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# REVEALED ALTRUISM 

BY JAMES C. COX, DANIEL FRIEDMAN, AND VJOLLCA SADIRAJ


#### Abstract

This paper develops a theory of revealed preferences over one's own and others' monetary payoffs. We introduce "more altruistic than" (MAT), a partial ordering over preferences, and interpret it with known parametric models. We also introduce and illustrate "more generous than" (MGT), a partial ordering over opportunity sets. Several recent discussions of altruism focus on two player extensive form games of complete information in which the first mover (FM) chooses a more or less generous opportunity set for the second mover (SM). Here reciprocity can be formalized as the assertion that an MGT choice by the FM will elicit MAT preferences in the SM and, furthermore, that the effect on preferences is stronger for acts of commision than acts of ommision by FM. We state and prove propositions on the observable consequences of these assertions. Then we test those propositions using existing data from investment games with dictator controls and Stackelberg games and new data from Stackelberg mini-games. The test results provide support for the theory of revealed altruism.


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## 1. Introduction

What are the contents of preferences? People surely care about their own material well-being, e.g., as proxied by income. Abstract theory and common sense have long recognized the possibility that in some contexts people also care about others' well-being, but until recently applied work has neglected that possibility. Evidence from the laboratory and field (as surveyed in Fehr and Gächter (2000) for example) has begun to persuade economists to develop specific models of how and when a person's preferences depend on others' material payoffs (Sobel, 2005).

Andreoni and Miller (2002) report "dictator" experiments in which a human subject decides on an allocation for himself and for some anonymous other subject while facing a linear budget constraint. Their analysis confirms consistency with the generalized axiom of revealed preference (GARP) for a large majority of subjects. They conclude that altruism can be modeled as utility maximizing behavior.

In this paper we take three further steps down the same path. First, we create non-linear opportunity sets. Such sets enable the subject to reveal more about the tradeoff between her own and another's income, e.g., whether her indifference curves have positive or negative slope, and whether the curvature is zero or positive. Second, we give the other subject an initial move that can be more or less

[^0]generous. This allows us to distinguish conditional altruism-positive and negative reciprocity -from unconditional altruism. It also allows us to clarify the observable consequences of convex preferences and of changed preferences. Third, we distinguish active from passive initial moves; i.e., we distinguish among acts of omission, acts of commission, and absence of opportunity to act, and their comparative impacts on reciprocity.

We begin in Section 2 by developing representations of preferences over own and others' income, and formalize the idea that one preference ordering is "more altruistic than" (MAT) another. We include the possibility of negative regard for the other's income; in this case $M A T$ really means "less malevolent than." Special cases include the main parametric models of other regarding preferences that have appeared in the literature.

Section 3 introduces opportunities and choices, and illustrates concepts with several two player games of complete information from the recent literature. Section 4 formalizes the idea that one opportunity set is more generous than (MGT) another, and then uses it to formalize reciprocity. Axiom R asserts that more generous choices by the first mover induce more altruistic preferences in the second mover. An interpretation urged in a previous paper (Cox, Friedman, and Gjerstad, 2004) is that preferences are emotional state-dependent, and the first mover's generosity induces a more benevolent (or less malevolent) emotional state in the second mover. Axiom S asserts that the reciprocity effect is stronger following an act of commission (upsetting the status-quo) than following an act of omission (upholding the status-quo), and that the effect on preferences is weakest when the first mover is unable to alter the status quo.

Section 5 reports some general theoretical propositions on the consequences of convex preferences and Axioms R and S . To illustrate the empirical content of the theory, we derive testable predictions for the well-known investment game, which features a complete MGT ordering of linear opportunity sets. We also derive testable predictions for Stackelberg duopoly games. These games are especially useful because a smaller output by the Stackelberg Leader induces a more generous opportunity set for the Follower, and because the opportunity sets have a parabolic shape that enables the Follower to reveal a wide range of positive and negative tradeoffs between own income and Leader's income. Some key predictions involve a variant game, called the Stackelberg mini-game, in which the Leader has only two alternative output choices, one of which is clearly more generous than the other.

The next three sections test the illustrative predictions on existing investment game data, on existing Stackelberg data, and on new Stackeberg mini-game data. Within the limitations of the data, the results are all consistent with the predictions. The last section summarizes and points to further empirical and theoretical work. All formal proofs and other mathematical details are collected in Appendix A. Instructions to subjects in the mini-Stackelberg game appear in Appendix B.

## 2. Preferences

Let $Y=\left(Y_{1}, Y_{2}, \ldots, Y_{N}\right) \in \mathfrak{R}_{+}^{N}$ represent the payoff vector in a game that pays each of $N \geq 2$ players a non-negative income. Admissible preferences for each player $i$ are smooth and convex orderings on the positive orthant $\Re_{+}^{N}$ that are strictly increasing in own income $Y_{i}$. The set of all admissible preferences is denoted $\mathfrak{P}$.

Any particular preference $\mathcal{P} \in \mathfrak{P}$ can be represented by a smooth utility function $u: \Re_{+}^{N} \rightarrow \Re$ with positive $i^{t h}$ partial derivative $\frac{\partial u}{\partial Y_{i}}=u_{Y_{i}}>0$. The other first partial derivatives are zero for standard selfish preferences, but we allow for the possibility that they are positive in some regions (where the agent is "benevolent") and negative in others (where she is "malevolent").

We shall focus on two-player extensive form games of complete information, and to streamline notation we shall denote own ("my") income by $Y_{i}=m$ and the other player's ("your") income by $Y_{-i}=y$. Thus preferences are defined on the positive quadrant $\Re_{+}^{2}=\{(m, y): m, y \geq 0\}$. The player's marginal rate of substitution $\operatorname{MRS}(m, y)=u_{m} / u_{y}$ is, of course, the negative of the slope of the indifference curve through the given point. Unfortunately, the MRS is not well defined at points where the agent is selfish, and diverges to $+\infty$ and back from $-\infty$ when we pass from slight benevolence to slight malevolence. Therefore it is convenient to work with willingness to pay, WTP $=1 / \mathbf{M R S}$, the amount of own payoff the agent is willing to give up in order to increase the other agent's payoff by a unit. WTP moves from slightly positive through zero to slightly negative when the agent goes from slight benevolence to slight malevolence.

What sort of factors might affect WTP? A prime candidate is relative income, as measured for non-zero allocations in $\Re_{+}^{2}$ by the ratio of other's income to own income, $d=y / m$. It is easily shown (see Appendix A.1) that only relative income $d$ matters for homothetic preferences, i.e., WTP is constant along each ray $R_{d}=$ $\{(t, t d): t>0\} \subset \Re_{+}^{2}$ when preferences are homothetic.

It is intuitive that WTP decreases in $d$, that is, I'm willing to pay less to increase your income when I'm relatively poor. The intuition is formalized in the convexity assumption imposed earlier. Recall that preferences are convex if each upper contour set (i.e., the set of allocations preferred to any given allocation) is a convex subset of $\Re_{+}^{2}$. A quantitative measure of convexity is provided by curvature of the indifference curves, using first and second partial derivatives of a utility function $u$ representing the preferences (e.g., Gray, 1997, pp.14-17). Curvature is the reciprocal of the radius of the circle that is second-order tangent to the indifference curve through a given point and is given by

$$
\begin{equation*}
\mathbf{K}=\frac{u_{m m} u_{y}^{2}-2 u_{m y} u_{m} u_{y}+u_{y y} u_{m}^{2}}{\left(u_{m}^{2}+u_{y}^{2}\right)^{3 / 2}} \tag{2.1}
\end{equation*}
$$

The convexity assumption is that $\mathbf{K} \leq 0$ at every point $\Re_{+}^{2}$. Negative $\mathbf{K}$ reflects decreasing WTP, e.g., the slope moves towards 0 as a benevolent player's own income increases along an indifference curve. Of course $\mathbf{K}=0$ in a region where indifference curves are straight lines, and more negative $\mathbf{K}$ means that the WTP changes more quickly with changes in relative income $d$ along an indifference curve.

Note that both WTP and $\mathbf{K}$ are intrinsic for preferences. That is, if we choose another utility function $w=h \circ u$ to represent the same preferences (so $h^{\prime}(t)>0$ $\forall t \in \operatorname{Range}(u)$ ), then using $w$ in the computations for WTP and K gives us the same values that we get using $u$.

We are now prepared to formalize the idea that one preference ordering on $\Re_{+}^{2}$ is more altruistic than another. Two different preference orderings $\mathcal{A}, \mathcal{B} \in \mathfrak{P}$ over income allocation vectors might represent the preferences of two different players, or might represent the preferences of the same player in two different emotional states (Cox, Friedman, and Gjerstad, 2004).

Definition 2.1. For a given domain $\mathfrak{D} \subset \Re_{+}^{2}$ we say that $\mathcal{A}$ MAT $\mathcal{B}$ on $\mathfrak{D}$ if $\mathbf{W T P}_{\mathcal{A}}(m, y) \geq \mathbf{W T P}_{\mathcal{B}}(m, y)$, for all $(m, y) \in \mathfrak{D}$.

The idea is straightforward. In the benevolence case (where utility is monotone increasing in $y$ ) more altruistic than (MAT) means that $\mathcal{A}$ has shallower indifference curves than $\mathcal{B}$ in $(m, y)$ space, so $\mathcal{A}$ indicates a willingness to pay more $m$ than $\mathcal{B}$ for a unit increase in $y$. In the malevolence case, WTP is less negative for $\mathcal{A}$, so it indicates a lesser willingness to pay for a unit decrease in $y$.

Of course, MAT is a partial ordering on $\mathfrak{P}$, not a total ordering, for nontrivial domains $\mathfrak{D}$ that contain more than a single point. When preferences are homothetic then it suffices to check for MAT on a thin subset of $\mathfrak{D}$, typically a single indifference curve. When no particular domain $\mathfrak{D}$ is indicated, the MAT ordering is understood to refer to the entire positive orthant $\mathfrak{D}=\Re_{+}^{2}$.

Example 2.2. Linear Inequality-averse Preferences (for $N=2$ only; Fehr and Schmidt, 1999). Let preferences $\mathcal{J}=\mathcal{A}, \mathcal{B}$ be represented by $u_{\mathcal{J}}(m, y)=(1+$ $\left.\theta_{\mathcal{J}}\right) m-\theta_{\mathcal{J}} y$, where

$$
\begin{array}{rlrl}
\theta_{\mathcal{J}} & =\alpha_{\mathcal{J}}, & & \\
& \text { if } m<y \\
& =-\beta_{\mathcal{J}}, & & \text { if } m \geq y
\end{array}
$$

and $0<\beta_{\mathcal{J}} \leq \alpha_{\mathcal{J}}, 0<\beta_{\mathcal{J}}<1$. Straightforwardly, $\mathcal{A}$ MAT B iff $\theta_{\mathcal{A}} \leq \theta_{\mathcal{B}}$.
Example 2.3. Nonlinear Inequality-averse Preferences (for $N=2$, Bolton and Ockenfels, 2000). Let preferences $\mathcal{J}=\mathcal{A}, \mathcal{B}$ be represented by $u_{\mathcal{J}}(m, y)=\nu_{\mathcal{J}}(m, \sigma)$, where

$$
\begin{aligned}
\sigma & =m /(m+y), & & \text { if } m+y>0 \\
& =1 / 2, & & \text { if } m+y=0
\end{aligned}
$$

It can be easily verified that $\mathcal{A} \operatorname{MAT} \mathcal{B}$ iff $\nu_{\mathcal{A} 1} / \nu_{\mathcal{A} 2} \leq \nu_{\mathcal{B} 1} / \nu_{\mathcal{B} 2}$.
Example 2.4. Quasi Maxi-min Preferences (for $N=2$, Charness and Rabin, 2002). Let preferences $\mathcal{J}=\mathcal{A}, \mathcal{B}$ be represented by

$$
\begin{aligned}
u_{\mathcal{J}}(m, y) & =m+\gamma_{\mathcal{J}}\left(1-\delta_{\mathcal{J}}\right) y, \quad \text { if } \quad m<y \\
& =\left(1-\delta_{\mathcal{J}} \gamma_{\mathcal{J}}\right) m+\gamma_{\mathcal{J}} y, \quad \text { if } \quad m \geq y
\end{aligned}
$$

and $\gamma_{\mathcal{J}} \in[0,1], \delta_{\mathcal{J}} \in(0,1)$. It is straightforward (although a bit tedious) to verify that $\mathcal{A}$ MAT $\mathcal{B}$ iff

$$
\gamma_{\mathcal{A}} \geq \gamma_{\mathcal{B}} \max \left\{\frac{1}{1+\left(\delta_{\mathcal{A}}-\delta_{\mathcal{B}}\right) \gamma_{\mathcal{B}}}, \frac{1-\delta_{\mathcal{B}}}{1-\delta_{\mathcal{A}}}\right\}
$$

Example 2.5. Egocentric Altrusim (CES) Preferences (Cox and Sadiraj, 2004). Let preferences $\mathcal{J}=\mathcal{A}, \mathcal{B}$ be represented by

$$
\begin{aligned}
u_{\mathcal{J}}(m, y) & =\frac{1}{\alpha}\left(m^{\alpha}+\theta_{\mathcal{J}} y^{\alpha}\right), & & \text { if } \quad \alpha \in(-\infty, 1) \backslash\{0\} \\
& =m y^{\theta_{\mathcal{J}}}, & & \text { if } \quad \alpha=0 .
\end{aligned}
$$

If $0<\theta_{\mathcal{B}} \leq \theta_{\mathcal{A}}$ then A MAT B. Verification is straightforward: WTP $\mathcal{J}_{\mathcal{J}}=$ $\theta_{\mathcal{J}}(m / y)^{1-\alpha}, \overline{\mathcal{J}}=\mathrm{A}, \mathrm{B}$ imply $\mathbf{W P} \mathbf{T}_{\mathcal{A}} / \mathbf{W T} \mathbf{P}_{\mathcal{B}}=\theta_{\mathcal{A}} / \theta_{\mathcal{B}} \geq 1$. "Egocentricity" implies $W T P(m, m) \leq 1$.

The exponent $\alpha \leq 1$ determines the curvature and hence the convexity of preferences. Straightforward algebra yields

$$
\mathbf{K}=\frac{\theta(\alpha-1) m^{\alpha+1} y^{\alpha+1}\left(m^{\alpha}+\theta y^{\alpha}\right)}{\left(\left(m^{\alpha} y\right)^{2}+\left(\theta m y^{\alpha}\right)^{2}\right)^{\frac{3}{2}}} \leq 0 .
$$

On a ray $R_{d}=\{(m, m d): m>0\}$ we have

$$
\mathbf{K}=\frac{(\alpha-1) d^{\alpha+1} \theta\left(d^{\alpha} \theta+1\right)}{m\left(d^{2}+\left(d^{\alpha} \theta\right)^{2}\right)^{\frac{3}{2}}} \leq 0
$$

Thus the curvature decreases (in absolute value) along a given ray as $1 / m$, i.e., $m \mathbf{K}$ is constant along the ray. Appendix A. 2 shows that the same is true for any homothetic preferences. Hence relative curvature $m \mathbf{K}$ sometimes is more useful than curvature $\mathbf{K}$.

The theoretical literature on social preferences has previously been characterized by special assumptions that appear to be inconsistent with the classical approach to preference (and demand) theory (Hicks, 1939; Samuelson, 1947). The preceding examples help to clarify this question. The linear and nonlinear inequality aversion models, quasi-maximin model, and egocentric altruism all have indifference curves that are convex to the origin. Thus they retain a central characteristic of classical preference theory: convex upper contour sets. Strict inequality aversion reverses the classical monotonicity ("more is preferred to less") assumption in the region where one's own income is smaller. For $\mathrm{N}=2$, the maxi-min property is implied by convexity and monotonicity. A preference for efficiency (or larger total of all agents' payoffs) is consistent with all of the preceding parametric models, and the revealed altruism model, in so far as WTP can be positive and less than the need to pay in the relevant opportunity set.

## 3. Opportunities

Define an opportunity set $F$ (or synonymously, a feasible set or budget set) as a convex compact subset of $\Re_{+}^{2} . F$ is closed (as a compact set in $\Re_{+}^{2}$ ) and therefore it contains its $\Re_{+}^{2}$-boundary, denoted $\partial F$; indeed $F$ is the convex hull of $\partial F$. Convexity of $F$ means that each boundary point has a supporting hyperplane (i.e., tangent line) defined by an outward-pointing normal vector, and $F$ is contained in a closed negative halfspace; see for example Rockafellar, 1970, pp.100. At some boundary points $X$ (informally called corners or kinks) the supporting hyperplane is not unique; examples will be noted later.

At regular boundary points there is a unique supporting hyperplane and the implicit function theorem guarantees a smooth function $f$ whose isoquant defines the boundary locally. The marginal rate of transformation MRT can be expressed as a ratio of the first partial derivatives, except when the tangent is vertical. We often need to work near vertical tangents, so we use the need to pay (NTP), defined as $\mathbf{N T P}(X)=1 / \mathbf{M R T}(X)=f_{y} / f_{m}$. The NTP is single-valued except at kinks and corners of the boundary, where its values lie in some interval.

Curvature can also be defined at regular boundary points, using the same formula (1) for $\mathbf{K}$ with $u$ replaced by $f$. Again $\mathbf{K}$ and NTP are intrinsic, independent of the
choice $f$ used to represent the boundary segment. Appendix A shows that convex opportunity sets have nonnegative curvature at all regular boundary points. ${ }^{1}$

Some examples may help fix ideas.
Example 3.1. Standard budget set. Let $F=\left\{(m, y) \in \Re_{+}^{2}: m+p y \leq I\right\}$ for given $p, I>0$. Then $\partial F$ consists of portions of the axes together with the line segment $\left\{(m, y) \in \Re_{+}^{2}: m+p y=I\right\}$, as shown in Figure 1. The NTP is $p$ along the budget line, is 0 along the $y$ axis and is $\infty$ along the $m$-axis. NTP assumes all values outside the interval $(0, p)$ at the corner $(0, I / p)$, and takes all values in the interval $[p,+\infty)$ at the corner $(I, 0) \in \partial F$. As usual, curvature is not defined at the corners, while at all regular boundary points $\mathbf{K}=0$.

Example 3.2. Ring test (Liebrand, 1994; see also Sonnemans, van Dijk and van Winden, 2005). Let $F=\left\{(m, y) \in \Re_{+}^{2}: m^{2}+y^{2} \leq R^{2}\right\}$ for given $R>0$, as shown in Figure 2. ${ }^{2}$ On the circular part of the boundary, NTP is $y / m$ and the curvature is $1 / R$.

Example 3.3. Ultimatum game (Güth, Schmittberger, and Schwarze, 1982). The responder's opportunities in the $\$ 10$ ultimatum game consist of two isolated points: the origin $(0,0)$ and the proposal $(x, 10-x)$. Since this set is not convex it doesn't qualify as an opportunity set by our definition.

Example 3.4. Investment game (Berg, Dickhaut, and McCabe, 1995). The First Mover (FM) and Second Mover (SM) each have an initial endowment of $\$ 10$. The FM sends an amount $s \in[0,10]$ to SM, who receives $3 s$. Then the SM returns an amount $r \in[0,3 s]$ to the FM, resulting in payoffs $m=10+3 s-r$ for SM and $y=10-s+r$ for FM. The FM's choice of $s$ selects the SM's opportunity set $F_{s}=\left\{(m, y) \in \Re_{+}^{2}: m \leq 10+3 s, 10-s \leq y \leq 10+2 s, m+y \leq 20+2 s\right\}$. Figure 3 shows $F_{s}$ for $s=3,9$. Curvature $\mathbf{K}=0$ on each segment of the boundary, and $\mathbf{N T P}=1$ on the segment of the boundary corresponding to $r \in(0,3 s)$.

Example 3.5. Moonlighting game (Abbink, Irlenbusch, and Renner, 2000). In this variant on the investment game, the FM sends $s \in[-5,10]$ to SM, who receives $g(s)=3 s$ for positive $s$ and $g(s)=s$ for negative $s$. Then the second mover transfers $t \in\left[\frac{-10+s}{3}, 10+g(s)\right]$ resulting in non-negative payoffs $m=10+g(s)-|t|$ and $y=10-s+t$ for positive $t$ and $y=10-s+3 t$ for negative $t$, as shown in Figure 4. The second mover's opportunity set is the convex hull of the four corner points $(m, y)=(0,0),(10+g(s)-(10-s) / 3,0),(10+g(s), 10-s)$, and $(0,20+2 s)$. The NTP along the boundary of the opportunity set is 1 above and $-1 / 3$ below the first mover's budget line, is 0 along the $y$ axis, and is $\infty$ along the $m$-axis. Again, curvature at all regular boundary points is $\mathbf{K}=0$.

Example 3.6. Power to take game (Bosman and van Winden, 2002). The "take authority" player chooses a take rate $\xi \in[0,1]$. Then the responder with income $I$ chooses a destruction rate $1-\delta$. The resulting payoffs are $m=(1-\xi) \delta I$ for the

[^1]responder and $y=\xi \delta I$ for the take authority. Thus, with free disposal the responder's opportunity set is the convex hull of three points: the origin $(0,0),(0, \xi I)$ and $((1-\xi) I, \xi I)$, as shown in Figure 5.

Example 3.7. Gift exchange labor markets (Fehr, Kirchsteiger, and Riedl, 1993). The employer offers a wage $w>0$ and the worker then chooses an effort level $e>0$ with a quadratic cost function $c(e)$. The final payoffs are $m=w-c(e)$ for the worker and $y=k e-w$ for the employer, where the productivity parameter $k=10$ in a typical game. With free disposal the worker's opportunity set is similar to the second mover's in the investment game, except that the northeastern boundary is a parabolic arc instead of a straight line of slope -1 . Also, if the employer offers a wage in excess of his endowment ( $=10$ in Figure 6 ) then the opportunity set includes part of the quadrant $[m>0>y]$. It is straightforward but a bit messy to extend the definition of opportunity set to include such possibilities.

Example 3.8. Sequential VCM public good game with two players. Each player has initial endowment $I$. FM contributes $c_{1} \in[0, I]$ to the public good. SM observes $c_{1}$ and then chooses his contribution $c_{2} \in[0, I]$. Each unit contributed has a return of $a \in(0.5,1]$, so the final payoffs are $m=I+a c_{1}-(1-a) c_{2}$ for SM and $y=I+a c_{2}-(1-a) c_{1}$ for FM. With free disposal for SM, SM's opportunity set is the convex hull of the four points $(m, y)=\left(I+a c_{1}, 0\right),\left(I+a c_{1}, I-(1-a) c_{1}\right),(a I+$ $\left.a c_{1},(1+a) I-(1-a) c_{1}\right)$ and $\left(0,(1+a) I-(1-a) c_{1}\right)$, as shown in Figure 7. Along the Pareto frontier, NTP is constant at $(1-a) / a$.

Example 3.9. Stackelberg Follower's Opportunity Set (e.g., Varian, 1992). Consider a duopoly with zero fixed cost, constant and equal marginal cost, and nontrivial linear demand. Without further loss of generality one can normalize so that the profit margin (price minus marginal cost) is $M=24-q_{L}-q_{F}$, where $q_{L} \in[0,24]$ is the Leader's output choice and $q_{F} \in\left[0,24-q_{L}\right]$ is the Follower's output to be chosen. Thus payoffs are $m=M q_{F}$ and $y=M q_{L}$. The Follower's opportunity set therefore is bounded by a parabolic arc opening towards the $y$-axis, as shown in Figure 8. A calculation of NTP and curvature appears in Appendix A.4. Unlike the earlier examples, the NTP varies smoothly from negative to positive values along the boundary of the opportunity set. Assuming free disposal by the Leader implies that the opportunity set is the convex hull of the arc in $\Re_{+}^{2}$.

## 4. Reciprocity

Reciprocity is key to our analysis. The idea is that more generous choices by one player induce more altruistic preferences in a second player. To formalize, consider a two person extensive form game of complete information in which the first mover (FM) chooses an opportunity set $C \in \mathcal{C}$, and the second mover (SM) chooses the payoff vector $(m, y) \in C$. Intuitively, opportunity set $G$ is more generous than (MGT) opportunity set $F$ if it is obtained by shrinking the $y$-axis (FM's potential payoffs) and stretching the $m$-axis (SM's potential payoffs), or by simply taking a larger set. We obtain a partial ordering MGT on the subsets of $\Re_{+}^{2}$ using

Definition 4.1. Opportunity set $G \subset \Re_{+}^{2}$ is more generous than opportunity set $F$ if $G$ contains $\mathbb{T} F$ for some smooth transformation $\mathbb{T}: \Re_{+}^{2} \longrightarrow \Re_{+}^{2}$ such that
$(m, y) \longmapsto(\rho(m), \vartheta(y))$ with $\rho(0) \geq 0 \geq \vartheta(0), \rho^{\prime}(m) \geq 1$ and $0 \leq \vartheta^{\prime}(y) \leq 1$. In this case we write $G$ MGT $F$.

Initially, the second mover knows the collection $\mathcal{C}$ of possible opportunity sets. Prior to her actual choice she learns the actual opportunity set $C \in \mathcal{C}$, and acquires preferences $\mathcal{A}_{C}$. Reciprocity is captured formally in
Axiom R. Let the first mover choose the actual opportunity set for the second mover from the collection $\mathcal{C}$. If $F, G \in \mathcal{C}$ and $G$ MGT $F$, then $\mathcal{A}_{G}$ MAT $\mathcal{A}_{F}$.

There is a traditional distinction between sins of commission (active choice) and sins of omission (retaining the status quo). Of course, sometimes there is no choice at all and the status quo cannot be altered. Intuitively, the second mover will respond more strongly to generous (or ungenerous) choices that overturn the status quo than to those that uphold it, and will respond least when the first mover has no real choice. ${ }^{3}$ These distinctions are captured formally in
Axiom S. Let the first mover choose the actual opportunity set $G$ for the second mover from a collection $\mathcal{C}$, with $G$ MGT $F$ for some $F \in \mathcal{C}$. Let $\mathcal{A}_{G}, \mathcal{A}_{G^{*}}$ and $\mathcal{A}_{G}$ o respectively denote the second mover's acquired preferences when $F$ is the status quo, when $G$ is the status quo, and when the choice set is the singleton $\mathcal{C}=\{G\}$. Then $\mathcal{A}_{G} \mathrm{MAT} \mathcal{A}_{G^{*}} \mathbf{M A T} \mathcal{A}_{G^{o}}$. On the other hand, if the first mover chooses the less generous set $F$, then the acquired preferences satisfy $\mathcal{A}_{F o}$ MAT $\mathcal{A}_{F *}$ MAT $\mathcal{A}_{F}$.

We will say that either Axiom holds strictly if the inequalities in the MAT definition (2.1) and the MGT definition (4.1) are strict.

It should be emphasized that the recent preference models noted in Examples (2.3-2.6) have no room for Axioms $R$ and $S$. In those models preferences are assumed fixed, unaffected by more or less generous opportunity sets chosen by the first mover. Actual choices by a first mover are not central even in the "reciprocity" models of Charness and Rabin (2002, Appendix), Falk and Fischbacher (2005), and Dufwenberg and Kiersteiger (2004). Those models focus on higher-order beliefs regarding other players' intentions (or, in Levine (1998), regarding other players' types). Cox, Friedman, and Gjerstad (2004) implicitly consider Axiom R, but only within the particular parametric family of CES utility functions noted in Example 2.2.

Natural MGT orderings are fairly common. For example, with the standard budget set in figure (3.1), an increase in own income $I$ together with a real increase in the price of transfers (so $I / p$ decreases) leads to a more generous budget set, as illustrated in figure 9. Indeed, $\rho^{\prime}(m)$ is simply the income ratio and and $\vartheta^{\prime}<1$ reflects the decrease in $I / p$. Likewise, it is clear from figures (3-7) that larger $s$ in the Investment and Moonlighting games, smaller take rate $\tau$ in the Power to Take game, larger $I$ in the Gift Exchange Labor Market and a larger contribution $c_{1}$ in the VCM public goods game all create MGT opportunities for the second mover.

But a few minutes study of those figures reveals that reciprocity and convexity will be difficult to disentangle. In the Investment game, for example, larger $s$ moves the second mover's initial endowment down along the dashed line, increasing his relative income. Indeed, the ray through the endowment point for $s=2$ has slope $d=8 / 16=1 / 2$, compared to $d=5 / 25=1 / 5$ for $s=5$. Hence if the

[^2]second mover's preferences are strictly convex and homothetic then a larger $s$ implies greater WTP even when those preferences are not at all affected by the first mover's more generous choice. The other games also conflate convexity and reciprocity. The underlying problem is that more generous choices by definition create better relative opportunities, hence lower $d$ and (by convexity) greater WTP.

## 5. Choice

As in standard revealed preference theory, our maintained assumption is that every player always chooses a most preferred point in the opportunity set $F$. By convexity, such points form a connected subset of the boundary of $F$, and if either preferences $\mathcal{A}$ or opportunities $F$ are strictly convex then that subset is a singleton, i.e., there is a unique choice $X \in \partial F$. In this case all points in $F \backslash\{X\}$ are revealed to be on lower $\mathcal{A}$-indifference curves than $X$. The empirical prediction is that no other bundle in $F$ will ever be chosen when $X$ is available.

Not all boundary points are candidates for choice in our set up. The first result is that, due to strict monotonicity in own payoff $m$, only "eastern" points will be chosen, since they have larger own payoff. To formalize, define the eastern boundary as $\partial_{E} F=\{(m, y) \in F: \forall x>m,(x, y) \notin F\}$. The North point $N_{F}$ and the South point $S_{F}$ are the points in $\partial_{E} F$ with respectively the largest and the smallest $y$ component.

Proposition 5.1. Suppose that either preferences $\mathcal{A}$ or the opportunity set $F$ are strictly convex, and let $X$ be the $\mathcal{A}$-chosen point in $F$. Then $X \in \partial_{E} F$.

All proofs are collected in Appendix A.
The next result shows that, as admissible preferences go from maximally malevolent through neutral to maximally benevolent under the MAT ordering, the player's choices trace out the Eastern boundary of the budget set from South to North. To put it another way, consider the ray of slope $d$. As $d$ increases from 0 to $\infty$, the intersection of the ray with the Eastern boundary traces out the chosen points. The notation $d_{X}$ indicates the slope of the ray through $X$, i.e., $d_{X}=y / m$ if $X=(m, y)$.

Proposition 5.2. Suppose that either preferences $\mathcal{A}$ and $\mathcal{B}$, or the opportunity set $F$, are strictly convex. Let $X_{\mathcal{A}}$ and $X_{\mathcal{B}}$ be the points in $F$ chosen when preferences are respectively $\mathcal{A}$ and $\mathcal{B}$. Then
(1) $\mathcal{B}$ MAT $\mathcal{A}$ implies $d_{X_{\mathcal{B}}} \geq d_{X_{\mathcal{A}}}$.
(2) If $X \in \partial_{E} F$ lies on a ray with slope between $d_{X_{\mathcal{A}}}$ and $d_{X_{\mathcal{B}}}$, then there are preferences $\mathcal{P}$ with $\mathcal{B}$ MAT $\mathcal{P}$ MAT $\mathcal{A}$ such that $X$ is the $\mathcal{P}$-chosen point in $F$.
(3) There are admissible preferences for which $S_{F}$ is the chosen point, and other preferences for which a point arbitrarily close to $N_{F}$ is the chosen point in $F$.

Propositions 5.1 and 5.2 deal with a fixed opportunity set. Often we need predictions of how an agent with given preferences will choose in a new opportunity set. Textbook revealed preference theory offers such predictions in the case of standard budget sets and convex monotone preferences. We will get weaker predictions because we deal with more general opportunity sets and with preferences that are
convex but not necessarily monotone in other's income $y$. The following example illustrates this.

Example 5.3. Figure 9 shows standard budget sets $F$ with $I=1, p=1$ (solid line) and $G$ with $I=2, p=4$ (dashed line). Suppose that a player with preferences $\mathcal{P}$ picks $X$ from $F$. What can we predict about his choice $W$ from $G$ ? If it happens that $X$ is in $G$ then textbook revealed preference theory tells us that $W$ is not in $F$; it must be on the segment of the $G$ budget line that lies outside $F$. Using homotheticity, we can strengthen the prediction: $W$ lies on the sub-segment between $Y=t X$ and the corner (as indicated in the Figure) as in the following Proposition.

Proposition 5.4. Let an agent with strictly convex and homothetic preferences $\mathcal{A}$ choose $X$ and $W$ from some opportunity sets $F$ and $G$, respectively. Let $Y=$ $t X \in \partial G$ be the most distant point from the origin on the ray through $X$ in the opportunity set $G$, and let $Z \in \partial_{E} G$ solve $\mathbf{N T P}_{\partial F}(X)=\mathbf{N T P}_{\partial G}(Z)$. Then, for preferences $\mathcal{A}$,
(1) if $X \in G$ then $W \in F^{c}$ or $W=X$,
(2) $d_{W} \leq d_{X}$ iff $\mathbf{N T P}{ }_{\partial F}(X) \leq \mathbf{N T P} \mathbf{P}_{\partial G}(Y)$, and
(3) $d_{W} \geq d_{Z}$ iff $d_{Z} \leq d_{X}$.

Propositions 1-3 do not invoke Axioms R and S . These axioms sometimes sharpen and sometimes weaken the predictions of standard revealed preference theory, as illustrated in the rest of this section.

Example 5.5. What happens in example 5.3 if preferences $\mathcal{A}$ are altered by the choice of $G$ over $F$ ? Were $G$ less generous than $F$, then reciprocity assumption $R$ would imply that the choice $W$ would shift southward, towards the corner $(2,0)$ of the budget set, i.e., the earlier prediction would hold a fortiori. However, $G$ MGT $F$ for reasons explained in the second to last paragraph of the previous section. Consequently, if the other player is responsible for the change in budget set, Axiom $R$ implies that $W$ will shift northward. The prediction reduces to saying that $W$ is north of the South corner. But this tells us nothing; no choice along the Eastern portion of $\partial G$ is ruled out. The problem here is that the reciprocity effect doesn't reinforce the usual substitution effect in revealed preference theory, but rather counteracts it and we have no indication which effect is stronger.

Sharper predictions often arise from closer examination of specific games. We illustrate first with the Investment game of example 3.4.

Proposition 5.6. Let the FM in the Investment game choose $F_{s}$ as the SM's opportunity set, and let $r(s)$ be the SM's response. Also let the same SM be given the same opportunity set $F_{s}$ in a dictator game, and let $r^{o}(s)$ be his response there. Assume that $\mathbf{W T P} \leq 1$ and $\partial \mathbf{W T P} / \partial \mathbf{m} \geq 0$. Then:
(1) convexity implies that $r^{o}(s)$ is increasing in $s$;
(2) Axiom $R$ implies that $r(s)$ is increasing in $s$;
(3) Axiom $S$ implies that $r(s) \geq r^{o}(s)$ for $s=0,1,2, \ldots, 10$.

The standard Stackelberg game in Example 3.9 is especially useful for our purposes. Figure 8 suggests (and Appendix A. 4 verifies) that smaller output choices by
the Stackelberg Leader create MGT opportunity sets for the Follower. By Axiom R we expect this to induce MAT preferences in the Follower. An observable consequence of such a shift is a smaller deviation $q_{D}$ of Follower's output $q_{F}$ from selfish best reply. But of course we must also take into account preference convexity, and also the changing curvature of the opportunity set. These effects are sorted out in the next proposition.

Proposition 5.7. In the standard Stackelberg game of example 3.9, let $Q_{D}\left(q_{L}\right)=$ $q_{F}-q_{F}^{o}$ be the deviation of the Follower's output choice $q_{F}$ from the selfish best reply $q_{F}^{o}=12-\frac{1}{2} q_{L}$ when the Leader chooses output $q_{L}$. One has

$$
\frac{d Q_{D}}{d q_{L}}=-\frac{1}{2} w-\frac{d w}{d q_{L}} q_{L}
$$

where $w=\mathbf{W T P}\left(M q_{F}, M q_{L}\right)$.
(1) If Follower's preferences $\mathcal{A}$ are fixed and linear then WTP of the optimal point is constant with respect to $q_{L}$ and $\frac{d Q_{D}}{d q_{L}}$ is positive if and only if preferences at the point are malevolent.
(2) If Follower's preferences $\mathcal{A}$ are fixed and convex, then WTP is decreasing in $q_{L}$ and $\frac{d Q_{D}}{d q_{L}}$ contains an additional term that is positive providing $q_{L} \leq 12$, $w \leq 1, w_{m} \geq 0$ and $w_{m}+w_{y} \geq 0$.
(3) If Follower's preferences satisfy Axiom $\mathbf{R}$ strictly, then WTP is decreasing in $q_{L}$ and $\frac{d Q_{D}^{r}}{d q_{L}}$ contains an additional positive term.
(4) If Follower's preferences satisfy Axiom $\mathbf{S}$ strictly then WTP is decreasing in $q_{L}$ and $\frac{d Q_{D}^{s}}{d q_{L}}$ has an additional positive (negative) term if the status quo is smaller (larger) than $q_{L}$.
Proposition 5.7 shows that an increase in $q_{L}$ has three different effects:

- A reciprocity effect, items 3-4 in the Proposition. If Axiom R holds strictly, then the less generous opportunity set decreases the Follower's WTP, increasing $q_{F}$ and $q_{D}$. Axiom S moderates or intensifies this effect, depending on the status quo.
- A preference convexity (or substitution) effect, item 2 in the Proposition. The choice point is pushed northwest, where (subject to some technical qualifications) WTP is less, again increasing $q_{F}$ and $q_{D}$.
- An opportunity set shape effect (in some ways analagous to an income effect), item 1. The curvature of the parabola decreases. Holding $w=$ WTP constant, $q_{D}$ increases when the Follower is malevolent $(w<0$, hence $q_{D}>0$ ), and decreases when the Follower is benevolent $(w>0$, hence $q_{D}<0$ ).
The technical qualifications for the preference convexity effect are not especially restrictive. In a sense, Leader choices $q_{L}>12$ are dominated: they produce choice sets for the Follower that are strict subsets of those produced by $q_{L} \leq 12$. (Note that $q_{L}=12$ is the monopoly as well as the Stackelberg output.) The condition $w \leq 1$ (also used in the previous Proposition) says that the Follower would not favor an inefficient adverse transfer; at the margin he loves his neighbor no more than himself. The condition $w_{m}+w_{y} \geq 0$ says that equal increases in income do not push preferences towards malevolence (and equal decreases do not push towards benevolence). The previous proposition uses an even weaker condition, $w_{m} \geq 0$. See the appendix for further discussion of alternative technical conditions.

A parametric example may clarify the logic. For given $q_{L} \in[0,24]$, the Follower's choice set is the parabola $\left\{(m, y): m=M q_{F}, y=M q_{L}, M=24-q_{L}-q_{F}, q_{F} \in\right.$ $\left.\left[0,24-q_{L}\right]\right\}$, with $N T P=-\frac{d m / d q_{F}}{d y / d q_{F}}=\frac{24-q_{L}-2 q_{F}}{q_{L}}$. Suppose that the Follower has fixed Cobb-Douglas preferences represented by $u(m, y)=m y^{\theta}$, so WTP is $\theta m / y=\theta q_{F} / q_{L}$. Solving $\mathbf{N T P}=\mathbf{W T P}$, one obtains $q_{F}=Q\left(q_{L} \mid \theta\right)=(24-$ $\left.q_{L}\right) /(2+\theta)$. Noting that the selfish best reply is $q_{F}^{o}=Q\left(q_{L} \mid 0\right)$, one obtains a closed form expression for the desired deviation, $q_{D}=-\frac{\theta}{4+2 \theta}\left(24-q_{L}\right)$. For fixed $\theta$ positive (benevolent preferences) or smaller than -2 (strongly malevolent preferences), the deviation is negative but increasing in the Leader's output; the opposite is true when $\theta$ is negative but larger than -2 (moderately malevolent). This is the combined impact of the convexity (or substitution) and shape (or income) effects noted above. Of course, reciprocity effects will decrease $\theta$ and hence increase $q_{D}$.

A direct way to test Axiom R is to manipulate the first mover's choice collection $\mathcal{C}$ in the laboratory so that a fixed opportunity set is more or less generous. For example, suppose restrictions on the Stackelber Leader's choice set make a given output choice $q^{*}$ the most generous (smallest) possible in one situation, and the same output $q^{*}$ the least generous (largest) possible in another situation. If a given Follower reacts differently in the two situations, it must be due to reciprocity effects, since by holding $q^{*}$ constant we have eliminated the convexity and shape effects. This is the idea behind the Stackelberg mini-game introduced in the last empirical section.

Corollary 5.8. Stackelberg Mini-Game. Let $x, s, z \in(0,24)$ be such that $x<s<z$. Suppose the Stackelberg Leader has restricted output choices $q_{L} \in\{x, s\}$ in a situation (a) and $q_{L} \in\{z, s\}$ in another situation (b). Let the Leader choose $s$ in both situations. If Follower's preferences satisfy Axiom $R$ then $W T P^{a}\left(M q_{F}, M s\right) \leq$ $W T P^{b}\left(M q_{F}, M s\right)$ and $Q_{D}^{a}\left(M q_{F}, M s\right) \geq Q_{D}^{b}\left(M q_{F}, M s\right)$.

## 6. Investment Game Data

We begin illustration of empirical applications with the Investment game of Example (3.4). Using a double-blind protocol, Cox (2004) gathers data from a one-shot investment game (Treatment A) with 32 pairs of FMs and SMs. Cox also reports parallel data (Treatment C) with another 32 pairs in which SMs are "dictators" with exactly the same opportunity sets given to them by the experimenter. In both treatments, the choices $s$ and $r$ are restricted to integer values but the conclusions of Proposition (5.6) still hold. Axiom S immediately implies that a SM with any particular $F_{s}$ would have more altruistic preferences in Treatment A than in Treatment C: $\mathcal{A}_{F_{s}}$ MAT $\mathcal{A}_{F_{s}}$, for $s=0,1, \ldots, 9$. Proposition (5.6) develops the testable predictions that the SM will return more to the FM when $s$ is larger in Treatment A, and for a given $s$ will return more in Treatment A than in Treatment C.

To test these predictions, construct the dummy variable $D=1$ for Treatment C data, so $D=0$ for Treatment A data. Regress the SM choice $r$ on the amount sent $s$ and its interaction with $D$, using the 2 -sided Tobit procedure to account for the limited range of SM choices in the 54 relevant observations $(r \in[0,3 s]) .{ }^{4}$ The estimated coefficient for $s$ is 0.58 ( $\pm$ standard error of 0.22 ) with one-sided $p$-value

[^3]of 0.006, consistent with the Axiom R prediction. The estimated coefficient for $D \times s$ is $-0.69( \pm 0.32, p=0.018)$, consistent with the Axiom S prediction.

We confirm the Axiom $S$ result by direct hypothesis tests on the most relevant subset of data, where $s=5$ (with 7 observations in each treatment) and $s=10$ (with 13 observations in each treatment). The Mann-Whitney and t-test both reject the null hypothesis of no difference between the amounts returned in favor of the strict Axiom S alternative hypothesis that returns are larger in Treatment A. The one-sided p-values for the t-test (respectively the Mann-Whitney test) are 0.027 (0.058) for the $s=5$ data and are $0.04(0.10)$ for the $s=10$ data.

## 7. Stackelberg Duopoly Data

The investment game data are consistent with the theory but they do not permit crisp tests of reciprocity because, among other limitations, (a) the opportunity sets are linear and hence can't reveal much about WTP, and (b) only one choice is observed per subject, precluding direct observation of changed preferences. Limitations (a) and (b) are overcome in the Stackelberg duopoly data of Huck, Müller, and Normann (2001, henceforth HMN).

The HMN data consist of 220 output choices $\left(q_{L}, q_{F}\right)$ by 22 FMs (or Leaders) choosing $q_{L} \in\{3,4,5, \ldots, 15\}$ randomly rematched for 10 periods each with 22 SMs (or Followers). As noted in Example (3.9) and elsewhere, the SM's choice $q_{F} \in$ $\{3,4,5, \ldots, 15\}$ determines payoffs $(m, y)$ within an opportunity set of discrete points on a parabolic arc. Specifically, payoffs are $m=M q_{F}$ and $y=M q_{L}$, where $M=24-q_{L}-q_{F}$ is the profit margin. The WTP can be inferred at a chosen point $\left(q_{L}, q_{F}\right)$ by the NTP at that point, $\left(24-2 q_{F}-q_{L}\right) / q_{L}$.

Recall that Proposition 5.7 predicts that the SM's output choice reveals a constant WTP if her preferences are linear and unaffected by the FM's output choice $q_{L}$. The corresponding deviation $Q_{D}$ from her selfish best reply output is linearly decreasing in $q_{L}$ if her preferences are benevolent. Convexity and Axiom R effects produce a revealed WTP that is decreasing (and $Q_{D}$ that is increasing) in $q_{L}$.

Table 1 reports tests of these predictions on the HMN data, omitting the 26 data points where the Proposition's hypothesis $q_{L} \leq 12$ fails. ${ }^{5}$ To check for asymmetric responses to large and small FM choices (relative to the Cournot choice $q_{L}=8$ ), we define the dummy variable $D P=1$ if $q_{L} \leq 8$. All columns in the Table report panel regressions with individual subject fixed effects. The first column, with dependent variable WTP $\times 100$, firmly rejects the hypothesis of benevolent linear and fixed preferences: the coefficient for $q_{L}$ is significantly negative, not positive. The second column, with dependent variable $Q_{D}$, confirms this result. We infer that $\mathrm{Q}_{D}$ is an increasing function of $F M$ output $q_{L}$, consistent with convexity and reciprocity. The last column reports that there is a stronger response to "greedy" FM choices in excess of the Cournot output 8 than to "generous" FM choices below or equal to output 8. Some supplementary regressions are noted in the Appendix, also consistent with Proposition 5.7.

[^4]| Dep.Variable | $W T P \times 100$ | $Q_{D}$ | $Q_{D}$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| $q_{L}$ | $-5.43 \pm 1.11^{0.000}$ | $0.35 \pm 0.06^{0.000}$ | $0.24 \pm 0.08^{0.002}$ |
| $D P \times q_{L}$ |  |  | $-0.10 \pm 0.05^{0.023}$ |
| constant | $28.87 \pm 11.01^{0.005}$ | $-2.13 \pm 0.58^{0.000}$ | $-0.81 \pm 0.87^{0.176}$ |
|  |  |  |  |

Table 1. Panel Regressions with fixed effects. Data consist of 194 choices by 22 Followers in HMN experiment when $q_{L}<13$. The coefficient point estimates are shown $\pm$ the standard error, with one-sided p-values in superscripts.

## 8. Stackelberg Mini-Game Data

The HMN data still do not permit tests of some of our most distinctive predictions. All FMs (Leaders) have the same choice set, eliminating variability that could help separate the convexity effect from the reciprocity effect. Also, due in part to differing experiences, SMs may have different views on the generosity of a given output choice $q_{L}$. In order to overcome these limitations while preserving the nice quadratic shape of the SM choice sets, we created a new version of the Stackelberg game that restricts FMs to binary choices.

In our Stackelberg mini-game, each subject in the FM role twice chooses $q_{L} \in$ $\{6,9\}$ and twice chooses $q_{L} \in\{9,12\}$ without feedback. Each subject in the SM role is then paired simultaneously with four different FMs and chooses an integer value of $q_{F} \in\{5,6, \ldots, 11\}$ with no feedback. The corresponding payoffs $(m, y)$ are clearly displayed. The final payoff is given by one of the four choices, selected randomly at the end of the session. The "double blind" procedures are detailed in the instructions to subjects, reproduced at the end of the appendix.

Figure (10) summarizes the data. More generous (smaller) choices by the FM seem to be associated with more altruistic or less malevolent (smaller) choices by the SM , but it is hard to tell from the figure whether the effect is significant. For example, there are only five observations at $q_{L}=6$. Most importantly, the scatterplot doesn't show which of the 24 subjects made which choices.

To infer how individual subjects respond to reciprocity concerns, we turn again to panel regressions with individual subject fixed effects. The second column in Table 2 reports that, consistent with Corollary 5.8 , SMs' average WTP decreased by almost 8 cents per dollar when $q_{L}=9$ was the less generous choice (indicated by $D 9=1$ ). The second column reports the same data in a different way: SM output choice increased by 0.34 on average, significant at the $p=0.016$ level (one-sided). Since the opportunity set $F_{9}$ is constant in these 72 data points, the result cannot be due to convexity or shape effects; it must be pure reciprocity. The last column of Table 2 reports regressions for $Q_{D}$ for the entire data set, using the additional dummy variables D6 (which takes value 1 if $\mathrm{q}_{L}=6,0$ otherwise) and D12 (which takes value 1 if $\mathrm{q}_{L}=12$, and 0 otherwise). The signs of all coefficient estimates are consistent with Axiom R and S, although (since it comes from only 5 non-zero observations), the D6 estimate is not significant.

|  | $\mathrm{WTP} \times 100$ <br> $\left(\mathrm{q}_{L}=9\right)$ | $\mathrm{Q}_{D}$ <br> $\left(\mathrm{q}_{L}=9\right)$ | $\mathrm{Q}_{D}$ |
| :---: | :---: | :---: | :---: |
| nobs $(\mathrm{gr})$ | $72(24)$ | $72(24)$ | $96(24)$ |
| $D 6$ |  |  | $-0.24 \pm 0.35^{0.242}$ |
| $D 9$ | $-7.65 \pm 3.05^{0.008}$ | $0.34 \pm 0.14^{0.008}$ | $0.33 \pm 0.15^{0.016}$ |
| $D 12$ |  |  | $0.37 \pm 0.20^{0.037}$ |
| constant | $-5.93 \pm 2.31^{0.007}$ | $0.27 \pm 0.10^{0.007}$ | $0.18 \pm 0.12^{0.068}$ |
|  |  |  |  |

TABLE 2. Panel Regressions with fixed effects for Stackelberg mini-game data. Entries are coefficient estimates $\pm$ standard errors with one-sided p-values in superscripts.

## 9. Discussion

Classic choice theory (e.g., Hicks, 1939; Samuelson, 1947) clarified and unified earlier work on how preferences and opportunities affect outcomes. The present paper applies those classic ideas to social preferences. We focus on willingness to pay (WTP), the reciprocal of the marginal rate of substitution between own income and others' income. Increasing WTP along indifference curves is simply convexity, and convex social preferences provide a unified account of several social motives previously considered separately, such as efficiency, maximin, and inequality aversion.

The same classic ideas also permit a unified definition of reciprocity. We say that one set of preferences is more altruistic than (MAT) another if it has a larger WTP at every point. We formalize reciprocity as a MAT-shift in preferences following more generous behavior by others. The definitions apply to malevolent (WTP < 0) as well as benevolent ( $\mathrm{WTP}>0$ ) preferences, and automatically combine positive and negative reciprocity

Convexity and reciprocity are quite different formally and conceptually, but we show that empirical work has a natural tendency to confound the two notions. The problem is simply that more generous behavior by a first mover tends to push the second mover's opportunities "southeast," towards larger income for the first mover and smaller income for the second mover. Convexity typically implies greater WTP as one pushes southeast, even when there is no MAT-shift in preferences.

Several theoretical propositions develop the observable consequences of convexity and reciprocity. We show that more northerly choices on the eastern boundary of an opportunity set reveal more altruistic (or less malevolent) preferences. For fixed preferences, choices in an opportunity set reveal bounds on preferences that we translate into bounds on choices in new opportunity sets. The last two theoretical propositions derive testable predictions for the well-known Investment game and the Stackelberg duopoly game.

Finally, to illustrate the theory, we examine two existing data sets and one new data set. Existing investment game data are consistent with convexity and reciprocity, and confirm that people respond more strongly to acts of commission than to default choices. Existing Stackelberg data confirm reciprocity/convexity effects and suggest a stronger negative response to greedy behavior than the positive response to generous behavior. The new Stackelberg mini-game data allow us
to separate convexity from reciprocity effects, and confirm that reciprocity has a significant impact.

Theoretical clarification sets the stage for further empirical work. One can now refine earlier empirical studies that examine altruism and reciprocity. Such work should shed light on the extent to which typical human preferences depart from selfishness, and to what extent they are altered by experiencing generous or selfish behavior.

Further theoretical work is also in order. In particular, Axiom $S$ invokes the status quo to distinguish between acts of commission and omission, and between generous and greedy acts. But what does it take for a choice to become generally recognized as the status quo, and what if an act has beneficial short run impact but is harmful in the long run? Answers to these and other questions await further theoretical development.

## Appendix A. Mathematical proofs and derivations

## A.1. Relative income sensitivity and homothetic preferences .

Lemma A.1. Preferences are homothetic on $\Re_{+}^{2}$ iff WTP is constant along every ray $R_{d}=\{(t, t d): t>0\} \subset \Re_{+}^{2}$.

Proof. By definition, preferences are homothetic iff they can be represented by a utility function $u(m, y)$ whose ratio of partial derivatives $u_{m} / u_{y}$ depends only on the ratio $m / y$, not on $m$ and $y$ separately. But the ratio $d=m / y$ is constant on the ray $R_{d}$ by construction.

## A.2. Relative curvature and homothetic preferences.

Lemma A.2. If preferences are homothetic then $m K(m, d m)$ is constant along the ray $R_{d}$.

Proof. It is well known that homothetic preferences can be represented by a utility function $u(m, y)$ that is homogenous of degree 1 , and that first (second) partial derivatives of homogeneous functions of degree 1 are homogeneous of degree 0 (1)(e.g. Varian, 1992, p.482). It then follows directly from equation (2.1) that $m K(m, m d)=K(1, d), \forall m>0$.

## A.3. Curvature sign of convex opportunity set.

Lemma A.3. A compact connected subset $F \subset \Re_{+}^{2}$ with non-empty interior is convex if and only if $K \geq 0$ at every regular point $X \in \partial F$.

## A.4. Stackelberg Follower's opportunity set. NTP and Curvature

The Follower's opportunity set $F\left(q_{L}\right)$ (with Follower free disposal) has eastern boundary $S\left(q_{L}\right)=\left\{(m, y): m=\left(24-q_{L}-q_{F}\right) q_{F}, y=\left(24-q_{L}-q_{F}\right) q_{L}, q_{F} \in[0,24-\right.$ $\left.\left.q_{L}\right]\right\}$. Along this boundary $\mathbf{N T P}=-\frac{d m / d q_{F}}{d y / d q_{F}}=\frac{24-q_{L}-2 q_{F}}{q_{L}}=\left(24-2 q_{F}\right) / q_{L}-1$. Thus NTP varies smoothly from negative to positive values as $q_{F}$ passes through $q_{F}^{o}=12-\frac{1}{2} q_{L}$, the selfish best response. The need to pay at corners $(0,0)$ and $\left(0,\left(24-q_{L}\right) q_{L}\right)$ respectively takes values from $\left(-\infty, 1-24 / q_{L}\right]$ and $\left[24 / q_{L}-1,+\infty\right)$.

Inserting the $S\left(q_{L}\right)$ expresions into ( $(2.1)$, one verifies that curvature along the eastern boundary is $K(m, y)=2 q_{L}^{-1}\left[\left(2 q_{F}+q_{L}-24\right)^{2}+q_{L}^{2}\right]^{-3 / 2}$. As usual, curvature is not defined at the two corners.

## MGT-ordering

We show that $F\left(q_{L}^{*}\right)$ MGT $F\left(q_{L}\right)$ for all $q_{L}>q_{L}^{*} \in(0,24)$.
Case 1. $q_{L}+q_{L}^{*} \leq 24$. Consider $\mathbb{T}: \Re_{+}^{2} \longrightarrow \Re_{+}^{2}$ s. t. $\rho(m)=\left(\frac{24-q_{L}^{*}}{24-q_{L}}\right)^{2} m$ and $\vartheta(y)=\frac{q_{L}^{*}\left(24-q_{L}^{*}\right)}{q_{L}\left(24-q_{L}\right)} y$. Since $q_{L}^{*}<q_{L}$ we have

$$
\rho^{\prime}(m)=\left(\frac{24-q_{L}^{*}}{24-q_{L}}\right)^{2}>1
$$

and together with $q_{L}^{*} \leq 24-q_{L}$ imply

$$
\left(q_{L}-q_{L}^{*}\right) 24>\left(q_{L}-q_{L}^{*}\right)\left(q_{L}+q_{L}^{*}\right)=q_{L}-q_{L}^{* 2}
$$

and therefore,

$$
\vartheta^{\prime}(y)=\frac{q_{L}^{*}\left(24-q_{L}^{*}\right)}{q_{L}\left(24-q_{L}\right)} \in(0,1) .
$$

Furthermore, $\mathbb{T} F\left(q_{L}\right) \subseteq F\left(q_{L}^{*}\right)$ since for all $P \in \mathbb{T} F\left(q_{L}\right), P \in F\left(q_{L}^{*}\right)$ as follows. $P \in \mathbb{T} F\left(q_{L}\right)$ implies $\exists q_{F} \in\left[0,24-q_{L}\right]$ s.t.

$$
\begin{aligned}
& P_{L}=\left(\frac{24-q_{L}^{*}}{24-q_{L}}\right)^{2}\left(24-q_{L}-q_{F}\right) q_{F}=\left(24-q_{L}^{*}-\frac{24-q_{L}^{*}}{24-q_{L}} q_{F}\right) \frac{24-q_{L}^{*}}{24-q_{L}} q_{F}, \text { and } \\
& P_{F}=\frac{q_{L}^{*}\left(24-q_{L}^{*}\right)}{q_{L}\left(24-q_{L}\right)}\left(24-q_{L}-q_{F}\right) q_{L}=\left(24-q_{L}^{*}-\frac{24-q_{L}^{*}}{24-q_{L}} q_{F}\right) q_{L}^{*}
\end{aligned}
$$

and therefore $P \in F\left(q_{L}^{*}\right)$ for Follower's output choice $\frac{24-q_{L}^{*}}{24-q_{L}} q_{F} \in\left[0,24-q_{L}^{*}\right]$.
Case 2. $q_{L}+q_{L}^{*}>24$. In this case $F\left(q_{L}\right) \subset F\left(q_{L}^{*}\right)$ and therefore $F\left(q_{L}^{*}\right)$ MGT $F\left(q_{L}\right)$.
A.5. Proposition 5.1. Suppose that either preferences $\mathcal{A}$ or the opportunity set $F$ are strictly convex, and let $X$ be the $\mathcal{A}$-chosen point in $F$. Then $X \in \partial_{E} F$.

Proof. Suppose that $X=(m, y) \notin \partial_{E} F$. Then by definition of $\partial_{E} F$ there exists $z>m$ such that $M=(z, y) \in F$. Positive monotonicity in own payoff implies that $M$ is strictly preferred to $X$, contradicting the hypothesis that $X$ is the $\mathcal{A}$-preferred point in $F$.
A.6. Proposition 5.2. Theoretical predictions for fixed opportunity set. Suppose that either preferences $\mathcal{A}$ and $\mathcal{B}$, or the opportunity set $F$, are strictly convex. Let $X_{\mathcal{A}}$ and $X_{\mathcal{B}}$ be the points in $F$ chosen when preferences are respectively $\mathcal{A}$ and $\mathcal{B}$. Then
(1) $\mathcal{B}$ MAT $\mathcal{A}$ implies $d_{X_{\mathcal{B}}} \geq d_{X_{\mathcal{A}}}$.
(2) If $X \in \partial_{E} F$ lies on a ray with slope between $d_{X_{\mathcal{A}}}$ and $d_{X_{\mathcal{B}}}$, then there are preferences $\mathcal{P}$ with $\mathcal{B}$ MAT $\mathcal{P}$ MAT $\mathcal{A}$ such that $X$ is the $\mathcal{P}$-chosen point in $F$.
(3) There are admissible preferences for which $S_{F}$ is the chosen point, and other preferences for which a point arbitrarily close to $N_{F}$ is the chosen point in $F$.

Proof. First, note that for homothetic and strict convex preferences $\mathcal{J}, \forall Y, Z \in$ $\partial_{E} F, d_{Y}<d_{Z}$ iff $\mathbf{W T P}_{\mathcal{J}}(Y)>\mathbf{W T P}_{\mathcal{J}}(Z)$. Denote $P$ the point of the indifference curve through $Y$ which is in the same ray with $Z$, i.e. $d_{Z}=d_{P}$, and write

$$
\begin{equation*}
\mathbf{W T P}_{\mathcal{J}}(Y)>\mathbf{W T P}_{\mathcal{J}}(P)=\mathbf{W T P}_{\mathcal{J}}(Z) \tag{A.1}
\end{equation*}
$$

where the inequality follows from strict convexity of $\mathcal{J}$ preferences whereas the equality from homotheticity of $\mathcal{J}$ preferences.

Part 1. Suppose that $\mathcal{B} \operatorname{MAT} \mathcal{A}$. Then $d_{X_{\mathcal{B}}} \geq d_{X_{\mathcal{A}}}$ as follows. If it were $d_{X_{\mathcal{B}}}<d_{X_{\mathcal{A}}}$ then $X_{\mathcal{A}}, X_{\mathcal{B}} \in \partial_{E} F$ from Proposition 5.1 and (A.1) imply

$$
\begin{aligned}
\boldsymbol{N T P}\left(X_{\mathcal{B}}\right) & =\mathbf{W T P}_{\mathcal{B}}\left(X_{\mathcal{B}}\right) \geq \mathbf{W T P}_{\mathcal{A}}\left(X_{\mathcal{B}}\right) \geq \mathbf{W T P}_{\mathcal{A}}\left(X_{\mathcal{A}}\right)=\mathbf{N T P}\left(X_{\mathcal{A}}\right) \\
& \geq \mathbf{N T P}\left(X_{\mathcal{B}}\right)
\end{aligned}
$$

where the equalities follow from the optimality of $X_{\mathcal{J}}$, for $\mathcal{J}$ preferences, $\mathcal{J}=\mathcal{A}, \mathcal{B}$; the last inequality follows from the positive curvature of $\partial_{E} F$, the first inequality follows from follows from $\mathcal{B} M A T \mathcal{A}$ whereas the second follows from (A.1). A contradiction is reached if either of preferences $\mathcal{A}, \mathcal{B}$ or the opportunity set $F$ is strictly convex since then either the second or the last inequality is strict.

Part 2. Let $X$ be given such that $d_{X_{a}}<d_{X}<d_{X_{b}}$. Let $w_{a}$ and $w_{b}$ denote continuous WTP functions for $\mathcal{A}$ and $\mathcal{B}$ preferences. If $w_{b}(X)=w_{a}(X)$ then $X$ is the chosen point for both $\mathcal{A}$ and $\mathcal{B}$ preferences. If $w_{b}(X)>w_{a}(X)$ then consider some preferences $\mathcal{P}$ such that, for all $Y, w_{p}(Y)=k w_{b}(Y)+(1-k) w_{a}(Y)$ where

$$
k=\frac{N T P(X)-w_{a}(X)}{w_{b}(X)-w_{a}(X)}
$$

There exists a P-utility function with $\mathrm{w}_{P}(Y)$ since the latter is continuous (Hurewicz, 1958, pp. $7-10$; see also Hurwicz and Uzawa, 1971). $\mathcal{B}$ MAT $\mathcal{P}$ MAT $\mathcal{A}$ (on points where $\mathcal{B}$ MAT $\mathcal{A}$ ) follows from $k$ between 0 and 1 (implied by $d_{X_{a}}<d_{X}<d_{X_{b}}$ ). $X$ is $\mathcal{P}$-chosen since straightforwardly, $w_{P}(X)=\operatorname{NTP}(X)$

Part 3. Linear preferences with $w$ going to $-\infty(+\infty)$ have the chosen point arbitrarily close to $S_{F}\left(N_{F}\right)$.
A.7. Proposition 5.4. Theoretical predictions for different opportunity set. Let an agent with strictly convex and homothetic preferences $\mathcal{A}$ choose $X$ and $W$ from some opportunity sets $F$ and $G$, respectively. Let $Y=t X \in \partial G$ be the most distant point from the origin on the ray through $X$ in the opportunity set $G$, and let $Z \in \partial_{E} G$ solve $\mathbf{N T P}_{\partial F}(X)=\mathbf{N T P}{ }_{\partial G}(Z)$. Then, for preferences $\mathcal{A}$,
(1) if $X \in G$ then $W \in F^{c}$ or $W=X$,
(2) $d_{W} \leq d_{X}$ iff $\mathbf{N T P} \mathbf{P}_{\partial F}(X) \leq \mathbf{N T P} \mathbf{P}_{\partial G}(Y)$, and
(3) $d_{W} \geq d_{Z}$ iff $d_{Z} \leq d_{X}$.

Proof.
Part 1. Standard revealed preferences theory.
Part 2. $d_{W} \geq d_{X}$ is equivalent with (a) $\mathbf{N T P}_{\partial G}(Y) \leq \mathbf{N T P}_{\partial G}(W)$ by convexity of $G$ and construction of $Y$; and (b) $\mathbf{W T P}(W) \leq \mathbf{W T P}(Y)$ from (A.1). On the other hand, by construction of $Y$ and homotheticity (c) $\mathbf{W T P}(\mathbf{Y})=\mathbf{W T P}(\mathbf{X})$, and (d) $\mathbf{W T P}(\mathbf{X})=\mathbf{N T P}_{\partial F}(\mathbf{X}), \mathbf{N T P}_{\partial G}(W)=\mathbf{W T P}(W)$ since $X, W$ are the most preferred points in respectively, $F, G . \mathbf{N T P}_{\partial G}(Y) \leq \mathbf{N T P}_{\partial F}(\mathbf{X})$ follows from (a)-(d) and transitivity.

Part 3. $d_{W}>d_{Z}$ iff $d_{Z}<d_{X}$. Referring to (A.1), WTP $(Y)<\mathbf{W T P}(Z)$ iff $\left(d_{X}=\right) d_{y}>d_{Z}$ and

$$
\begin{aligned}
\boldsymbol{W T P}(Y) & =\mathbf{W T P}(X)=\mathbf{N T P}_{\partial F}(X)=\mathbf{N T P}_{\partial G}(Z) \leq \mathbf{N T P}_{\partial G}(W)=\mathbf{W T P}(W) \\
& <\mathbf{W T P}(Z)
\end{aligned}
$$

where the first and the third equalities are true by construction of $Y$ and $Z$, the second and the fourth equalities follow from the optimality of $X$ and $W$, whereas the first and the last inequalities are, for convex opportunity sets and preferences, equivalent with $d_{W}>d_{Z}$.
A.8. Proposition 5.6. Investment Game. Let the FM in the Investment game choose $F_{s}$ as the SM's opportunity set, and let $r(s)$ be the SM's response. Also let the same SM be given the same opportunity set $F_{s}$ in a dictator game, and let $r^{o}(s)$ be his response there. Assume that $\mathbf{W T P} \leq 1$ and $\mathbf{W T P} \mathbf{P}_{m} \geq 0$. Then:
(1) convexity implies that $r^{o}(s)$ is increasing in $s$;
(2) Axiom $R$ implies that $r(s)$ is increasing in $s$;
(3) Axiom $S$ implies that $r(s) \geq r^{o}(s)$ for $s=0,1,2, \ldots, 10$.

Proof. Parts 1 and 2. To streamline notation, let $w(s)=W T P(m(s), y(s))$, where $m(s)=10+3 s-r(s)$ and $y(s)=10-s+r(s)$. By hypothesis, $w \leq 1$ and (a) $w_{m} \geq 0$. By Lemma A. 4 below, strict convexity implies that $w_{m} w-w_{y}>0$. It follows that (b) $w_{m}-w_{y}>0$. Since $\mathbf{N T P}=1$, a constant, along the eastern boundary of the opportunity set, the first order condition for optimality requires $w(s)$ also to remain constant. Therefore

$$
\begin{aligned}
0=\frac{d w}{d s} & =w_{m} \frac{d m}{d s}+w_{y} \frac{d y}{d s} \\
& =\left[(3-d r / d s) w_{m}+(-1+d r / d s) w_{y}\right] \\
& =2\left[w_{m}\right]+(1-d r / d s)\left[w_{m}-w_{y}\right]
\end{aligned}
$$

The bracketed expressions are positive by (a) and (b) above, so we must have $d r / d s>1$ for choices not at the corner. For corner choices and weak convexity, the argument allows only to conclude that $r(s)$ is nondecreasing.

Part 3. Follows directly from Axiom S.

Lemma A.4. Strict convexity implies that $w_{m} w-w_{y}>0$.

Proof. Substitute

$$
w=\frac{u_{y}}{u_{m}}, w_{m}=\frac{u_{y m} u_{m}-u_{m m} u_{y}}{u_{m}^{2}}, w_{y}=\frac{u_{y y} u_{m}-u_{m y} u_{y}}{u_{m}^{2}}
$$

in the formula for $K_{w}$ to get

$$
K_{w}=\frac{\left(w_{y}-w_{m} w\right)}{\left(1+w^{2}\right)^{3 / 2}}<0
$$

A.9. Proposition 5.7. Stackelberg Duopoly Game. In the standard Stackelberg game of example 3.9, let $Q_{D}\left(q_{L}\right)=q_{F}-q_{F}^{o}$ be the deviation of the Follower's output choice $q_{F}$ from the selfish best reply $q_{F}^{o}=12-\frac{1}{2} q_{L}$ when the Leader chooses output $q_{L}$. One has

$$
\frac{d Q_{D}}{d q_{L}}=-\frac{1}{2} w-\frac{d w}{d q_{L}} q_{L}
$$

where $w=\mathbf{W T P}\left(M q_{F}, M q_{L}\right)$.
(1) If Follower's preferences $\mathcal{A}$ are fixed and linear then WTP of the optimal point is constant with respect to $q_{L}$ and $\frac{d Q_{D}}{d q_{L}}$ is positive if and only if preferences at the point are malevolent.
(2) If Follower's preferences $\mathcal{A}$ are fixed and convex, then WTP is decreasing in $q_{L}$ and $\frac{d Q_{D}}{d q_{L}}$ contains an additional term that is positive providing $q_{L} \leq 12$, $w \leq 1, w_{m} \geq 0$ and $w_{m}+w_{y} \geq 0$.
(3) If Follower's preferences satisfy Axiom $\mathbf{R}$ strictly, then WTP is decreasing in $q_{L}$ and $\frac{d Q_{D}^{r}}{d q_{L}}$ contains an additional positive term.
(4) If Follower's preferences satisfy Axiom $\mathbf{S}$ strictly then WTP is decreasing in $q_{L}$ and $\frac{d Q_{D}^{s}}{d q_{L}}$ has an additional positive (negative) term if the status quo is smaller (larger) than $q_{L}$.

Proof. The $F O C$ is $\left(w\left(q_{F}, q_{L}\right)=\right) W T P\left(M q_{F}, M q_{L}\right)=N T P=\frac{24-2 q_{F}}{q_{L}}-1$, which can be rewritten as

$$
\begin{equation*}
q_{F}=12-\frac{w\left(q_{F}, q_{L}\right)+1}{2} q_{L} \tag{A.2}
\end{equation*}
$$

and therefore $Q_{D}$ satisfies

$$
\begin{equation*}
Q_{D}=-\frac{w\left(q_{F}, q_{L}\right)}{2} q_{L} \tag{A.3}
\end{equation*}
$$

Part 1. Linear preferences If Follower's preferences are fixed and linear with $W T P=w$ then differentiation of (A.3) with respect to $q_{L}$ gives

$$
\frac{d Q_{D}}{d q_{L}}=-\frac{w}{2}
$$

Part 2.Convex Preferences If Follower's preferences are fixed and convex then

$$
\frac{d Q_{D}}{d q_{L}}=-\frac{w\left(q_{F}, q_{L}\right)}{2}-\frac{q_{L}}{2} \frac{d w\left(q_{F}, q_{L}\right)}{d q_{L}}
$$

We shall show that $\frac{d w\left(q_{F}, q_{L}\right)}{d q_{L}}$ is negative. Indeed

$$
\begin{aligned}
\frac{d w\left(q_{F}, q_{L}\right)}{d q_{L}} & =w_{m} \frac{d m}{d q_{L}}+w_{y} \frac{d y}{d q_{L}} \\
& =w_{m}\left(\left(-1-\frac{d q_{F}}{d q_{L}}\right) q_{F}+M \frac{d q_{F}}{d q_{L}}\right)+w_{y}\left(\left(-1-\frac{d q_{F}}{d q_{L}}\right) q_{L}+M\right)
\end{aligned}
$$

which after substituting $M=24-q_{L}-q_{F}, q_{F}=12-\frac{w\left(q_{F}, q_{L}\right)+1}{2} q_{L}$ and $\frac{d q_{F}}{d q_{L}}=$ $-\frac{w\left(q_{F}, q_{L}\right)+1}{2}-\frac{d w\left(q_{F}, q_{L}\right)}{d q_{L}} q_{L}$ and solving for $\frac{d w\left(q_{F}, q_{L}\right)}{d q_{L}}$ we get

$$
\frac{d w\left(q_{F}, q_{L}\right)}{d q_{L}}=\frac{B}{A}
$$

where

$$
A=2+\left[w_{m} w-w_{y}\right] q_{L}^{2}
$$

and

$$
B=24\left(w_{y}-w_{m}\right)+q_{L}(1-w)\left(w_{m}-w_{y}+w w_{m}-w_{y}\right)
$$

As in the proof of the previous proposition and Lemma (A.4), we have $w_{y}-w_{m} \leq 0$ due to convexity, $w \leq 1$ and $w_{m} \geq 0$.

Suppose $q_{L} \leq 12$. Then $2 q_{L} \leq 24$ and $w_{y}-w_{m} \leq 0$ imply

$$
B<-\left(w^{2} w_{m}-2 w w_{y}+w_{m}\right) q_{L}
$$

If $w \in[0,1]$ then the expression in brackets is non-negative (write it as $w\left(w w_{m}-\right.$ $\left.w_{y}+w_{m} / w-w_{y}\right)$ which is larger than $\left.2 w\left(w w_{m}-w_{y}\right) \geq 0\right)$. If however $w<0$ then the term in brackets is positive since $w_{y}^{2}-w_{m}^{2} \leq 0$. To see this, recall that $w_{y}+w_{m} \geq 0$ and $w_{y}-w_{m} \leq 0$.

Part 3. Axiom $R$ Effect. Let $w^{r}\left(q_{F}, q_{L}\right)$ denote $W T P$ for changed preferences as in Axiom $R$. Then

$$
Q_{D}^{r}=-\frac{w_{r}\left(q_{F}, q_{L}\right)}{2} q_{L}
$$

for all $q_{L}$, and

$$
\begin{aligned}
\frac{d Q_{D}^{r}}{d q_{L}} & =-\frac{w^{r}\left(q_{F}, q_{L}\right)}{2}-\frac{q_{L}}{2} \frac{d w^{r}\left(q_{F}, q_{L}\right)}{d q_{L}} \\
& =-\frac{w\left(q_{F}, q_{L}\right)}{2}-\frac{w^{r}\left(q_{F}, q_{L}\right)-w\left(q_{F}, q_{L}\right)}{2}-\frac{q_{L}}{2} \frac{d w^{r}\left(q_{F}, q_{L}\right)}{d q_{L}}
\end{aligned}
$$

From Axiom $R$ the second term is positive and similarly as in part 2 the third term is positive if induced preferences are benevolent $\left(w^{r} \geq 0\right)$ or malevolent with $w_{y}^{r}+w_{m}^{r} \geq 0$.

Part 4. Axiom $S$ Effect.
Let $w^{s}\left(q_{F}, q_{L}\right)$ denote $W T P$ for changed preferences as in Axiom $S$ Then

$$
Q_{D}^{s}=-\frac{w^{s}\left(q_{F}, q_{L}\right)}{2} q_{L}
$$

is smaller (larger) than $Q_{D}^{r}$ if the status quo is smaller (larger) than $q_{L}$, and

$$
\frac{d Q_{D}^{s}}{d q_{L}}=-\frac{w^{s}\left(q_{F}, q_{L}\right)}{2}-\frac{q_{L}}{2} \frac{d w^{s}\left(q_{F}, q_{L}\right)}{d q_{L}}
$$

has an additional positive (negative) term if the status quo is smaller (larger) than $q_{L}$.
A.10. Alternative Regressions. The last proof suggests alternative specifications for the HMN regressions. Let $a=w_{m} w-w_{y}, b=w_{y}-w_{m}$ and $c=$ $(1-w)\left(w_{m}-w_{y}+w w_{m}-w_{y}\right)$. Recall from the proof that $a \geq 0, b \leq 0$ and $c \geq 0$ for all $q_{L}$, and

$$
\begin{aligned}
\frac{d w\left(q_{F}, q_{L}\right)}{d q_{L}} & =\frac{24 b}{2+a q_{L}^{2}}+\frac{c}{2+a q_{L}^{2}} q_{L} \\
\frac{d Q_{D}}{d q_{L}} & =-\frac{w\left(q_{F}, q_{L}\right)}{2}-\frac{12 b}{2+a q_{L}^{2}} q_{L}-\frac{c}{2\left(2+a q_{L}^{2}\right)} q_{L}^{2}
\end{aligned}
$$

The first order Taylor expansion $Q_{D}\left(q_{L}\right) \approx C+\frac{d Q_{D}}{d q_{L}} q_{L}$ then suggests fitting $Q_{D}$ to a cubic expression in $q_{L}$,

$$
\begin{equation*}
Q_{D}=\beta_{0}+\beta_{1} q_{L}+\beta_{2} q_{L}^{2}+\beta_{3} q_{L}^{3}+u_{i}+\varepsilon_{i} . \text { Predictions are: } \beta_{2}>0, \beta_{3}<0 \tag{A.4}
\end{equation*}
$$

The predicted coefficient signs then are $\beta_{2}>0, \beta_{3}<0$, consistent with the results reported in table (3) below. Likewise, $\frac{d w}{d q_{L}} \approx \gamma_{1}+\gamma_{2} q_{L}$ suggests the quadratic specification $W T P=\beta_{0}+\beta_{1} q_{L}+\beta_{2} q_{L}^{2}+u_{i}+\varepsilon_{i}$. The predictions $\beta_{2}>0, \beta_{1}<0$ are consistent with the results but are not significant in this specification. Allowing asymmetric responses to Leader choices more or less generous than Cournot produces more significant estimates, as reported in the last column.

| Dep.Variable | $Q_{D}$ | $W T P \times 100$ | $W T P \times 100$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| $q_{L}$ | $-2.13 \pm 0.93^{0.013}$ | $-6.07 \pm 5.24^{0.124}$ | $-3.98 \pm 5.19^{0.222}$ |
| $D P \times q_{L}$ |  |  | $2.62 \pm 0.92^{0.003}$ |
| $q_{L}^{2}$ | $0.29 \pm 0.11^{0.005}$ | $0.08 \pm 0.27^{0.385}$ | $0.09 \pm 0.26^{0.365}$ |
| $q_{L}^{3}$ | $-0.01 \pm 0.004^{0.005}$ |  |  |
| constant | $4.41 \pm 2.63^{0.048}$ | $28.26 \pm 24.87^{0.129}$ | $0.39 \pm 26.29^{0.494}$ |
|  |  |  |  |

TABLE 3. Panel Regressions with fixed effects. Data consist of 220
choices by 22 Followers in HMN experiment. One-sided p-values
are reported and $\pm$ refers to standard error.

## Appendix B. Instructions

## Welcome

This is an experiment about decision-making. You will be paid a $\$ 5$ participation fee plus an additional positive or zero amount of money determined by the decisions that you and the other participants make, as explained below. Payment is in cash at the end of the experiment. A research foundation has provided the funds for this experiment.

## No Talking Allowed

Now that the experiment has begun, we ask that you do not talk. If you have a question, please raise your hand and an experimenter will approach you and answer your question in private.

## A Monitor and Two Groups

A monitor will be selected randomly from among those of you who came here today. The rest of you have been divided randomly into two groups, called the First Mover Group and the Second Mover Group.

## Complete Privacy

The experiment is structured so that no one - not even the experimenters, the monitor, and the other subjects - will ever know your personal decision in the experiment. You collect your cash payment from a staff person in the Economics Department office who has no other role in the experiment. Your payment is in a sealed envelope with a code letter (A, B, C, etc). Your privacy is guaranteed because neither your name nor your student ID number will appear on any decision records. The only identifying mark on the decision forms will be a code letter known only to you. You will show your code letter to the staff person and nobody else will see it. The experimenters will not be in the department office when you collect you cash payment. This procedure is used to protect your privacy.

## The Idea of the Game

The game involves two players, called the First Mover (FM) and the Second Mover (SM), in the roles of producers of an identical good. Each decides how much to produce. The profit for each player is the number of units he decides to produce times price, net of cost. The price of the good decreases as total production increases. If you and the other player produce too much, you will drive down the price and your profits. Of course, if you don't produce much you won't have many units to sell. To simplify your task, the profits will be calculated for you and shown in an easy-to-read table. Your cash payment will include the profit you earn in one round of the game. The round will be selected randomly at the end of the experiment.

## Game Details

Each round the FM chooses between two possible amounts to produce, as shown in a table with two rows. The SM sees the choice of the FM, and then decides among seven possible amounts to produce, as shown in seven columns of the same table. The table shows the profits for both players. The FM's profit is shown in italics in the lower left corner of each box, and the SM's profit is shown in bold in the upper right corner. For example, in Table 1 below, if FM chooses Output=6 and SM then chooses Output=4, then FM's profit is 84 and SM's profit is 56 .

Table 1

|  | SM's Choice of Output Quantity: |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ |
| FM's Output=6 | $84^{56}$ | $78^{65}$ | $72^{72}$ | $66^{77}$ | $60^{80}$ | $54^{81}$ | $48^{80}$ | $42^{77}$ |
| FM's Output=9 | $99^{44}$ | $90^{50}$ | $81^{54}$ | $72^{56}$ | $63^{56}$ | $54^{54}$ | $45^{50}$ | $36^{44}$ |

## Different Subject Pairs in Every Decision

Each First Mover and each Second Mover will make four decisions. But the pairing of First Movers with Second Movers will be different in every decision. This means that you will interact with a DIFFERENT person in the other group in every decision that you make.

## Experiment Procedures and the Monitor

At the beginning of the experiment, the monitor will walk through the room carrying a box containing unmarked, large manila envelopes. Each subject in the First Mover Group will take one of these envelopes from the box. This envelope will contain the experiment decision forms and a code letter.
After the First Movers have made their decisions, they return the experiment decision forms to their large manila envelopes and then walk to the front of the room and deposit the envelopes in the box on the table. It is very important that the First Movers do NOT return their code letters to the large manila envelopes, because they will need them to collect their payoffs.
After all First Movers have deposited their envelopes in the box, the Monitor will take the box to another room in which the experimenters will sort the decision forms and place them in the correct large manila envelopes for the Second Movers. The experimenters will also put code letters in the envelopes for the Second Movers. Next, the Monitor will walk through the room carrying a box containing unmarked, large manila envelopes. Each subject in the Second Mover Group will take one of these envelopes from the box. This envelope will contain the experiment decision forms and a code letter.
After the Second Movers have made their decisions, they return the experiment decision forms to their large manila envelopes and then walk to the front of the room and deposit the envelopes in the box on the table. It is very important that the Second Movers do NOT return their code letters in the large manila envelopes because they will need them to collect their payoffs.
After all Second Movers have deposited their envelopes in the box, the Monitor will take the box to another room in which the experimenters will record the profits and cash payments determined by the subjects' decisions.

## A Roll of a Die Determines Which Decision Pays Money

Although you will make four decisions, only one will pay cash. Which of these decisions will pay cash will be determined by rolling a six-sided die. The experimenter will roll the die in front of you and the monitor will announce which of the numbered sides has ended up on top. The first number from 1 to 4 that ends up on top will determine the page number of the decision that pays cash.
The monitor's cash payment will be the average of all First Movers and Second Movers payments.

## Be Careful

Be careful in recording your decisions. If a First Mover forgets to circle one of the rows in the table, or circles both rows on the same decision page, then it will be impossible to ascertain what decision the First Mover made. In that case, the First Mover will get paid 0 and the Second Mover will get paid 60 if that decision page is selected for payoff by the roll of the die. If a Second Mover doesn't circle a column, then it will be impossible to ascertain what decision the Second Mover made. In that case, the Second Mover will get paid 0 and the First Mover will get paid 60 if that decision page is selected for payoff by the roll of the die.

## Pay Rates

For each point of profit you earn, the experimenter will put a fixed number of dollars in your envelope. This fixed number is called the pay rate and is written on the
board at the front of the room. Today's pay rate is $\$ 0.25$, which means that every participant earns 25 cents for each profit point shown in the table.

## Frequently Asked Questions

Q1: Exactly how are profits calculated in the Tables?
A: Price is 30 minus the sum of FM output and SM output. Marginal cost is 6 . Profit is output times (price minus marginal cost). But you don't have to worry about doing the calculation; the Tables do it for you.

Q2: Who will know what decisions I make?
A: Nobody else besides you; that is the point of the private envelopes etc. The experimenters are only interested in knowing the distribution of choices for FMs and SMs , not in the private decisions of individual participants.

Q3: Is this some psychology experiment with an agenda you haven't told us?
A: No. It is an economics experiment. If we do anything deceptive, or don't pay you cash as described, then you can complain to the campus Human Subjects Committee and we will be in serious trouble. These instructions are on the level and our interest is in seeing the distribution of choices made in complete privacy.

## Any More Questions?

If you have any questions, please raise your hand and an experimenter will approach you and answer your question in private. Make sure that you understand the instructions before beginning the experiment; otherwise you could, by mistake, mark a different decision than you intended.

## Quiz

(1) In Table 2 below, what are the two possible output choices for the FM?
(2) $\overline{\bar{D} o e s}$ the SM see the FM's choice? (Y or N)
(3) In Table 2, can the SM choose:
(a) Output=5? __( Y or N$)$
(b) Output $=7 ?_{--}(\mathrm{Y}$ or N$)$
(c) Output=12? ${ }_{--}(\mathrm{Y}$ or N$)$
(4) Suppose the FM chooses the top row (Output $=9$ ) in Table 1 and the SM chooses a middle column (Output $=8$ ).
(a) How many points will the FM get? __ points
(b) How much money is that if this is the decision that pays money?

$$
\$
$$

(c) How much will the SM get in this case? __ points, $\$$
(5) In the previous question, if SM chose Output $\overline{=9}$ instead of $\overline{\text { Output }}=8$,
(a) how many more or fewer points would the SM get?
_ more/fewer points
(b) how many more or fewer points would the FM get? more/fewer points
(6) If the $\overline{\mathrm{FM}}$ chooses the top row, what is the maximum number of points that the SM can get? $\qquad$ the minimum number?
(7) If the FM chooses the bottom row, what is the maximum number of points that the SM can get? the minimum number?
(8) Will the SM ever be able to tell which person made any FM choice? (Y or N )
(9) Will the FM ever be able to tell which person made any SM choice? (Y or N )
(10) Will the experimenter ever be able to tell who made any choices? (Y or N )

Table 2

|  | SM's Choice of Output Quantity: |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| FM's Output=9 | $99^{44}$ | $90^{50}$ | $81^{54}$ | $72^{56}$ | $63{ }^{56}$ | $54^{54}$ | $45^{50}$ | $36{ }^{44}$ |
| FM's Output=12 | $96{ }^{32}$ | $84^{35}$ | $72{ }^{36}$ | $60{ }^{35}$ | $48{ }^{32}$ | $36{ }^{27}$ | $24^{20}$ | $12^{11}$ |

## References

[1] Abbink, K., B. Irlenbusch, and E. Renner (2000): "The Moonlighting Game: An Empirical Study on Reciprocity and Retribution," Journal of Economic Behavior and Organization 42, 265-77.
[2] Andreoni, J. and J. Miller (2002): "Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism," Econometrica 70, 737-53.
[3] Berg, J., J. Dickhaut, and K. McCabe (1995): "Trust, Reciprocity, and Social History," Games and Economic Behavior 10, 122-42.
[4] Bolton, G. E. and A. Ockenfels (2000): "ERC: A Theory of Equity, Reciprocity, and Competition," American Economic Review 90, 166-193.
[5] Bosman, R. and F. van Winden (2002), "Emotional Hazard in a Power-to-Take Experiment," Economic Journal 112, 147-69.
[6] Charness, G. and M. Rabin (2002), "Understanding Social Preferences with Simple Tests," Quarterly Journal of Economics 117, 817-869.
[7] Cox, J. C., D. Friedman and S. Gjerstad (2004): "A Tractable Model of Reciprocity and Fairness," University of Arizona, Department of Economics, Discussion Paper.
[8] Cox, J. C. and V. Sadiraj (2004): "Direct Tests of Models of Social Preferences and a New Model," University of Arizona Department of Economics, Discussion Paper.
[9] Dufwenberg, M. and G. Kirchsteiger (2004): "A Theory of Sequential Reciprocity," Games and Economic Behavior 47, 268-98.
[10] Falk, A. and U. Fischbacher (2005), "A Theory of Reciprocity," Games and Economic Behavior, in press.
[11] Fehr, E. and S. Gaechter (2000): "Fairness and Retaliation: The Economics of Reciprocity," Journal of Economic Perspectives 14, 159-181.
[12] Fehr, E., G. Kirchsteiger, and A. Riedl (1993): "Does Fairness Prevent Market Clearing? An Experimental Investigation," Quarterly Journal of Economics 108, 437-460.
[13] Fehr, E. and K. M. Schmidt (1999): "A Theory of Fairness, Competition, and Cooperation," Quarterly Journal of Economics 114, 817-868.
[14] Gray, A. (1997): "Curvature of Curves in the Plane," §1.5 in [http://www.amazon.com/exec/obidos/ASIN/0849371643/ref=nosim/weisstein-20](http://www.amazon.com/exec/obidos/ASIN/0849371643/ref=nosim/weisstein-20) Modern Differential Geometry of Curves and Surfaces with Mathematica, 2nd ed. Boca Raton, FL: CRC Press, pp. 14-17.
[15] Guth, W., R. Schmittberger, and B. Schwarze (1982): "An Experimental Analysis of Ultimatum Barga:ining," Journal of Economic Behavior and Organization, 4, 367-88.
[16] Hicks, J. (1939). Value and Capital. Oxford: Clarendon Press.
[17] Huck, S., W. Muller and H. Normann (2001): "Stackelberg beats Cournot: On collusion and efficiency in experimental markets," Economic Journal 111,749-766.
[18] Hurewicz, W., (1958): Lectures on Ordinary Differential Equations. New York: Wiley.
[19] Hurwicz, L., and H. Uzawa (1971): "On the Integrability of Demand Functions," in J.S.Chipman et al. (eds.), Studies in the Mathematical Foundations of Utility and Demand Theory. New York: Harcourt, Brace \& World.
[20] Levine, D. (1998): "Modeling Altruism and Spitefulness in Experiments," Review of Economic Dynamics, 1, 593-622.
[21] Liebrand, W.B.G. (1984): "The effect of social motives, communication and group sizes on behavior in an n-preson multi stage mixed motive game," European Journal of Social Psychology, 14, 239-264.
[22] Rockafellar (1970): Convex Analysis, Princeton University Press.
[23] Samuelson, P. (1947): Foundations of Economics Analysis. Cambridge, Mass.,Harvard University Press.
[24] Sobel, J. (2005): "Interdependent Preferences and Reciprocity," Journal of Economic Literature, 43, 392-436.
[25] Sonnemans, J, F. van Dijk and F. van Winden (2005): "On the Dynamics of Social Ties Structures in Groups," Journal of Economic Psychology, forthcoming.
[26] Varian, H. R. (1992): "Microeconomic Analysis," third ed., Norton.


Figure 1. Standard Budget Set.


Figure 2. Ring Test Budget Set.


Figure 3. Investment Game, Second Mover's Opportunity Set.


Figure 4. Moonlighting Game, Second Mover's Opportunity Set.


Figure 5. Power to Take Game, Second Mover's Opportunity Set.


Figure 6. Gift Exchange Labor Markets, Second Mover's Opportunity Set.


Figure 7. Sequential VCM Public Goods Game with two players ( $a=0.75$ ).


Figure 8. Stackelberg Duopoly Game, Follower's Opportunity Set.


Figure 9. Illustration of Example 5.1.


Figure 10. Data from Stackelberg mini-game. Deviations $Q_{D}$ of SM actual output choice from the selfish best response are shown in four situations. From left to right, the situations are: FM chose $q_{L}=6$ from $\{6,9\}$, FM chose $q_{L}=9$ from $\{9,12\}$, FM chose $q_{L}=9$ from $\{6,9\}$, and FM chose $q_{L}=12$ from $\{9,12\}$. The size of the dot reflects the fraction of observations in each situation. The number of observations is shown in parentheses.


[^0]:    Date: Working Paper, November 2005.
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[^1]:    ${ }^{1}$ Why positive curvature for convex opportunity sets but negative curvature for convex preferences? One answer is that the opportunity set is the (intersection of) lower contour sets of the $f$-functions, while convex preferences refer to the upper contour sets of $u$.
    ${ }^{2}$ In the original studies $\partial F=\left\{(m, y) \in \Re^{2}: m^{2}+y^{2}=R^{2}\right\}$.

[^2]:    ${ }^{3}$ This intuition goes back at least to Adam Smith's Theory of Moral Sentiments, 1759, p. 181.

[^3]:    ${ }^{4}$ The five observations for each treatment in which $s=0$ are not used in the estimation for two reasons: (a) since the SM opportunity set is a singleton, there is nothing for a theory about SM

[^4]:    choices to explain; and (b) since the left $(r=0)$ and right $(r=3 s)$ censors in the Tobit estimation are equal, the estimation algorithm would not be well defined.
    ${ }^{5}$ The results are substantially unchanged when the data points for $q_{L}>12$ are included.

