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## On the cyclicity of real wages and wage differentials

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# On The Cyclicalities of Real Wages and Wage Differentials\*

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## Abstract

We show that two models of the labor market, a Walrasian model and a labor contracting model, both have an approximate dynamic factor structure. We use this result to motivate our empirical approach to estimating the cyclical properties of real wages, which does not impose any structure between real wages and observed cyclical indicators. In particular, we employ a Bayesian dynamic factor model and longitudinal microdata to estimate common latent factors driving real wages. We find that the comovement of real wages is related to a common factor that exhibits a mild correlation with the national unemployment rate. Our findings indicate that overall, roughly half of the wages move procyclically while half move countercyclically. In addition, we find that the estimated common factor can explain only a small portion of wage variability. We conclude that these facts are inconsistent with the prediction of a Walrasian labor market model, but consistent with the prediction of a labor contracting model. Finally, our findings suggest that although skilled and unskilled wages are driven by different common skill factors, these factors cannot explain a significant portion of wage variability.

*JEL Classification Codes:* C11, C32, C33, E32, J31

*Keywords:* Wages, Wage Differentials, Business Cycles, Bayesian Analysis

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# 1 Introduction

The cyclical nature of real wages allows us to differentiate between competing theories of the labor market. However, there has been empirical evidence put forth both for and against real wages exhibiting procyclical behavior. In this paper we provide new evidence on the cyclical nature of real wages using longitudinal microdata in conjunction with a new econometric approach, that of a dynamic factor model. The dynamic factor model searches directly for the largest common cycle in wage data, alleviating the problem of defining the cycle as any particular macroeconomic variable. The use of individual-level micro data allows us to determine whether the cyclical nature of wages is specific to a certain subset of individuals, which alleviates the problem of composition bias. Our main objective is to investigate whether real wages comove over the business cycle, and whether and in what extent their dynamic properties are consistent with the predictions of a Walrasian or an implicit contracts model. The factor model also allows us to disentangle the cyclical properties of wages for skilled (college) and unskilled (no college) workers. To do so, we employ a dynamic latent factor model in which real wages respond to common as well as skill-specific factors.

Beaudry and DiNardo (1991) show that adding appropriate lagged values of the unemployment rate (cyclical indicator) in a wage equation reduces substantially the degree of wage cyclical nature. This raises an issue, not only about the choice of the cyclical indicator but also about the structure of the relationship between real wages and the cyclical indicator imposed by the econometrician. We postulate that if real wages comove with the business cycle then this must be reflected on a common, and possibly unobserved, factor. Specifically, our dynamic factor model is motivated by the fact that if real wages exhibit a systematic relationship with the business cycle, then there should be a common factor which drives the movement of real wages in the same direction and accounts for a large portion of their variability. In addition, if the cyclical properties of real wages for skilled and unskilled workers are not alike, then there should exist skill-specific factors characterized by distinct dynamics.

We find that the common factor, which is estimated quite precisely, exhibits a correlation with the national unemployment rate in the order of 0.64 (0.20 in its first difference, as used in the previous studies). The latter indicates that the comovement of real wages derived from the unemployment rate might be biased to some extent. Moreover, we find that overall, real wages exhibit responses with different signs to a given change in the common factor. We provide evidence that only the wages of skilled workers exhibit comovement and show that indeed there are two additional distinct dynamic factors driving the real wages of skilled and unskilled workers. Finally, we demonstrate that our results are more consistent with the predictions of an implicit contracts model rather than those of a Walrasian model.

The Bayesian dynamic factor model we employ is part of an emerging literature on developing techniques to estimate factor models on large datasets (ours has a cross-section of over two thousand workers). We make a technical contribution to this literature by developing a method to apply large scale factor models to unbalanced panel-time series datasets. Following Otrok and Whiteman (1998), Kose, Otrok and Whiteman. (2003) we proceed with an explicitly Bayesian approach for estimating the parameters and the factors.

Before estimating the factor model we first show that two competing theories of the labor market impose a structure on the relationship between real wages and the business cycle that is in fact the form of an approximate dynamic factor model. The first model, a Walrasian model, implies that only productivity affects real wages. The second model, an implicit contracts model, implies that real wages depend not only on productivity, but also on an insurance component that results from bargaining between worker and firm. Over the business cycle the two elements move in opposite directions; marginal productivity is procyclical whereas the insurance component is countercyclical. Depending on what effect dominates real wages exhibit procyclical, acyclical or countercyclical behavior.<sup>1</sup> We show that these two models have different implications for both the relationship between individual

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<sup>1</sup>The theoretical background of implicit contracts lies in the work of Bailey (1974), Azariadis (1975, 1976) and Gordon (1974).

wages and the common factor, as well as the quantitative importance of the factor itself. The dynamic factor analysis in this paper is then a direct test of the neoclassical labor market model. It has two advantages over a direct estimation of the structural models. First, we can consider a large panel of workers of different types to see if the neoclassical implications hold for ‘most’ individuals, or for at least a subset of workers. Second, our test of the model will not lead to a rejection simply because some other feature of the structural model (such as the consumption Euler equation) rejects the RBC model.

Our findings suggest that real wages behave in a manner more consistent with models of labor contracting. This is in the line of the findings presented by Cooley and Ogaki (1996) who show that the time series properties of real wages are compatible with Walrasian models only in the long-run, whereas in the short-run they are better explained by an optimal labor contract model.<sup>2</sup> We find that the real wages of a majority of skilled workers tend to move in the same direction after a movement in the common factor. For unskilled workers we find that the real wages of roughly half the workers move in one direction, while half move in the opposite direction. We show that while a labor contracting model does not exclude any of these observed responses, a Walrasian model does, since real wages correspond solely to marginal productivities which are positively correlated with the business cycle. Further evidence for the labor contracting model is provided by the quantitative implications of the model. The labor contracting model implies that two components of the real wages, one capturing productivity and the second an insurance motive, offset one another after a movement in the common factor. Thus, we expect that the common factor should not be quantitatively significant if this model is largely correct. Our empirical results support this idea, as the estimated common factor is not significant. Furthermore, we show that even skill-specific factors do not appear to be quantitatively significant. To test the sensitivity of our results we introduce gender as well as race factors. We find that our results remain

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<sup>2</sup>Similar results to Cooley and Ogaki are reported by Osano and Inoue (1991), Beaudry and DiNardo (1991,1995) and Ham and Reilly (2002) who contrast and test Walrasian and labor contacting models. While the Walrasian models perform poorly in testing, the contracting models cannot be rejected by the data.

robust while the additional factors do not explain a substantial portion of wage variability. As previous studies found, there is a considerable amount of individual-specific heterogeneity in wage data which cannot be captured by a small number of control variables.

Our results are distinct from the existing literature in the use of the individual level data coupled with the dynamic factor model. At the same time our work is part of a long history of studying the cyclical behavior of wages and it is useful to briefly review some of the main contributions in the literature. The literature begins with Dunlop (1938) and Tarshis (1939), who conducted the earliest empirical studies on real wage cyclicality. They found that Keynes's view, in the *General Theory*, that real wages move countercyclically is not borne statistically. A simple average measure of real wages does not appear to move systematically over the business cycle. Thus, many leading macroeconomists have accepted the acyclical behavior of real wages as a stylized fact of the business cycle.<sup>3</sup>

Several studies, beginning from Stockman (1983), questioned the validity of the average measure of wages and stressed the importance of controlling for composition bias in obtaining accurate measurements of wage cyclicality. The idea is that during recessions workers in the lower tail of the skill distribution are more likely to be laid off and thereby the average wage might be countercyclically biased.<sup>4</sup> This argument implies that "true" and "spurious" movements in real wages may not be disentangle by a simple average measure. Since then, several studies estimate econometric models using disaggregated data to control for composition and aggregation effects. In particular, real wages are regressed on the unemployment rate, as an indicator of the business cycle, and other worker-specific characteristics. Among others, these studies, include the work of Bils (1985), Keane, Moffitt and Runkle (1988), Beaudry and DiNardo (1991), Solon, Barsky, and Parker (1994) and Ziliak, Wilson and Stone (1999). Bils, Solon et al. and Ziliak et al. find that wage acyclicity is simply a statistical illusion

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<sup>3</sup>For instance, the acyclicity of real wages is reported in Lucas (1977), Greenwald and Stiglitz (1988), Mankiw (1989), Blanchard and Fischer (1989), Hall and Taylor (1991), Christiano and Eichenbaum (1992), Gomme and Greenwood (1995), Boldrin and Horvath (1995), Rebelo and King (2000).

<sup>4</sup>Heckman and Sedlacek (1985) find evidence of composition bias in the manufacturing sector.

and that real wages are strongly procyclical. Bils however, finds that the impact of composition bias is not particularly large and argues that wage procyclicality is due to the inclusion of overtime earnings. Contrary to the previous studies, Keane et al report that real wages are mildly procyclical after controlling for sample selection bias. Beaudry and DiNardo (1991) extend the wage equation employed by Bils (1985) and Solon et al. (1994) by adding lagged values of the cyclical indicator to test their theory of implicit contracts. Their finding is that when appropriate lagged values of the cyclical indicator are taken into consideration the contemporaneous correlation between real wages and the business cycle goes to zero.

Previous studies have found that the real wages of skilled workers exhibit different low frequency variation than that of the real wages of unskilled workers. Katz and Murphy (1992), find that this behavior can be explained by different demand shifts for skilled and unskilled labor. Motivated by those findings, Acemoglu (1998), develops a theoretical framework to show that the wage gap between skilled and unskilled workers as well as the changes in the demand for skills are due to skilled-biased technological change which is determined endogenously. Krusell, Ohanian, Rios-Rull and Violante (2000), report empirical evidence showing that wage differentials are due to the existence of capital-skill complementarity which is present in the production process. These theoretical and empirical arguments have direct implications in building alternative theories of the labor market. These theories must also be consistent with the cyclical behavior of wage differentials and thus, knowledge of cyclical facts of skilled versus unskilled wages is essential. We find some evidence of distinct cycles for these two groups of workers before the mid 1980's.

The remainder of the paper is organized as follows. The next section provides the link between competing theories of the labor market and our dynamic factor model. Section 3 presents a description of our dataset which is extracted from the National Longitudinal Surveys (NLS). Section 4, introduces the model and section 5 lays out our econometric framework and methodology. Section 6 presents our results and section 7 concludes.



## 2 Two Theories of Wage Dynamics

### 2.1 A Neoclassical Model

Our dynamic factor model is motivated by a standard real business cycle model augmented with a model of measurement error induced by the agency gathering data. This motivation follows directly from the work of Sargent (1989). We start with a ‘textbook’ real business cycle model, that of King, Plosser and Rebelo (1988), which specifies preferences, technology and budget constraints. Using standard parametric functional forms for preferences and technology the model can be log-linearized and solved.<sup>5</sup> As is well known the solution of this model takes the form of a state law of motion and set of decision rules for observable variables:

$$\mathbf{S}_{t+1} = \Phi \mathbf{S}_t + \mathbf{E}_{t+1} \quad (2.1)$$

$$\mathbf{Y}_t = \mathbf{H} \mathbf{S}_t \quad (2.2)$$

The first system of equations describes the dynamic evolution of the vector of state variables and exogenous shocks, such as capital and technology. The second system of equations are the decision rules, linking the vector of endogenous choices,  $\mathbf{Y}_t$ , to the current state vector,  $\mathbf{S}_t$ . Typical decision variables are labor effort and consumption. Of course, the real wage would appear in  $\mathbf{Y}_t$  as well.

The real wage of the representative agent in this model is highly procyclical as the wage is equal to the marginal product of labor. To clarify this implication we follow the conventional way to decentralize the Pareto optimal equilibria of the model by assuming spot-competitive labor markets. Let the utility of agent  $i$ ,  $U^i$ , be defined over consumption,  $C_{it}$ , and work effort,  $H_{it}$  such that  $U_C^i > 0$ ,  $U_{CC}^i < 0$ ,  $U_H^i < 0$  and  $U_{HH}^i < 0$ , where subscripts denote derivatives. Following the common assumption of RBC models, neutral technology is the

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<sup>5</sup>Typically one assumes CRRA utility, Cobb-Douglas production, AR(1) technology shocks and a linear capital accumulation equation

main driving force of business cycle fluctuations. Let  $\psi_i(\theta_t)$  denote the agent's marginal productivity which is an increasing function of technology  $\theta_t$ . The intratemporal efficiency condition derived from an RBC model is<sup>6</sup>

$$\frac{-U_H^i(C_{it}, H_{it})}{U_C^i(C_{it}, H_{it})} = \psi_i(\theta_t), \quad (2.3)$$

This condition results from the agent equating his marginal rate of substitution between consumption and leisure to the real wage, while firms choose labor such that the marginal product of labor equals the real wage. The spot-market equilibrium then implies that real wages equal marginal productivities. Over the business cycle, since both consumption and work effort are procyclical, their marginal rate of substitution will be procyclical as well. Consequently, under spot-competitive labor markets we expect that over the business cycle there is a common (macro) component,  $\theta$ , driving the real wages of all agents, and that these wages move in the same direction. Note that this is true even with heterogeneity in risk aversion (or labor elasticities).

Our extension of this model assumes that we do not get to observe the 'true' real wage. Instead, we have many noisy observations on individual wages from this competitive spot labor market. The noise is induced by a data-gathering agency which must survey individuals to find out their wages. These survey data are riddled with errors, both recall errors from the agents and statistical errors from the agency itself. Our second system of equations then becomes:

$$\mathbf{Y}_t = \mathbf{H}\mathbf{S}_t + \mathbf{U}_t \quad (2.4)$$

where  $\mathbf{U}_t$  represents the measurement error and the  $\mathbf{Y}_t$  vector contains the full set of individuals surveyed.

The empirical model we will use in this paper, a dynamic factor model, is motivated directly from equations 2.1 and 2.4. These equations take the same general form as a dynamic

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<sup>6</sup>For the sake of simplicity we omit shocks other than  $\theta_t$  from our notation.

factor model. To make this link concrete consider the dynamic factor representation for a vector of wage data  $\mathbf{y}_t$ :

$$\mathbf{y}_t = \mathbf{b}\mathbf{f}_t + \boldsymbol{\varepsilon}_t \quad (2.5)$$

where  $\mathbf{b}$  is a  $N \times K$  matrix of factor loadings. The factor  $\mathbf{f}_t$  is assumed to follow an autoregressive process:<sup>7</sup>

$$\mathbf{f}_t = \boldsymbol{\phi}^f(L)\mathbf{f}_{t-1} + \mathbf{u}_t^f \quad (2.6)$$

where  $L$  denotes the lag operator.

It is clear from comparing equations 2.1 and 2.4 with equations 2.5 and 2.6 that the dynamic factor model takes the same form as the linearized solution to the real business cycle model with measurement error. Were one to simulate data from the RBC model and estimate a factor model on the simulated data, the estimated dynamic factor would then be the common technology shock in the business cycle model. When we turn to actual data, if the neoclassical labor market embodying this model is largely correct, then when we estimate the factor model on wage data we should have two key results. First, as long as the wage data are not dominated by measurement errors, the common factor should be quantitatively important for explaining real wage dynamics. Second, wages should all respond with the same sign to this common factor since in the business cycle model all wages respond positively to changes in productivity.

## 2.2 A Wage Contracting Model

Our second labor market model is based on an alternative way to decentralize the Pareto optimal equilibria by considering a model where agents trade labor contracts. In such a model, wages and employment are specified in a contract which is the outcome of dynamic bargaining between workers and firms. The contract,  $\{w^i(\theta_t), H^i(\theta_t)\}$ , consists of an hourly

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<sup>7</sup>This model can easily be extended to a model with multiple factors for wages of workers with different skill levels. We will do this in a subsequent section.

wage rate and hours of work that are contingent on the future state of technology. The contract is such that the efficiency condition 2.3 holds, but the hourly wage rate is not necessarily equal to  $\psi_i(\theta_t)$ . The hourly wage not only responds to changes in productivity but also provides insurance to risk averse agents against business cycle fluctuations.<sup>8</sup> Contrary to the spot market case, under reasonable assumptions, in equilibrium the wage will not be strongly correlated with productivity. This is due to the fact that the wage embodies an insurance component which minimizes their fluctuations. Furthermore, a given change in  $\theta$  may induce the wages of some agents to increase while others to decrease. Hence, responses of different signs to a given change in the common component are consistent with the theory of implicit contracts. To illustrate these two points, we provide a simple example where consumption equals labor earnings that is,  $C_{it} = w_t^i H_{it}$ , and the agents differ in terms of their aversion toward risk. Assuming separable CRRA preferences, condition 2.3 can be solved for the equilibrium wage (see Boldrin and Horvath (1995)):<sup>9</sup>

$$w_t^i = \delta_i [\psi_i(\theta_t)]^{\frac{1}{\lambda_i}} \left[ \frac{T - H_{it}}{H_{it}} \right] \quad (2.7)$$

where  $\delta_i > 0$ ,  $\lambda_i$  is the agent's coefficient of risk aversion and  $T$  is the worker's total time endowment. (Note that the linearized version of equation 2.7 would enter the decision rules 2.2 or 2.4 in the state space system describing the model dynamics.) In this case, the equilibrium wage is comprised of two components, productivity and insurance (which is the ratio of leisure to labor). Productivity is strongly procyclical whereas the insurance component is countercyclical because hours of work are procyclical. The latter offsets the increases (decreases) in productivity and thus, wages do not appear to respond strongly to technology shocks. Notice that parameter  $\lambda_i$  controls the elasticity of the hourly wage to the

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<sup>8</sup>The idea is based on the assumption that capital markets are inadequate to fully buffer the agents' consumption against adverse shocks.

<sup>9</sup>The same condition for the equilibrium wage can be derived when preferences are nonseparable. In that case however, parameter  $\lambda_i$  is the within period elasticity of substitution between consumption and leisure (see Pourpourides (2008)).

marginal product of labor fluctuations. Depending on the value of  $\lambda_i$ , for some individuals the effect of the insurance component may dominate the effect of productivity and thereby, the change in their wage, in response to an increase in  $\theta$ , will have a negative sign. The more risk averse an agent is the more likely she/he is to have a negative wage response to an increase in  $\theta$ . To summarize, the contracting model first implies that real wages will not exhibit a strong common cycle, implying that any common dynamic factor should have little explanatory power for real wage fluctuations. Second, if there is heterogeneity in preferences than the model predicts that the factor loading coefficients in the dynamic factor model will have both positive and negative signs.

### 3 The Data

Our data on hourly wages are taken from the National Longitudinal Survey, which is a nationally representative sample of 12,686 men and women born in the years 1957 through 1964. All respondents were interviewed annually from 1979 to 1994. We use the time series from 1979 to 1993 and collect information from the survey on employment, wages and sociodemographic characteristics.<sup>10</sup>

The advantage of the NLS panel data set is that it avoids problems related to having a changing work force and enables us to control for various worker characteristics. Unlike the Michigan's Panel Study of Income Dynamics (PSID), where the hourly wage in a given year is the ratio of the annual income to the annual hours of work, in the NLS the respondents directly report their hourly rate of pay in the week of the interview. Thus, the advantage

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<sup>10</sup>The text of question for the years 1979 to 1993 asks the respondents to report amount earned that includes tips, overtime and bonuses before deductions. The hourly rate of pay in survey year 1994 is calculated a little differently. Respondents are first asked if they are paid hourly; if so, then that reported hourly wage is used in the created hourly rate. Presumably, this hourly wage does not include tips, overtime and bonuses. Otherwise, if the respondents report other than an hourly wage, then they are asked for earnings that include tips, overtime, and bonuses (just as in the years 1979-1993) from which hourly rate of pay is created. Given that there is a difference in methodology for 1994 we exclude this year from our sample.

of using NLS over PSID is that hourly wages are less contaminated by recall bias.<sup>11</sup> <sup>12</sup> We accept only those respondents that meet the following restrictions: 1) Must be at least 18 years old at the interview date; 2) Are not self-employed; 3) There must be at least 7 years of available time series observations; 4) Are not enrolled in school the last 2 years of the sample period.

After removing the respondents who do not meet our criteria our sample contains 2,123 individuals and 31,845 person-year observations. We provide further analysis of our sample by classifying individuals into 8 broadly defined categories on the basis of skills, gender and race. We define skilled workers as those having at least a college degree and unskilled workers as the remainder of the sample. Race is defined based on the information provided by NLS, which classifies the respondents into three race groups, Hispanic, black and non-black/non-Hispanic. We group the sample into two main categories. One category consists of blacks and Hispanic and the other one consists of the remainder of the sample, which is assumed to be largely non-minority. A detailed description of the composition of our sample can be found in Table 1. The wage measure is deflated by the Consumer Price Index (CPI) to provide a real wage measure normalized in terms of 1983 CPI dollars. The data are log-first-differenced and demeaned before estimation.<sup>13</sup>

One potential issue that we face is that our dataset is an unbalanced panel as missing observations constitute 27.7% of the sample. Missing observations arise in the NLS because either the respondent is not interviewed or he/she is enrolled at school or he/she is unemployed. Wage observations where respondents are enrolled at school but at the same time report a positive wage rate are treated as missing observations. (Information about missing observations for each category can be found in Table 1.) One approach to solving this prob-

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<sup>11</sup>The reported hourly wage refers to the respondent's current or most recent job at the time of the interview. In the NLS survey the current or most recent job is referred to as job #1 which, after 1982, is nearly always the CPS job.

<sup>12</sup>We do not use the newest NLS survey of 1997 because it is still in progress and a shorter sample period is currently available.

<sup>13</sup>This treatment of the data is the same form as the log deviations from steady-state that would come from a RBC model.

lem is to simply drop the time series containing missing observations. Since this significantly reduces the sample size, and may induce a selection bias, we take an alternative approach. We treat the missing observations as random variables and estimate them as part of our econometric model. Our methodology for estimating the missing observations is described in section 5.1.

## 4 The Dynamic Factor Model

To estimate the cyclical properties of real wages we use a dynamic factor model along the lines of Sargent and Sims (1977), Stock and Watson (1989) and Kose et al. (2003). This statistical model differs from the models traditionally employed to estimate wage cyclicity. In previous work, wages are associated with cyclical indicators (eg. the unemployment rate). Of course, if one chooses the ‘wrong’ cyclical indicator the results will be biased towards finding acyclical wages. The factor model, by definition, extracts the largest common cycle(s) in the wage data. Hence, we are finding the maximum possible amount of cyclicity in the wage data. Our model then gives the best possible chance to the theories in favor of cyclical wages.

To be concrete, let  $\mathbf{y}_t$  be a vector of real wages for  $N$  individuals at time  $t$ . Then,  $\mathbf{y}_t$  can be explained by a vector  $\mathbf{f}_t$  of  $K$  common factors and a vector  $\boldsymbol{\varepsilon}_t$  of  $N$  individual-specific noise terms. We assume that  $\mathbf{f}_t$  and  $\boldsymbol{\varepsilon}_t$  evolve according to the following autoregressions:

$$\mathbf{f}_t = \boldsymbol{\phi}^f(L) \mathbf{f}_{t-1} + \mathbf{u}_t^f \quad (4.1)$$

and

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\phi}(L) \boldsymbol{\varepsilon}_{t-1} + \mathbf{u}_t \quad (4.2)$$

where  $\boldsymbol{\phi}^f(L)$  and  $\boldsymbol{\phi}(L)$  are  $K \times K$  and  $N \times N$  matrices of polynomials in the lag operator, respectively. The vectors of disturbances  $\mathbf{u}_t^f$  and  $\mathbf{u}_t$  are assumed to be zero mean and

normally distributed with

$$E\left(\mathbf{u}_t^f \mathbf{u}_\tau^{f'}\right) = \begin{cases} \mathbf{M}^f & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise} \end{cases} \quad \text{and} \quad E\left(\mathbf{u}_t \mathbf{u}_\tau'\right) = \begin{cases} \mathbf{M} & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise} \end{cases}$$

where  $\mathbf{M}^f$  and  $\mathbf{M}$  are diagonal matrices. In other words, the factors are independent from each other and the individual-specific noise terms are independent across individuals. The statistical model for  $\mathbf{y}_t$  is

$$\mathbf{y}_t = \mathbf{b}\mathbf{f}_t + \boldsymbol{\varepsilon}_t \quad (4.3)$$

where  $\mathbf{b}$  is a  $N \times K$  matrix of factor loadings.

We focus our attention in characterizing the dynamic effects of three factors. The common dynamics of real wages across all individuals are captured by the common factor  $f^c$ . The factors  $f^s$  (where  $s$ = skilled or unskilled) drive the wages of a subset of individuals with the same skill level. Thus, having panel data on  $N$  individuals, each observed for  $T$  time periods, our model for the real wage of individual  $i$  is

$$y_{i,t} = b_{c,i}f_t^c + b_{s,i}f_t^s + \varepsilon_{i,t} \quad (4.4)$$

for  $i = 1, 2, \dots, N$ ;  $s$ =skilled or unskilled;  $t = 1, \dots, T$

where  $b_{j,i}$  is the ‘factor loading’ that captures the sensitivity of the wage of worker  $i$  to factor  $j$ . The corresponding idiosyncratic error  $\varepsilon_{i,t}$  follows a  $p_i$ -order autoregression:

$$\varepsilon_{i,t} = \phi_{i,1}\varepsilon_{i,t-1} + \phi_{i,2}\varepsilon_{i,t-2} + \dots + \phi_{i,p_i}\varepsilon_{i,t-p_i} + u_{i,t} \quad (4.5)$$

where  $\phi_{i,j}$  represents the exposure of the idiosyncratic error to its  $j$ th lag and  $u_{i,t} \sim \text{iid}N(0, \sigma_i^2)$ .



Likewise, the law of motion of factor  $j$  is given by the  $AR(q_i)$  process:

$$f_t^j = \phi_{fk,1}f_{t-1}^j + \phi_{fk,2}f_{t-2}^j + \dots + \phi_{fk,q_i}f_{t-q_i}^j + u_{k,t}^f \quad (4.6)$$

for  $k = c, s$

where  $\phi_{fk,j}$  represents the exposure of factor  $k$  to its  $j$ th lag and  $u_{k,t}^f \sim \text{iid}N(0, \sigma_{f,k}^2)$ .

## 5 Estimation

We estimate the factors and the parameters of the econometric model 4.4 – 4.6 using the Bayesian approach developed in Otrok and Whiteman (1998). We simulate from the joint posterior of the parameters and factors using a Markov Chain Monte Carlo algorithm. The main part of their procedure is a Gibbs sampler that sequentially draws the parameters conditional on the factors, and then the factors conditional on the parameters.<sup>14</sup>

Since the covariance matrix  $\mathbf{M}$  is diagonal, conditional on the factors, the system 4.4 consists of  $N$  independent regression models. Hence, conditional on the factors, we use Chib and Greenberg’s (1994) procedure to draw the regression parameters separately for each equation. Since the model has 2,123 equations this feature of their procedure makes the estimation feasible for our dataset. A full derivation and description of the relevant conditional densities can be found in Otrok and Whiteman (1998).

The (conjugate) prior densities for  $\mathbf{b}_i$ ,  $\phi_i$ ,  $\phi_{fk}$  and  $\sigma_i^2$  are chosen to be the same as those used in Otrok and Whiteman (1998). Specifically, the prior for the factor loadings  $\mathbf{b}_i$  is Gaussian with zero mean and precision (1/variance) equal to 0.01. The persistence parameters of the innovation and factor processes  $\phi_i$  and  $\phi_{fk}$  are also Gaussian with zero mean and precision equal to 0.85 for all lags. The prior of the idiosyncratic innovation variance  $\sigma_i^2$  is an *inverted gamma*  $\sim (\alpha/2, \beta/2)$  with  $\alpha = 6$  and  $\beta = 0.001$ . These priors are

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<sup>14</sup>The scales of the factor loadings are separately identified from those of the factors by normalizing the variances of the factors to a constant, as is common in the literature

fairly diffuse and the main results are not very sensitive to values of prior parameters around the ones chosen.

## 5.1 Missing Observations

Our dataset poses a technical problem due to missing observations for wages in some years for many of the survey respondents. Instead of omitting the time series we assume that the missing observations are random variables and we estimate these missing observations as part of our econometric model. We do so by first deriving the distribution of the missing data points conditional on the parameters and factors. This distribution depends on both cross-sectional information as well as the time series data before and after the missing observation. Intuitively, the distribution depends on both a ‘forecast’ and ‘backcast’ of the missing observation using the univariate time series data itself, and the parameters governing the dynamics of the time series. It also includes cross-sectional information: the factor loading is used along with the factor itself to ‘predict’ the missing value. Our procedure combines both types of information. A direct way to do this is by applying the Kalman filter and then smoothing the means and the variances by backward induction. Details of the procedure are in the Appendix.

Our Gibbs sampler then has three blocks. In block one we condition on factors and model parameters to draw the missing observations (for those time series with missing data). Then, in block two we treat the missing data drawn in block one as data and draw the model parameters. Finally, conditional on the drawn missing data and parameters we draw the factors. The procedure is repeated 5000 times after an initial burnin of 500 draws.

## 6 Empirical Results

Our primary interest is to provide answers to three questions: First, do real wages exhibit a systematic relationship with the business cycle? Second, is the behavior of real wages

consistent with the prediction of a Walrasian or a labor contracting model? Third, are the wages of skilled and unskilled workers subject to a significantly different degree of cyclical variation? To answer the first and second questions we focus on the importance of the common factor in equation 4.4 as well as the signs of the wage responses to a change in the common factor. To answer the third question we focus on the characteristics of the dynamic behavior of the skill factors and their relative contribution in accounting for real wage fluctuations.

Since the factors (common and skill specific) are estimated simultaneously, the skill factors are capturing the comovement in a specific skill group conditional on comovement already accounted for by the factor common to all wages. That is, skilled (or unskilled) wages may comove simply because all wages comove. Our model determines instead how much comovement there is in skilled wages that is not common to wages of all skill levels. This conditioning is important, as it alleviates the danger of looking only at, say, the wages of skilled workers, and mistakenly concluding that skilled wages have a common cycle, when that cycle is in fact common to a wider array of individuals.

## 6.1 The Dynamic Factors

Figure 1 presents the mean of the posterior distribution of the factors along with corresponding 95 percent posterior coverage intervals. The bounds of the confidence intervals are tight which shows that the factors are estimated quite precisely. The common factor is characterized by the peaks of 1983 and 1990 and the trough of 1987. The peaks occur at roughly the same time that NBER recessions occur. In particular, the peak of the 1983 lags the NBER recession of the 1982 whereas the peak of 1990 leads the NBER recession of 1991. The variable used by the previous studies as an indicator of the business cycle is the first difference of the annual national unemployment rate. In fact, our common factor exhibits a mildly positive correlation of 0.64 with the level of the national unemployment

rate and a much smaller correlation of 0.20 with its the first difference.<sup>15</sup> It is the case that our estimates suggest that macroeconomic conditions are relevant, at least to some extent, for the cyclical behavior of real wages. However, even though the level of the unemployment indicator captures a portion of real wage cyclical, assuming that unemployment is the common wage cycle biases the estimated comovement of real wages as the estimated common cycle is not simply the unemployment rate.

The skill-specific factors appear less cyclical than the aggregate factor and have distinct dynamics from each other. The correlation coefficient between the skilled and the unskilled factors is 0.26 which signifies that real wages embody a distinct component which is specific to skills.<sup>16</sup> The correlation coefficient between the skill factors and the unemployment rate is almost zero. Both factors exhibit substantial variation until 1985 and relatively smooth afterwards.

To examine whether common fluctuations are more persistent than skill specific fluctuations we report the first-order autocorrelation coefficients of the factors. Our estimates indicate that aggregate common fluctuations are highly persistent just like the unemployment rate. The common fluctuations of unskilled wages are also highly persistent with an autocorrelation coefficient of 0.68. Contrary to the common and the unskilled factors, the skilled factor exhibits a negative autocorrelation of -0.21 which suggests that it is weakly mean reverting. The differing dynamics of the skilled factor suggests that there are forces unique to skilled workers driving their wages. If we interpret this in light of our theoretical models, then this would suggest skill-specific productivity shocks. We do not push this interpretation very hard though, since we will see that these factors are not quantitatively important.

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<sup>15</sup>The NLS interviews usually take place around March and thus the reported wages better correspond to that period. The correlations are slightly bigger if March's unemployment rate is used rather than the annual rate. Specifically, the correlation between the level of the unemployment rate and the common factor is 0.73 while the correlation with its first difference is 0.41.

<sup>16</sup>The assumption in the econometric model is that the innovations between the two skill factors is zero. However, this assumption is not imposed in the estimation so the skill factor can be correlated if the data so indicate. We do impose that that aggregate factor is orthogonal to the two skill factors.

Next we examine the direction to which a change in each of the factors affects real wages. Figure 2 displays the cumulative distribution functions (CDFs) of the factor loadings. The CDFs illustrate that roughly half of real wages in our sample respond positively to the factors while the other half respond negatively. Thus, overall there is no distinct pattern of the responses of real wages to the common factors. Figure 3 indicates however that the majority of the real wages of skilled workers (about 75%) respond negatively to a given change in the common factor thus, exhibiting a higher degree of comovement relative to that of the wages of unskilled workers.<sup>17</sup>

As discussed in section 2, a neoclassical model of the labor market would imply that all wages respond with the same sign to the common factor (which would be interpreted as technology). On the other hand, a wage contracting model with some heterogeneity in preferences predicts that the factor loading coefficients differ in sign and magnitude. Our results for the signs of the wage responses suggest that the labor market is better characterized by a model of labor contracting rather than a Walrasian model. Although skilled wages tend to move modestly in the same direction after a change in the common factor, such behavior is also consistent with the labor contracting model. What is more, as will be shown in the following subsection, the common factor cannot explain a significant portion of wage variability either for skilled or unskilled wages, which is precisely the prediction of an implicit contracts model.

Our finding that skilled wages exhibit a higher degree of comovement than unskilled wages, coupled with our finding that skilled and unskilled factors exhibit distinct dynamics indicates that skilled and unskilled wages exhibit a different degree of cyclical variation. This result seems to stand in contrast to the finding of Keane and Prasad (1993) who find that skilled and unskilled workers are subject to essentially the same degree of cyclical variation in wages. However, the quantitative significance of the factors we estimate appears to be

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<sup>17</sup>Recall that with a factor model the signs of the factors and factor loadings are not separately identified. The key point is the percent of wages that move in the same direction, not the positive or negative aspect of the comovement.

small, so we prefer to focus more on the Walrasian versus Contracting labor models debate, rather than differences across skill levels.

## 6.2 Quantitative Significance of Wage Factors

To examine the quantitative significance of the cyclical factors we estimate the contribution of each of them to the overall variability of observables. Since the factors and the idiosyncratic component are orthogonal to each other it is straightforward to partition the variance of each observable into the fraction that is due to each of the underlying factors and the idiosyncratic component. The variance of observable  $i$  can be written as (by applying the Var operator to equation 4.4)

$$\text{var}(y_{i,t}) = (b_i^c)^2 \text{var}(f_t^c) + (b_i^s)^2 \text{var}(f_t^s) + \text{var}(\varepsilon_{i,t}) \quad (6.1)$$

Then, the fraction of the volatility explained by factor  $j$  is

$$\frac{(b_i^j)^2 \text{var}(f_t^j)}{\text{var}(y_{i,t})} \quad (6.2)$$

Reporting the full posterior distributions of all 2,123 posteriors is infeasible, so instead we report information on the distribution of the posterior means of the 2,123 variance decompositions. (In most cases that we examined the posterior coverage intervals were tightly concentrated about the mean.) Figure 4 displays frequencies and CDFs of variance decompositions across the skilled, the unskilled and the whole sample. Table 2 presents analytically the number of individuals falling in each interval of variance shares attributable to each of the factors and the idiosyncratic component.

The common factor explains, on average, no more than 9% of the variance of real wages. We obtain similar results when we examine the impact of the factor separately on skilled and unskilled wages. Overall, the common factor accounts for 20% or less of the wage

variability for 88% of the workers in the sample. The share of variance attributable to the common factor exceeds 50% for only 1% of the workers. In other words, the wages of only 1% of the respondents are overwhelmingly influenced by common economic conditions, as reflected through the dynamic factor. These results show that the factor plays a relatively minor role in accounting for wage movements over the business cycle. Consequently, the explanatory power of the common factor is inadequate to justify claims for strong procyclical or countercyclical movements of real wages. This finding is also consistent with the prediction of the labor contracting model. Recall that the equilibrium wage under the contract is driven by a procyclical (marginal productivity) and a countercyclical (insurance) component. The latter implies that two offsetting effects reduce the response of the wage to a given change in the common factor, reflecting a quantitatively insignificant common factor.

Likewise, the skill factor explains, on average, no more than 10% of wage variability and accounts for 20% or less of wage variability for 84% of the workers in the sample. Those findings reinforce the evidence of previous studies which show that skilled and unskilled wages face essentially the same degree of cyclical variation.

These results are also inconsistent with a fixed nominal wage contract model. If we augment the neoclassical model in section 2.1 with a model where nominal wages are set for a fixed number of periods, then we would find that at least half, or a quarter of the wages, depending on the nominal wage contract length, would depend almost completely on the common factor. For example, if we have nominal wages fixed for 1 period, and half of workers get to change wages in a given period, then our common factor would find that more than half of the workers respond to the common factor.<sup>18</sup>

Notably, the idiosyncratic component is an important factor of wage fluctuations. It can explain more than 70% of wage variability for 78% of the workers. It is possible that this

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<sup>18</sup>If productivity were iid then exactly 50 percent of the individuals would be driven by the common factor, but since there is serial correlation in productivity, wages in adjacent periods would be related to each other, which would be picked up by the dynamics in the factor. This would lead to more than 50 percent of the sample having a quantitatively important response to the common factor .

residual may include the effects of characteristics such as gender and race. To examine the robustness of our main results we extend our model by including gender and race factors. Specifically, we assume that there is a specific factor driving the wages of male workers and a separate factor driving the wages of female workers. As for the race characteristics we follow the NLS classification and assume two broadly defined race factors, one driving the wages of blacks and hispanics and another driving the wages of the remainder. We call the latter group nonminority and the former group minority. For instance, in this setting, the real wage of a skilled female worker who belongs to a minority group is driven by five factors, one that drives the wages of all workers, one that drives the wages of all skilled workers, one that drives the wages of all female workers, one that drives the wages of all minority workers and finally a factor that is specific to the worker. We find that the gender and the race factors have little to no explanatory power and do not change our main results. Thus, they are not retained in the final statistical model. The result that there is a significant amount of individual-specific heterogeneity which cannot be explain by small number of factors is no different from findings of previous studies.<sup>19</sup>

## 7 Concluding Remarks

The cyclical behavior of real wages has long been a central issue in macroeconomics. Our contribution to this literature is to use a dynamic factor model with longitudinal data to find the largest possible common cycle in real wages. We first show that the factor model itself is motivated directly from two RBC models with alternative theories of the labor market. The virtue of the dynamic factor framework is that we need not subject the full range of implications of the RBC model to a test, rather we focus on implications for the labor market. It also allows us to use longitudinal micro data from the NLS to control for composition and aggregation effects. Our model allows us to analyze the degree and the

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<sup>19</sup>For instance, the  $R^2$  of the wage equation estimated by Bils (1985) is in the order of 2%.



nature of the comovement of real wages across the entire population as well as separately for skilled and unskilled workers. It also enables us to quantify the contribution of each factor in wage variability.

We find that the common factor is mildly correlated with the national unemployment rate which is the common component of real wages assumed by some previous studies. This indicates that macroeconomic conditions do have an impact on real wages, though the impact is quantitatively small. We then demonstrate that implicit contract models are more appropriate in understanding the time series properties of individual real wages. First, we find that roughly half the wages in our sample are procyclical and half are countercyclical. Such pattern cannot be generated by a neoclassical model of the labor market because wages correspond only to marginal productivities which are positively correlated with the business cycle. On the other hand, wage responses of different signs over the business cycle are possible in a model where firms and heterogeneous workers trade (implicit) labor contracts. Second, variance decompositions show that, on average, the common factor accounts for only a small fraction of wage fluctuations. The latter is consistent with a labor contracting model where the wage is composed of two offsetting elements. Our findings also suggest that although skilled and unskilled wages are driven by different common skill factors, these factors cannot explain a significant portion of wage variability.

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## Appendix: Factor Models with Unbalanced Panels

In this appendix we describe the procedure for estimating the missing observations. This procedure forms one block of our Gibbs sampler. In block one we draw the parameters conditional on factors and missing data. In block two we draw the factors conditional on parameters and missing data. In block three we draw the missing data conditional on parameters and factors. In essence, we fill in the missing observations of the unbalanced panel using information in both the model and available data. It is this last block that we describe in this appendix. The first two blocks are described in Otrok and Whiteman (1998).

Let  $\xi_{i,t} = \phi_{i,1}\xi_{i,t-1} + \dots + \phi_{i,p_i}\xi_{i,t-p_i} + u_{i,t}$  where  $\xi_{i,t} = y_{i,t} - b_{c,i}f_t^c - b_{s,i}f_t^s$ . Then, the following state space system is obtained:

$$\mathbf{y}_{i,t} = \mathbf{A}'_i \mathbf{x}_t + \mathbf{H}' \boldsymbol{\xi}_{i,t} + \mathbf{w}_{i,t} \quad (\text{A1})$$

$$\boldsymbol{\xi}_{i,t+1} = \mathbf{F}_i \boldsymbol{\xi}_{i,t} + \mathbf{v}_{i,t+1} \quad (\text{A2})$$

where

$$\mathbf{y}_{i,t} = y_{i,t}, \boldsymbol{\xi}_{i,t} = \begin{bmatrix} \xi_{i,t} & \xi_{i,t-1} & \dots & \xi_{i,t-p_i+1} \end{bmatrix}', \mathbf{x}_t = \begin{bmatrix} 1 & f_{t-1}^c & \dots & f_{t-q_i}^c & f_{t-1}^s & \dots & f_{t-q_i}^{sk} \end{bmatrix}'$$

$$\mathbf{w}_{i,t} = b_{c,i} u_{c,t}^f + b_{s,i} u_{s,t}^f, \mathbf{v}_{i,t} = \begin{bmatrix} u_{i,t} & \mathbf{0}_{1 \times (p_i-1)} \end{bmatrix}', \mathbf{A}'_i = \mathbf{B}_i \cdot \boldsymbol{\Phi}, \mathbf{B}_i = \begin{bmatrix} b_{c,i} & b_{s,i} \end{bmatrix}$$

$$\mathbf{H}' = \begin{bmatrix} 1 & \mathbf{0}_{1 \times (p_i-1)} \end{bmatrix}, \boldsymbol{\Phi} = \begin{bmatrix} \phi_{c,1} & \dots & \phi_{c,q_i} & 0 & \dots & 0 \\ 0 & \dots & 0 & \phi_{s,1} & \dots & \phi_{s,q_i} \end{bmatrix}, \mathbf{F}_i = \begin{bmatrix} \phi_{i,1} & \dots & \phi_{i,p_i-1} & \phi_{i,p_i} \\ \mathbf{I}_{(p_i-1) \times (p_i-1)} & & & \mathbf{0}_{(p_i-1) \times 1} \end{bmatrix}$$

The variance matrix of  $\mathbf{v}_{i,t}$  is

$$E(\mathbf{v}_{i,t} \mathbf{v}'_{i,\tau}) = \mathbf{Q}_i = \begin{cases} \begin{bmatrix} \sigma_i^2 & 0 & \dots & 0 \\ 0 & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \vdots \\ 0 & \dots & \dots & 0 \end{bmatrix} & \text{for } t = \tau \\ \mathbf{0}_{p_i \times p_i} & \text{otherwise} \end{cases}$$

Consequently, the system (A1) – (A2) satisfies the following conditions:

1.  $E(\mathbf{w}_{i,t}^2) = b_{c,i}^2 \sigma_{f,c}^2 + b_{s,i}^2 \sigma_{f,s}^2 = \mathbf{R}_i$
2.  $E(\mathbf{w}_{i,t} \mathbf{w}_{i,\tau}) = \mathbf{0}$ , and  $E(\mathbf{v}_{i,t} \mathbf{w}_{i,\tau}) = \mathbf{0}$  for all  $t$  and  $\tau$

Equations (A1) and (A2) are the observation and state equations, respectively. The recursion of the Kalman filter begins with  $\widehat{\boldsymbol{\xi}}_{i,0|0}$  which denotes the unconditional mean of  $\boldsymbol{\xi}_{i,1}$ , where  $\widehat{\boldsymbol{\xi}}_{i,0|0} = E(\boldsymbol{\xi}_{i,1}) = \mathbf{0}$ . The associated Mean Square Error (MSE) is  $\mathbf{P}_{i,0|0} = \boldsymbol{\Sigma} = E(\boldsymbol{\xi}_{i,1} \boldsymbol{\xi}'_{i,1})$  where  $\boldsymbol{\Sigma} = \mathbf{F} \boldsymbol{\Sigma} \mathbf{F}' + \mathbf{Q}$ . To enable the recursion steps we replace missing observations with values drawn from the distribution of the data,<sup>20</sup>

<sup>20</sup>Alternatively, instead of drawing a value from  $L(\cdot)$ , we can merely skip the updating equations by assuming that  $\widehat{\boldsymbol{\xi}}_{i,\tau|\tau} = \widehat{\boldsymbol{\xi}}_{i,\tau|\tau-1}$  and  $\mathbf{P}_{i,\tau|\tau} = \mathbf{P}_{i,\tau|\tau-1}$ . The results do not change significantly under this alternative.

$$L(y_{i,t}/\phi, \xi_{i,t}, \dots, \xi_{i,t-p}) = (2\pi\sigma_i^2)^{-1/2} \exp \left\{ -\frac{1}{2\sigma_i^2} (y_{i,t} - \hat{y}_{i,t/t-1})^2 \right\}$$

where  $\hat{y}_{i,t/t-1} = y_{i,t} - \xi_{i,t} + \phi_{i,1}\xi_{i,t-1} + \dots + \phi_{i,p}\xi_{i,t-p}$ . The transition from  $\hat{\xi}_{i,t-1|t-1}$  and  $\mathbf{P}_{i,t-1|t-1}$  to  $\hat{\xi}_{i,t|t}$  and  $\mathbf{P}_{i,t|t}$  is given by the following set of equations<sup>21</sup>

$$\begin{aligned} \hat{\xi}_{i,t|t-1} &= \mathbf{F}_i \hat{\xi}_{i,t-1|t-1} \\ \mathbf{P}_{i,t|t-1} &= \mathbf{F}_i \mathbf{P}_{i,t-1|t-1} \mathbf{F}'_i + \mathbf{Q}_i \\ \hat{y}_{i,t|t-1} &= \mathbf{A}'_i \mathbf{x}_t + \mathbf{H}'_i \hat{\xi}_{i,t|t-1} \\ \hat{\xi}_{i,t|t} &= \hat{\xi}_{i,t|t-1} + \mathbf{P}_{i,t|t-1} \mathbf{H} (\mathbf{H}' \mathbf{P}_{i,t|t-1} \mathbf{H} + \mathbf{R}_i)^{-1} (\mathbf{y}_t - \hat{y}_{i,t|t-1}) \\ \mathbf{P}_{i,t|t} &= \mathbf{P}_{i,t|t-1} - \mathbf{P}_{i,t|t-1} \mathbf{H} (\mathbf{H}' \mathbf{P}_{i,t|t-1} \mathbf{H} + \mathbf{R}_i)^{-1} \mathbf{H}' \mathbf{P}_{i,t|t-1} \end{aligned}$$

Since our goal is to form an inference about the value of  $\xi_{i,t}$  based on the full set of time series we compute the smoothed estimate  $\hat{\xi}_{i,t|T}$  and the corresponding MSE,  $\mathbf{P}_{i,t|T}$ , by conditioning on next period's observation that is,  $\hat{\xi}_{i,t|T} = \hat{\xi}_{i,t|t} + \mathbf{J}_{iT} (\hat{\xi}_{i,t+1|T} - \hat{\xi}_{i,t+1|t})$  and  $\mathbf{P}_{i,t|T} = \mathbf{P}_{i,t|t} + \mathbf{J}_{iT} (\mathbf{P}_{i,t+1|T} - \mathbf{P}_{i,t+1|t}) \mathbf{J}'_{iT}$  where  $\mathbf{J}_{iT} = \mathbf{P}_{i,t|t} \mathbf{F}'_i \mathbf{P}_{i,t+1|t}^{-1}$ .<sup>22</sup> Wherever there is a missing observation, in each loop of the Markov chain, we replace it with  $y_{i,t}^* = \xi_{i,t}^{*1} + b_{c,i} f_t^c + b_{s,i} f_t^s$  where  $\xi_{i,t}^{*1}$  is the first element of the drawing  $\xi_{i,t}^*$  from  $N(\hat{\xi}_{i,t|T}, \mathbf{P}_{i,t|T})$ . The values for the missing observations are drawn right after the completion of steps 1 and 2 of the estimation procedure.

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<sup>21</sup>The formulas were directly taken from Hamilton's (1994) time series textbook. For more details concerning the algorithm refer to Hamilton pp. 377-381.

<sup>22</sup>Refer to Hamilton (1994) pp.394-397.

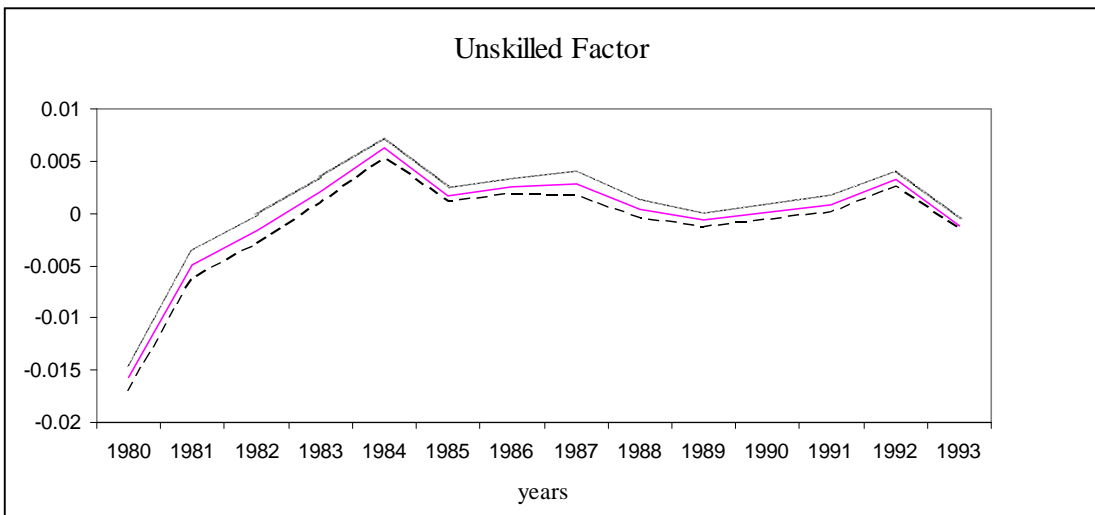
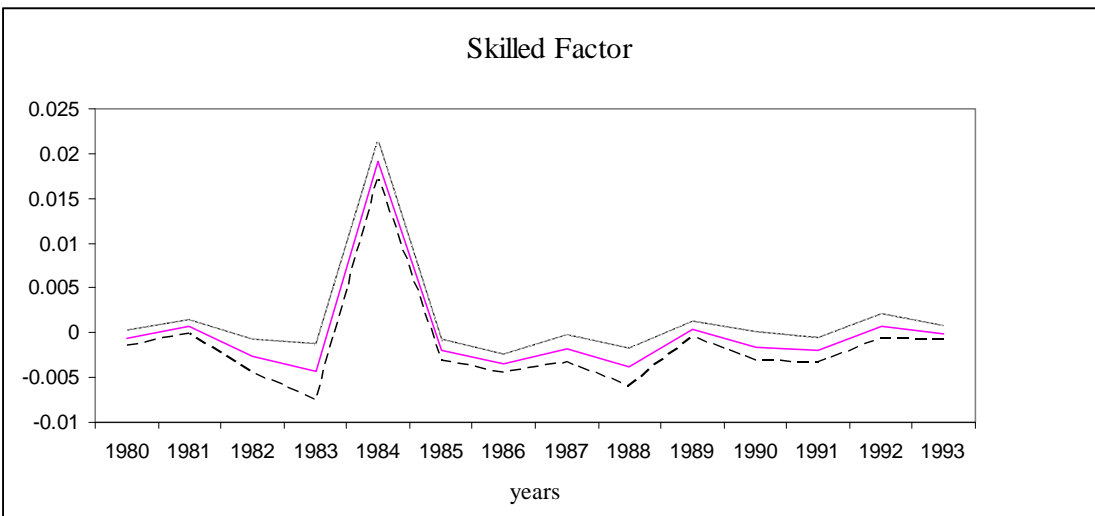
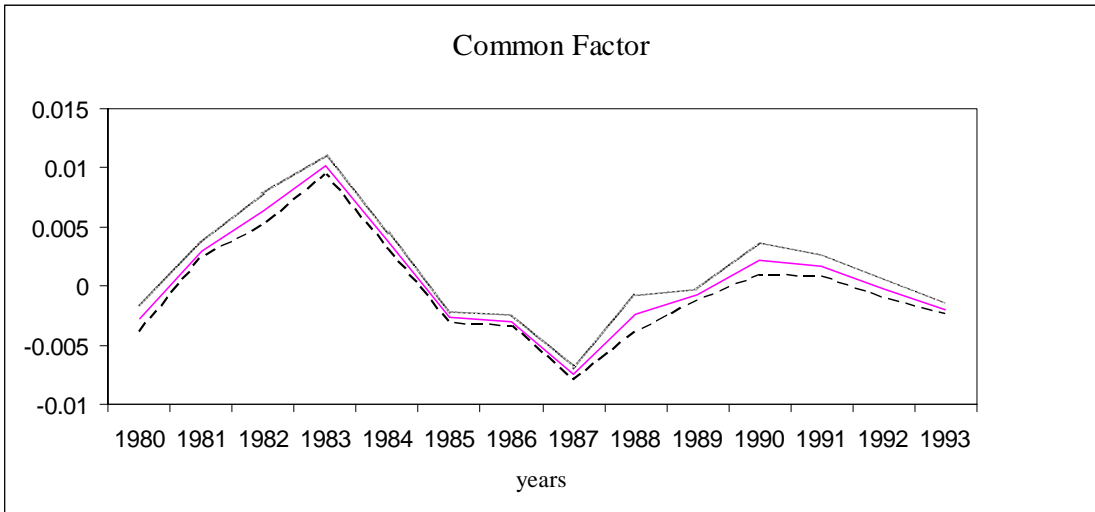


Figure 1 Dynamic Factors (means, upper and lower bounds), 1980-1993

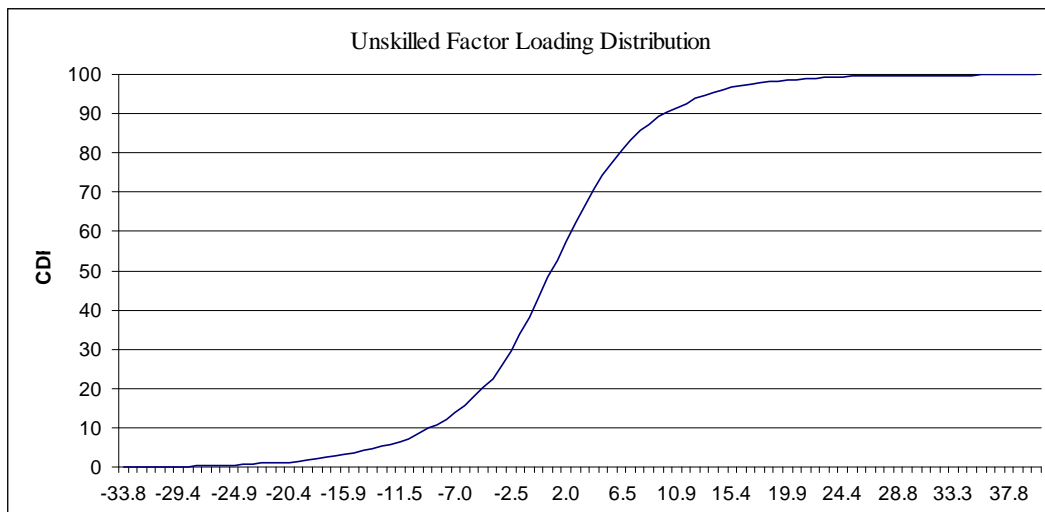
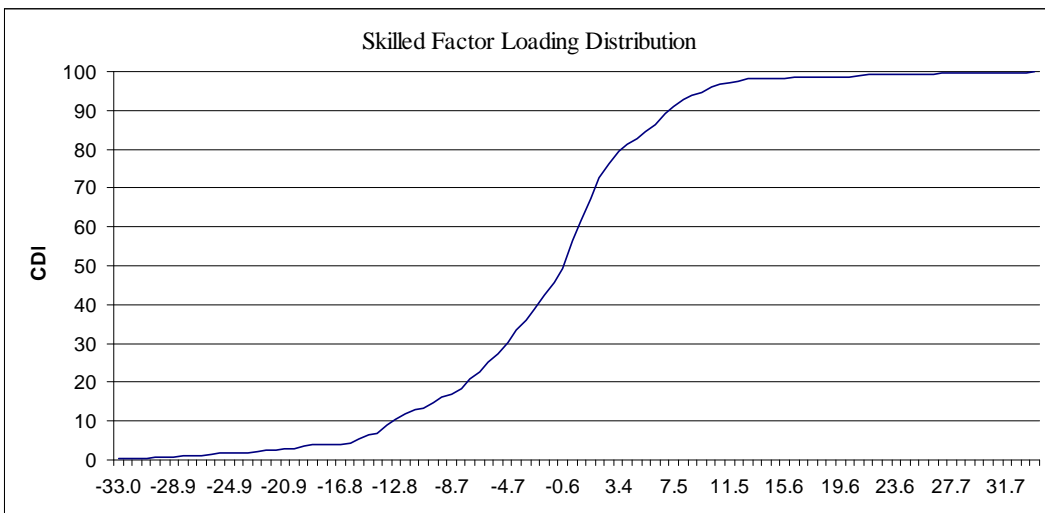
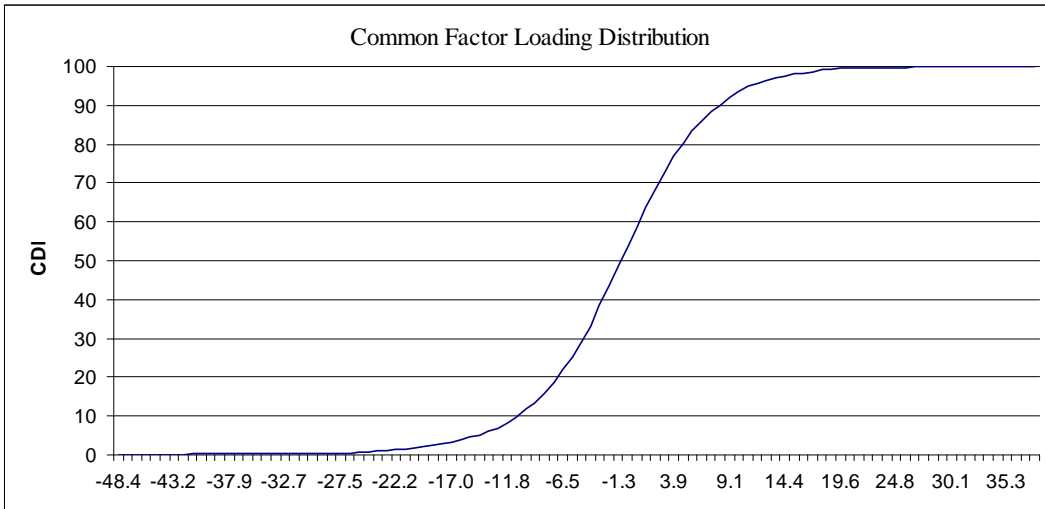


Figure 2 Factor Loading Distributions



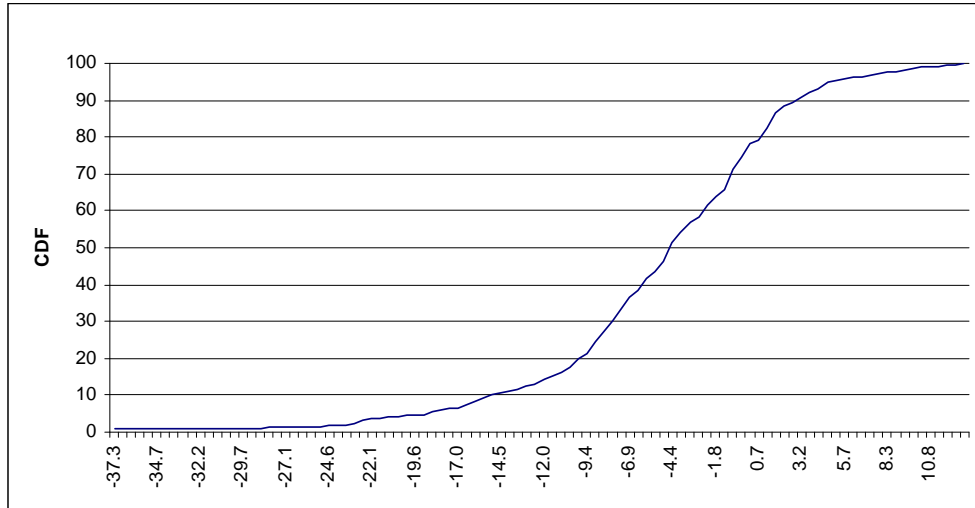


Figure 3 Skilled Worker Common Factor Loading Distribution

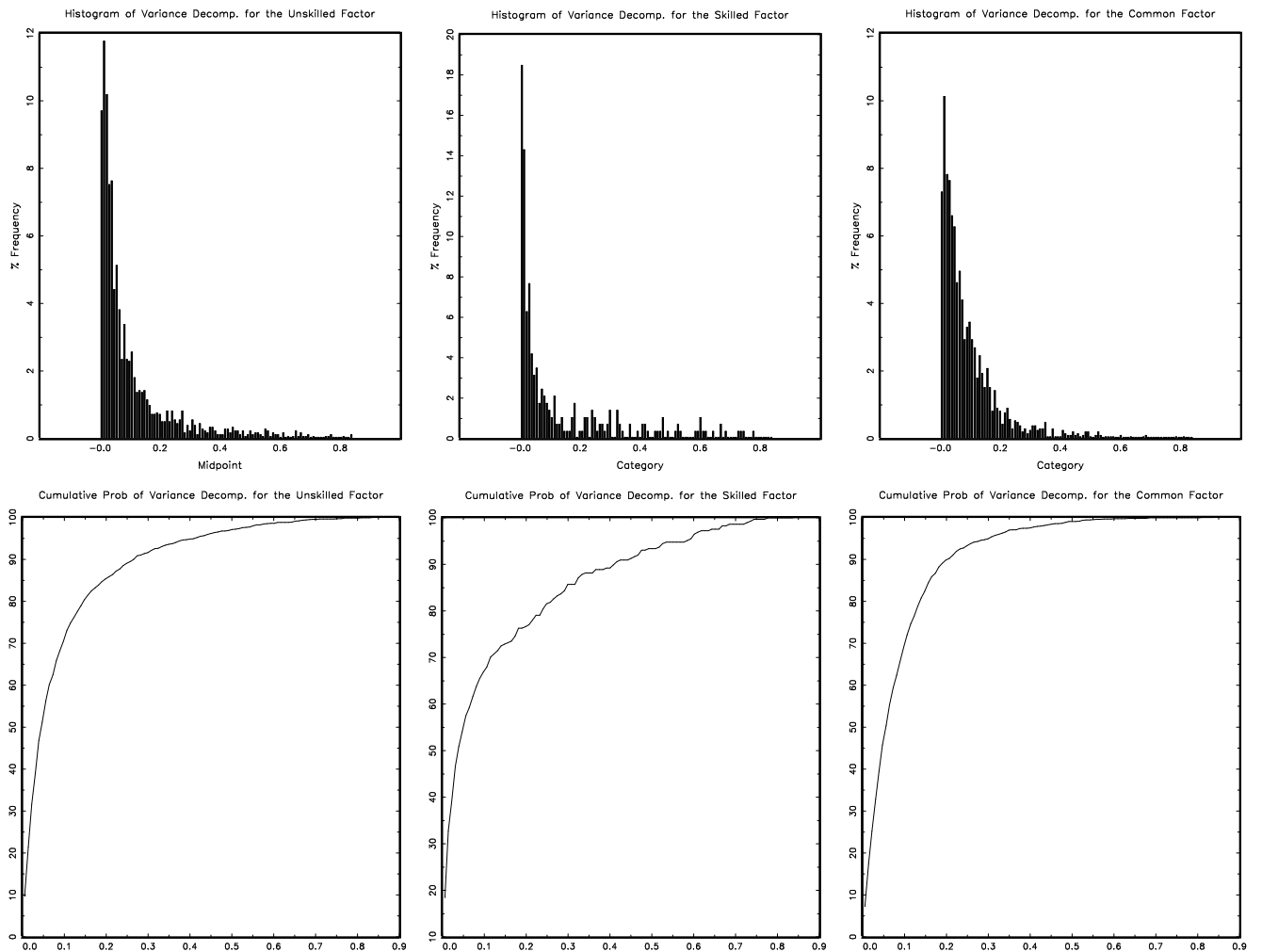


Figure 4 Variance Decompositions for the Factors

Table 1: Composition of the Sample

category	# of people	% in sample	% of missing obs.
skilled males minority	40	1.90	0.86
skilled males nonminority	89	4.19	1.80
skilled females minority	60	2.82	1.16
skilled females nonminority	98	4.61	1.99
unskilled males minority	528	24.90	6.30
unskilled males nonminority	452	21.30	4.88
unskilled females minority	428	20.14	5.70
unskilled females nonminority	428	20.14	5.08
<i>aggregate</i>	2123	100.0	27.77
males	1109	52.23	13.84
females	1014	47.77	13.93
skilled	287	13.52	5.81
unskilled	1836	86.48	21.96
minority	1056	49.74	14.02
nonminority	1067	50.26	13.75

Table 2: Variance Decompositions\*

decomp. (%)	common factor						skill factor						idiosyncratic factor					
	# of workers			% in sample			# of workers			% in sample			# of workers			% in sample		
	sk	un	tot	sk	un	tot	sk	un	tot	sk	un	tot	sk	un	tot	sk	un	tot
0-10	194	1262	1456	67.60	68.74	68.58	190	1285	1475	66.20	69.99	69.48	0	5	5	0	0.27	0.24
10-20	48	394	442	16.72	21.46	20.82	30	282	312	10.45	15.36	14.70	6	8	14	2.09	0.44	0.66
20-30	24	92	116	8.36	5.01	5.46	25	115	140	8.71	6.26	6.59	10	31	41	3.48	1.69	1.93
30-40	9	46	55	3.14	2.51	2.59	11	58	69	3.83	3.16	3.25	6	43	49	2.09	2.34	2.31
40-50	10	23	33	3.48	1.25	1.55	12	41	53	4.18	2.23	2.50	19	55	74	6.62	3	3.49
50-60	2	13	15	0.70	0.71	0.71	8	30	38	2.79	1.63	1.79	26	80	106	9.06	4.36	4.99
60-70	0	4	4	0	0.22	0.19	7	17	24	2.44	0.93	1.13	21	145	166	7.32	7.90	7.82
70-80	0	1	1	0	0.05	0.05	4	5	9	1.39	0.27	0.42	30	256	286	10.45	13.94	13.47
80-90	0	1	1	0	0.05	0.05	0	3	3	0	0.16	0.14	44	518	562	15.33	28.21	26.47
90-100	0	0	0	0	0	0	0	0	0	0	0	0	125	695	820	43.55	37.85	38.62

\* sk = skilled, un = unskilled, tot = total