

Flacher, David; Jennequin, Hugues

Conference Paper

Access regulation and geographic deployment of a new generation infrastructure

19th Biennial Conference of the International Telecommunications Society (ITS): "Moving Forward with Future Technologies: Opening a Platform for All", Bangkok, Thailand, 18th-21th November 2012

Provided in Cooperation with:

International Telecommunications Society (ITS)

Suggested Citation: Flacher, David; Jennequin, Hugues (2012) : Access regulation and geographic deployment of a new generation infrastructure, 19th Biennial Conference of the International Telecommunications Society (ITS): "Moving Forward with Future Technologies: Opening a Platform for All", Bangkok, Thailand, 18th-21th November 2012, International Telecommunications Society (ITS), Calgary

This Version is available at:

<https://hdl.handle.net/10419/72537>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

**Proceedings of the 19th ITS Biennial Conference 2012
Bangkok, Thailand**

**Access regulation and geographic deployment of a new
generation infrastructure**

By

David Flacher and Hugues Jennequin

Access regulation and geographic deployment of a new generation infrastructure

David Flacher* and Hugues Jennequin[†]

Abstract

This article addresses the impact of regulatory policy on levels of infrastructure deployment and derived welfare in the telecommunications sector. The model considers two potentially coexisting and partially competing techniques (the “old” ADSL - Asymmetric Digital Subscriber Line - technique) - and the “new” FTTH - Fibre To The Home - one). Competition is supposed to be high on the ADSL market because of already existing regulation. We assume that two types of operators are competing in order to provide FTTH services: those that build and operate the new infrastructures (OP_{f1}) and those that just buy access to them (OP_{f2}). In our model, the level of investment is decided at stage 1 and the access price is decided at stage 2. At stage 3, OP_{f1} and OP_{f2} compete *à la Cournot*. This common framework allows us to show that the “regulation defining access price in order to maximise infrastructure deployment” is strictly equivalent to the case in which “no regulation applies”. We also derive from the model that these two types of regulation induce higher social welfare, but lower numbers of FTTH consumers than “cost-oriented access regulation”. Finally, we show that the level of infrastructure deployment (as well as social welfare and number of FTTH consumers) will be at its highest if both investment and access price decisions are taken by the regulator. This suggests that the social optimum will be achieved through a call-for-tender process including deployment and access prices requirements.

Keywords: Access regulation; geographic deployment; network industries; telecommunications; investment.

JEL: L43, L51, L96, R58.

1 Introduction

The first phase of liberalisation in the telecommunications market occurred in the late 1990s in many countries. It was aimed at introducing competition in

*Université Paris 13, CEPN - CNRS - 99 avenue Jean-Baptiste Clément - 93430 Villetaneuse - FRANCE - david@flacher.fr - Tel. : +33 1 45 81 69 92 (*corresponding author*)

[†]Université de Rouen, CARE - 3, avenue Pasteur - 76186 Rouen Cedex 1 - FRANCE - hugues@jennequin.eu

previously monopoly markets. To achieve this, and since national incumbents generally have retained ownership of infrastructure built before liberalisation, National Regulatory Agencies (NRAs) have usually been created. They apply asymmetric and sector specific regulation: national incumbents have specific obligations in order to prevent them from deterring access. One of the most important interventions is access regulation which allows new entrants to access the incumbent's network at a regulated (and generally cost-oriented) price. This type of regulation is due to the high levels of fixed and sunk costs of rolling out a network infrastructure, and also due to the uncertainties for potential entrants as to market shares and accumulation of a customer base.

Around ten years later, the situation had changed. First, some competitors have developed major market shares and, in all countries, competition has increased. Second, even though full duplication of the original network is not complete, all operators - incumbents and competitors - face a much more symmetric situation for rolling out the New Generation Access (NGA), such as Fibre-To-The-Home (FTTH) networks¹. These new conditions were integrated in the recent "review" of the European telecommunications framework, adopted on 25 November 2009².

However, since the fixed sunk costs for building these new networks are high, this has raised concerns among regulators about the geographic extension of NGA deployment in a free market. Since deployment is much more costly in low-density areas, one of the questions being addressed by the regulators is whether to subsidise the network in less profitable areas (e.g. organising beauty contests between operators or leaving public institutions to organise deployment)³. In this article, we study and compare the efficiency of a range of

¹ FTTH is the most complete network since it brings the fibre network right to the consumer.

² The framework includes Directives 2009/136/CE and 2009/140/CE and Regulation (EC) No 1211/2009, OJEU, 18.12.2009. Although, according to this framework, asymmetric remedies can be reinforced to enable access to the historical infrastructure, the new situation which is aimed at fostering NGA network, is taken into account. This is the case of mandatory infrastructure sharing and taking account of investment risks in the regulation: "In order to ensure investment in new technologies in underdeveloped regions, electronic communications regulation should be consistent with other policies, such as State aid policy, cohesion policy or the aims of wider industrial policy" (Directive 2009/140/EC of the European Parliament, (9), p. L337/38). "National regulatory authorities should, when imposing obligations for access to new and enhanced infrastructures, ensure that access conditions reflect the circumstances underlying the investment decision, taking into account, inter alia, the roll-out costs, the expected rate of take up of the new products and services and the expected retail price levels. [...] When imposing remedies to control prices, national regulatory authorities should seek to allow a fair return for the investor on a particular new investment project. In particular, there may be risks associated with investment projects specific to new access networks [...]" (Directive 2009/140/EC of the European Parliament, (55) and (57), p. L337/43).

³ See, e.g., the decision of the European Commission of 30.9.2009, N331/2008 concerning deployment of NGA in the French department of "Hauts-de-Seine": "Under the EC Treaty rules on state aid, the European Commission has approved public co-financing of the roll-out of a passive, neutral and open broadband network covering the entire French department of Hauts-de-Seine, including the non-profitable areas. The Commission concluded that the public funding amounting to €59 millions would be used to offset the cost of complying with the obligations of a service of general economic interest imposed following an open and transparent tendering procedure, and did not therefore constitute state aid. [...]" (EC press

regulatory policies in the context of geographic deployment of NGA networks. We consider a dynamic framework with two types of operators. In one, the main operators are already active in the market using the “old technology” (say the ADSL network)⁴. These main operators are facing fairly similar opportunities in building a new network (say the FTTH network). The other type of operator is the new entrants, attracted by the new market opportunities but unwilling to be integrated upstream. These “virtual network operators” (VNO) rely on the access offered by the main operators, with or without the constraints of regulation. We define a game in which the operators decide on the geographical coverage before competing *à la Cournot* and the regulator can decide to introduce different types of intervention. In the model competition is supposed to be established in the ADSL market⁵ and, in areas where FTTH is available, the consumer can choose between subscribing to the ADSL or FTTH network, which are imperfect substitutes.

Relying on this framework, we first compare the case in which the access price is “unregulated” (and thus is defined on a commercial basis), with the case of “cost-oriented regulation”. We show that the former is more efficient (in terms of welfare and geographical deployment). However, welfare in the telecommunications market is not necessarily the State’s only objective: a larger deployment of FTTH, for instance, might produce externalities for the whole economy, justifying regulation focused primarily on geographic coverage. Consequently, we introduce access price regulation that maximises FTTH coverage. We find, somewhat unexpectedly, that this is equivalent to the “unregulated” case. Finally, we compare the previous results to optimal sector specific regulation, i.e. where not only access price, but also the level of geographic coverage are set, in order to maximise welfare. This approach is consistent with a beauty contest that includes requirements such as minimum level of network deployment and possible subsidies⁶. We show that this regulation leads to the best results in terms not only of level of welfare but geographic coverage (and is better with subsidies than without).

The article is organised as follows: In Section 2, we review the literature. Section 3 presents the model. In Section 4 we analyse and compare the impacts of various regulation policies on investment behaviour and welfare. We offer some concluding remarks in Section 5.

release, IP/09/1391, Brussels, 30.9.2009).

⁴ ADSL is a technique that was used to upgrade the original copper wire network, to allow broadband Internet. Compared to FTTH, ADSL is considered an old generation wireline network.

⁵ Even if the network is not fully duplicated, this assumption is acceptable if we consider that the “historical” network has to remain regulated on a cost-oriented basis and possibly separated from the incumbent. This allows us to study the effects of regulation of FTTH deployment without distortions caused by profits derived from the ADSL network. The inclusion of the ADSL network in the model is supposed only to have an impact on consumer choice.

⁶ E.g., France, Norway and Sweden have a binding coverage requirement for firms that have been granted UMTS - Universal Mobile Telecommunications System (third generation mobile network) licences. The case of deployment of a subsidised FTTH infrastructure in Hauts-de-Seine (France) has been cited in Footnote 3.

2 Related literature

The impact of sector specific regulation on operators' investment, and especially on geographical deployment of new networks, has been rather overlooked in the literature. To our knowledge, Valletti et al. [2002], Hoernig [2006] and Foros and Kind [2003] are the main contributions analysing the geographical inequalities of the infrastructure deployment. Both these works deal with regulation of retail prices and coverage constraints. The first shows that the effects of regulation policy depend very much on the "basket" of measures taken by the NRA. They stress that by linking otherwise unrelated geographic areas, "uniform pricing" may lead to lower coverage by the incumbent and the new entrants while a "minimum coverage constraint" induces higher retail prices. Also considering a "universal service" approach, Foros and Kind [2003] conclude that competition reduces welfare under uniform retail pricing. These authors show that this effect can be offset by fixing "a coverage requirement prior to competition" that solves the co-ordination problems⁷. However, these contributions do not deal with the impact of wholesale price regulation (access regulation), which is one of the most debated issue among NRAs and operators around the world.

Most of the literature dealing with access regulation is relatively new and addresses the question of static and dynamic efficiency of regulation, but does not take account of the issues related to geographical coverage. It highlights that while a low access price may favour competition (and thus lower retail prices) within a static point of view, it may also reduce the incentives to invest in upgrading or deploying new networks. Therefore, it will reduce any dynamic efficiency gains. This literature could be interpreted indirectly in terms of geographic coverage although optimal regulation is not seen as a way to manage geographic deployment of the networks in these models⁸. The corresponding literature adopts two main approaches⁹.

The first deals with *service-based competition* (SBC), where competition depends on the access allowed to competitors and new entrants to the incumbent's network. The question then becomes whether or not the incumbent will have an incentive to invest in improving its infrastructure in the face of the (asymmetric) regulation. Foros [2004] and Kotakorpi [2006] show that, even if new entries can encourage incumbents to invest in upgrades to their infrastructure¹⁰, regulating access charge below the level resulting from free commercial bargaining would have a negative impact on investment incentives. Other research,

⁷ This requirement corresponds to a wider geographic coverage than that based on a free market.

⁸ Indeed, thresholds beyond which operators would decide to invest are related to deployment costs. Also, in these models, cost parameters can be interpreted in relation to the various densities of the geographic areas. However, it might be difficult to derive conclusions in terms of welfare exploiting the literature in this way.

⁹ For a more detailed literature review, see Guthrie [2006], Gans [2007] and Cambini and Jiang [2009]. In particular, Guthrie [2006] points to the need to consider more deeply the impact of access regulation on investment and on welfare.

¹⁰ This is the case when access charges are not too low and when the new entrants' quality of service (or the consumer's willingness to pay for their service) is higher than that of the incumbent.

particularly work based on “real options” theory, points out that the situation for incumbents is more risky than that of new entrants: Jorde et al. [2000] and Pindyck [2007], for instance, show that a regulated access charge is a disincentive to the incumbent to invest. All these contributions support the existence of a trade-off between promotion of competition, and incumbent’s deployment (or upgrade to) the network. However, Klumpp and Su [2010] consider the alternative that dynamic efficiency can be consistent with static allocative efficiency if the access price is set according to the principle of “revenue neutrality”. Indeed, a *revenue-neutral open access* may motivate the incumbent to invest more in quality improvements as competition increases. In Klumpp and Su’s model, competition and investment (in the incumbent’s network quality) are complementary. The access price prescribed is a linear access tariff, chosen after the investment is made and before the downstream outputs are chosen, in order that firms pay for the upstream investment in proportion to their downstream market shares. This access pricing internalizes extension of the industry size due to the incumbent’s investment and to the competition. Consequently, competition in the retail market raises the incumbent’s return on its investment, thus leading to stronger incentives to invest.

The other approach deals with *facility-based competition* (FBC), focusing mainly on the investment incentives of new entrants. In this field, Bourreau and Dogan [2005] show that it can be profitable for incumbents to set a low access charge in order to reduce the entrants’ incentives to invest. Consequently, if FBC is considered socially desirable, the access charge must be set at a high enough level by the regulator. However, policies fostering FBC are usually decided after a first phase of SBC policies. Indeed many NRA, especially in the European Union (EU), have implemented the “ladder of investment” theory (Cave and Vogelsang, 2003, De Bijl and Peitz, 2004, Cave, 2006). According to this theory, NRA should first set the incumbent’s access charge low, in order to stimulate entry and help new entrants to achieve a critical mass of users. In a second step, the NRA increases the access charge in order to persuade the new operators to “climb” the ladder of investment by deploying their own infrastructures. Avenali et al. [2010] made a contribution to this literature by focusing on the first step of the investment ladder: They assume that rather than being unavoidable (as in Bourreau and Dogan, 2005), deployment of the new entrant’s own infrastructure depends on its capacity to build a customer base, which will increase its reputation and brand loyalty, and on the access policy. Like Bourreau and Dogan, 2005, Avenali et al. conclude that access charges should increase over time in order to incite competitors to climb the investment ladder. They conclude also that, in order to induce investment from both early and late entries, NRA should implement a system of differentiated access prices for all sequential entries depending on the time and entry period. On this same issue, the investment ladder, Bourreau et al. [2010] highlights the imperative for the regulator to increase access prices in order to remove entrants’ replacement effects and encourage the rolling out of the infrastructure.

In most of the articles cited above, investment is related to the upgrade of the existing network or the duplication of a network by new entrants, leading

mainly to horizontal differentiation. However, FBC should be considered in a different way in the context of the deployment of networks based on new and vertically differentiated technologies. As in the case of FTTH networks, the expected quality is supposed to be much higher than the quality of the previous wireline network (ADSL). Also, if the market and costs make the roll out of a new network profitable, we can consider, for the first time since the beginning of the liberalisation process¹¹, that the incumbent and its competitors are facing almost symmetric conditions. This situation revises the regulatory approach characterised by asymmetric regulation between the incumbent (owner of the former public infrastructure) and competitors (relying on the regulator's decision to access the network or on pure duplication of the existing historical network). With FTTH, all operators can invest; the NRA needs to decide the access rules before investment takes place.

Within this framework, Gans [2001], Hori and Mizuno [2006, 2009] and Vareda and Hoernig [2007] propose models inspired by the literature on innovation races (Fudenberg and Tirole [1985] and Katz and Shapiro [1987]). The main question they address is the impact of access regulation on the timing of investment. Due to technical process, investment cost is supposed to be lower if investment is made later. However, delayed investment reduces the total potential profits for the operator for the new infrastructure. Moreover, firms risk being preceded by a competitor. Therefore, depending on the situation, each firm must decide whether to wait (choosing the investment date that will maximise net profit value) or preempting the market. In this context, the literature shows that the access regulation can incite firms to deploy their networks at socially optimal dates. Brito et al. [2010] take account of the coexistence of an old and a new infrastructure, and the (in)ability of regulators to commit to a policy (i.e. to decide on access rules before operators invest). They show that, depending on the circumstances, two-part access tariffs or a regulation moratorium can solve the dynamic consistency problem characterising the trade-off between static and dynamic efficiency in the context of rolling out a NGA network. However, they consider that the old network is phased out when the new network is deployed, which does not allow the effects of competition between likely coexisting generations of networks to be studied. Also, in their model, the investment is a lump sum: all the territory is covered if the investment is profitable, otherwise no investment is made.

As far as we know and with the exception of Lestage and Flacher [2009]'s work¹², this article is the first attempt to study and model the impact of access

¹¹ Perhaps with the exception of the mobile networks which were poorly developed before liberalisation, even among incumbents.

¹² Lestage and Flacher [2009] emphasises that fostering competition (or avoiding monopolisation) and promoting investment and geographical coverage are conflicting objectives. In terms of geographic coverage, regulatory tools dealing with infrastructure sharing are shown to be preferable to standard access price regulation. The present article is based on a completely different model and addresses complementary questions. For instance: access regulation is considered beyond cost-oriented pricing, different types of technologies and agents are introduced, and the question of an optimal regulation policy is addressed by comparing the welfare from different cases.

regulation on the geographical deployment of NGA networks, by endogenizing the corresponding level of investment. It may also be the first attempt to model regulation and investment issues with coexisting generations of networks¹³ and the first article dealing with both coverage constraints and access regulation¹⁴. It is also one of the first models to include different types of actors (network operators, virtual operators) and be placed in a new, symmetrical context, in which the incumbent and its competitors may both need to invest into NGA network.

3 The model

3.1 Assumptions

We assume that in the retail market, two generations of infrastructures can coexist: the “old” one (ADSL), already deployed, which provides a “standard” level of Internet access to households; and the “new” one (FTTH), which provides high level broadband access. Contrary to Foros and Kind [2003]’s assumption, we introduce the more realistic assumption that ADSL and FTTH services are imperfect substitutes¹⁵. In our model, the main concern is not competition between operators of the same type, but competition between techniques and between operators of different types¹⁶. For this reason we model representative operators for each type of technique and behaviour.

3.1.1 Consumers and operators

Consumers As in Foros [2004] and in Kotakorpi [2006], the consumer’s basic willingness to pay (WTP) for the Internet service is denoted by s and assumed to be uniformly distributed on $[0, 1]$. We assume also that the distribution is the same for all geographical areas¹⁷. If, for a consumer of type s , the utility and price associated with operators i and j are $(U_i(s), p_i)$ and $(U_j(s), p_j)$ respectively, this consumer will choose operator $k \in \{i, j\}$ if and only if $U_k - p_k > U_l - p_l$, with $l \in \{i, j\}$ and $l \neq k$. He is thus indifferent to either operator’s services when $U_k - p_k = U_l - p_l$.

ADSL operators Formally, we assume that regulation has reached its aim of fostering competition in the ADSL market and that the market is mature¹⁸. In

¹³ Foros and Kind [2003], for instance, in their model, consider the broadband and narrowband markets to be separate.

¹⁴ The articles mentioned above deal with retail price regulation and coverage constraints.

¹⁵ In Foros and Kind [2003]’s model, narrowband and broadband are not considered substitutes (which is less realistic than in our approach).

¹⁶ Foros and Kind [2003], Valletti et al. [2002] and Faulhaber and Hogendorn [2000] do not introduce coexisting techniques in their models. While Foros and Kind [2003] deals with the number of operators active in the market, Valletti et al. [2002]’s model relies on two differentiated operators, not characterised by different behaviours or processing conditions.

¹⁷ This assumption is similar to the assumption in Foros and Kind [2003].

¹⁸ This implies that profits are equal to zero for all firms in this market. Any price increase is supposed to induce the entry of new competitors, which cancels out potential extra profit.

this situation, innovating in the ADSL framework is not longer profitable¹⁹ and profits are equal to zero.

We assume that N vertically differentiated ADSL operators (OP_{ci}) are competing in the retail market. This differentiation increases consumers' valuation for the OP_{ci} 's service with β_i . We normalise to zero differentiation by the most "basic" operator ($\min\beta_k = 0$). The willingness to pay for the OP_{ci} 's ADSL service, of a type $s \in [0, 1]$ consumer is then $s_i = s + \beta_i - p_{ci}$. If the N ADSL operators are active in the market, quality-adjusted prices need to be the same: for all i, j , $s + \beta_i - p_{ci} = s + \beta_j - p_{cj}$. Let $P_c = \min(p_{ck})$. This implies that for all i , $p_{ci} = P_c + \beta_i$. The ADSL market being competitive, $P_c = \alpha_c \in [0, 1]$ and $\alpha_{ci} = \alpha_c + \beta_i$, $i \in \{1 \dots N\}$, where the α_{ci} are the (constant) marginal costs of each OP_{ci} ²⁰. Consequently, we will have ADSL subscribers only if $P_c \in [0, 1]$.

FTTH operators Rolling out an FTTH infrastructure needs specific knowledge and skills, a large enough initial market share (in order to reduce the risks), and financial capacity (in order to bear the very large investment). Only a very few operators meet these requirements. They are already active in the ADSL market and have duplicated part of the incumbent infrastructure. They are now faced with almost similar opportunities in the building of a new network. The other operators will be obliged to buy access from them. We capture this difference by modelling two representative operators. The first (OP_{f1}) represents operators that are able to invest in network deployment²¹. The second (OP_{f2}), represents the VNOs²².

OP_{f1} introduces a "qualitative break" by deploying the FTTH infrastructure,: For a consumer of type "s", the utility becomes $\delta s + \gamma_{fi}$, where $\delta > 1$ and $\gamma_{fi} \geq 0$ is a differentiation parameter associated with OP_{fi} , $i \in \{1, 2\}$. FTTH can thus be seen as a "radical" innovation (captured by δ). Since liberalisation and sector specific regulation are supposed to allow only efficient entries, the competitor (OP_{f2}) must provide services at least as efficient as the existing ones supplied by OP_{f1} if it wants to have any chance of competing in the retail market (i.e. $\gamma_{f2} \geq \gamma_{f1}$). To simplify, let $\gamma_{f1} = 0$ and $\gamma_{f2} = \gamma \geq 0$.

If both the FTTH operators are active, for all $s \in [0, 1]$, $\delta s - p_{f2} = \delta s + \gamma - p_{f1}$ where p_{fi} is the price of OP_{fi} 's service. Let $P_f = p_{f1}$. The condition becomes $p_{f2} = P_f + \gamma$.

In considering that the ADSL market is mature, we assume that this is the case for all differentiated goods provided in this market.

¹⁹ This assumption also considers profits are negligible compared to the potential profit from FTTH services. Note also that zero profit can be the result of strong regulation in a less competitive environment. In all cases, we note that the "old" infrastructure property is not relevant in this context.

²⁰ $\alpha_c = \alpha_{ck}$ with $k = \operatorname{argmin}\beta_i$.

²¹ Choosing one representative operator the main operators has the advantage that it avoids the issue of duplication of the network. Indeed, duplication would appear inefficient in this model since the capacity of an FTTH network is such that it can accommodate all operators willing to be active in the market. This problem has been avoided by other scholars. For instance, Foros and Kind [2003]'s model simply abstracts from fixed costs.

²² The distinction between "real" and "virtual" operators in a context dealing with NGA deployment is similar to that in Brito et al. [2010].

We assume that OP_{f1} will never provide below cost access to its infrastructure: $P_f \geq \alpha_f$.

Coexistence of ADSL and FTTH and coexistence of OP_{f1} and OP_{f2} A consumer chooses among ADSL and FTTH services and among the different operators, by comparing together the $s + \beta_i - p_{ci}$, $i \in \{1 \dots N\}$ and the $\delta s + \gamma_j - p_{fj}$, $j \in \{1, 2\}$.

The consumer (s_0) that is indifferent between ADSL and FTTH techniques is characterised by $s_0 - P_c = \delta s_0 - P_f$. Thus, we can write that $s_0 = (P_f - P_c)/(\delta - 1)$. If $s < s_0$, consumer will (strictly) prefer ADSL. If $s > s_0$, he will (strictly) prefer FTTH.

Lemma 1. *On areas covered by FTTH network, i) $P_f > \delta\alpha_c$ if and only if there are ADSL consumers; ii) $P_f - \alpha_c < \delta - 1$ if and only if there are FTTH consumers.*

Proof. Assume ADSL and FTTH coexist. It is then easy to show that $P_f > \delta\alpha_c \iff s_0 > P_f/\delta \iff s_0 > \alpha_c$. We derive that $P_f > \delta\alpha_c$ if and only if ADSL consumers exist. Similarly, $P_f - \alpha_c < \delta - 1$ if and only if FTTH consumers exist²³. \square

3.1.2 Investment

In this article, and unlike the work of Avenali et al. 2010 and many other of the authors mentioned in Section 2, the investment variable is not constant, but is endogenized and used to model geographic coverage. We assume that μD^2 is the fixed cost to serve a percentage D of the population. (μ is the cost for full coverage of the population). This means that covering densely populated areas is much cheaper than covering rural areas. To be realistic, we suppose that μ is sufficiently high that, in any case, $D < 1$.

3.1.3 Additional assumptions

In order to simplify the model, each retail price for a given service is supposed to be the same for all geographical areas²⁴. In addition, we suppose in the remainder of this article that the marginal cost α_f is negligible compared to the cost of infrastructure deployment (μD^2), and we normalise it to zero (as in

²³ Note that this lemma is valid only for a contestable market. If not, it is possible to have only FTTH consumers and $P_f > \delta\alpha_c$. This would be the case if FTTH operators remain profitable even after lowering their prices so that $P_f \leq \delta\alpha_c$, meaning that FTTH operators could discourage ADSL entries. Nevertheless, we will consider that the lemma is valid even in non-perfectly contestable markets. Indeed, on the one hand, prices are not very flexible (because of the contracts between operators and customers), reducing the threat to potential ADSL entrants. On the other hand, the control exerted by the competition authorities should prevent such behaviours.

²⁴ As stressed by Foros and Kind [2003], prices may be set the same in all geographical areas even without any legal requirement: "The reason is that [...] arbitrage opportunities strictly reduce the providers' ability to set prices that vary with where people live" (p. 218). This applies to mobile and ADSL services in almost all countries.

Avenali et al. 2010). We assume also that the marginal cost for ADSL operators is equal to zero ($\alpha_c = 0$). Finally, we assume that the differentiation between OP_{f1} and OP_{f2} is not overly important ($\gamma < \frac{\delta-1}{6}$). We can then consider that OP_{f2} is an efficient entry, but that it is not too disruptive of the market, as stressed by proposition 2.

3.2 Profit, demand and welfare functions

3.2.1 Profit functions

For ADSL operators, profits are equal to zero²⁵. Recall that we have assumed that the normal profits of ADSL operators are null. The profit functions for the FTTH operators are given by:

$$\begin{cases} \Pi_{f1} = p_{f1} q_{f1} + w_{f1} q_{f2} - \alpha_f (q_{f1} + q_{f2}) - \mu D^2 \\ \Pi_{f2} = (p_{f2} - w_{f1}) q_{f2} \end{cases}$$

The facility-based firm (OP_{f1}) is active in both the wholesale and the retail segments, while the service-based firm (OP_{f2}) earns profits only from the retail segment. q_{f1} and q_{f2} are the quantities offered by OP_{f1} and OP_{f2} respectively, w_{f1} is the wholesale price charged by OP_{f1} to OP_{f2} . α_f is the marginal cost per user and is assumed to be constant. We assume also that $w_{f1} \geq \alpha_f$, i.e. that the marginal cost of providing FTTH access to OP_{f2} is covered by the wholesale price.

3.2.2 Demand and welfare functions

As the proposition 2 show, the only case that is relevant is where ADSL and FTTH are present in the areas covered by the FTTH network. In our model, the distribution of consumers' WTP is the same in the areas covered by the FTTH infrastructure, and in the areas where only ADSL services are available. The total numbers of ADSL and FTTH consumers respectively are given by:

$$\begin{cases} Q_c = (s_0 - P_c) + (1 - D) (1 - s_0) = (1 - \alpha_c) - D \left(1 - \frac{P_f - P_c}{\delta - 1}\right) \\ Q_f = D * (1 - s_0) = D \left(1 - \frac{P_f - P_c}{\delta - 1}\right) \end{cases}$$

It is easy to derive the inverse demand functions for FTTH operators:

$$\begin{cases} p_{f1} = P_f = (\delta - 1) + \alpha_c - (\delta - 1) \frac{Q_f}{D} \\ p_{f2} = P_f + \gamma \end{cases}$$

²⁵ Note that, in this model, regulation is not "technologically neutral" since ADSL can remain implicitly under cost-based asymmetrical regulation (in order to maintain competition in the ADSL market) while a range of policies can apply to FTTH suppliers. Even were neutrality our goal, our implicit assumption would seem more consistent with the reality, especially within the new European framework.

The welfare function (W) is the sum of ADSL and FTTH consumers' surplus (CS_c and CS_f) and the operators' profits. We can write:

$$\begin{aligned} CS_c &= \frac{D}{2} (s_0 - P_c)^2 + \frac{1-D}{2} (1 - P_c)^2 \\ CS_f &= D \left[\frac{\delta}{2} (1 - s_0^2) - P_f (1 - s_0) \right] \\ W &= CS_c + CS_f + \Pi_{f1} + \Pi_{f2} \end{aligned}$$

Proof. Denoting pdm_{xi} as the market shares of operator i of type $x \in \{c, f\}$ (c for ADSL and f for FTTH), CS_x and W are deduced easily from:

$$\begin{aligned} CS_c &= \sum_{i=1}^N \int_{P_c}^{s_0} pdm_{ci} [(s + \beta_i) - (P_c + \beta_i)] ds \\ &\quad + (1-D) \sum_{i=1}^N \int_{s_0}^1 pdm_{ci} [(s + \beta_i) - (P_c + \beta_i)] ds \\ CS_f &= D \sum_{j=1}^2 \int_{s_0}^1 pdm_{fj} [(\delta s + \gamma_i) - (P_f + \gamma_i)] ds \end{aligned}$$

3.3 The stages of the game

We model a three-stage game with the following timing structure²⁶:

- Stage 1: OP_{f1} chooses the investment level (D).
- Stage 2: The regulator or OP_{f1} determines the access price (w_{f1}). Whether the regulator intervenes, depends on the regulation policies implemented.
- Stage 3: OP_{f1} and OP_{f2} compete à la Cournot²⁷.

We consider here that both ADSL and FTTH are active in the areas covered by the FTTH network. From lemma 1, we derive that $0 < P_f < \delta - 1$. We assume that firms will not sell their services below marginal cost in either the retail or wholesale markets. Since the only interesting case is the one where investment in FTTH deployment is strictly positive, we can summarise the previous conditions and constraints:

$$\forall i \in \{1, 2\} \quad q_{fi} \geq 0 \quad \begin{cases} p_{f1} \geq 0 \\ w_{f1} \geq 0 \\ p_{f2} \geq w_{f1} \\ D > 0 \end{cases} \quad 0 < P_f < \delta - 1$$

²⁶ As Brito et al. [2010] emphasise, this type of game considers cases in which NRA cannot commit before the investment decision. In Sections 4.3 and 4.4, we consider cases when NRA can commit. In Section 4.3, we assess the impact of deciding the level of the access charge before the investment decision. In Section 4.4, the game is reduced to a two-stage game, and the level of investment decided by the regulator.

²⁷ In most of the literature a Cournot competition framework is used.

It then is possible to derive a proposition from stage 3 of the game.

Proposition 2. *If $\gamma < \frac{\delta-1}{6}$, (i) both the ADSL and FTTH services remain active in the areas covered by the FTTH network; (ii) the facility-based operator (OP_{f1}) always remains active in the retail market (and thus never becomes a purely wholesale access operator) and: (iii)*

$$\begin{cases} \text{if } w_{f1} < \gamma + \frac{1}{2}(\delta - 1) & \begin{cases} q_{f1} = \frac{D}{3(\delta-1)}(\delta - 1 - \gamma + w_{f1}) & > 0 \\ q_{f2} = \frac{D}{3(\delta-1)}(\delta - 1 + 2\gamma - 2w_{f1}) & > 0 \end{cases} \\ \text{if } w_{f1} \geq \gamma + \frac{1}{2}(\delta - 1) & \begin{cases} q_{f1} = q_{f1}^m = \frac{D}{2} \\ q_{f2} = 0 \end{cases} \end{cases}$$

Proof. See Appendix A. □

4 Regulation policies and investment

4.1 Unregulated access price

One of the regulator's options is to leave the operator OP_{f1} to decide the wholesale access price on a commercial basis.

4.1.1 Access price determination (stage 2)

When $w_{f1} \in [0, \gamma + \frac{1}{2}(\delta - 1)[$, both FTTH operators are active ($q_{f1} > 0$ et $q_{f2} > 0$) and we have:

$$\frac{\partial \Pi_{f1}}{\partial w_{f1}} = 0 \iff w_{f1}^* = \frac{2\gamma}{5} + \frac{1}{2}(\delta - 1) \in [0, \gamma + \frac{1}{2}(\delta - 1)[$$

We check that $\frac{\partial^2 \Pi_{f1}}{\partial w_{f1}^2} = -\frac{10}{9} \frac{D}{\delta-1} < 0$ at the critical point. We check also that $\Pi_{f1}(w_{f1} = w_{f1}^*) > \Pi_{f1}(w_{f1} = 0)$. Moreover, when $w_{f1} \geq \gamma + \frac{1}{2}(\delta - 1)$, OP_{f1} is alone on the market and we can show that $\forall w_{f1} \geq \gamma + \frac{1}{2}(\delta - 1)$ $\Pi_{f1}(w_{f1} = w_{f1}^*) > \Pi_{f1}(w_{f1})$. Consequently, w_{f1}^* is the value of w_{f1} chosen by OP_{f1} .

4.1.2 Investment decision (stage 1)

We deduce the level of investment maximising OP_{f1} 's profit:

$$\frac{\partial \Pi_1}{\partial D} = 0 \iff D^* = \frac{1}{8\mu(\delta-1)} \left[(\delta - 1)^2 + \frac{4\gamma^2}{5} \right] > 0$$

The second-order condition is fulfilled since $\frac{\partial^2 \Pi_1}{\partial D^2} = -2\mu < 0$ at the critical point. Finally, we have:

$$\begin{cases} q_{f1}^* = \frac{1}{16\mu(\delta-1)^2} \left[(\delta-1)^2 + \frac{4\gamma^2}{5} \right] \left[\delta - 1 - \frac{2\gamma}{5} \right] \\ q_{f2}^* = \frac{\gamma}{20\mu(\delta-1)^2} \left[(\delta-1)^2 + \frac{4\gamma^2}{5} \right] \\ Q_f^* = \frac{1}{16\mu(\delta-1)^2} \left[(\delta-1)^2 + \frac{4\gamma^2}{5} \right] \left[\delta - 1 + \frac{2\gamma}{5} \right] \\ w_{f1}^* = \frac{2\gamma}{5} + \frac{\delta-1}{2} \\ D^* = \frac{1}{8\mu(\delta-1)} \left[(\delta-1)^2 + \frac{4\gamma^2}{5} \right] \\ W^* = \frac{112\gamma^4 + (\delta-1)[125(\delta-1)^3 + 50\gamma(\delta-1)^2 + (240\gamma^2 + 2000\mu)(\delta-1) + 40\gamma^3]}{4000\mu(\delta-1)^2} \end{cases}$$

In the “unregulated case”, both FTTH geographic coverage and welfare increase when OP_{f2} is more efficient in its vertical differentiation (i.e. when γ increases). This is because OP_{f1} benefits from the higher quality of OP_{f2} through the access charge. Coverage and welfare also increase when FTTH is seen by consumers as being a more radical innovation (i.e. δ is higher) and when the cost of building the network (μ) decreases.

4.2 Cost-oriented regulation (“regulation 1”)

Cost-oriented regulation is the most common remedy applied by NRA and has provoked much debate on the nature of the costs that should be considered. For simplicity, as in Avenali et al. [2010], Fors [2004], Kotakorpi [2006], in this section we only consider marginal costs.

4.2.1 Access price determination (stage 2)

The NRA regulates w_{f1} in order to maximise welfare (W). In order to specify that we are dealing with “regulation 1”, the notations are indexed by “1”. We use a similar convention to distinguish the other types of regulation. When $w_{f1} \in [0, \gamma + \frac{1}{2}(\delta-1)[$, both OP_{f1} and OP_{f2} are active and:

$$\frac{\partial W}{\partial w_{f1}} = 0 \iff w_{f1,1}^* = -(5\gamma + \delta - 1) < 0$$

And $\frac{\partial^2 W}{\partial w_{f1}^2} = -\frac{D}{9(\delta-1)} < 0$ at the critical point. Therefore, the maximum on $[0, \gamma + \frac{1}{2}(\delta-1)[$ is obtained for $w_{f1,1}^* = 0$.

We check also that $\forall w_{f1} \geq \gamma + \frac{1}{2}(\delta-1) \quad W(w_{f1} = w_{f1,1}^*) > W(w_{f1})$. Consequently, $w_{f1}^* = 0$ is the value of w_{f1} chosen by the regulator.

4.2.2 Investment decision (stage 1)

At stage 1, OP_{f1} maximises its profit in order to determine the level of investment:

$$\frac{\partial \Pi_{f1}}{\partial D} = 0 \iff D_1^* = \frac{1}{18\mu(\delta-1)} \left[(\delta-1-\gamma)^2 \right] > 0$$

The second-order is fulfilled since $\frac{\partial^2 \Pi_{f1}}{\partial D^2} = -2\mu < 0$ at the critical point. Finally, we have:

$$\begin{cases} q_{f1,1}^* = \frac{1}{54\mu(\delta-1)^2} (\delta-1-\gamma)^3 \\ q_{f2,1}^* = \frac{1}{54\mu(\delta-1)^2} (\delta-1-\gamma)^2 (\delta-1+2\gamma) \\ Q_{f,1}^* = \frac{1}{54\mu(\delta-1)^2} (\delta-1-\gamma)^2 [2(\delta-1)+\gamma] \\ w_{f1,1}^* = 0 \\ D_1^* = \frac{1}{18\mu(\delta-1)} [\delta-1-\gamma]^2 \\ W_1^* = \frac{10\Gamma^4 + (\delta-1)[7(\delta-1)^3 - 4\Gamma(\delta-1)^2 + (162\mu - 3\gamma^2)(\delta-1) - 10\Gamma^3]}{324\mu(\delta-1)^2} \end{cases}$$

In the “cost-oriented” regulation, the FTTH geographic coverage and welfare decrease when OP_{f2} is more efficient in its vertical differentiation (i.e. when γ increases). This is explained by the fact that the market share of OP_{f2} increases, reducing OP_{f1} 's profit without any compensation through the access charge. Conversely, geographic coverage and welfare increase when FTTH is seen by consumers to be a more radical innovation (i.e. δ is higher) and when the cost of building the network (μ) decreases.

Comparing the “unregulated case” and the case of “cost-oriented regulation” (regulation 1) leads to Proposition 3.

Proposition 3. *The level of investment is higher with no regulation than with cost-based regulation ($D^* > D_1^*$). Results are similar for the “welfare” ($W^* > W_1^*$). Conversely, there are more consumers of FTTH services with cost-based regulation than with no regulation ($Q_f^* < Q_{f,1}^*$).*

Proof. See Appendix B. □

This proposition is in line with the intuitions and results in the literature: cost-based regulation, based on reducing the access price, reduces the incentives to invest in the deployment of a new infrastructure. This has a negative impact on dynamic efficiency ($W^* > W_1^*$), but a positive effect on static efficiency (since $Q_f^* < Q_{f,1}^*$).

4.3 Access price regulation maximising geographic coverage (“regulation 2”)

In this section, we assume that the objective of the regulator is to maximise network deployment by regulating the access price. The game then becomes:

- Stage 1: the regulator determines the access price in order to maximise the investment level (D);
- Stage 2: OP_{f1} determines D ;
- Stage 3: OP_{f1} and OP_{f2} compete à la Cournot.

4.3.1 Investment decision (stage 2)

OP_{f_1} determines the level of investment (D) that maximises its profit, given the access price (w_{f_1}).

When $w_{f_1} \in [0, \gamma + \frac{1}{2}(\delta - 1)[$, both OP_{f_1} and OP_{f_2} are active ($q_{f_1} > 0$ and $q_{f_2} > 0$) and:

$$\frac{\partial \Pi_{f_1}}{\partial D} = 0 \iff D_2^* = \frac{1}{18\mu(\delta-1)} \left[(\delta - 1 - \gamma)^2 + 5w_{f_1} (\delta - 1 - w_{f_1} + \frac{4\gamma}{5}) \right] > 0$$

We check that $D_2^* > 0$ ²⁸ and that $\frac{\partial^2 \Pi_{f_1}}{\partial D^2} = -2\mu < 0$ at the critical point.

When $w_{f_1} \geq \gamma + \frac{1}{2}(\delta - 1)$, OP_{f_1} is a monopoly on the FTTH retail market and $D_2^* = \frac{\delta-1}{8\mu} > 0$.

4.3.2 Access price determination (stage 1)

At stage 1, the regulator determines the access price in order to maximise FTTH network deployment. If the regulator chooses $w_{f_1,2}^* \in [0, \gamma + \frac{1}{2}(\delta - 1)[$, i.e. a value of w_{f_1} that allows both OP_{f_1} and OP_{f_2} to be active ($q_{f_1} > 0$ and $q_{f_2} > 0$), we have:

$$\frac{\partial D}{\partial w_{f_1}} = 0 \iff w_{f_1,2}^* = \frac{2\gamma}{5} + \frac{\delta-1}{2} \in]0, \gamma + \frac{1}{2}(\delta - 1)[$$

The second-order condition is fulfilled: $\frac{\partial^2 W}{\partial w_{f_1}^2} = -\frac{5}{9(\delta-1)\mu} < 0$ at the critical point. Since $D(w_{f_1} = w_{f_1,2}^*) = \frac{1}{8\mu(\delta-1)} \left[(\delta - 1)^2 + \frac{4\gamma^2}{5} \right] > D(w_{f_1} = 0)$ and since it is easy to check that $D(w_{f_1} = w_{f_1,2}^*) > D(w_{f_1})$, for all $w_{f_1} > \gamma + \frac{1}{2}(\delta - 1)$, we can deduce that $w_{f_1} = w_{f_1,2}^* \in]0, \gamma + \frac{1}{2}(\delta - 1)[$ is the access price chosen by the regulator. Finally, we have:

$$\begin{cases} q_{f_1,2}^* = \frac{[4\gamma^2 + 5(\delta-1)^2][(5(\delta-1) - 2\gamma)]}{400\mu(\delta-1)^2} \\ q_{f_2,2}^* = \frac{[4\gamma^2 + 5(\delta-1)^2]\gamma}{100\mu(\delta-1)^2} \\ Q_{f,2}^* = \frac{[4\gamma^2 + 5(\delta-1)^2][5(\delta-1) + 2\gamma]}{400\mu(\delta-1)^2} \\ w_{f_1,2}^* = \frac{2\gamma}{5} + \frac{\delta-1}{2} \\ D_2^* = \frac{1}{8\mu(\delta-1)} \left[(\delta - 1)^2 + \frac{4\gamma^2}{5} \right] \\ W_2^* = \frac{112\gamma^4 + (\delta-1)[125(\delta-1)^3 + 50\gamma(\delta-1)^2 + (240\gamma^2 + 2000\mu)(\delta-1) + 40\gamma^3]}{4000\mu(\delta-1)^2} \end{cases}$$

Somewhat unexpectedly, we note that the equilibrium is the same as in the unregulated case, as highlighted in Proposition 4.

²⁸ D_2^* is a quadratic function of the variable w_{f_1} . Its discriminant is strictly positive and its roots are $\frac{2\gamma}{5} + \frac{\delta-1}{2} \pm \frac{3}{10}\sqrt{4\gamma^2 + 5(\delta-1)^2}$. When $\gamma \in [0, \frac{\delta-1}{4}]$, the lowest root is strictly negative and the highest is higher than $\delta - 1$ and higher therefore than $\gamma + \frac{1}{2}(\delta - 1)$. We can easily derive that $D_2^* > 0$.

Proposition 4. *Regulation determining the access price in order to maximise FTTH network deployment, and an unregulated market both produce the same equilibrium: $q_{f1}^* = q_{f1,2}^*$, $q_{f2}^* = q_{f2,2}^*$, $w_{f1}^* = w_{f1,2}^*$, $D^* = D_2^*$ and $W^* = W_2^*$.*

4.4 Access price and geographic coverage regulation maximising welfare (“regulation 3”)

In this section, we consider the optimal sector specific regulation, i.e. that would not only set the access price, but also would decide the level of geographic coverage in order to maximise welfare. This approach is consistent with a beauty contest and includes requirements such as minimum level of network deployment, and possible subsidies²⁹. We also distinguish the cases where public subsidies can be provided to the firm (Section 4.4.1), and where the firm faces budget constraints (Section 4.4.2). Finally, we compare the previous results to the optimal sector specific regulation. The game can be summarised as follows:

- Stage 1: Access price and level of investment are set by the regulator simultaneously in order to maximise welfare.
- Stage 2: OP_{f1} and OP_{f2} compete *à la Cournot*.

4.4.1 The case without budgetary constraints (a subvention is provided to OP_{f1} to fill the deficit) - First-best regulation

In the case in which a subvention can be provided to OP_{f1} in order to cancel out a budgetary constraint, there is no constraint on the sign of Π_{f1} .

When $w_{f1} \in [0, \gamma + \frac{1}{2}(\delta - 1)[$, $\frac{\partial W}{\partial w_{f1}} = 0 \iff w_{f1} < 0$. Thus $w_{f1,3,S}^* = 0$ is the value of w_{f1} that maximises W on $[0, \gamma + \frac{1}{2}(\delta - 1)[$ (whatever the level of investment, D). From $\frac{\partial W}{\partial D} = 0$, we derive that

$$D_{3,S}^* = \frac{1}{36\mu(\delta-1)} \left[11\gamma^2 + 8(\delta-1)\gamma + 8(\delta-1)^2 \right] > 0.$$

Let $W_{3,S}^*$ be the corresponding value of W .

When $w_{f1} \geq \gamma + \frac{1}{2}(\delta - 1)$, OP_{f2} is foreclosed. In this case, only D has to be determined:

$$\frac{\partial W}{\partial D} = 0 \iff D = \frac{3(\delta-1)}{16\mu} > 0$$

²⁹ The possibility of subsidising the network is not a purely theoretical issue: For instance, it is allowed in the telecommunications framework even though “public support should be given by means of open, transparent and competitive procedures.” (Directive 2009/140/EC of the European Parliament, (10), p. L337/38). The *Community guidelines for the application of state aid rules in relation to rapid deployment of broadband networks* (2009/C 235/04) present the conditions under which each member state can subsidise a service of a general economic interest (SGEI) within the meaning of Article 86(2) of the Treaty. These subsidies generally are accompanied by coverage constraints. These rules have been applied in several cases (see, e.g., footnote 3).

We check that $\frac{\partial W}{\partial D} = -2\mu < 0$. We can then check that the corresponding value of W is lower than $W_{3,S}^*$. Consequently, $(w_{f1,3,S}^*, D_{3,S}^*)$ is chosen by the regulator if there is no budgetary constraint. In this case, $\Pi_{f1} < 0$: OP_{f1} has to be subsidised. Finally, we have:

$$\begin{cases} q_{f1,3,S}^* = \frac{[11\gamma^2 + 8(\delta-1)(\delta-1+\gamma)][\delta-1-\gamma]}{108\mu(\delta-1)^2} \\ q_{f2,3,S}^* = \frac{[11\gamma^2 + 8(\delta-1)(\delta-1+\gamma)][\delta-1+2\gamma]}{108\mu(\delta-1)^2} \\ Q_{f,3,S}^* = \frac{[11\gamma^2 + 8(\delta-1)(\delta-1+\gamma)][2(\delta-1)+\gamma]}{108\mu(\delta-1)^2} \\ w_{f1,3,S}^* = 0 \\ D_{3,S}^* = \frac{1}{36\mu(\delta-1)} [8(\delta-1)(\delta-1+\gamma) + 11\gamma^2] \\ W_{3,S}^* = \frac{121\gamma^4 + (\delta-1)[64(\delta-1)^3 + 128\gamma(\delta-1)^2 + (240\gamma^2 + 648\mu)(\delta-1) + 176\gamma^3]}{1296\mu(\delta-1)^2} \end{cases}$$

In this case, geographic coverage and welfare increase when OP_{f2} is more efficient in its vertical differentiation (i.e. when γ increases). This is because, by extending geographic coverage, the regulator is able to capture growth opportunities in the consumer surplus, which exceed the loss of profit in rural areas. In addition, geographic coverage and welfare also increase when FTTH is seen by consumers to be a more radical innovation (i.e. higher δ), and when the cost of building the network (μ) decreases.

4.4.2 The case with budgetary constraints (no subvention provided to OP_{f1}) - Second-best regulation

In the case in which OP_{f1} receives no subvention, we need to take into account the constraint imposed by $\Pi_{f1} \geq 0$ ³⁰. In the following, absence of subvention is denoted by \bar{S} .

Let $\mathcal{L} = W + \lambda w_{f1} + \theta \Pi_{f1}$ be the Lagrangian function associated with:

$$\begin{cases} \text{Max } W \\ w_{f1} \geq 0, \Pi_{f1} \geq 0 \end{cases} \quad (1)$$

. The result of this is that the access price is $w_{f1,3,\bar{S}}^* \in]0, \gamma + \frac{1}{2}(\delta-1)[$ so that both OP_{f1} and OP_{f2} are active in the FTTH retail market. The corresponding values of D , Q_f and W are denoted $D_{3,\bar{S}}^*$, $Q_{f,3,\bar{S}}^*$ and $W_{3,\bar{S}}^*$. Explicit values of the main variables cannot be calculated, but we obtain and can prove some interesting results. They are provided in Proposition 5 and proved in Appendix C.

4.5 Comparison of regulation policies

Comparing the results obtained with the regulation policies studied in the previous sections, we obtain Proposition 5.

³⁰ The main proofs are given in the Appendix C.

Proposition 5. (i) Depending on the construction of the regulation policy, welfare is higher with “regulation 3” (Access price and investment regulation maximising welfare) than under the other regulations studied in this article. This is true for first-best regulation (the case without budgetary constraint) and also for second-best regulation (the case with budgetary constraint): $W_{3,S}^* > W_{3,\bar{S}}^* > W^* = W_2^* > W_1^*$. (ii) We obtain also that the level of investment is always higher with “regulation 3” than with other types of regulation (“regulations 1 and 2”) and with the “unregulated case”. It is highest in the absence of a budgetary constraint: $D_{3,S}^* > D_{3,\bar{S}}^* > D^* = D_2^* > D_1^*$. (iii) Finally, we can show that the number of FTTH consumers is higher under “regulation 3” and always higher if there are no budgetary constraints: $Q_{f,3,S}^* > Q_{f,3,\bar{S}}^* > Q_{f,1}^* > Q_f^* = Q_{f,2}^*$.

Proof. See Appendix D. □

The different regulation policy constructions show that the best regulation in terms of welfare is to regulate access price and to regulate the level of geographical deployment. This can be done through a beauty contest with specific requirements. Less obvious, and particularly interesting, is that access price and investment regulation maximising welfare, lead to the highest levels of geographical deployment. Consequently, regardless of whether the regulator is concerned about welfare in the telecommunications market or by the level of geographical deployment of FTTH network (for more general economic or social reasons), implementation of this policy appears to be the most relevant. This remains true if we introduce a budgetary constraint even if it is welfare is lower compared to welfare in first-best regulation. As far as we know, this result might be of great importance since current regulatory policy does not apply such an approach to deployment of FTTH.

5 Concluding remarks

We analysed the impact of different types of access price regulation on the geographic deployment of NGA networks. To achieve this, we built a framework integrating realistic dimensions, such as the coexistence of different and competing techniques (the old - ADSL - and the new - FTTH), and different types of operators (the main operators which have the same opportunity to roll out the new infrastructure and virtual operators whose activity depends on the access price).

We showed that, consistent with the static versus dynamic trade-off problem, “cost-based regulation” increases consumer demand, but reduces the incentives to deploy a new network and reduces social welfare compared to the “unregulated” case. Therefore, whether the regulator focuses on social welfare or geographic coverage, absence of regulation is more effective than cost based regulation. We showed that geographic coverage cannot be increased more than in the case of no regulation by regulating over the access price. Indeed, we highlighted that to do so in order to maximise geographic coverage leads to a

strictly equivalent equilibrium than in the unregulated case. Finally, we considered the case in which both access price and investment are regulated in order to maximise welfare (which thus can be considered as the “optimal” regulation in terms of welfare). We derived that this regulation leads to the highest welfare and also to the highest investment in geographic deployment. We showed also that, even if adding a budgetary constraint leads to lower investment and welfare, this conclusion remains true for second-best regulation, compared to the other policy options.

These results are significant in terms of the current concerns of NRA: while regulation is being discussed in relation to encouraging investment in FTTH deployment, we have proposed a framework to understand the impact of a range of policy options, including traditional “unregulated” and “cost-based” regulation. In questioning these traditional options, we suggest that geographic coverage requirements should be implemented in combination with access price regulation. This would produce both higher welfare and higher deployment and resolve the dilemma over static and dynamic efficiency.

Among the perspectives for further research, we suggest that the question of “nonuniform” access pricing should be addressed in order that regulatory remedies can be adapted to areas. As stressed by Foros and Kind [2003] and Valletti et al. [2002], infrastructure duplication may reduce deployment of the network to only the most densely populated areas. Therefore, the regulatory problem induced by network duplication needs more analysis. The trade-off between new technologies (especially mobile versus fixed telecommunications) could be integrated into the model, taking account of the complementarities and substitutabilities between corresponding networks. Finally, some parameters of the model could be endogenized by introducing other types of investments than those involved in rolling out the infrastructure, e.g. R&D investment, which has an impact on the quality of the network and on differentiation.

A Proof of Proposition 2

A.1 Quantities and profits

When $q_{f1} > 0$ **and** $q_{f2} > 0$, equilibrium quantities at stage 3 in the FTTH competitive segment are:

$$\begin{cases} \frac{\partial \Pi_{f1}}{\partial q_{f1}} = 0 \Leftrightarrow q_{f1} = \frac{D}{3(\delta-1)} (\delta - 1 - \gamma + w_{f1}) \\ \frac{\partial \Pi_{f2}}{\partial q_{f2}} = 0 \Leftrightarrow q_{f2} = \frac{D}{3(\delta-1)} (\delta - 1 + 2\gamma - 2w_{f1}) \end{cases} \quad (2)$$

We can easily check that $\frac{\partial \Pi_{f_i}^2}{\partial q_{f_i}^2} = -2\frac{\delta-1}{D} < 0$ at the critical points. Since $\gamma < \frac{\delta-1}{6} < (\sqrt{2}-1)(\delta-1)$, we have that $q_{f1} > 0$ et $p_{f1} > 0$. We note that $q_{f2} > 0$ and $p_{f2} > w_{f1}$ if and only if $w_{f1} < \gamma + \frac{1}{2}(\delta-1)$. In this case, we also

have $0 < P_f < \delta - 1$ ³¹. We can deduce Π_{f1} and Π_{f2} . **When** $q_{f1} > 0$ **and** $q_{f2} = 0$, OP_{f2} is foreclosed from the market and the equilibrium is provided by:

$$\frac{\partial \Pi_{f1}}{\partial q_{f1}} = 0 \iff q_{f1}^m = \frac{D}{2}$$

We can check that $\frac{\partial \Pi_{f1}^2}{\partial q_{f1}^2} = -2\frac{\delta-1}{D} < 0$ at the critical point. Since $0 < P_f < \delta - 1$, the remaining condition that must be fulfilled is $D > 0$. We easily derive Π_{f1} .

A.2 Proof of (i)

If FTTH were to foreclose ADSL service in the areas covered by the FTTH network, we would have $P_f = 0$ (lemma 1). Let q_{fi}^{ADSL} and Π_{fi}^{ADSL} , $i \in \{1, 2\}$, denote the quantity and profit of OP_{fi} when ADSL is foreclosed by FTTH service in the areas covered by the FTTH network. In this case, we can easily show that:

$$\left\{ \begin{array}{l} w_{f1} < \gamma \Rightarrow q_{f1}^{ADSL} = 0 \quad q_{f2}^{ADSL} = D \quad \left\{ \begin{array}{l} \Pi_{f1}^{ADSL} = w_{f1}D - \mu D^2 \\ \Pi_{f2}^{ADSL} = (\gamma - w_{f1})D \end{array} \right. \\ w_{f1} = \gamma \Rightarrow q_{f1}^{ADSL} = 0 \quad q_{f2}^{ADSL} \leq D \quad \left\{ \begin{array}{l} \Pi_{f1}^{ADSL} = w_{f1}q_{f2}^{ADSL} - \mu D^2 \\ \Pi_{f2}^{ADSL} = 0 \end{array} \right. \\ w_{f1} > \gamma \Rightarrow q_{f1}^{ADSL} \leq D \quad q_{f2}^{ADSL} = 0 \quad \Pi_{f1}^{ADSL} = \Pi_{f2}^{ADSL} = 0 \end{array} \right.$$

When $w_{f1} \in [0, \gamma] \subset [0, \gamma + \frac{1}{2}(\delta - 1)[$, we can check easily that³²:

$$\left\{ \begin{array}{l} \Pi_{f1} - \Pi_{f1}^{ADSL} = \frac{D(\delta-1)}{9} \left[(\delta - 1 - \gamma)^2 - 5w_{f1}^2 - 4w_{f1}(\delta - 1 - \gamma) \right] > 0 \\ \left\{ \begin{array}{l} \Pi_{f2} - \Pi_{f2}^{ADSL} = \frac{D}{9(\delta-1)} \left[(\delta - 1)^2 + 4(w_{f1} - \gamma)^2 + 5(\delta - 1)(w_{f1} - \gamma) \right] > 0 \\ \text{if } w_{f1} < \gamma \\ \Pi_{f2} - \Pi_{f2}^{ADSL} = \Pi_{f2} > 0 \\ \text{if } w_{f1} = \gamma \end{array} \right. \end{array} \right.$$

Consequently, the case in which prices are higher, leaving space for the ADSL service, is more profitable for both operators.

When $w_{f1} > \gamma$, profits are equal to zero for both OP_{f1} and OP_{f2} when ADSL service is discouraged. Here, again, the operators prefer to raise their prices, leaving room for ADSL.

Finally, in all cases the ADSL service remains active (together with FTTH) in the areas covered by the FTTH network.

³¹ The constraints depend only on w_{f1} and on the parameters of the model (but not of the level of investment D). We will thus need to check later that $\begin{cases} w_{f1} \geq 0 \\ D > 0 \end{cases}$.

³² The signs are obtained by studying the variations of the functions, given that $\gamma < \frac{\delta-1}{6}$. Since $w_{f1} < \gamma < \gamma + \frac{1}{2}(\delta - 1)$, in the equations, Π_{fi} designates the profit of OP_{fi} when $q_{f1} > 0$ and $q_{f2} > 0$.

A.3 Proof of (ii) and (iii)

If both OP_{f_1} and OP_{f_2} are not active in the retail market, we need to determine which of the operators remains. $\Pi_{f_1}^{i,m}$ is the profit of OP_{f_1} when OP_{f_i} $i \in \{1, 2\}$ is the remaining operator on the retail market and $q_{f_i}^m$ the corresponding number of FTTH consumers. If $i = 2$, $q_{f_1} = 0$ and $q_{f_2} > 0$. At stage 3, from $\frac{\partial \Pi_i}{\partial q_{f_2}} = 0$ we obtain that $q_{f_2}^m = \frac{D}{2(\delta-1)}(\delta - 1 + \gamma - w_{f_1})$, with $\frac{\partial \Pi_2^2(q_{f_2}^m)}{\partial q_{f_2}^2} = -2\frac{\delta-1}{D} < 0$. We derive that $q_{f_2} > 0$ if and only if $w_{f_1} < \gamma + (\delta - 1)$. If this condition is fulfilled, we can deduce $\Pi_{f_1}^{f_2,m}$. Similarly, we can derive $\Pi_{f_1}^{f_1,m}$ (without any condition on w_{f_1}). When $w_{f_1} < \gamma + (\delta - 1)$, we can write that $\Pi_{f_1}^{f_2,m} - \Pi_{f_1}^{f_1,m} = \frac{D}{4(\delta-1)} \left[-2w_{f_1}^2 + 2(\delta - 1 + \gamma)w_{f_1} - (\delta - 1 + \gamma)^2 \right]$. Since $\gamma < \frac{\delta-1}{6}$, the discriminant of the polynomial is strictly negative and thus $\Pi_{f_1}^{f_2,m} < \Pi_{f_1}^{f_1,m}$. Consequently, OP_{f_1} always prefers to be a monopoly on the retail market rather than being purely a wholesale access operator. We showed in Section A.1 that OP_{f_1} and OP_{f_2} are both active in the FTTH retail market for $w_{f_1} \in [0, \gamma + \frac{1}{2}(\delta - 1)[$. Then, for $w_{f_1} \in [\gamma + \frac{1}{2}(\delta - 1), \gamma + (\delta - 1)[$, only one FTTH operator is active on the retail market. However, as we have just shown, OP_{f_1} does not prefer to be purely a wholesale access operator; thus, if it decides to be active, it forecloses OP_{f_2} (see conditions stressed in Section A.1). Consequently, OP_{f_1} is the only operator in the retail market. Finally, for $w_{f_1} \geq \gamma + (\delta - 1)$, the conditions below show that, even as a monopolist, it is no longer profitable to be active in the market OP_{f_2} . summarising and using the calculus in Section A.1, we immediately find the assertions (ii) and (iii) of the proposition³³.

B Proof of Proposition 3

$D^* - D_1^* = \frac{1}{360\mu(\delta-1)} \left[16\gamma^2 + 40(\delta - 1)\gamma + 25(\delta - 1)^2 \right] > 0$. Let $k = \frac{\gamma}{\delta-1}$, with $k \in [0, \frac{\delta-1}{4}[$. We show that $W^* - W_1^* = \frac{A(\delta-1)^2}{324000\mu}$ where $A = 3125 + 8050k + 22440k^2 + 13240k^3 - 928k^4$. Since $\frac{d^2A}{dk^2} > 0$ on $[0, \frac{\delta-1}{4}[$ and $\frac{dA}{dk}(k=0) > 0$, we derive that $\frac{dA}{dk} > 0$. A is thus a growing function with $A(k=0) > 0$. Consequently, $W^* - W_1^* > 0$. Finally, we show that $Q_f^* - Q_{f,1}^* = \frac{B(\delta-1)}{400\mu}$ where $B = -392k^3 + 20k^2 + 1210k - 775$. Since $\frac{dB}{dk} > 0$ and $B(k = \frac{1}{4}) < 0$, we get $Q_f^* - Q_{f,1}^* < 0$.

³³ Note also that when both OP_{f_1} and OP_{f_2} are active ($w_{f_1} \in [0, \gamma + \frac{1}{2}(\delta - 1)[$): if $w_{f_1} < \frac{\delta-1}{2} - \frac{\gamma}{5}$, OP_{f_1} prefers to be a monopolist in the retail market (but OP_{f_2} is active... It is obvious, therefore, that the access price will never be chosen in this range if decided by OP_{f_1} on a commercial basis). If $w_{f_1} = \frac{\delta-1}{2} - \frac{\gamma}{5}$, OP_{f_1} is indifferent between being a monopolist and sharing the retail market with OP_{f_2} . If $w_{f_1} > \frac{\delta-1}{2} - \frac{\gamma}{5}$, OP_{f_1} prefers to share the market with OP_{f_2} .

C Proofs for "regulation 3"

Case $w_{f1} \in [0, \gamma + \frac{1}{2}(\delta - 1)[$. If $\theta = 0$, the resolution of the Lagrangian equation (associated with equation 1) is reduced to the resolution in Section 4.4.1. But in this case, there is no solution that matches the constraints. We thus have $\theta \neq 0$ and $\Pi_{f1} = 0$. Then, either $\lambda \neq 0$ and thus $w_{f1,3,\bar{S}}^{**} = 0$ and $D_{3,\bar{S}}^{**} = \frac{[\gamma - (\delta - 1)]^2}{9\mu(\delta - 1)} > 0$. Or $\lambda = 0$ and we have to solve:

$$\begin{cases} \Pi_{f1} = 0 \\ \frac{\partial W}{\partial w_{f1}} + \theta \frac{\partial \Pi_{f1}}{\partial w_{f1}} = 0 \\ \frac{\partial W}{\partial D} + \theta \frac{\partial \Pi_{f1}}{\partial D} = 0 \end{cases}$$

From $\Pi_{f1} = 0$, we can deduce that $D = \frac{-5w_{f1}^2 + [4\gamma + 5(\delta - 1)]w_{f1} + [\gamma - (\delta - 1)]^2}{9\mu(\delta - 1)}$. Replacing D by this expression into W , we obtain \widetilde{W} . $\frac{\partial \widetilde{W}}{\partial w_{f1}} = 0 \Rightarrow w_{f1} = w_{f1,3,\bar{S}}^*$ where $w_{f1,3,\bar{S}}^* \in]0, w_{f1,3,0}[$ and $w_{f1,3,0} = \frac{7\gamma}{10} + \frac{7(\delta - 1)}{12} - \frac{\sqrt{324\gamma^2 + 60\gamma(\delta - 1) + 145(\delta - 1)^2}}{60} \in]0, \gamma + \frac{(\delta - 1)}{2}[$. We also obtain that $\widetilde{W}(w_{f1,3,\bar{S}}^*) > W(w_{f1,3,\bar{S}}^{**}, D_{3,\bar{S}}^{**})$. Indeed, putting $D = \frac{-5w_{f1}^2 + [4\gamma + 5(\delta - 1)]w_{f1} + [\gamma - (\delta - 1)]^2}{9\mu(\delta - 1)}$ into W , we obtain $\widetilde{W}(w_{f1})$.

$$\frac{\partial^2 \widetilde{W}}{\partial w_{f1}^2} = \frac{-1}{3\mu(\delta - 1)^2} \left[10w_{f1}^2 - \frac{1}{3}(42\gamma + 35(\delta - 1))w_{f1} + \frac{1}{3}(12\gamma^2 + 9(\delta - 1)^2 + 24\gamma(\delta - 1)) \right]$$

Since $\gamma < \frac{\delta - 1}{4}$, $\frac{\partial^2 \widetilde{W}}{\partial w_{f1}^2} = 0$ has two positive roots: $w_{f1} = \frac{7}{12}(\delta - 1) + \frac{7}{10}\gamma \pm \frac{1}{60}\sqrt{145(\delta - 1)^2 + 60\gamma(\delta - 1) + 324\gamma^2}$, denoted respectively $w_{f1,3,0}$ and $w'_{f1,3,0}$ ($w_{f1,3,0} < w'_{f1,3,0}$). We can easily check that $\gamma + \frac{\delta - 1}{2} \in]w_{f1,3,0}, w'_{f1,3,0}[$. As a consequence, $\frac{\partial^2 \widetilde{W}}{\partial w_{f1}^2} < 0$ on $[0, w_{f1,3,0}[$ and $\frac{\partial^2 \widetilde{W}}{\partial w_{f1}^2} > 0$ on $]w_{f1,3,0}, \gamma + \frac{\delta - 1}{2}]$. We can easily check that:

$$\frac{\partial \widetilde{W}}{\partial w_{f1}}(w_{f1} = 0) = \frac{1}{18\mu(\delta - 1)^2} \left(2(\delta - 1 + \gamma)^3 + 4\gamma(\delta - 1)^2 + 7\gamma^2(\delta - 1) \right) > 0.$$

Moreover, $\frac{\partial \widetilde{W}}{\partial w_{f1}}(w_{f1} = w_{f1,3,0}) = -\frac{A \cdot B + C}{194400\mu(\delta - 1)^2} < 0$, with

$$A = \sqrt{145(\delta - 1)^2 + 60\gamma(\delta - 1) + 324\gamma^2},$$

$$B = 290(\delta - 1)^2 + 120\gamma(\delta - 1) + 648\gamma^2 \text{ and}$$

$$C = 66050(\delta - 1 + \gamma)^3 + 3630\Gamma(\delta - 1 + \gamma)^2 + 1984\gamma^3 - 22170\gamma^2(\delta - 1).$$

By developing the expression, we can isolate the following terms:

$$120\gamma(\delta - 1)A + (18150 + 3630)\gamma^2(\delta - 1) > (120\sqrt{324} + 21780)\gamma^2(\delta - 1) > 22170\gamma^2(\delta - 1).$$

In addition, $\frac{\partial \widetilde{W}}{\partial w_{f1}}(w_{f1} = \gamma + \frac{\delta - 1}{2}) = \frac{-1}{24\mu}(2\Gamma + \delta - 1) < 0$. Consequently, by continuity of $\frac{\partial \widetilde{W}}{\partial w_{f1}}$, $\left\{ \frac{\partial \widetilde{W}}{\partial w_{f1}} = 0, w_{f1} \in [0, \gamma + \frac{\delta - 1}{2}] \right\}$ has only one solution denoted $w_{f1,3,\bar{S}}^* \in]0, w_{f1,3,0}[$, which is also a maximum of \widetilde{W} . Finally, since \widetilde{W}

is growing on $\left[0, w_{f1,3,\bar{S}}^*\right]$ and since $\widetilde{W}(w_{f1}) - W(w_{f1,3,\bar{S}}^{**} = 0, D_{3,S}^*) = 0$ when $w_{f1} = 0$, we have $\widetilde{W}(w_{f1,3,\bar{S}}^*) > W(w_{f1,3,\bar{S}}^{**}, D_{3,S}^*)$.

Case $w_{f1} \geq \gamma + \frac{1}{2}(\delta - 1)$.

W is independent of w_{f1} and $\frac{\partial W}{\partial D} = 0 \Rightarrow D = \frac{3(\delta-1)}{16\mu}$. With this value of D , $\Pi_{f1} > 0$. Moreover, $W\left(D = \frac{3(\delta-1)}{16\mu}\right) > W\left(D = \frac{\delta-1}{4\mu}\right)$, where $D = \frac{\delta-1}{4\mu}$ corresponds to the value of D when $\Pi_{f1} = 0$. And we can show that $\widetilde{W}(w_{f1,3,\bar{S}}^*) > W\left(D = \frac{3(\delta-1)}{16\mu}\right)$. Indeed:

$$\widetilde{W}(w_{f1,3,0}) - W\left(D = \frac{3(\delta-1)}{16\mu}\right) = \frac{A.E + F}{23328000\mu(\delta-1)^2}$$

Where $E = \left(12100(\delta-1)^3 + 43560\Gamma(\delta-1)^2 + 6480\Gamma^2(\delta-1) + 23328\Gamma^3\right)$, and $F = 112800\Gamma(\delta-1)^3 + 887040\Gamma^2(\delta-1)^2 + 419904\Gamma^4 + 155520(\delta-1)\Gamma^3 - 109625(\delta-1)^4$.

This expression is strictly positive because, developing the expression shows that $A.B > 12100\sqrt{145}(\delta-1)^4 > 109625(\delta-1)^4$. Since \widetilde{W} is decreasing on $\left[w_{f1,3,\bar{S}}^*, w_{f1,3,0}\right]$, $\widetilde{W}(w_{f1,3,\bar{S}}^*) - W\left(D = \frac{3(\delta-1)}{16\mu}\right) > 0$. Finally, since for $\widetilde{W}(w_{f1,3,\bar{S}}^*) > W(w_{f1,3,\bar{S}}^{**} = 0, D_{3,S}^*)$ and $\widetilde{W}(w_{f1,3,\bar{S}}^*) > W\left(D = \frac{3(\delta-1)}{16\mu}, w_{f1} \geq \gamma + \frac{1}{2}(\delta-1)\right)$, $w_{f1,3,\bar{S}}^*$ is the value of w_{f1} chosen by the regulator in order to maximise welfare under budgetary constraint

D Proof of Proposition 5

(i) Relying on Propositions 3 and 4, we get $W^* = W_2^* > W_1^*$. By construction of “regulation 3 without budgetary constraint” $W_{3,S}^* > W^* = W_2^* > W_1^*$. Necessarily, welfare is lower when a “budgetary constraint” is introduced into “regulation 3”: $W_{3,S}^* > W_{3,\bar{S}}^*$. Finally, comparing the “unregulated access price” and “regulation 3 under budgetary constraint” which determine w_{f1} and D respectively by maximising profits (under $\Pi_{f1} \geq 0$) and by maximising welfare (under $\Pi_{f1} \geq 0$), necessarily leads to $W_{3,\bar{S}}^* > W^*$. (ii) Similarly, we obtain that $D^* = D_2^* > D_1^*$. It

is then easy to show that $D_{3,S}^* - D^* = \frac{74\gamma^2 + 35(\delta-1)^2 + 80\gamma(\delta-1)}{360\mu(\delta-1)} > 0$. $D_{3,\bar{S}}^* - D_2^* = \left[\frac{1}{3240\mu(\delta-1)} \left[18\gamma\sqrt{324\gamma^2 + 60\gamma(\delta-1) + 145(\delta-1)^2} + 320(\delta-1)^2 + \left(5\sqrt{324\gamma^2 + 60\gamma(\delta-1) + 145(\delta-1)^2} - 120\Gamma \right) (\delta-1) \right] \right] > 0$.

Indeed, the only negative term is the last one, but by developing the expression we can show that $18\gamma\sqrt{324\gamma^2 + 60\gamma(\delta-1) + 145(\delta-1)^2} - 120\Gamma(\delta-1) > (18\sqrt{145} - 120)\gamma(\delta-1) > 0$. Moreover

$D_{3,S}^* - D_{3,\bar{S}} = \frac{1}{36\mu(\delta-1)} \left[20w_{f1}^2 - (20(\delta-1) + 16\gamma)w_{f1} + 4(\delta-1)^2 + 16\gamma(\delta-1) + 7\gamma^2 \right]$

 (where $D_{3,\bar{S}}$ is the function of w_{f1} which represents FTTH investment in the case of a budgetary constraint). The discriminant of the polynomial ($\Delta = 5(\delta-1)^2 - 40\gamma(\delta-1) - 19\gamma^2$) is a polynomial in γ , which has one negative and one positive root. The positive one (denoted γ') is in $]0, \frac{\delta-1}{4}[$. Consequently, $\Delta < 0$ and $D_{3,S}^* - D_{3,\bar{S}} > 0$ for $\gamma \in]\gamma', \frac{\delta-1}{4}[$. For $\gamma \in]0, \gamma']$, $D_{3,S}^* - D_{3,\bar{S}} = 0$ there are two positive roots (denoted here w'_{f1} and w''_{f1} , with $w'_{f1} < w''_{f1}$). For $\gamma \in]0, \gamma']$, since w'_{f1} is an increasing function of γ , $D_{3,S}^* - D_{3,\bar{S}} > 0$ for each $w_{f1} \in [0, w'_{f1}(\gamma=0)[$. Taking $\gamma = 0$, $w'_{f1} = \frac{5+\sqrt{5}}{10}(\delta-1)$. We calculate $\frac{\partial \widetilde{W}}{\partial w_{f1}}(w_{f1} = w'_{f1}) = -\frac{5+\sqrt{5}}{180\mu}(\delta-1) < 0$ and derive that $w'_{f1} > w_{f1,3,\bar{S}}^*$. Therefore, $D_{3,S}^* - D_{3,\bar{S}} > 0$ (iii) Similarly, we obtain $Q_{f,1}^* > Q_f^* = Q_{f,2}^*$ and show that $Q_{f,3,S}^* - Q_{f,1}^* = \frac{[3\gamma^2 + 4\gamma(\delta-1) + 2(\delta-1)^2][2(\delta-1) + \gamma]}{36\mu(\delta-1)^2} > 0$. Calculating $Q_{f,3,S}^* - Q_{f,3,\bar{S}}$ (where $Q_{f,3,\bar{S}}$ is a function of w_{f1} , which represents the total number of FTTH consumers in the case of budgetary constraint), we show that $\frac{\partial(Q_{f,3,S}^* - Q_{f,3,\bar{S}})}{\partial w_{f1}} = \frac{1}{108\mu(\delta-1)} \left[-60w_{f1}^2 + 2(60(\delta-1) + 36\gamma)w_{f1} - (12\gamma^2 + 36(\delta-1)^2 + 60\gamma(\delta-1)) \right]$ has two positive roots ($\frac{3}{5}\gamma + \delta - 1 \pm \frac{1}{5}\sqrt{4\gamma^2 + 5\gamma(\delta-1) + 10(\delta-1)^2}$): one is above $\gamma + \frac{\delta-1}{2}$ (which is impossible in our model) and the other (say w'_{f1}), is below this value. Consequently, $Q_{f,3,S}^* - Q_{f,3,\bar{S}}$ is decreasing on $]0, w'_{f1}[$ and increasing on $]w'_{f1}, \gamma + \frac{\delta-1}{2}[$. Since $\gamma < \frac{\delta-1}{4}$, it is then easy to show that $f(\gamma) = [Q_{f,3,S}^* - Q_{f,3,\bar{S}}]_{w_{f1}=w'_{f1}}$ is an increasing function of $\gamma \in [0, \frac{\delta-1}{4}[$ and that $f(0) > 0$. Therefore, $Q_{f,3,S}^* - Q_{f,3,\bar{S}} > 0$ on $]0, w_{f1,3,0}[\subset]0, \gamma + \frac{(\delta-1)}{2}[$ and $Q_{f,3,S}^* > Q_{f,3,\bar{S}}^*$. Finally, we calculate $Q_{f,3,\bar{S}} - Q_{f,1}^*$ and $\frac{\partial(Q_{f,3,\bar{S}} - Q_{f,1}^*)}{\partial w_{f1}} = \frac{1}{9\mu(\delta-1)} \left[5w_{f1}^2 - (6\gamma + 10(\delta-1))w_{f1} + 3(\delta-1)^2 + \gamma^2 + 5\gamma(\delta-1) \right]$ has two positive roots: one is above $\gamma + \frac{\delta-1}{2}$ (which is impossible in our model) and the other (say w'_{f1}) is below this value. Consequently, $Q_{f,3,\bar{S}} - Q_{f,1}^*$ is increasing on $]0, w'_{f1}[$ and decreasing on $]w'_{f1}, \gamma + \frac{\delta-1}{2}[$. Given that $\gamma < \frac{\delta-1}{4}$, it is easy to show that for $w_{f1} = 0$, and for $w_{f1} = \gamma + \frac{\delta-1}{2}$, $Q_{f,3,\bar{S}} - Q_{f,1}^* > 0$. Therefore, $Q_{f,3,\bar{S}} - Q_{f,1}^* > 0$ on $]0, w_{f1,3,0}[\subset]0, \gamma + \frac{(\delta-1)}{2}[$ and $Q_{f,3,\bar{S}}^* > Q_{f,1}^*$.

References

Alessandro Avenali, Giorgio Matteucci, and Pierfrancesco Reverberi. Dynamic access pricing and investment in alternative infrastructures. *International Journal of Industrial Organization*, 28(2):167 – 175, 2010.

- Marc Bourreau and Pinar Dogan. "build or buy" strategies in the local loop. *European Economic Review*, (49):173–199, 2005.
- Marc Bourreau, Pinar Dogan, and Matthieu Manant. A critical review of the "ladder of investment" approach. *Telecommunications Policy*, forthcoming, 2010.
- Duarte Brito, Pedro Pereira, and João Vareda. Can two-part tariffs promote efficient investment on next generation networks? *International Journal of Industrial Organization*, 28(3):323–333, 2010.
- Carlo Cambini and Yanyan Jiang. Broadband investment and regulation: A literature review. *Telecommunications Policy*, 33(10/11):559 – 574, 2009.
- Martin Cave. Encouraging infrastructure competition via the ladder of investment. *Telecommunications Policy*, 30(3-4):223–37, April-May 2006.
- Martin Cave and Ingo Vogelsang. How access pricing and entry interact. *Telecommunications Policy*, 27(10-11):717–27, November-December 2003.
- Paul W.J. De Bijl and Martin Peitz. Unbundling the local loop : one-way access and imperfect competition. Discussion Paper 25, Tilburg University, Tilburg Law and Economic Center, 2004.
- Gerald R. Faulhaber and Christiaan Hogendorn. The market structure of broadband telecommunications. *Journal of Industrial Economics*, 48(3), September 2000.
- Oystein Fors. Strategic investment with spillovers, vertical integration and foreclosure in the broadband access market. *International journal of industrial organization*, (22):1–24, 2004.
- Oystein Fors and Hans Jarle Kind. The broadband access market: Competition, uniform pricing and geographical coverage. *Journal of Regulatory Economics*, 23(3):215–35, May 2003.
- Drew Fudenberg and Jean Tirole. Preemption and rent equalization in the adoption of new technology. *The Review of Economic Studies*, 52(3):383–401, 1985.
- Joshua Gans. Regulating private infrastructure investment: Optimal pricing for access to essential facilities. *Journal of Regulatory Economics*, 20(2):167–189, 2001.
- Joshua S. Gans. Access pricing and infrastructure investment. In Ralf Dewenter and Justus Haucap, editors, *Access pricing: Theory and practice*, chapter 2. Elsevier Science, Amsterdam, 2007.
- Graeme Guthrie. Regulating infrastructure: The impact on risk and investment. *Journal of Economic Literature*, 44(4):925 – 972, 2006.

- Steffen Hoernig. Should uniform pricing constraints be imposed on entrants? *Journal of Regulatory Economics*, 30:199–216, 2006.
- Keiichi Hori and Keizo Mizuno. Access pricing and investment with stochastically growing demand. *International Journal of Industrial Organization*, 24(4):795–808, July 2006.
- Keiichi Hori and Keizo Mizuno. Competition schemes and investment in network infrastructure under uncertainty. *Journal of Regulatory Economics*, 35(2):179–200, April 2009.
- Thomas M. Jorde, J. Gregory Sidak, and David J. Teece. Innovation, investment and unbundling. *Yale Journal on Regulation*, 17(1):1–37, 2000.
- Michael L. Katz and Carl Shapiro. R and d rivalry with licensing or imitation. *The American Economic Review*, 77(3):402–420, 1987.
- Tilman Klumpp and Xuejuan Su. Open access and dynamic efficiency. *American Economic Journal: Microeconomics*, 2(2):64–96, 2010.
- Kaisa Kotakorpi. Access price regulation, investment and entry in telecommunications. *International Journal of Industrial Organization*, (24 (5)):1013–1020, 2006.
- Romain Lestage and David Flacher. Le déploiement des réseaux d'accès à très haut débit: régulation et inégalités géographiques. *Revue d'Économie Industrielle*, (127):15–45, 2009.
- Robert S. Pindyck. Mandatory unbundling and irreversible investment in telecom networks. *Review of Network Economics*, 6(3):274 – 298, 2007.
- Tommaso M. Valletti, Steffen Hoernig, and Pedro P. Barros. Universal service and entry: The role of uniform pricing and coverage constraints. *Journal of Regulatory Economics*, 21(2):169–190, 2002.
- Joao Varela and Steffen Hoernig. The race for telecoms infrastructure investment with bypass: Can access regulation achieve the first-best? Technical Report 6203, CEPR, 2007.