

Herwartz, Helmut; Plödt, Martin

**Conference Paper**

Sign restrictions and statistical identification under volatility breaks --  
Simulation based evidence and an empirical application to monetary  
policy analysis

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik  
- Session: Multiple Time Series Analysis, No. C13-V1

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Herwartz, Helmut; Plödt, Martin (2014) : Sign restrictions and statistical identification under volatility breaks -- Simulation based evidence and an empirical application to monetary policy analysis, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2014: Evidenzbasierte Wirtschaftspolitik - Session: Multiple Time Series Analysis, No. C13-V1, ZBW - Deutsche Zentralbibliothek für Wirtschaftswissenschaften, Leibniz-Informationszentrum Wirtschaft, Kiel und Hamburg

This Version is available at:

<https://hdl.handle.net/10419/100326>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Sign restrictions and statistical identification under volatility breaks – Simulation based evidence and an empirical application to monetary policy analysis\*

HELMUT HERWARTZ<sup>†</sup>      MARTIN PLÖDT<sup>‡</sup>

February 26, 2014

## Abstract

Apart from a priori assumptions on instantaneous or long run effects of structural shocks, sign restrictions have become a prominent means for structural vector autoregressive (SVAR) analysis. Moreover, second order heterogeneity of systems of times series can be fruitfully exploited for identification purposes in SVARs. We show by means of a Monte Carlo study that taking statistical information into account offers a more accurate quantification of the true structural relations. In contrast, resorting only to commonly used sign restrictions bears a higher risk of failing to recover these structural relations. As an empirical illustration we employ the statistical and the sign restriction approach in a stylized model of US monetary policy. By combining identifying information from both approaches we strive for improved insights into the effects of monetary policy on output. Our results point to a decline in real GDP after a monetary tightening at an intermediate horizon.

**JEL classification:** C32, E47

**Keywords:** Vector autoregression, identification, sign restrictions, volatility, simulation.

---

\*Financial support by Deutsche Forschungsgemeinschaft (He 2188/3-2) is gratefully acknowledged.

<sup>†</sup>Department of Economics, Georg-August-University Göttingen, Platz der Göttinger Sieben 5, 37073 Göttingen, Germany. E-mail: *hherwartz@uni-goettingen.de*

<sup>‡</sup>Kiel Institute for the World Economy, Hindenburgufer 66, 24105 Kiel, Germany. E-mail: *martin.ploedt@ifw-kiel.de*

# 1 Introduction

Structural vector autoregressive (SVAR) modeling has become a widely used tool in empirical macroeconomics. Representing the innovations to the VAR system independently and identically distributed Gaussian structural shocks, however, cannot be recovered from reduced form residuals without further assumptions. Against this background, imposing zero restrictions on some instantaneous effects (Sims (1980)) or on long-run effects of the shocks (Blanchard and Quah (1989)) has been suggested for identification. More recently, the imposition of theory-based sign restrictions upon impulse responses has become one of the most popular approaches to either fully identify all structural relations or, in a more ‘agnostic’ scenario, to leave some room for rotation based identification of a few not directly restricted structural relations.<sup>1</sup> Albeit often seen as relatively mild restrictions, identification by means of sign restrictions also provoked a critical discussion (see Fry and Pagan (2007, 2011) and Paustian (2007)). Particularly with regard to a quantitative interpretation of the results, the prevailing focus on median impulse responses might often be misleading, as pointed out by Jääskelä and Jennings (2011), Kilian and Murphy (2009) and Inoue and Kilian (2013). While using sign restrictions may offer useful information on the structural interplay of variables within a dynamic system, it is fair to notice that a unique structural decomposition of the covariance matrix of reduced form error terms does not exist. To arrive at unique impulse responses by means of sign restrictions an analyst willingly relies on censored simulation outcomes. In light of biases invoked by censoring, one may critically recast the opting for sign restrictions, at least, if further data based information is available that could be used for identification purposes.

Numerous recent empirical evaluations of macroeconomic systems have uncovered a general tendency of decreasing macroeconomic risks as a characteristic of a period beginning in the mid of the 1980s and lasting for about or more than two decades (Perez-Quiros and McConnell (2000), Kahn, McConnell and Perez-Quiros (2002)). The period of the so-called ‘Great Moderation’ is characterized by a general mitigation of second order moment levels and dynamics. As shifts in the covariances of reduced form vector innovations have become a stylized fact at least for mature economies, it appears worthwhile to shed more light on the potential of volatility shifts in identifying structural relations.

Rigobon (2003), Lanne and Lütkepohl (2008a) and others have proposed a method

---

<sup>1</sup>See Fry and Pagan (2011) for a summary of empirical studies employing sign restrictions. Faust (1998), Uhlig (2005), Mountford (2005) and Dedola and Neri (2007) are, among others, examples in the field of monetary policy, Peersman (2005) for the analysis of economic fluctuations, and Mountford and Uhlig (2009) for the analysis of fiscal policy shocks.

that exploits changes in the (unconditional) (co)variance of residuals for a unique identification of shocks.<sup>2</sup> On the one hand, the possibility to exploit statistical properties for identification purposes is advantageous, since eventual incompatibility of conventional approaches with the data becomes testable. On the other hand, however, shocks identified in a data driven manner by means of statistical arguments may often lack a straightforward economic interpretation. Herwartz and Lütkepohl (2013) therefore recommend to combine information extracted from the data with economic nonsample information. Despite the fact that changes in volatility are a common characteristic of many macroeconomic and financial variables, such statistical approaches have been rarely adopted in empirical studies.

In this work we assess the relative merits of the purely theory-based and the statistical identification that rests on shifts in second order moments. It consists of two main parts. In the first place, employing a dynamic stochastic general equilibrium (DSGE) model we subject both identification schemes to a simulation-based comparison. The Monte Carlo design can be thought of as mimicking the identification strategies followed by two fictive analysts. While one analyst throughout employs a set of a priori sign restrictions for identification, the other employs changes in the volatility structure of the residuals if such changes have been diagnosed by means of a suitable pretest. In cases where the volatility approach is not applicable, (s)he proceeds with the former theory-based approach. This setup allows to investigate numerous interesting aspects of performance and informational content of the rival identification strategies. Firstly, we investigate the accuracy of both identification strategies by means of a new and intuitive evaluation criterion. Secondly, we assess under which circumstances and how often the statistical approach might be feasible. Thirdly, we compare the outcome of imposing an agnostic and a fully restricted sign pattern. Fourthly, we investigate to what extent omitted variable biases could change the relative performance of both strategies for structural analysis. The simulation exercises show that resorting only to sign restrictions bears a higher risk of failing to recover structural relations, while impulse response patterns identified by means of (co)variance shifts offer more precise measures of the true dynamics.

In the second place, we apply both identification approaches to a standard low-dimensional model of US monetary policy. In this vein and similar to a recent study by Lütkepohl and Netšunajev (2013), we are able to test the theoretically motivated

---

<sup>2</sup>See Rigobon (2003) for a simple intuition of the method. Note that also other approaches using statistical identification exist. For instance, Lanne and Lütkepohl (2010) assume mixed normal distributed model innovations to identify the shocks and impulse responses.

restrictions concerning the signs of the impulse responses.<sup>3</sup> We conclude that common restrictions used to identify demand, supply and monetary policy shocks are not rejected by the data. Hence, these a priori beliefs can be helpful for an economic interpretation of the more precise responses resulting from an identification with statistical means. Based on this diagnosis we examine the impact of a monetary tightening on real GDP. In contrast to previous agnostic sign restriction approaches our results suggest a significant decline in real GDP at an intermediate horizon.

In the next section we provide a brief summary of the two identification approaches. Section 3 provides the simulation setup and the respective identifying assumptions. The evaluation criterion and simulation results are discussed in section 4. The empirical example is provided in section 5. Section 6 concludes.

## 2 Identification of structural shocks

For purposes of exposition consider a  $K$ –dimensional vector autoregression of order  $p$

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + B \varepsilon_t, \quad t = 1, \dots, T. \quad (1)$$

In (1)  $\varepsilon_t$  denotes the vector of structural shocks which are typically presumed serially and cross sectionally uncorrelated with mean zero and unit covariance matrix. Thus,  $u_t = B \varepsilon_t$  can be considered as reduced form residuals. These reduced form residuals can be estimated by means of standard OLS or ML estimators. Presuming the structural shocks to be multivariate Gaussian,  $\varepsilon_t \sim iidN(0, I_K)$ , it is well known that the structural shocks  $\varepsilon_t$  cannot be recovered from reduced form estimates, since the reduced form covariance is robust under rotations of  $\varepsilon_t$ , i.e.

$$\Sigma_u = Cov[u_t] = BB' = BQQ'B', \quad \text{with } Q \neq I_K \text{ and } QQ' = I_K.$$

As a consequence, the impacts of the structural shocks  $\varepsilon_t$  on the variables in the system  $y_t$  cannot be identified without further assumptions. In the following we describe two recent approaches to identify the columns of  $B$  that differ with respect to the exploited additional information.

---

<sup>3</sup>Lütkepohl and Netšunajev (2013) model changes in volatility by means of a Markov switching mechanism and test the identification assumptions of Kilian and Murphy (2009) in a model of the crude oil market.

## 2.1 Identification by means of sign restrictions

One potential candidate for the matrix  $B$  is given, for instance, by a Choleski factor  $C$  of the estimated reduced form covariance matrix  $\hat{\Sigma}_u$ . However, this popular decomposition implies a recursive dynamic structure that might be hard to justify in many cases. Multiplying  $C$  with some rotation matrix  $Q$  one obtains another candidate generating orthogonal shocks. The decomposition by means of  $CQ$  yields distinct, generally nonrecursive, responses of the variables. The sign restriction approach consists of generating a broad set of distinct impulse responses in this vein, and subsequently discarding those that do not satisfy a given theory-based sign pattern.<sup>4</sup>

To determine matrices  $Q$  one may consider the product of the  $K(K-1)/2$  distinct forms of Givens rotation matrices. For instance, in case  $K = 3$  (the model dimension considered in the Monte Carlo study in section 3 and the empirical example in section 5) one may choose

$$Q(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) \\ 0 & \sin(\theta_1) & \cos(\theta_1) \end{bmatrix} \times \begin{bmatrix} \cos(\theta_2) & 0 & -\sin(\theta_2) \\ 0 & 1 & 0 \\ \sin(\theta_2) & 0 & \cos(\theta_2) \end{bmatrix} \times \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (2)$$

where  $0 \leq \theta_i \leq \pi$ ,  $i = 1, 2, 3$ . A large set of  $Q(\theta)$  matrices and, subsequently, of impulse responses can be determined by drawing values for the elements in  $\theta = (\theta_1, \theta_2, \theta_3)'$  from a uniform distribution,  $\theta_i \sim U[0, \pi]$ ,  $i = 1, 2, 3$ . A particular impulse response pattern is accepted if it is in line with a priori specified sign restrictions. Otherwise it is discarded. The sampling procedure is repeated until a prespecified number of successful draws (in this study 1,000) is obtained. In accordance with the literature, we resort to the median and the 16% and 84% quantile of the distribution of the accepted impulse responses for structural analysis.

## 2.2 Identification by means of changes in volatility

Rigobon (2003), Rigobon and Sack (2003) and Lanne and Lütkepohl (2008a, 2008b) show that distinguishing  $m = 1, 2, \dots, M$  covariance states of VAR residuals may allow to obtain structural shocks from the estimated reduced form residuals. For simplicity of exposition assume that there is one permanent volatility break at period  $T_B$  such that

<sup>4</sup>See, for instance, Fry and Pagan (2007) for a more detailed explanation of this approach.

$M = 2$  and

$$E(u_t u_t') = \begin{cases} \Sigma_1 & \text{for } t = 1, \dots, T_B - 1 \\ \Sigma_2 & \text{for } t = T_B, \dots, T. \end{cases} \quad (3)$$

The two covariance matrices  $\Sigma_1$  and  $\Sigma_2$  can be decomposed into  $\Sigma_1 = WW'$  and  $\Sigma_2 = W\Psi W'$ , where  $W$  is a  $(K \times K)$  matrix and  $\Psi$  is a diagonal matrix with diagonal elements  $\psi_{kk} > 0$ ,  $k = 1, \dots, K$ . The crucial condition for a unique decomposition (apart from changes in sign and ordering of the shocks) is that the diagonal elements of  $\Psi$  have to be distinct. It is worthwhile to mention, however, that this assumption can be subjected to statistical testing. Setting  $B = W$  obtains unique structural shocks  $\varepsilon_t = W^{-1}u_t$ . Accordingly, these shocks have an identity covariance matrix in the period before the break (regime 1) and a diagonal covariance matrix  $\Psi$  in the period after the break (regime 2). Potential ambiguity of structural effects due to sign changes is easily avoided by switching all signs of a column of  $W$  in case the corresponding diagonal element has a negative sign. This is without loss of generality, as it only implies that one studies the effects of positive shocks. The specific ordering of the shocks is, however, an important issue for their economic labeling later on (see section 5).

Volatility based identification can easily be extended to cases of more than two regimes, however, for systems with  $M > 2$  the decomposition mentioned below the definition in (3) is restrictive, i.e. testable by means of common likelihood based tests. Put differently, the assumption of regime invariant responses to structural shocks can be contrasted against the data for multi-regime systems with  $M > 2$ , while it is ‘just identifying’ in case  $M = 2$ . Details on the estimation procedure are given in the Appendix.

### 3 Simulation setup and identifying assumptions

In this section we introduce the data generating process (DGP) used for the Monte Carlo simulations, the postulated volatility shifts of the artificial time series, and the specific assumptions made for the identification of structural shocks.

#### 3.1 Data generating process

For simulation purposes we employ a simple 3-equation DSGE model that has been widely used as a baseline framework for monetary policy analysis (see, amongst others, Gertler, Gali and Clarida (1999), Carlstrom, Fuerst and Paustian (2009), and Castelnuovo (2012a, 2012b)). The consideration of trivariate systems is also common practice in the SVAR

literature. The log-linearized version of the model reads as

$$x_t = \gamma E_t x_{t+1} + (1 - \gamma)x_{t-1} - \delta_x(r_t - E_t \pi_{t+1}) + \omega_{x,t}, \quad (4)$$

$$\pi_t = (1 + \alpha\beta)^{-1} \beta E_t \pi_{t+1} + (1 + \alpha\beta)^{-1} \alpha \pi_{t-1} + \kappa x_t + \omega_{\pi,t}, \quad (5)$$

$$r_t = \tau_r r_{t-1} + (1 - \tau_r)(\tau_\pi \pi_t + \tau_x x_t) + \omega_{r,t}, \quad (6)$$

$$\omega_{\bullet,t} = \rho_\bullet \omega_{\bullet,t-1} + \varepsilon_{\bullet,t}, \bullet \in \{x, \pi, r\}, t = 1, 2, \dots, T, \quad (7)$$

where  $x_t$ ,  $\pi_t$  and  $r_t$  denote the output gap, inflation and the nominal interest rate, respectively, and  $E_t$  indicates expectations formed at period  $t$ . Accordingly, the equations (4) to (6) represent a New Keynesian IS equation, a hybrid New Keynesian Phillips curve, and a Taylor rule with interest rate smoothing. First order autoregressive (AR(1)) shock processes are summarized in (7), with subscripts  $\bullet \in \{x, \pi, r\}$  indicating a demand shock, a supply shock and a monetary policy shock, respectively.

The employed parameter settings correspond to common calibration assumptions drawn from the macroeconomic literature. In particular, we set the discount factor  $\beta = 0.99$ , the slope of the Phillips curve  $\kappa = 0.05$ , the parameter governing the indexation to past inflation  $\alpha = 0.5$ , the parameter governing the impact of the ex-ante real interest rate  $\delta_x = 0.1$ , and the weight allocated to expectations of future output  $\gamma = 0.5$ . The policy parameters in the Taylor rule are given by  $\tau_\pi = 1.8$  and  $\tau_x = 0.5$ , the smoothing parameter is  $\tau_r = 0.6$ . Finally, the autoregressive parameters in (7) are set to  $\rho_x = \rho_\pi = \rho_r = 0.5$ .

Given equilibrium determinacy, the model in (4) to (7) can be formulated as

$$\begin{bmatrix} x_t \\ \pi_t \\ r_t \end{bmatrix} = \Phi \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \\ r_{t-1} \end{bmatrix} + \mathbf{B} \begin{bmatrix} \omega_{x,t} \\ \omega_{\pi,t} \\ \omega_{r,t} \end{bmatrix}, \quad (8)$$

where, by implication, the matrices  $\Phi$  and  $\mathbf{B}$  read as

$$\Phi = \begin{bmatrix} 0.74 & -0.09 & -0.16 \\ 0.13 & 0.44 & -0.06 \\ 0.24 & 0.30 & 0.53 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 2.32 & -0.48 & -0.41 \\ 0.72 & 2.32 & -0.22 \\ 0.98 & 1.57 & 0.76 \end{bmatrix}. \quad (9)$$

The system in (8) has a finite order structural VAR representation,

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \mathbf{B} \varepsilon_t, \quad (10)$$

where  $y_t = (x_t, \pi_t, r_t)'$ . Defining  $F = \text{diag}(\rho_x, \rho_\pi, \rho_r)$  the autoregressive parameter ma-



trices in (10) are  $A_1 = \Phi + \mathbf{B}\mathbf{F}\mathbf{B}^{-1}$  and  $A_2 = -\mathbf{B}\mathbf{F}\mathbf{B}^{-1}\Phi$ .<sup>5</sup> The elements in  $\mathbf{B}$  represent the instantaneous effects of the shocks on the variables. Note that this matrix implies a unique pattern of signs corresponding to each structural shock. This feature will be relevant for the applicability of sign restrictions later on. Furthermore, it should be stressed that the specification in (10) formalizes the residuals as linear functions of the structural shocks, while the empirical analysis delivers unique estimates only for the reduced form residuals  $u_t = \mathbf{B}\varepsilon_t$ .

### 3.2 Volatility shifts

By assumption, the DGP in (10) exhibits a change in the (unconditional) volatility of the shocks. This phenomenon is quite common in numerous macroeconomic data (see, for instance, Sensier and van Dijk (2004)). Similar to Cavaliere, Rahbek and Taylor (2010) we presume that  $\varepsilon_t = V_t e_t$ , where  $e_t$  is a  $K$ -dimensional Gaussian process with mean zero and identity covariance matrix. Moreover,  $V_t$  is a time-dependent diagonal matrix,  $V_t = \text{diag}(v_{1t}, v_{2t}, \dots, v_{Kt})$ . We allow for shock processes with a single variance break ( $M = 2$ ) and generate the true shifts to occur at  $T_B = [0.5T]$ , where  $[z]$  denotes the integer part of  $z$ . In practice an analyst will likely rely on historical information or statistical evidence to determine the time location of a change in volatility. For this purpose (s)he might resort to common break point diagnostics (see, for instance, Inclan and Tiao (1994)). Given that such a test is consistent, the timing error diminishes as the sample size  $T$  increases. In the simulations we mimic this situation by setting the presumed (co)variance shift to occur at a random time instance

$$T_B^* = \left\lceil \left( 0.5 + \frac{0.5}{\sqrt{T}} g \right) T \right\rceil, \quad g \sim N(0, 1). \quad (11)$$

Hence, for  $T \rightarrow \infty$  the break point chosen by the analyst coincides with the true one, i.e.  $T_B^* \rightarrow T_B = [0.5T]$ . To be explicit with regard to the simulated (co)variance patterns we set  $V_t = I_K$  for the pre-break period throughout. Two main scenarios of post-break (co)variance patterns are distinguished with  $V_t$  switching either to some random  $V^{(r)}$  or to fixed second order moments  $V^{(f)}$ . Thereby we examine the effects of alternative magnitudes of the volatility shifts that are specified next.

- Shift to random (co)variances ( $V_t = V^{(r)}$ ,  $t \geq T_B$ )

Randomizing the extent of actual (co)variance shifts in a particular simulation experiment the diagonal elements of  $V^{(r)}$  are drawn from a uniform distribution,  $v_k^{(r)} \sim U[1, 3]$ ,  $k = 1, \dots, K$ . Given the support of this distribution, the simulated

<sup>5</sup>For details, see Ravenna (2007), Carlstrom, Fuerst and Paustian (2009) and Castelnovo (2012b).

scenarios cover (i) both marked and mild shifts in second order moments and/or (ii) states where pretesting the distinctness of elements in  $\Psi$  is more or less powerful. Thus, the generation of random (co)variances mimics a broad range of empirically relevant scenarios.

- Shift to fixed (co)variances ( $V_t = V^{(f)}$ ,  $t \geq T_B$ )

Within the fully controlled variance regime we set  $V^{(f)} = \text{diag}(3, 2, 1)$  for all simulation experiments. In contrast to the randomly drawn magnitudes before, this choice implies that the volatility shifts are markedly non-proportional throughout. While one may expect the pretesting for volatility shifts a priori to be more powerful for this case of shifts to fixed (co)variances, it is worthwhile to mention that at the system level average (expected) standard errors are identical across the two shift scenarios  $V^{(f)}$  and  $V^{(r)}$ .

Note that Cavaliere, Rahbek and Taylor (2010) also consider volatility shifts of size 3 standard deviations and argue that such values are empirically plausible. Aside from changes in the volatility of the shocks, the dynamic structure of the DGP is time-invariant. Hence, the resulting impulse responses do not differ across varying volatility regimes. Such time-invariance is supported in some related studies (see Christiano, Eichenbaum and Evans (1999), Sims and Zha (2006), and the discussion in Lanne and Lütkepohl (2008a)) but not uncontroversial. We return to this issue in the empirical application.

### 3.3 Imposed identifying restrictions

The pattern of the sign restrictions imposed on the responses of the variables is summarized in Table 1. Consistent with the theoretical model a demand shock  $\varepsilon_x$  leads to a positive response of all three variables. A supply shock  $\varepsilon_\pi$  invokes a positive response of inflation and the interest rate and a negative response of the output gap. Moreover, the monetary policy shock  $\varepsilon_r$  is identified by requiring output and inflation to fall and the interest rate to rise. All sign restrictions on the responses apply to the impact effect.

**Table 1:** Imposed sign restrictions in the simulation exercise.

Variable	Shock		
	$\varepsilon_x \rightarrow$	$\varepsilon_\pi \rightarrow$	$\varepsilon_r \rightarrow$
$x$	+	−	−/?
$\pi$	+	+	−
$r$	+	+	+

As argued by Uhlig (2005), a particular merit of the sign restriction approach is the

option to impose restrictions only on a subset of responses, and let the data decide about some effects of central interest. In this regard we concentrate on the response of the output gap to monetary policy and leave this response unconstrained as indicated by ‘?’ in Table 1. The policy shock is still uniquely identified by its negative and positive impact effect on inflation ( $\pi_t$ ) and the interest rate ( $r_t$ ), respectively. In the Monte Carlo study it will be of interest to what extent the outcome of this more agnostic procedure differs from outcomes based on the full set of sign restrictions.

### 3.4 Further comments on the simulation setup

We generate  $S = 1,000$  sets of time series for each simulation experiment. The length of a simulated process is set to  $T + 100$  and the first 100 observations of each series are discarded to immunize simulation outcomes against initial conditions. Three alternative sample sizes,  $T = 200, 500$  and  $T = 1,000$ , are considered. These choices are also motivated by a sample size of  $T > 500$  in the empirical application provided in section 5. Since the intention of the study is to compare the identification accuracy of both identification strategies, we assume that the true lag order of the VAR model is known in advance and abstract from potential biases due to an inaccurate dynamic specification. As stated above, we also assume that an analyst imposes the correct pattern of signs and the (almost) precise break date. These assumptions might be justified if the analyst only imposes sign restrictions that are robust across various calibrations and models, and the break date is gathered from consistent statistical pretesting. Moreover, with regard to the ordering of the statistically identified shocks, the diagonal elements of the matrix  $\widehat{\Psi}$  are ordered according to the ranks of the diagonal elements in  $V^{(r)}$  or  $V^{(f)}$ . The columns of  $\widehat{W}$  are adjusted accordingly.

For a given time series an important issue is to address if identification by means of changes in volatility is applicable. Note again that the diagonal elements of the matrix  $\widehat{\Psi}$  have to be distinct for a unique determination of the structural shocks. Therefore, for each process, we apply a formal pretest to check for equality of the diagonal elements of  $\widehat{\Psi}$ , i.e. we separately test the null hypotheses  $H_0 : \psi_{kk} = \psi_{ll}$  for  $k, l \in [1, \dots, K], k \neq l$ . The pretests are performed at a 10% significance level by means of Wald statistics that do not require additional likelihood optimizations. If all null hypotheses are jointly rejected, we proceed with identification based on the diagnosis of volatility breaks. If the data do not allow for an identification by means of statistical criteria, we employ the sign restriction approach. Thus, the outcome of this (hybrid) identification strategy comprises results of both approaches in some weighted form.

## 4 Evaluation criterion and simulation results

In this section we evaluate the accuracy of the impulse response estimates resulting from both identification strategies. For this purpose we first introduce an evaluation criterion that may reasonably apply to both identification approaches. Before we discuss simulation results in more detail, a few particular simulation scenarios are highlighted that are thought to correspond to particular stances of econometric identification in practice.

### 4.1 Evaluation of structural impulse responses

To compare the outcomes from the two identification strategies, we presume that the ultimate aim of both methods is a most accurate description of a system's responses to structural shocks on impact and over time. Targeting at a close approximation of true impulse response functions, it is natural to compare both identification approaches graphically. To summarize all results in one figure, we use the following simple criterion. In a first step, an acceptance range around the known, true dynamics of the DGP is constructed. For this purpose we specify for each DSGE model parameter an interval around the specific value previously chosen for the DGP in (4) to (7) (see Table 2). Following Canova and Paustian (2011) we assume a uniform distribution over each interval, randomly draw a new set of parameter values and determine the impulse responses arising from this new calibration at periods  $h$ ,  $h = 0, 1, \dots, 15$ . Subsequently, we construct an area containing 90% of these artificial responses based on 10,000 draws of distinct sets of parameter values. With these 'areas of acceptance' (AoA) at hand, we check if the impulse response point estimates resulting from the volatility based identification scheme and the median response derived from the sign restriction approach are located within the defined AoA. This is performed for all  $S = 1,000$  Monte Carlo replications of a particular experiment, and finally we report the frequency of responses located within the AoA.

The intervals documented in Table 2 are specified such that they include theoretically plausible values and are sufficiently small to result in an AoA that provides a meaningful and straightforward measure for the accuracy of estimated impulse response functions. To get accepted, responses have to deliver the correct sign (at least for the first periods) and to be quantitatively similar to the true ones.<sup>6</sup> The evaluation criterion takes into account that point estimates resulting from the volatility based identification scheme and

---

<sup>6</sup>As we will see later, our defined AoA does not cross the zero line within the first periods. Therefore the chosen sign restrictions (Table 1) are robust across the different sets of parameter values. Instead of the intervals documented in Table 2 we also experimented with intervals based on the 5% and 95% quantile of the posterior densities from a Bayesian estimation of the DSGE model. However, this often results in relatively broad acceptance ranges and eventually lacks a clear-cut sign pattern for the on impact effects of the structural shocks.

**Table 2:** DGP parameter values and interval for AoA.

Parameter	DGP calibration	Interval for AoA
$\beta$	0.99	0.99
$\alpha$	0.5	[0.4, 0.6]
$\kappa$	0.05	[0.03, 0.07]
$\gamma$	0.5	[0.4, 0.6]
$\delta_x$	0.1	[0.05, 0.15]
$\tau_x$	0.5	[0.3, 0.7]
$\tau_\pi$	1.8	[1.6, 2.0]
$\tau_r$	0.6	[0.4, 0.8]
$\rho_x$	0.5	[0.4, 0.6]
$\rho_\pi$	0.5	[0.4, 0.6]
$\rho_r$	0.5	[0.4, 0.6]

the median response functions of the sign restriction approach are not directly comparable.<sup>7</sup>

Figure 1 shows the true impulse responses and the AoA. We emphasize that each structural shock is normalized such that its impact effect on one variable is equal to unity.<sup>8</sup> In the same vein we also normalize the outcome of both identification schemes. Note that while in the framework of the statistical approach the impulse response are based on structural shocks that have unit variance in regime 1, the responses resulting from the sign restriction approach are based on shocks that have unit variance for the entire sample period. Therefore, the adopted standardization ensures that the simulation results are not tampered by a different scale of the shocks.

[Insert Figure 1 about here.]

## 4.2 Alternative scenarios

As mentioned before, we implement several distinct simulation scenarios, which are summarized in Table 3. The scenarios differ with respect to the magnitudes of the volatility shifts and the sample size. Scenarios I, II, and III refer to cases with shifts to random

<sup>7</sup>Furthermore, our results are not affected by using the median response function itself instead of selecting the particular impulse response that is closest to the median response function (‘median target’) as proposed by Fry and Pagan (2011). Using a Bayesian framework, Inoue and Kilian (2013) have recently proposed an alternative method that addresses the problem of summarizing the evidence from the standard sign restriction approach by evaluating the posterior of sign-identified models. However, in this paper we only focus on the standard approach and leave an additional comparison with this new method for future research.

<sup>8</sup>We achieve this by dividing each column of the matrix of instantaneous effects by the absolute value of the respective main diagonal element of this column.

(co)variances ( $V_t = V^{(r)}$ ,  $t \geq T_B$ ), i.e. the magnitude of the volatility shift of each time series at time  $T_B = [0.5T]$  is randomly determined by drawing from the uniform distribution over the interval  $[1, 3]$ . Scenarios IV, V, and VI refer to cases where the magnitude of the volatility shifts at time  $T_B$  is fixed across simulations at  $V_t = V^{(f)} = \text{diag}(3, 2, 1)$ ,  $t \geq T_B$ . The marginal impact of distinguished sample sizes becomes evident when contrasting scenarios I, IV ( $T = 200$ ); II, V ( $T = 500$ ); and III, VI ( $T = 1,000$ ).

We choose the scenarios II and V ( $T = 500$ ) for the purpose of benchmarking with regard to shifts to random (co)variances ( $V^{(r)}$ ) and shifts to fixed (co)variances ( $V^{(f)}$ ), respectively. For both cases simulation results are shown for all responses to all shocks. With regard to the remaining scenarios we focus on the response of output to a monetary policy shock emphasizing that the analysis of this particular impulse response function is at the core of this work.<sup>9</sup> Beyond that, we consider potential biases for the identification of a monetary policy shock that may arise as a consequence of an omitted variable.

**Table 3:** Overview of simulation scenarios.

Shift	$T = 200$	$T = 500$	$T = 1,000$
$V^{(r)}$	Scenario I (Fig. 5)	Scenario II (Fig. 2)	Scenario III (Fig. 5)
$V^{(f)}$	Scenario IV (Fig. 5)	Scenario V (Fig. 3, 4)	Scenario VI (Fig. 5)

## 4.3 Simulation results

Simulation results are depicted in Figure 2 to Figure 6. For each identification strategy we display the frequency of impulse responses that are located within the defined AoA at periods  $h = 0, 1, \dots, 15$ . Note again that the frequency of accepted impact responses reported in the panels along the main diagonal is unity by construction. They should therefore not be interpreted as evidence supporting either one of the employed identification strategies.

### 4.3.1 Shifts to random (co)variances

We first consider the simulation results based on scenario II ( $V^{(r)}$ ,  $T = 500$ ), which are summarized in Figure 2. Identification by means of changes in volatility is only applicable in around 450 out of  $S = 1,000$  replications while in all other replications we have to resort to sign restrictions. It is therefore not surprising to observe only a small difference in results between the identification strategy that targets to employ statistical information and the pure sign restriction approach. Overall, however, one may diagnose a slight lead of the volatility based identification in that respective impulse responses are more frequent

<sup>9</sup>The complete set of simulation results is available from the authors upon request.

close to the true functional patterns in comparison with median responses obtained from the sign restriction approach. Impulse responses based on the sign restriction approach are outside the AoA in almost all cases at the impact period  $h = 0$ , with the reaction of the interest rate  $r_t$  to a supply shock  $\varepsilon_\pi$  being the only exception. At higher horizons, the performance of the impulse responses based on the sign restriction approach appears somewhat more satisfactory. Nevertheless, in this scenario ( $V^{(r)}$ ,  $T = 500$ ), we often fail to recover the true dynamics due to the lack of sufficient statistical information in the data, and the relatively poor accuracy of median responses drawn from the sign restriction approach.

[Insert Figure 2 about here.]

### 4.3.2 Shifts to fixed (co)variances

Results for scenario V ( $V^{(f)}$ ,  $T = 500$ ) are displayed in Figure 3. Apparently, the shift to fixed (co)variances implies a stronger change in relative variances of the shocks compared with the previous scenario ( $V^{(r)}$ ,  $T = 500$ ). This allows for a unique determination of the structural shocks by statistical means in around 98% of the replications. Overall, the strategy to exploit diagnosed (co)variance shifts performs considerably better than the pure sign restriction approach even though, for instance, only around 40% of the responses of inflation to a monetary policy shock are inside the AoA at the impact period  $h = 0$ . The instantaneous response of the output gap ( $x_t$ ) to a monetary policy shock ( $\varepsilon_r$ ) is now satisfactorily detected in around 70% of replications. In contrast, the distinct shifts in volatility appear to have little impact on the results of the pure sign restriction approach. Yet, the reaction of the interest rate  $r_t$  to the shock with the strongest shift in volatility ( $\varepsilon_x$ ) is now correctly diagnosed in almost all cases when applying the full set of sign restrictions. Interestingly, simulating with post-break (co)variances  $V^{(f)}$  in comparison with  $V^{(r)}$  requires up to three times as many draws of rotation matrices (indexed by  $\theta_i$ ) to obtain the desired number of responses that are in line with the specified sign pattern.

For both scenarios II and V ( $T = 500$ ), altogether, there appear to be only little differences between the median responses from the agnostic and the fully restricted sign restriction approach. Remarkably, this also holds true for the agnostically unspecified reaction of the output gap ( $x_t$ ) to the monetary policy shock ( $\varepsilon_r$ ). Yet, it seems likely that the range of responses in the agnostic setup is much wider, complicating the decision about the impact of the structural shock. Only if both, the 16% and the 84% quantile of the distribution of the accepted set of impulse responses have the same sign at the respective period, the results are usually regarded to indicate the direction of the effect. To further

investigate this issue we calculate the frequencies of cases where these signs are identical and correct. As displayed in the upper right hand side panel of Figure 4, one would be noncommittal concerning the proper direction of the unconstrained response of the output gap ( $x_t$ ) to restrictive monetary policy shocks ( $\varepsilon_r$ ) in almost all cases. This finding is also in line with simulation results in Castelnovo (2012a).<sup>10</sup> The potential failure of the agnostic setup to provide a clear qualitative conclusion concerning the effect of monetary policy on output will deserve further consideration when discussing the empirical example in section 5.

[Insert Figure 3 and Figure 4 about here.]

### 4.3.3 Alternative sample sizes

Consider next the marginal impacts of a smaller and a larger sample size relative to the  $T = 500$  baseline scenarios. In doing so we focus on the response of output to a monetary policy shock. Respective results for scenario I ( $V^{(r)}$ ,  $T = 200$ ) are displayed in the upper left panel of Figure 5. Again, most median responses from the pure sign restriction approach are outside the AoA during the first periods after the shock. As expected, the smaller sample size compared with scenario II ( $V^{(r)}$ ,  $T = 500$ ) goes along with power loss when pretesting distinctness of the diagonal elements in  $\hat{\Psi}$ . Specifically, statistical information can be exploited for identification in around 30% of the cases. Nevertheless, using statistical properties obtains a somewhat larger frequency of impulse responses located in the neighborhood of the true functions.

The lower left hand side panel of Figure 5 illustrates the results for scenario IV ( $V^{(f)}$ ,  $T = 200$ ). Pretesting now hints at the potential of volatility based identification in 87% of the replications. Notably, this frequency is higher as it has been for scenario II ( $V^{(r)}$ ,  $T = 500$ ). Power gains in pretesting translate into a more frequent detection of quantitatively accurate effects of monetary policy on output. The combined identification strategy recovers the true dynamics to a satisfactory degree in around 43% of the cases while relying merely on sign restrictions, none of the median responses based on the full or on the agnostic set of sign restrictions fulfills the defined criterion at the impact period.

For scenarios III and VI we use a sample size of  $T = 1,000$ . The upper and lower right hand side panels of Figure 5 display the results based on volatility shifts  $V^{(r)}$  and  $V^{(f)}$ , respectively. In both cases, the quantitative accuracy of responses gathered from sign restrictions generally resembles that for previous scenarios. The median responses are still at odds with the true dynamics at the impact period but more often located within

<sup>10</sup>Results in Paustian (2007) indicate that the agnostic sign restriction approach might pin down the correct sign of an unconstrained impulse response of special interest, but the number of restrictions and the variance of the specific shock have to exceed settings that are typical for applied work.



the AoA at a longer horizon. Under scenario III, the volatility shifts can be exploited in around 62% of all replications. Under scenario VI, identification by means of changes in volatility is applicable in each Monte Carlo replication and delivers responses that come very close to the true ones in the vast majority of replications.

[Insert Figure 5 about here.]

#### 4.3.4 The case of an omitted variable

In practical applications SVAR models are often small dimensional to respect the postulate of model parsimony in light of finite sample information. In consequence, empirical analysis often proceeds under the potential threat that important variables have been left out from the analysis. In this subsection, therefore, we aim at investigating to what extent an omitted variable could change the relative performance of both strategies for structural analysis. For instance, interactions between financial markets and the real economy might constitute an important factor for the transmission of monetary policy and other shocks, which could be overlooked when working with a standard trivariate system. For this reason we suppose that the true DGP is a 4-equation DSGE model featuring macro-finance interactions (Castelnuovo and Nisticò (2010), Castelnuovo (2013)), as it is discussed in some more detail in Appendix C. We then check how the quantitative accuracy of both strategies is affected when still considering a 3-dimensional SVAR comprising output, inflation and the interest rate.<sup>11</sup>

Collecting the contemporaneous relations in the 4-equation model, the matrix  $\mathbf{B}$  is given by

$$\mathbf{B} = \begin{bmatrix} 2.14 & -0.70 & -0.49 & 0.46 \\ 0.63 & 2.27 & -0.22 & 0.07 \\ 0.79 & 1.30 & 0.62 & 0.81 \\ -0.22 & -0.50 & -0.30 & 1.66 \end{bmatrix}. \quad (12)$$

The impact effect of demand/supply/monetary shocks on the output gap, inflation and the interest rate are given in the upper left  $3 \times 3$  block of  $\mathbf{B}$ . The sign pattern used to identify these three shocks remains unchanged.<sup>12</sup> Note that a monetary policy shock

<sup>11</sup>Clearly, there might be good reasons to use an even richer DGP and, hence, investigate the consequences of several omitted variables. We choose the model given in Appendix C since it is closely related to the small-scale baseline DGP outlined in (4) to (7) but still allows for interactions that might be crucial for analysing economic activity. An additional investigation using a large-scale model is beyond the scope of this paper.

<sup>12</sup>However, when considering a three variable system only, the sign restrictions used for the identification of a demand shock are equal to the signs of the impact effect of a financial shock  $\varepsilon_s$  to the output gap, inflation, and the interest rate (fourth column, first three elements). It is therefore not possible to differentiate between a demand shock and a financial shock without any additional assumptions.

(unexpected increase in the interest rate) now has a stronger negative effect on output and inflation as it has been the case for the 3-dimensional DGP introduced in section 3.1.

The simulation exercise with the omitted variable basically resembles the previous scenario V ( $V^{(f')} = \text{diag}(3, 2, 1, 1)$ ,  $T = 500$ ). We focus again on the response of the output gap to a monetary policy shock. The left hand side panel of Figure 6 shows the AoA based on the extended model, and the right hand side panel of Figure 6 shows respective simulation results. Confirming former results for scenario V, none of the sign-identified responses is located within the AoA at the impact period  $h = 0$ . In contrast to the outcome of scenario V, identification by means of changes in volatility mostly fails to accurately recover the somewhat stronger negative response of output. Still, however, in the extended model the frequency of statistically identified impulse responses located within the AoA is generally in excess of the frequency of sign-identified responses.<sup>13</sup>

[Insert Figure 6 about here.]

#### 4.3.5 Simulation summary

In summary, the Monte Carlo results show that median responses mostly fail to recover quantitative features of the true impulse response patterns. This supports the findings of Jääskelä and Jennings (2011) who state that median responses to a monetary policy shock do not coincide with the corresponding true impulse responses generated from a small open economy DSGE model. In addition, our results indicate that the agnostic approach often does not offer a clear qualitative statement about an unrestricted response. A setting with more variables and, consequently, more restrictions could indeed be suitable to improve the accuracy of an identification scheme but might also imply a higher risk of a priori imposing an incorrect economic structure. As expected, the feasibility of the volatility approach depends on the relative size of the volatility shifts and the length of the sample at hand. Since median impulse responses are drawn from censored simulations it is not surprising that the accuracy of this identification scheme does not gain from increasing sample information. Thus, given that both volatility shifts and sample information are of reasonable size, it appears worthwhile to exploit the potential of volatility shifts in identifying structural relations. Moreover, the omission of relevant variables provides a severe obstacle to an accurate evaluation of impulse response patterns. While this holds true for both identification approaches compared in the Monte Carlo study, identification

<sup>13</sup>Since a 4-dimensional VAR(2) could be approximated by a 3-dimensional VAR( $p$ ) with  $p > 2$ , we also estimated the empirical model composed of output, inflation and the interest rate using  $p = 4$  lags. For both strategies the results regarding the on impact responses remain virtually unchanged. Results are available upon request.

by means of statistical criteria still obtains closer approximations of the true dynamics in comparison with median responses based on the imposition of sign restrictions.

We emphasize that our results do not suggest that one approach outmatches the other when it comes to applied work. Contrary to shocks identified by means of sign restrictions, it is not obvious to give statistically identified shocks an accurate economic interpretation. This leads to the question if and how an analyst could make use of information from both approaches. We further consider this issue in the context of an empirical application.

## 5 A small model of US monetary policy

In the following we illustrate the utilization of both identification techniques in an empirical application concerning the US economy. To remain closely related to the DGP employed in the previous sections, we stick to a model comprising real GDP, a measure of the level of prices and an interest rate. This set of variables is commonly used in the empirical and theoretical literature to analyze the effects of monetary policy shocks on the economy. In our context, such a parsimonious model also allows the imposition of sign restrictions that are supported by standard textbook models.

We employ an extended version of a data set at monthly frequencies that has been previously used by Bernanke and Mihov (1998) and Uhlig (2005), starting in 1954M07 and ending in 2008M09 (651 observations). In line with these authors, we interpolate real GDP with industrial production and the GDP deflator with a consumer price and a producer price index by means of the approach in Chow and Lin (1971).<sup>14</sup> The data set ends in 2008M09 to focus on monetary policy before the financial crisis. The vector of endogenous variables  $y_t$  in our SVAR model consists of the log of real GDP, scaled by 100 ( $q_t$ ), the log of the GDP deflator, scaled by 100 ( $p_t$ ), and the federal funds rate ( $i_t$ ). We choose a lag length of  $p = 17$  (Uhlig (2005):  $p = 12$ ) based on standard information criteria and the LM test for absence of serial correlation and also include intercept terms.

It should be stressed again that we assume changes only to occur in the volatility of the structural shocks while the coefficient matrices  $A_1, \dots, A_p$  and the matrix  $B$ , which relates the reduced form disturbances to the structural shocks, remain unaffected. We note that these assumptions are controversial, especially as the sample period includes

---

<sup>14</sup>The data source is the FRED database of the Federal Reserve Bank of St. Louis. We use the series GDPC1 for real GDP, GDPDEF for the GDP Deflator, FEDFUNDS for the federal funds rate, INDPRO for industrial production, CPIAUCSL for the consumer price index, and PPIFGS for the producer price index. See also Uhlig (2005).

both, the high inflation episode of the 1970s and the ‘Great Moderation’ period.<sup>15</sup> Though a time-invariant dynamic structure of the model finds some support in the literature (see section 3.2 for references), amongst others Boivin and Giannoni (2006) and Bacchiocchi and Fanelli (2012) argue against constant SVAR parameters. Noticing eventual changes in the dynamic specification, however, we stay with the assumption of time-invariance in this regard for two reasons. First, it allows for a better comparison with Uhlig (2005). Second, it is worthwhile to point out that both approaches to identification build on dynamic stability such that both might suffer ‘symmetrically’ from an eventual misspecification of the VAR dynamics. Next, we apply sign restrictions and covariance diagnosis for the identification of structural shocks.

We impose sign patterns that are analogous to the agnostic scenario in the simulation study discussed in section 3.2 and displayed in Table 4. Considering monthly data, we decide to put restrictions not just on the impact effects but to restrict responses for one quarter (i.e. in periods  $h = 0, 1, 2$ ). Accordingly, three structural shocks are distinguished, an aggregate demand shock  $\varepsilon_d$  (e.g. a shock in government spending or a shift of aggregate consumption), an aggregate supply shock  $\varepsilon_s$  (e.g. composed of a productivity shock or an unexpected increase of the oil price) and a monetary policy shock  $\varepsilon_m$  (unexpected increase in the interest rate).

**Table 4:** Imposed sign restrictions for the analysis of US monetary policy.

Variable	Shock		
	$\varepsilon_d \rightarrow$	$\varepsilon_s \rightarrow$	$\varepsilon_m \rightarrow$
$q$	+	−	?
$p$	+	+	−
$i$	+	+	+

We focus on the effect of a monetary policy shock on real GDP where the size of the monetary policy shock is normalized to an increase by 25 basis points. The left hand side panels of Figure 7 provide the median impulse responses and the 16% and 84% quantiles derived from the sign restriction approach over a support of  $h = 0, 1, 2, \dots, 24$  periods. We stress that our results are very similar to those of Uhlig (2005) though they are based on a lower-dimensional system with only three variables and an extended sample period. The range of the unrestricted responses of real GDP to a monetary policy shock is rather wide and covers positive and negative effects, with the median response implying a positive reaction of real GDP at least for several periods after the impact. Uhlig (2005)

<sup>15</sup>Our assumptions therefore correspond to the so-called ‘good-luck’ view (e.g. Stock and Watson (2003)).

therefore concludes that neutrality of monetary policy is not at odds with the data. However, Castelnovo (2012a) and the simulation based evidence in section 4.3 indicate that such an unclear result is not unlikely even if one applies the sign restriction approach to data that are generated under non-neutrality of monetary policy. Relying exclusively on sign restrictions in this example might therefore be unsatisfactory. Given that there are volatility changes in the data, it appears worthwhile to employ the statistical identification approach for a deeper investigation.

As to postwar US data, Boivin and Giannoni (2006) and Bacchiocchi and Fanelli (2012) consider a single break in the year 1979. This is also consistent with Lanne and Lütkepohl (2008a) providing empirical evidence for a regime change in 1979M10. We therefore set a single break point  $T_B=1979M10$ . Thus, we examine a model with two volatility regimes, where the corresponding covariance matrices obey the decompositions  $\Sigma_1 = WW'$  and  $\Sigma_2 = W\Psi W'$ . The estimated matrices (with standard errors in parentheses) are

$$\widehat{W} = \begin{bmatrix} 0.067 & -0.474 & -0.023 \\ (0.018) & (0.021) & (0.049) \\ 0.026 & 0.022 & -0.152 \\ (0.012) & (0.022) & (0.007) \\ 0.355 & 0.035 & 0.073 \\ (0.016) & (0.031) & (0.044) \end{bmatrix}, \widehat{\Psi} = \begin{bmatrix} 1.827 & 0 & 0 \\ (0.207) & & \\ 0 & 0.406 & 0 \\ & (0.046) & \\ 0 & 0 & 0.789 \\ & & (0.089) \end{bmatrix}. \quad (13)$$

Wald statistics for testing the equality of the diagonal elements of  $\widehat{\Psi}$  are documented in Table 5. Generally, the null hypotheses of identical diagonal elements in  $\widehat{\Psi}$  are to be rejected at common significance levels. Thus, there is considerable evidence that the impact matrix is identified and, hence, the structural shocks are to be uncovered by statistical properties of the data.

**Table 5:** Test for equality of diagonal elements of  $\widehat{\Psi}$ .

$H_0$	Wald test	$p$ -value
$\psi_{11} = \psi_{22}$	45.20	0.0000
$\psi_{11} = \psi_{33}$	21.34	0.0000
$\psi_{22} = \psi_{33}$	14.61	0.0001

Since the columns of  $W$  and the corresponding diagonal elements of  $\Psi$  can be permuted without affecting the decompositions  $\Sigma_1 = WW'$  and  $\Sigma_2 = W\Psi W'$ , the labeling of the shocks is unclear yet. However, we are able to contrast the sign restrictions specified in Table 4 with the signs of the elements in  $\widehat{W}$  in (13). If the theoretically founded prior beliefs were in line with the data, the signs used to define a specific shock should be

reflected in one of the columns of this matrix. Considering the sign pattern of the columns of  $\widehat{W}$ , it can be seen that it indeed supports the assumptions made regarding a demand, supply and monetary policy shock.<sup>16</sup> In line with a demand shock, the impact effects of the first shock (first column of  $\widehat{W}$ ) are all positive. The second shock leads to a positive response of the price level and the interest rate, and to a negative response of real GDP (second column of  $\widehat{W}$ ). Thus, the second shock conforms with the definition of a supply shock. The third shock increases the interest rate and decreases the price level (third column of  $\widehat{W}$ ), thereby matching the assumptions relating to an unexpected tightening of monetary policy. Summarizing these arguments, we conclude that the theory-based a priori assumptions do not conflict with the data in our model.

[Insert Figure 7 about here.]

Based on this first result the assumptions in Table 4 offer a reasonable device for subsequent economic interpretation of the statistically identified shocks. Drawing on statistically identified shocks for economic analysis might be desirable not only in light of the ambiguous outcome of the sign restriction approach with respect to the effect of monetary policy on economic activity (see the left hand side panel of Figure 7). Simulation results discussed in section 4 also show that the point estimates of the volatility approach are more precise, and for this reason more appropriate for a quantitative assessment by means of impulse responses or variance decompositions.<sup>17</sup>

Interestingly, the impact effect of the third statistically identified shock (which we now label as a monetary policy shock) on real GDP is slightly negative. This negative reaction of real GDP to an increase in the interest rate is at odds with the unrestricted median response resulting from the sign restriction approach, but accords with conventional theoretical views. Beyond that, the estimated matrices in (13) are such that the structural shocks are scaled to unit variance in regime 1. This allows for an interpretation of the diagonal elements of the matrix  $\widehat{\Psi}$  as relative variances of the structural shocks in regime 2 (1979M10–2008M09) versus regime 1 (1954M07–1979M09). Therefore, the second regime in the model can be associated with a relatively lower volatility of supply and monetary policy shocks and a relatively higher volatility of demand shocks.

Next we look at the impulse response functions resulting from the volatility approach, which are displayed in the right hand side panels of Figure 7. As before, the size of the

<sup>16</sup>We have already permuted the columns of  $\widehat{W}$  and the corresponding diagonal elements of  $\widehat{\Psi}$  in (13) to facilitate comparisons with the sign pattern in Table 4. Clearly, this does not affect our analysis.

<sup>17</sup>It should be emphasized that in contrast to the standard sign restriction approach the statistical approach also offers interpretable confidence bounds. For methods of constructing error bands for impulse response functions of SVARs with sign restrictions see Moon, Schorfheide, Granziera and Lee (2011).

monetary policy shock is normalized to an increase by 25 basis points. The confidence intervals around the point estimates indicate that at least on impact the negative reaction of real GDP is not clear-cut.<sup>18</sup> However, there seems to be a significantly negative reaction of real GDP at an intermediate horizon. Remarkably, this is fully in line with the theoretical considerations in Inoue and Kilian (2013). Inoue and Kilian (2013) argue in favor of a sign restriction on the response of real GDP after 6 months while leaving the short-run and the long-run response unrestricted. They show that their resulting model yields more plausible dynamics. We therefore regard the statistically identified impulse response functions as evidence in favor of this restriction.

To sum up, based on the volatility approach empirical evidence suggests a negative response of real GDP to a rise in the interest rate, which becomes more pronounced after about six periods. In contrast, the sign restriction approach obtains quite ambiguous results. Analysts who wish to use sign restrictions in a monetary policy model may thus consider to follow the approach in Inoue and Kilian (2013), and impose sign restrictions at time instances within the second or third quarter after the monetary impulse.

## 6 Conclusions

Identification procedures relying on zero restrictions and on sign restrictions have been subjected to various criticisms. Besides the fact that imposing false restrictions can severely distort the results, the simulation results in this work further indicate that focusing on median responses derived from the sign restriction approach often leads to inaccurate conclusions with regard to the quantitative impact of distinct structural shocks. Under the condition that the relative volatility shifts and the available samples are of reasonable size, impulse responses quantified by means of volatility shifts prove to be more precise.

We therefore argue that it is useful to amend a SVAR analysis by checking if information from the data can be exploited for identification purposes. Using a stylized 3-dimensional model of US monetary policy we illustrate how identifying information from both approaches can be beneficially combined. First, we employ statistical informa-

<sup>18</sup>The bootstrapped confidence intervals in the right hand side panel of Figure 7 are obtained along the lines described in Benkwitz, Lütkepohl and Wolters (2001). We use Hall's percentile method in setting up the intervals and choose the 16% and 84% quantile of the bootstrap distribution based on 1,000 replications. We take directly the estimators of the parameters obtained from maximizing (15) and, hence, omit the iteration process. Furthermore, the initially obtained estimate of  $W$  is used as the starting value in the optimization step while the elements in  $\hat{\Psi}$  are held constant.

tion to evaluate commonly used sign restrictions. Second, we use these theoretical beliefs for an economic interpretation of the more precise responses resulting from the statistical identification under volatility breaks. Our results point to a decline in real GDP after a monetary tightening at an intermediate horizon. This finding is in line with previous theoretical considerations in the literature, but could not have been recovered by employing an agnostic sign restriction approach.

The identification of structural shocks is an ongoing issue in the SVAR literature with new approaches developing in several dimensions. On the one hand, Inoue and Kilian (2013) propose a method to enhance structural analysis based on sign restrictions, which is also applicable if the data do not allow for identification by means of statistical diagnostics. Evaluating the quantitative accuracy of this new sign restriction-based method might constitute an interesting topic for future research. On the other hand, Bacchiocchi and Fanelli (2012) have recently proposed a method that allows volatility changes to also affect the structural parameters of the SVAR model by directly combining statistical identification with some theory-driven restrictions.



## References

- BACCHIOCCHI, E., AND L. FANELLI (2012): “Identification in structural vector autoregressive models with structural changes,” Departmental Working Papers 2012-16, Department of Economics, Management and Quantitative Methods at Università degli Studi di Milano.
- BENKWITZ, A., H. LÜTKEPOHL, AND J. WOLTERS (2001): “Comparison of Bootstrap Confidence Intervals for Impulse Responses of German Monetary Systems,” *Macroeconomic Dynamics*, 5, 81–100.
- BERNANKE, B. S., AND I. MIHOV (1998): “Measuring Monetary Policy,” *The Quarterly Journal of Economics*, 113(3), 869–902.
- BLANCHARD, O. J., AND D. QUAH (1989): “The Dynamic Effects of Aggregate Demand and Supply Disturbances,” *American Economic Review*, 79(4), 655–73.
- BOIVIN, J., AND M. P. GIANNONI (2006): “Has Monetary Policy Become More Effective?,” *The Review of Economics and Statistics*, 88(3), 445–462.
- CANOVA, F., AND M. PAUSTIAN (2011): “Business cycle measurement with some theory,” *Journal of Monetary Economics*, 58(4), 345–361.
- CARLSTROM, C. T., T. S. FUERST, AND M. PAUSTIAN (2009): “Monetary policy shocks, Choleski identification, and DNK models,” *Journal of Monetary Economics*, 56(7), 1014–1021.
- CASTELNUOVO, E. (2012a): “Monetary Policy Neutrality: Sign Restrictions Go to Monte Carlo,” ”Marco Fanno” Working Papers 0151, Dipartimento di Scienze Economiche ”Marco Fanno”.
- (2012b): “Monetary Policy Shocks and Cholesky-VARs: An Assessment for the Euro Area,” Working paper.
- (2013): “Monetary policy shocks and financial conditions: A Monte Carlo experiment,” *Journal of International Money and Finance*, 32(C), 282–303.
- CASTELNUOVO, E., AND S. NISTICÒ (2010): “Stock market conditions and monetary policy in a DSGE model for the U.S,” *Journal of Economic Dynamics and Control*, 34(9), 1700–1731.

- CAVALIERE, G., A. RAHBK, AND A. R. TAYLOR (2010): “Testing for co-integration in vector autoregressions with non-stationary volatility,” *Journal of Econometrics*, 158(1), 7 – 24, Twenty Years of Cointegration.
- CHOW, G. C., AND A.-L. LIN (1971): “Best Linear Unbiased Interpolation, Distribution, and Extrapolation of Time Series by Related Series,” *The Review of Economics and Statistics*, 53(4), 372–75.
- CHRISTIANO, L. J., M. EICHENBAUM, AND C. L. EVANS (1999): “Monetary policy shocks: What have we learned and to what end?,” in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford, vol. 1 of *Handbook of Macroeconomics*, chap. 2, pp. 65–148. Elsevier.
- DEDOLA, L., AND S. NERI (2007): “What does a technology shock do? A VAR analysis with model-based sign restrictions,” *Journal of Monetary Economics*, 54(2), 512–549.
- FAUST, J. (1998): “The robustness of identified VAR conclusions about money,” *Carnegie-Rochester Conference Series on Public Policy*, 49(1), 207–244.
- FRY, R., AND A. PAGAN (2007): “Some Issues in Using Sign Restrictions for Identifying Structural VARs,” NCER Working Paper Series 14, National Centre for Econometric Research.
- (2011): “Sign Restrictions in Structural Vector Autoregressions: A Critical Review,” *Journal of Economic Literature*, 49(4), 938–60.
- GERTLER, M., J. GALI, AND R. CLARIDA (1999): “The Science of Monetary Policy: A New Keynesian Perspective,” *Journal of Economic Literature*, 37(4), 1661–1707.
- HERWARTZ, H., AND H. LÜTKEPOHL (2013): “Structural Vector Autoregressions with Markov Switching: Combining Conventional with Statistical Identification of Shocks,” *Journal of Econometrics*, forthcoming.
- INCLAN, C., AND G. C. TIAO (1994): “Use of Cumulative Sums of Squares for Retrospective Detection of Changes of Variance,” *Journal of the American Statistical Association*, 89(427), pp. 913–923.
- INOUE, A., AND L. KILIAN (2013): “Inference on Impulse Response Functions in Structural VAR Models,” *Journal of Econometrics*, forthcoming.
- JÄÄSKELÄ, J. P., AND D. JENNINGS (2011): “Monetary policy and the exchange rate: Evaluation of VAR models,” *Journal of International Money and Finance*, 30(7), 1358 – 1374.

- KAHN, J. A., M. M. MCCONNELL, AND G. PEREZ-QUIROS (2002): “On the causes of the increased stability of the U.S. economy,” *Economic Policy Review*, (May), 183–202.
- KILIAN, L., AND D. MURPHY (2009): “Why Agnostic Sign Restrictions Are Not Enough: Understanding the Dynamics of Oil Market VAR Models,” CEPR Discussion Papers 7471, C.E.P.R. Discussion Papers.
- LANNE, M., AND H. LÜTKEPOHL (2008a): “Identifying Monetary Policy Shocks via Changes in Volatility,” *Journal of Money, Credit and Banking*, 40(6), 1131–1149.
- (2010): “Structural Vector Autoregressions With Nonnormal Residuals,” *Journal of Business & Economic Statistics*, 28(1), 159–168.
- LANNE, M., AND H. LUETKEPOHL (2008b): “A Statistical Comparison of Alternative Identification Schemes for Monetary Policy Shocks,” Economics Working Papers ECO2008/23, European University Institute.
- LÜTKEPOHL, H., AND A. NETSUNAJEV (2013): “Disentangling Demand and Supply Shocks in the Crude Oil Market: How to Check Sign Restrictions in Structural VARs,” *Journal of Applied Econometrics*, forthcoming.
- MOON, H. R., F. SCHORFHEIDE, E. GRANZIERA, AND M. LEE (2011): “Inference for VARs Identified with Sign Restrictions,” NBER Working Papers 17140, National Bureau of Economic Research, Inc.
- MOUNTFORD, A. (2005): “Leaning into the Wind: A Structural VAR Investigation of UK Monetary Policy,” *Oxford Bulletin of Economics and Statistics*, 67(5), 597–621.
- MOUNTFORD, A., AND H. UHLIG (2009): “What are the effects of fiscal policy shocks?,” *Journal of Applied Econometrics*, 24(6), 960–992.
- PAUSTIAN, M. (2007): “Assessing Sign Restrictions,” *The B.E. Journal of Macroeconomics*, 7(1).
- PEERSMAN, G. (2005): “What caused the early millennium slowdown? Evidence based on vector autoregressions,” *Journal of Applied Econometrics*, 20(2), 185–207.
- PEREZ-QUIROS, G., AND M. M. MCCONNELL (2000): “Output Fluctuations in the United States: What Has Changed since the Early 1980’s?,” *American Economic Review*, 90(5), 1464–1476.
- RAVENNA, F. (2007): “Vector autoregressions and reduced form representations of DSGE models,” *Journal of Monetary Economics*, 54(7), 2048–2064.

- RIGOBON, R. (2003): “Identification Through Heteroskedasticity,” *The Review of Economics and Statistics*, 85(4), 777–792.
- RIGOBON, R., AND B. SACK (2003): “Measuring The Reaction Of Monetary Policy To The Stock Market,” *The Quarterly Journal of Economics*, 118(2), 639–669.
- SENSIER, M., AND D. VAN DIJK (2004): “Testing for Volatility Changes in U.S. Macroeconomic Time Series,” *The Review of Economics and Statistics*, 86(3), 833–839.
- SIMS, C. A. (1980): “Macroeconomics and Reality,” *Econometrica*, 48(1), 1–48.
- SIMS, C. A., AND T. ZHA (2006): “Were There Regime Switches in U.S. Monetary Policy?,” *American Economic Review*, 96(1), 54–81.
- STOCK, J. H., AND M. W. WATSON (2003): “Has the Business Cycle Changed and Why?,” in *NBER Macroeconomics Annual 2002, Volume 17*, NBER Chapters, pp. 159–230. National Bureau of Economic Research, Inc.
- UHLIG, H. (2005): “What are the effects of monetary policy on output? Results from an agnostic identification procedure,” *Journal of Monetary Economics*, 52(2), 381–419.

## A Estimation of $\mathbf{B}$ by means of changes in volatility

For given model orders  $M = 2$  and  $p$  the estimation of the SVAR model is initiated with a common OLS estimation to obtain reduced form residual estimates  $\hat{u}_t$ . For details on the following multistep iterative estimation procedure see also Lanne and Lütkepohl (2008a):

1. Reduced form residuals  $\hat{u}_t$  are used to determine covariance estimates

$$\hat{\Sigma}_1 = \frac{1}{T_B - 1} \sum_{t=1}^{T_B-1} \hat{u}_t \hat{u}_t' \quad \text{and} \quad \hat{\Sigma}_2 = \frac{1}{T - T_B + 1} \sum_{t=T_B}^T \hat{u}_t \hat{u}_t'. \quad (14)$$

2. The concentrated log likelihood

$$\begin{aligned} \log L_H = & -\frac{T_B - 1}{2} \left[ \log |WW'| + \text{tr} \left( \hat{\Sigma}_1 (WW')^{-1} \right) \right] \\ & - \frac{T - T_B + 1}{2} \left[ \log |W\Psi W'| + \text{tr} \left( \hat{\Sigma}_2 (W\Psi W')^{-1} \right) \right] \end{aligned} \quad (15)$$

is maximized to obtain ML-estimates  $\widehat{W}$  and  $\widehat{\Psi}$ . As starting values for the optimization we use  $W = \left( T^{-1} \sum_{t=1}^T \hat{u}_t \hat{u}_t' \right)^{1/2} + W^0$  and  $\Psi = I_K$ , where the ‘matrix square root’ is obtained from an eigenvalue decomposition,  $W^0$  is a matrix of small random numbers, and  $I_K$  is the  $K$ -dimensional identity matrix.

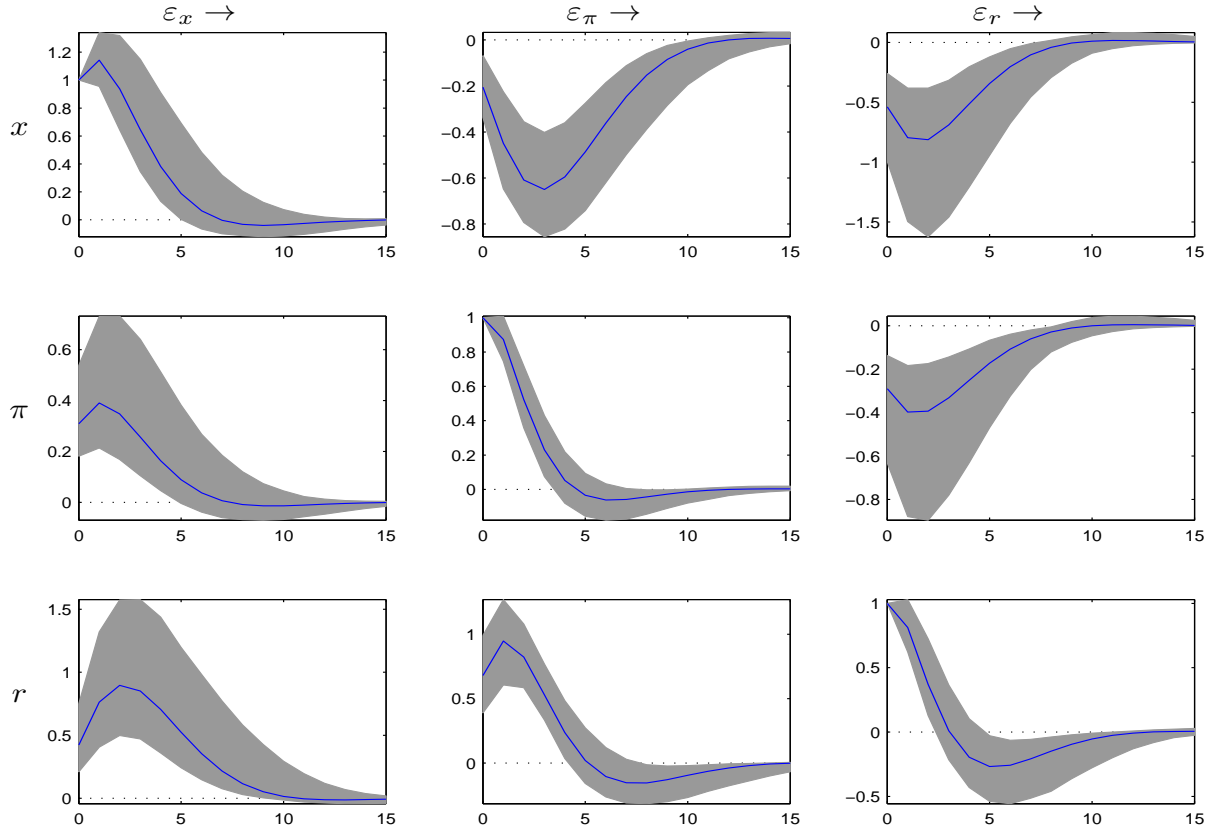
3. Conditional on  $\widehat{W}$  and  $\widehat{\Psi}$ , GLS estimators for the dynamic parameters read as

$$\begin{aligned} \widehat{\Upsilon} &= \text{vec} \left[ \hat{A}_1, \hat{A}_2, \dots, \hat{A}_p \right] \\ &= \left[ \sum_{t=1}^{T_B-1} \left( Z_t Z_t' \otimes \left( \widehat{W} \widehat{W}' \right)^{-1} \right) + \sum_{t=T_B}^T \left( Z_t Z_t' \otimes \left( \widehat{W} \widehat{\Psi} \widehat{W}' \right)^{-1} \right) \right]^{-1} \\ &\quad \times \left[ \sum_{t=1}^{T_B-1} \left( Z_t \otimes \left( \widehat{W} \widehat{W}' \right)^{-1} \right) y_t + \sum_{t=T_B}^T \left( Z_t \otimes \left( \widehat{W} \widehat{\Psi} \widehat{W}' \right)^{-1} \right) y_t \right], \end{aligned} \quad (16)$$

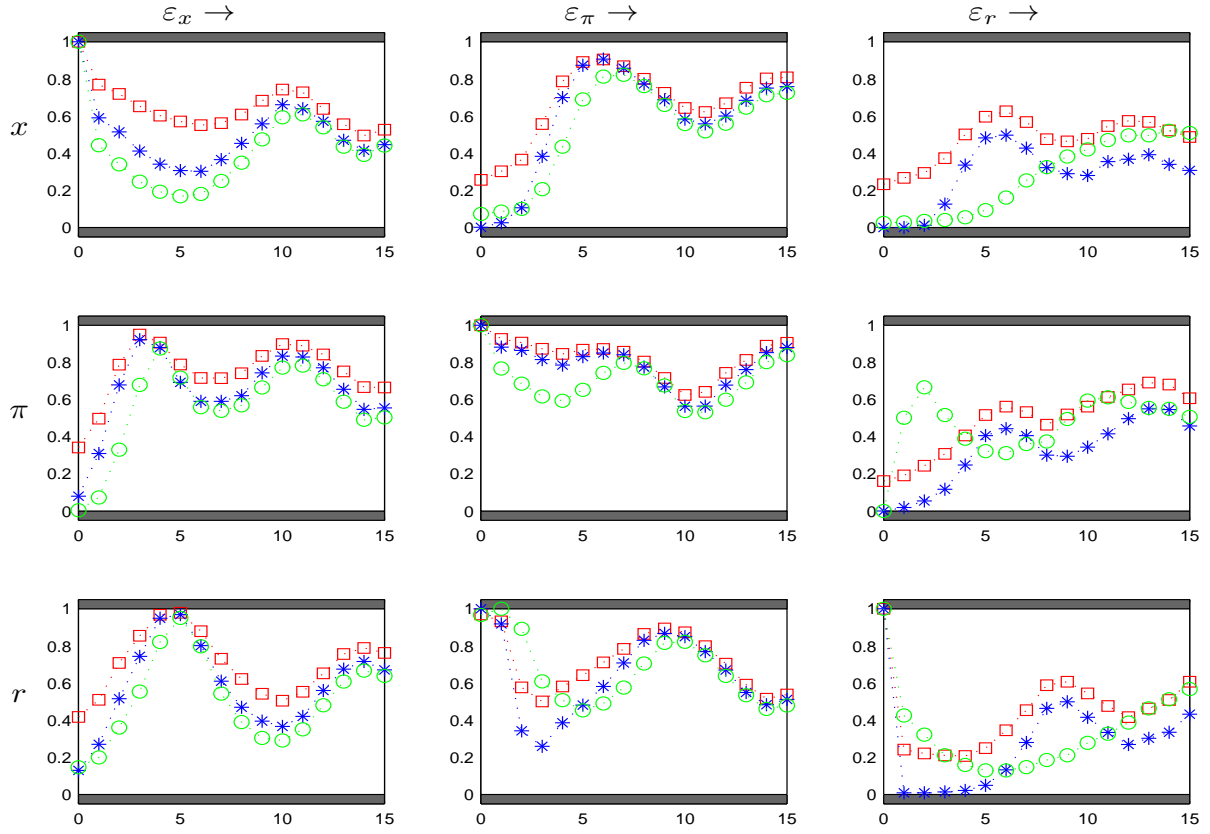
where  $Z_t' = [y_{t-1}', y_{t-2}', \dots, y_{t-p}']$ . The GLS estimates in  $\widehat{\Upsilon}$  are used to update the covariance estimates  $\hat{\Sigma}_1$  and  $\hat{\Sigma}_2$  in (14) by means of residuals  $\hat{u}_t = y_t - (Z_t' \otimes I_K) \widehat{\Upsilon}$ .

Steps 1. to 3. are iterated until convergence of  $\widehat{W}$ ,  $\widehat{\Psi}$  and  $\widehat{\Upsilon}$ . Second order properties of the estimates in  $\widehat{W}$  and  $\widehat{\Psi}$  can be obtained from the inverse Hessian of the log likelihood function.

## B Figures: Simulation experiment

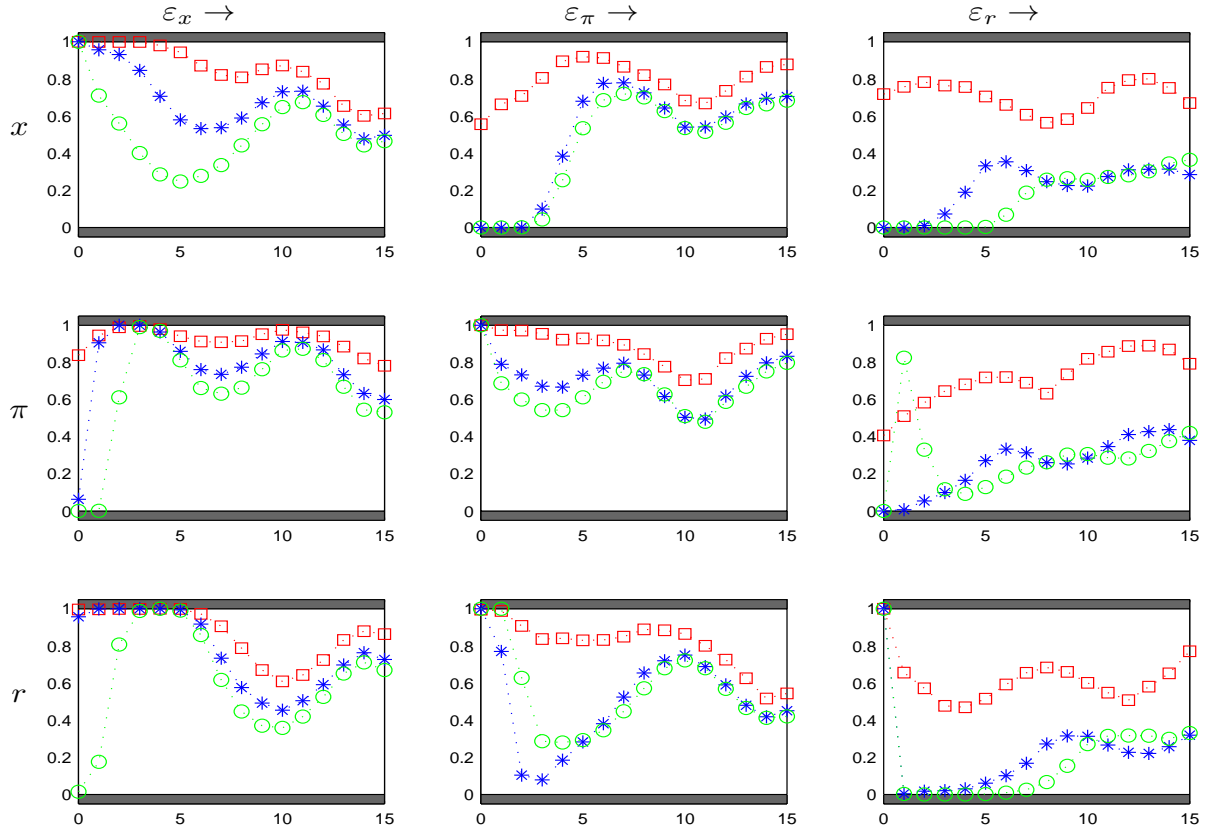


**Figure 1:** True impulse responses (—) and defined AoA (grey shaded) of the baseline 3-equation DSGE model. AoA illustrates the 90% set based on 10,000 different calibrations. Variables  $x$ ,  $\pi$ ,  $r$  ( $\varepsilon_x$ ,  $\varepsilon_\pi$ ,  $\varepsilon_r$ ) denote the output gap, inflation and the nominal interest rate (demand shock, supply shock and monetary policy shock).



**Figure 2:** Scenario II ( $V^{(r)}, T = 500$ ): Frequency of impulse responses located within the defined AoA at periods  $h = 0, 1, \dots, 15$ .

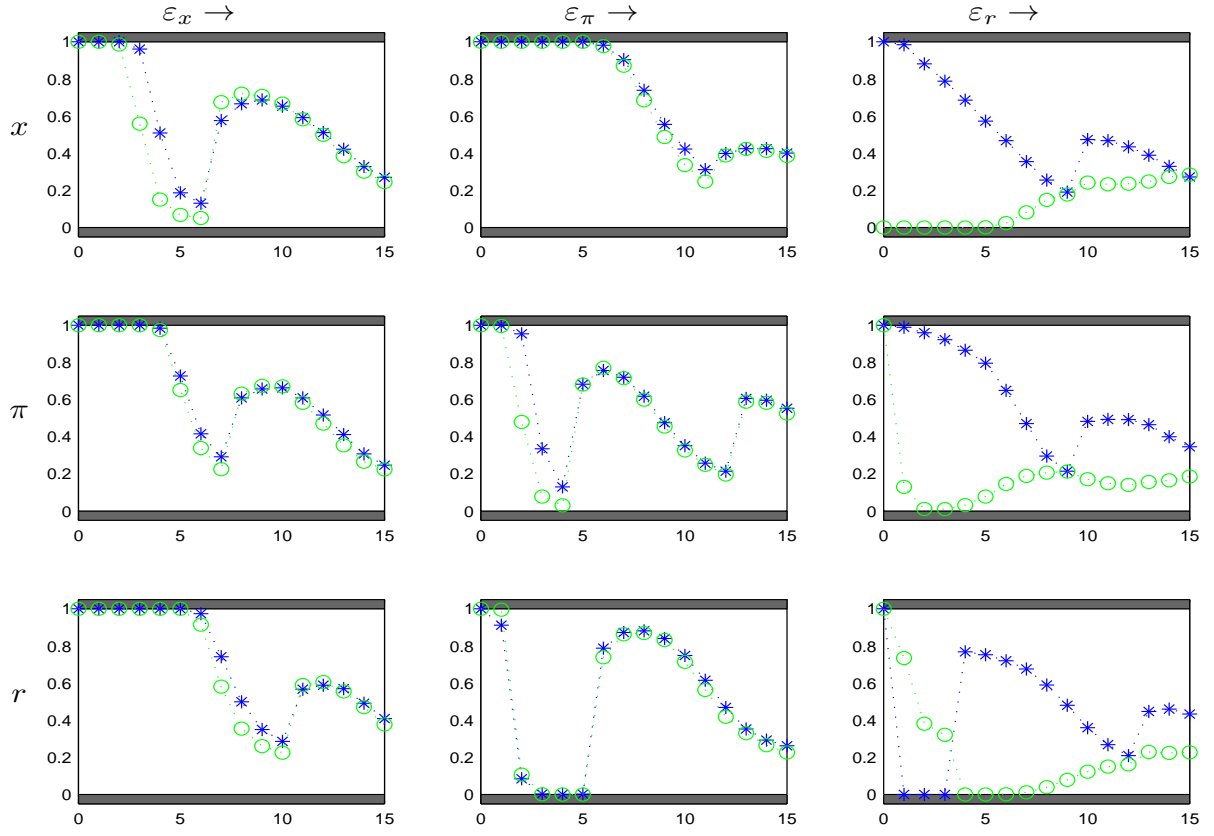
( $\square$ ) Identification by means of changes in volatility (if applicable, or else sign restrictions). Volatility approach is applicable in 45% of the cases. ( $*$ ) Identification by means of sign restrictions only. ( $\circ$ ) Sign restrictions on subset of responses (agnostic). On average, one out of 69 (20) draws of rotation matrices (see equation (2)) obtains responses that satisfy the fully restricted (agnostic) sign pattern. For further notes see Figure 1.



**Figure 3:** Scenario V ( $V^{(f)}, T = 500$ ). Frequency of impulse responses located within the defined AoA at periods  $h = 0, 1, \dots, 15$ .

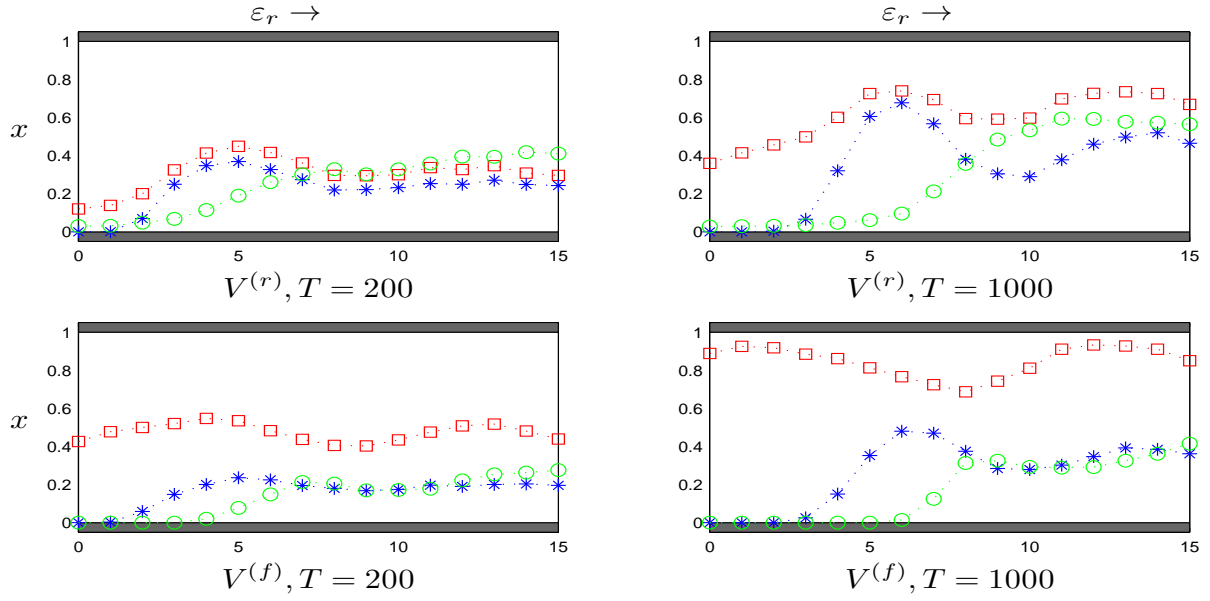
Volatility approach is applicable in 98% of the cases. On average, one out of 205 (35) draws of rotation matrices (see equation (2)) obtains responses that satisfy the fully restricted (agnostic) sign pattern. For further notes see Figure 2.





**Figure 4:** Scenario V ( $V^{(f)}$ ,  $T = 500$ ). Identification by means of sign restrictions: Sign checks for ranges between the 16% and 84% quantiles of the distribution of the accepted set. Frequency of identical and correct signs.

(\*) Sign restrictions on all responses. (o) Sign restrictions on subset of responses (agnostic).



**Figure 5:** Overview of alternative scenarios. Frequency of impulse responses located within the defined AoA at periods  $h = 0, 1, \dots, 15$ . Response of output to a monetary policy shock.

*Upper panels:* Scenarios with volatility shift  $V^{(r)}$ . Scenario I ( $T = 200$ ): Volatility approach applicable in 29% of the cases. Scenario III ( $T = 1,000$ ): Volatility approach applicable in 62% of the cases.

*Lower panels:* Scenarios with volatility shift  $V^{(f)}$ . Scenario IV ( $T = 200$ ): Volatility approach applicable in 87% of the cases. Scenario VI ( $T = 1,000$ ): Volatility approach applicable in all cases. For further notes see Figure 2.

## C A 4-equation DSGE model

To analyze the effects of a potentially important omitted variable, we extend the baseline DGP provided in section 3.1 by a law of motion of an indicator of financial soundness  $s_t$ . The model was initially proposed by Castelnovo and Nisticò (2010) and has been recently employed in a Monte Carlo experiment by Castelnovo (2013). We refer to these authors for all details and just quote the relevant equations, i.e.

$$x_t = \gamma E_t x_{t+1} + (1 - \gamma)x_{t-1} - \delta_x(r_t - E_t \pi_{t+1}) + \psi_x s_t + \omega_{x,t}, \quad (17)$$

$$\pi_t = (1 + \alpha\beta)^{-1} \beta E_t \pi_{t+1} + (1 + \alpha\beta)^{-1} \alpha \pi_{t-1} + \kappa x_t + \omega_{\pi,t} \quad (18)$$

$$r_t = \tau_r r_{t-1} + (1 - \tau_r)(\tau_\pi \pi_t + \tau_x x_t + \tau_s s_t) + \omega_{r,t}, \quad (19)$$

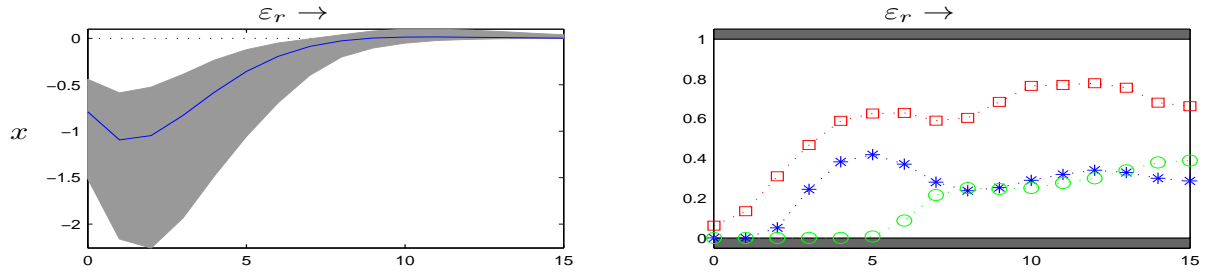
$$s_t = \beta E_t s_{t+1} + \lambda_s E_t x_{t+1} - \delta_s(r_t - E_t \pi_{t+1}) + \omega_{s,t}, \quad (20)$$

$$\omega_{\bullet,t} = \rho_\bullet \omega_{\bullet,t-1} + \varepsilon_{\bullet,t}, \bullet \in \{x, \pi, r, s\}. \quad (21)$$

Apart from its own representation in (20) financial soundness  $s_t$  enters the monetary policy reaction function (19) and the IS equation (17). The model again has a VAR representation of order 2. Table 6 summarizes the calibration of the additional parameters of the extended model. The values of the additional parameters closely correspond to the estimates in Castelnovo (2013). The values of all other parameters and the intervals around these values remain unchanged (see Table 2). The left hand side panel of Figure 6 shows the resulting true impulse response of output to a monetary policy shock and the corresponding new AoA. Note that this AoA is only affected by the specific values of the additional parameters as documented in Table 6. The right hand side panel of Figure 6 reports the frequency of impulse responses that fulfill the evaluation criterion.

**Table 6:** Additional parameter values of the extended model.

Parameter	$\psi_x$	$\tau_s$	$\lambda_s$	$\delta_s$	$\rho_s$
DGP calibration	0.2	1.0	0.05	0.1	0.5

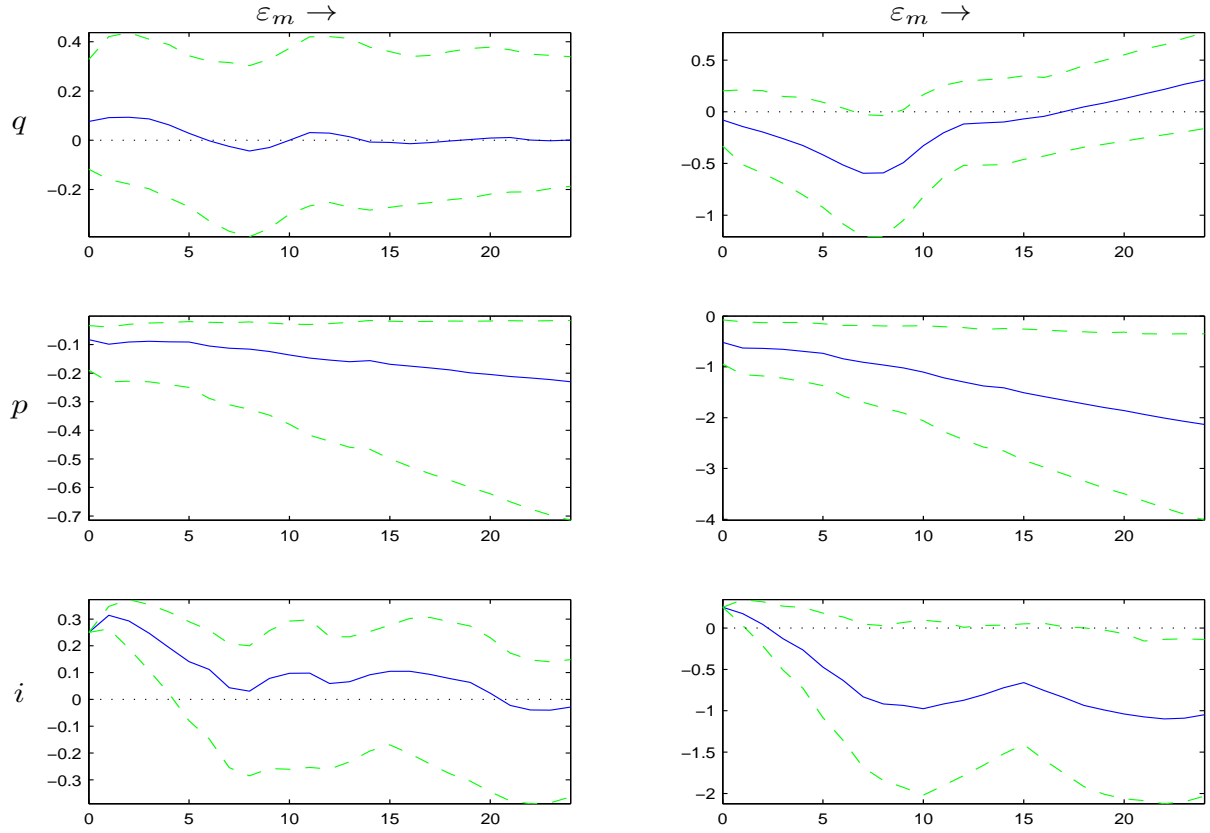


**Figure 6:** *Left:* True impulse response of output to a monetary policy shock (—) and defined AoA (grey shaded) based on the extended model. AoA illustrates the 90% set based on 10,000 different calibrations.

*Right:* Results for the extended model. Scenario V':  $T = 500$ , Volatility shift  $V^{(f')}$ . Frequency of impulse responses located within the defined AoA at periods  $h = 0, 1, \dots, 15$ .

( $\square$ ) Identification by means of changes in volatility (if applicable, or else sign restrictions). Volatility approach applicable in 98% of the cases. ( $*$ ) Identification by means of sign restrictions only. ( $\circ$ ) Sign restrictions on subset of responses (agnostic).

## D Figures: Empirical application



**Figure 7:** Impulse responses to a monetary policy shock  $\varepsilon_m$ . *Left hand side panels:* Identification by means of sign restrictions (agnostic). Median impulse responses and 16% and 84% quantiles. *Right hand side panels:* Identification by means of changes in volatility. Impulse responses with 68% confidence bounds. Variables  $q$ ,  $p$ ,  $i$  denote real GDP, GDP Deflator and federal funds rate.