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## Venture Capital Financing of Innovation Races

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# Venture Capital Financing of Innovation Races\*

Preliminary and incomplete

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## Abstract

This paper analyzes innovation races in a moral hazard setting. I develop a model where two competing entrepreneurs, financed by a single venture capitalist, work independently on the same project. The venture capitalist cannot observe the allocation of funds she provides, which creates a moral hazard problem. I identify properties of the optimal financing contract and compare them to two benchmarks: single entrepreneur and first best. In particular, if the entrepreneurs differ in their skills, the relevant question is whether it is profitable to employ the less skilled entrepreneur in addition to the more skilled one. I show that if the entrepreneurs do not differ in their skills too much, financing both of them indeed helps to alleviate the moral hazard problem. Moreover, it might be profitable to finance the less skilled entrepreneur, in addition more skilled one, even in cases where the former would not be employed in the first best.

*Keywords:* venture capital, moral hazard, optimal contract, patent races, stopping time

*JEL Classification:* D92, G24, L26, O31

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# 1 Introduction

The classical approach in the literature on patent races is to model firms as being run by their owner and to assume away any agency problems within these firms.<sup>1</sup> This approach ignores an important fact that in many cases the financing and allocation decisions in R&D process are separated. Such a situation may arise within firms but it is especially important for grant agencies and venture capital funds.

Venture capital funds are usually directed to projects of uncertain quality, where neither time nor financial recourses needed for successful completion of the project are known *ex ante*. As a rule, venture capitalists are actively involved in monitoring firms in their portfolio. Nevertheless, they can rarely control perfectly whether resources are allocated efficiently, since such control may require expertise that often only an entrepreneur himself possesses. This creates a moral hazard problem: entrepreneurs tend to misallocate the funds provided by the venture capitalist. In particular, they may divert part of funds for their own uses, or may allocate them into activities that have high personal return but create little market value.<sup>2</sup>

The venture capital literature has extensively discussed contractual arrangements that can be used in order to alleviate the moral hazard problem. These are, for instance, convertible securities (Sahlman 1990; Kaplan and Stromberg 2003) and monitoring mechanisms (Gompers 1995). On the one hand, these mechanisms are efficient in mitigating the agency conflict. On the other hand, they are costly, complicated, and in some circumstances they are not feasible at all — most obviously, if the capital markets are not sufficiently developed to allow the use of complicated securities schemes. This creates obstacles for efficient funding of R&D.

As a novel feature this paper proposes a simple mechanism, namely competition between portfolio firms, that can be used to mitigate the agency conflict. I show that a venture capitalist can use competition between portfolio entrepreneurs in order to improve their incentives and thus her own profit. In particular, I investigate a model of patent races in a moral hazard setting, where financing decision (made by the venture capitalist) and the allocation decisions (made by two competing agents) are separated. There is a single venture capitalist financing (at most) two entrepreneurs. The venture capitalist carries all the research costs and rewards the entrepreneurs if they succeed.

I assume that the venture capitalist has full commitment and proposes a contract to the entrepreneurs. The contract specifies the maximal financing horizon and rewards for each entrepreneur if he succeeds. I identify properties of the optimal contract and contrast them two benchmarks, namely single agent and first best. It is known that

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<sup>1</sup>See, for instance, Loury (1979), Lee and Wilde (1980) Reinganum (1981), Grossman and Shapiro (1987), Malueg and Tsutsui (1997).

<sup>2</sup>See Gompers and Lerner (2004), p. 174 for a discussion of agency costs in venture capital financing.

when financing only a single entrepreneur, due to the agency costs, the venture capitalist always “underinvests” in the sense that the optimal financing horizon is inefficiently short (Bergemann and Hege 2002). However, I show that this is not necessarily true in the case of competing agents. In particular, when facing agent with different skills, there is still “underinvestment” into the more skilled one, but there may be “overinvestment” into the less skilled one.

In order to obtain intuition for the above results, let me compare the patent race to the single agent case. I identify two effects of competition. First, there is a *scale effect*: Employing two entrepreneurs increases the chances of innovation and thus it takes on average a shorter time achieve a success. Second, and more importantly, there is a *disciplining effect*: The fear that the competitor wins the patent race limits the incentives of each agent to divert funds for own uses. While the scale effect is also present in the first best, the disciplining effect occurs only under moral hazard. In particular, I show that when facing entrepreneurs with different skills, it may be profitable for the principal to employ the low skilled entrepreneur in order to discipline the high skilled one, although in terms of success probability the contribution of the former is minor. His presence, however, disciplines the high skilled entrepreneur and limits the rent which he can extract from the principal. The disciplining effect may be so strong that such employment occurs even though the less skilled entrepreneur would not be employed in the first best. This is in stark contrast to the case of a single entrepreneur.

The observation that venture capital firms, grant agencies and similar institutions may find it profitable to finance an innovation race between two or more entrepreneurs is well supported by casual empirical evidence. The venture capital division of Vulcan Inc., a corporation owned by Microsoft co-founder Paul Allen, has contracted three competing agencies for the project Halo, aimed at the development of the problem-solving software.<sup>3</sup> National Archives and Records Administration of the USA awarded two contracts to the competing firms to develop an Electronic Record Archives, a new system of record keeping.<sup>4</sup> National Institutes of Health (USA) routinely finances competing research teams working on the same problem.<sup>5</sup>

Despite the numerous evidence that financing of competing projects plays an important role in the venture capital and the grant funding, the literature on venture capital has up to now paid little attention to this potent mechanism of mitigating the agency problem. The only exception is Levitt (1995), who analyzes the problem of a principal employing two agents. The principal’s payoff depends on the best of agents’ outputs. Unlike in the present model, the author investigates a static situation and only allows

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<sup>3</sup>See [www.projecthalo.com](http://www.projecthalo.com) for details.

<sup>4</sup>For details see [www.diglib.org/preserve/ERA2004.htm](http://www.diglib.org/preserve/ERA2004.htm).

<sup>5</sup>Recent example involves two large scale competing studies which independently revealed a gene responsible for multiple sclerosis. Both studies were supported by a grant from NIH. For details see [www.ninds.nih.gov/news\\_and\\_events/press\\_releases/pressrelease\\_two\\_new\\_MS\\_genes.htm](http://www.ninds.nih.gov/news_and_events/press_releases/pressrelease_two_new_MS_genes.htm).

for symmetric agents. Moreover his results rely on the fact that production technologies of the agents are interdependent, and the model is not able to deliver predictions for the case of independent technologies. Both these issues are addressed in this paper.

The modelling approach in this paper is closely related to that of Bergemann and Hege (1998, 2002, 2005). They investigate the decision of a venture capitalist who finances a single entrepreneur under uncertainty about the quality of the project and investments needed for its successful realization. Bergemann and Hege (1998) analyze a model in which the quality of a project is not known and has to be resolved through a costly experiment. Their main result is that agency costs lead to inefficiently early stopping of the project. In their second paper Bergemann and Hege (2005) extend these results and analyze the difference between relationship financing and arm-length financing. Finally, in the third model Bergemann and Hege (2002) investigate the value of staged financing. The authors show that the use of financing rounds (stages) allows to increase the funding horizon and to make it closer to the socially optimal horizon. I use the framework of Bergemann and Hege to study the patent race in the moral hazard setting.

Another related paper is Schmidt (1997) who studies the incentives of a manager, who operates on the competitive product market. He identifies a “threat-of-liquidation” effect of competition. As the product-market competition increases, the manager is induced to spend more effort, because otherwise the profits of his firm fall below the critical value and the firm will be liquidated. This effect is somewhat similar to the disciplining effect in this paper. In Schmidt (1997), however, the competitive environment is exogenously given. I, to the contrary, assume that the venture capitalist can determine the extent of the disciplining effect by employing two agents and specifying in contracts how long each of the competitors will stay in the game.

Finally, let me point out that the idea that competition positively affects incentives of the agents (i.e., relaxes their incentive constraint) has already been applied in various settings, like yardstick competition (Tirole 1997, pp. 41–42) or design of team incentives (Holmström 1982). I contribute to this literature by investigating the effect of competition on incentives in the dynamic framework, where only the winner’s output matters to a principal. I show, that in this framework, competition has a positive incentive effect even when the research technologies are independent (unlike in yardstick competition).

The structure of this paper is the following. I first analyze the basic setup with a single entrepreneur in Section 2. In Section 3 I then analyze the innovation race between two entrepreneurs. Section 4 concludes. Proofs of all proposition can be found in the Appendix.

## 2 Model: Single entrepreneur

I start by analyzing a simple benchmark model with single agent, which is essentially the model Bergemann and Hege (2002) but with just a single stage to complete. In the following section I then present the model with two competing agents.

There is an entrepreneur (agent, he) with no wealth of his own. The entrepreneur has an idea (a project) how to solve a particular problem. For instance, he tries to find a cure against a disease. Financing of the project is done by a venture capitalist (principal, she) who provides all necessary funds. The project, if successful, generates a prize with value  $R$ . The prize is to be divided between the venture capitalist and the entrepreneur.

I first consider the model in discrete time and later I make a transition to continuous time. The entrepreneur and the venture capitalist are risk neutral individuals with common discount rate  $r$  (per period). In order to successfully complete the project in some period, the entrepreneur needs to allocate an amount  $c$  (provided by the venture capitalist) into the project. In that case the outcome of the R&D is stochastic and I model the innovation process as a Bernoulli trial, where the project is completed by entrepreneur  $k$  in the current period with probability  $p_i$ . With probability  $1 - p_i$  the entrepreneur does not succeed and needs to invest further. Expected profit from a single stage then becomes  $p_i R - c$ .<sup>6</sup> I assume that the probability of success in each period is independent across time.<sup>7</sup> The entrepreneur's probability of success  $p_i$  (his type) is common knowledge.

Following Bergemann and Hege (2002), I assume that the venture capitalist has the bargaining power. She proposes a contract to the agent and retains the residual payoff from the project. This can be justified, for instance, when there is a competitive market for innovative projects and a limited supply of venture capital. Furthermore, the outside option of the entrepreneur is normalized to zero. As the entrepreneur does not incur any costs of his own, the participation constraint is satisfied whenever the expected rewards are positive.

The funds are provided by the principal, but the allocation decisions are made by the entrepreneur. The allocation of funds in this model is subject to a moral hazard: In each period the entrepreneur faces a choice between allocating the funds into R&D and divert them for private uses. The venture capitalist is not able to observe the allocation decision. All she can observe is a success (completion of the project) or an absence of success, which can either mean that an entrepreneur has invested money but failed, or that he has diverted it. The venture capitalist is, therefore, willing to finance R&D only

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<sup>6</sup>In the specification of the costs I depart from Bergemann and Hege (2002) who consider costs depending on the success probability. While this does not play a role in the single agent case, it will become relevant for competing agents. There I keep the costs constant and assume that the agents differ with respect to the success probability, reflecting different skills or experience.

<sup>7</sup>The innovation process in this game can be interpreted as following. The entrepreneur owns a coin (representing a project) with probability  $p_i$  of achieving a "head" (success). In order to make one toss, the entrepreneur has to pay a prescribed amount of money  $c$ .

if he can ensure that funds are allocated truthfully in each period of time. That is, the venture capitalist needs to suggest such a reward to the entrepreneur, so that he prefers to allocate the funds to R&D, rather than to divert them. Since the allocation of funds is not observable, the reward, which the agent receives according to the contract, has to be conditioned on observable outcomes: the event and the time of success.

Much like Bergemann and Hege (2002) I consider deterministic stopping time contracts, where the principal commits to a maximal financing horizon for the entrepreneur and discards him, should he make no invention until that time. More specifically, the contract specifies a non-negative stopping time  $T_i$  and a schedule of rewards  $s_i(t)$  (where  $0 \leq t \leq T_i$ ), which have the following meaning. The principal commits to finance entrepreneur until time  $T_i$ . Reward  $s_i(t)$  specifies the payoff of the entrepreneur, if he makes the invention at time  $t \leq T_i$ . When no invention is made by time  $T_i$ , the principal discards the entrepreneur without any payments.<sup>8</sup> I will also say that agent  $i$  is *financed* or *is employed* if his stopping time is positive.

I assume that the principal can choose the financing horizon and can commit to it. This means that if the maximum time allowed for experimentation elapses without success, then the entrepreneur will be irrevocably abandoned. Commitment to stop financing the project is an important punishment mechanism, that allows to decrease the compensation of the entrepreneur and therefore to increase the profit of the venture capitalist, compared to a situation with no commitment.<sup>9</sup> While in the present model there is no endogenous mechanism, which would make the ex-ante commitment credible ex-post, the commitment power can be justified by assuming that the principal is wealth-constrained. This assumption is well supported by the evidence about practice of the venture-capital funds. According to Inderst and Münnich (2003), the venture capital funds are normally close-ended, which means that funds are raised once from the investors and are directed into the portfolio of projects afterwards. The partnership agreements, which govern the venture capital funds, often contain a covenant that limits a possibility of the venture capitalist to raise further investments. Likewise, the partnership agreements restrict ability of the venture capitalist to transfer investments across projects and across different funds, run by the same partners. The wealth-constrained venture capitalist can credibly commit to limit resources directed to each of his portfolio projects and hence can commit to the finite financing horizon. In the present model, this commitment can

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<sup>8</sup>One could also consider contracts where the entrepreneur is also paid in other instances, for example, when the stopping time has elapsed but no success was made. However, such contracts are clearly suboptimal, and will, thus, be omitted.

<sup>9</sup>Note that in the case when the principal cannot commit to stopping the project after the maximal allowed time has elapsed, she will finance the entrepreneur infinitely long (once the financing has started). This follows from a simple argument that after the financing horizon has elapsed but no success was made, the principal is willing to start the game from the beginning, as if the world were in time  $t = 0$ . Indeed, all costs that the venture capitalists has already incurred are sunk, and the game has not changed since the venture capitalist made his optimal decision at  $t = 0$ .

be understood as the following. Ex ante, the venture capitalist is able to calculate the optimal period of time, during which he is willing to finance the project. He then commits a corresponding amount of money for this project and commits all other resources to his other portfolio projects. The partnership agreements restrict the ability of the venture capitalist to raise additional funds and, therefore, the commitment to stop the project is credible.<sup>10</sup>

I continue by deriving the value optimal functions of the principal and the entrepreneur. Let me fix the stopping time  $T_i$ . I will first specify the value of the venture, which is the expected profit of the principal with no moral hazard in place. As the second step, I derive the optimal reward scheme and compute the corresponding expected reward of the entrepreneur. The venture capitalist's profit is then difference between the expected value of the venture and the expected costs. In the last step I maximize the venture capitalist's profit and derive the optimal stopping time.

### Value of the venture (first best)

The value of the venture specifies the expected value of the project in the absence of moral hazard, i.e., in the first best. Let  $V_i(t)$  denote the expected value of the project in period  $t \in [0, T_i]$ . The value of the venture in such case satisfies the following recursive formula

$$V_i(t) = p_i R + \frac{1 - p_i}{1 + r} V_i(t + 1) - c \quad (1)$$

and consists of three terms. The last term,  $c$ , represents costs of a trial that the principal needs to provide to the entrepreneur. With probability  $p_i$  the entrepreneur makes a discovery with value  $R$  in period  $t$ . With probability  $(1 - p_i)$  the entrepreneur fails, and the value of the project in period  $t$  is the discounted value of the project in period  $t + 1$ , that is  $V_i(t + 1)/(1 + r)$ . The sequence of values is the given by the solution of the difference equation (1) together with terminal condition  $V_i(T_i + 1) = 0$ . This condition states that there is no continuation after time  $T_i$  (if no innovation was made before) and the value of the project is therefore zero.

Following Bergemann and Hege (2002) I consider transition to continuous time (see Appendix for details). The innovation process then becomes a Poisson process with arrival rate  $p_i$  (which can now take any non-negative value) and the above difference equation becomes a differential equation. In order to avoid confusion, I will call  $p_i$  the entrepreneur's *innovation rate*.

Solving the resulting differential equation I obtain the following expression for value

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<sup>10</sup>More realistic approach to model the venture capital process is to assume the venture capitalist have some prior beliefs about quality of the project. If the project fails to succeed, the venture capitalist becomes pessimistic and will eventually abandon the project. This mechanism ensures that the project will be stopped in finite time and is extensively discussed in Bergemann and Hege (1998, 2005).



function in period  $t = 0$ :

$$\tilde{V}^i = V_i(0) = \frac{p_i R - c}{r + p_i} (1 - e^{-(r+p_i)T_i}). \quad (2)$$

The expression for the value function consists of two factors. The first factor represents the expected payoff from the investment, discounted with a composite discount rate which combines time discount  $r$  and the uncertain arrival of success. The second factor shows how the value of the project decreases with time of discovery.

Observe that  $V^i$  is increasing in  $T_i$  whenever the inequality

$$p_i > \frac{c}{R} \quad (3)$$

is satisfied. Then in the first best, the principal would employ the agent forever. On the other hand, if (3) holds with the opposite inequality, then  $\tilde{V}^i$  is decreasing in  $T_i$ . In such a case, there is no investment whatsoever in the first best.

### Incentive contracts under moral hazard

Now I derive the entrepreneur's rewards. In each period of time the entrepreneur faces a choice between diverting the funds provided by the venture capitalist for private needs (an action called *shirk*), and investing them into the project (an action called *work*). Incentive compatibility then requires that in order to motivate entrepreneur to allocate funds into research and development, the venture capitalist has to promise him a reward which is at least as large as the stream of rent that the entrepreneur can receive by diverting the funds.<sup>11</sup> The incentive to divert funds arises from two sources. First, the entrepreneur obtains immediate payoff of  $c$ . Second, he ensures that financing of the project will continue in the next period. By investing the entrepreneur cuts himself off the future stream of rent.

Let me denote  $E_i(t)$  the (present value of) expected rewards of the entrepreneur in period  $t$ . The intertemporal incentive compatibility constraint at time  $t = 0, 1, \dots, T_i$  is

$$E_i(t) = p_i s_i(t) + \frac{1 - p_i}{1 + r} E_i(t + 1) \geq c + \frac{1}{1 + r} E_i(t + 1). \quad (4)$$

The left-hand side of the inequality in (4) represents the expected utility of the entrepreneur, if he *works* in period  $t$ . The entrepreneur can achieve a success with probability  $p_i$ , which gives him reward  $s_i$ , or fail with probability  $1 - p_i$ , in which case he continues to the next period with discounted expected payoff  $E_i(t + 1)/(1 + r)$ . The right-hand side of (4) represents the expected utility of the entrepreneur if he *shirks* in period  $t$ . The

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<sup>11</sup>As already pointed out, any contract promising non-negative expected reward satisfies the participation constraint, as the entrepreneur's outside option is normalized to 0.

two terms correspond to the two incentive to divert the funds. The first term represents the immediate payoff of  $c$ . The second term corresponds to the continuation value of potential future rewards.

The principal aims at paying the entrepreneur the lowest reward that will force the him to invest the funds rather than consume them. Under such reward scheme, (4) holds with equality, i.e.,  $s_i(t) = c/p_i + E_i(t+1)/(1+r)$ , and I obtain the difference equation

$$E_i(t) = c + \frac{1}{1+r} E_i(t+1). \quad (5)$$

The terminal condition is again  $E_i(T_i+1) = 0$ , as the continuation value after time  $T_i$  is zero if no innovation was made before. Considering transition to continuous time, I obtain a differential equation that can be easily solved (see again Appendix for details). The value function of the entrepreneur at time  $t = 0$  is then

$$\tilde{E}^i = E_i(0) = \frac{c}{r} (1 - e^{-rT_i}). \quad (6)$$

The expression for the value function consists of two factors. The first factor represents the expected reward, discounted with the discount rate. The second factor shows how the entrepreneur's reward decreases with time of discovery, as the value of the stream of rewards the entrepreneur can obtain by diverting the funds shrinks.

### Optimal stopping

The principal's profit is simply the difference between the value of the venture and the expected reward of the entrepreneur:

$$\tilde{\Pi}^i = \tilde{V}^i - \tilde{E}^i = \frac{p_i R - c}{r + p_i} (1 - e^{-(r+p_i)T_i}) - \frac{c}{r} (1 - e^{-rT_i}). \quad (7)$$

The intuition suggests that due to additional costs, namely entrepreneur's rewards, the principal should shorten the stopping time compared to the first best. Solving for the optimal stopping time I obtain the following proposition.

**Proposition 1.** *With a single agent, the optimal stopping time is finite and equal to*

$$\sigma_i \equiv \begin{cases} \frac{1}{p_i} \log \left( p_i \frac{R}{c} - 1 \right), & \text{if } p_i > 2c/R, \\ 0, & \text{if } p_i \leq 2c/R. \end{cases} \quad (8)$$

**Corollary 1.** *Financing an agent in the first best is a necessary condition for financing him under moral hazard.*

The proposition in particular implies that the agent is financed (i.e., the optimal

stopping time is positive) if and only if  $p_i > 2c/R$ . The corollary then follows by comparing this condition to the condition (3) under first best. Consistently with the intuition, the threshold for  $p_i$  where the agent is employed is higher under moral hazard. At the same time, once the agent is employed, his stopping time is finite under moral hazard as opposed to the infinite stopping time under the first best. On the other hand, when  $c/R < p_i \leq 2c/R$ , the agent would be employed in the first best, but not under moral hazard (where the stopping time is zero).

Now, if the agent is indeed financed, i.e.,  $p_i > 2c/R$ , then I can see from (8) that the optimal stopping time is increasing in the prize  $R$  and decreasing in the cost  $c$ . However, the comparative statics with respect to  $p_i$  is not monotone: There is a critical value  $\xi$ , such that  $\sigma_i$  is increasing in  $p_i$  when  $p_i < \xi c/R$ , and  $\sigma_i$  is decreasing in  $p_i$  when  $p_i > \xi c/R$ . This critical value can be found as the maximizer of the function  $\psi(x) = 1/x \cdot \log(x - 1)$  and can be evaluated numerically as  $\xi \approx 4.5911$  (where  $\psi(\xi) \approx 0.2784$ ).

The intuition for this non-monotonicity is as follows. When  $p_i$  is small, the innovation arrives at a small rate. Thus, it is rather unlikely that the innovation will be achieved soon, and the marginal value of stopping later is large. On the other hand, when  $p_i$  is large, the innovation arrives a high rate. The large likelihood of innovation lowers the marginal value from extending the stopping time. Once this marginal value falls below the costs, i.e., the amount the agent could divert during the extended time, the venture capitalist starts to shorten the stopping time. In the limit case  $p_i \rightarrow \infty$ , the optimal stopping time  $\sigma_i$  converges to 0.<sup>12</sup>

Finally, let me discuss some comparative statics. It is straightforward to see from (7) that for a fixed  $T_i$ , the profit is increasing in the prize  $R$  and decreasing in costs  $c$ . Moreover, it is increasing in  $p_i$ .<sup>13</sup> These effects are, due to *Envelope theorem*, preserved for the optimal profit, under the optimal stopping time. In particular, despite the stopping time being non-monotone in  $p_i$ , the venture capitalist's profit is increasing in  $p_i$ . As a result, if the venture capitalist could choose between two entrepreneurs with different innovation rates (reflecting different skills), she would choose the more skilled one (with a higher  $p_i$ ).

### 3 Innovation race between two entrepreneurs

Now I analyze the case when two entrepreneurs (denoted 1 and 2) are employed. I keep the same structure of the model as in the previous section and let entrepreneur  $i$ 's probability of success be  $p_i$  (with  $i = 1, 2$ ). The probability may in general differ for both entrepreneurs, reflecting different skills or experience. As already discussed in Section 2,

<sup>12</sup>This follows from  $\lim_{p \rightarrow \infty} 1/p \cdot \log p = 0$ .

<sup>13</sup>The derivative  $\partial \Pi_i / \partial p_i = (rR + c)/(r + p_i)^2 \cdot (1 - e^{-(r+p_i)T_i}) + (p_i R - c)/(r + p_i) \cdot T_i e^{-(r+p_i)T_i}$  is clearly positive for  $T_i > 0$ .

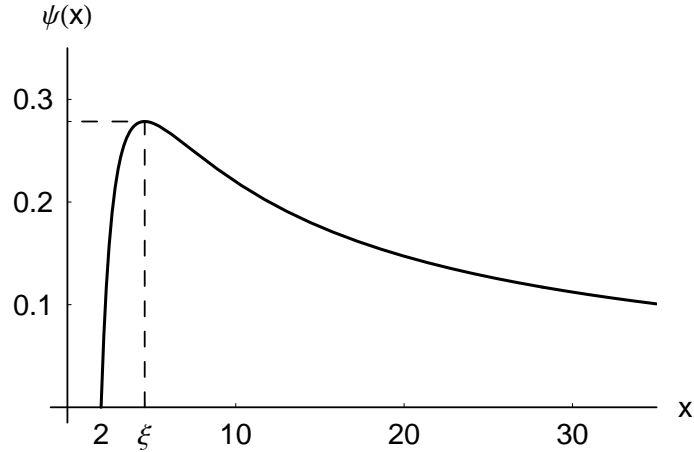


Figure 1: Function  $\psi(x) = 1/x \cdot \log(x - 1)$ .

I assume that agents are heterogeneous only with respect to the success probability, but have the same costs (see footnote 6). This allows for clear ranking of the agents. Following the literature on patent races (Lee and Wilde 1980; Reinganum 1981) I assume that the probability of success in each period is independent across the entrepreneurs and across time. I also assume that the first innovation yields monopoly power over the product. Therefore, there is no reason to continue financing an entrepreneur, once the innovation has been made.<sup>14</sup>

Compared to the case of single entrepreneur there are two additional effects. First, there is a *scale effect*, as the probability of success as well as the costs is now larger larger number of trials each period. Second, and more importantly, there is a *disciplining effect*. By consuming the funds rather than investing them, each entrepreneur faces a risk that his rival wins the prize. This lowers the expected present value of his future consumption from diverting the funds and, therefore, limits the option of each entrepreneur to deviate and to consume the funds. Hence, competition makes it cheaper for the venture capitalist to meet the incentive compatibility constraints of the entrepreneurs.

A contract can again condition only on observable outcomes, which are now the event and the time of success, and the identity of the winner. I again consider stopping time contracts, where the principal commits to a maximal financing horizon for each entrepreneur and discards him should he make no invention until that time. More specifically, the contract will specify for each entrepreneur ( $i = 1, 2$ ) a non-negative stopping time  $T_i$  and a schedule of rewards  $s_i(t)$  (where  $0 \leq t \leq T_i$ ) with the following meaning. The principal commits to finance the entrepreneur  $i$  for at most  $T_i$  periods. Reward  $s_i(t)$  specifies the payment to entrepreneur  $i$  if he makes the invention at time  $t \leq T_i$  and he

<sup>14</sup>Bergemann and Hege (2002) analyze a model with a single entrepreneur and  $N$  stages. However, for more than one entrepreneur the analysis of the multistage game becomes extremely complicated. In spite of this limitation, the present model enables me to illustrate the importance of competition between the entrepreneurs in venture capital financing.

is the first to make the invention.<sup>15</sup> When entrepreneur  $i$  makes no invention by time  $T_i$ , he is discarded without any payments. Should the rival's stopping time (denoted as  $T_{-i}$ ) be higher, the venture capitalist will continue to finance the rival for another  $T_{-i} - T_i$  periods.

As argued in the previous section, if only a single entrepreneur is financed, the venture capitalist prefers to employ the more skilled one (with a higher probability of success). Now I am, in particular, interested in the question whether the venture capitalist should in addition to the more skilled entrepreneur employ the less skilled one. The reason for doing so may be exactly the disciplining effect. Despite the small contribution to the probability of success, the presence of the less skilled entrepreneur relaxes the high skilled entrepreneur's incentive compatibility and thus enables to lower his rewards.

I start by deriving the value functions of the principal and the entrepreneurs given stopping times  $T_1$  and  $T_2$ . Much like in the case of a single agent, I first specify the value of the venture, which is the expected profit of the principal with no moral hazard in place. As the second step, I derive the optimal reward scheme and compute the corresponding expected rewards of the entrepreneurs. Her profit is then difference between the expected value of the venture and the expected costs. In the final step, I analyze the principal's maximization problem as optimal stopping with respect to two stopping times  $T_1$  and  $T_2$ .

### Value of the venture (first best)

Let me denote  $T = \min\{T_1, T_2\}$  the time for which both entrepreneurs are financed and let entrepreneur  $k$  be the entrepreneur with a higher stopping time, i.e.,  $T_k \geq T_{-k} = T$ . Let  $V(t)$  denote the expected value of the project. First, I derive the value of the project recursively starting with periods  $t \in [T + 1, T_k]$  where only a single entrepreneur  $k$  is employed (i.e., after entrepreneur  $-k$  has been abandoned). The value of the venture again satisfies the recursive formula (1) with terminal condition  $V(T_k + 1) = 0$ . Continuing in the recursion, I can compute the value  $V(T + 1)$  at time  $T + 1$ . This then serves as a terminal condition for the value function when both entrepreneurs are still present. Second, let me focus on the periods where both entrepreneurs are employed:  $t \in [0, T]$ . The derivation of this value function is similar. It differs only in the probability of success, which is now  $1 - (1 - p_1)(1 - p_2)$ . Thus, for  $t \in [0, T]$  I have the recursive equation

$$V(t) = [1 - (1 - p_1)(1 - p_2)]R + \frac{(1 - p_1)(1 - p_2)}{1 + r} V(t + 1) - 2c. \quad (9)$$

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<sup>15</sup>If both entrepreneurs innovate in the same period, tie-breaking rule may play a role in discrete time. However, after transition to continuous time, it becomes a zero probability event. Thus, tie-breaking rule will not matter. For the computations below I simply assume that each entrepreneur wins the race with probability  $\frac{1}{2}$ .

After transition to continuous time, I obtain two differential equations: one for  $t \in [0, T]$  and another for  $t \in [T, T_k]$ . Solving these equations, I obtain the value of the venture (see Appendix for details)

$$W = V(0) = V^0 + V^k e^{-(r+p_1+p_2)T}, \quad (10)$$

where

$$V^0 = \frac{(p_1 + p_2)R - 2c}{r + p_1 + p_2} (1 - e^{-(r+p_1+p_2)T}), \quad (11)$$

$$V^k = V(T) = \frac{p_k R - c}{r + p_k} (1 - e^{-(r+p_k)(T_k-T)}). \quad (12)$$

The value of the venture  $W$  is now composed from two terms. The first term,  $V^0$ , is the value if the venture capitalist would finance both entrepreneurs up to time  $T$ . Observe that this term resembles the value (2) in the case of single agent with adjusted success probability, as the arrival rate of an innovation is now  $p_1 + p_2$ . The second term is the continuation value from keeping agent  $k$  longer, discounted by  $e^{-rT}$  and multiplied by the probability  $e^{-(p_1+p_2)T}$  that no innovation is made until time  $T$ .

Analyzing value function  $W$ , I may derive the optimal stopping time in the first best.

**Proposition 2.** *Assume that  $p_1 \geq p_2$ . Then the following statements hold in the first best:*

- (i) *If  $p_1 \leq c/R$ , then no agent is financed ( $T_1 = T_2 = 0$ ).*
- (ii) *If  $rp_2/(r + p_1 - p_2) \leq c/R < p_1$ , then only agent 1 is financed and he is financed forever ( $T_1 = \infty, T_2 = 0$ ).*
- (iii) *If  $c/R < rp_2/(r + p_1 - p_2)$ , then both agents are financed forever ( $T_1 = T_2 = \infty$ ).*

Much like in the case of a single agent, the stopping time for each agent is infinite or zero. Moreover, the condition for financing agent 1 is identical with the condition for financing him as single. Thus, financing agent 1 as single is sufficient and necessary condition for financing him in competition (against an inferior agent 2). On the other hand, because  $rp_2/(r + p_1 - p_2) \leq p_2$  (when  $p_1 \geq p_2$ ), employing agent 2 as single is necessary but not sufficient for employing him under competition.

To see the intuition behind this result, let me compare the benefit from agent 2 in the cases when nobody is employed yet and in the case when agent 1 is employed already. Assume that agent  $i$  makes the invention (i.e., the first Poisson hit occurs) at time  $t_i$ . While the marginal benefit from agent 2 is the same in both cases in instances where agent 1 makes the invention later (i.e.,  $t_1 \geq t_2$ ), it is clearly higher in the latter case in instances where agent 1 makes the invention earlier (i.e.,  $t_1 < t_2$ ) and wins the race.

Due to the lower marginal benefit, the principal is more reluctant to employ agent 2 in addition to agent 1.

### Incentive contracts under moral hazard

As the next step I determine the expected payoffs of the entrepreneurs. I start with time  $t \in [T + 1, T_k]$  when only entrepreneur  $k$  is employed. By the same procedure as in the single entrepreneur case, the expected payoff of entrepreneur  $k$  follows the difference equation (5) with terminal condition  $E_k(T_k + 1) = 0$ . Using the recursion, I can compute the value  $E_k(T + 1)$  at time  $T + 1$  that then serves as terminal condition for entrepreneur  $k$ . The terminal condition for the rival (denoted  $-k$ ) who is abandoned at time  $T$  is then  $E_{-k}(T + 1) = 0$ .

Now I analyze the entrepreneurs' expected payoffs in the periods when both are employed,  $t \in [0, T]$ . Following Mookherjee (1984), the principal aims to implement the outcome where  $(work, work)$  in every period  $t \in [0, T]$  is subgame perfect Nash equilibrium. The intertemporal incentive compatibility constraint for agent  $i \in \{1, 2\}$  then is:

$$\begin{aligned} E_i(t) &= p_i(1 - p_{-i})s_i(t) + \frac{1}{2}p_1p_2s_i(t) + \frac{(1 - p_1)(1 - p_2)}{1 + r} E_i(t + 1) \\ &\geq c + \frac{1 - p_{-i}}{1 + r} E_i(t + 1). \end{aligned} \quad (13)$$

The inequality requires that  $work$  is a best response to  $work$ . The left-hand side of (13) represents the expected utility of entrepreneur  $i$ , if he  $works$  in period  $t$ , given that the rival  $works$ . If entrepreneur  $i$  achieves a success while his rival does not, which occurs with probability  $p_i(1 - p_{-i})$ , entrepreneur  $i$  earns his reward  $s_i(t)$ . If there is a tie (i.e., both make the invention in this period), which occurs with probability  $p_1p_2$ , I assume that he earns this share with probability  $\frac{1}{2}$ .<sup>16</sup> Last, if nobody wins, with probability  $(1 - p_1)(1 - p_2)$ , the entrepreneur will receive further financing with present value  $E_i(t + 1)/(1 + r)$ . The right-hand side of (13) represents the expected utility of entrepreneur  $i$ , if he  $shirks$  in period  $t$ , given that the rival  $works$ . In that case he receives a certain payoff of  $c$  and ensures continuation if the rival does not win the race, which occurs with probability  $1 - p_{-i}$ .

Compared to the case with only one entrepreneur, future funding of each entrepreneur is uncertain and depends on the fact that another entrepreneur has not yet reached success. Therefore, competition softens the incentive compatibility constraint and makes it less costly for the venture capitalist to satisfy it. I refer to this effect as the *disciplining effect*. The venture capitalist aims at paying each entrepreneur the minimal share which will induce  $(work, work)$  in every period to be subgame perfect Nash equilibrium — so

<sup>16</sup>As noted above, tie breaking rules do not matter after transition to continuous time.

that the constraint (13) is binding. Proceeding the same way as above (see Appendix for details), I obtain the expected rewards

$$F = E_1(0) + E_2(0) = E^{10} + E^{20} + E^k e^{-(r+p-k)T}, \quad (14)$$

where

$$E^{i0} = \frac{c}{r + p_i} (1 - e^{-(r+p-i)T}), \quad (15)$$

$$E^k = \frac{c}{r} (1 - e^{-r(T_k - T)}). \quad (16)$$

Much like for the value of the venture, the entrepreneurs' expected rewards can be decomposed into three terms. The first and the second term represent the entrepreneurs' expected rewards when they are financed until time  $T$ . The third term is the continuation value for entrepreneur  $k$  who is financed longer, discounted by  $e^{-rT}$  and multiplied by the probability  $e^{-p-kT}$  that the rival does not innovate until time  $T$ .

### Optimal stopping time

Principal's profit is simply again the difference between the value of the venture and expected rewards of both entrepreneurs:

$$\Pi = W - F = V^0 - E^{10} - E^{20} + (V^k - E^k e^{p_k T}) e^{-(r+p_1+p_2)T}. \quad (17)$$

This profit is to be maximized with respect to the agents' stopping times. Thus, using the above notation, for  $T_k \geq T_{-k} = T$ , the venture capitalist maximizes her profit  $\Pi$  as given by (17) with respect to stopping times  $T_k$  and  $T$ , and also with respect to the choice which entrepreneur is financed for a longer time,  $k \in \{1, 2\}$ . Let me use the asterisk to denote the optimal stopping times ( $T_1^*$  and  $T_2^*$ ). The following proposition compares these stopping times to the case where an agent is financed alone.

**Proposition 3.** *Agent  $i$ 's optimal stopping time is bounded from above by  $\sigma_i$  (for both  $i = 1, 2$ ). Moreover, in optimum, this bound is attained for the agent with a longer stopping time.*

**Corollary 2.** *Financing an agent when being single is a necessary condition for financing him in competition.*

For the intuition let me first consider the agent  $k$  with a longer stopping time. The stopping time is determined by the equality of marginal value of the venture and marginal costs (agent's value). However, both these marginals are affected by the rival's stopping time in the same way — by a factor representing the probability that the rival makes



no invention before his own stopping time. Thus, the optimal stopping time for agent  $k$  is the same as if the agent was alone (i.e., rival's stopping time was zero). Now, the intuition for the agent with a shorter stopping time is not so straightforward. Due to the presence of the rival, both the marginal value of the venture as well as marginal costs are lower than in the case of single agent. In the proof I in fact show that the effect on the costs is stronger. Therefore, the principal prefers a shorter stopping time than if the agent is single.

**Proposition 4.** *Assume that  $p_1 \geq p_2$ . Then the following statements hold in optimum:*

- (i) *If  $p_1 \leq 2c/R$ , then no agent is financed.*
- (ii) *If  $rp_2/(r + p_1 - p_2) \leq 2c/R < p_1$ , then agent 1 is financed.*
- (iii) *If  $2c/R < rp_2/(r + p_1 - p_2)$ , then both agents are financed.*

**Corollary 3.** *Assume that the agents are symmetric ( $p_1 = p_2$ ). Then both of them are financed if and only if each of them would be financed as single.*

The above proposition is in spirit similar to Proposition 2 for the first best. However, while Proposition 2 provides a full characterization of financing for both agents, there is no claim about financing agent 2 in statement (ii) of Proposition 4. In that respect, statement (iii) only provides a sufficient but not a necessary condition for financing agent 2. In particular, this condition only requires the principal's profit to be increasing in  $T$  when  $T$  is close to zero. However, as the profit may not be in general concave in  $T$ , such a condition is only sufficient.<sup>17</sup>

Finally, I illustrate the disciplining effect under competition. To do so, I compare financing of two agents in the first best and under moral hazard. Recall from Corollary 1 that financing an agent as single in the first best is a necessary condition for financing him under moral hazard. The reason for this is the negative effect of the costs. These costs need to be incurred by the principal in order to provide the agent with incentives to work. Although such costs are also present under competition, they are lower due to the disciplining effect. This effect might be so strong that under moral hazard it is profitable to add the second agent who would not be added in the first best.

**Proposition 5.** *Let me fix the parameters  $R$ ,  $c$ , and  $r$ . If  $p_1$  is large enough, then there exists an open set of values of  $p_2$  (such that  $p_2 < p_1$ ) where only agent 1 is financed in the first best, but both agents are financed under moral hazard.*

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<sup>17</sup>Numerical computations for specific parameter values show that it might be indeed the case that the principal's profit is maximized by some interior  $T \in (0, T_k)$  and, at the same time, the profit is decreasing for  $T$  close to zero.

The striking consequence of the above proposition is that financing agent 2 in the first best is not anymore a necessary condition for financing him under moral hazard. This is in sharp contrast with the case of only single agent (Corollary 2). As already mentioned, the reason for this contrast is the disciplining effect of competition which is present under competition, but not in the case of a single agent.

## 4 Conclusion

The main argument of this paper is that competition can be used by the venture capitalist as an effective cure against the moral hazard, in a situation where the allocation of funds by the entrepreneurs is not observable. Hence, competition serves as a “natural” mechanism that allows to improve the efficiency of research and development. The existence of such mechanism is particularly important in those cases, where the use of complicated security schemes, developed in the venture capital literature, is difficult or not possible at all.

In terms of empirical implications this result suggests that in particular grant and government agencies, as well as venture capital firms that are active in less developed capital markets, could use competition between portfolio projects as a mechanism of mitigating the agency problems. Indeed, the former, usually do not use complicated security schemes either due to the lack of expertise or following the established practices.<sup>18</sup> The latter, on the other hand, might find it difficult to use such schemes on the capital market which is not sufficiently developed to accommodate them.

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<sup>18</sup>Consider for example the practise of NIH. The grant is usually split into several budget periods (analogy of stages in the present model). Within each stage the financing is provided on a cash request basis, where the cash is transferred to the grantee’s account based on his need. See [grants.nih.gov/grants/managing\\_awards.htm](http://grants.nih.gov/grants/managing_awards.htm).

## A Appendix: Proofs

*Derivation of the value functions in Section 2.* I derive the value function of the principal. Consider a period of length  $\Delta$  and assume that  $\Delta p_i$  is the success probability,  $\Delta r$  is the discount rate, and  $\Delta c$  is the cost per period. Then equation (1) can be rewritten as:

$$V_i(t) = \Delta p_i R + \frac{1 - \Delta p_i}{1 + \Delta r} V_i(t + \Delta) - \Delta c.$$

After rearranging, dividing the expression by  $\Delta$  and taking the limit  $\Delta \rightarrow 0$  I receive the following differential equation:

$$-\dot{V}_i(t) + (r + p_i)V_i(t) = p_i R - c. \quad (18)$$

Solving the differential equation with terminal condition  $V_i(T_i) = 0$ , I obtain

$$V_i(t) = (p_i R - c)e^{(r+p_i)t} \int_t^{T_i} e^{-(r+p_i)\tau} d\tau = \frac{p_i R - c}{r + p_i} (1 - e^{-(r+p_i)(T_i-t)}). \quad (19)$$

Setting  $t = 0$  then gives  $V_i(0)$  which as in (2).

To derive value functions of entrepreneurs  $E_i(t)$  I use the same approach. With period length  $\Delta$ , difference equation (5) becomes

$$E_i(t) = \Delta c + \frac{1}{1 + \Delta r} E_i(t + \Delta).$$

This yields the following differential equation

$$-\dot{E}_i(t) + rE_i(t) = c, \quad (20)$$

with terminal condition  $E_i(T_i) = 0$ . Solving the problem I obtain

$$E_i(t) = ce^{rt} \int_t^{T_i} e^{-r\tau} d\tau = \frac{c}{r} (1 - e^{-r(T_i-t)}). \quad (21)$$

Setting  $t = 0$  then gives (6). □

*Proof of Proposition 1.* Computing the derivative of the principal's profit with respect to stopping time  $T_i$  I obtain

$$\frac{\partial \Pi^i}{\partial T_i} = (p_i R - c)e^{-(r+p_i)T_i} - ce^{-rT_i} = (p_i R - c - ce^{p_i T_i})e^{-(r+p_i)T_i}.$$

When  $p_i R \leq 2c$ , this is clearly negative for any  $T_i > 0$ . Thus, the venture capitalist's profit is decreasing in  $T_i$  and the optimal stopping time is zero. On the other hand,

consider the case  $p_i R > 2c$ . Then  $(p_i R - c)/c > 1$  and  $\log[(p_i R - c)/c] > 0$ . The venture capitalist's profit is increasing when  $T_i < 1/p_i \cdot \log[(p_i R - c)/c]$  and decreasing when  $T_i > 1/p_i \cdot \log[(p_i R - c)/c]$ . Summing up, the optimal stopping time is equal to  $\sigma_i$  as specified in the lemma.  $\square$

*Derivation of the value functions in Section 3.* As noted in the main text, for  $t \in [T + 1, T_k]$ , the value function follows the same difference equation as in the single agent case. Thus, after transition to continuous time, I again obtain the differential equation (18) with terminal condition  $V(T_k) = 0$ . Its solution is then again of the form (19). Setting  $t = T$  I obtain the value  $V^k = V(T)$  as defined in (12).

Now I derive the value function of the principal for  $t \in [0, T]$ . When  $\Delta$  is the length of the time period, equation (9) can be rewritten as:

$$V(t) = [1 - (1 - \Delta p_1)(1 - \Delta p_2)]R + \frac{(1 - \Delta p_1)(1 - \Delta p_2)}{1 + \Delta r} V(t + \Delta) - 2\Delta c.$$

After rearranging, dividing the expression by  $\Delta$  and taking  $\Delta \rightarrow 0$  I obtain the following differential equation:

$$-\dot{V}(t) + (r + p_1 + p_2)V(t) = (p_1 + p_2)R - 2c.$$

Solving this differential equation with terminal condition  $V(T) = V^k$  yields

$$\begin{aligned} V(t) &= [(p_1 + p_2)R - 2c]e^{(r+p_1+p_2)t} \int_t^T e^{-(r+p_1+p_2)\tau} d\tau + V^k e^{-(r+p_1+p_2)(T-t)} \\ &= \frac{(p_1 + p_2)R - 2c}{r + p_1 + p_2} (1 - e^{-(r+p_1+p_2)(T-t)}) + V^k e^{-(r+p_1+p_2)(T-t)}. \end{aligned}$$

Setting  $t = 0$  then gives  $V(0)$  which indeed satisfies (10)–(12).

To derive value functions of entrepreneurs  $E_i(t)$  I use the same approach. After transition to continuous time, I again have differential equation (20) for  $t \in [T, T_k]$  with terminal condition  $E_k(T_k) = 0$ . Its solution is then as specified in (21) and setting  $t = T$  I obtain the value  $E^k = E_k(T)$ .

Now consider  $t \in [0, T]$ . The principal pays the entrepreneur the lowest possible reward  $s_i(t)$ , i.e., such that the incentive compatibility constraint (13) hold with equality. When  $\Delta$  is the length of the time period, I obtain

$$E_i(t) = \Delta c + \frac{1 - \Delta p_{-i}}{1 + \Delta r} E_i(t + \Delta).$$

This yields the following differential equation

$$-\dot{E}_i(t) + (r + p_{-i})E_i(t) = c,$$

with terminal condition  $E_k(T) = E^k$  for  $i = k$  and terminal condition  $E_{-k}(T) = E^{-k} = 0$  for  $i = -k$ . Solving the problem I obtain

$$\begin{aligned} E_i(t) &= ce^{(r+p_{-i})t} \int_t^T e^{-(r+p_{-i})\tau} d\tau + E^i e^{-(r+p_{-i})(T-t)} = \\ &= \frac{c}{r + p_{-i}} (1 - e^{-(r+p_{-i})(T-t)}) + E^i e^{-(r+p_{-i})(T-t)}. \end{aligned}$$

Setting  $t = 0$  then gives

$$E_k(0) = E^{k0} + E^k e^{-(r+p_{-k})T}, \quad E_{-k}(0) = E^{-k0},$$

with  $E^{k0}$  and  $E^{-k0}$  defined in (15). □

*Proof of Proposition 2.* The principal maximizes the value  $W$  as given by (10).

Consider first the case  $p_2 \leq p_1 \leq c/R$ . Clearly, in this case all three values  $V^0$ ,  $V^1$ , and  $V^2$  are non-positive. Thus, the highest value that can be achieved is zero, by choosing  $T_1 = T_2 = 0$ . This proves statement (i).

Second, assume that  $p_1 > c/R$ . I show that in optimum the entrepreneur with better skills is financed (weakly) longer, i.e.,  $T_1 \geq T_2$ . To see this observe that  $V^k$  is increasing whenever  $p_k > c/R$ , as

$$\frac{\partial V^k}{\partial p_k} = \frac{rR + c}{(r + p_k)^2} (1 - e^{-(r+p_k)(T_k-T)}) + (p_k R - c).$$

Thus,  $V^1 \geq V^2$ , which also implies that  $V^0 + V^1 e^{-(r+p_1+p_2)} \geq V^0 + V^2 e^{-(r+p_1+p_2)}$ . Therefore, the venture capitalist would indeed prefer to give entrepreneur 1 a larger financing horizon and thus  $k = 1$ .

Moreover, observe also that  $V^0$  does not depend on  $T_k = T_1$  and  $V^1$  is positive and increasing in  $T_1$ . Thus, in optimum  $T_1 = \infty$  and  $V^1 = (p_1 R - c)/(r + p_1)$ . Now, taking the derivative with respect to  $T$ , I obtain

$$\frac{\partial W}{\partial T} = \frac{rp_2 R - (r + p_1 - p_2)c}{r + p_1} e^{-(r+p_1+p_2)T}. \quad (22)$$

If, in addition,  $rp_2/(r + p_1 - p_2) < c/R$ , then the derivative is negative and  $W$  is decreasing in  $T$ . Thus, the venture capitalist would not finance entrepreneur 2 ( $T_2 = 0$ ), achieving value  $W = (p_1 R - c)/(r + p_1)$ . This proves statement (ii).

On the other hand, if  $rp_2/(r+p_1-p_2) > c/R$ , then (22) is positive and  $W$  is increasing in  $T$ . In that case both entrepreneurs will be financed infinitely long ( $T_1 = T_2 = \infty$ ), yielding value  $W = [(p_1 + p_2)R - 2c]/(r + p_1 + p_2)$ . This proves statement (iii) and completes the proof.  $\square$

*Proof of Proposition 3.* The proof is based on the following lemmas. Their proofs follow the current proof.

**Lemma 1.**  $T_k^* = \max\{T^*, \sigma_k\}$ .

**Lemma 2.**  $T_i^* \leq \sigma_i$  for both  $i = 1, 2$ .

**Lemma 3.** If  $T_1^* = T_2^* = T^*$  in optimum, then  $\sigma_1 = \sigma_2 = T^*$ .

The first claim of the proposition is identical to Lemma 2, whereas the second claim follows from Lemmas 1 and 3.  $\square$

*Proof of Lemma 1.* Let me fix any  $T = T_{-k} \geq 0$  and maximize  $\Pi$  with respect to  $T_k$  subject to the constraint  $T_k \geq T$ . As none of the values  $V^0$ ,  $E^{10}$ , and  $E^{20}$  depends on  $T_k$ , the derivative of  $\Pi$  with respect to  $T_k$  has a rather simple form

$$\frac{\partial \Pi}{\partial T_k} = \frac{\partial}{\partial T_k} (V^k - E^k e^{p_k T}) e^{-(r+p_1+p_2)T} = (p_k R - c - c e^{p_k T_k}) e^{-(r+p_k)T_k - p_{-k}T}. \quad (23)$$

Observe that the derivative is positive when  $0 \leq T_k < \sigma_k$  and negative when  $T_k > \sigma_k$ .<sup>19</sup> Let me distinguish two cases. First, if  $T \geq \sigma_k$ , then  $\Pi$  is decreasing in  $T_k$  for all  $T_k \in [T, \infty)$ . Thus, maximum is attained for  $T_k = T$ . Second, if  $T < \sigma_k$ , then the above observation implies that the maximum is attained for  $T_k = \sigma_k$ . Summing up, I obtain that it is optimal to choose  $T_k = \max\{T, \sigma_k\}$ .  $\square$

*Proof of Lemma 2.* Taking the derivative of  $\Pi$  with respect to the stopping time  $T$  yields

$$\begin{aligned} \frac{\partial \Pi}{\partial T} &= (p_{-k}R - c) e^{-(r+p_1+p_2)T} - c e^{-(r+p_k)T} \\ &\quad - \frac{p_{-k}(p_k R - c)}{r + p_k} (1 - e^{-(r+p_k)(T_k - T)}) e^{-(r+p_1+p_2)T} + \frac{p_{-k}C}{r} (1 - e^{-r(T_k - T)}) e^{-(r+p_{-k})T} \\ &= [p_{-k}R - c - c e^{p_{-k}T} - p_{-k}(V^k - E^k e^{p_k T})] e^{-(r+p_1+p_2)T}. \end{aligned} \quad (24)$$

Now assume to the contrary and without loss of generality that  $T_2^* > \sigma_2$  in optimum. Observe that due to Lemma 1 is it not possible that  $T_2^* > T_1^*$ , as then I would have  $T_2^* > \max\{T^*, \sigma_2\}$ . Thus  $T_1^* \geq T_2^* = T^*$  and  $k = 1$  in optimum.

<sup>19</sup>Note that this also holds when  $\sigma_k = 0$ .

I show that the derivative (24) is negative for  $T_1 = T_1^*$  and  $T = T^* = T_2^*$  (i.e.,  $k = 1$  and  $-k = 2$ ), which gives a contradiction, as the principal could benefit from slightly decreasing the stopping time  $T$ . By assumption  $T^* = T_2^* > \sigma_2$ , which implies that  $p_2R - c - ce^{p_2T} < 0$  for  $T = T^*$ . It is therefore sufficient to show that  $V^1 - E^1 e^{p_1T} \geq 0$  for  $T_1 = T_1^*$  and  $T = T^*$ .

Let me discuss two cases. First, if  $T^* \geq \sigma_1$ , it follows from Lemma 1 that  $T_1^* = T^*$ . Then I have from (12) and (16) that  $V^1 = E^1 = 0$ , and thus  $V^1 - E^1 e^{p_1T} = 0$ . Second, assume that  $T^* < \sigma_1$ . Then it follows from Lemma 1 that  $T_1^* = \sigma_1$ . Moreover, it follows from (23) that  $V^1 - E^1 e^{p_1T}$  is increasing in  $T_1$  for  $T_1 \in [T^*, \sigma_1]$ . Thus,  $V^1 - E^1 e^{p_1T}|_{T_1=\sigma_1} > V^1 - E^1 e^{p_1T}|_{T_1=T^*} = 0$ . This completes the proof.  $\square$

*Proof of Lemma 3.* As  $T_1^* = T_2^*$ , it follows from Lemma 1 that  $T_i^* \geq \sigma_i$  for both  $i = 1, 2$ . Moreover, due to Lemma 2 I have  $T_i^* \leq \sigma_1$  for  $i = 1, 2$ . Thus,  $T_1^* = \sigma_1$  and  $T_2^* = \sigma_2$ . The claim then follows directly.  $\square$

*Proof of Proposition 4.* (i) If  $p_1 \leq 2c/R$ , then I have  $\sigma_1 = \sigma_2 = 0$  and due to Proposition 3, no agent is financed.

(ii) As  $p_1 > 2c/R$ , it follows from Proposition 1 that the principal prefers financing agent 1 to financing no agent. Moreover, recall from the discussion below Proposition 1 that the principal's optimal profit is increasing in  $p_i$ . Thus, due to the assumption  $p_1 \geq p_2$ , the principal prefers financing only agent 1 to financing only agent 2. Therefore, in optimum, either only agent 1 is financed, or both agents are financed.

(iii) It follows from  $p_1 \geq p_2$  that  $rp_2/(r + p_1 - p_2) \leq p_1$ . Thus, I also have  $2c/R < p_1$ , which due to part (ii) means that agent 1 is financed in the optimum. Assume to the contrary that agent 2 is *not financed* in addition to agent 1. In this case, it follows from Proposition 1 that  $T_1^* = \sigma_1$  and  $T_2^* = T^* = 0$ . Now recall that  $(p_1R - c)e^{p_1\sigma_1} = c$  and evaluate the derivative (24) when  $k = 1$  and  $T_1 = \sigma_1$ :

$$\begin{aligned} & \left. \frac{\partial \Pi}{\partial T} \right|_{T_1=\sigma_1} \\ &= e^{-(r+p_1+p_2)T} \left[ p_2R - c - ce^{p_2T} - p_2 \left( \frac{p_1R - c}{r + p_1} - \frac{c}{r} e^{p_1T} + \frac{p_1c}{r(r + p_1)} e^{-r\sigma_1} e^{(r+p_1)T} \right) \right]. \end{aligned}$$

For  $T = 0$  I obtain

$$\begin{aligned} \left. \frac{\partial \Pi}{\partial T} \right|_{T_1=\sigma_1, T=0} &= p_2R - 2c - p_2 \left( \frac{p_1R - c}{r + p_1} - \frac{c}{r} + \frac{p_1c}{r(r + p_1)} e^{-r\sigma_1} \right) \\ &= \frac{p_2c}{r(r + p_1)} \left[ r^2 \frac{R}{c} + 2r + p_1(1 - e^{-r\sigma_1}) \right] - 2c. \end{aligned} \tag{25}$$

This is clearly positive when  $R/c > 2(r+p_1-p_2)/(rp_2)$ . Thus, the principal can benefit by providing a small stopping time  $T$  to agent 2. This contradicts the assumption  $T^* = 0$ .  $\square$

*Proof of Proposition 5.* Consider  $p_1 > 2c/R$  so that agent 1 would be financed as single. First recall from Proposition 2 that in such a case only agent 1 is financed in the first best if and only if

$$p_2 \leq \frac{r+p_1}{rR/c+1}. \quad (26)$$

Note that for  $p_1 > 2c/R$ , the right hand side of (26) is smaller than  $p_1$ . Moreover, it follows from (25) that a sufficient condition for financing both agents under moral hazard is<sup>20</sup>

$$\frac{2r(r+p_1)}{r^2R/c+2r+p_1(1-e^{-r\sigma_1})} \leq p_2. \quad (27)$$

I show that for  $p_1$  large enough, the left-hand side of (27) is smaller than the right-hand side of (26), which is equivalent to

$$p_1(1-e^{-r\sigma_1}) \geq r^2R/c. \quad (28)$$

In such a case there is indeed an open set of values of  $p_2$  where both (27) and (26) hold. Then agent 2 is financed under moral hazard, but not in the first best.

To complete the proof, let me analyze the left-hand side of (28) when  $p_1 \rightarrow \infty$ . Recall that then  $\sigma_1 \rightarrow 0$ , and thus  $1-e^{-r\sigma_1} \rightarrow 0$ . In addition, I have

$$\lim_{p_1 \rightarrow \infty} \frac{p_1(1-e^{-r\sigma_1})}{\log(p_1R/c-1)} = \lim_{\sigma_1 \rightarrow 0} \frac{1-e^{-r\sigma_1}}{\sigma_1} = r,$$

where the first equality follows from the definition of  $\sigma_1$  and the second equality follows from *L'Hôpital's rule*. This implies that the left-hand side of (28) is of the order  $\log p_1$  when  $p_1 \rightarrow \infty$ , while its right-hand side is a constant. Therefore, (28) indeed holds when  $p_1$  is large enough.  $\square$

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<sup>20</sup>Instead of the condition provided in Proposition 4, (ii), I use now this stronger (but also more complex) condition (27).



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