\omega\eta}^2 + \sigma_\omega^2}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2)(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2} s_\eta \end{aligned} \quad (9)$$

and

$$\begin{aligned} \mathbb{E}[\omega | s_\omega, s_\eta] &= \sigma_\omega^2 \frac{\sigma_{\varepsilon_\eta}^2 + \sigma_\eta^2 - \sigma_\eta^2 \rho_{\omega\eta}^2}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2)(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2} s_\omega \\ &\quad + \sigma_{\varepsilon_\omega}^2 \frac{\sigma_\eta \sigma_\omega \rho_{\omega\eta}}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2)(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2} s_\eta. \end{aligned} \quad (10)$$

Consider first the case where at least one of the variances is zero and the other one finite, e.g.,  $\sigma_{\varepsilon_\eta}^2 = 0$ . In this case disagreement is obvious: the Sender does not use signal  $s_\omega$  at all while the Receiver wishes to take that into account. Likewise for the case where  $\sigma_{\varepsilon_\omega}^2$  is zero.

Next, consider the case where  $\sigma_{\varepsilon_\eta}^2$  and  $\sigma_{\varepsilon_\omega}^2$  are both positive and finite. (9) and (10) are identical for all  $s_\omega$  and  $s_\eta$  if and only if

$$\sigma_{\varepsilon_\eta}^2 \rho_{\omega\eta} \sigma_\omega \sigma_\eta = \sigma_\omega^2 (\sigma_{\varepsilon_\eta}^2 + \sigma_\eta^2 - \sigma_\eta^2 \rho_{\omega\eta}^2)$$

and

$$\sigma_\eta^2 (\sigma_{\varepsilon_\omega}^2 - \sigma_\omega^2 \rho_{\omega\eta}^2 + \sigma_\omega^2) = \sigma_\eta \sigma_\omega \rho_{\omega\eta} \sigma_{\varepsilon_\omega}^2.$$

This requires that

$$\sigma_\eta^2 (1 - \rho_{\omega\eta}^2) = \left( \frac{\rho_{\omega\eta} \sigma_\eta}{\sigma_\omega} - 1 \right) \sigma_{\varepsilon_\eta}^2$$



and

$$\sigma_\omega^2 (1 - \rho_{\omega\eta}^2) = \left( \frac{\sigma_\omega \rho_{\omega\eta}}{\sigma_\eta} - 1 \right) \sigma_{\varepsilon_\omega}^2.$$

A necessary condition for these two conditions to hold is that  $\rho_{\omega\eta} \geq \frac{\sigma_\omega}{\sigma_\eta}$  and  $\rho_{\omega\eta} \geq \frac{\sigma_\eta}{\sigma_\omega}$ . Since  $\rho_{\omega\eta} \in [-1, 1]$ , this implies that  $\rho_{\omega\eta} = \frac{\sigma_\eta}{\sigma_\omega} = \frac{\sigma_\omega}{\sigma_\eta}$  and therefore,  $\rho_{\omega\eta}^2 = 1$  and  $\sigma_\eta = \sigma_\omega$ .

Consider finally the limiting cases where one of the variances goes out of bounds. Applying l'Hôpital's rule to (9) and (10), we get in the limit as  $\sigma_{\varepsilon_\eta}^2 \rightarrow \infty$

$$\mathbb{E}[\omega|s_\omega] = \frac{\sigma_\omega^2}{\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2} s_\omega \quad \text{and} \quad \mathbb{E}[\eta|s_\omega] = \frac{\rho_{\omega\eta} \sigma_\omega \sigma_\eta}{\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2} s_\omega,$$

so that

$$\mathbb{E}[\omega|s_\omega] \equiv \mathbb{E}[\eta|s_\omega] \quad \Leftrightarrow \quad \rho_{\omega\eta} \sigma_\eta = \sigma_\omega.$$

Likewise, for the case where  $\sigma_{\varepsilon_\omega}^2 \rightarrow \infty$ , we get

$$\mathbb{E}[\omega|s_\eta] = \frac{\rho_{\omega\eta} \sigma_\omega \sigma_\eta}{\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2} s_\eta \quad \text{and} \quad \mathbb{E}[\eta|s_\eta] = \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2} s_\eta,$$

so

$$\mathbb{E}[\omega|s_\eta] \equiv \mathbb{E}[\eta|s_\eta] \quad \Leftrightarrow \quad \rho_{\omega\eta} \sigma_\omega = \sigma_\eta.$$

■

**Proof of Theorem 1.** The moments of the distribution of  $\theta$  are given by

$$\begin{aligned} \theta &= \mathbb{E}[\eta|s_\omega, s_\eta] = \alpha s_\omega + \beta s_\eta \\ \mathbb{E}[\theta] &= \alpha \mathbb{E}[s_\omega] + \beta \mathbb{E}[s_\eta] = 0 \\ \text{Var}(\theta) &= \alpha^2 (\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2) + 2\alpha\beta \rho_{\omega\eta} \sigma_\omega \sigma_\eta + \beta^2 (\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) \\ &= \frac{\sigma_{\varepsilon_\omega}^2 \sigma_\eta^4 + \sigma_{\varepsilon_\eta}^2 \sigma_\eta^2 \sigma_\omega^2 \rho_{\eta\omega}^2 - \sigma_\eta^4 \sigma_\omega^2 \rho_{\eta\omega}^2 + \sigma_\eta^4 \sigma_\omega^2}{\sigma_{\varepsilon_\omega}^2 \sigma_{\varepsilon_\eta}^2 + \sigma_{\varepsilon_\omega}^2 \sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2 \sigma_\omega^2 - \sigma_\eta^2 \sigma_\omega^2 \rho_{\omega\eta}^2 + \sigma_\eta^2 \sigma_\omega^2}, \end{aligned}$$

where we have used that

$$\begin{aligned} \alpha &= \sigma_{\varepsilon_\eta}^2 \sigma_\eta \sigma_\omega \frac{\rho_{\omega\eta}}{\sigma_{\varepsilon_\omega}^2 \sigma_{\varepsilon_\eta}^2 + \sigma_{\varepsilon_\omega}^2 \sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2 \sigma_\omega^2 - \sigma_\eta^2 \sigma_\omega^2 \rho_{\omega\eta}^2 + \sigma_\eta^2 \sigma_\omega^2} \\ \beta &= \sigma_\eta^2 \frac{\sigma_{\varepsilon_\omega}^2 - \sigma_\omega^2 \rho_{\eta\omega}^2 + \sigma_\omega^2}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2)(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - (\rho_{\omega\eta} \sigma_\omega \sigma_\eta)^2}. \end{aligned}$$

Moreover,

$$\begin{aligned} Cov(\omega, \theta) &= \mathbb{E}[\omega\theta] = \mathbb{E}[\alpha\omega s_\omega + \beta\omega s_\eta] = \alpha\sigma_\omega^2 + \beta\rho_{\omega\eta}\sigma_\omega\sigma_\eta \\ &= \sigma_\eta\sigma_\omega\rho_{\omega\eta} \frac{\sigma_{\varepsilon_\eta}^2\sigma_\omega^2 + \sigma_\eta^2\sigma_{\varepsilon_\omega}^2 + \sigma_\eta^2\sigma_\omega^2 - \sigma_\eta^2\sigma_\omega^2\rho_{\omega\eta}^2}{(\sigma_\omega^2 + \sigma_{\varepsilon_\omega}^2)(\sigma_\eta^2 + \sigma_{\varepsilon_\eta}^2) - \rho_{\omega\eta}^2\sigma_\omega^2\sigma_\eta^2} \end{aligned}$$

The statement then follows from solving  $Cov(\omega, \theta) = Var(\theta)$  for the underlying noise structure. ■

**Proof of Theorem 2.** We first derive expected payoffs in any partitional equilibrium demonstrating that the expressions in the text provide upper bounds on expected utilities.

Part i): expected utility

In a partitional equilibrium, upon receiving a message that is sent by types  $\theta \in [\theta_{i-1}, \theta_i]$ , the receiver takes action

$$x_i^* = \arg \max_x \int_{\theta_{i-1}}^{\theta_i} u^R(x - \omega) f(\omega | \theta \in [\theta_{i-1}, \theta_i]) d\omega.$$

With a slight abuse of notation, we denote the optimal action as  $x_i^*$  also in the limiting case where  $\theta_{i-1} = \theta_i$ . For the quadratic case, we have

$$x_i^* = \mathbb{E}[\omega | \theta \in [\theta_{i-1}, \theta_i]]$$

and, the Sender's expected utility,  $\sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} \int u^S(x_i^* - \eta) f(\eta | \theta) d\eta f(\theta) d\theta$ , can be written as

$$\begin{aligned} & - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} \int (x_i^* - \theta + \theta - \eta)^2 f(\eta | \theta) d\eta f(\theta) d\theta \\ &= - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} \int ((x_i^* - \theta)^2 + 2(x_i^* - \theta)(\theta - \eta) + (\theta - \eta)^2) f(\eta | \theta) d\eta f(\theta) d\theta \\ &= - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} (x_i^* - \theta)^2 f(\theta) d\theta - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} \int (\theta - \eta)^2 f(\eta | \theta) d\eta f(\theta) d\theta \\ &= - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} (x_i^* - \theta)^2 f(\theta) d\theta + Var(\theta) - Var(\eta) \end{aligned}$$

where the second and third equation follow from  $\theta = \mathbb{E}[\eta | \theta]$ , which implies also that  $Cov(\theta, \eta) = Var(\theta)$ .

Likewise, the Receiver's expected utility,  $\sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} \int u^R(x_i^* - \omega) f(\omega|\theta) d\omega f(\theta) d\theta$ , takes the form

$$\begin{aligned}
& - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} \int (x_i^* - \mathbb{E}[\omega|\theta] + \mathbb{E}[\omega|\theta] - \omega)^2 f(\omega|\theta) d\omega f(\theta) d\theta \\
= & - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} \int ((x_i^* - \mathbb{E}[\omega|\theta])^2 + 2(x_i^* - \mathbb{E}[\omega|\theta])(\mathbb{E}[\omega|\theta] - \omega) + (\mathbb{E}[\omega|\theta] - \omega)^2) f(\omega|\theta) d\omega f(\theta) d\theta \\
= & - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} (x_i^* - \mathbb{E}[\omega|\theta])^2 f(\theta) d\theta - \left( \frac{Cov(\theta, \omega)^2}{Var(\theta)^2} Var(\theta) - 2 \frac{Cov(\theta, \omega)^2}{Var(\theta)} + Var(\omega) \right) \\
= & - \sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} (x_i^* - \mathbb{E}[\omega|\theta])^2 f(\theta) d\theta + \frac{Cov(\theta, \omega)^2}{Var(\theta)^2} Var(\theta) - Var(\omega).
\end{aligned}$$

Part ii): bargaining

Using Theorem 1, we can write

$$Cov(\omega, \theta) = \sigma_\eta \sigma_\omega \rho_{\omega\eta} \frac{\frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2} + \frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2} + 1 - \rho_{\omega\eta}^2}{\left(1 + \frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2}\right) \left(1 + \frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2}\right) - \rho_{\omega\eta}^2}$$

and

$$Var(\theta) = \sigma_\eta^2 \frac{\frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2} + \frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2} \rho_{\omega\eta}^2 + 1 - \rho_{\omega\eta}^2}{\left(1 + \frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2}\right) \left(1 + \frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2}\right) - \rho_{\omega\eta}^2}.$$

Let  $a \equiv \frac{\sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2}$  and  $b \equiv \frac{\sigma_{\varepsilon_\eta}^2}{\sigma_\eta^2}$ .

Since  $Cov(\omega, \theta)$  is decreasing in  $a$  and  $b$ , we have  $Cov(\omega, \theta) \leq \sigma_\eta \sigma_\omega \rho_{\omega\eta} = Cov(\omega, \eta)$ . Furthermore, the upper bound is attained for  $a = 0$  or  $b = 0$ .

$Var(\theta)$  becomes independent of  $a$  for  $b = 0$ . Therefore, we cannot shift surplus from Sender to Receiver for  $b = 0$ . Let  $Var(\theta; b)$  denote the variance of  $\theta$  for  $a = 0$  as a function of  $b$ .  $Var(\theta; b)$  is decreasing in  $b$ . For  $b = 0$ ,  $Var(\theta; 0) = \sigma_\eta^2$ . In the limit as  $b \rightarrow \infty$  we have  $\lim_{b \rightarrow \infty} Var(\theta; b) = \rho_{\omega\eta}^2 \sigma_\eta^2$ .

Thus, there exists a solution to the equation  $Var(\theta) = Cov(\omega, \eta)$  iff

$$\sigma_\eta^2 > \sigma_\eta \sigma_\omega \rho_{\omega\eta} > \rho_{\omega\eta}^2 \sigma_\eta^2$$

which is equivalent to

$$\frac{1}{\rho_{\omega\eta}} > \frac{\sigma_{\omega}}{\sigma_{\eta}} > \rho_{\omega\eta} > 0.$$

■

## 9 References

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