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# Investigating the Zero Lower Bound on the Nominal Interest Rate Under Financial Instability\*

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## Abstract

This paper studies the effects of three financial shocks in the economy: a net-worth shock, an uncertainty or risk shock, and a credit-spread shock. We argue that only the latter can push the nominal interest rate against its zero lower bound. Further, a recessionary shock to the net worth or the credit spread generates a positive response for loans, which is counter-intuitive during an economic downturn. Finally, we find that there is an optimal commitment period for the central bank to keep the nominal interest rate equal to zero (forward guidance) after a financial turmoil. Beyond that optimal period, the volatility of inflation and output rise quick and sharply. Thus, an excessive forward guidance policy may destabilize the economy.

**Keywords:** Zero Lower Bound; Financial Accelerator; Financial Shocks.

**JEL Classification:** E31; E44; E52; E58.

## Resumen

Este artículo investiga los efectos de tres choques financieros en la economía: un choque a las ganancias de capital, un choque de incertidumbre o riesgo, y un choque a la prima crediticia. Argumentamos que sólo éste último puede empujar la tasa de interés nominal hacia su cota inferior de cero. Además, un choque recesivo a las ganancias de capital o a la prima crediticia genera una respuesta positiva en los préstamos, lo cual es contra intuitivo durante una recesión económica. Por último, encontramos que existe un periodo de compromiso óptimo para que el banco central mantenga la tasa nominal de interés igual a cero (*guía futura*) durante una turbulencia financiera. Más allá de éste periodo óptimo, la volatilidad de la inflación y del producto aumentan rápida y bruscamente. Por lo tanto, una *guía futura* excesiva puede desestabilizar la economía.

**Palabras Clave:** Cota Inferior de Cero; Acelerador Financiero; Choques Financieros.

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# 1 Introduction

The global financial crisis of 2007-2008 has prompted macroeconomists to acknowledge the risks that *imperfect* financial markets pose for the real economy.<sup>1</sup> In particular, aggregate-fluctuations scholars now seek to understand how financial shocks propagate to the real economy and what can be done to moderate a financial-melting tsunami. Within a general-equilibrium framework, considerable work on this domain has been done recently.<sup>2</sup> However, most of studies omit one major aspect of the recent crisis, namely that the short-run nominal interest rate in major advanced economies has met record low levels since the wake of the crisis (for instance, U.S., U.K., or the Euro Area have endured near-zero rates since 2009; Japan’s experience is even longer).<sup>3</sup>

Indeed, we examine the effects of several financial shocks on the real economy, while we impose the zero lower bound (ZLB, hereafter) constraint on the nominal interest rate. Our framework builds on a New-Keynesian model with state-of-the-art real and nominal frictions. The model is upgraded with the financial accelerator of Bernanke, Gertler, and Gilchrist (1999), where frictions appear between the financial intermediary and the borrowing productive sector. We include in our analysis three financial shocks: an equity or net-worth shock that destroys the value of borrowers’ collateral; a risk shock that increases the uncertainty of borrowers’ venture projects; and a credit-spread shock that increases the cost of credit independently of borrowers financial health. The first two shocks originate from the demand side of the credit market since they are directly related to borrowers’ conditions. The third one is a supply-side shock that restricts credit regardless borrowers’ characteristics. It can be thus thought as a reduced-form shock that summarizes problems within the financial sector itself.

We perform our analysis in three steps. First, we study whether the propagation dynamics of each of the three financial disturbances are strong enough to push the nominal interest rate towards its zero-floor. We thus compute the probability that the ZLB constraint binds when the economy is hit by one particular shock. Second, we compare the effects of the three financial shocks in two monetary policy regimes: in “normal” times and when the ZLB binds. And finally, we explore an unconventional policy that aims to stimulate an economy immersed in a liquidity trap, which has

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<sup>1</sup>In general equilibrium, a financial sector affects the real economy when financial intermediation is imperfect. There are different ways to introduce frictions in the credit market. For instance, Bernanke et al. (1999) and Kiyotaki and Moore (2000) assume asymmetric information and borrowing constraints between banks and firms. Dib (2010) and Gerali et al. (2010) assume frictions among banks. And Gertler and Karadi (2011), Gertler and Kiyotaki (2010), and Christiano and Ikeda (2013) include agency costs between depositors and banks.

<sup>2</sup>See Nolan and Theonissen (2009), Christiano *et al.*, (2013), Gilchrist and Zakrajšek (2012), Jermann and Quadrini (2012), among others.

<sup>3</sup>An exception is De Fiore and Tristani (2012), who investigate the optimal response of monetary policy (conventional and unconventional) to a financial shock in presence of the ZLB constraint. However, they do not compare the propagation dynamics for different types of financial shocks.

A joint environment embedding these two features, i.e. the propagation of different types of financial shocks and the presence of near-zero interest rates, deserves thus a thorough exploration. The present study contributes to fill this gap.

been induced by a financial disruption. In particular, we assume that the central bank promises to keep the nominal interest rate at very low levels for a period longer than recommended by its usual policy rule.<sup>4</sup> This policy is known as forward guidance, and its benefits and limitations have been addressed by Eggertsson and Woodford (2003) and more recent papers.<sup>5</sup>

Following Christiano et al. (2014), the model is calibrated to a typical advanced economy. Our results are the following. First, we argue that only the credit-supply shock is likely to set the nominal interest rate equal to zero. The other two shocks lack propagation power to do so. The reason is that investment and consumption co-move after a credit-spread shock, while their directions diverge after a net-worth shock or a risk shock. Amano and Shukayev (2012) find similar results when comparing the credit-spread shock to other non-financial shocks. We extend their analysis to a financial-accelerator model, which allows us to compare the propagation dynamics of different financial disturbances and their associated ZLB-inducing probabilities. Second, we show that the three shocks affect differently the path of loans. The net-worth and the credit-spread shocks imply a countercyclical response for loans, while the risk shock predicts a cyclical response. This result is invariant to the monetary policy regime. Christiano, Motto, and Rostagno (2014) emphasize that only the risk shock implies loan dynamics that are consistent with the data, while the net-worth shock does not. We extend Christiano et al. analysis in two ways: first, we show that the credit-spread shock has also counter intuitive loan dynamics; and second, we study all financial shocks in a ZLB regime. Putting these results together, we infer that a combination of various financial shocks might be necessary to explain all features of aggregate dynamics observed during the Great Recession. Finally, our last result refers to the effectiveness of forward guidance at moderating the recession. After a credit-spread shock, we show that there exists an optimal commitment period for the central bank to keep the interest rate at the ZLB for more time than prescribed by its policy rule. In our exercise, inflation and output volatilities are minimized if the central bank announces 5 quarters of extra liquidity. In contrast, keeping the interest rate low for more periods destabilizes the economy very quickly. Our results confirm Eggertsson and Woodford (2003) findings about the theoretical benefits of forward guidance, this time tested in a financial-accelerator model. However, our results suggest that the consequences of miscalculating the optimal commitment period can be very serious for economic stability.

The remainder of the paper is organized as follows. Section 2 describes the baseline model. Section 3 discusses the calibration and solution strategy. Section 4 computes the ZLB-inducing probabilities for each financial shock, and compares their associated aggregate dynamics. Section 5 presents the implementation of forward guidance. The final section concludes.

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<sup>4</sup>An alternative policy for helping the economy would be a fiscal stimulus. Such policy has been treated in a model with financial frictions by Carrillo and Poilly (2013).

<sup>5</sup>Del Negro et al. (2012) show that a typical DSGE model over-estimates the effects of forward guidance (see also Carlstrom et al., 2012, and Levin et al., 2010). This result can be explained by the excessive response of the long-term bond yield in the model to the policy announcement. Nonetheless, the authors show that forward guidance still helps the recovery when they correct for the response of the long-term interest rate.

## 2 The Model

Our framework is based on the workhorse New-Keynesian model enriched with frictions on the credit market (as in Bernanke et al., 1999). As such, the model features several real and nominal rigidities that close the gap between the model's predictions and the data. We assume that households have consumption habits; that prices and wages are sticky (as in Calvo, 1983) and partially indexed to past inflation; that adjustment costs are levied on investment; and that the nominal interest rate is constraint by its zero lower bound. In addition, financial frictions arise from asymmetric information between entrepreneurs (borrowers) and the financial intermediary (lender): the latter pays a monitoring cost to observe the entrepreneur's realized return, while borrowers observe it for free. Agency costs result in an negative relationship between the external finance premium and the value of entrepreneurs' collateral or net worth. In this environment, a recessionary shock decreases asset prices, thus reducing the value of entrepreneurs collateral. As the cost of borrowing increases, investment demand plummets, and asset prices drop again, leading to a new round of reductions in investment. The financial accelerator mechanism amplifies, indeed, the effects of shocks.<sup>6</sup>

### 2.1 Households

**Preferences.** The economy is inhabited by a continuum of differentiated households, indexed by  $i \in [0, 1]$ . A typical household selects a sequence of consumption and savings that are invested in a financial intermediary. Households differ by the specific labor type they are endowed with, which gives them monopolistic power to set their own wage. Household  $i$ 's objective is to maximize her lifetime utility, given by

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t - \varpi c_{t-1}) - V(\ell_{i,t}^h) \right\},$$

subject to the sequence of constraints

$$c_t + d_t \leq w_{i,t} \ell_{i,t}^h + \exp(\varepsilon_{t-1}) R_{t-1} \frac{d_{t-1}}{1 + \pi_t} + \frac{\Upsilon_t}{P_t} + \frac{\mathcal{A}_t}{P_t} + \text{div}_t, \quad (1)$$

where  $E_t$  is the expectation operator conditional to the information available in period  $t$ ;  $\beta \in (0, 1)$  is the subjective discount factor and  $\varpi \in [0, 1)$  is the habit parameter;  $c_t$  is real consumption;  $P_t$  is the price of final goods;  $1 + \pi_t = P_t/P_{t-1}$  is the gross inflation rate;  $w_{i,t} \equiv W_{i,t}/P_t$  and  $\ell_{i,t}^h$  denote the real wage and the labor supply of type- $i$  household's at period  $t$ ;  $d_t$  are real deposits held at the financial intermediary in period  $t$  and maturing in period  $t + 1$ ;  $R_t$  is the risk free interest rate set by the central bank; and finally,  $\text{div}_t$ ,  $\Upsilon_t$  and  $\mathcal{A}_t$  denote real profits redistributed by monopolistic firms, nominal net lump-sum transfers from the government, nominal lump-sum transfers from entrepreneurs, respectively. The term  $\varepsilon_t$  denotes a shock that affects the spread between the risk free interest rate and the rate of return on private assets. The gross nominal interest rate perceived

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<sup>6</sup>For the rest of the document, we denote a hatted variable, like  $\hat{a}_t$ , as its deviation from the deterministic steady state. A variable with neither a hat nor a time subscript, like  $a$ , denotes its steady-state level. The full derivations of the model are available upon request.

in deposits is therefore  $\exp(\varepsilon_t)R_t$ . A positive innovation of this shock increases the return on savings, prompting households to consume less and save more. This shock is similar to the risk-premium shock introduced by Smets and Wouters (2007). We assume that it follows an autoregressive process, such as

$$\begin{aligned} \varepsilon_t &= \rho_\varepsilon \varepsilon_{t-1} + \epsilon_{\varepsilon,t}, & \text{(Credit-spread shock)} \\ \text{where } \rho_\varepsilon &\in (0, 1) \text{ and } \epsilon_{\varepsilon,t} \sim iid(0, \sigma_\varepsilon). \end{aligned}$$

The log-linearized first-order conditions with respect to  $c_t$  and  $d_t$  are

$$(1 - \beta\varpi) \sigma \hat{\lambda}_t = \beta b E_t \{\hat{c}_{t+1}\} - (1 + \beta\varpi^2) \hat{c}_t + b \hat{c}_{t-1}. \quad (2)$$

$$\hat{\lambda}_t = E_t \left\{ \hat{\lambda}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1} + \varepsilon_t \right\}. \quad (3)$$

where  $\sigma^{-1} \equiv -\mathbb{U}_{cc}c/\mathbb{U}_c$  is the inter-temporal elasticity of substitution and  $\lambda_t$  is the Lagrangian multiplier associated to the budget constraint. Equation (2) defines the marginal utility of consumption. Equation (3) is the Euler equation and states that the marginal sacrifice of a consumption unit must equal the marginal benefit of consuming this unit plus a compensation driven by the real interest rate on savings.

**Wage Setting.** A typical household  $i$  acts as a monopoly supplier of type- $i$  labor. Following Erceg et al. (2000), a competitive labor intermediary aggregates the set of differentiated labor inputs into a single labor input  $\ell_t^h$  using a Dixit-Stiglitz technology

$$\ell_t^h = \left( \int_0^1 \left[ \ell_{i,t}^h \right]^{(\theta_w - 1)/\theta_w} di \right)^{\theta_w / (\theta_w - 1)}, \quad (4)$$

where  $\theta_w > 1$  is the elasticity of substitution between any two labor types. The associated aggregate nominal wage obeys  $W_t = \left( \int_0^1 W_{i,t}^{1-\theta_w} di \right)^{1/(1-\theta_w)}$ . Following Calvo (1983), it is assumed that at each point in time only a fraction  $1 - \alpha_w$  of the households re-optimize their nominal wage. The remaining households simply index their wages to past inflation at rate  $\gamma_w \in (0, 1)$ . Since the household is a monopoly supplier, it internalizes the demand for its labor when setting its wage. The optimization program yields the log-linearized wage setting equation

$$\hat{\pi}_t^w - \gamma_w \hat{\pi}_{t-1} = \frac{(1 - \alpha_w)(1 - \beta\alpha_w)}{\alpha_w(1 + \omega_w\theta_w)} \left[ \omega_w \hat{\ell}_t^h - \hat{\lambda}_t - \hat{w}_t \right] + \beta E_t \{ \hat{\pi}_{t+1}^w - \gamma_w \hat{\pi}_t \}, \quad (5)$$

where  $\omega_w^{-1} \equiv \mathbb{V}_\ell / \ell^h \mathbb{V}_{\ell\ell}$  denotes the Frisch elasticity of labor supply and the gross wage inflation is defined by  $\hat{\pi}_t^w \equiv \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t$ .

## 2.2 Entrepreneurs

Consider a continuum, mass one, of risk neutral entrepreneurs indexed by  $e \in [0, 1]$ . At the end of period  $t$ , type- $e$  entrepreneur buys the capital stock  $k_{e,t}$  at price  $Q_t$ . The entrepreneur finances her capital purchases with own internal funds and a loan borrowed from the risk neutral financial intermediary.

**Capital Returns.** At time  $t+1$ , entrepreneurs rent their capital  $k_{e,t}$  to intermediate firms at the real rental rate  $z_{t+1}$ . After production, they sell the un-depreciated capital to a capital producer at price  $Q_{t+1}$ . Entrepreneurs are also requested to pay a fee  $\text{cap}_{t+1}$  to the capital producer, which is intended to cover for capital adjustment costs. The *undistorted* gross nominal rate of returns on capital,  $R_{t+1}^k$ , equals<sup>7</sup>

$$R_{t+1}^k = \frac{P_{t+1}z_{t+1} + (1 - \delta - \text{cap}_{t+1})Q_{t+1}}{Q_t}, \quad (6)$$

where  $\text{cap}_t \equiv \frac{\vartheta}{2} \left( \frac{i_{t+1}}{k_t} - \delta \right)^2 - \vartheta \left( \frac{i_{t+1}}{k_t} - \delta \right) \frac{i_{t+1}}{k_t}$ , with  $\vartheta > 0$  (for further details, see the capital producer problem in the online appendix of Carrillo and Poilly, 2013, available in any of the authors website). Capital returns perceived by entrepreneurs,  $\tilde{R}_t^k$ , are also distorted by the credit-spread shock,  $\varepsilon_t$ , so  $\tilde{R}_t^k = R_t^k \exp(-\varepsilon_{t-1})$ . A positive innovation of  $\varepsilon_t$  reduces the value of capital and shifts downwards investment demand. Along with its effects on savings, an increase of  $\varepsilon_t$  induces a co-movement between investment and consumption (see Equation 3).<sup>8</sup>

**Optimal Financial Contract.** At the end of time  $t$ , type- $e$  entrepreneur acquires (nominal) debt equal to  $B_{e,t} = Q_t k_{e,t} - N_{e,t}$ , where  $N_{e,t}$  is its (nominal) net worth. As in Townsend (1979) and Bernanke et al. (1999), it is assumed that every single entrepreneurs investing project is subject to a idiosyncratic shock,  $\omega_{e,t+1}$ . The shock is a random variable distributed log normal with mean 1 and variance  $\sigma_{\omega,t}^2 > 0$ .<sup>9</sup> Both the entrepreneur and the lender do not observe  $\omega_{e,t+1}$  when they sign the loan contract. As Christiano et al. (2014), we allow the variance  $\sigma_{\omega,t}^2$  to vary over time, which reflects changes in the risk of default of entrepreneurs projects. An increase in  $\sigma_{\omega,t}^2$  widens the distribution of  $\omega_{e,t+1}$ , which makes the likelihood of success more uncertain. We assume the shock follows the process

$$\begin{aligned} \log(\sigma_{\omega,t}) &= \rho_\sigma \log(\sigma_{\omega,t-1}) + (1 - \rho_\sigma) \log(\sigma_\omega) + \epsilon_{\sigma,t}, & \text{(Risk shock)} \\ \text{where } \rho_\sigma &\in (0, 1), \sigma_\omega > 0, \text{ and } \epsilon_{\sigma,t} \sim iid(0, \sigma_\sigma). \end{aligned}$$

In time  $t+1$ , and given a threshold value  $\bar{\omega}_{e,t+1}$ , an entrepreneur repays its (real) debt,  $b_{e,t}$ , at the gross rate  $r_{e,t+1}^L$  if  $\omega_{e,t+1} > \bar{\omega}_{e,t+1}$ . Then threshold and the loan rate are jointly defined as

$$\bar{\omega}_{e,t+1} \tilde{r}_{e,t+1}^k q_t \tilde{k}_{e,t} = r_{e,t+1}^L b_{e,t}, \quad (7)$$

where  $\tilde{r}_{e,t+1}^k$  is the real gross rate of capital returns. In the case where  $\omega_{e,t+1} < \bar{\omega}_{e,t+1}$ , the entrepreneur declares bankruptcy. In such a case, the lender pays a monitoring cost to audit

<sup>7</sup>For the sake of exposition, some dynamic equations are presented in their non-linear form. All log-linearized equations of the model are provided in the appendix.

<sup>8</sup>In Smets and Wouters (2007) frictionless-financial-market model, households hold both deposits and capital goods. Therefore, a no-arbitrage condition naturally creates a spread between the risk free rate and the rate of return on capital. In a model with financial frictions, there is a separation between deposit holders (households) and capital owners (entrepreneurs), so the no-arbitrage condition no longer holds. Consequently, we need to include the risk premium shock in both the household and entrepreneur problems (see Equation 6).

<sup>9</sup>This shock is an i.i.d. random variable across time and types, with a continuous and once-differentiable c.d.f.,  $F(\omega)$ , over a non-negative support.



the entrepreneur and gets to keep all of the entrepreneur's realized returns. This truth-telling mechanism prevents borrowers from misreporting their returns to fake a bankruptcy. For simplicity, the monitoring cost is a proportion  $\mu \in [0, 1]$  of the realized gross payoff to the entrepreneur's capital, i.e.,  $\mu\omega_{e,t+1}^k \tilde{r}_{e,t+1} q_t \tilde{k}_{e,t}$ , where  $q_t \equiv Q_t/P_t$  is the relative price of capital.

The optimal lending contract consists in choosing  $k_t$  and  $\bar{\omega}_{t+1}$  in order to maximize an entrepreneur's expected returns subject to the participation constraint of the lender (because of symmetry, we drop the type- $e$  subindex<sup>10</sup>), i.e.

$$\max_{\tilde{k}_t, \bar{\omega}_{t+1}} \text{E}_t \left[ (1 - \Gamma(\bar{\omega}_{t+1})) \tilde{r}_{t+1}^k q_t \tilde{k}_t \right], \quad (8)$$

subject to

$$\text{E}_t \left[ (\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})) \tilde{r}_{t+1}^k q_t \tilde{k}_t \right] \geq \text{E}_t \left[ r_t (q_t \tilde{k}_t - n_t) \right], \quad (9)$$

where  $\Gamma(\bar{\omega}_{t+1})$  and  $\mu G(\bar{\omega}_{t+1})$  represent the expected gross share of profits going to the lender, and the expected monitoring costs, respectively.<sup>11</sup> Equation (9) states that the lender participates in the contract as long as she is assured to receive an expected loan return equal to the opportunity costs of her funds. Since it is assumed that the lender can perfectly diversify the risk associated with the loan, its relevant opportunity cost is represented by the economy real risk free rate  $r_t$ . Let  $\tilde{r}_t = \text{E}_t\{r_{t+1}^k/r_t\}$  be the expected discounted return on capital. The first-order conditions of the above problem imply that, in equilibrium, the discounted return on capital, which denotes the external finance premium, equals the marginal cost of external finance. In log-linear terms, we have

$$\widehat{\tilde{r}}_t = \chi \left[ \widehat{q}_t + \widehat{\tilde{k}}_t - \widehat{n}_t \right] + \varepsilon_t, \quad (10)$$

where  $\chi \geq 0$  is the elasticity of  $\widehat{\tilde{r}}_t$  to a measure of leverage ( $\frac{q_t k_t}{n_t}$ ). All else equal, the external finance premium (or simply the credit spread or the cost of credit) increases whenever the net worth falls. The reason is that a lower collateral raises the probability of loan default and so the lender demands a higher premium to compensate this risk. This relationship is the key feature of the financial accelerator model.

**Aggregate Net Worth.** The real aggregate net worth of entrepreneurs,  $n_t$ , is composed by their real aggregate capital gains,  $v_t$ , and their aggregated real wage  $w_t^e$ .<sup>12</sup> At the end of period  $t$ , the  $n_t$  is given by:

$$n_t = \gamma_t v_t + w_t^e \quad (11)$$

<sup>10</sup>It is assumed that entrepreneurs have a linear utility in consumption and are subject to similar aggregate shocks, implying that  $\tilde{k}_{t+1} = \int \tilde{k}_{e,t+1} de$ ,  $r_{e,t+1}^k = r_{t+1}^k$ ,  $\bar{\omega}_{e,t+1} = \bar{\omega}_{t+1} \forall e$ .

<sup>11</sup>See Bernanke et al. (1999) and the online appendix of Carrillo and Poilly (2013) for further details.

<sup>12</sup>Following Bernanke et al. (1999) and Carlstrom and Fuerst (1997), entrepreneurs participate in the general labor market by supplying one unit of labor every period. This salary helps new entrepreneurs to start up their projects with a minimum set of collateral.

The term  $\gamma_t$  denotes the probability of survival of an entrepreneur in a given period. Indeed,  $1 - \gamma_t$  represents the odds that an entrepreneur exits the economy and loose all of its capital gains.<sup>13</sup> This assumption prevents entrepreneurs to accumulate enough wealth to be fully self-financed, and thus keep the credit market active at all times. We assume that the parameter  $\gamma_t$  follows the process

$$\begin{aligned} \log(\gamma_t) &= (1 - \rho_\gamma) \log(\gamma) + \rho_\gamma \log(\gamma_{t-1}) + \epsilon_{\gamma,t}, & \textbf{(Net-worth shock)} \\ \text{where } \rho_\gamma &\in (0, 1), \text{ and } \epsilon_{\gamma,t} \sim iid(0, \sigma_\gamma). \end{aligned}$$

Real capital gains,  $v_t$ , equal gross revenues from capital holdings from  $t - 1$  to  $t$  less borrowing repayments

$$v_t = \tilde{r}_t^k q_{t-1} k_{t-1} (1 - \mu G(\bar{\omega}_t)) - r_{t-1} (q_{t-1} \tilde{k}_{t-1} - n_{t-1}). \quad (12)$$

## 2.3 Capital Producer

At time  $t$ , capital producers sell to entrepreneurs the capital stock  $k_t$ , which has been built by combining investment goods,  $i_t$ , and un-depreciated capital:

$$k_t = (1 - \delta) k_{t-1} + i_t - \frac{\vartheta}{2} \left( \frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1}, \quad (13)$$

where  $\vartheta > 0$  controls the size of the adjustment cost on capital accumulation. In equilibrium, the relative price of capital,  $q_t$ , is given by  $\hat{q}_t = \vartheta \delta [\hat{i}_t - \hat{k}_{t-1}]$  and the law of motion (13) is  $\hat{k}_t = \hat{k}_{t-1} (1 - \delta) + \delta \hat{i}_t$ .

## 2.4 Firms

### 2.4.1 Final Goods Producers

The final good,  $y_t$ , used for consumption and investment, is produced in a competitive market by combining a continuum of intermediate goods indexed by  $j \in [0, 1]$ , via the Dixit-Stiglitz production function

$$y_t = \left( \int_0^1 y_{j,t}^{\frac{\theta_p - 1}{\theta_p}} dj \right)^{\frac{\theta_p}{\theta_p - 1}}, \quad (14)$$

where  $y_{j,t}$  denotes the overall demand addressed to the producer of intermediate good  $j$  and  $\theta_p$  is the elasticity of demand for a producer of intermediate good. The maximization of profits yields typical demand functions

$$y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_p} y_t, \quad \text{with } P_t = \left( \int_0^1 P_{j,t}^{1-\theta_p} dj \right)^{\frac{1}{1-\theta_p}}, \quad (15)$$

where  $P_{j,t}$  denotes the price of intermediate good produced by firm  $j$ .

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<sup>13</sup>It is assumed, though, that the rate of birth of entrepreneurs equals its mortality rate, in order to keep constant the number of entrepreneurs. Entrepreneurs that exit at time  $t$ , consume their residual net worth such as  $c_t^e = (1 - \gamma_t) \varrho v_t$ , where the complementary fraction  $(1 - \varrho)$  is lump-sum transferred to households.

### 2.4.2 Intermediate firms

**Production technology.** Type- $j$  intermediate firms produce differentiated goods by combining labor and capital,  $\ell_{j,t}$  and  $k_{j,t-1}$ , respectively. Capital services are rented from entrepreneurs. Type- $j$  firm's total labor input,  $\ell_{j,t}$  is composed by household labor,  $\ell_{j,t}^h$ , and entrepreneurial labor,  $\ell_{j,t}^e$ , according to  $\ell_{j,t} = [\ell_{j,t}^h]^\Omega [\ell_{j,t}^e]^{1-\Omega}$ . Type- $j$  intermediate good is produced with the following constant return to scale technology

$$y_{j,t} = \ell_{j,t}^{1-\alpha} k_{j,t-1}^\alpha. \quad (16)$$

Each monopolistic firm determines its capital and labor demand in order to minimize its real cost, subject to its production technology, taking  $w_t$ ,  $w_t^e$  and  $z_t$  as given. Accordingly, labor and capital demands satisfy

$$\hat{w}_t = \hat{s}_t + \hat{y}_t - \hat{\ell}_t^h, \quad (17)$$

$$\hat{w}_t^e = \hat{s}_t + \hat{y}_t, \quad (18)$$

$$\hat{z}_t = \hat{s}_t + \hat{y}_t - \hat{k}_{t-1}. \quad (19)$$

where  $\hat{s}_t$  is the the real marginal cost.

### 2.4.3 Price Setting.

As Calvo (1983), we assume that each period a monopolistic firm faces a constant probability,  $1-\alpha_p$ , of being able to re-optimize its price. Firm  $j$  takes the demand function (15) into account when setting its price. Additionally, it takes into consideration the possibility that this price remains for more than one period. If the firm cannot re-optimize its price, the latter is indexed to past inflation at rate  $\gamma_p \in (0, 1)$ . The following New Keynesian Phillips curve can be thus derived:

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \frac{(1-\alpha_p)(1-\beta\alpha_p)}{\alpha_p} \hat{s}_t + \beta \mathbb{E}_t \{ \hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t \}. \quad (20)$$

## 2.5 Resource Constraint

The production of the final good is allocated to investment, total private consumption by households and entrepreneurs, public spending, and to monitoring costs paid by lenders

$$y_t = i_t + c_t + c_t^e + g_t + \mu G(\bar{\omega}_t) r_t^k q_{t-1} \tilde{k}_t,$$

where  $g_t$  denotes government expenditures. For simplicity, we assume that government spending are financed with lump-sum taxes.

## 2.6 Monetary Policy

$R_t$ , the *gross* nominal interest rate implemented by the central bank, satisfies a zero-lower-bound constraint of the form:

$$R_t = \max(1, R_t^{not}). \quad (21)$$

where the desired (notional) nominal interest rate,  $R_t^{not}$  in gross terms, obeys the rule

$$\hat{R}_t^{not} = \rho_R \hat{R}_{t-1}^{not} + (1 - \rho_R) [a_\pi \hat{\pi}_t + a_{\Delta y} \Delta \hat{y}_t], \quad (22)$$

where  $\rho_R \in (0, 1)$  is a smoothing parameter;  $a_p$  is the elasticity of  $R_t^{not}$  to the inflation gap (the difference of inflation from its target value), and finally,  $a_y$  is the elasticity of  $R_t^{not}$  to output growth.

## 2.7 Equilibrium

In the symmetric equilibrium, all entrepreneurs, households and firms are identical and make the same decisions. In addition, equilibrium on the labor market yields  $\int_0^1 \ell_{j,t} dj = \ell_t^h$ . The symmetric equilibrium is characterized by an allocation  $\{y_t, c_t, i_t, \ell_t, k_t, n_t\}$  and a sequence of price and co-state variables  $\{\pi_t, r_t, r_t^k, q_t, \pi_t^w, z_t, \lambda_t, \bar{\omega}_t\}$  that satisfy the optimality conditions in each sector, the monetary policy rule, and the stochastic shocks.

## 3 Calibration and solution strategy

### 3.1 Calibration

The model parameters are calibrated to fit the quarterly frequency. Table 1 describes the calibrated values for parameters related to households, firms, and economic authorities. Most of values from the households and production sectors are borrowed from Christiano et al. (2014), who fit their model to U.S. data. Values from the financial sector are taken from Bernanke et al. (1999).

[ insert Table 1 here ]

The subjective discount factor,  $\beta$ , is set to 0.995, which entails a annual real interest rate of 2 per cent. The Frisch parameter,  $\omega_w^{-1}$ , is set to unity. The degree of habit consumption,  $\varpi$ , is set to 0.74, while the inter-temporal elasticity of substitution,  $\sigma^{-1}$ , is set to 1.

Regarding production, the capital share in the intermediate sector,  $\alpha$ , is set to 0.4; the rate of capital depreciation,  $\delta$ , is equals 0.025. The capital adjustment cost,  $\vartheta$ , is calibrated to 17, following Christiano et al. (2011). Concerning price setting, we assume that the elasticity of substitution between intermediate goods,  $\theta_p$ , is set to 6, which implies a price mark-up of 20 per cent. Similarly, the elasticity of substitution between labor types,  $\theta_w$ , is set to 21, which translates into a wage mark-up of 5 per cent. The degrees of price and wage rigidities,  $\alpha_p$  and  $\alpha_w$ , are set equal to 0.74 and 0.81 respectively, implying average durations between price or wage re-optimization of about one year. Price and wage indexation parameters,  $\gamma_p$  and  $\gamma_w$ , are set to 0.10 and 0.51, respectively. In terms of monetary policy, the interest rate smoothing parameter,  $\rho_R$ , is calibrated to 0.85; the elasticity of the notional interest rate with respect to inflation,  $a_\pi$ , is set to 2.40; and the elasticity of the interest rate with respect to output growth,  $a_y$ , is set to 0.36. These values follow once more the estimations of Christiano et al. (2014). Finally, the steady-state share of government purchases in total output is calibrated to 0.20, which roughly corresponds to the last decade historical average.

We now turn to the parameters related to the financial sector. The share of entrepreneurial wages in terms of income is set to 0.01, implying a value of  $\Omega = 0.9833$ . The steady-state share of capital investment that is financed by the entrepreneur’s net worth,  $x = \tilde{k}/n$ , is calibrated to 2, meaning that the steady-state leverage ratio amounts to 50 per cent. The steady-state external finance premium,  $\tilde{r} = r^k/r$ , is set to  $1.02^{0.25}$ . Finally, the annual business failure rate,  $F(\bar{\omega})$ , is set to 3 per cent. It is assumed that the idiosyncratic productivity shock,  $\omega_t$ , has a log-normal distribution with positive support, and an unconditional expectation equal to 1. These moments help to determine the steady-state survival probability of entrepreneurs,  $\gamma$ , which is set to 0.9843, the monitoring costs to realized payoffs ratio,  $\mu$ , which amounts to 0.1175, the steady-state variance of the entrepreneurs’ idiosyncratic shock,  $\sigma_\omega$ , which is equal to 0.2763, and the steady-state idiosyncratic threshold is set to 0.4983.

Finally, the shocks are calibrated as follow. For the credit-spread shock, we have  $\rho_\varepsilon = 0.95$ , as in Fernández-Villaverde (2010); for the net-worth and risk shocks, we borrow from Christiano et al. (2014), who set  $\rho_\gamma = 0$  and  $\rho_\sigma = 0.97$ . The size of the shocks vary with different exercises, and are indicated in the following sections.

### 3.2 Solution Strategy

We assume that at time  $t = 0$ , the economy is hit by a negative financial shock which pushes the nominal interest rate towards its zero floor. Given its non-linear nature, the ZLB constraint prevent us to use a standard solution method. We thus adopt a piecewise-linear approach, similar to Bodenstein, Erceg and Guerrieri (2009), and Christiano *et al.* (2011). In particular, we use an extended deterministic path over the linearized model equations, where we make sure that the ZLB constraint is satisfied at all times. We then apply a shooting algorithm to determine the duration of the liquidity trap, which is endogenous. We encourage the reader to consult Carrillo and Poilly (2013) and its related online appendix (available at any of the authors’ webpage) for further details.<sup>14</sup>

## 4 Financial Shocks in a Liquidity Trap

In this section, we investigate the effects of three financial shocks in the economy. The net-worth shock and the risk shock originate on the demand side of the credit market. They respectively affect the value of entrepreneur’s collateral and its idiosyncratic likelihood of default. In contrast, the credit-spread shock is a supply-side disturbance that restricts credit despite the current state of the economy. It can be thought as a reduced-form shock that summarizes problems within the financial sector itself.

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<sup>14</sup>Alternative solution methods exist, such as a collocations, spline functions, or projections (see Nakov, 2008; De Fiore and Tristani, 2012; and Fernández-Villaverde et al., 2012). However, they can be very costly in terms of computation time for complex models like ours. We also disregard equilibria driven by self-fulfilling beliefs (like in Mertens and Ravn, 2011).

Our analysis proceeds as follows. We first ask how likely is it that the economy falls into a liquidity trap. We argue that only the credit-spread shock can push the central-bank interest rate towards its zero floor. A similar result is found in Amano and Shukayev (2012), although they only compare the credit-spread shock to other non-financial shocks. Next, we explore the dynamics of an economy hit by a positive credit-spread shock in two regimes: when the zero bound binds and when it does not. Finally, as we are still interested in the consequences of the other two shocks, we study the partial impulse responses of the net-worth and risk shocks in the two aforementioned interest-rate regimes.

#### 4.1 What financial shock causes a liquidity trap?

We answer this question by computing the probability that the ZLB binds for each of the three financial shocks considered. What interests us is the propagation dynamics of each shock. So, we normalize the standard deviation of all financial disturbances, such as an one-standard-deviation shock increases the external finance premium by 125 basis points on impact, in annual terms.<sup>15</sup> That is, we assume that all shocks have, on average, the same initial effect on the credit spread of the economy. Similar to Amano and Shukayev (2012),<sup>16</sup> we draw 10,000 random innovations from a normal distribution and we use them to compute a similar number of series for each of the financial shocks.<sup>17</sup> Then, we count the number of series for which the nominal interest rate equals zero for at least one period. We obtain the following probabilities: 1.24 % for the credit-spread shock, 0 % for the net-worth shock, and 0 % for the risk shock. Put it differently, the zero bound binds on average every 20 years ( $= \frac{1}{1.24\%} \div 4$  quarters) for the credit-spread shock, whereas for the other shocks it never binds. Figure 1 displays the distribution of the nominal interest rate at the quarter where a financial shock reaches its peak propagation effect. Peak effects are reached after six quarters for the credit-spread shock, and three quarters for the other two shocks.

[ insert Figure 1 here ]

The figure shows that the nominal interest rate lies in the interval 0-0.25 %, in annual terms, about 2.5 % of the times for the credit-spread shock (or once every 10 years), and never for the net-worth shock and the risk shock. Amano and Shukayev (2012) obtain similar findings for the credit-spread shock. For the other two shocks, we do not know of any other study that computes their ZLB-inducing probabilities. Given their near-zero probabilities of the demand-side financial shocks, we argue that only a supply-side shock gives more chances for the nominal interest rate to hit its zero floor. The reason is that a credit-spread shock implies a co-movement between investment and

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<sup>15</sup>Certainly, the 1.25 % is an *ad hoc* choice. However, the size of this normalizing constant does not affect the qualitative direction of our simulations. On the upside, we are able to mimic Amano and Shukayev (2012)'a results for the credit-spread shock.

<sup>16</sup>These authors identify and estimate the dynamics of the credit-spread shock and include it in a model which features other aggregate shocks. Although their approach is robust, they do not include other type of financial shocks in their analysis.

<sup>17</sup>We have 10,000 impulse-response series for each of the shocks, each one featuring a single innovation in period 1.

consumption, whereas the other considered shocks affect mainly investment and have a counter-intuitive effect on consumption. Figure 2 illustrates this point, where a one-standard-deviation innovation is displayed for each type of financial perturbation. Notice that the external finance premium rises from 4 % in annual terms, its steady state level, towards 5.25 % at impact for all the three shocks.

[ insert Figure 2 here ]

As shown in the figure, investment and consumption co-move only for the credit-spread shock, whereas consumption actually rises in the short-run for the other two shocks. A credit-spread shock achieves co-movement because, on the one hand, it reduces the returns of capital so investment demand falls, and, on the other, it increases the returns of savings so private consumption falls. The net-worth and the risk shocks certainly reduce investment, but they fail to discourage consumption. In fact, consumption soars because, in general equilibrium, a fall in investment demand must be met by a decrease in savings. The latter implies that the real interest rate must fall to discourage households to save, who use their spare resources for consumption. All in all, it is more likely that the ZLB binds with a credit-spread shock as the two main components of aggregate demand fall simultaneously.

## 4.2 Model's Dynamics

In this section, we assume that the ZLB binds due to a credit-spread shock. First, we analyze the aggregate dynamics associated with this shock in two regimes: when the interest rate moves freely, and when the ZLB constraint is imposed. We then analyze the effects of the net-worth and risk shocks by looking at their *partial* impulse responses in each of the two interest-rate regimes. We assume that these shocks hit the economy only after the credit spread shock has occurred. Looking at the partial IRFs allow us to compare the marginal effects on endogenous variables of all the three shocks.

### 4.2.1 Credit-Spread Shock

Let the spread between the expected returns on private assets and the risk free rate widens (i.e. a positive innovation on  $\varepsilon_t$ ). It affects both household's inter-temporal decisions and the expected return on capital. Gilchrist et al. (2009) and Fernández-Villaverde (2010) also assume that the external finance premium is driven by an stochastic component, and interpret it as a credit-supply shock. Figure 3 compares the impulse response functions (IRFs, hereafter) of this shock in two cases: the non-ZLB regime, and the ZLB regime.

[ insert Figure 3 here ]

We first consider the ZLB regime. The size of the shock is deliberately chosen so that the ZLB binds at period 1 (see constraint 21).<sup>18</sup> The monetary policy instrument stays at zero for 12 quarters, until the economy show signs of recovery. A positive innovation of  $\varepsilon_t$  drives the external finance premium up (see Equation 10). As credit market conditions worsen, investment and the price of capital plummet. Despite this recessionary trend, this shock generates a countercyclical response of loans! Similar to Christiano et al. (2014), the intuition is as follows: the reduction in the price of capital is temporary, suggesting that the return on capital will increase in the future (see Equation 6). Higher expected returns on capital increases the expected value of the project, stimulating in turn the demand for loans. It is worth noticing that effective loans are demand-driven in the financial-accelerator model. In fact, our perfectly competitive financial intermediary is willing to provide as many loans as entrepreneurs demand, under the condition that the latter pay the current external finance premium (which ensures that the participation constraint of the lender is satisfied at all times).<sup>19</sup> Also, notice that investment and consumption co-move for this shock, as already noticed above.

We now focus on the non-ZLB regime. As expected, Figure 3 shows that in this case the recession is damped. This result is not surprising since the monetary authorities are now allowed to use their instrument to stimulate private spending and promote the recovery. Consequently, the presence of a liquidity trap makes the recovery harder to reach, but it does not change the direction in the response of any endogenous variable.

#### 4.2.2 Net-Worth Shock

We now assume a reduction in the entrepreneurs survival probability ( $\gamma_t$ ), which can be interpreted as an exogenous decrease in the entrepreneur’s net worth value. This shock directly deals with a shift in the demand for capital through a lower aggregate purchasing power of entrepreneurs. Christiano et al. (2014) refer to it as an “equity shock”. As mentioned above, it is unlikely that the ZLB constraint binds for a reasonable size of this shock. So, in order to fix this issue, we assume that the credit-spread shock drives the nominal interest rate towards the ZLB, and that the net-worth shock occurs once the ZLB is in place. Figure 4 compares the *partial* IRFs of selected variables in response to a negative shock on the survival probability,  $\gamma_t$ .<sup>20</sup> It is worth noticing that the partial IRFs isolate the effect of the net-worth shock by showing marginal effects, which allows for the comparison with the previous exercise.<sup>21</sup>

[ insert Figure 4 here ]

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<sup>18</sup>The size of the credit spread shock is set to  $\sigma_\varepsilon = 0.012$ .

<sup>19</sup>A different response of loans might be obtained in a model which explicitly describes the behavior of banks, setting some frictions to the supply of loans.

<sup>20</sup>Precisely, we compute the partial IRFs as the difference between the IRFs to both financial shocks and the IRFs to the net worth shock. Since the partial IRFs are the result of a difference, they are expressed in percentage points and not in percent deviations from its steady-state level.

<sup>21</sup>The size of the shock is set to  $-0.0081$ , which is equal, in absolute value, to the estimated standard deviation of the equity shock in Christiano et al. (2014).



In “normal times” (i.e., in the non-ZLB regime), the model predicts that the nominal interest rate drops for several quarters. In addition, the negative financial shock reduces the demand for capital which drives its price down. Through the financial accelerator mechanism, the drop in the value of the collateral rises the external finance premium and depresses investment, turning the economy into a recession. As emphasized by Christiano et al. (2014), this shock generates a counter-cyclical demand for loans. As explained above, when the returns on capital is expected to rise, loan demand increases. Regarding consumption, it rises in the medium-run, as in Christiano et al. (2014). The reason that explains the lack of co-movement between investment and consumption for the demand-side financial shocks is stated in section 4.1.

Let now assume that the economy is in a liquidity trap (the ZLB regime). In that case, a negative net-worth shock generates also more volatility. The deflationary effects of this shock generates a strong rise in the real interest rate. Consequently, the demand for capital drops by more, as for the price of capital. As a result, the response of loans is slightly more counter-cyclical. In addition, investment and output fall sharply, and now also consumption falls. So, investment and consumption co-move only during on the ZLB regime. However, it has been established that a net-worth shock on its own cannot generate a liquidity trap.

### 4.2.3 Risk Shock

The last financial shock we consider is a rise in volatility of the idiosyncratic productivity shock ( $\sigma_\omega$ ). A risk shock widens the entrepreneurs’ returns distribution and eventually worsens credit conditions (the quality of entrepreneurs’ projects is hardly distinguishable). This shock has been analyzed by Christiano et al. (2014) in the absence of a liquidity trap. As for the net worth shock, the transmission mechanisms of this shock are such that a liquidity trap cannot be reached with only this shock. Figure 5 compares the *partial* IRFs of this shock in normal times and in a presence of a liquidity trap.

[ insert Figure 5 here ]

Consider the non-ZLB regime. An increase in projects uncertainty makes harder for the lender to distinguish whether an entrepreneur might default or not. The latter translate into a rise in the lending contract threshold,  $\bar{\omega}_t$ , and consequently, more entrepreneurs default. This facts have two main effects: first, the risk premium increases, and second, the demand of loans fall, and is now pro-cyclical. As in the previous demand-side shock, inflation initially drops, following the marginal cost and output. Interestingly, our result differ from Christiano et al. (2014) regarding the effect of the risk shock on consumption. We observe a medium-run increase in consumption when risk is rising. The difference might be explained by the presence of a variable utilization rate of capital and investment adjustment costs (rather than capital adjustment costs) in their paper.

We now turn to the ZLB regime. In that case, the real interest rate strongly increases due to deflationary pressures. As previously, consumption is reduced, at least in the short-run. In addition, since the nominal interest rate is stuck to it zero floor, the risk shock generates a much larger increase

in the credit spread, leading to a strong reduction in investment and a large recession.

## 5 The Credit-Spread shock and Forward Guidance

This section is motivated by the findings of Eggertsson and Woodford (2003). In an extensive discussion, these authors explore alternative policies that the monetary authority may employ when dealing with a liquidity trap. In particular, they explore *forward guidance*, a policy in which the central bank announces and keeps the nominal interest rate at very low levels for longer time than prescribed by an strict inflation targeting rule. The argument, accordingly, is that when agents expect a period of abundant liquidity, they smooth consumption and plan investment better.

We focus on the credit-spread shock since it constitutes a good candidate to generate a liquidity trap. In the initial scenario, recall that the Taylor rule prescribes that the interest rate should be positive after 13 periods. In the alternative policy, the central bank announces and keeps the nominal interest at its ZLB for more quarters than recommend by its Taylor rule. Our aim here is to investigate how inflation and output volatility react to the news. We thus perform an heuristic evaluation of this policy using a representative loss function that may represent the preferences of the central bank,<sup>22</sup> of the form

$$\sum_{t=0}^{\infty} \beta^t \left( \hat{\pi}_t^2 + \lambda \left( \hat{y}_t - \hat{y}_t^f \right)^2 \right). \quad (23)$$

In this numerical exercise, we assume that deviations of output are as important as the deviations of inflation from its target value, thus  $\lambda = 1$ . Notice as well that the output gap is defined with respect to the level of output that would prevail in the absence of nominal rigidities,  $y_t^f$ . Finally, we compute the loss function at convergence (we set the end period 300 quarters after the shock). The upper panels of Figure 6 display the responses of output and inflation gaps to the financial shock when the ZLB constraint is binding for 13, 18 and 21 quarters. The lower panel of the figure provides the value of the loss function (23) for different durations of the liquidity trap.

[ insert Figure 6 here ]

It clearly appears that the loss function is convex with respect to the number of periods the constraint is binding. This convexity suggests that there exists an optimal commitment period for the central bank to keep the interest rate at low levels. The welfare losses are minimized when the central bank commits to keeping low the interest rate for 18 periods, i.e. 5 quarters more than prescribed by its Taylor rule. Beyond that point, the value of the loss function increases, implying that inflation and output become too volatile.

This last result is illustrated in the upper panel of Figure 6. Precisely, the commitment period alters significantly the response of real variables. Let consider the optimal commitment period

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<sup>22</sup>This standard loss function is commonly used in the literature, which is referred to be derived from a second-order Taylor expansion from the utility function of the representative household. In the derivation of the optimal monetary policy, Oda and Takashi (2008) interprets the managing expectations policy as the “zero interest rate commitment”. We adopt here a more elementary approach since we are not interested in optimal monetary policy.

(5 quarters). Since agents expect a low interest rate for a long period of time, they respond to the adverse economic conditions by smoothing their consumption and investment. This yields a lower deflation, which in turn produce a smaller increase in the real interest rate. On the one hand, a smaller deflation rises by less firm's real debt, through the debt-deflation channel (Fisher, 1933). On the other hand, a damped increase in the real interest rate also reduces the service of the debt. These two effects – which can be observed in Equation (12) – stop the net worth to fall deeply. Although the economy still experiences a recession but volatilities are smaller than in the benchmark case. When the commitment period is larger than 5 quarters, the monetary policy avoids the recession but at a price of a stronger volatility in the inflation and output gaps. The bottom line of this analysis is that there exists an optimal commitment period that the central bank must consider in order to provide abundant liquidity, which is in line with the analysis of Eggertsson and Woodford (2003). Further than this period, the economy destabilizes very quick.

## 6 Conclusion

This paper studies the propagation of different types of financial shocks in the economy, given the presence of the ZLB constraint on the nominal interest rate. In a financial-accelerator model, we look at a net-worth shock, a risk shock, a credit-spread shock. The first two shocks originate from the demand side of the credit market, while the third one is a supply-side shock that creates a gap between the interest rate of private assets and the risk free rate. We first ask which one of these shocks is strong enough to bind the ZLB. We find that only the credit-supply shock is likely to do so, as the other two shocks lack transmission channels. The credit-spread shock causes investment and consumption to move in similar directions, whereas for the other shocks investment falls but consumption booms. We also find that, out of the three shocks, only the risk shock implies a pro-cyclical response of loans, which is typical of a recession. For the other two shocks, loans are counter-cyclical, which is at odds with the data. Finally, we show that, after a negative financial shock, there exists an optimal commitment period for the central bank to keep the interest rate at the ZLB for more time than prescribed by its policy rule. However, the volatility of inflation and output rise quick and sharply after this optimal period. Thus, forward guidance may be destabilizing when not used properly.

As it is widely known, the vulnerability of an ill-behaved banking sector was a crucial determinant of the global financial crisis. In our model, we have focused on the traditional financial accelerator model to study the transmission of shocks, while assuming that the financial intermediary is fully insured and perfectly competitive. Given the potential of the credit-spread shock to generate a liquidity trap, a natural step in the analysis of the propagation of financial shocks is to include an imperfect banking sector, in which the credit-spread shock is microfounded. This modification will prove very valuable for the evaluation of anti-recessionary policies, and it might help to obtain a response of loans more in accordance with the data. We leave this topic for future research.

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## 7 Appendix

The log-linear model can be sum up as

$$(1 - \beta b) \sigma \hat{\lambda}_t = \beta b \mathbf{E}_t \{ \hat{c}_{t+1} \} - (1 + \beta b^2) \hat{c}_t + b \hat{c}_{t-1}, \quad (24)$$

$$\hat{\lambda}_t - \hat{R}_t - \varepsilon_t = \mathbf{E}_t \{ \hat{\lambda}_{t+1} - \hat{\pi}_{t+1} \} \quad \text{and} \quad \hat{r}_t = \hat{R}_t - \mathbf{E}_t \{ \hat{\pi}_{t+1} \}. \quad (25)$$

$$\hat{\pi}_t^w - \gamma_w \hat{\pi}_{t-1}^w = \frac{(1 - \alpha_w)(1 - \beta \alpha_w)}{\alpha_w(1 + \omega_w \theta_w)} \left[ \omega_w \hat{\ell}_t^h - \hat{\lambda}_t - \hat{w}_t \right] + \beta \mathbf{E}_t \{ \hat{\pi}_{t+1}^w - \gamma_w \hat{\pi}_t^w \}, \quad (26)$$

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t. \quad (27)$$

$$\hat{y}_t = (1 - \alpha) \hat{\ell}_t + \alpha \hat{k}_t \quad \text{and} \quad \hat{\ell}_t = \Omega \hat{\ell}_t^h. \quad (28)$$

$$\hat{w}_t = \hat{s}_t + \hat{y}_t - \hat{\ell}_t^h, \quad \hat{w}_t^e = \hat{s}_t + \hat{y}_t, \quad \text{and} \quad \hat{z}_t = \hat{s}_t + \hat{y}_t - \hat{k}_t \quad (29)$$

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \frac{(1 - \alpha_p)(1 - \beta \alpha_p)}{\alpha_p} \hat{s}_t + \beta \mathbf{E}_t \{ \hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t \}, \quad (30)$$

$$\hat{x}_t = \hat{q}_t + \hat{k}_t - \hat{n}_t, \quad \text{and} \quad \hat{r}_t = \mathbf{E}_t \{ \hat{R}_{t+1}^k \} - \varepsilon_t - \hat{R}_t. \quad (31)$$

$$(x - 1) \hat{b}_t = x(\hat{q}_t + \hat{k}_t) - \hat{n}_t. \quad (32)$$

$$\hat{r}_{t-1} = f_0 f_1 \hat{\omega}_t + f_3 \sigma_{\omega,t} + \hat{\pi}_t, \quad (33)$$

$$\mathbf{E}_t \hat{r}_t = \chi \hat{x}_t + f_5 \mathbf{E}_t \{ \sigma_{\omega,t+1} \} + \mathbf{E}_t \hat{\pi}_{t+1} + \varepsilon_t \quad (34)$$

where  $f_0, f_1, f_3, f_5$ , and  $\chi$  are reduced-form parameters.

$$\hat{n}_t = n_0(\hat{v}_t + \gamma_t) + [1 - n_0] \hat{w}_t^e, \quad (35)$$

$$\hat{v}_t + \hat{\pi}_t = v_0 \left[ \hat{R}_t^k + \hat{q}_{t-1} + \hat{k}_{t-1} \right] - [v_0 - 1] \left[ \hat{R}_{t-1} + \hat{b}_{t-1} + \gamma_b \hat{\pi}_t \right] - v_1 \hat{\omega}_t - v_2 \sigma_{\omega,t}, \quad (36)$$

$$\frac{c^e}{k} \hat{c}_t^e = \frac{c^e}{k} \hat{v}_t - v_3 \gamma_t, \quad (37)$$

where  $n_0, v_0, v_1, v_2$  and  $v_3$  are reduced-form parameters.

$$\hat{R}_t^k = \hat{\pi}_t + \hat{z}_t \left[ \frac{z}{r^k} \right] + \hat{q}_t \left[ \frac{1 - \delta}{r^k} \right] - \hat{q}_{t-1} + (\hat{i}_t - \hat{k}_{t-1}) \left[ \frac{\vartheta \delta^2}{r^k} \right],$$

$$\hat{k}_t = \hat{k}_{t-1} (1 - \delta) + \delta \hat{i}_t, \quad (38)$$

$$\hat{q}_t = \vartheta \delta \left[ \hat{i}_t - \hat{k}_{t-1} \right]. \quad (39)$$

$$\hat{y}_t = \hat{c}_t \frac{c}{y} + \hat{i}_t \frac{i}{y} + \hat{g}_t \frac{g}{y} + \hat{c}_t^e \frac{c^e}{y} + \left[ \hat{r}_t^k + \hat{q}_{t-1} + \hat{k}_{t-1} \right] \left[ \mu G(\bar{\omega}) r^k \frac{k}{y} \right] + \hat{\omega}_t \bar{\omega} \mu G_\omega(\bar{\omega}) r^k \frac{k}{y} + \sigma_{\omega,t} \sigma_\omega \mu G_\omega(\bar{\omega}) r^k \frac{k}{y}. \quad (40)$$

$$\hat{R}_t^{not} = \rho_R \hat{R}_{t-1}^{not} + (1 - \rho_R) \left[ a_\pi \hat{\pi}_t + a_{\Delta_y} \Delta \hat{y}_t \right], \quad (41)$$

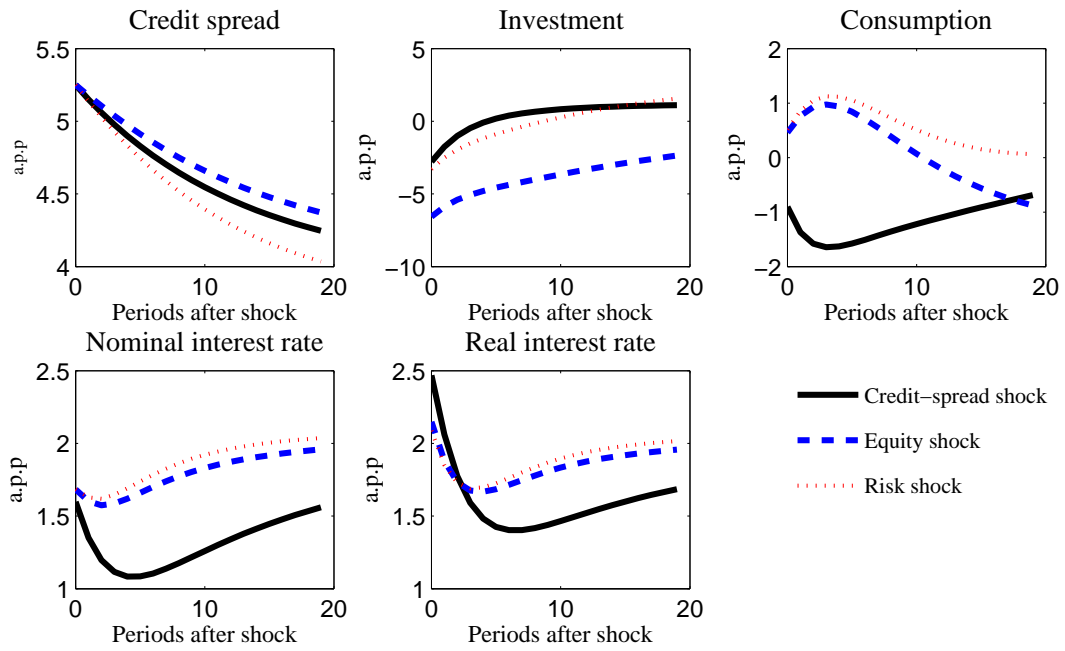


Figure 2: Paths for investment and consumption after three different financial shocks.

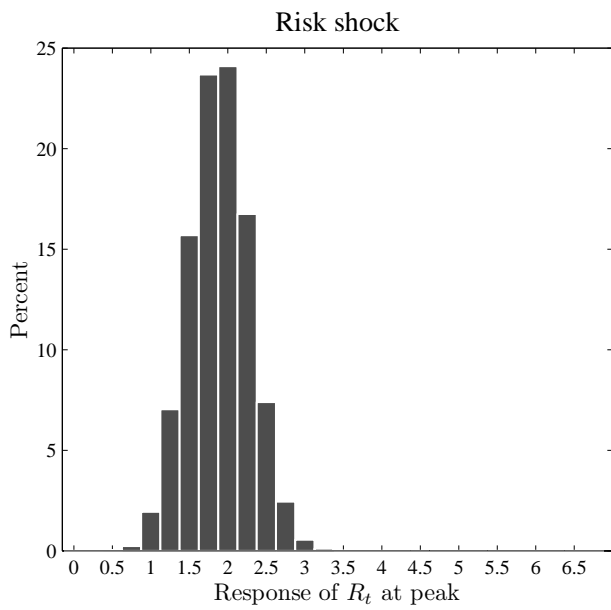
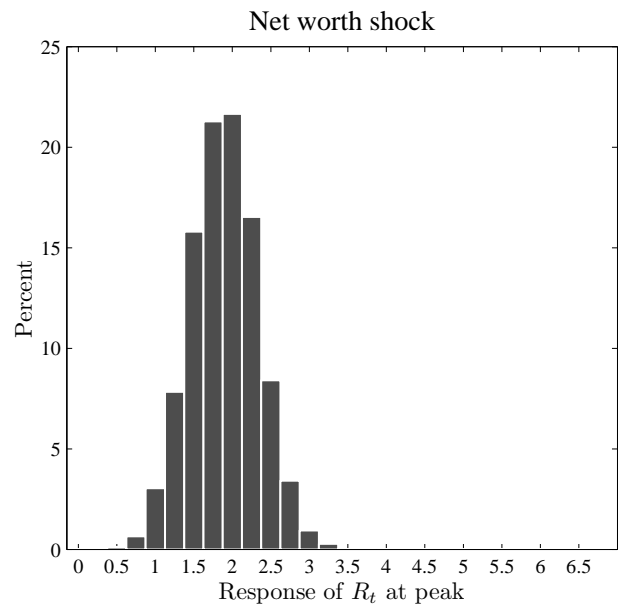
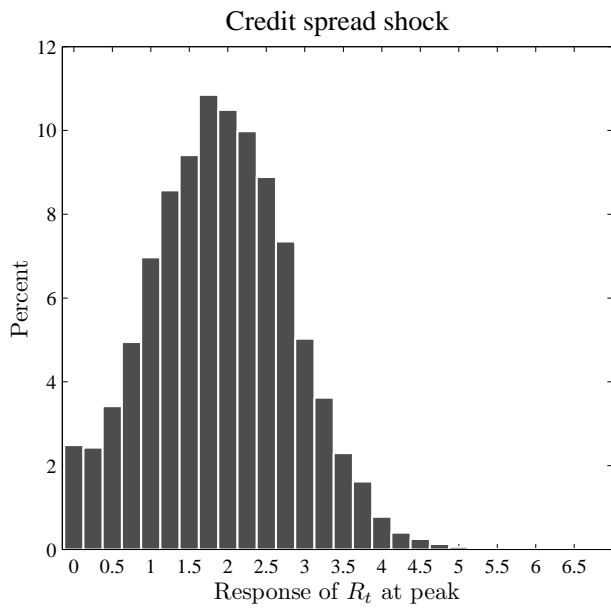


Figure 1: Distribution of the response of the nominal interest rate at its peak effect for 10,000 draws for the three financial shocks.



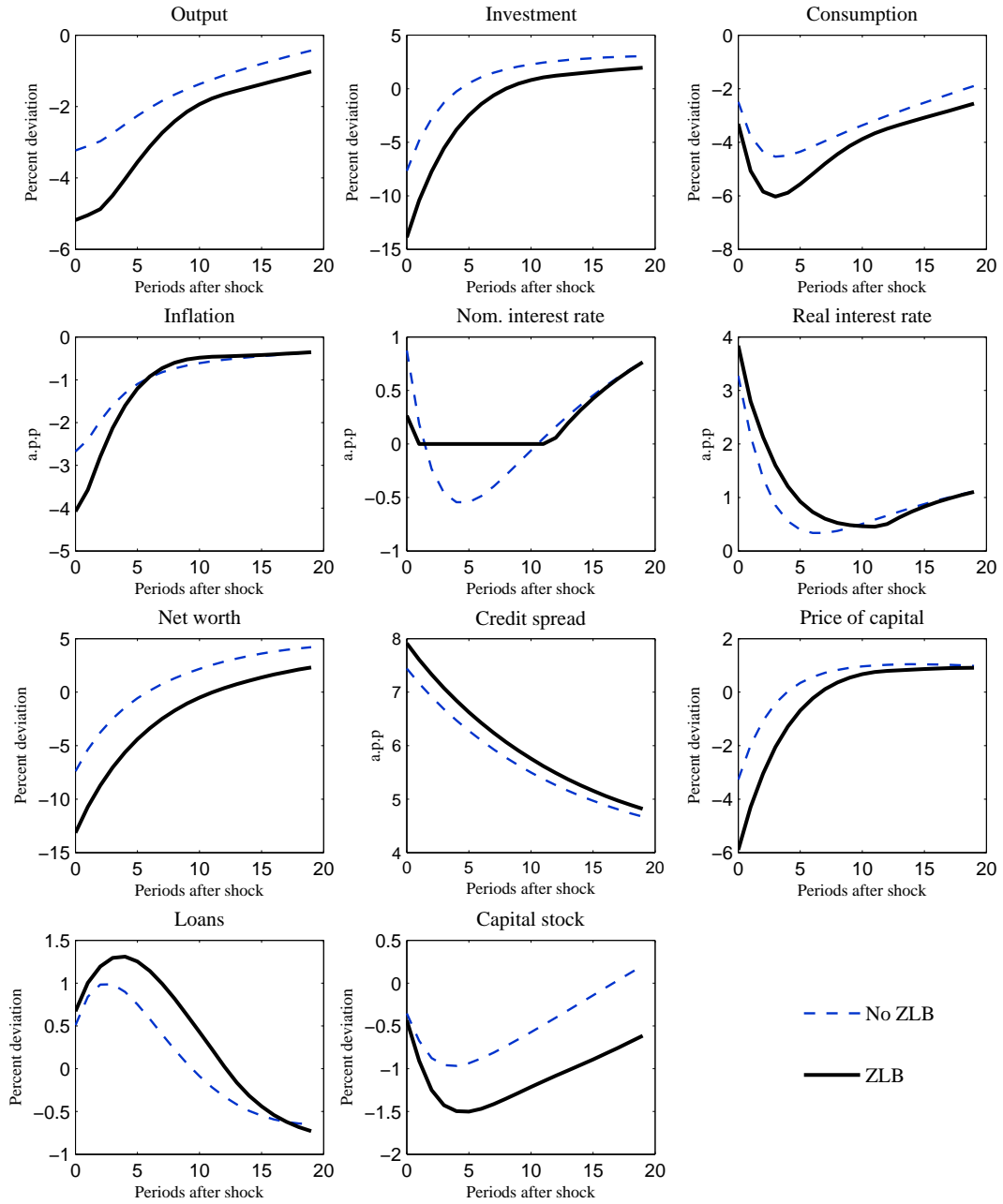


Figure 3: Impulse responses for the credit-spread shock, selected variables.

Table 1. Calibrated Parameters

<b>Preferences and Technology</b>		Value
$\beta$	Discount factor	0.995
$\varpi$	Degree of habit consumption	0.74
$\sigma$	Inv. of the elasticity of intertemp. substitution	1.00
$\omega_w$	Elasticity of labor disutility	1.00
$\phi$	Elasticity of value added wrt capital	0.40
$\delta$	Capital depreciation rate	0.025
$\vartheta$	Capital adjustment cost	17.00
<b>Nominal Rigidities</b>		
$\theta_p$	Elasticity of substitution of goods	6.00
$\alpha_p$	Degree of price stickiness	0.74
$\gamma_p$	Degree of price indexation	0.10
$\theta_w$	Elasticity of substitution of labor	21.00
$\alpha_w$	Degree of wage stickiness	0.81
$\gamma_w$	Degree of wage indexation	0.51
<b>Fiscal and Monetary Policy</b>		
$g/y$	Share of government expenditure in output	0.20
$\rho_R$	Interest rate smoothing	0.85
$a_p$	Elasticity of the interest rate wrt inflation	2.40
$a_y$	Elasticity of the interest rate wrt output gap	0.36
<b>Financial Accelerator Mechanism</b>		
$\Omega$	Share of household labor in aggr. labor	0.9833
$x$	Steady-state ratio of capital to net worth	2.00
$\check{r}$	Steady-state risk spread	1.02 <sup>0.25</sup>
$\gamma$	Survival rate of entrepreneurs	0.9843
$\bar{\omega}$	Threshold value of idiosyncratic shock	0.4983
$\sigma_\omega$	Standard error of idiosyncratic shock	0.2763
$\mu$	Monitoring cost	0.1174

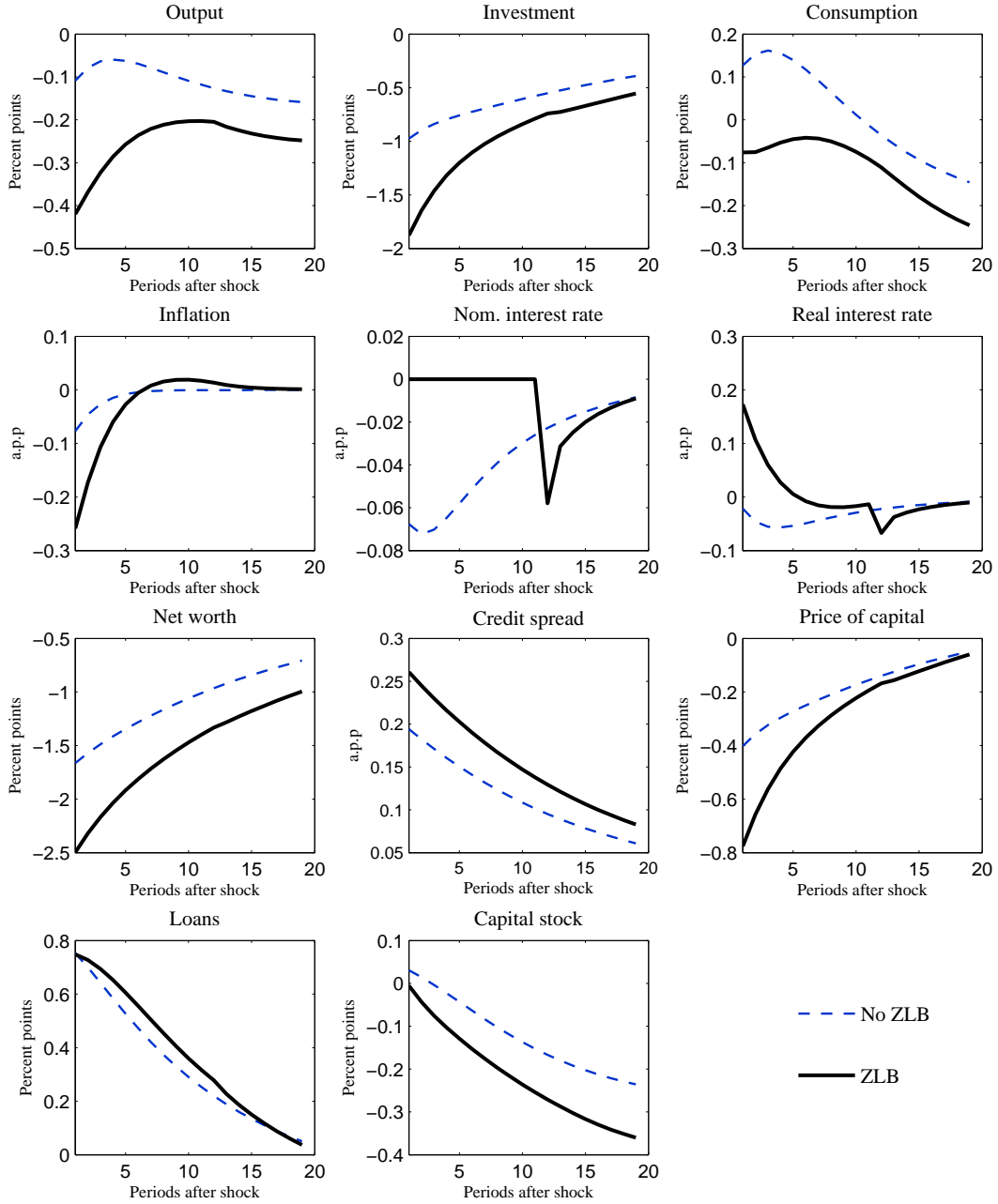


Figure 4: Partial impulse responses for the net-worth shock, selected variables.

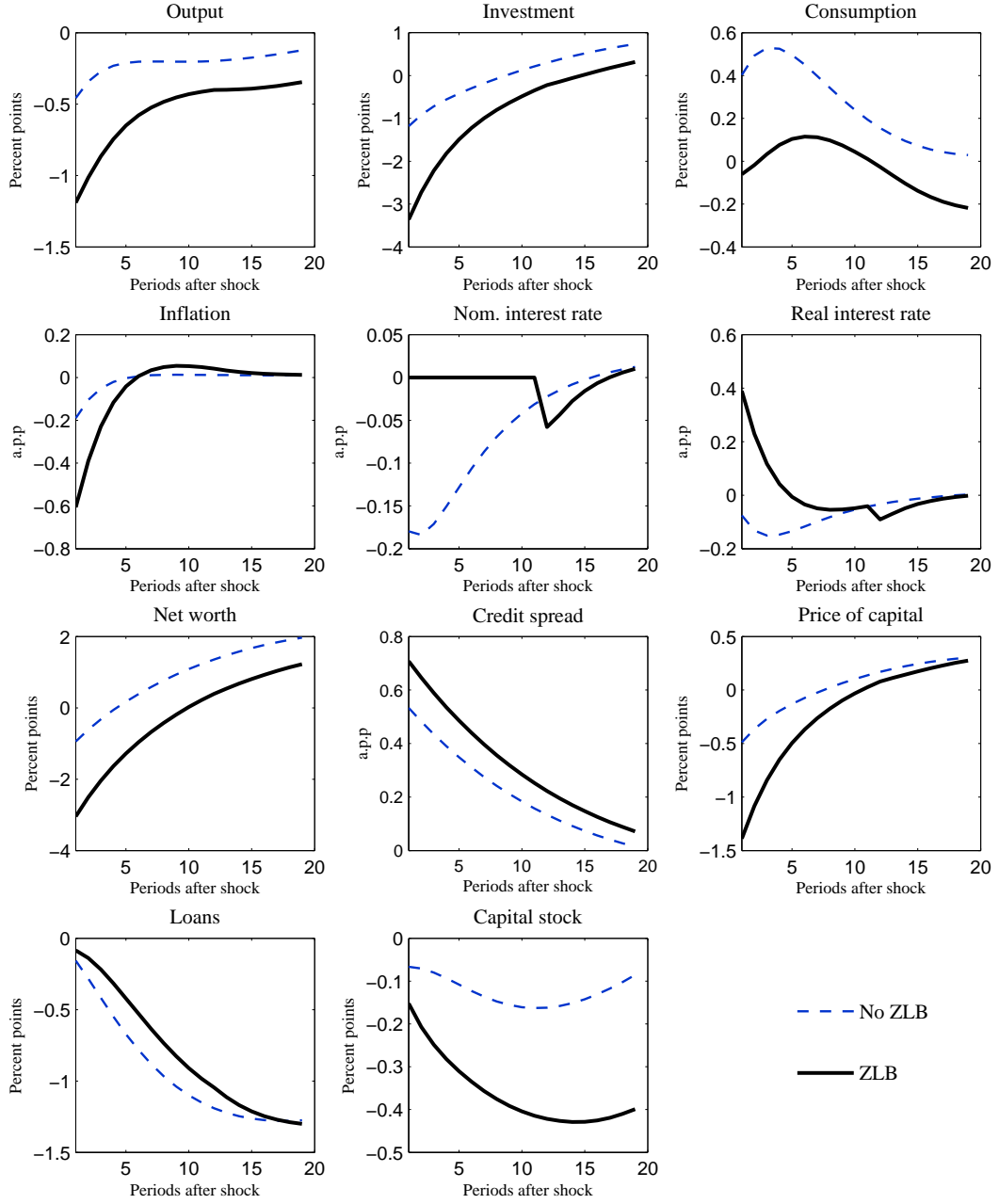


Figure 5: Partial impulse responses for the risk shock, selected variables.

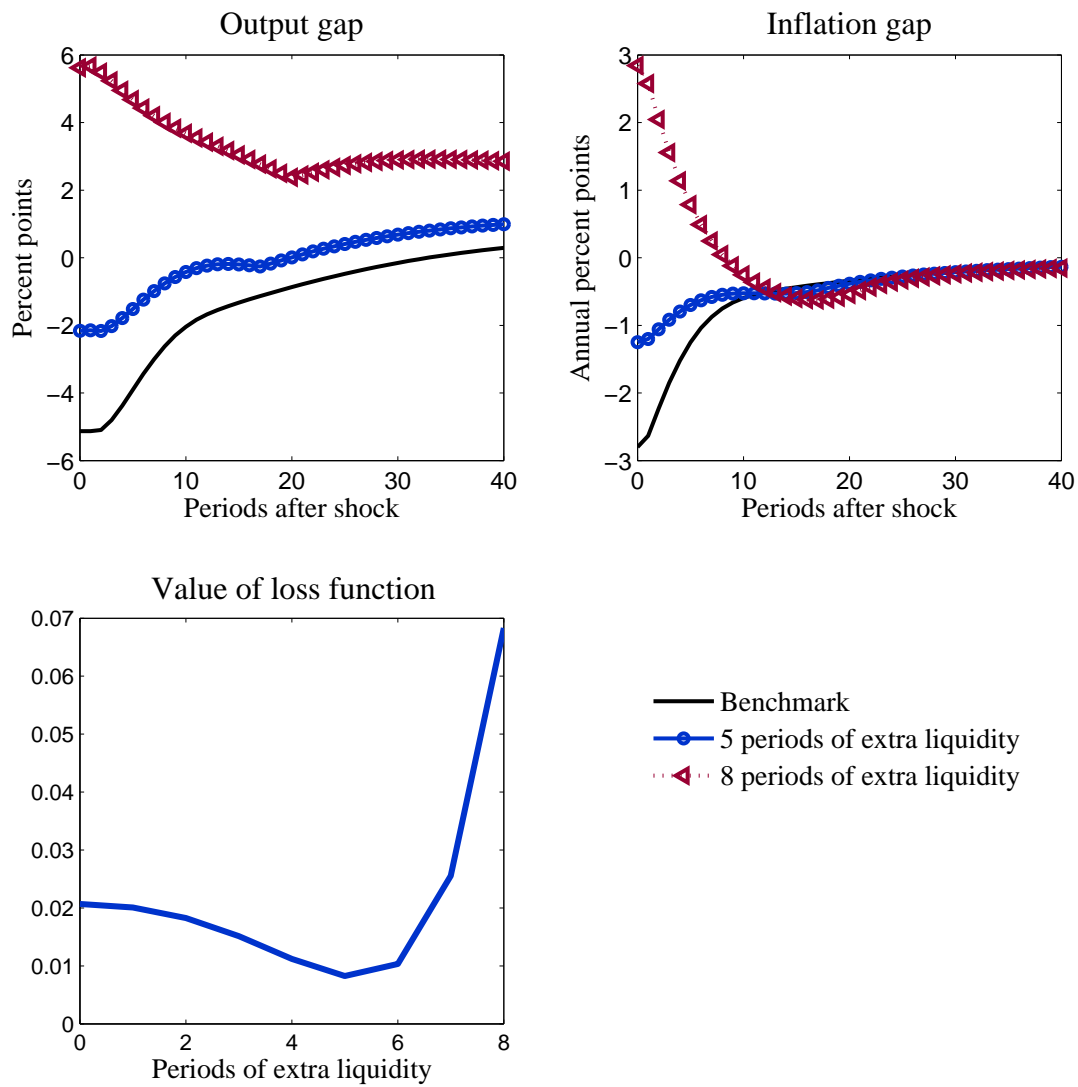


Figure 6: *First row.* Impulse responses for the output gap and the inflation gap for selected extra-liquidity commitments. *Second row.* Value of the central bank loss function, which achieves its minimum for a commitment of 5 extra quarters of low interest rates.