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MIDAS regressions with time-varying parameters: An application to corporate bond spreads and GDP in the Euro area*

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Abstract

Mixed-data sampling (MIDAS) regressions allow to estimate dynamic equations that explain a low-frequency variable by high-frequency variables and their lags. To account for temporal instabilities in this relationship, this paper discusses an extension to MIDAS with time-varying parameters, which follow random-walk processes. The non-linear functional forms in the MIDAS regression necessitate the use of non-linear filtering techniques. In this paper, the Particle filter is used to estimate the time-varying parameters in the model. Simulations with time-varying DGPs help to assess the properties of the estimation approach. A real-time application to the relationship between daily corporate bond spreads and quarterly GDP growth in the Euro area shows that the leading indicator property of the spreads ahead of GDP has diminished during the recent crisis. During that period, corporate bond spreads rather seem to be coincident indicators of GDP growth.

JEL Classification Codes: C51, C53, E37

Keywords: Mixed-data sampling, time-varying parameters, Particle filter, corporate bond spreads, real activity

1 Introduction

Mixed-data sampling (MIDAS) regressions allow to directly estimate dynamic equations with data sampled at different frequencies. If the differences in sampling frequencies are

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huge, functional lag polynomials are employed in order to keep the number of parameters to be estimated small, see Ghysels, Sinko, and Valkanov (2007). MIDAS is basically a reduced form single-equation approach and complements more complicated dynamic models to be estimated in state-space form, see the discussion in Bai, Ghysels and Wright (2013).

Recent applications of MIDAS regressions can be found in the financial literature, see for example Ghysels, Rubia, and Valkanov (2009) as well as Engle, Ghysels, and Sohn (2013) in the context of volatility forecasting and the link between macro fundamentals and volatility. In the macroeconomic literature, applications are often related to forecasting, see the recent contributions by Monteforte and Moretti (2012), Andreou, Ghysels, and Kourtellis (2013), and Kuzin, Marcellino, and Schumacher (2011, 2013), as well as the surveys in Armesto, Engemann, and Owyang (2010), and Andreou, Ghysels, and Kourtellis (2011).

A general difficulty in macro applications potentially suitable for the MIDAS approach is the inherent temporal instability in the relationship between GDP and many indicators. Many papers in the literature document that business cycle indicators typically possess information content over a certain time period, but have low predictive power in other periods, see, amongst others, Stock and Watson (2003, 2004), Banerjee and Marcellino (2006), D’Agostino, Giannone, and Surico (2006), Campbell (2007), and Kuzin, Marcellino, and Schumacher (2013).

To account for temporal instability, this paper extends the MIDAS approach to account for time-varying parameters. In particular, we let the model parameters evolve as random walks. Below, the approach will be called time-varying parameter (TVP-) MIDAS. In the literature, we can find at least three related approaches: Carriero, Clark, and Marcellino (2012) estimate a Bayesian mixed-frequency regression model with random-walk parameters. Compared to their work based on Bayesian shrinkage priors on the high-frequency lags, this paper is based on MIDAS weighting, which introduces a non-linearity in the equation. Galvao (2013) proposes smooth transition (ST-) MIDAS, where a transition function governs the change in some parameters of the MIDAS regression. Guerin and Marcellino (2013) propose Markov-Switching (MS-) MIDAS. Both approaches allow for recurrent switches between a small number of regimes over time. In contrast, the random-walk specification of the model parameters used in this paper implies that time variation does not occur around regimes and that parameters evolve more freely, see the general discussion of pros and cons of random-walk time variation in Primiceri (2005). This approach might be helpful in case recurrent regimes cannot be identified properly in the data, or if unprecedented one-time events lead to structural breaks. Another motivation of the random-walk time variation might come from a limited availability of data, in particular, short time series, which do not contain enough information to disentangle recurrent regimes. The empirical exercise below will face this difficulty. In practice, we can expect that the performance of the alternative approaches will depend on the data and the

kind of temporal instabilities in it. For example, in times of a crisis that evolves differently as crises in the past, one might favour more general time variation. With respect to the current crisis in many industrialized countries since 2008, some observers regard it as a "This time is different" (Reinhard and Rogoff, 2011) case, see also Ng and Wright (2013). On the other hand, typical business cycle regimes might be better captured with the MS- or ST-MIDAS. Following these arguments, the different approaches might be regarded as complementary tools. Compared to the standard MIDAS, the TVP variant could be used as a tool to check whether the implicit assumption of temporal stability in the standard MIDAS regression is met or not.

To estimate TVP-MIDAS, we have to account for the non-linearity of the model. Standard MIDAS is estimated by non-linear least squares (NLS), which is not applicable in the presence of non-linear functional forms with time variation. Instead, we cast the MIDAS equation and equations for time variation in the parameters together in state-space form and estimate the states by the Particle filter. Since the TVP-MIDAS has not been employed in the literature before, we provide simulation results that assess the properties of the filter under time-varying data-generating processes (DGPs).

To illustrate the properties of TVP-MIDAS further, we carry out an empirical exercise for the Euro area, where we investigate the leading properties of corporate bond spreads for GDP growth. In the recent literature, corporate bond spreads, defined as the difference in yields between various corporate bonds and government securities of comparable maturity, are regarded as good leading indicators of economic activity. Recent evidence for the US by Gilchrist, Yankov, and Zakrajsek (2009) and Faust, Gilchrist, Wright, and Zakrajsek (2013) indicates that corporate bond spreads contain substantial predictive power for real economic activity. Andreou et al. (2013) also find similar evidence for the US, taking into account the high-frequency nature of the spreads in time-constant MIDAS regressions. Based on different methods, Ng and Wright (2013) and Ng (2013) report declines in the predictive performance of spreads during the recent crisis in the US. For the Euro area, there is also increased interest in the information content of corporate bond spreads, see Buchmann (2011), Nicoletti, Passaro (2012), and Gilchrist, Mojon (2013). However, the data availability is in general worse in the Euro area compared to the US, as only short time series for corporate spreads are available. We expand on the existing Euro area results and discuss the time-varying information content of corporate bond spreads for GDP growth.

The paper proceeds as follows: In Section 2, time-varying parameter MIDAS is introduced. Section 3 discusses how we can assess the leading properties of high-frequency indicators in MIDAS. Section 4 discusses the estimation with the Particle filter, and Section 5 contains a small simulation exercise to evaluate how well time-varying DGPs can be estimated from simulated data. Section 5 contains the empirical application to corporate bond spreads and GDP growth in the Euro area. Section 6 concludes.

2 Time-varying parameter MIDAS regressions

The time-varying parameter approach expands on the standard MIDAS regression by Ghysels et al. (2007) based on the exponential Almon lag with normalized weights, see Marcellino and Guerin (2013) and Galvao (2013) for other MIDAS variants with time variation. In the MIDAS regression, we explain the low-frequency variable y by the high-frequency spread sp . If we assume random-walk time variation in MIDAS parameters, we obtain the following set of equations:

$$y_t = \gamma + \beta_t \sum_{k=0}^K c(k, K, \theta_{1,t}, \theta_{2,t}) L^{j/m} sp_t^{(m)} + \varepsilon_t \quad (1)$$

$$c(k, K, \theta_{1,t}, \theta_{2,t}) = \frac{\exp(\theta_{1,t}k^1 + \theta_{2,t}k^2)}{\sum_{j=0}^K \exp(\theta_{1,t}j^1 + \theta_{2,t}j^2)} \quad (2)$$

$$\theta_{i,t} = \theta_{i,t-1} + \epsilon_{i,t} \text{ for } i = 1, 2 \text{ with } \theta_{2,t} < 0 \quad (3)$$

$$\beta_t = \beta_{t-1} + \epsilon_{\beta,t} \quad (4)$$

with K lags of the high-frequency variable $sp_t^{(m)}$, which is sampled m times more often than y_t . Time index t refers to the available time periods of the low-frequency variable y . The difference in sampling frequencies m between y and sp depends on the data at hand. For example, Andreou et al. (2013) and Galvao (2013) mix quarterly GDP growth with daily data, which leads to a mixed-frequency problem with $m = 60$ times more daily observations of $sp_t^{(60)}$ than y_t . Equation (1) relates y_t in period t to high-frequency lags of $sp_t^{(60)}$. These lags of the high-frequency variable are denoted by $L^{1/m} sp_t^{(m)}$, where the lag operator is defined as $L^{1/m} sp_t^{(m)} = sp_{t-1/m}^{(m)}$. Thus, high-frequency time periods are fractions of low-frequency periods t , see Ghysels et al. (2007).

In (1), γ is a constant, and β_t is a regression coefficient, which is multiplied by weighted lags of the high-frequency variable $sp_t^{(m)}$. The weights (2) are defined as the exponential Almon lag weight function $c(k, K, \theta_{1,t}, \theta_{2,t})$ where $k = 0, \dots, K$, see Ghysels et al. (2007). Furthermore, they are normalized to sum to one. The parsimonious exponential Almon specification $c(k, K, \theta_{1,t}, \theta_{2,t})$ in (2) allows to approximate $K + 1$ lag coefficients by the chosen parametric form with two shape parameters $\theta_{1,t}$ and $\theta_{2,t}$. Following (3), the shape parameters evolve as random-walks as well as the regression coefficient β_t in equation (4).

The stability of the weight function is ensured by the negative of the leading coefficient, in particular, the restriction $\theta_{2,t} < 0$, see Lütkepohl (1981). To impose this restriction, we estimate the model with a newly defined variable $\theta_{2,t}^*$ by using $\theta_{2,t} = -\exp(\theta_{2,t}^*)$ in (1). The additional equation $\theta_{2,t}^* = \theta_{2,t-1}^* + \epsilon_{2,t}$ replaces the equation for $\theta_{2,t}$ in (3), see Judge

et al. (1985):

$$c(k, K, \theta_{1,t}, \theta_{2,t}^*) = \frac{\exp(\theta_{1,t}k^1 - \exp(\theta_{2,t}^*)k^2)}{\sum_{j=0}^K \exp(\theta_{1,t}j^1 - \exp(\theta_{2,t}^*)j^2)} \quad (5)$$

$$\theta_{1,t} = \theta_{1,t-1} + \epsilon_{1,t}, \quad \theta_{2,t}^* = \theta_{2,t-1}^* + \epsilon_{2,t} \quad (6)$$

The exponential Almon function is known to be quite flexible and can take various shapes even with two parameters, see Ghysels et al. (2007). By construction of $c(k, K, \theta_{1,t}, \theta_{2,t}^*)$, the weights sum to one. If we leave this constraint aside, the weights are equal to the exponential Almon weights that have been introduced to the distributed lag literature by Lütkepohl (1981).¹

The errors ε_t , $\epsilon_{\beta,t}$, $\epsilon_{1,t}$, and $\epsilon_{2,t}$ are assumed to be mutually independent and follow normal distributions with mean zero and variance σ_ε^2 and σ_β^2 , σ_1^2 , and σ_2^2 , respectively.

Potential extensions to the TVP-MIDAS model include autoregressive terms as in Clements and Galvao (2008) or Andreou et al. (2013). Further refinements include stochastic volatility, which can be introduced to MIDAS as in Carriero et al. (2012).

3 Leading indicators in MIDAS regressions

The main use of TVP-MIDAS below will be the empirical assessment of the leading properties of business cycle indicators. For this purpose, we need a definition of leading indicators. A useful definition is provided in Box and Jenkins (1976): If a lag of a predictor sp receives non-zero weight in a regression, changes in y will be anticipated by changes in sp , and we can call sp_t a leading indicator. Related to the MIDAS equation (1) above, we can investigate the importance of the high-frequency lags $L^{k/m}sp_t^{(m)}$ for $k = 0, \dots, K$. Typically a graphical inspection of the estimated weights helps to assess, which weights receive a lot of weight and which don't.

In addition, as a summary statistic of the leading properties in the MIDAS equation (1) above, we can compute the high-frequency lead of the indicator, which receives the maximum weight according to

$$\frac{\partial c(k, K, \theta_{1,t}, \theta_{2,t}^*)}{\partial k} = 0 \Leftrightarrow k_{\max c,t} = -\frac{\theta_{1,t}}{2(-\exp(\theta_{2,t}^*))}. \quad (7)$$

Due to the unimodal shape with two coefficients $\theta_{1,t}$ and $\theta_{2,t}^*$, we generally obtain a unique solution.² The lead with maximum weight $k_{\max c,t}$ is measured in terms of high-frequency lags, for example, days in the case of $m = 60$ in case we mix quarterly and daily data.

¹Other weighting functions include Beta weights in Ghysels et al. (2007), non-exponential Almon in Drechsel and Scheufele (2013), penalized changes in weights, see Breitung, Elengikal, and Roling (2013), as well as unrestricted weights in Foroni, Marcellino, and Schumacher (2013).

²If both $\theta_{1,t}$ and $\theta_{2,t}^*$ converge to zero, we obtain equal weights and cannot identify a lead with maximum weight. This flat-weight case is discussed in Andreou et al. (2010).

Due to the time-varying nature of the model in (1), (5), (6) and (4), $k_{\max c,t}$ can change over time. Furthermore, we can obtain positive and negative signs of $k_{\max c,t}$. A positive sign implies a lead of the indicator ahead of GDP growth, whereas a negative sign implies a lagging indicator.³ A value of $k_{\max c,t} = 0$ implies that the indicator is rather coincident.

Note that the computation of the lead with maximum weight $k_{\max c,t}$ only involves the weight function $c(k, K, \theta_{1,t}, \theta_{2,t}^*)$. A significant effect of a lag of s on y is only established, if the regression coefficient β_t is significantly different from zero, which has to be tested.

Compared to regressions based on time-aggregated data, for example quarterly data only, the MIDAS regression allows a more detailed assessment of the leading indicator. The lead $k_{\max c,t}$ represents a high-frequency statistic, whereas the lead which attains maximum weight in a time-aggregated regression is a low-frequency (e.g. quarterly) statistic. Thus, we have the opportunity to assess leading behavior of indicators more detailed with MIDAS than with time-aggregated regressions, depending on the uncertainty of the estimates.

4 Estimation by the Particle filter

The TVP-MIDAS in equations (1), (5), (6) and (4) can be cast in a non-linear state-space form with the three-dimensional state vector $(\beta_t, \theta_{1,t}, \theta_{2,t}^*)$. The transition equation in this model is fully linear. Only the observation equation is non-linear in $\theta_{1,t}$ and $\theta_{2,t}^*$ due to the exponential Almon lags, see equation (1).

Whereas Linear TVP models are typically estimated using the standard Kalman filter, we need another solution for the non-linear TVP-MIDAS. In the applications below, I will make use of the Particle filter by Liu and West (2001). The Particle filter is a Monte Carlo method for nonlinear filtering, which is based on sequential versions of the importance sampling paradigm. This technique amounts to simulating samples of states under an proposal distribution and then approximating the target distributions, which usually impossible to sample from, by weighting these samples using appropriately defined importance weights.

The propagation of the samples makes use of the state-space dynamics in the models, and the weights for the particles are updated against new incoming data. In this the sense, these algorithms can be implemented sequentially. By defining appropriately a sequence of proposal distributions, it is not necessary to regenerate the population of samples from scratch upon the arrival of each new observation.

In this literature, Liu and West (2001) propose to sequentially resample and propagate particles associated with the states as well as model parameters simultaneously. In our case, we have to estimate the variances of the shocks σ_ε^2 and σ_β^2 , σ_1^2 , and σ_2^2 as well as the

³Note that $k_{\max c,t}$ can obtain negative values despite the fact that the MIDAS regression only considers $k = 0, \dots, K$ lags of the indicator on the right-hand side. The reason is that $k_{\max c,t}$ is obtained from an analytical solution based on the estimated θ_1 and θ_2 .

constant. Estimation of these parameters proceeds in a sequential way in a similar way as the states. A further attraction of the filter is its generality as it can be implemented in any state-space model, see Lopes and Tsay (2011) for a review. The details of the algorithm are described in the Appendix.

Note that the state-space system is non-linear in the states $\theta_{1,t}$ and $\theta_{2,t}^*$. Thus, it is not conditionally linear and necessitates the use of approximate filters. However, the Particle filter could be made more efficient, if the linear state variables are marginalized. This so-called Rao-Blackwellization can be implemented by running the standard Kalman filter to the linear subsystem conditional on the linear states. In the TVP-MIDAS framework above, this extension is left for future work.

5 Simulations

To get an impression of the quantitative properties of the Particle filter and TVP-MIDAS, some simulation results are provided. In particular, we want to investigate to what extent time variation in the DGP can be detected by the estimated model. We simulate $R = 500$ datasets based on DGPs defined in the TVP-MIDAS equations above for y_t with $T = 200$. The daily indicator s_t follows an AR(1) process with intermediate autocorrelation according to

$$sp_t = 0.5sp_{t-1/m} + \sqrt{0.5}\varepsilon_{sp,t} \quad (8)$$

where $\varepsilon_{sp,t}$ is $N(0, 1)$ distributed. We assume $m = 60$, thus mimicking mixed-data sampling of quarterly and daily data with periods $t = 1/60, 2/60, \dots, T - 1/60, T$ in equation (8).

The endogenous variable y_t is simulated for periods $t = 1, 2, 3, \dots, T$ as defined in (1), (5), (6) and (4). For the parameters of TVP-MIDAS, we consider different time-varying weight shapes through $\theta_{1,t}$ and $\theta_{2,t}^*$ and time-varying information contents of the indicator sp_t for y_t through β_t . In the weight function, the maximum lag order is set to $K = 240$ days. To define alternative shapes of the weight function and different regression coefficients, we specify two different DGPs:

1. DGP 1: In this DGP, the coefficient β_t is decreasing over time, implying that the information content of the indicator for y_t goes down over time given normalized weights. In detail, we define

$$\beta_t = \begin{cases} 0.5 & \text{for } t = 1, \dots, T/4 \\ -0.003 + \beta_{t-1} & \text{for } t = T/4 + 1, \dots, T \times 3/4 \\ 0.2 & \text{for } t = T \times 3/4 + 1, \dots, T \end{cases} \quad (9)$$

With respect to the shape, we keep the $\theta_{1,t}$ and $\theta_{2,t}^*$ fixed over time. The weight function has a hump shape weights pattern with a maximum at lag $k = 200$ days obtained from $\theta_{1,t} = 0.133$ and $\theta_{2,t}^* = -0.00033$ for all $t = 1, \dots, T$. The resulting

shape of the weight function is shown in the bottom panels of Figures 1 and 2 below in green color.

2. DGP 2: The coefficient β_t is again decreasing over time as in DGP 1. In this DGP, however, the shape of the weights also change over time. In particular, we simulate that the lead of the indicator $k_{\max c, t}$ is going down by fixing $\theta_{2, t}^* = -0.00033$ for all t as before, but $\theta_{1, t}$ is declining according to

$$\theta_{1, t} = \begin{cases} 0.133 & \text{for } t = 1, \dots, T/4 \\ -0.0011 + \theta_{1, t-1} & \text{for } t = T/4 + 1, \dots, T \times 3/4 \\ 0.027 & \text{for } t = T \times 3/4 + 1, \dots, T \end{cases} \quad (10)$$

This corresponds to a lead with maximum weight at time $t = T \times 3/4 + 1$ of $k_{\max c, t} = 40$ days, which has declined from $k_{\max c, t} = 200$ days at $t = T/4$.

The constant in the equation is set to $\gamma = 0.2$ for all t . The residual $\varepsilon_{y, t}$ is normally distributed with mean zero and the variance is fixed such that the population R^2 in the DGP is equal to 0.66. Given these DGPs, we can simulate $\{\{sp_t\}_{t=1}^T, \{y_t\}_{t=1}^T\}_{r=1}^R$. For each replication, the Particle filter is used to estimate TVP-MIDAS. To initialize the filter, we estimate a time-constant MIDAS regression over the first part of the sample $t = 1, \dots, T/4$, whose point estimates provide the starting values of the particle sets. The initial variances of the shocks in the time-varying parameter equations are all set to 10^{-5} . The residual covariance provides a starting value for σ_ε^2 in the Particle filter. From the sampled particle distributions, we compute the mean across particles to obtain a point estimate of the following quantities: The time-varying parameter β_t as well as the weights $c(k, K, \theta_{1, t}, \theta_{2, t}^*)$ and the corresponding maximum weight lead $k_{\max c, t}$.

To assess the filter estimates for both DGPs, we compare the 5-th, 50-th and 95-th percentiles of the simulated distributions across the R replications for estimated β_t as well as the weights $c(k, K, \theta_{1, t}, \theta_{2, t}^*)$ and $k_{\max c, t}$ against the true quantities in the DGP as defined above. In the two Figures 1 and 2 below, the true quantities are shown in green, whereas the median as well as the 5-th and 95-th percentiles of the estimated distributions are shown in blue.

In the simulations, we find no huge biases. The true parameters are close to the median of the distributions of estimates. The percentiles indicate that the estimation uncertainty is not too big. When it comes to tracking the time-varying parameters in the DGP, there is always some lagged adjustment in the estimates over those time periods with a change in the parameters. However, the magnitude of the deviations is quite small. Overall, the parameter estimates of TVP-MIDAS with the Particle filter follow the time variation in the true parameters closely.

Figure 1: Simulation results, DGP 1

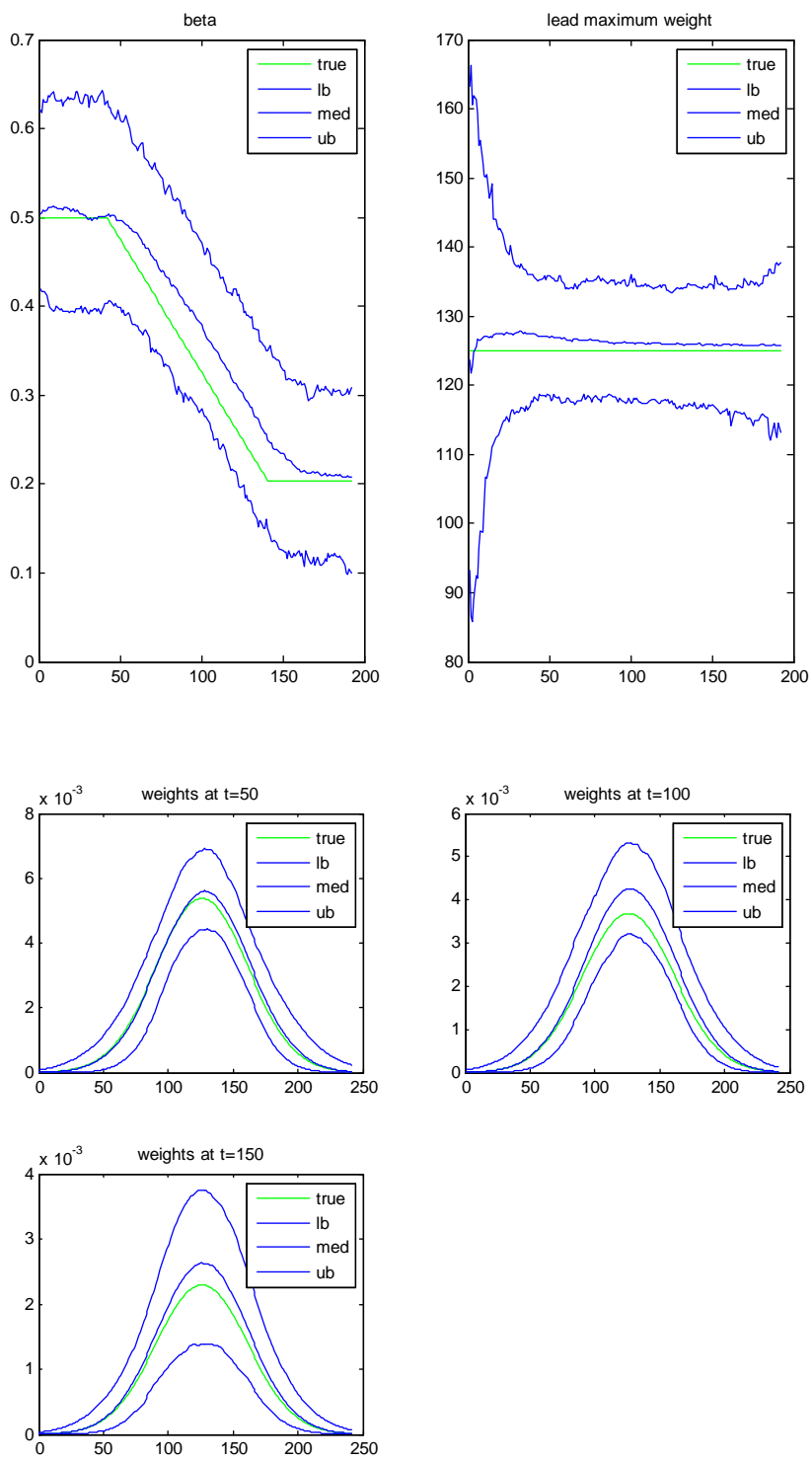
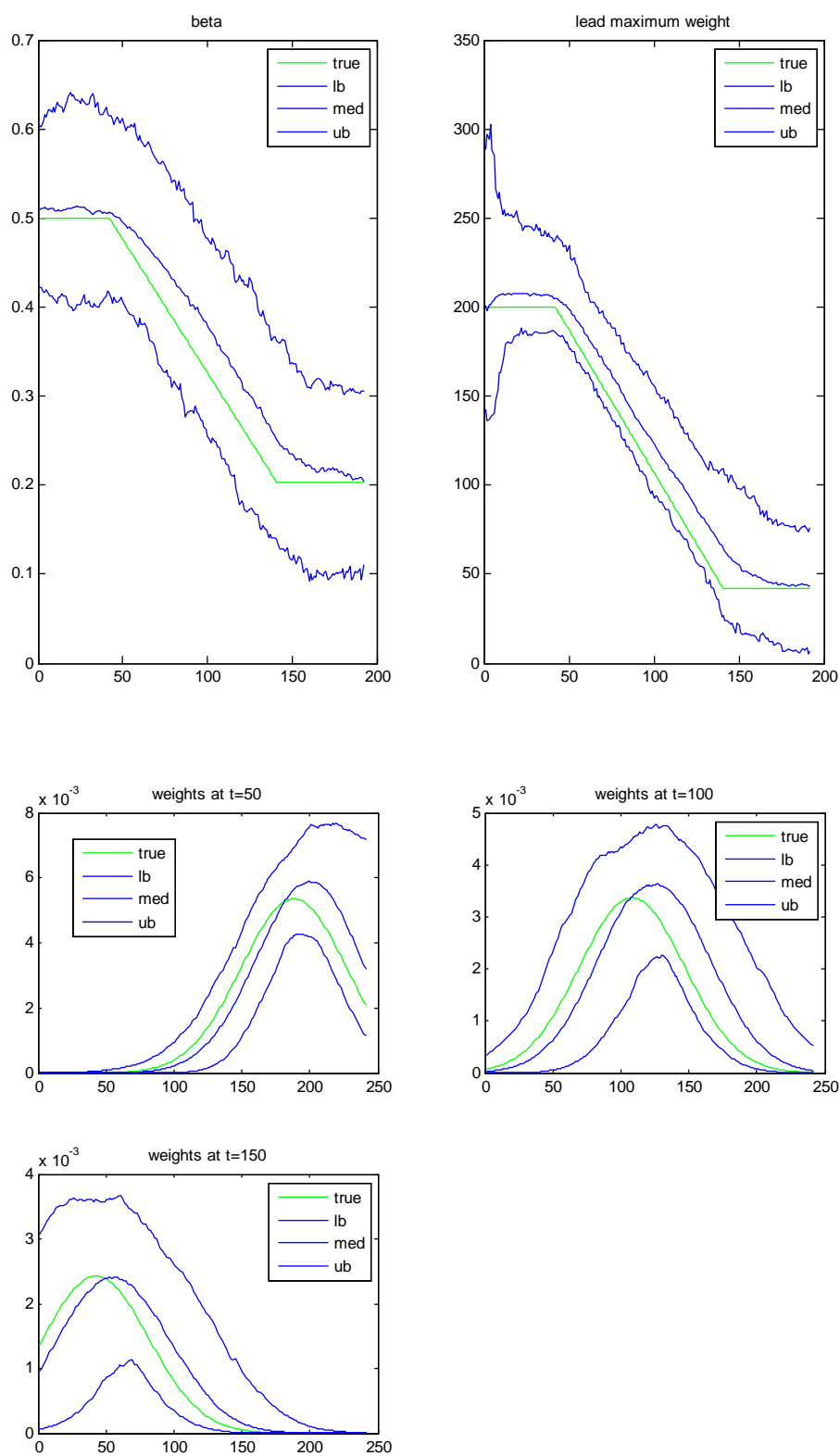


Figure 2: Simulation results, DGP 2



6 Empirical application: Corporate bond spreads and GDP growth in the Euro area

What kind of empirical relationship can we expect from our results? There are various ways to establish a theoretical motivation for the important role of corporate bond spreads for real economic activity. An increase in corporate bond spreads might reflect a reduction in the expected present value of corporate cash flows leading to a cyclical downturn. Using a modification of the q -theory of investment, Philippon (2009) shows how bond spreads can be used to construct a market-based measure of Tobin's q . Increasing credit spreads can also reflect disruptions in the supply of credit resulting from the worsening in the quality of corporate balance sheets or of financial intermediaries that supply credit to the firms. Thus, the financial accelerator by Bernanke et al. (1999) also provides a motivation for the relationship between corporate bond spreads and real activity. Following these two lines of arguments, we can expect bond spreads to be counter-cyclical and leading the business cycle. Below we address whether this is the case in Euro area data following Buchmann (2011), Nicoletti, Passaro (2012), and Gilchrist, Mojon (2013).

6.1 Data

The GDP time series is quarterly Gross Domestic Product for the Euro area at market prices (chain-linked volumes), transformed to quarter-on-quarter growth rates. The time series is also working day and seasonally adjusted, and has been taken from the ECB Statistical Data Warehouse. The sample starts in 1997Q1 and ends in 2012Q4.

The corporate bond yield series used below are option-adjusted spreads provided by Merrill Lynch. The indices cover the financial sector (banks and insurance companies), utility and industrials. Bond spreads are then calculated from corporate bond yields and the relevant benchmark government bond yield series by Merrill Lynch. The corporate spreads are available for different rating classes and maturity buckets: In the data available here, the rating classes include AAA, AA, A, BBB. The maturity buckets range from 1-3, 3-5, 5-7, and 7-10 years. We end up with 20 different yield spreads with all combinations of rating and maturity buckets. The spread dataset is chosen as in Nicoletti, Passaro (2012), and Buchmann (2011). However, in contrast to that work, we directly use the spreads sampled at daily frequency without time aggregation.

6.2 Model and filter setup

The empirical estimation results are based on a specification of TVP-MIDAS, where the absolute term in the equation is also time-varying. Furthermore, the equation contains

an autoregressive term with coefficient λ_t :

$$y_{t+1} = \gamma_t + \alpha_t y_t + \beta_t \sum_{j=0}^K \frac{\exp(\theta_{1,t} j^1 - \exp(\theta_{2,t}^*) j^2)}{\sum_{k=0}^K \exp(\theta_{1,t} k^1 - \exp(\theta_{2,t}^*) k^2)} L^{j/m} sp_t^{(m)} + \varepsilon_{t+1} \quad (11)$$

$$\theta_{1,t} = \theta_{1,t-1} + \epsilon_{1,t}, \theta_{2,t}^* = \theta_{2,t-1}^* + \epsilon_{2,t} \quad (12)$$

$$\beta_t = \beta_{t-1} + \epsilon_{\beta,t}, \gamma_t = \gamma_{t-1} + \epsilon_{\gamma,t}, \alpha_t = \alpha_{t-1} + \epsilon_{\alpha,t} \quad (13)$$

As before, the shocks ε_t , $\epsilon_{\beta,t}$, $\epsilon_{1,t}$, $\epsilon_{2,t}$, $\epsilon_{\lambda,t}$, and $\epsilon_{\gamma,t}$ are assumed to follow independent normal distributions with mean zero and variances σ_y , σ_β , σ_1 , σ_2 , σ_α , and σ_γ , respectively. Given this model and data, I want to investigate the relationship between the different corporate bond spreads and GDP growth in the Euro area. To this end, the estimated paths of the regression coefficient β_t , the weights $c(k, K, \theta_{1,t}, \theta_{2,t}^*)$, and the lead with maximum weight $k_{\max c,t}$ from the Particle filter are examined. Further results are provided in the Appendix. The results contain the mean over 5000 particles as well as the 5-th and 95-th percentiles of the particle distribution. In the estimation, we use $K = 315$ days. To initialize the Particle filter, we again use a MIDAS regressions without time variation in the parameters as above in the simulations.

6.3 Empirical results

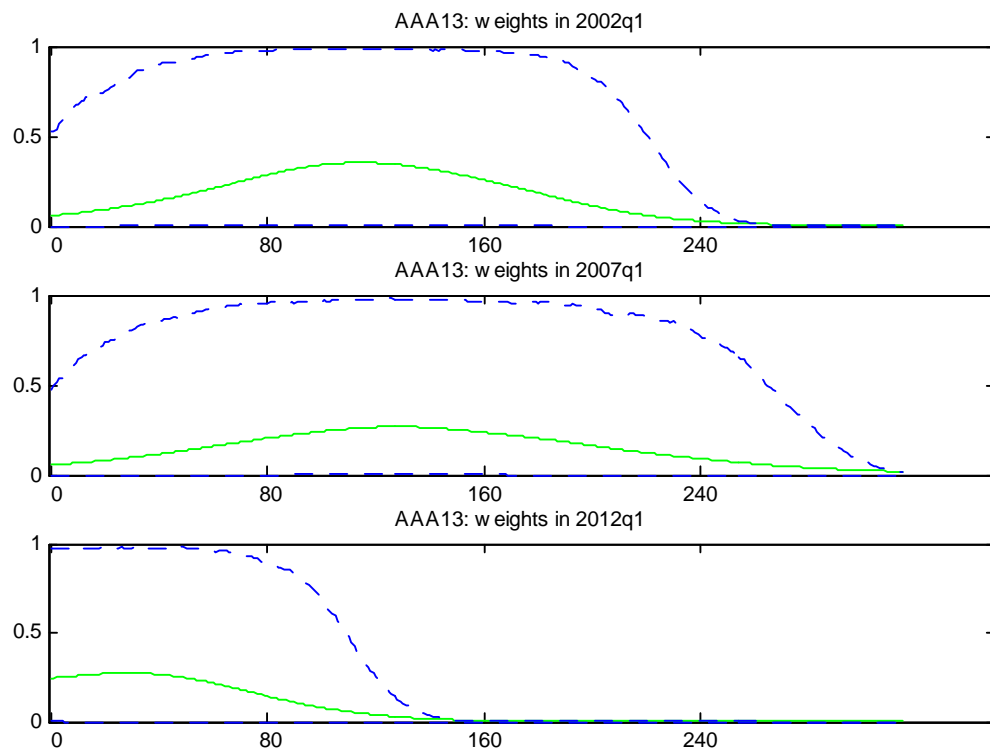
The first set of results is based on one representative corporate spread, namely the AAA spread with maturity bucket 1-3 years. The rest of the spreads will be covered in the later parts of the text. In Figure 3, the estimation results for the regression coefficient β_t are shown. The green line is the mean over the particles, and dotted blue lines are the boundaries of the 90% confidence interval derived from the particle distribution. The Figure shows the expected negative sign of the parameter over the whole sample. Until 2008, the parameter increases in absolute terms from -0.2 to -0.5 and returns to -0.3 afterwards. Apart from these movements, the parameter is in all the periods significantly different from zero, thus indicating a time-varying but relevant relationship between the spread and GDP growth. The width of the confidence interval is considerable, indicating severe estimation uncertainty.

In Figure 4, the evolution of the weights $c(k, K, \theta_{1,t}, \theta_{2,t}^*)$ is shown for three points in time, namely, 2002Q1, 2007Q1 and 2012Q1. In each period, the weights are shown as a function of the lags $k = 0, \dots, 315$. The green line is the mean over the particles of the weights, and dotted lines are the boundaries of the 90% confidence interval. In 2002Q1 and 2007Q1, the weights have a maximum at lag 112, which is hardly different from zero. In 2002Q1, lags greater than 260 days are equal to zero with high probability, whereas in 2007Q1, the uncertainty is very high across all lags. In 2012Q1, the peak of the weights has shifted towards zero, and the lower 5% bound lies on the zero line. Lags greater than 150 days are equal to zero with high probability. Thus, the effect of the spreads on GDP

Figure 3: Evolution of the estimates of the regression coefficient over time



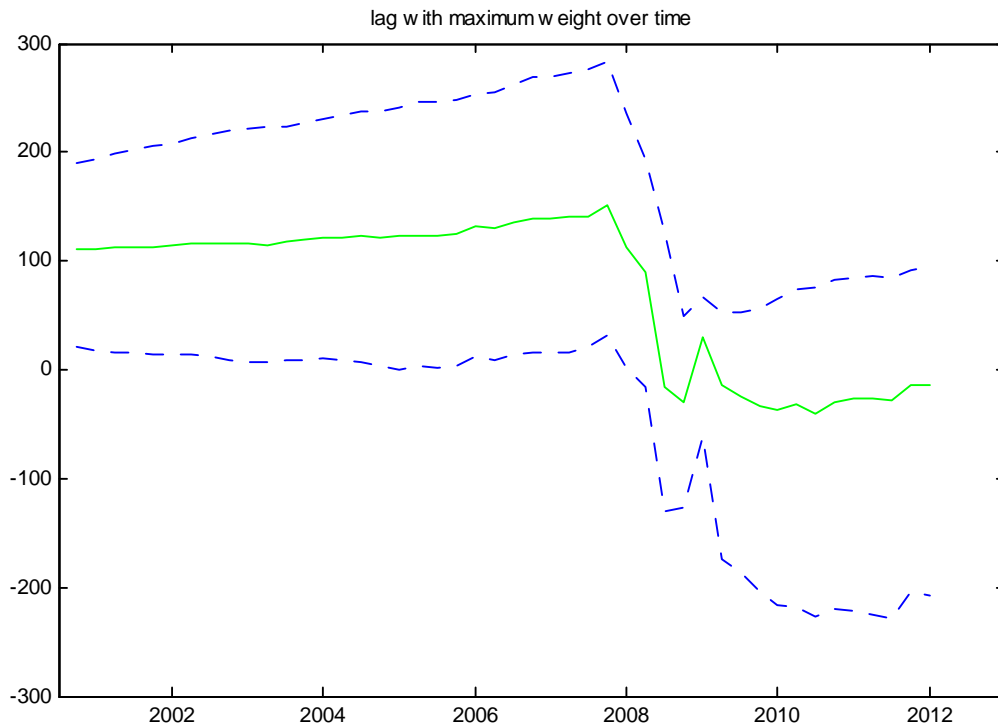
Figure 4: Evolution of the Almon lag weights over time



growth has become more concentrated on a smaller set of lags in 2012Q1 compared to the periods 2002Q1 and 2007Q1. In general, the Figures show a considerable uncertainty surrounding the weights.

To assess the lead of the spread ahead of GDP growth further, the lead at maximum weight introduced in Section 3 is shown in Figure 5. The vertical axis in the Figure reflects daily leads or lags of the indicator relative to GDP growth. Positive numbers indicate leads of the daily indicator ahead of GDP growth, negative numbers a lag. Prior to 2008,

Figure 5: Evolution of the lead with maximum weight over time



the lead with maximum weight $k_{\max c,t}$ is about one hundred days, where the confidence interval excludes zero. During and after 2008, the lead of the AAA spread declines sharply and is insignificantly different from zero. Thus, during the recent crisis, the spread has turned from a leading into a coincident indicator of GDP, following the definition of Box and Jenkins (1976).

In the Appendix we provide further results regarding the evolution of the other parameters in the estimated TVP-MIDAS model. The results indicate that time variation comes mostly from the weights, whereas the other parameters do not vary as much. To summarize the empirical findings with the AAA spread and 1-3 years maturity, we can state that there is a negative relationship between the spread and GDP growth. However, there is considerable uncertainty as to the exact weighting of high-frequency lags. Furthermore, during the recent crisis, the lead ahead of GDP growth has diminished.

Now, we turn to the rest of the spreads. The estimation results for the regression coefficient β_t and the lead with maximum weight $k_{\max c,t}$ are summarized for all the spreads in Table 1. In particular, estimation results are shown for two periods before and after

Table 1: beta and lead with maximum weight over time

	β_t				$k_{\max c,t}$			
	2002Q1		2012Q1		2002Q1		2012Q1	
AAA 1-3	-0.25	(-0.40,-0.07)	-0.33	(-0.42,-0.24)	112.3	(15.7,199.3)	-26.8	(-224.9,86.3)
AAA 3-5	-0.28	(-0.42,-0.11)	-0.34	(-0.44,-0.23)	107.3	(11.9,206.1)	-75.5	(-192.6,50.0)
AAA 5-7	-0.24	(-0.40,-0.06)	-0.24	(-0.33,-0.17)	111.3	(12.1,215.2)	-65.2	(-193.8,51.2)
AAA 7-10	-0.22	(-0.40,-0.04)	-0.01	(-0.07,0.04)	105.6	(11.5,198.4)	101.6	(13.5,189.1)
AA 1-3	-0.31	(-0.54,-0.14)	-0.43	(-0.55,-0.31)	120.4	(33.7,211.8)	-35.5	(-151.1,57.1)
AA 3-5	-0.08	(-0.21,0.02)	0.00	(-0.10,0.09)	115.1	(16.2,208.1)	107.7	(-5.9,300.5)
AA 5-7	-0.24	(-0.41,-0.06)	-0.24	(-0.38,-0.15)	117.9	(19.1,220.2)	-10.6	(-123.4,83.2)
AA 7-10	-0.23	(-0.39,-0.06)	-0.28	(-0.38,-0.16)	122.9	(22.1,231.7)	-74.2	(-193.3,41.4)
A 1-3	-0.25	(-0.42,-0.08)	-0.32	(-0.43,-0.19)	100.4	(7.1,201.2)	-45.2	(-135.7,34.9)
A 3-5	-0.21	(-0.38,-0.05)	-0.24	(-0.34,-0.14)	90.5	(-7.4,191.8)	-62.1	(-181.2,58.6)
A 5-7	-0.26	(-0.42,-0.09)	-0.42	(-0.51,-0.33)	89.5	(-5.1,191.2)	-80.1	(-178.4,16.3)
A 7-10	-0.24	(-0.41,-0.07)	-0.38	(-0.48,-0.24)	109.6	(19.4,206.4)	-71.8	(-176.3,19.9)
BBB 1-3	-0.20	(-0.35,-0.03)	-0.06	(-0.17,0.03)	93.3	(1.8,189.4)	7.1	(-141.9,196.7)
BBB 3-5	-0.27	(-0.42,-0.09)	-0.26	(-0.41,-0.10)	112.2	(32.3,205.1)	-28.2	(-120.3,65.1)
BBB 5-7	-0.20	(-0.38,-0.03)	-0.11	(-0.16,-0.05)	88.6	(-3.1,190.2)	19.2	(-28.3,68.2)
BBB 7-10	-0.18	(-0.36,-0.01)	-0.01	(-0.16,0.06)	100.1	(12.4,195.8)	309.4	(-12.6,476.3)

The table contains estimation results for the regression coefficient and the lead with maximum weight, estimated at different points in the sample. The first entry in a row is the name of the spread, followed by the mean across the particles, whereas the 90% confidence interval bounds are shown in brackets.

the beginning of the recent crisis, namely 2002Q1 and 2012Q1. In each row of the Table, the first entry is the name of the spread, followed by the mean of β_t and $k_{\max c,t}$ across the particles, whereas the 90% confidence interval bounds are shown in brackets. When looking at the coefficient β_t , we can see that it is in almost all of the cases (apart from one) significant in quarter 2002Q1, whereas it is insignificant in twelve out of twenty cases in the period 2012Q1. The lead with maximum weight $k_{\max c,t}$ is significantly different from zero in all but four cases in 2002Q1, but significant in 2012Q1 in only one case.

Hence, the results are relatively robust across the different maturities and quality groups of the spreads. The relationship between the spreads and GDP has weakened during the crisis, but is still significantly negative in the majority of cases. At the same time, however, the spreads have lost their lead ahead of GDP. Before the crisis, we estimate a lead of the indicator of broadly 100 days, which has vanished after 2008.

7 Conclusions

This paper discusses mixed-data sampling (MIDAS) regressions with time-varying parameters (TVP-MIDAS). This approach introduces random-walk time variation to the

MIDAS approach and thus complements the literature that has recently been focused on regime switching or smooth transition MIDAS models, see Marcellino and Guerin (2013) and Galvao (2013). To estimate the non-linear model, the Particle filter with sequential parameter estimation following Liu and West (2001) is applied. Simulations show that the Particle filter can estimate the parameters well even if the DGP varies over time in different ways.

As an empirical application of TVP-MIDAS, we investigate the leading indicator properties of corporate bond spreads for GDP growth in the Euro area over the last ten years, including the recent crisis period. In the results, we find a decline in the lead of corporate bond spreads for GDP growth in the Euro area during the recent crisis. In other words, corporate bond spreads have been leading indicators of real activity before the crisis, but during the crisis they have rather become coincident indicators. The results are in line with findings by Ng and Wright (2013) and Ng (2013) for the US, where also a decline in the leading properties of corporate bond spreads has been found.

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A The Particle filter by Liu and West (2001)

Assume we have a state-space model with the vector of observables y_t and states x_t . Time-varying parameters are part of the state. A vector of parameters which is constant over time is denoted as ϕ . The goal of the filter by Liu and West (2001) is to approximate the posterior

$$p(x_t, \phi | D_t) \quad (14)$$

for both the states x_t and the parameters ϕ . The period- t information is summarized in $D_t = \{D_{t-1}, y_t\}$. The state-space model is specified at each period t by the observation and transition densities

$$p(y_t | x_t, \phi), p(x_t | x_{t-1}, \phi), \quad (15)$$

respectively, based on the state-space model equations and underlying distributions of shocks. Sequential Monte Carlo methods aim at sequentially updating Monte Carlo sample approximations to the sequences of $p(x_t, \phi | D_t)$, represented by a discrete sample of points and weights. On observing the new data y_{t+1} , a new sample $p(x_{t+1}, \phi | D_{t+1})$ is needed.

To motivate the steps of the filter, Liu and West (2001) rely on the decomposition

$$p(x_{t+1}, \phi | D_{t+1}) \propto p(y_{t+1} | x_{t+1}, \phi) p(x_{t+1} | \phi, D_t) p(\phi | D_t) \quad (16)$$

Samples from the first two terms can be obtained by the auxiliary Particle filter proposed by Pitt and Shephard (1999). To sample from $p(\phi | D_t)$, Liu and West (2001) assume an artificial random-walk time variation for ϕ with little variance. As this adds noise to the estimation, kernel smoothing is applied to reduce the variance of the estimates, see step 3 below. Thus, the filter by Liu and West (2001) sequentially provides particles as well as parameter samples. In order to follow the evolution of the fixed-parameter values, an index t is added to ϕ below. The filter works as follows:

1. Given the set of particles, parameters, and weights for period $t - 1$

$$\{x_{t-1}^{(j)}, \phi_{t-1}^{(j)}, w_{t-1}^{(j)}\}_{j=1}^P \quad (17)$$

compute the state and parameter quantities

$$\mu_t^{(j)} = E(x_t | x_{t-1}^{(j)}, \phi_{t-1}^{(j)}), \quad (18)$$

$$m_{t-1}^{(j)} = a\phi_{t-1}^{(j)} + (1-a)\bar{\phi}_{t-1}, \quad (19)$$

where $\bar{\phi}_{t-1} = \frac{1}{P} \sum_{j=1}^P \phi_{t-1}^{(j)}$ and $a = \sqrt{1-h^2}$ with $h^2 = 1 - ((3\delta - 1)/2\delta)^2$. The discount factor δ has to be fixed by the model user. Typical values range between 0.95 to 0.99, see Liu and West (2001). Both in the simulations and empirical applications, we use $\delta = 0.95$. $\mu_t^{(j)}$ is an auxiliary estimate of the state x_t as in the filter

proposed by Pitt and Shephard (1999), see step 2 below. $m_{t-1}^{(j)}$ helps to retain the mean of the fixed parameters posterior in the normal mixture in step 3 below.

2. Sample an integer k from the set $\{1, \dots, P\}$ with probabilities

$$g_t^{(j)} \propto w_{t-1}^{(j)} p(y_t | \mu_t^{(j)}, m_{t-1}^{(j)}) \quad (20)$$

using the observation density.

3. Sample a new parameter from the k -th normal kernel density

$$\phi_t^{(k)} \sim N(\cdot | m_{t-1}^{(k)}, h^2 V_{t-1}), \quad (21)$$

where h^2 is defined in step 1. The variance V_{t-1} is defined as $V_{t-1} = \frac{1}{P} \sum_{j=1}^P (\phi_{t-1}^{(j)} - \bar{\phi}_{t-1})(\phi_{t-1}^{(j)} - \bar{\phi}_{t-1})'$. The parameters $m_{t-1}^{(k)}$ and h^2 are chosen such that the approximation of ϕ has the kernel form $p(\phi | D_{t-1}) \approx \sum_{j=1}^P w_{t-1}^{(j)} N(\cdot | m_{t-1}^{(j)}, h^2 V_{t-1})$.

4. Sample a new state particle

$$x_t^{(k)} \sim p(\cdot | x_{t-1}^{(k)}, \phi_t^{(k)}), \quad (22)$$

using $\phi_t^{(k)}$ from step 3 and the transition density.

5. Evaluate the weight corresponding to the k -th particle by

$$w_t^{(k)} \propto \frac{p(y_t | x_t^{(k)}, \phi_t^{(k)})}{p(y_t | \mu_t^{(k)}, m_{t-1}^{(k)})} \quad (23)$$

6. Repeat steps 2 to 5 for P particles with $\{x_t^{(k)}, \phi_t^{(k)}, w_t^{(k)}\}_{k=1}^P$. Then, go to step 1 again for $t = 1, \dots, T$.

Having obtained samples from $\{\{x_t^{(k)}, \phi_t^{(k)}\}_{k=1}^P\}_{t=1}^T$, we can follow the evolution the states and fixed-parameter estimates by computing first and second moments over particle indices $k = 1, \dots, P$. To assess the uncertainty surrounding the mean, we can use the percentiles of the particle distribution to obtain confidence intervals.

For simplicity, the filter description above contained only y_t as data. However, including exogenous variables as necessary in the TVP-MIDAS regression is straight-forward to implement. We can also modify the model structure easily to switch certain model objects between fixed and time-varying. For example, we could add stochastic volatility, which becomes a time-varying state, see Lopes and Tsay (2011).

B Further empirical results

In Figures 6 and 7, we show the evolution of the full set of parameters in the model apart from the weights, which are discussed above in the main text. In addition to β_t ,

Figure 6: Evolution of the time-varying parameters, spread AAA with maturity 1-3 years

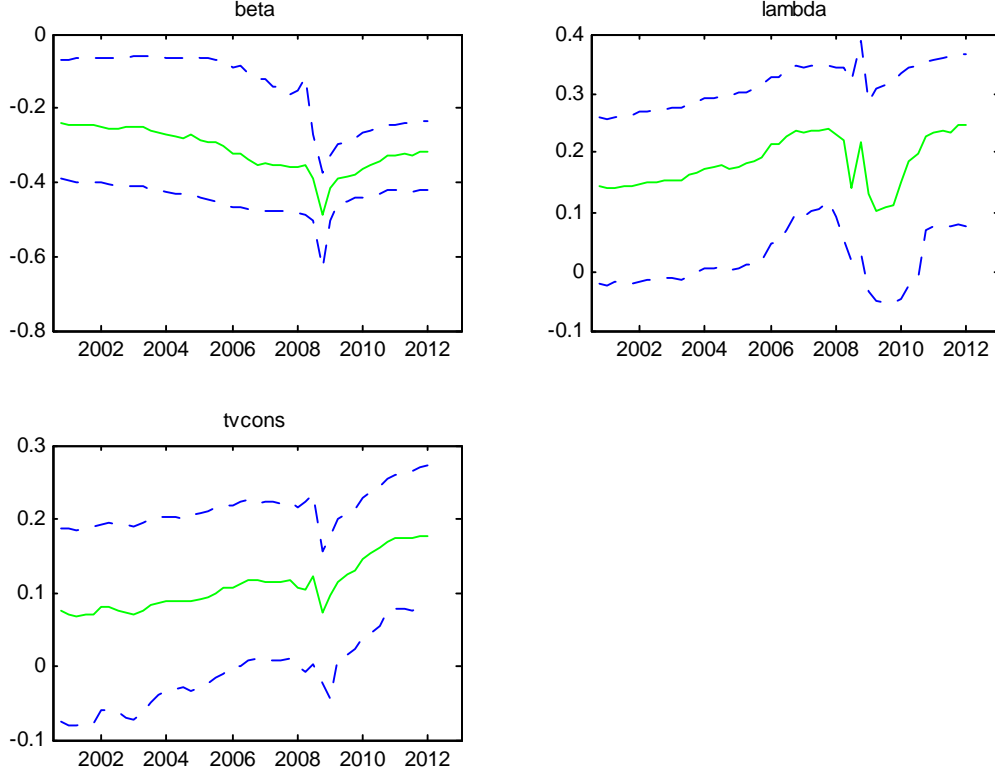


Figure 6 contains the evolution of the parameters γ_t and λ_t from the MIDAS regression in equation (11). The results show a slight upward trend in the constant from 0.09 to 0.19. The autoregressive parameter changes between 0.15 and 0.25, indicating a low degree of persistence in GDP. Disruptions in the evolution of these parameters happen in the culmination of the crisis period 2008 and 2009.

The evolution of the covariances σ_y , σ_β , σ_1 , σ_2 , σ_λ , and σ_γ are shown in Figure 7. We can again see some erratic movements in 2008 and 2009, but the estimates are relatively stable and stabilize again after this period.

Figure 7: Evolution of the variances parameters, spread AAA with maturity 1-3 years

