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From the Loser to the Winner - How Trade Liberalization can lead to Leapfrogging between Countries

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From the Loser to the Winner – How Trade Liberalization can lead to Leapfrogging between Countries

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Abstract

How shifts in the economic leadership between countries can occur has been widely debated not only since the recent catch up of China in several sectors. However, there is no adequate theoretical model analyzing this question in the light of trade liberalization. This paper is the first one to address productivity leapfrogging between two countries using a heterogeneous firms trade framework.

In the model, firms’ R&D investments determine their expected productivity draw. In one country firms face lower R&D costs. Before trade liberalization, the sector productivity and the competition intensity is higher in this country. However, when trade liberalization occurs, fiercer competition can more than offset the investment advantage. Hence, firms from the disadvantaged country may invest relatively more in R&D than firms from the advantaged country. Consequently, the laggard country can become the leader in terms of sector productivity after trade liberalization. The results of the model highlight open markets in combination with innovations by firms as the necessary requirement for leapfrogging between two countries.

JEL-Classification: F12, F13

Keywords: Heterogeneous Firms; Leapfrogging; R&D investments; Trade liberalization

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1 Introduction

Empirical observations show a lot of convergence or even overtaking in terms of economic performance of industries among countries over the time (Landes, 1999). Examples are the East Asian Tigers like South Korea, Taiwan and Hong-Kong during the last 30 years or the present catch up of China in several sectors (Bernhofen et al., 2011; Adams et al., 2006; Nelson and Pack, 1999). International trade plays thereby an important role (Noland, 2012; Cameron et al., 2005; Bernard and Jones, 1996). Several reasons can be imagined why this is the case. For example, international trade allows firms to import sophisticated technology that would not be available in their domestic country (Amiti and Konings, 2007). Or international competition as well as access to foreign markets can stimulate innovations of firms (Bustos, 2011; Lawrence and Weinstein, 2001).

However, trade models accounting for the mentioned empirical facts are still scarce. This has already been recognized a while ago by Motta et al. (1997). They argue that intra-industry trade between different productive countries have gained surprisingly little attention in theoretical models, and in most of those models assuming asymmetries the productivity gap will even increase with international economic integration, e.g. in the seminal contribution on trade and growth by Grossman and Helpman (1991).

This critic seems still be appropriated when heterogeneous firms models with intra-industry trade in horizontal differentiated products are considered. These new models, starting with Melitz (2003) and Bernard et al. (2003), assume firm heterogeneity according to productivity. One main novelty compared to former intra-industry trade models like Krugman (1980) is that the productivity of a sector is endogenously determined and depends on the degree of trade openness. The early heterogeneous firms trade models do not take any technology differences between countries into account. Hence, trade liberalization leads in all countries to a similar increase of the sector productivity due to a reallocation of resources from less productive exiting firms to more productive exporting firms.

Since recently, some heterogeneous firms models take asymmetries in the technology potential between countries into account by assuming exogenously given different productivity distributions of firms (Falvey et al., 2011; Okubo, 2009; Demidova, 2008). Trade liberalization influences the sector productivity of each country again only via a selection effect. The competition increases to a greater extent in the leading country and this leads to a stronger drop out of unproductive firms there. As a result, the productivity difference on a sector level between the country, which was leading before the trade liberalization, and the laggard country will even be larger when trade becomes

\[ \text{The literature on heterogeneous firms in international economics is reviewed among others by Melitz and Redding (2012) and with focus on the impact of trade liberalization by Melitz and Trefler (2012).} \]
liberalized.\footnote{The impact of trade liberalization on sector productivity is not directly addressed in these models, because they focus on welfare. However, the result of an increased difference in sector productivity due to trade liberalization can easily be shown by using the proofs of Melitz (2003) on the impact of trade liberalization on sector productivity.}

Other recent heterogeneous firms models consider within firm productivity adjustments and analyze the impact of trade liberalization on the industry level (Unel, 2013; Long et al., 2011; Atkeson and Burstein, 2010). However, almost all of these models are limited to symmetric countries. The only model that takes asymmetric countries into account is to the best of the authors knowledge Unel (2013).\footnote{Ederington and McCalman (2008) assume also asymmetric countries in a technology adaption model. However, they focus on the speed of adaption.} In this model, firms can adapt to a better technology, whereby the adaption costs differ between the countries. Trade liberalization leads to a larger fraction of firms that adapt to the better technology. More adaption takes place in the country with lower adaption costs. As a result, the laggard country can never become the leading one after trade liberalization. Hence, the criticism raised by Motta et al. (1997), which proclaims in theoretical models the for whatever reasons technologically advantaged country will even become more advantaged after trade liberalization, still applies to heterogeneous firms trade models.

The aim of this paper is to develop a model, in which one country can leapfrog another country with respect to sector productivity as a result of trade liberalization. In doing so it extends Melitz (2003) to consider endogenous innovations as proposed by Rutzer (2013) and assumes additionally asymmetric countries. As in Rutzer (2013), firms can choose between two $R&D$ technologies, high and low. This choice determines the distribution from which a firm draws its productivity. The high $R&D$ technology leads to a productivity draw, which first order stochastically dominates a draw in the case of the low $R&D$ technology. The investment requirements for a particular $R&D$ technology are lower in one country than in the other country. An interpretation can be that firms from one country have access to a better basic knowledge stock, which is exogenous to a sector, and allows firms to cheaper conduct applied innovations.\footnote{Keller (2004) provides an overview why knowledge stocks can differ among countries.}

As long as trade openness is low, total sales of each firm are in both countries too low to invest in the high $R&D$. However, the lower $R&D$ requirements lead to more entry of firms in the technologically advanced country. Similar to Falvey et al. (2011) or Demidova (2008) the result is a tougher domestic market competition in the technologically leading country. This leads to a stronger selection of unproductive firms. Eventually, the aggregate sector productivity is higher in the country with lower $R&D$ investment requirements. Now trade becomes liberalized. In accordance with empirical evidence by Bustos (2011) or Lileeva and Treffer (2010), the improved market access increases in both countries the incentives of firms to invest in the high $R&D$ technology. But
the domestic market competition is tougher in the leading country. As a result, the expected profits of an investment in high R&D are lower in the technologically leading country. Hence, firms in the country with higher innovation costs will under certain conditions choose the high R&D technology as trade becomes liberalized. At the same time firms from the country with notable lower labor requirement in both kinds of R&D will still find it optimal to invest in the low R&D technology.

In contrast to other heterogeneous firms models and technological difference between two countries like Falvey et al. (2011) or Demidova (2008), the sector productivity of each country is not only affected by selection but also by R&D investments of firms. This has a fundamental impact, because the increased innovation investments in the laggard country can more than compensate for the stronger selection effect in the leading country. Hence, the laggard country can become the new leader in terms of sector productivity.

The mechanism for overtaking is alike to the seminal leapfrogging model of Brezis et al. (1993). In their Ricardian trade model, the lagging country invests in new techniques because of lower wages and less experience in the old technology, whereas the leading country will not. However, the responsible shocks are fundamentally different between both models. In Brezis et al. (1993) a new major technology appears exogenously. In the model of this paper, a deeper international market integration leads firms in the laggard country to invest more in R&D and gain a higher productivity.

Few other models consider also leapfrogging between two countries as a result of trade liberalization (Moraga-Gonzalez and Viaene, 2005; Cabrales and Motta, 2001; Motta et al., 1997). In those partial equilibrium oligopoly models firms use the same production technology in both countries but can decide on the quality of their products. Open up to trade can lead to a convergence or even overtaking in the supplied quality of one country relative to another one. However, these models only permit a partial equilibrium analyze and focus on quality leapfrogging in the produced goods. The model at hand instead proposes a framework in which trade liberalization can lead to leapfrogging in terms of sector productivity between countries in a full blown general equilibrium model.

The remainder of this paper is structured as follows. The formal model is introduced next by first deriving the R&D-choice methodology of firms. Afterwards it is analyzed how trade liberalization affects the R&D choice and how it differs between the two countries. In a next step, the results on firm level are used to determine the impact

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5 The selection effect covers in Melitz (2003) style models the competition intensity on the product market and the labor market (Potin, 2009). Therefore, another explanation is a higher real wage rate in the initially leading country. Investments in the high R&D can then be in cost terms cheaper in the laggard country.

6 Other leapfrogging models with technology shocks and fully integrated markets are Desmet (2002) within a dynamic Heckscher-Ohlin model or recently Furukawa (2012) on perpetual leapfrogging between countries.
of trade liberalization on the countries' sector productivity. Subsequently, the results of the model are embedded into the literature. The paper closes with key findings and possible further research.

2 The Model

There are two countries, foreign and home. Foreign is marked by an asterisk whenever it is required to distinguish between the two countries. In addition, there are a differentiated goods sector and a homogenous goods sector. The differentiated goods consist of a standard Dixit-Stiglitz CES demand structure. Labor is the only factor of production. It is immobile across countries. But it is mobile between the homogenous good and the differentiated goods sector. Each country produces the freely tradeable homogenous good under constant returns to scale, whereby one unit of labor is needed to produce one unit of output. This leads to same nominal wages of one in both countries as long as there is incomplete specialization. The countries differ only with respect to $R&D$ costs in the differentiated goods sector.

2.1 Firms

We solve the decision process of an entrant recursive. After an entrant has drawn its productivity, it decides which market to serve and what price to set. From there on, the steps are equivalent to other heterogeneous firms models with asymmetric countries and upper tier Cobb-Douglas preferences and CES preferences on the differentiated goods like Chaney (2008) or Demidova (2008). They derive domestic profits of a home firm with a productivity $z$ above a productivity threshold $z_D$ as

$$
\pi_d(z) = \frac{\mu R}{\sigma} \left( P z^\frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma - 1}{\sigma}} - f_d,
$$

where $\mu R$ denotes aggregate revenue in home earned in the differentiated sector, $P$ the aggregate price index in home, $\sigma$ the constant elasticity of substitution between two consumed varieties, and $f_d$ the fixed costs required to remain in the domestic market.\(^7\)

The corresponding domestic profits of a firm in foreign with a productivity $z$ above the

\(^7\)Melitz (2003) assumes symmetric countries. Therefore, he does not need a homogenous good to get same nominal wages among countries. Here, consumers have Cobb-Douglas utility and use a share of $\mu$ of their income to buy the differentiated goods and $1 - \mu$ to buy the homogenous good. The introduction of a freely traded homogenous good is standard in the literature to avoid different nominal wages in the case of asymmetric countries. The size of the share has no influence on the productivity in the differentiated goods sector of a country. This result follows immediately, because sector productivity is in standard CES heterogeneous firms models independent of aggregate income. For a detailed discussion on this point see Akerman and Forslid (2009).
threshold $z_D^*$ are

$$\pi_d^*(z) = \frac{\mu R^*}{\sigma} \left( P^* z \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} - f_d.$$  \hspace{1cm} (2)$$

The domestic profits between foreign firms and home firms differ due to a different price index $P^* \neq P$ and a different aggregate revenue $R^* \neq R$. Only firms having a high enough productivity to earn at least zero profits remain in the market. The required productivity to earn at least zero profits in the corresponding domestic market is implicitly defined as $\pi_d(z_D) = 0$ for home firms and $\pi_d^*(z_D^*) = 0$ for foreign firms. By using (1) and (2) this leads to

$$z_D = \left( \frac{f_d \sigma}{\mu R} \right)^{\frac{1}{\sigma - 1}} \frac{\sigma}{(\sigma - 1)P}; \quad z_D^* = \left( \frac{f_d \sigma}{\mu R^*} \right)^{\frac{1}{\sigma - 1}} \frac{\sigma}{(\sigma - 1)P^*}. \hspace{1cm} (3)$$

The cut-off productivity $z_D$ summarizes the competition intensity in the home market and $z_D^*$ in the foreign market. As usual in monopolistic competition models, each firm takes the competition intensities as given. In addition, a firm that exports has to pay fixed costs $f_x$ and variable trade costs of Iceberg type $\tau \geq 1$ per unit shipped. The export profits of a home firm with productivity $z$ larger than some threshold $z_X$ or foreign firm with a productivity $z$ above $z_X^*$ can be written as

$$\pi_x(z) = \frac{\mu R^*}{\sigma} \left( P^* z \frac{\sigma - 1}{\tau \sigma} \right)^{\sigma - 1} - f_x; \quad \pi_x^*(z) = \frac{\mu R}{\sigma} \left( P z \frac{\sigma - 1}{\tau \sigma} \right)^{\sigma - 1} - f_x.$$  \hspace{1cm} (4)$$

The export market cut-off is defined as $\pi_x(z_X) = 0$ for home firms and as $\pi_x^*(z_X^*) = 0$ for foreign firms. Only firms with a productivity to earn at least zero profits in the export-market will export. The home export market cut-off $z_X$ depends only on the competition intensity in foreign, and the foreign export cut-off $z_X^*$ on the competition intensity in home. This can be seen by using (3) and (4):

$$z_X = \frac{z_D^*}{\phi}; \quad z_X^* = \frac{z_D}{\phi}, \hspace{1cm} (5)$$

with $\phi = \frac{1}{\tau} \left( \frac{f_d}{f_x} \right)^{\frac{1}{\sigma - 1}}$ as a measure of trade openness. Autarky would be $\phi = 0$ and free trade $\phi = 1$. As in most heterogeneous firms trade models, it is assumed that all international active firms sell also their good in the domestic market. Hence, each threshold productivity to earn exactly zero domestic profits has to be at least as low as the threshold productivity to earn exactly zero export profits, $z_X \geq z_D$ and $z_X^* \geq z_D^*$. This assumption implies together with (5) a maximal difference between $z_D$ and $z_D^*$.
according to
\[
\frac{z^*_D}{z_D} = \phi, \text{ if } z_D > z^*_D \quad \text{or} \quad \frac{z^*_D}{z^*_D} = \phi, \text{ if } z^*_D > z_D. \tag{6}
\]

If the competition intensities differ a lot between home and foreign (large difference between \(z_D\) and \(z^*_D\)), trade can not be liberalized by much without violating the above assumption. Otherwise low productive firms may find it only profitable to export, because less stiff export market competition would more than offset the trade costs.

A prospective entrant has to decide how much to invest in R&D before enter into the market. The level of R&D investments determine from which Pareto distribution an entrant draws its productivity. The decision is constraint for convenience to a low and a high level of R&D investments. This covers an essential aspect of firm R&D. It is build on a given knowledge stock of a sector, which reflects for example basic research. Such a knowledge or technology is in general available to all firms of a specific sector within an economy. Furthermore, it is the basis for subsequent applied firms’ R&D (Cohen and Levinthal, 1989). In the model, the knowledge stock is exogenously given and summarized by the two R&D choices:
\[
f_i(z|z \geq z_0) = \frac{\theta_i z_0^{\theta_i}}{z^{\theta_i+1}}, \tag{7}
\]
with \(i \in \{L, H\}\). In both countries the lowest possible productivity draw is \(z = z_0\) with \(z_0 \geq 0\). The \(\theta_H\)-distribution has the lower shape parameter, \(\theta_H < \theta_L\). Therefore, it first order stochastically dominates the \(\theta_L\)-distribution. It is widespread in the literature to model R&D as a choice among different distributions ranked according to the stochastic domination criterion (Addessi et al., 2014; Bagwell and Staiger, 1994; Nelson, 1982). As a result, the superior \(\theta_H\)-distribution yields unambiguously higher productivity draws \(z\) than the inferior \(\theta_L\)-distribution. However, not every draw from the better distribution is higher than from the worse distribution (Mas-Colell et al., 1995).

An entrant in home has to invest \(F_L\) units of labor, if it chooses a low level of R&D and \(F_H > F_L\) if it chooses a high level. This is similar to Rutzer (2013). In addition, foreign entrants have only access to an inferior knowledge stock. Hence, it is more difficult to conduct R&D in foreign than in home, captured by \(k_L F_L\) and \(k_H F_H\), with \(k_L > 1\) and \(k_H \geq 1\), units of labor a foreign entrant has to invest to gain the low or the high R&D technology, respectively. For example, Nelson (1982) argues knowledge is central to R&D outcomes, because it enables to search (more) efficiently: ”Stronger knowledge implies (in this sense) a lower expected cost [...] of achieving an advance of given magnitude ” [p. 459]. If insufficient knowledge is available, even much effort

---

8This is only for simplifying reasons. Rutzer (2013) shows that the qualitative results do not change if entrants can draw from a larger number of Pareto distributions, which differ according to the first order stochastic dominance criterion.
may not lead to a successful innovation. In the model, it implies for foreign entrants higher investments in R&I to gain the same expected productivity draw as home entrants. Reasons for different knowledge stocks across countries can be the quality of institutions, the level of infrastructure or human capital (Unel, 2013).

A home entrant seeks to maximize its expected firm value with respect to the R&I investments:

$$V(z_D, z^*_D, \phi) = \max \{V_i(z_D, z^*_D, \phi) - F_i\}. \tag{8}$$

The superscript $i \in \{L, H\}$ indicates the distribution and

$$V_i(z_D, z^*_D, \phi) = \frac{1}{1 - F_i(z_D)} \int_{z_D}^\infty \pi_d(z, z_D) f_i(z) dz \ldots$$

$$+ \frac{1}{1 - F_i(z_X(z^*_D, \phi))} \int_{z_X(z^*_D, \phi)}^\infty \pi_x(z, z^*_D, \phi) f_i(z) dz \tag{9}$$

is the corresponding expected firm value without the sunk entry costs. The first term is the expected profits earned in the domestic market and the second term the expected profits earned in the export market. A foreign entrant makes an equivalent decision:

$$V^*_j(z_D, z^*_D, \phi) = \max \{V^*_j(z_D, z^*_D, \phi) - k_j F_j\}, \tag{10}$$

with $j \in \{L; H\}$. Each possible expected firm value $V^*_j$ is equivalent to that of a home entrant (9) with inverted cut-off productivities. The explicit functional form of the entrant value (8) in the case of investing in the $i$-level of R&I can be derived by using the expected domestic profits ((3) in (2) and put this together with (7) in the first term of (9)) and the expected export profits ((5) in (4) and put this together with (7) in the second term of (9)), as

$$V_i(z_D, z_X, \phi) = B_i(f_d \left( \frac{z_D}{z_0} \right)^{-\theta_i} + f_x \left( \frac{z^*_D}{z_0 \phi} \right)^{-\theta_i}). \tag{11}$$

$B_i = \frac{\sigma - 1}{\theta_i - \sigma + 1}$ summarizes the expected profits of a single firm in a market without taking the impact of the general equilibrium values into account. An entrant’s expected profit net of entry costs is in general higher in the case of superior R&D than in the case of inferior one, because $B_H \left( \frac{z_k}{z_0} \right)^{-\theta_H} > B_L \left( \frac{z_k}{z_0} \right)^{-\theta_L}$. This inequality is due to $z_k \geq z_0$, with $z_k \in \{z_D, z^*_D\}$ and $\theta_H < \theta_L \rightarrow B_H > B_L$. The same steps lead to the expected value of a foreign entrant as

$$V^*_j(z_D, z^*_X, \phi) = B_j(f_d \left( \frac{z^*_D}{z_0} \right)^{-\theta_j} + f_x \left( \frac{z_D}{z_0 \phi} \right)^{-\theta_j}). \tag{12}$$
2.2 Free entry equilibrium

The aim is to determine simultaneously the competition intensity in home $z_D$ and foreign $z_D^*$ and the optimal R&D choice of entrants in home and foreign. In doing so the competition intensities influence the R&D choices and the R&D choices of all firms together influence $z_D$ and $z_D^*$. The solution method is in principal the same as in Rutzer (2013). However, the optimal R&D choice may now differ between home entrants and foreign entrants. In equilibrium, the expected entrant value has be equal to zero for home as well as foreign entrants due to free entry. The competition intensities in home and foreign can be determined by using the expected value of a home entrant (11) and a foreign entrant (12) and set each of them equal to the corresponding R&D investment costs:

\[
\max \left\{ B_i \left( \left( \frac{z_D}{z_0} \right)^{-\theta_i} + \phi^{\theta_i} \left( \frac{z_D^*}{z_0} \right)^{-\theta_i} \right) - F_i \right\} = 0, \tag{13}
\]

\[
\max \left\{ B_j \left( \left( \frac{z_D^*}{z_0} \right)^{-\theta_j} + \phi^{\theta_j} \left( \frac{z_D}{z_0} \right)^{-\theta_j} \right) - k_j F_j \right\} = 0. \tag{14}
\]

The fixed costs of domestic market participation $f_d$ and export market participation $f_x$ are both set to one. This leads to a simpler notation without influencing the general results, as long as trade liberalization is only interpreted as a reduction of the variable trade costs $\tau$.\(^9\) In addition, the minimal possible productivity draw $z_0$ can be set equal to one without loss of generality.\(^10\) An equilibrium comes about if (8), (10), (13) and (14) are fulfilled at the same time.

In addition, the equation system has a solution if the following two conditions derived in the Appendix are fulfilled:

\[
\left( \frac{B_j (\phi^{-\theta_j} + \phi^{\theta_j})}{k_j F_j} \right)^{\frac{1}{\theta_j}} > \left( \frac{2B_i}{F_i} \right)^{\frac{1}{\theta_i}}, \tag{15}
\]

and

\[
\left( \frac{B_i (1 + \phi^{\theta_i})}{F_i} \right)^{\frac{1}{\theta_i}} > \left( \frac{B_j (1 + \phi^{\theta_j})}{k_j F_j} \right)^{\frac{1}{\theta_j}}. \tag{16}
\]

\(^9\)However, the model would not work if there are no fixed costs, $f_d = f_x = 0$. Because CES-demand functions have usually no prohibitive price. Hence, in such a case even the most unproductive entrant would remain in the market after it got knowledge about the drawn productivity. This entrant would set a very high price and sell a tiny amount in each market.

\(^10\)Each free entry condition can be divided by the minimal possible productivity $z_0$. The adjusted sunk investment costs for home entrants are then $F_i = F_i z_0^{-\theta_i}$ and for foreign entrants $k_j F_j = k_j F^*_j z_0^{-\theta_j}$. Each cost is scaled by the constant $z_0^{\theta_i}$, which can be set equal to one without influencing the results qualitatively.
Here the minimal possible productivity draw \( z_0 \) has already been set equal to 1. Furthermore, if a solution exists, it is unique. The proof is again neglected to the Appendix. These two conditions ensure also that no specialization takes place. Why? \( z_D \) and \( z^*_D \) are endogenously driven by a positive mass of entrants. Entry takes place until the expected entry value equals in both countries the sunk investment costs. Hence, if the home and foreign expected entrant value can be zero at the same time, then there are a positive number of firms from both countries in the market. Otherwise one country would fully specialize in the production of the homogenous good. In this country no entry in the differentiated goods sector would take place, because at the equilibrium competition intensity the expected value of an entrant would be lower than zero for both kinds of \( R&D \). \(^{11}\)

2.3 Equilibrium previous to trade liberalization

In the beginning both countries are in autarky or at some low level of bilateral trade openness. If at this initial level of trade openness, home and foreign entrants find it optimal to invest in a low level of \( R&D \), it is possible to solve the equation system (13) and (14) explicit for \( z_D \) and \( z^*_D \) by setting \( \theta_i = \theta_j = \theta_L \):

\[
\begin{align*}
    z^L_D &= \left( \frac{B_L(1 - \phi^{2L})}{F_L(1 - k_L \phi^{\theta_L})} \right) ^{\frac{1}{\theta_L}}, \\
    z^*_D &= \left( \frac{B_L(1 - \phi^{2L})}{F_L(k_L - \phi^{\theta_L})} \right) ^{\frac{1}{\theta_L}}.
\end{align*}
\]

The superscript "\( L \)" indicates that in both countries the general equilibrium values are defined by low \( R&D \) investments of all entrants. The existence of such an equilibrium can be easily shown. Each entrant’s value declines monotonically in the \( R&D \) investment costs: \( \frac{\partial V_i}{\partial F_i} < 0 \) and \( \frac{\partial V^*_j}{\partial F^*_j} < 0 \). Thus, an inferior \( R&D \) equilibrium exists in any case, as long as the labor requirement for superior \( R&D \), \( F_H \) and \( k_H F_H \), are high enough in home and foreign, respectively. The necessary conditions are\(^{12}\)

\[
\begin{align*}
    F_H > B_H \left( \frac{F_L(1 - k_L \phi^{\theta_L})}{B_L(1 - \phi^{2L})} \right) ^{\frac{\theta_H}{\theta_L}} + B_H \phi^{\theta_H} \left( \frac{F_L(k_L - \phi^{\theta_L})}{B_L(1 - \phi^{2L})} \right) ^{\frac{\theta_H}{\theta_L}}, \\
    k_H F_H > B_H \left( \frac{F_L(k_L - \phi^{\theta_L})}{B_L(1 - \phi^{2L})} \right) ^{\frac{\theta_H}{\theta_L}} + B_H \phi^{\theta_H} \left( \frac{F_L(1 - k_L \phi^{\theta_L})}{B_L(1 - \phi^{2L})} \right) ^{\frac{\theta_H}{\theta_L}}.
\end{align*}
\]

\(^{11}\)The case of complete specialization in a Melitz (2003) model with asymmetric countries according to the technological potential is described in Demidova (2008) on p. 1450.

\(^{12}\)These conditions are obtained by setting \( V_H(z_D, z^*_D, \phi = 0) < 0 \) (11) and \( V_L(z_D, z^*_D, \phi = 0) \geq 0 \) (8) as well as \( V^*_H(z_D, z^*_D, \phi = 0) < 0 \) (12) and \( V^*_L(z_D, z^*_D, \phi = 0) \geq 0 \) (10).
They reduce in autarky to
\[ F_H > B_H \left( \frac{F_L}{B_L} \right) ^{\theta_H} \theta_L \] and
\[ k_H F_H > B_H \left( \frac{k_L F_L}{B_L} \right) ^{\theta_H}. \] In this case, home as well as foreign entrants find it under autarky optimal to invest in a low level of R&D.

2.4 How does trade liberalization affects the R&D investment decision of firms?

In general there are two trade liberalization scenarios possible. A reduction of the fixed export costs \( f_x \) or a decline of the variable trade costs \( \tau \). In this paper the focus is solely on the impacts of a lower \( \tau \), which is equivalent to a larger \( \phi \) (see (5)).

Lower variable trade costs affect the R&D decision of firms due to better export perspectives. At the same time it increases the competition in each market. The competition intensities in home and foreign are determined by the free entry conditions. However, it is not possible to derive an explicit solution for \( z_D \) and \( z_D^* \) as soon as the optimal R&D choice differ between home and foreign entrants. In this case the exponent of \( z_D \) and \( z_D^* \) is no longer the same in the home and the foreign free entry condition. How a change in variable trade costs affects both competition intensities can nevertheless be analyzed by total differentiate each free entry condition, (13) and (14), as

\[
-\theta_i z_D ^{-\theta_i -1} dz_D - \theta_i \phi \theta_i z_D ^{-\theta_i } dz_D^* + \theta_i \phi \theta_i z_D ^{-\theta_i -1} d\phi = 0, 
\]

\[
-\theta_j z_D^* ^{-\theta_j -1} dz_D^* - \theta_j \phi \theta_j z_D^* ^{-\theta_j } dz_D + \theta_j \phi \theta_j z_D^* ^{-\theta_j -1} d\phi = 0. 
\]

By solving (21) after \( \phi \) and putting the result in (22), it follows:

\[
dz_D^* = \xi^{-1} dz_D, \tag{23}
\]

with \( \xi = \frac{z_D ^{\theta_j + \theta_j - \theta_j + 1}}{z_D ^{\theta_j + \theta_j - \theta_j - 1}} > 0^{13} \) and \( Z = z_D / z_D^* \). In addition, using this equation in (22) the differential \( d\phi \) can be written as

\[
d\phi = (\phi z_D ^{-1} + z_D ^{\theta_j - \theta_j + 1} \phi ^{-\theta_j + 1} \xi) dz_D. \tag{24}
\]

The term in brackets is clearly positive. Hence, if trade becomes more open (\( d\phi > 0 \)), the home cut-off \( z_D \) increases (\( dz_D > 0 \)). In addition, the foreign cut-off increases also due to (23). As a result, trade liberalization leads to an increase of the home as well as the foreign competition intensity.

Now it is analyzed, how trade liberalization influence the R&D choice of entrants. The

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13\( \text{Proof: The nominator is always positive, because } \phi ^{-1} \geq 1 \text{ and } Z > 1 \text{ as long as } k_H, k_L > 1. \) In addition, the denominator has its minimal value at the maximal possible value of \( Z \), which is \( Z_{\max} = \phi ^{-1} \text{ due to (6). In this case the denominator is zero.} \)
analysis is done in three steps. First, it is examined whether the incentive to invest in a high level of R&D increases in at least one country as trade becomes liberalized. This is necessary, because otherwise no entrant would ever invest in the superior R&D. Second, it is analyzed in which country entrants find it optimal to invest at first, that means at a lower level of trade liberalization, in a high level of R&D. Third, it is analyzed whether investing in a high level of R&D in one country from a certain level of trade openness on determines a new stable equilibrium.

As long as trade is rarely liberalized, entrants in both countries find it optimal to invest in a low level of R&D as long as the above conditions are fulfilled, (19) and (20). The R&D choice function of a home entrant (8) can be written by using (11) as

$$V(z_D, z^*_D, \phi) = \max \left\{ B_L((z_D)^{-\theta_L} + \phi^\theta_L(z_D^*)^{-\theta_L}) - F_L ; B_H((z_D)^{-\theta_H} + \phi^\theta_H(z^*_D)^{-\theta_H}) - F_H \right\}. \tag{25}$$

The first argument represents the expected value of an investment in a low level of R&D. The expected value of such an investment is zero due to free entry for any level of trade openness, as long as the competition in the market is driven by home firms that have also invested in a low level of R&D (see (21) with $\theta_i = \theta_L$). The second argument represents the expected value if an entrant decides to invest in a high level of R&D. Its value is negative in autarky as long as the conditions (19) and (20) are fulfilled. In addition, its value changes as trade costs decline:

$$dV_H(z^L_D, z^*_L_D, \phi) = B_H(-\theta_H z^L_D z^L_D - \theta_H \phi^\theta_L z^*_L_D z^L_D + \theta_H \phi^\theta_H z^*_L_D z^L_D \phi) > 0, \tag{26}$$

with $dz_D > 0 \ (23)$ and $dz^*_D > 0 \ (24)$. As long as the expected value to invest in a high level of R&D is negative, entrants will invest in a low level of R&D. Entry of such firms makes the competition stiffer in both markets. The tougher competition reduces the incentives to invest in a high level of R&D (first and second term of equation (26)). At the same time, the better access to the foreign market leads to an increased incentive to invest in a high level of R&D (third term of (26)). It is shown in the Appendix that the better access to the foreign market makes more than offsets the negative effect of a stronger competition in both markets. Hence, the incentive of a home entrant to invest in a high level of R&D increases as trade becomes liberalized.

Furthermore, the R&D choice function of a foreign entrant can be derived by using equation (12) as

$$V^*(z^L_D, z^*_D, \phi) = \max \left\{ B_L((z^*_L)^{-\theta_L} + \phi^\theta_L(z^L_D)^{-\theta_L}) - k_L F_L ; B_H((z^*_L)^{-\theta_H} + \phi^\theta_H(z^L_D)^{-\theta_H}) - k_H F_H \right\}. \tag{27}$$

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The expected value of an investment in a low level of R&D remains again zero for any level of trade openness as long as the competition intensity is determined by foreign firms that have invested in a low level of R&D (first argument). But the expected value for a foreign entrant to invest in a high level of R&D increases as trade becomes liberalized:

\[ dV^*_H(z^L_D, z^*_D, \phi) = B_H(-\theta_H z^*_D z_D^{-\theta_H-1} d\phi - \theta_H \phi^{\theta_H} z^*_D z_D^{-\theta_H-1} d\phi + \theta_H \phi^{\theta_H-1} z_D^{-\theta_H} d\phi) > 0. \]  

(28)

It is negatively affected by a stronger competition in both markets (first and second term) and positively affected by the better access to the home market (third term). It is shown in the Appendix that the total effect is again positive.

In the second part of the analysis it will now be investigated in which country entrants find it at first optimal to invest in the superior R&D after exceeding a certain level of trade openness; whereby firms in the other country will still remain at the inferior one. This question can be analyzed by subtracting the expected value of an investment in a high level of R&D of an home entrant (second argument of (25)) from that of a foreign entrant (second argument of (27)):

\[ V_H(z^L_D, z^*_D, \phi) - V^*_H(z^L_D, z^*_D, \phi) = \cdots B_H((1 - \phi^{\theta_H})(z^*_D)^{-\theta_H} - (z^L_D)^{-\theta_H}) - F_H(1 - k_H). \]  

(29)

If both expected values increase with lower trade costs and at least one of the two expected values turn positive within the possible range of trade liberalization the difference between both of them reveals, which of them will turn first positive as trade costs decline. The equations (26) and (28) show that the incentive to invest in a high level of R&D increases for home as well as foreign entrants in the presence of trade liberalization. The second condition implies that the benefits of a high level of R&D relative to a low level of R&D more than out weigh the higher investment costs. This is the case as long as at the maximal level of trade openness entrants in at least one country find it optimal to invest in a high level of R&D. It can be shown that as long as foreign has no advantage in R&D investments, \( k_L, k_H \geq 1 \), the competition intensity will always be larger in home than in foreign: \( z_D \geq z^*_D \) (the proof is neglected to the Appendix). Hence, from (6) it follows that trade openness can maximal be \( \phi^{\max} = z^*_D z^*_D \). By using this in (25), home entrants find it optimal to invest in a high level of R&D at the maximal level of trade openness if \( F_H < 2B_H \left( \frac{F_L(k_L - (\phi^{\max})^\theta_L)}{B_L(1 - (\phi^{\max})^\theta_L)} \right)^{\theta_H} \), where \( \phi^{\max} = \frac{1}{k_L + \sqrt{k_L^2 - 1}} \) \((6) \) with \((17) \) and \((18)\)), is fulfilled. In addition, foreign entrants find it optimal at the maximal level of trade openness to invest in a high level
of R&D if

\[ k_H F_H < B_H (1 + \phi^{2\theta_H}) \left( \frac{F_L (k_L - (\phi^{\text{max}})^{\theta_L})}{B_L (1 - (\phi^{\text{max}})^{2\theta_L})} \right)^{\frac{\theta_H}{\theta_L}} \]  

(30)
is fulfilled. This condition can be derived by using \( \phi^{\text{max}} = \frac{z_D}{z_D} \) in (27).

From now on it is assumed that the investment cost \( F_H \) is low enough that at least in one country it is optimal to invest in a high level of R&D within the possible range of trade costs. The difference function (29) indicates in this case, which entrants will first invest in a high level of R&D as trade becomes liberalized. The two possibilities that can occur are depicted in figure 1. If the difference function is larger than zero, entrants from home will invest at a lower level of trade openness into a high level of R&D than foreign ones (left figure). If the value is instead negative, entrants from foreign will invest at a lower level of trade openness in a high level of R&D than entrants from home (right figure). The sign of the difference function (29) depends on the magnitude of both foreign disadvantages, \( k_H \) and \( k_L \). Its value is zero in the case that the labor requirements for each particular R&D investment are of same magnitude in home and foreign, \( k_H = k_L = 1 \). The model is in this case identical to the symmetric country model of Rutzer (2013) and entrants will behave similarly in both countries.

But what happens if foreign entrants have a disadvantage in either one or both kinds of R&D? This can be analyzed by total differentiate the difference function (29) with

\[ V_H - V')_H (26) > 0 \]

\[ V_H (22) \]

\[ V_H (24) \]
respect to $k_H$ and $k_L$:

\[
d[V_H(z_D^L, z_D^L, \phi) - V_H^*(z_D^L, z_D^L, \phi)] = ...
\]

\[
... = -\frac{\theta_H B_H F_L (1 - \phi^{\theta_H})}{\theta_L B_L (1 - \phi^{\theta_L})} (\phi^{\theta_L} (z_D^L)^{\theta_H - 1} + (z_D^L)^{\theta_H - 1}) dk_L + F_H dk_H
\]  

(31)

Here the explicit values of $z_D^L$ (17) and $z_D^L$ (18) have been used to total differentiate the function (29). The term before the differential $dk_L$ is negative, because the cut-offs (17) and (18) are larger than zero. Hence, the first term in the second row is negative if $dk_L > 0$. This means the value of the difference function is lower the larger is the foreign disadvantage to invest in a low level of R&V. In addition, the second term is positive if $dk_H > 0$. Hence, the difference function is larger the larger is the foreign disadvantage to invest in a high level of R&V.

As a result, if foreign entrants have no disadvantage in investing in the superior R&V ($k_H = 1$), the value of the difference function is negative as soon as foreign have a disadvantage in investing in a low level of R&V ($k_L > 1$). Then foreign entrants will be at first that invest in the superior R&V as trade becomes liberalized. In addition, the investment requirement for the superior R&V can be larger in foreign than in home, $k_H > 1$. Nevertheless, it can be that foreign entrants will invest in the superior R&V at a lower level of trade openness than home entrants. Furthermore, there exists a relationship between both foreign disadvantages with regard to the R&V investment decision. The larger the disadvantage of investments in a low level of R&V is (higher $k_L$), the larger can the disadvantage of an investment in a high level of R&V be (higher $k_H$) and foreign entrants will still invest at fist in a high level of R&V. This means that although foreign entrants may have an absolute disadvantage in both kinds of R&V, they can still be the first one which invest in the superior R&V. The necessary requirement is that the absolute disadvantage of foreign entrants in investing in a high level of R&V has to be less than the absolute disadvantage in a low level of R&V: $k_H < k_L$. The proof can be found in the Appendix. However, it is not possible to derive an explicit expression for the sufficient requirement. In other words, foreign needs at least a comparative advantage in investments in a high level of R&V relative to investments in a low level of R&V. Otherwise home entrants will always be the first one that invest in a high level of R&V as trade becomes liberalized.

What is the intuition behind this result? Entrants in both countries will invest in a low level of R&V as long as trade costs are high. Labor requirement for this kind of R&V are supposed to be lower in home than in foreign. As a consequence more firms enter in home than in foreign into the market. This leads to a tougher competition in the home market compare to the foreign market, as it can be seen by dividing both
general equilibrium values with each other ((17) divided by (18)): \( z_D^L = \left( \frac{k_L - \phi \theta L}{1 - k_L \phi \theta L} \right)^{\frac{1}{\theta L}} > 1. \) (32)

The competition intensity in home is stiffer than in foreign as long as home has an absolute advantage in low R&D investments, \( k_L > 1 \). The profitability of investments in a high level of R&D relative to a low level of R&D increases in both countries as trade becomes liberalized. For the expected profits of an entrant does the domestic market account more than export market (see (13) and (14)). Hence, the attractiveness to invest in a high level of R&D increases less in home than in foreign due to a tougher domestic competition. A trade-off occurs, which is shown mathematically in equation (31): on the one hand labor requirement to invest in a high level of R&D is lower in home than in foreign. On the other hand the expected profits of an investment in a high level of R&D net of investment requirements are lower in home than in foreign.

The ultimately result is that after trade liberalization investments in a high level of R&D can solely become profitable in the disadvantaged foreign country. A similar discussion on this point can be found in Rutzer (2013). In that paper it is explained in more detail, why entrants find it profitable at all to invest in a low level of R&D.

The argument is the same as in this paper.

### 2.5 Equilibrium after trade liberalization

In the previous chapter it has been shown that entrants from at least one country find it optimal to invest in a high level of R&D after a sufficient degree of trade liberalization. Thereby it was assumed that the competition intensity adjusts such that the free entry condition is fulfilled for entrants investing in a low level of R&D. This is correct as long as the expected value for each entrant to invest in a high level of R&D is lower than zero. In this case no entrant will invest in a high level of R&D and hence no firm is in the market that have invested in a high level of R&D.\(^\text{15}\) But as soon as entrants find it optimal to invest in a high level of R&D, free entry implies that the competition intensity has to adjust until the expected profits of investments in a high level of R&D are again zero. However, this is only an equilibrium if at this competition intensity the expected profits of investments in a low level of R&D are lower than zero. Otherwise entry of firms that have invested in a low level of R&D would take place and this would change again the competition intensity in the market. This section aims

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\(^\text{14}\)See Melitz (2003) pp. 1715 – 1716 for a more profound discussion on how the endogenous productivity cut-off and the real wage rate of a country depends on each other.

\(^\text{15}\)Rutzer (2013) gives an intuition why in such an equilibrium no entrant invests in a high level of R&D, although such an investment leads always to a higher benefit when the investment costs are not taken into account.
therefore, to analyze whether investments in a high level of R&D defines a new stable equilibrium after trade liberalization. This is the case if (13) and (14) as well as (8) and (10) are fulfilled at the same time. The analyze focuses thereby only on the case when foreign entrants find it optimal to invest at first in a high level of R&D after trade liberalization. The reason is simple. As it will be seen afterward, only in this case leapfrogging can occur.

Under rarely liberalized trade investments in a low level of R&D are optimal as long the parameter conditions (19) are fulfilled. In addition, it was shown above that the expected value of investments in a high level of R&D increases and at the same time the expected value of investments in a low level of R&D remains constant at zero due to free entry as trade becomes liberalized. Hence, as long as the parameter conditions (30) are fulfilled, there exists a particular level of trade openness $\phi = \phi^*$ at which a foreign entrant is indifferent between both kind of R&D investments. Formally, at $\phi^*$ both arguments of (27) have the same value:

$$V_H^*(z_D, z_D^*, \phi = \phi^*) = V_L^*(z_D, z_D^*, \phi = \phi^*) = \ldots = V_L^*(z_D^L, z_D^*, \phi = \phi^*) = V_H^*(z_D^L, z_D^*, \phi = \phi^*) = 0.$$  (33)

The last line results from the fact that at $\phi^*$ the cut-off values $z_D$ and $z_D^*$ are the same irrespective whether foreign entrants invest in a high or a low level of R&D. Why? Because at this particular level of trade openness the entrant value to invest in a high level of R&D is zero in the case that the free entry equilibrium is defined by low R&D investments. At the same time its value would also be zero if all foreign firms would have invested in a high level of R&D. The higher investment benefits of a high level of R&D are at this level of trade openness just offset by the higher sunk investment costs. But this implies that the competition intensity would be the same at this particular level of trade openness irrespective whether the economy consists of firms that have invested in a low- or high level of R&D. The formal proof is shown in the Appendix.

Now trade liberalization continues. Assume for a moment that the foreign economy consists of firms that have all invested in a low level of R&D. The expected profit to invest in a high level of R&D increases further (see (28)) and its value is therefore above zero. Each profit maximizing foreign entrant will invest in a high level of R&D, because the expected value to invest in a low level of R&D is still zero. But then the economy consists no longer of firms that have all invested in a low level of R&D. Hence, this cannot be a stable equilibrium as soon as $\phi > \phi^*$. Now assume that the foreign economy consists of firms that have all invested in a high level of R&D. Free entry implies that in equilibrium the expected profit to invest in a high level of R&D is zero. In addition, the value of a foreign entrant to invest in a low level of R&D

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decreases in this case as trade becomes liberalized:

$$\frac{\partial V^*_L(z^L_D, z^H_D, \phi)}{\partial \phi} = -z^L_D\theta_L - z^H_D\theta_L - z^H_D - z^L_D\phi^\theta_L d\phi < 0. \quad (34)$$

The formal proof can be found in the Appendix. Due to (33), the value to invest in a low level of R&D is at $\phi = \phi^*$ zero in the case that all other foreign firms have invested in a high level of R&D. Hence, the expectation value to invest in a low level of R&D is lower than zero for levels of trade openness $\phi > \phi^*$. In the new free entry equilibrium the competition intensity is stiffer than it would be the case if all firms have invested in a low level of R&D. At this new competition intensity the expected value of a foreign entrant that decides to invest in a low level of R&D would be negative. Hence, no entrant invests in a low level of R&D. It follows that for trade openness levels of $\phi > \phi^*$ the values of the foreign R&D choice function are

$$V^*_H(z^L_D, z^H_D, \phi > \phi^*) = 0 > V^*_L(z^L_D, z^H_D, \phi > \phi^*). \quad (35)$$

The stable equilibrium is then determined by foreign firms that have all invested in a high level of R&D. In addition, the expected value to invest in a high level of R&D is zero, because increased R&D profits are offset by more firm entry.

What is with home entrants? The values of the two arguments of the R&D choice function of home entrants (25) are at $\phi^*$

$$V^*_H(z^L_D, z^H_D, \phi = \phi^*) = V^*_H(z^L_D, z^H_D, \phi = \phi^*)... < V^*_L(z^L_D, z^H_D, \phi = \phi^*) = V^*_L(z^L_D, z^H_D, \phi = \phi^*) = 0 \quad (36)$$

The equalities result due to the above described fact that at $\phi^*$ the competition intensity is the same irrespective whether foreign firms have invested in a high or low level of R&D: $z^H_D = z^H_D$. Hence, at this particular trade openness level the foreign R&D choice does in equilibrium not impact the home economy. In addition, as long as the difference function (29) is negative, the left-hand side of (36) has to be lower than zero due to $V^*_H(z^L_D, z^L_D, \phi = \phi^*) = 0$ (see (33)). Hence, a home entrant that invests at $\phi^*$ in a high level of R&D would have a negative expected value. Investments in a low level of R&D remains therefore the optimal choice of home entrants.

But the incentive of a home entrant to invest in a high level of R&D continues to increase as trade becomes further liberalized (see Appendix for proof), although the competition intensity in foreign increases now more than it would be the case if foreign firms have invested in a low level of R&D:

$$\frac{\partial V^*_H(z^L_D, z^H_D, \phi)}{\partial \phi} = B_H\theta_H(-z^L_D\theta_H - z^H_D\theta_H - z^H_D - z^L_D\phi^\theta_H d\phi > 0. \quad (37)$$
At the same time the higher expected profits of an investment in a low level of R&D due to a better access to the foreign market is again offset by more entry. The expected value of a home entrant to invest in a low level of R&D remains unchanged at zero as trade becomes more liberalized. Hence, a home entrant is therefore also at some trade openness $\phi^{home} > \phi^*$ indifferent between both kind of R&D investments:

$$ V_L(z_D^L, z_D^{*H}, \phi = \phi^*) = V_H(z_D^L, z_D^{*H}, \phi = \phi^{home}). $$

(38)

Same arguments as above show that investments in a low level of R&D would lead to a negative expected profit for a home entrant as soon as trade is liberalized beyond $\phi^{home}$:

$$ V_H(z_D^H, z_D^{*H}, \phi > \phi^{home}) = 0 > V_L(z_D^H, z_D^{*H}, \phi > \phi^{home}), $$

(39)

because the expected value to invest in a low level of R&D will in this case decline

$$ \frac{\partial V_L(z_D^H, z_D^{*H})}{\partial \phi} = B_L \theta_L (-z_D^{\theta_L-1} d z_D - z_D^{*\theta_L-1} \phi^{\theta_L} d z_D + \phi^{\theta_L-1} z_D^{*\theta_L} d \phi) < 0. $$

(40)

The proof is again neglected to the Appendix. Hence, as soon as trade openness is $\phi > \phi^{home}$ home entrants will find it also optimal to invest in a high level of R&D.\footnote{The necessary condition on the parameter values is that trade liberalization beyond $\phi^{home}$ do not violate the assumption that all firms that export are also sell in the domestic market. This is fulfilled as long as $\phi^{max} \leq \frac{z_D}{z_D} = \frac{1}{k_H + \sqrt{k_H^2 - 1}}$ is fulfilled (using (6), (17), and (18) with index H instead of L).}

Taken together, as soon as trade is liberalized until some level between $\phi^*$ and $\phi^{home}$, there exists a new stable equilibrium at which foreign entrants find it optimal to invest in a high level of R&D and home entrants still remain at a low level of R&D. The unequal R&D investment decision of home and foreign entrants have also an important impact on the industry productivity of the two countries. How it is affected, and, under which requirements leapfrogging occur, will be analyzed in the following paragraph.

2.6 Trade liberalization and Leapfrogging

Following Brezis et al. (1993), leapfrogging occurs if the leading and laggard position with regard to industry productivity of the two countries are reversed. Similar to Brezis et al. (1993), the leading country is defined as the country with the higher productivity. The analysis focuses only on the case in which foreign entrants invest at a lower level of trade openness in a high level of R&D than home firms: $\phi^* < \phi^{home}$. If home firms would invest first in a high level of R&D, leapfrogging can never occur. Instead of, the results would be similar to Demidova (2008).

In the case of CES-demand and heterogeneous firms the industry productivity can
be measured either using a quantity aggregation or using a revenue-based aggregation (Bernard et al., 2011). Following Melitz (2003), in the model the industry productivity of home measured as the output-weighted average productivity of firms at the factory gate is:

\[ a = \left( \frac{1}{1 + p_X} [\tilde{z}(z_D) + p_X \tilde{z}_X(z_D^*)] \right)^{\frac{1}{\sigma - 1}}, \]  

(41)

with \( \tilde{z}(z_D) = \int_{z_D}^{\infty} \frac{z^{\sigma - 1}}{\sqrt{2\pi}} e^{-\frac{(z - \theta)^2}{2\sigma^2}} dz \) as the average productivity of all home firms, \( \tilde{z}_X(z_D^*) = \int_{z_D^*}^{\infty} \frac{z^{\sigma - 1}}{\sqrt{2\pi}} e^{-\frac{(z - \theta)^2}{2\sigma^2}} dz \) as the average productivity of home export firms, \( \frac{1}{1 + p_X} \) as the share of domestic-output of home firms relative to the total output of home firms and \( \frac{p_X}{1 + p_X} \) as the share of export-output of home firms relative to total output of home firms.\(^{18}\) The industry productivity of foreign firms, \( a^* \), is equivalent defined.

By using (7), the home sector productivity is in the case of Pareto-distributed productivity draws

\[ a^{\sigma - 1} = \frac{z_D^{\sigma - 1} + p_X \left( \frac{z_D}{\theta} \right)^{\sigma - 1}}{1 + p_X} A, \]  

(42)

with \( A = \frac{\theta^{\sigma - 1}}{\theta^{\sigma - 1} + 1} \) and \( p_X = \frac{1 - F(\tilde{z}_X)}{1 - F(\tilde{z}_D)} = \left( \frac{z_D}{\tilde{z}_X} \right)^{\theta} = \left( \frac{z_D}{\tilde{z}_D} \right)^{\theta} \phi^{\theta} \) as the fraction of export firms to all firms in home. The corresponding foreign sector productivity is

\[ a^{*\sigma - 1} = \frac{z_{D^*}^{\sigma - 1} + p_X^* \left( \frac{z_D}{\theta^*} \right)^{\sigma - 1}}{1 + p_X^*} A^*, \]  

(43)

with \( A^* = \frac{\theta^{\sigma^* - 1}}{\theta^{\sigma^* - 1} + 1} \) and \( p_X^* = \left( \frac{z_D}{\tilde{z}_D^*} \right)^{\theta^*} \phi^{\theta^*} \) as the fraction of foreign export firms to all foreign firms. In general, the sector productivities can differ due to the Melitz (2003)-selection effect, \( z_D \) and \( z_{D^*} \), and the distribution shape effect, \( \theta \) and \( \theta^* \).

Firms from both countries find it optimal to choose low R&D investments in the case of minor trade openness. Hence, as long as \( \phi < \phi^* \), the shape parameter is the same in both countries, \( \theta = \theta_L \) in home and \( \theta^* = \theta_L \) in foreign. In addition, if foreign entrants have to invest the same amount to participate at the productivity draw, \( k_L = 1 \), the competition intensities are of same magnitude in both countries, \( z_D = z_{D^*}^* \). As a result

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\(^{17}\)Measuring aggregate productivity through quantity weights can be "at the firm gate" or at the consumers. The measurement at the consumers takes the productivity reducing losses in form of iceberg trade costs \( \tau \) into account. As Melitz (2003) points out the aggregate productivity has to be based "at the firm gate" to induce always an increase of the aggregate productivity by trade liberalization.

\(^{18}\)In a symmetric Melitz (2003) model this is also the average productivity of all goods available to a consumer in a country. In the model at hand this is in general not the case, because the implied firm productivities of exports of one country are not the same as of imports of the same country.
both countries are equal productive, \(a = a^*\) (this can be seen immediately by comparing (42) and (43)). This is the Melitz (2003) result in the case of Pareto distributed firms. However, as soon as foreign entrants suffer from higher labor requirement to invest in a low level of R&D, \(k_L \geq 1\), the competition is more intense in home than in foreign, 
\[ Z = z_D/z^*_D > 1 \text{ (see (32)).} \]
Consequently, home is more productivity than foreign, which can be seen by dividing (42) through (43):
\[
\left( \frac{a}{a^*} \right)^{\sigma-1} = \frac{Z^\theta(1 + \phi^{2\theta} Z^{-\sigma+1}) + \phi^\theta + \phi^\theta Z^{2\theta - \sigma + 1}}{Z^\theta(Z^{-\sigma+1} + \phi^{2\theta}) + \phi^\theta + \phi^\theta Z^{2\theta - \sigma + 1}} > 1, \tag{44}
\]
because \(Z^{-\sigma+1} < 1\). The lower entry barrier in home attracts more entrants in home than in foreign. As a consequence, competition in the home market is stronger and entrants need a higher productivity to survive. The stronger selection of unproductive entrants leads to a higher sector productivity in home than in foreign. Hence, using the above definition, home is the initial leading country.

Now trade becomes gradual liberalized till \(\phi^*\). The previous section shows that entrants from the laggard foreign find it under certain conditions optimal to invest in a high level of R&D and at the same time entrants from the leading home still remain at a low level of R&D investments. As soon as foreign entrants invest in a high level of R&D, they draw their productivity from the Pareto distribution with a lower shape parameter, \(\theta_H < \theta_L\). In general, a larger shape parameter affects the sector productivity negatively:
\[
\frac{\partial a^*}{\partial \theta^*} = A^* a^* d\theta^* + A^* \left( \frac{z_D}{\phi} \right)^{\sigma-1} (1 + p_X h) - p_X z^*_D (z^*_D - 1 + p_X z^*_D) d\phi^* \tag{45}
\]
\[
\text{→ } \frac{\partial a^*}{\partial \theta^*} = \left( A^* a^* + p_X^* A^* \left( \frac{z_D}{\phi} \right)^{\sigma-1} - z^*_D \right) \frac{(1 + p_X^*)^2}{(1 + p_X)^2} d\theta^* < 0. \tag{46}
\]
The derivative is negative, because \(\frac{\partial A^*}{\partial \theta^*} < 0\), \(p_X^* < 0\) and \(\left( \frac{z_D}{\phi} \right)^{\sigma-1} > z^*_D\). Furthermore, as it was shown in the previous section, the home and foreign market competition intensities are not affected. As a result the selection effect remains unchanged in both countries. The foreign sector productivity jumps up, as soon as foreign entrants invest in a high level of R&D, \(a^*_H > a^*_L\). At the same time, the home sector productivity remains unchanged at \(\phi^*\). Hence, the foreign country has gained in productivity terms relative to the home country through an increase in R&D investments by its entrants.

The new relative sector productivity under consideration of foreign investments in a
A high level of $R&D$ is

$$
\left( \frac{a_L}{a_H} \right)^{\sigma-1} = \frac{A_L}{A_H} \left( \frac{Z^{\theta_H}(1 + \phi^{\theta_L+\theta_H}Z^{\theta_L-\theta_H-\sigma+1}) + \phi^{\theta_H} + \phi^{\theta_L}Z^{\theta_H+\theta_L-\sigma+1}}{Z^{\theta_H}(Z^{-\sigma+1} + \phi^{\theta_L+\theta_H}Z^{\theta_L-\theta_H}) + \phi^{\theta_H} + \phi^{\theta_L}Z^{\theta_H+\theta_L-\sigma+1}} \right),
$$

(47)
as long as $\phi^{\text{home}} > \phi \geq \phi^*$. The increased investments by foreign entrants can even lead that foreign overtakes home in terms of productivity. This is the case as soon as the foreign sector productivity is larger than the home one: $a_H > a_L$. Although it is not possible to give an exact range of parameter values for which leapfrogging occurs, it is possible to show it for a special case. Assume the knife edge case when foreign has no disadvantage to invest in a high level of $R&D$, $k_H = 1$. In this case foreign entrants will invest at a particular level of trade openness $\phi^*$ in a high level of $R&D$ and home entrants will not, as long as $k_L > 1$. At $\phi^*$ the relative selection effect is $Z = \left( \frac{k_L - \phi^* \sigma_H}{1-k_L \phi^* \sigma_H} \right)^{\theta_L} > 1$, because $z_D(\theta_L, \theta_L, \phi^*) = z_D(\theta_L, \theta_H, \phi^*)$ and $z_D(\theta_L, \theta_L, \phi^*) = z_D(\theta_L, \theta_H, \phi^*)$ (see (33)). Further trade liberalization leads to an increased incentive of home entrants to invest in a high level of $R&D$ (see (37)). Hence, home entrants find it also optimal to invest in a high level of $R&D$ as soon as trade becomes liberalized until a particular level of trade openness $\phi^{\text{home}}$. The relative selection effect $Z$ is one for $\phi \geq \phi^{\text{home}}$ if $k_H = 1$.\footnote{If entrants from both countries find it optimal to invest in a high level of $R&D$, by using (13) and (14) the relative selection effect is $Z = \left( \frac{k_H - \phi^* \sigma_H}{1-k_H \phi^* \sigma_H} \right)^{\theta_H}$. Its value is one for $k_H = 1$.} However, at a trade openness level $\phi = \phi^{\text{home}} - \epsilon$, with $\epsilon > 0$ and $\epsilon \to 0$, the selection effect is approximately one but home firms have not yet invested in a high level of $R&D$. The relative sector productivity between home and foreign (47) can be at this particular level of trade openness written as

$$
\left( \frac{a_L}{a_H} \right)^{\sigma-1} = \frac{A_L}{A_H} < 1.
$$

(48)

It is lower than one, because $A_H > A_L$. Hence, foreign has a higher sector productivity than home as soon as trade openness exceeds a particular level $\phi$ in the interval $\phi \in [\phi^*, \phi^{\text{home}}]$. As a result, leapfrogging takes place. However, once trade becomes liberalized beyond $\phi^{\text{home}}$, home entrants find it also optimal to invest in a high level of $R&D$. As a result, both countries have the same industry productivity for trade openness levels between $\phi^{\text{home}}$ and $\phi^{\text{max}}$.

### 2.7 How does the results depend on $k_H$, $k_L$ and $\theta_H$

Based on the knife-edge case described above, it can be analyzed, how changes in some fundamental parameters influence the possibility of leapfrogging. In particular the analysis focuses on the parameters covering the foreign investment disadvantage,
\( k_L \) and \( k_H \), and the parameter summarizing benefits of a high investment in \( R&D \), \( \theta_H \). Different values of \( \theta_L \) will not to be considered, because the aim is to compare the sector productivity between the two countries. As soon as entrants in one country find it optimal to invest in a high level of \( R&D \) and in the other country not, the impact of a high \( R&D \) investment on industry sector productivity is relative to the outcome of a low \( R&D \) investment. Hence, to analyze changes in \( \theta_L \) an \( \theta_H \) would be two sides of the same coin. Again, it is assumed throughout the complete analysis that foreign entrants find it optimal to invest in a high level of \( R&D \) and home entrants in a low level of \( R&D \).

If \( k_H \) is larger, foreign entrants have to invest more to gain the high \( R&D \) level. Hence, entry becomes less attractive. It implies a less fierce competition in the foreign market, summarized by a lower general equilibrium value \( z^*_D \). As a result, entry in home becomes more attractive due to a less fierce competition in the corresponding export market (i.e. for home firms the foreign market is the export market). This leads in equilibrium to a stronger competition in the home market, summarized by a higher general equilibrium value \( z_D \).

Take all effects together, the home sector productivity is affected by a larger foreign disadvantage according to

\[
\frac{\partial a}{\partial k_H} = \left( \frac{1}{1 + p_X} \right) \frac{d}{dz_D} \left( \frac{p_X}{1 + p_X} \right) + \frac{p_X}{1 + p_X} \frac{d}{dz_D} \left( \frac{\partial X(z_D)}{\partial z_D} \right) + \frac{1}{1 + p_X} \frac{d}{dz_D} \left( \frac{\partial X(z_D)}{\partial z_D} \right) > 0. \tag{49}
\]

Less strong competition in the foreign market enables less productive home firms to export profitably. This has two effects. First, it affects the fraction of export firms and therewith the weight of export firms to all home firms \((a)\). The fraction of home export firms to all home firms increases, \( dp_X > 0 \). Second, it leads to a lower average productivity of home exporters \((b)\), because \( dz_D^* < 0 \). In addition, the stronger competition in the home market requires a higher productivity of a home firm to remain in the domestic market, \( dz_D > 0 \). As a result the average productivity of all home firms will be larger \((c)\). The overall effect is positive if the industry productivity is measured at the firm gate.\(^{20}\) Proof see Appendix.

In addition, the average productivity of foreign firms is affected by a larger foreign disadvantage.\(^{20}\) Otherwise the total effect of a larger foreign disadvantage on the home productivity is negative, because the efficiency loss that the increased fraction of home firms that export induces is larger than the increased domestic productivity of home firms.
disadvantage according to

\[
\frac{\partial a}{\partial k_H} = \left( \frac{1}{1+p_X} \right)' \hat{z}'(z_D) + \left( \frac{p_X}{1+p_X} \right)' \hat{z}_X^*(z_D) dp_X^* + \frac{p_X}{1+p_X} \hat{z}_X^*(z_D) dz_D + \frac{1}{1+p_X} \hat{z}^\theta_*(z_D^* dz_D^* < 0
\]

(50)

Compare to home each adjustment goes now in the opposite direction. The fraction of foreign export firms is lower, \(dp_X^* < 0\), because competition in the home market is tougher. The stronger competition implies also a higher average productivity of foreign exporters. In addition, average productivity of all foreign firms is lower due to less competition in their domestic market, \(dz_D^* < 0\). The overall effect on the productivity of foreign firms is negative no matter whether the export productivity is measured at the firm gate or at the consumers. Again, the proof is neglected to the Appendix.

What does this imply for leapfrogging? If foreign has a higher disadvantage, the industry productivity is larger in home and at the same time lower in foreign. Hence, based on the case of \(k_H = 1\) at which foreign always leapfrogs home, it can be said that if the foreign disadvantage \(k_H\) is not too large, foreign still leapfrogs home. However, if the disadvantage is rather large, foreign does not overtake home even if foreign firms invest in a high level of \(R&D\) and home firms not. The selection effect that induces unproductive firms to exit is in such a case too less in the foreign market compared to the home market. As a result, the presence of a higher amount of well productive foreign firms compared to well productive home firms cannot compensate the weaker selection effect.

Changes in the investment benefits of a high level of \(R&D\) influences the home sector productivity according to

\[
\frac{\partial a}{\partial \theta_H} = \left( \frac{1}{1+p_X} \right)' \hat{z}(z_D) + \left( \frac{p_X}{1+p_X} \right)' \hat{z}_X(z_D) dp_X + \frac{p_X}{1+p_X} \hat{z}(z_D) dz_D + \frac{p_X}{1+p_X} \hat{z}_X(z_D^*) dz_D^* > 0.
\]

The effects are similar to the previous analysis of equation (49). It is shown in the Appendix that in the case of lower benefits of a high level of \(R&D\) investments (larger \(\theta_H\)), the competition intensity is stiffer in home and less stiff in foreign. Hence, the qualitative impacts are the same as in the case of a larger disadvantage to invest in a high level of \(R&D\). As a result, home sector productivity is higher in the case that \(R&D\) benefits of investments in a high level of \(R&D\) are lower.
How the foreign sector productivity is impacted can be seen from

\[ \frac{\partial a^*_{\theta_H}}{\partial \theta} = \left( \left( \frac{1}{1 + p_X} \right) \hat{a}^*(z_D) + \left( \frac{p'_X}{1 + p_X} \right) \hat{a}'^*(z_D) \right) dp^*_X + \frac{1}{1 + p_X} \hat{a}_X^*(z_D)dz_D + \frac{p'_X}{1 + p_X} \hat{a}_X^*(z_D)dz_D + ... \]

(51)

Competition effect, same as above: \( < 0 \)

\[ ... + \left( A^* a^* + p'_X A^* \frac{z_D^{\sigma - 1} - z^\sigma_D^{\sigma - 1}}{(1 + p_X^*)^2} \right) d\theta^*. \]

Distribution effect, same as equation (45): \( < 0 \)

A lower benefit leads to a less fierce competition in the foreign market and at the same time to a more fierce competition in the home market. The qualitative effects through the competition channel are the same as in the case of a larger foreign disadvantage, \( k_H \). In total it affects the foreign productivity negatively. In addition, there is a second effect. A higher \( \theta_H \) implies that investments in a high level of R&D results in lower expected productivity draws of entrants (i.e. entrants draw from a Pareto-distribution with a lower tail). As a result, a lower number of well productive foreign firms exists in equilibrium. This affects the foreign sector productivity negatively. Hence, the overall effect on the foreign sector productivity is negative. Thus, it is less likely that foreign leapfrogs home as a consequence of trade liberalization if the benefits of a high investment in R&D are lower.

What happens when foreign has a larger disadvantage of investments in a low level of R&D, \( k_L \)? The home cut-off productivity is larger and the foreign cut-off productivity lower (see (17) and (18)). By using this result in the foreign R&D choice function (27), it implies a higher value of the second argument. The first argument remains at the same time unaffected at zero. Hence, foreign entrants will invest at a lower level of trade openness in a high level of R&D. As soon as foreign entrants will invest in a high level of R&D the size of the disadvantage \( k_L \) becomes meaningless.\(^{21}\) However, if the question is on the sector productivity of each country at a particular level of trade openness, it can be interesting to consider different values of \( k_L \). Foreign firms may have invested at a particular level of trade openness in a high level of R&D in the case of a large disadvantage and not in the case of a low disadvantage. Hence, the foreign sector productivity can be very different at a same level of trade openness as long as trade is not liberalized to a level at which for any considered disadvantage \( k_L \) foreign entrants invest in a high level of R&D.

Taken together, leapfrogging of foreign is more likely the lower is the disadvantage to

\(^{21}\)The foreign disadvantage in low levels of R&D, \( k_L \), is nevertheless one key variable to ensure that foreign entrants find it at a lower level of trade openness optimal to invest in a high level of R&D than home firms. For more details see equation (29) and the subsequent paragraph.
invest in a high level of R&D and the higher is the benefit of such an investment, i.e. the lower is $k_H$ and the larger is the difference between $\theta_L$ and $\theta_H$. In contrast, differences in the disadvantage of investments in a low level of R&D, $k_L$, have no impact on the aggregate level as soon as foreign entrants find it optimal to invest in a high level of R&D.

3 Discussion of Related Research

In this section the results are embedded into the related literature. The aim is to highlight several essential findings. On this basis an attempt is made to give a conclusion on the impact of trade liberalization on sector productivity when countries differ in their technology potential.

First of all, how does trade liberalization affects the sector productivity in heterogeneous firms models, in which firms cannot conduct productivity enhancing investments? The general case is provided by Demidova (2008). In Demidova (2008) firms from two countries draw their productivity from not closer specified distributions that are ordered in terms of the hazard rate stochastic dominance (HRSD) criterion. The competition is stronger in the country, in which firms draw from the better distribution. As a result, the selection of unproductive firms is stronger in this country. In addition, more well productive firms exist in this country. Both effects together imply for all degrees of trade openness that the country with the exogenously given better distribution has also a larger sector productivity compared to the other country. The result of Demidova (2008) holds for any possible firm productivity distribution. This leads to an important conclusion: If endogenous adjustments to trade liberalization is only caused through a selection of the least productive firms, the country with exogenous given better technology potential will never lose its leading position in sector productivity.

It implies a change of productivities within firms as the necessary requirement for leapfrogging. This is in accordance with arguments on how to keep productivity growth in lagging countries on a sustainably high level, so that it can lead to catch-up.

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22 HRSD implies first order stochastic dominance, but the reverse is not always true (Demidova, 2005). Hence, all results derived by Demidova (2008) are also valid in models where entrants from one country draw from a distribution that first order stochastic dominates the distribution from another country. This is the case in models where distributions of Pareto kind between two countries only differ exogenously according to the minimum possible productivity draw of, e.g. Bernard et al. (2011); Falvey et al. (2011); Okubo (2009); Melitz and Ottaviano (2008).

23 Demidova (2008) does not derive this result explicitly, because she focuses on welfare. However, it can be easily shown by combining general proofs of Melitz (2003) on the impact of trade liberalization on sector productivity with the results of Demidova (2008).

24 It could be argued that an asymmetric change of the technology potential caused by public R&D investments can also lead to leapfrogging. However, this would be the same as in the model of Demidova (2008) if the technology potential of the disadvantaged country would exogenously increased to a level above the technology potential of the advantaged country.
up or overtaking (Landes, 1990). In general, it requires the creation and use of new knowledge. The question is in which manner. Unel (2013) proposes to the best of the author’s knowledge the only general equilibrium model which considers within firm productivity adjustment in an asymmetric country setting. He proposes a binary technology choice of firms in a Melitz (2003) model with Pareto distributed firms. Countries are asymmetric with respect to the technology adjustment costs of firms. Firms can adapt to a better technology after resolving uncertainty about their drawn productivity. If a firm switches to the superior technology, it leads to a known reduction of its marginal costs and an increase in its fixed costs. More firms adapt to the better technology in the country with lower adaption costs. As a consequence, the competition intensity is more fierce in this market and leads to a stronger selection of unproductive firms. At the same time, the fraction of firms adjusting their technology is also larger. Hence, the productivity distribution of firms in this country HRSD the distribution in the other country. In contrast to Demidova (2008) this domination is now endogenously caused by different firm behavior with regard to technology upgrading. However, the aggregate result is similar to Demidova (2008): the country with an exogenously given advantage, in this case lower technology upgrading costs, will also be the leading one with respect to sector productivity for every possible degree of trade openness.

What is the major difference between the model of Unel (2013) and the model presented in this paper causing the very different outcomes? It is the assumption about the timing of productivity enhancing measures of firms. In Unel (2013) a firm can upgrade its technology after knowing its productivity. The uncertainty of such measures is rather low. Hence, as argued in Rutzer (2013), such technology adoption can be seen as short/medium term decisions of firms. As a result, in the model of Unel (2013) the difference in the competition intensities are an outcome of the adaption decisions of firms and not the other way around. In contrast, in the model at hand, a firm has to decide how much to invest in innovations before knowing its productivity. The uncertainty of such measures is rather high. Hence, firms’ investments in innovations aiming to develop an entire new production process can be seen as long term decisions (Rutzer, 2013). Furthermore, the difference in the competition intensities between the two countries, which is exogenous from the perspective of a firm, leads to different R&D investment decisions between home and foreign firms (and not the other way around as in Unel (2013)).

This distinction between a short run and a long run perspective suits well with growth models and leapfrogging (Furukawa, 2012; Brezis et al., 1993). Brezis et al. (1993) claim that usually technology improves gradually over time through constantly en-

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25 Heterogeneous firms is a necessary requirement for within firm productivity changes in models of monopolistic competition. If firms are homogenous, trade liberalization does not affect firms' profits. Hence, it is not possible in homogenous firms models with monopolistic competition to consider productivity enhancing investments of firms in response to trade liberalization (Neary, 2004).
hancing already known methods. This improvements are usually larger in the country with an established advantage in a technology. However, once in a while a complete new superior technology becomes available for whatever reasons. The initially laggard country may adapt to the superior technology and the leading country not. As a result, overtaking can take place in the long run. Yet, these models do not consider trade liberalization. However, the results of the recent model suggests, the degree of trade openness plays an important role whether firms will find it profitable at all to invest in a new and better technology, since access to international markets matters for the profitability of innovations.

![Diagram](image)

**Figure 2:** Productivity leapfrogging in a schematic representation based on results of Unel (2013), Brezis et al. (1993) and the model of this paper.

By combining the main results of the previous mentioned models, the following general picture on productivity leapfrogging between two countries can be drawn. It is summarized in figure 2. Initially, firms in one country are more experienced than firms from another country in the available technology. This country will also be the leading one in terms of sector productivity. Now trade becomes liberalized. Shortly after trade liberalization, a couple of firms in each country upgrade their production process. The fraction of upgrading firms is larger in the leading country than in the laggard country. As a result, the competition intensity increases more in the leading country, which leads to a stronger selection of unproductive firms. Hence, in the short run the sector productivity between the leading country and the laggard country will diverge even more. Now, a major technology shock happens as in Brezis et al. (1993). By investing more in R&D it allows firms now to conduct entire new innovations based on this new technology. What happens? Following Brezis et al. (1993) and the paper at hand, foreign firms will invest in this new R&D technology and home firms not, because the expected revenue to cost ratio of such innovations is only in the laggard

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26 This conclusion should be taken with caution, because it is a merge of results from different models. If all assumptions would be integrated in a single framework, it could lead to trade-offs not considered within this non-mathematical argumentation.
country higher than the ratio of the old R&D technology. One reason can be that firms in the leading country are more experienced in the old production processes than firms in the laggard country. As a result, these firms face a stronger competition and higher wages.\textsuperscript{27} Hence, to invent complete new production processes is less profitable in the leading country. In the long run, firms in the laggard country will change to new production processes, whereby firms from the leading country will still produce in the old fashioned way. Finally, this lack of innovations in the leading country and at the same time realized innovations in the laggard country can lead to leapfrogging. However, if trade would have not been liberalized in the first place, in both countries the expected revenue to cost ratio for the superior R&D technology would be lower than of the inferior R&D. Contrary to Brezis et al. (1993), no firm would invest in the new technology. This leads to a very important result: The necessary condition for leapfrogging are open markets. This fit in with oligopoly trade models and quality leapfrogging (Moraga-Gonzalez and Viaene, 2005; Motta et al., 1997). As pointed out in the introduction, these models, however, differ strongly in the scope and the focus from the general equilibrium models discussed here.

3.1 Conclusion

This paper presents one idea how trade liberalization can change the ranking of countries with respect to sector productivity by considering innovation investments of firms. It turns out that due to a tougher competition, firms in the previous to trade liberalization laggard country will under certain conditions invest in the superior R&D technology as trade becomes liberalized. At the same time, firms from the initially leading country may still find it optimal to invest in the inferior R&D technology. The necessary condition is a lower disadvantage to invest in the superior R&D technology compared to investments in the initially used inferior R&D technology. Furthermore, the larger the investment disadvantage for the inferior technology or the lower the disadvantage for the superior R&D technology, the more likely is this investment pattern. As a consequence, trade liberalization influences the aggregate level in both countries through a reallocation of resources towards more productive firms and at the same time by an upward rotation of the laggard country’s productivity distribution. This effect is caused by the higher R&D investments, which leads to more innovations and therefore to a larger number of productive firms. The reallocation effect is always stronger in the previous to trade liberalization leading country. Nevertheless, the previous to trade liberalization laggard country can become the economic leader after trade liberalization.

\textsuperscript{27}Other arguments proposed by Economic historians are the willingness of laggard countries to catch up combined with weariness in leading countries (Landes, 1999, 1990). Additionally, they argue that leading countries may have accumulated institutional rigidities, which hinder them to adapt to major technological innovations.
tion, because the increased innovations can more than outweigh the weaker reallocation effect. This is more likely, the lower is the disadvantage of the laggard country to conduct superior R&D and the higher is the benefits of it.

The main message of this paper is, in contrary to other asymmetric country models with heterogeneous firms, a country can loose in the long run its initial productivity advantage in the presence of trade liberalization. Even more important, the initial better position of a country is the reason why it may loose its leading position as trade becomes liberalized, since it causes a (relative) lack of innovations.

By combining these results with the growth and leapfrogging literature as well as the literature on heterogeneous firms and trade liberalization, the following general conclusions have been made: Open markets are the necessary requirement for leapfrogging. Furthermore, during trade liberalization periods, the initial productivity gap may at first increase due to asymmetric technology upgrading. However, in a long run perspective, firms in the laggard country may focus more on developing and using new technologies than firms from the leading country. This can lead to leapfrogging.

Interesting areas for future research can be to analyze other kinds of country asymmetries, e.g. difference in the preferences of consumers. Or study whether incentives to pursue industrial policies by governments exist, for example by subsidizing innovations. This could help to shed some additional light on what kind of factors are crucial, and which role government interventions play, to explain leapfrogging of countries as a result of trade liberalization.
.1 Existence of an equilibrium

Figure 3: Home and foreign free entry conditions in dependency of the cut-off productivities.

An equilibrium exists if the home and foreign free entry conditions solved after $z_D$ intersect at least one time between the the lowest possible foreign cut-off, $z_{D}^* = \phi z_D$, and largest possible foreign cut-off, $z_{D}^* = z_D$. It follows from equation (13),

$$z_{D}^{\text{home}} = \left( \frac{F_i}{B_i} - \phi \theta_i (z_{D}^{\text{home}})^{-\theta_i} \right)^{-\frac{1}{\theta_i}}$$ (52)

and from (14),

$$z_{D}^{\text{foreign}} = (\phi^{-\theta_j} (\frac{k_j F_j}{B_j} - (z_{D}^{\text{foreign}})^{-\theta_j}))^{-\frac{1}{\theta_j}}.$$ (53)

First, at $(z_{D}^*)_{\text{min}} = \phi z_D$, the home free entry condition (13) is:

$$\frac{F_i}{B_i} = z_D^{-\theta_i} + \phi \theta_i z_D^{-\theta_i} \phi^{-\theta_i} = 2z_D^{-\theta_i}$$

$$\rightarrow z_{D}^{\text{home}} = \left( \frac{2B_i}{F_i} \right)^{\frac{1}{\theta_i}}$$ (54)
and the foreign free entry condition (14) is

\[
\frac{k_j F_j}{B_j} = z_D^{-\theta_j} + \phi_j z_D^{-\theta_j} = (\phi^{-\theta_j} + \phi^\theta_j) z_D^{-\theta_j}
\]

(55)

\[
\rightarrow z^\text{foreign}_D = \left( \frac{B_j (\phi^{-\theta_j} + \phi^\theta_j)}{k_j F_j} \right)^{\frac{1}{\pi_j}}
\]

(56)

Hence, \( z^\text{foreign}_D > z^\text{home}_D \) if

\[
\left( \frac{B_j (\phi^{-\theta_j} + \phi^\theta_j)}{k_j F_j} \right)^{\frac{1}{\pi_j}} > \left( \frac{2B_i}{F_i} \right)^{\frac{1}{\pi_i}}
\]

(57)

Second, at \( z^*_D = z_D \) the home free entry condition (13) is

\[
\frac{F_i}{B_i} = z_D^{-\theta_i} + \phi^\theta_i z_D^{-\theta_i} \rightarrow (1 + \phi^\theta_i) = z_D^{-\theta_i}
\]

\[
\rightarrow z^\text{home}_D = \left( \frac{B_i (1 + \phi^\theta_i)}{F_i} \right)^{\frac{1}{\pi_i}}
\]

The foreign free entry condition (14) is

\[
\frac{k_j F_j}{B_j} = z_D^{-\theta_j} + \phi_j z_D^{-\theta_j} \rightarrow (1 + \phi^\theta_j) = z_D^{-\theta_j}
\]

\[
\rightarrow z^\text{foreign}_D = \left( \frac{B_j (1 + \phi^\theta_j)}{k_j F_j} \right)^{\frac{1}{\pi_j}}
\]

(58)

Hence, \( z^\text{foreign}_D < z^\text{home}_D \) if

\[
\left( \frac{B_i (1 + \phi^\theta_i)}{F_i} \right)^{\frac{1}{\pi_i}} > \left( \frac{B_j (1 + \phi^\theta_j)}{k_j F_j} \right)^{\frac{1}{\pi_j}}
\]

An equilibrium exists if the parameter values fulfill the requirements (57) and (58).

This conditions are the same as (15) and (16), which are the requirements for no specialization takes place.

.2 Uniqueness of the equilibrium

The equilibrium is unique, if the equation system (13) and (14) has only one solution.

This is similar to show that over the complete possible range of \( z_D \) and \( z^*_D \), the relative change of one equation is larger than of the other equation. The total differentials of
This equation system are
\[-\theta_i z_D^{-\theta_i-1} dz_D - \theta_i \phi^{\theta_i} z_D^{s-\theta_i-1} dz_D^* = 0, \tag{59}\]
\[-\theta_j z_D^{s-\theta_j-1} dz_D^* - \theta_j \phi^{\theta_j} z_D^{s-\theta_j-1} dz_D = 0. \tag{60}\]

This leads to
\[\frac{dz_D}{dz_D^*} = -\phi^{\theta_i} Z_{\theta_i}^{\theta_i+1}, \tag{61}\]
\[\frac{dz_D^*}{dz_D} = -\phi^{-\theta_j} Z_{\theta_j}^{\theta_j+1}. \tag{62}\]

There exists at most one intersection point if either (61)<(62) or (61)> (62) for all possible values of $z_D$ and $z_D^*$. It turns out that
\[\phi^{-\theta_j} Z_{\theta_j}^{\theta_j+1} > \phi^{\theta_i} Z_{\theta_i}^{\theta_i+1}, \tag{63}\]
because by using the minimal value of $Z$, $Z^\min = 1$, and the maximal value of $Z$, $Z^\max = \phi^{-1}$, in this inequality it follows:
\[\phi^{-\theta_j} > \phi^{\theta_i}, \quad \phi^{-\theta_j+\theta_i} > \phi^{\theta_i+\theta_j}, \tag{64}\]
due to $0 \leq \phi < 1$. Furthermore, the inequality (63) changes with an increase in $Z$ according to
\[(\theta_j + 1)\phi^{-\theta_j} Z_{\theta_j}^{\theta_j+1} > (\theta_i + 1)\phi^{\theta_i} Z_{\theta_i}^{\theta_i+1}. \tag{65}\]
The left-hand side is larger, if $\theta_i < \theta_j$, because $\frac{\theta_j + 1}{\theta_i + 1}\phi^{\theta_j} - \theta_i > 1 > Z_{\theta_i}^{\theta_i} - \theta_j$. If $\theta_i > \theta_j$, the left-hand side or the right-hand side can be larger, depending on the value of $Z$. However, there exists maximal one reversing over the entire range of $Z$, because $\frac{\theta_j + 1}{\theta_i + 1}\phi^{\theta_j} - \theta_i < Z_{\theta_i}^{\theta_i} - \theta_j$: The right-hand side increases monotonically in $Z$ and the left-hand side remains constant. This means, if at the minimal and the maximal value of $Z$, the inequality (63) is fulfilled, it has to be fulfilled at any other possible value of $Z$, too. This is the case, as equation (64) shows. Hence, (61)> (62) \[\square\] This proofs, if there exists an equilibrium, it is the only equilibrium.

.3 Proof of $z_D > z_D^*$ if $k_L; k_H > 1$ and $z_D \geq z_D^*$ if $k_H = 1$

As long as entrants in both countries find it optimal to choose the same level of $R&D$, the proof is simply equation (32).\[28\]

The proof is more complicated if entrants in foreign

\[28\] In the case of high investments in both countries, equation (32) would be compound of $k_H$ instead of $k_L$ and $\theta_H$ instead of $\theta_L$.
find it optimal to invest in a high level of R&D. This is the case as long as

\[ V_H^* \geq 0 : B_H(z_D^{s-\theta_H} + \phi^{\theta_H}z_D^{-\theta_H}) - k_H F_H \geq 0, \]  
\[ V_H < 0 : B_H(z_D^{-\theta_H} + \phi^{\theta_H}z_D^{s-\theta_H}) - F_H < 0. \]

(66)

(67)

Now assume that \( z_D = z_D^* \). If \( V_H^* \geq 0 \) → \( B_H(z_D^{-\theta_H} + \phi^{\theta_H}z_D^{-\theta_H}) - k_H F_H \geq 0 \) implies that \( B_H(z_D^{-\theta_H} + \phi^{\theta_H}z_D^{-\theta_H}) - F_H > 0 \). Hence, \( V_H < 0 \) and \( k_H \geq 1 \) cannot be if \( z_D = z_D^* \). What is if \( z_D^* > z_D \)? Each entrant value to invest in a high level of R&D changes as \( z_D \) and \( z_D^* \) changes according to

\[ dV_H = B_H(-\theta_H z_D^{-\theta_H-1}dz_D - \theta_H \phi^{\theta_H} z_D^{-\theta_H-1}dz_D^*), \]  
\[ dV_H^* = B_H(-\theta_H z_D^{s-\theta_H-1}dz_D^* - \theta_H \phi^{\theta_H} z_D^{-\theta_H-1}dz_D). \]

(68)

(69)

Assume initially \( z_D = z_D^* \). In this case to have \( z_D^* > z_D \), it is needed that \( dz_D^* > dz_D \). Using \( z_D = z_D^* \) in (68) and (69) leads to

\[ B_H(-\theta_H z_D^{-\theta_H-1}dz_D - \theta_H \phi^{\theta_H} z_D^{-\theta_H-1}dz_D^*) > B_H(-\theta_H z_D^{s-\theta_H-1}dz_D^* - \theta_H \phi^{\theta_H} z_D^{-\theta_H-1}dz_D), \]

as long as \( dz_D^* > dz_D \). Hence, \( z_D^* > z_D \) implies \( V_H > V_H^* \). Therefore, the above conditions (66) and (67) cannot be fulfilled. This means it is not possible that foreign entrants invest in a high level of R&D and home entrants not and at the same time \( z_D^* > z_D \). In addition, (32) implies if entrants in both countries invest in the same level of R&D, the cut-offs are either similar, \( z_D = z_D^* \), if \( k_L, k_H = 1 \), or the home cut-off is larger, \( z_D > z_D^* \), if \( k_L, k_H > 1 \).

.4 Change of the R&D choice functions due to trade liberalization

As long as firms in both countries invest in the same level of R&D it is possible to solve explicit for the competition intensities in home and foreign. In this case the equation
(26) can be written by using (17) and (18) in (25) as

\[
\frac{\partial V_H(z^L_H, z^L_D, \phi)}{\partial \phi} = \theta_H B_H \left( \frac{F_L(1-k_L\phi^L)}{B_L(1-\phi^{2^L})} \right) \frac{\theta^L_H\phi^L}{\phi^L} + \frac{F_L\phi^{L-1}(2\phi^{\theta_L} - k_L - k_L\phi^{2^L})}{B_L(1-\phi^{2^L})^2} + \cdots
\]

\[
\Delta \text{ Home domestic market}
\]

\[
\frac{\partial V_b(z^L_H, z^L_D, \phi)}{\partial \phi} = \theta_H B_H \left( \frac{F_L(1-k_L\phi^L)}{B_L(1-\phi^{2^L})} \right) \frac{\theta^L_H\phi^L}{\phi^L} + \frac{F_L\phi^{L-1}(-2\phi^{\theta_L} + k_L + k_L\phi^{2^L})}{B_L(1-\phi^{2^L})^2} > 0
\]

\[
\Delta \text{ Home export market}
\]

The positive inequality results due to several facts. First of all note that \((b) = -(d)\). Furthermore, term \((b)\) is for all possible \(\phi\) negative and term \((d)\) positive as long as \(k_L > 1\).\(^\text{29}\) In addition, because of \(\theta_L > \theta_H\) it follows that \((c) > (a)\). Therewith the requirement that less productive home firms sell only in their domestic market, formally \(z_X > z_D\), is also fulfilled. This requirement bounds the maximal foreign disadvantage to \(k_L < \frac{\phi^{\theta_L} + \phi^{2^L}}{2}\), as it can be seen by setting term \((a)\) equal to zero. Otherwise the home cut-off \(z_D\) goes to \(\infty\) for highly liberalized trade and a simultaneous fulfill of both free entry conditions, home and foreign, is no longer possible.

The same is possible for the equation (28). It can be written by using (17) and (18) in (27) as

\[
\frac{\partial V_b^*(z^L_H, z^L_D, \phi)}{\partial \phi} = \theta_H B_H \left( \frac{F_L(k_L - \phi^{\theta_L})}{B_L(1-\phi^{2^L})} \right) \frac{\theta^L_H\phi^L}{\phi^L} + \frac{F_L\phi^{L-1}(2k_L\phi^{\theta_L} - \phi^{2^L} - 1)}{B_L(1-\phi^{2^L})^2} + \cdots
\]

\[
\Delta \text{ Foreign domestic market}
\]

\[
\frac{\partial V_b(z^L_H, z^L_D, \phi)}{\partial \phi} = \theta_H B_H \left( \frac{F_L(1-k_L\phi^L)}{B_L(1-\phi^{2^L})} \right) \frac{\theta^L_H\phi^L}{\phi^L} + \frac{F_L\phi^{L-1}(1 + \phi^{2^L} - 2k_L\phi^{\theta_L})}{B_L(1-\phi^{2^L})^2} \geq 0
\]

\[
\Delta \text{ Foreign export market}
\]

As before, \((c') > (a')\) due to the fact that \(z_X^* > z_D^*\). In addition, the relationship between the two other terms is \((e) = -(f)\). The term \((f)\) is negative for any possible degree of trade openness. Proof: the maximal value of \(k_L^{\max}\) is determined by the maximal relative selection effect (32) as \(k_L^{\max} = \frac{\phi^{\theta_L} + \phi^{2^L}}{2}\). Use this in term \((e)\) leads to

\[
\frac{F_L\phi^{L-1}(\phi^{\theta_L} + \phi^{2^L} - \phi^{2^L} - 1)}{B_L(1-\phi^{2^L})^2} = \frac{F_L\phi^{L-1}(1 + \phi^{2^L} - \phi^{2^L} - 1)}{B_L(1-\phi^{2^L})^2} = 0 \quad \square
\]

Hence, the positive

\(^\text{29}\) At autarky \(\phi = 0\) the term in brackets in the nominator of \((b)\) is \(k_L > 1\). The term becomes larger when trade gets liberalized: \(\frac{\partial}{\partial \phi} = 2\theta_L k_L \phi^{2^L - 1} - 2\theta_L \phi^{\theta_L - 1} > 0\). Hence, it is positive for all \(0 \leq \phi \leq 1\).
term in the second line is larger or equal to the negative term in the first line.

The following paragraph proofs that

\[
\frac{\partial V_L^*(\theta_H, \theta_L)}{\partial \phi} < 0,
\]
\[
\frac{\partial V^*_L(\theta_H, \theta_H)}{\partial \phi} < 0,
\]
\[
\frac{\partial V_L(\theta_L, \theta_H)}{\partial \phi} < 0,
\]
\[
\frac{\partial V_H(\theta_L, \theta_H)}{\partial \phi} > 0,
\]
\[
\frac{\partial V_H(\theta_H, \theta_H)}{\partial \phi} > 0.
\]

It is not possible to derive an explicit solution for \(z_D^*\) and \(z_D\), as soon as the investment decision differs between both countries. Hence, the proofs are done by analyzing each total differential. The free entry conditions in case that foreign firms choose a high level of \(R&D\) and home firms a low level of \(R&D\) are

\[
F_L = B_L[z_D^{-\theta_L} + \phi^{\theta_L} z_D^{-\theta_L}],
\]
\[
k_H F_H = B_H[z_H^{-\theta_H} + \phi^{\theta_H} z_H^{-\theta_H}].
\]

How a change in trade openness affects the cut-offs \(z_D\) and \(z_D^*\) can be analyzed by total differentiate each free entry condition:

\[
-\theta_L z_D^{-\theta_L - 1} dz_D - \theta_L \phi^{\theta_L} z_D^{-\theta_L - 1} dz_D^* + \theta_L \phi^{\theta_L - 1} z_D^{-\theta_L} d\phi = 0,
\]
\[
-\theta_H z_D^{-\theta_H - 1} dz_D^* - \theta_H \phi^{\theta_H} z_D^{-\theta_H - 1} dz_D + \theta_H \phi^{\theta_H - 1} z_D^{-\theta_H} d\phi = 0.
\]

By solving (72) after \(\phi\) and putting this into (72), it follows:

\[
dz_D = \frac{Z^\theta_H \phi^{-\theta_H} - 1}{Z^{-1}(Z^{-\theta_L} \phi^{-\theta_L} - 1)} dz_D^*
\]

with \(Z = z_D/z_D^*\). Or in case that home firms and foreign firms have invested in a high level of \(R&D\):

\[
dz_D = \frac{Z^\theta_H \phi^{-\theta_H} - 1}{Z^{-1}(Z^{-\theta_H} \phi^{-\theta_H} - 1)} dz_D^*
\]

In addition, using (72) the differential \(d\phi\) is

\[
d\phi = \phi z_D^{-1} dz_D + z_D^{\theta_H} z_D^{-\theta_H - 1} \phi^{-\theta_H + 1} dz_D^*.
\]
The R&D choice function of home rms in case that home rms have chosen a low level of R&D and foreign rms a high level of R&D is:

\[ V_H(\theta_L, \theta_H) = B_H(z_D^{-\theta_H} + \phi^{\theta_H} z_D^{-\theta_H}) - F_H. \]

It changes by a change of \( \phi \) according to

\[ dV_H = -z_D^{-\theta_H-1}dz_D - z_D^{\theta_H-1} \phi^{\theta_H}dz_D + \phi^{\theta_H-1} z_D^{-\theta_H}d\phi. \]

By using (74) and (75) this derivative can be written as

\[ dV_H = (\phi^{\theta_H} - Z^{-\theta_H})(\frac{Z^\theta_H - \phi^{\theta_H}}{\phi^{\theta_H}}) + (Z^{\theta_H} - \phi^{\theta_H})(Z^{-\theta_L - \theta_H} - 1)dz_D. \tag{76} \]

The value of this derivative declines in \( Z \): \( \frac{\partial dV_H}{\partial Z} = \theta_H Z^{-\theta_H-1} - \theta_L Z^{-\theta_H-\theta_H} < 0 \). In addition, a higher \( Z \) let the value of this derivative increase, but at the highest possible \( Z^{\text{max}} = \phi^{-1} \) its value is negative: \( \frac{\partial dV_H}{\partial Z} = \theta_H^{\theta_H} - \phi^{\theta_H} < 0 \). Hence, the change of the derivative (76) is negative for all possible \( Z \). From this result it follows that \( dV_H \) (76) has its largest value at \( Z = 1 \) and its lowest at \( Z^{\text{max}} = \phi^{-1} \). Evaluate the derivative at \( Z^{\text{max}} \) leads to

\[ dV_H \rightarrow_{Z=\phi^{-1}} = -\phi^{\theta_H} + \phi^{\theta_H} + \phi^{-\theta_H+\theta_L} \phi^{\theta_L} - \phi^{\theta_H-\theta_H}, \]

\[ \rightarrow dV_H \rightarrow_{Z=\phi^{-1}} = \phi^{-\theta_H} - \phi^{\theta_H} > 0. \]

Hence, the home R&D choice function to invest in a high level of R&D in the case that foreign firms have invested in a high level or R&D increases with trade liberalization:

\[ \frac{\partial V_H(\theta_L, \theta_H)}{\partial \phi} > 0. \]

The R&D choice function of foreign firms to invest in a low level of R&D given that all other foreign firms have invested in a high level of R&D and home firms in a low level of R&D is

\[ V_L^*(\theta_L, \theta_H) = B_L(z_D^{\theta_L} + \phi^{\theta_L} z_D^{\theta_L}) - k_L F_L. \]

It changes with a change of \( \phi \) according to

\[ dV_L^* = -z_D^{\theta_L-1}dz_D - z_D^{\theta_L-1} \phi^{\theta_L}dz_D + \phi^{\theta_L-1} z_D^{-\theta_L}d\phi. \]
By using (75), this differential can be written as

\[ dV_L^* = -z_D^{s-\theta_L-1}dz_D^* - z_D^{s-\theta_L-1}\phi_L dz_D + \phi_L^{-1}z_D^{s-\theta_L}dz_D + z_D^{s-\theta_L-1}\phi_L^{-\theta_H+\theta_L}dz_D^*. \]

Both terms of \( dz_D \) cancel each other out and the differential reduces to

\[ dV_L^* = (-z_D^{s-\theta_L-1} + \left(\frac{z_D}{z_D^{s-\theta_L}}\right)^{\theta_H} z_D^{s-\theta_L-1}\phi_L z_D^{s-\theta_H-1}\phi_L^{-\theta_H+\theta_L})dz_D^*. \]

The term in front of the differential \( dz_D^* \) is negative, because

\[ 1 > \left(\frac{z_D}{z_D^{s-\theta_L}}\right)^{\theta_H} \phi_L^{-\theta_H+\theta_L}. \]

The overall sign of \( dV_L^* \) depends on the sign of \( dz_D^* \). In the case that home firms have chosen a high level of \( R&\)D investment it follows by using (73) in (75)

\[ d\phi = (\phi z_D^{-1} Z^{\theta_H} \phi_{-\theta_H} - 1) + z_D^{\theta_H} z_D^{s-\theta_H-1}\phi_L^{-\theta_H+1})dz_D^*. \]

and in the case of a low level of \( R&\)D investments, it follows by using (74) in (75)

\[ d\phi = (\phi z_D^{-1} Z^{\theta_H} \phi_{-\theta_H} - 1) + z_D^{\theta_H} z_D^{s-\theta_H-1}\phi_L^{-\theta_H+1})dz_D^*. \]

In both cases the fraction term is positive. Hence, if \( d\phi > 0 \rightarrow dz_D^* > 0 \). As a result it follows that \( \frac{\partial V_L^*}{\partial \phi} < 0 \): the incentive of foreign firms to invest in a low level of \( R&\)D decreases, as trade becomes more liberalized.

The incentive of home firms to invest in a low level of \( R&\)D in the case that foreign firms as well as home firms have invested in a high level of \( R&\)D changes with trade liberalization according to

\[ dV_L(\theta_H, \theta_H) = -z_D^{s-\theta_L-1}dz_D - z_D^{s-\theta_L-1}\phi_L dz_D + \phi_L^{-1}z_D^{s-\theta_L}d\phi. \]

By using (73) and (74), it follows

\[ dV_L(\theta_H, \theta_H) = ((Z^{\theta_L} \phi_{\theta_L} - 1)(Z^{\theta_H} \phi_{-\theta_H} - 1) + ... \]

\[ + \phi_L^{-1} Z^{\theta_L} (Z^{\theta_H} \phi_{-\theta_H} - 1)(Z^{-\theta_H} \phi_{-\theta_H} - 1))dz_D^*. \]  

To analyze the sign of this derivative it is enough to focus on the term \( m = ((Z^{\theta_L} \phi_{\theta_L} - 1) + \phi_L^{-1} Z^{\theta_L} (Z^{\theta_H} \phi_{-\theta_H} - 1))dz_D^*, \) because \( Z^{\theta_H} \phi_{-\theta_H} - 1 > 0 \). This term increases in \( Z \):

\[ \frac{\partial m}{\partial z} = \theta_L \phi_L Z^{\theta_L} + \phi_L^{-\theta_H} (\theta_L - \theta_H) Z^{\theta_H} - \theta_L \phi_L Z^{\theta_L} > 0, \]
because \( \theta_L > \theta_H \). Hence, it has its maximal value at \( Z^{\text{max}} \). Furthermore, at \( Z^{\text{max}} \) the value of the derivative (77) is \( dV_L(\theta_H, \theta_H) = 0 \). This can be easily seen by replacing \( Z \) through \( \phi^{-1} \) in (77). Together with (78), it follows
\[
\frac{\partial V_L(\theta_H, \theta_H)}{\partial \phi} < 0. \tag{79}
\]

**Proof of a larger \( k_H \) leads to a stronger relative selection effect \( Z \)**

How a change in \( k_H \) affects the cut-offs \( z_D \) and \( z_D^* \) can be analyzed by total differentiate each free entry condition (71):
\[
-\theta_L z_D^{-\theta_L-1} dz_D - \theta_L \phi^{\theta_L} z_D^{\theta_L-1} dz_D^* = 0, \tag{80}
\]
\[
-z_D^{\phi^\theta_H-1} dz_D - \phi^{\theta_H} z_D^{\phi^\theta_H-1} dz_D = \frac{F_H}{\theta_H B_H} dk_H. \tag{81}
\]

Use (80) in (81) leads to
\[
(1 - \phi^{\theta_H + \theta_L} Z^{\theta_L - \theta_H}) dz_D = \frac{\phi^{\theta_L} F_H}{B_H \theta_H} z_D^{\phi^\theta_H+1} Z^{\phi^\theta_H+1} dk_H. \tag{82}
\]

The right-hand side is larger than zero, as long as \( dk_H > 0 \). Hence, the left-hand side has also to be larger than zero. This is the case as long as \( dz_D > 0 \), because \( 0 < Z^{\theta_L - \theta_H} \leq 1 \) and \( 0 \leq \phi^{\theta_H + \theta_L} \leq 1 \to (1 - \phi^{\theta_H + \theta_L} Z^{\theta_L - \theta_H}) \geq 0 \). Hence, if \( dk_H > 0 \to dz_D > 0 \). By using this result in equation (80), it follows \( dz_D^* < 0 \). Hence, the relative selection effect \( Z = \frac{z_D}{z_D^*} \) increases in \( k_H \): \( \Delta k_H > 0 \to \Delta Z > 0 \), because \( dz_D > 0 \) and \( dz_D^* < 0 \).

**How does a larger \( k_H \) influences home and foreign sector productivity**

The home sector productivity can be written as
\[
\tilde{Z} = \frac{1}{1 + p_X} \tilde{z}(z_D) + \frac{p_X}{1 + p_X} \tilde{z}_X(z_D^*). \tag{83}
\]

Its total differential is
\[
d\tilde{Z} = \left( \frac{1}{1 + p_X} \right)' \tilde{z}(z_D) dz_D + \left( \frac{p_X}{1 + p_X} \right)' \tilde{z}_X(z_D^*) dp_X + \frac{1}{1 + p_X} \tilde{z}'(z_D) dz_D + \frac{p_X}{1 + p_X} \tilde{z}_X'(z_D^*) dz_D^*.
\]
By using
\[
\left( \frac{1}{1 + pX} \right)' = - \left( \frac{pX}{1 + pX} \right)' = - \frac{1}{(1 + pX)^2},
\]
\[
dz_D = - \left( \frac{z_D}{z_D^*} \right)^{\theta_{L+1}} \phi^{\theta_L} dz_D^* \text{ see (80)},
\]
\[
\tilde{z}(z_D) = A z_D^{\sigma - 1} \text{ see (41)},
\]
\[
\tilde{z}'(z_D) = (\sigma - 1) A z_D^{\sigma - 2},
\]
\[
\tilde{z}_X(z_D^*) = \tau^{1 - \sigma} A \left( \frac{z_D^*}{\phi} \right)^{\sigma - 1} \text{ see (41)},
\]
\[
\tilde{z}_X'(z_D^*) = \tau^{1 - \sigma} (\sigma - 1) A \left( \frac{z_D^*}{\phi} \right)^{\sigma - 2} \phi^{-1}.
\]

In addition, by using \( p_X = \phi^{\theta_L} \left( \frac{z_D}{z_D^*} \right)^{\theta_L} \) (see (42)), it follows
\[
dp_X = \phi^{\theta_L} \left( \frac{\theta_L z_D^{\theta_{L-1}} z_D^{\sigma \phi L} \sigma_{D - D} - \theta_L z_D^{\sigma \phi L} z_D^{\sigma_{L-1} \phi} z_D^*}{z_D^*} \right) dz_D^*
\]
\[
\to dp_X = \theta_L p_X \left( \frac{z_D^{\sigma_{D-2}} z_D^* - z_D^{\sigma_{L-2}}}{z_D^*} \right) dz_D^*
\]
\[
\to dp_X = \theta_L p_X \left( \frac{\theta_L z_D^{\sigma_{L-1}} - \sigma_{L+1}}{z_D^*} \phi^{\theta_L} \right) dz_D^*
\]
\[
\to dp_X = - \theta_L p_X \frac{1 + p_X}{z_D^*} dz_D^*
\]

Use all in (82) leads to
\[
d\tilde{Z} = - \left( - \frac{1}{(1 + pX)^2} A z_D^{\sigma - 1} + \frac{1}{(1 + pX)^2} A \left( \frac{z_D^*}{\phi} \right)^{\sigma - 1} \theta_L p_X \frac{1 + p_X}{z_D^*} dz_D^* \right)
\]
\[
- \frac{1}{1 + p_X} (\sigma - 1) A z_D^{\sigma - 2} \left( \frac{z_D}{z_D^*} \right)^{\theta_{L+1}} \phi^{\theta_L} dz_D^* + (\sigma - 1) A z_D^{\sigma - 2} \frac{1}{z_D^*} \phi^{\theta_L} dz_D^*.
\]

It can be reduced to
\[
d\tilde{Z} = A (\theta_L - \sigma + 1) p_X \left( \frac{z_D^{\sigma_{D-2}}}{z_D^*} - z_D^{\sigma_{D-2} \phi^{\sigma + 1}} \right) dz_D^*.
\]

We know from (82), \( dz_D^* < 0 \). Hence, \( d\tilde{Z} > 0 \) if the second term is larger than the first term:
\[
\frac{z_D^{\sigma_{D-2}}}{z_D^*} \phi^{\sigma + 1} > \frac{z_D^{\sigma_{D-1}}}{z_D^*}.
\]
The right-hand side increases in \( z_D \). The minimal possible value of \( z_D \) is \( z_D^{\text{min}} = z_D^* \) and the maximal value \( z_D^{\text{max}} = z_D^*\phi^{-1} \). Use these results in (84), its value is at \( z_D^{\text{min}} \)

\[
\frac{\sigma - 2}{\sigma - 1} \phi^{-1} + 1 > \frac{\sigma - 1}{\sigma - 1} \phi^{-1}
\]

and at \( z_D^{\text{max}} \):

\[
\frac{\sigma - 2}{\sigma - 1} \phi^{-1} + 1 = \frac{\sigma - 1}{\sigma - 1} \phi^{-1}.
\]

Hence, (84) is fulfilled for any possible value of \( z_D \) and \( z_D^* \). This leads to the final result:

\[
\frac{\partial \tilde{Z}}{\partial k_H} = \begin{cases} 
> 0, & \text{if } z_D < z_D^{\text{max}}; \\
= 0, & \text{if } z_D = z_D^{\text{max}}. 
\end{cases}
\] (85)

Furthermore, by applying the same steps as above in the case of the home sector productivity to the foreign sector productivity, one can show that it changes with an increase in \( k_H \) according to

\[
d\tilde{Z}^* = \left(-\frac{1}{(1 + p_X^*)^2} A^* z_D^{\sigma - 1} + \frac{1}{(1 + p_X^*)^2} A^* \left(\frac{z_D}{\phi}\right)^{\sigma - 1}\right) dp_X^* - \left(\frac{\sigma - 1}{1 + p_X^*} A^* z_D^{\sigma - 2} - \frac{z_D}{z_D^*} \frac{\theta}{\phi} \frac{(\sigma - 1)}{1 + p_X^*} A^* \left(\frac{z_D}{\phi}\right)^{\sigma - 2} \phi^{-1 + \theta_L + \theta_H} \left(\frac{z_D}{z_D^*}\right)^{\theta_L + 1}\right) dz_D^* < 0.
\] (86)

The foreign sector productivity is lower if the disadvantage \( k_H \) is larger, because both terms are negative. The term in the first line is negative, because \( A^* \left(\frac{z_D}{\phi}\right)^{\sigma - 1} > A^* z_D^{\sigma - 1} \) and \( dp_X^* < 0 \). In addition, the term in the second line is negative, because

\[
\frac{\sigma - 1}{1 + p_X^*} A^* z_D^{\sigma - 2} > \left(\frac{z_D}{z_D^*}\right)^{\theta_H} \frac{(\sigma - 1)}{1 + p_X^*} A^* \left(\frac{z_D}{\phi}\right)^{\sigma - 2} \phi^{-1 + \theta_L + \theta_H} \left(\frac{z_D}{z_D^*}\right)^{\theta_L + 1}
\]

\[
\Rightarrow z_D^{\sigma - 2} > z_D^{\sigma - 2} \phi^{1 + \theta_L + \theta_H - \sigma} \left(\frac{z_D}{z_D^*}\right)^{\theta_L + 1 - \theta_H + \sigma - 2}
\]

\[
\Rightarrow 1 > \phi^{1 + \theta_L + \theta_H - \sigma} \left(\frac{z_D}{z_D^*}\right)_{\theta_L + 1 - \theta_H + \sigma - 2}.
\]

The last inequality results from \( z_D \geq z_D^* \) and \( z_D^{\text{max}} = \phi^{-1} \). Hence, the largest value of the right-hand side can be \( \phi^{1 + \theta_L + \theta_H - \sigma} \phi^{-\theta_L - 1 + \theta_H - \sigma + 2} = \phi^{2 + \theta_H - 2\sigma} < 1 \), because \( \theta_H > \sigma \) and \( 0 \leq \phi < 1 \). Take both results together it follows \( \frac{\partial \tilde{Z}^*}{\partial k_H} < 0 \).

This result holds also with taking the lost in transit caused by the Iceberg costs \( \tau \).
into account as long as the difference between $z_D$ and $z_D^*$ is not too large. In this case the productivity implied in exports of home and of foreign are not too different. The larger the difference between $z_D$ and $z_D^*$, the higher is the home relative to foreign weighted export productivity. Iceberg trade costs reduce the export productivity by a multiplicative constant. The total amount of efficiency loss is therefore larger in home than in foreign, the larger the difference between $z_D$ and $z_D^*$. Hence, if the difference between $z_D$ and $z_D^*$ is already high, a further increase of $k_H$ can lead to a reduction of the difference between $\tilde{Z}$ and $\tilde{Z}^*$.

Proof of a larger $\theta_H$ leads to a fiercer competition in home and a less strong competition in foreign

How a change in $\theta_H$ affects the cut-offs $z_D$ and $z_D^*$ can be analyzed by total differentiate each free entry condition (71):

$$\left( B'_H (z_D^{s-\theta_H} + \phi \theta_H z_D^{s-\theta_H}) - B_H \log(z_D^*)z_D^{s-\theta_H} - \phi \theta_H \log(z_D)z_D^{s-\theta_H} + \phi \theta_H \log(\phi)z_D^{s-\theta_H} \right) d\phi ... - \theta_H z_D^{s-\theta_H-1}dz_D - \theta_H \phi \theta_H z_D^{s-\theta_H-1}dz_D = 0,$$

$$-\theta_L z_D^{s-\theta_L-1}dz_D - \theta_L \phi \theta_L z_D^{s-\theta_L-1}dz_D = 0. \tag{87}$$

It follows from equation (88)

$$dz_D = -\phi \theta_L \left( \frac{z_D^*}{z_D^s} \right)^{\theta_L+1} dz_D^*. \tag{89}$$

Use this in equation (87) leads to

$$\left( B'_H (z_D^{s-\theta_H} + \phi \theta_H z_D^{s-\theta_H}) - B_H \log(z_D^*)z_D^{s-\theta_H} - \phi \theta_H \log(z_D)z_D^{s-\theta_H} + \phi \theta_H \log(\phi)z_D^{s-\theta_H} \right) d\phi ... - \theta_H z_D^{s-\theta_H-1}dz_D - \theta_H \phi \theta_H z_D^{s-\theta_H-1}dz_D = 0 \tag{90}$$

The expression in the first line in front of $d\phi$ is negative, because $B'_H < 0$, $\log(\phi) < 0$ and $log(z_D^*) > 0$ as well as $log(z_D) > 0$. Hence, the second line has to be positive to fulfill this equation. The second line increases in $z_D$. Hence, this line has its maximal value at $z_D^{max} = \phi^{-1}z_D^*$, which is

$$(-z_D^{s-\theta_H-1} + \phi \theta_H z_D^{s-\theta_H-1})dz_D^*.$$
The expression in front of $dz_D^*$ is negative, because $0 \leq \phi^{2\theta_H} < 1$. As a result, the foreign productivity cut-off has to decrease, $dz_D^* < 0$, in order to get the expression in the second line of (90) positive and therewith the total equation equal to zero. By using this result in (89), it follows $dz_D > 0$. This proofs an increase of the relative selection effect $Z = \frac{z_H}{z_D}$ in $\theta_H$: $\Delta \theta_H \to \Delta Z > 0$.

.5 Cut-off values at $\phi^*$

Suppose the competition intensities in home and foreign are defined by a low level of R&D investment of firms in both countries. This implies:

$$V^*(z^L_D, z^{*L}_D, \phi = \phi^*) = B_L(z^{-\theta_L}_D + \phi^{\theta_L} z^{*-\theta_L}_D) - k_L F_L = 0,$$  \hspace{1cm} (91)

$$V(z^L_D, z^{*L}_D, \phi = \phi^*) = B_L(z^{*-\theta_L}_D + \phi^{\theta_L} z^{-\theta_L}_D) - F_L = 0.$$  \hspace{1cm} (92)

Furthermore, definition of $\phi^*$ implies

$$V^*_H(z^L_D, z^{*H}_D, \phi = \phi^*) = B_H((z^L_D)^{-\theta_H} + \phi^{\theta_H}(z^{*H}_D)^{-\theta_H}) - k_H F_H = 0.$$  \hspace{1cm} (93)

Hence, a foreign entrant is at $\phi = \phi^*$ indifferent between both kinds of R&D investments, which can be seen by comparing this equation with (91). Now suppose all foreign entrants invest in a high level of R&D. This implies:

$$V^*(z^L_D, z^{*H}_D, \phi = \phi^*) = B_H((z^L_D)^{-\theta_H} + \phi^{\theta_H}(z^{*H}_D)^{-\theta_H}) - k_H F_H = 0,$$  \hspace{1cm} (94)

$$V(z^L_D, z^{*H}_D, \phi = \phi^*) = B_L((z^H_D)^{-\theta_L} + \phi^{\theta_L}(z^{*H}_D)^{-\theta_L}) - F_L = 0.$$  \hspace{1cm} (95)

Since equation (93)=(94) and (92)=(95) at $\phi = \phi^*$, it follows $z^L_D = z^H_D$ and $z^{*L}_D = z^{*H}_D$. Hence, at $\phi = \phi^*$ each endogenously defined competition intensity is of the same value, regardless of foreign firms’ R&D investment decisions.

.6 Proof if $k_L = k_H > 1$ home firms will conduct superior R&D investments at a lower level of trade liberalization than foreign firms

First of all, we set $k_L = k_H = k$. The difference function (29) is then

$$V_H(z^L_D, z^{*L}_D) - V^*_H(z^L_D, z^{*L}_D) = B_H(1 - \phi^{\theta_H})((z_D)^{-\theta_H} - (z^{*}_D)^{-\theta_H}) - F_H(1 - k).$$
By using the explicit cut-off values, (17) and (18), it can be written as

$$= B_H (1 - \phi_H) \left( \left( \frac{F_L (1 - k \phi_L)}{B_L (1 - \phi^2 L)} \right)^{\theta_H / \theta_L} - \left( \frac{F_L (k - \phi_H)}{B_L (1 - \phi^2 L)} \right)^{\theta_H / \theta_L} \right) + F_H (k - 1). \quad (96)$$

The first term is negative for all possible values of $k > 1$, $\phi^{-1} \geq k > 1$, because $z_D > z^*_D$ (see (29)). The second term is positive. Home firms will invest before foreign firms in a high level of $R&D$ if the total value of (96) is positive. This will be proved in the following. To summarize, it will be shown that the negative term of the difference function is convex in $k$ for any possible value of $k$ ($1 \leq k \leq k^{max} = \phi^{-1}$). In addition, its value is at $k = 1$ identical to and at the maximal possible value of $k$ lower than the value of the positive first term. Hence, the negative term of (96) can never be in absolute terms larger than the positive term. An illustration is given in figure (4).

First, the value of the difference function (96) changes in $k$ according to

$$\frac{\partial (V_H (z_D, z^{*L}_D) - V^*_H (z^{L}_D, z^{*L}_D))}{\partial k} = ...$$

$$= -\frac{\theta_H F_L B_H (1 - \phi_H)}{\theta_L B_L (1 - \phi^2 L)} \left[ \phi^{\theta_L} L \left( \frac{F_L - k F_L \phi_L}{B_L (1 - \phi^2 L)} \right)^{\theta_H / \theta_L} + \left( \frac{k F_L - F_L \phi_L}{B_L (1 - \phi^2 L)} \right)^{\theta_H / \theta_L} \right] + F_H$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 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\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \q
The second derivative of the difference function with respect to $k$ is

$$\frac{\partial^2}{\partial^2 k} \left( V_H(z^L_D, z^{*L}_D) - V^*_H(z^L_D, z^{*L}_D) \right) = \ldots$$

$$= -\frac{\theta_L - \theta_H}{\theta_L} \frac{\phi^2}{\phi^2 L} B_H (1 - \phi^2 L) \left[ \phi^2 \left( \frac{B_L (1 - \phi^2 L)}{F_L - k F_L \phi^2} \right)^2 \right] \ldots$$

$$= \left[ \frac{B_L (1 - \phi^2 L)}{k F_L - F_L \phi^2} \right] \left( \frac{B_L (1 - \phi^2 L)}{F_L - k F_L \phi^2} \right) \ldots \right]. \quad (98)$$

The first term in the square bracket is positive and the second term negative. In addition, the multiplicative term in front of the bracket is negative, because $\theta_L > \theta_H$. For values of $k$ close to one, the second term in square brackets is larger than the first one. The overall value of the function is then positive. However, for large values of $k$ the first term is larger than the second one. This can be seen by evaluating it at the maximal possible value of $k_{max} = \phi^{-\theta_L}$. The first term tends towards infinity:

$$\left( \frac{B_L (1 - \phi^2 L)}{\theta_L} \phi^2 \right) \left( \frac{B_L (1 - \phi^2 L)}{\theta_L} \right) \rightarrow \infty. \quad \text{Hence, the negative part of the difference function is convex.}$$

Second, the value of the difference function (96) evaluated at the maximal possible $k$, $k_{max} = \phi^{-\theta_L}$, is larger than zero:

$$V_H(z^L_D, z^{*L}_D) - V^*_H(z^L_D, z^{*L}_D) \bigg|_{k = k_{max}} = \left( \frac{F_L}{B_L} \right)^{\frac{\theta_H}{\theta_L} \phi^2 \theta_H + \phi^{-\theta_L} - 1} F_H \frac{1}{B_H} > 0 \quad (99)$$

From (19) it follows, $\frac{F_H}{B_H} \geq \left( \frac{F_L}{B_L} \right)^{\frac{\theta_H}{\theta_L} \phi^2 \theta_H}$. In addition, $\phi^{-\theta_L} - 1 > \phi^{-\theta_H}$ (because $\phi^{-\theta_L} - 1 > \phi^{-\theta_H} - 1 \rightarrow \phi^{-\theta_H} > \phi^{-\theta_L}$).

Hence, the difference function can never be lower than zero, if the absolute disadvantage of both kinds of $R&D$ investments is the same, $k_L = k_H = k > 0$. If the foreign disadvantage is even larger for the superior $R&D$ than for the inferior $R&D$, $k_H > k_L$, the second term of (99) will also be larger compared to the case with $k_H = k_L = k$. As a result, the value of the difference function is also positive for any $k_H > k_L$. 

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References


