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Explaining the Aggregate Price Level  
with Keynes's Principle of Effective  
Demand

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# Explaining the Aggregate Price Level with Keynes's Principle of Effective Demand

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## Abstract

The conventional wisdom about Keynes's Principle of Effective Demand is that it states something about quantities. It is widely held that the Principle determines the levels of output and employment in a world not governed by Say's Law. This paper argues that the Principle of Effective Demand goes beyond this to explain not only 'real' activity levels but also the aggregate price level. A variant of the Post Keynesian *D/Z*-model is brought together with Marxian reproduction schemes to derive this result.

*Key words:* Effective demand, multiplier, Post Keynesianism, *D/Z*-model, reproduction schemes

*JEL classifications:* B14, B22, E20, E31

## Introduction

The conventional wisdom about Keynes's Principle of Effective Demand is that it states something about quantities – or 'quantity reactions', respectively. Most economists would probably endorse Milgate's claim:

The formal proposition is that saving and investment are brought into equality by variations in the level of income (output). This is the Principle of Effective Demand (MILGATE, 1982, p. 78).

The famous 'Keynesian cross' (SAMUELSON, 1948, Chapter 12), which has crucially shaped economists' imagination of how 'quantity reactions' might work, is in 'real' terms: there are no prices in that model. Yet, when Keynes introduced the Principle of Effective Demand in

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Chapter 3 of the *General Theory* (KEYNES, 1973a, p. 25), he did so in terms of two *nominal* functions,  $D$  and  $Z$ , which he calls ‘expected proceeds’ and ‘the aggregate supply price’, respectively. These two functions intersect at what Keynes calls the point of effective demand.

This paper aims at disentangling the price and quantity components of the two functions  $D$  and  $Z$  in order to show that the Principle of Effective Demand explains both of them, separately and individually. By ‘explain’ we mean that – under certain assumptions about the ‘given’ and ‘independent’ variables Keynes enumerates on p. 245 of the *General Theory*<sup>1</sup> – the Principle of Effective Demand will produce a precise theoretical forecast for both the level of output and the price level.

The expedience of such an approach might be disputed on the grounds that Keynes himself has expressed his dislike for the concept of the aggregate price level. He writes: “(T)he well-known, but unavoidable, element of vagueness which admittedly attends the concept of the general price-level makes this term very unsatisfactory for the purpose of causal analysis” (KEYNES, 1973a, p. 39). Still, he kept using this concept throughout the *General Theory*. Certainly, THIRLWALL (1999, p. 369) was right when, writing ‘as Keynes’ in review of the ‘Second Edition of the General Theory’ (HARCOURT/RIACH 1996), he stated: “I ended up by measuring real income in terms of wage units; ... I now think I made the analysis unnecessarily complicated, and could just as well have used a price index. No one cares about wage units any more, and ... I was in any case lax in applying the concept of wage units consistently. In chapter 21 on ‘The Theory of Prices,’ I took the aggregate price level for analysis without a second thought”.

Although the aggregate price level has not been in the center of interest of Post Keynesian research, it has also not gone unnoticed. In one of his early papers, Paul Davidson has criticized the “typical Keynesian multiplier analysis” for ignoring “the problem of changing price

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<sup>1</sup> Keynes takes as ‘given’ “the existing skill and quantity of available labour, the existing quality and quantity of available equipment, the existing technique, the degree of competition, the tastes and habits of the consumer, the disutility of different intensities of labour and of the activities of supervision and organisation, as well as the social structure including the forces, other than our variables set forth below [i.e., the independent variables], which determine the distribution of income” (KEYNES, 1973a, p. 245). His ‘independent’ variables are “the propensity to consume, the schedule of the marginal efficiency of capital and the rate of interest” (*ibid.*). In this context Keynes mentions that to classify a factor as ‘given’ does not mean that this factor is assumed to be *constant*. This is important since the provisional inclusion of the nominal wage rate among the ‘given’ variables by Keynes (cf. KEYNES, 1973a, p. 27) has been interpreted by many as an indication that his theory relied on the assumption of wage rigidity. Cf. HARTWIG (2004a, pp. 87-89) for a refutation of that view.

and wage levels as output expands or contracts” (DAVIDSON, 1962a, p. 738). By insisting that Keynes’s is a flex-price model, Davidson dissents from the bulk of the ‘typical Keynesian analysis’ that maintains that Keynes relied on fixed (or rigid) prices. Consequently, DAVIDSON (1999, p. 584) and others (e.g. DUTT, 1987, p. 276, DEPREZ, 1996, pp. 129-130) rejected LEIJONHUFVUD’s (1986, p. 52) claim that the essence of the ‘Keynesian revolution’ consisted in the reversal of the “Marshallian ranking of price- and quantity-adjustment speeds” by Keynes.<sup>2</sup> These authors showed that in Keynes – as in Marshall – disequilibria between supply and demand are cured by price reactions in the short – or ‘ultra short’ (SKOTT, 1989, p. 63) – period. Section 1 of this paper will add weight to this argument.

In contrast to the Post Keynesian analysis of price movements so far mentioned, this paper will distinguish the price component in  $Z$ , which we will label the ‘supply price level’, from the price component in  $D$ , referred to as the ‘demand price level’. This distinction has hardly ever been made in the literature so far because Post Keynesians have mostly maintained that these two price levels must be equal under all circumstances. This view will be rejected in section 1; and a variant of the  $D/Z$ -model will be developed that distinguishes between the demand and supply price levels. We will argue that supply prices are merely notional magnitudes whereas demand prices are ‘real’ in the sense that they are observable in the marketplace. The two price levels are not identical for all conceivable levels of employment – as in most of the previous Post Keynesian literature – but only at the point of effective demand.

This prompts the question how this, so to speak, ‘equilibrium’ price level – the price level for which  $D$  equals  $Z$  – is determined. Since (as has been said and will be detailed below) supply prices are merely notional, it is the demand price level that we must, above all, explain. This task has not yet been accomplished. Even Edward J. Amadeo – who is one of the few to discriminate between supply prices and demand prices, and whose 1989 book is still the most comprehensive account of the subject – treats the demand price level as an exogenous variable (cf. AMADEO, 1989, p. 104).

Section 2 of the paper intends to fill this gap and to explain the aggregate demand price level. Extending earlier work (HARTWIG, 2004b), we will argue that Keynes’s macroeconomic theory implies an equilibrium proportionality between the consumption goods sector and the investment goods sector, and that it is the demand price level which ensures that the point of effective demand coincides with this equilibrium proportionality. Marxian reproduction schemes are brought into play to derive this result.

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<sup>2</sup> Leijonhufvud later stepped back from this interpretation (cf. LEIJONHUFVUD, 1974), but it is still alive in the New Keynesian school.

# 1 Keynes's Principle of Effective Demand – as in Chapter 3 of the *General Theory*

Following CHICK (1983, pp. 16-21), this paper argues that, in order to fully understand the Principle of Effective Demand, we have to visualize the economic process as a sequence of production periods. Admittedly, the concept of production periods is not prominent in the *General Theory*. LINDAHL (1939), for example, and HICKS (1982, 1985) rather than Keynes have portrayed the economic process as a succession of production periods.<sup>3</sup> Conceptually, the production period is characterized by the length of time that an entrepreneur is bound by her employment decisions taken at the beginning of that period. Theorizing in terms of production periods amounts to adopting *ex-ante ex-post* analysis as introduced by MYRDAL (1939). Plans made at the outset of the period are compared with results realized at its end.

Now, we know that Keynes has distanced himself from the *ex-ante ex-post* method in a letter to Ohlin (KEYNES, 1973b). Writing in 1937, Keynes pointed out that the production periods of individual firms “are all of different length and overlap one another”, a property that seemed to be inconsistent with the idea of a macroeconomic production period. But Keynes probably overstated the difficulties inherent in the concept of production periods. The rules of collective bargaining and legal regulations concerning the beginning and the end of the accounting year tend to bring the production periods of individual firms into line.<sup>4</sup> Another solution to the problem of establishing a macroeconomic production period is to make it very short so that the individual firms’ periods won’t overlap any more. Keynes considered this possibility in the *General Theory* where he writes: “*Daily* here stands for the shortest interval after which the firm is free to revise its decision as to how much employment to offer. It is, so to speak, the minimum effective unit of economic time” (KEYNES, 1973a, p. 47). This quote shows that Keynes’s theory requires the division of time into periods so that plans can be compared with realized results. Such comparisons are necessary because, as GNOS (2004) puts it, “the principle of effective demand presupposes the possibility of a discrepancy between supply and demand”.

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<sup>3</sup> FONTANA (2004, p. 79) notes: “Besides, as he [Hicks] explains, when agents make decisions they have in mind a stage-by-stage temporal frame. It was not only for theoretical convenience but also for the realism of the study that period analysis had to be considered superior to continuous analysis.”

<sup>4</sup> Nell presents the same argument somewhat differently: “‘Continuous output’ should not be overstressed. Even under Mass Production the seasons, traditional holidays and social customs provide a framework that sets definitive marketing dates toward which manufactures aim. ... So, while under continuous production there need be no common starting and finishing points, these will often exist, nevertheless” (NELL, 1998, p. 205).

Entrepreneurs decide at the beginning of the production period how much to produce during that period, and they deduce from this decision how much employment to offer. From the definition of the production period follows that they are not able to revise these decisions during the production period. So, if we abstract from inventory adjustments, we see that disequilibria *must* be cured by price reactions in the short term, i.e. the production period. There are simply no other mechanism available to entrepreneurs (cf. also HARCOURT, 2001, p. 118, HARTWIG, 2004b, p. 328).

It is the Principle of Effective Demand that guides entrepreneurs' decisions how much to produce. Their cost conditions together with the aim to maximize profits are reflected in their supply function ( $Z$ ). The price component inherent in the  $Z$ -function is a 'level of aspiration'. It is not the market price level an entrepreneur *expects* (as in the bulk of the Post Keynesian literature on  $Z$ ), but the proceeds he or she *must have* for the last unit of output at each level of employment to satisfy the profit maximizing condition. This unit supply price will grow with employment under conditions of decreasing marginal returns to labor.<sup>5</sup>

The formula for the  $Z$ -curve thus reads:

$$Z = P^s \cdot Y(N) = \frac{dN}{dY} \cdot w \cdot Y(N) \quad (2)$$

$Z$  is calculated by multiplying each conceivable output quantity (which is dependent on employment) by the (supply) price that maximizes profits. There is no element of uncertainty – or expectation-building – involved as long as the entrepreneur is certain about her cost conditions. The supply price is *not* the price expected to rule in the market, nor is the supply price, multiplied by the corresponding output quantity, a measure for expected demand (or expected sales). The supply price is the purely hypothetical price that is just sufficient to maximize profits for each volume of employment. This hypothetical price is then compared to the price that is really expected to rule in the market, i.e. the demand price. Both prices – multiplied by the same output quantity – can be compared to assess whether an expansion of employment and output will be worth while. This the essence of the Principle of Effective Demand.

The supply functions of individual firms can be aggregated straightforwardly (cf. DAVIDSON, 1987), which yields the aggregate supply function of the economy:  $Z = \phi(N)$ . Due

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<sup>5</sup> On the assumption that labor is the only variable input in the production process, the supply price is given by:

$P^s = w \cdot \frac{dN}{dY}$ , with  $w$  = nominal wage rate,  $N$  = employment, and  $Y$  = value added, This, of course, results from profit maximization (cf. also CHICK, 1983, p. 66).



to diminishing returns, the quantity component of  $Z$  grows at a decreasing rate while the price component grows at an increasing rate. Altogether,  $Z$  could be a straight line – for instance, when the production function is given by  $Y = \alpha N^\beta$ .<sup>6</sup> If, on the other hand,  $Z$  is assumed to be convex in a diagram with aggregate employment as abscissa and aggregate proceeds as ordinate (as below in Figure 8), this implies that with rising employment the profit share will grow as  $Z$  departs more and more from a straight line depicting the wage bill.<sup>7</sup>

For Keynes, Say's Law does not hold because of fundamental uncertainty in a monetary production economy (cf. KEYNES, 1973c). Therefore, each entrepreneur is forced to form expectations about how much he or she might be able to sell. This leads to (to quote VICKERS, 1987) “the producer's expected demand curve”. But contrary to VICKERS (1987, p. 98), the price level implicit in the expected demand curve need not be equal to the price level implicit in the supply curve at the same level of employment. This claim is a decisive shortcoming of the prevalent Post Keynesian interpretation. WEINTRAUB (1958, p. 32), who is the ‘founding father’ of  $D/Z$ -analysis, writes for instance:

(I)t should be apparent, however, that embedded in each point on the aggregate-demand function,  $D$ , will be the same prices that are found in  $Z$  at corresponding  $N$ -points.

His idea is that, with rising employment, people will earn more and can afford to pay a higher price. So we get a “family of Marshallian industry demand curves” (WEINTRAUB, 1958, p. 32, cf. also DAVIDSON, 2002, pp. 35-36) in a diagram with employment as abscissa and with the price level as ordinate – each of them being defined for a certain money income. The higher the supply-price and associated employment levels are, the farther to the right lies the respective industry demand curve. We can read people's “intended outlays” off these curves for each supply price level, join the points up, and get a “demand-outlay function” WEINTRAUB, 1958, p. 31). This function is then transferred to the aggregate employment/aggregate proceeds diagram to yield the  $D$ -curve.

Although Weintraub spends some time arguing that the slopes of the two curves  $D$  and  $Z$  (so constructed) could deviate from each other, formally, they cannot. Because both curves have been defined as equal to  $P^s \cdot Y(N)$ , they could only deviate from each other if we assume a different production function underlying each of them. Or, having assumed a profit share rising with employment to derive the convex form of the  $Z$ -curve, we now would have to assume the opposite to establish that  $D$  is concave (which is the usual presentation, cf. also

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<sup>6</sup> Cf. DAVIDSON, 1962b, p. 454.

<sup>7</sup> Keynes seems to assume that the profit share rises with employment, cf. KEYNES, 1973a, p. 17, fn 1.

Figure 8 below). No solution can be reached along these lines. It's the failure to distinguish clearly between the supply price level and the demand price level that leads into this trap.

The price level implicit in the supply curve is a 'level of aspiration', and it shifts with employment. The price level implicit in the expected demand curve, however – the proceeds each individual entrepreneur thinks he or she will be able to receive for a unit of output –, is independent of the level of employment she offers. Each entrepreneur has to form an expectation with respect to the price that can be enforced for her product on the (at least to some degree) competitive markets that characterize contemporary economies. These expectations inform numberless price-setting decisions which result in the aggregate (demand) price level. The remainder of this paper is concerned with the 'fundamentals' that determine the demand price level and thus serve as the basis for the expectation-building on the part of the entrepreneurs.

Contrary to the supply curves, the individual demand curves cannot be aggregated straightforwardly. Since no single producer expects her own proceeds to be negatively influenced if she cuts back employment, the producer's expected demand curve should be a horizontal line in a graph with her own offers of employment as abscissa and her own expected proceeds as ordinate. But although it is true that no entrepreneur will expect to sell more just because she employs more people, each entrepreneur will expect to sell more if she expects *aggregate* employment to be higher in the next production period because each entrepreneur knows that in this case aggregate demand will be higher. If we interpret the employment quantity with regard to the individual entrepreneur's D-curve as the share of expected aggregate employment for an individual firm, then the D-curve of every single firm (as well as the aggregate D-curve) will be strictly concave. – There is a range of conceivable total employment levels along with the specific share of an individual firm. The expected proceeds of each firm grow with this share, but due to diminishing marginal returns to labor, (real) income and also sales proceeds are expected to grow at a decreasing rate.<sup>8</sup> So we have two cases: if the entrepre-

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<sup>8</sup> Few scholars have so far considered this solution. It is implied in KOENIG (1980, pp. 437, 454) and explicit in CASAROSA (1981, p. 192). But Casarosa believes the solution to be "completely incompatible with the theory of the firm operating in an atomistic (let alone perfectly competitive) market", and claims that "the notion that the expected demand function is the producers' estimate of the expenditure function is clearly a theoretical aberration which has strangely survived" (*ibid.*). Casarosa is right that the notion of firms forming *ex ante* expectations about their market share is incompatible with the microeconomic theory of the small firm operating under perfect competition, cf. also ASIMAKOPOULOS (1991, pp. 43-44). But Keynes – who was concerned with the real world – did not have such firms in mind. In his theory, firms are not 'atomistic', but also not powerful enough to

neers have some macroeconomic insights, the aggregate demand curve  $D = f(N)$  (which is expectation-dependent) will be strictly concave. If they have no such insights, it will be a horizontal line.<sup>9</sup>

As long as  $D > Z$ , the entrepreneurs could increase profits by expanding production. Therefore, they fix their labor demand at the point where the  $D$ - and  $Z$ -curves intersect – “for it is at this point that the entrepreneurs’ expectation of profits will be maximised” (KEYNES, 1973a, p. 25). This point of intersection, the point of effective demand, contains all information about price, output, and employment levels for the next production period. One might conceive of this point as an equilibrium, but it is not some kind of ‘market equilibrium’. It is a point where the entrepreneurs’ expectations and aspirations concerning different things, e.g. prices, costs, profits, demand etc. are mutually consistent (cf. also MILLAR, 1972, p. 607). Note that, although the concepts of price and employment are different for both curves (supply price level as a ‘level of aspiration’ dependent on costs *versus* demand price level as an estimated enforceable price level for the firms’ output; employment level of the firms *versus* employment level as the firms’ share of expected total employment), they are all mutually consistent at this point of intersection. The supply price level equals the demand price level, and the firms’ *de facto* employment equals the entrepreneurs’ expectation as to how much of total employment is attributable to the respective firms.

## 2 Explaining the Aggregate Price Level

HARTWIG (2004b) has introduced a ‘structural view’ of the Keynesian multiplier. Traditionally, the multiplier is either interpreted as a dynamic process (or quantity reaction of output) or as a logical relation (or ratio) between income and investment expenditure. The structural view, however, holds the multiplier to be an *equilibrium condition* that describes a specific

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dictate the price. They have to form expectations about the price for their products they can enforce and about the market share that might be attributable to them.

<sup>9</sup> As to the shapes of  $D$  and  $Z$  the following should be noted:  $Z$  may be a linear function of  $N$  even under decreasing marginal returns (see above). It will be a straight line under constant returns, with the slope given by the wage rate. In this case, the  $D$ -curve, too, will be a linear function of  $N$ , with the slope given by the marginal propensity to consume multiplied by the demand price level and by the (constant) marginal product of labor. The slopes of  $D$  and  $Z$  depend crucially on the assumptions made about the ‘production function’ – in other words: about the marginal product of labor. Note also that the case of increasing returns cannot be handled well with the  $D/Z$ -diagram because if the  $Z$ -curve becomes concave, and the  $D$ -curve becomes convex, then the two curves do not intersect, and there is no Point of Effective Demand.

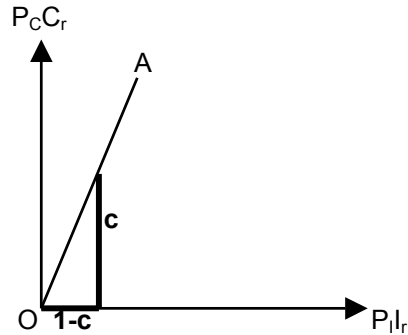
proportionality between the two economic ‘departments’ –the consumption goods sector and the investment goods sector – that is necessary for ‘completely successful reproduction’. ‘Completely successful reproduction’ means that the whole output of both departments is sold for its value – or price of production as defined by MARX (1973b, Ch. 9) – during a production period so that (at the same time) all *ex ante* plans have been fulfilled at the end of the period. HARTWIG (2004b) proves that under certain simplifying assumptions the ‘equilibrium proportion of departments’ – i.e. the proportion that allows for completely successful reproduction – is *Department I : Department II* as  $1 : c/(1-c)$ .<sup>10</sup> Keynes was referring to this proportion, albeit without specifying it, when he wrote: “(T)here is always a formula ... relating the output of consumption-goods which it pays to produce to the output of investment-goods; and I have given attention to it in my book under the name of the *multiplier*” (KEYNES, 1973c, p. 121). The significance of the ‘structural multiplier’ is that it provides the entrepreneurs of the consumption goods sector with the ‘formula’ (mentioned by Keynes) to estimate *ex ante* the total ‘emanation’ (in value terms) of an expected (additional) investment expenditure into the consumption goods department. The “consumption industries .. advance *pari passu* with the capital-goods industries” (KEYNES, 1973a, p. 122) if the net value added of the two departments expands in the proportion *Department I : Department II* as  $1 : c/(1-c)$ . If the departments do ‘advance *pari passu*’, we have a situation of ‘macroeconomic balance’; and reproduction will be ‘completely successful’. The economically important variable is not so much the investment multiplier  $1/(1-c)$ , but the multiplier  $c/(1-c)$ . It designates the proportion of *Department II* relative to *Department I* that is necessary for completely successful reproduction and that must be sustained in an expanding economy to ensure the identity between saving and investment.

If we combine the ‘structural multiplier’ with the *D/Z*-model of the Principle of Effective Demand, we are equipped to determine the demand price level. A couple of figures will be presented in order to show this. In Figure 1,  $P_I$  is the price-of-production level (in Marx’s sense) in the investment goods sector,  $I_r$  is the real output of this sector,  $P_C$  is the price-of-production level in the consumption goods sector, and  $C_r$  is the real output of consumption goods. The line OA (with a slope of  $c/(1-c)$ ) depicts the equilibrium proportion of departments.

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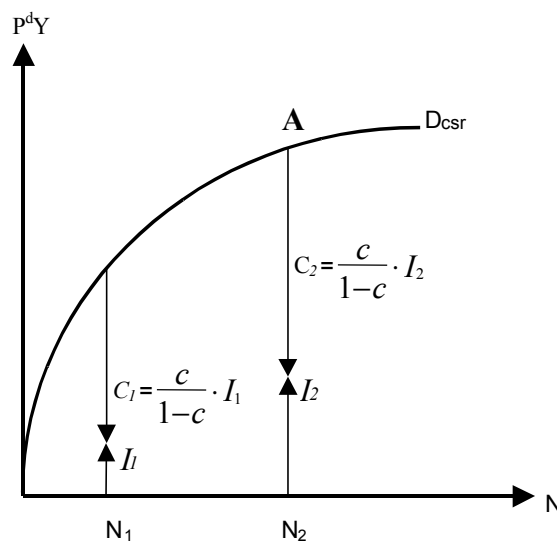
<sup>10</sup> In terms of MARX (1973a, Chapters 20–21) *Department I* produces investment goods, and *Department II* produces consumption goods.  $c$  denotes the marginal propensity to consume.

**Figure 1:** The equilibrium proportion of departments



In Figure 2, this proportion has been translated into a demand curve. Note that the ‘aggregate demand curve under completely successful reproduction’ ( $D_{csr}$ ) is not to be confused with the D-curve introduced in section 1.  $D_{csr}$  reflects the equilibrium proportion of departments at different levels of employment. If employment grows along the X-axis, the part of aggregate demand that is dependent on employment ( $D_I$  in Keynes’s Chapter 3-notation) will grow, but – as has been noted before – with a decreasing rate due to diminishing marginal returns.  $D_{csr}$  is so drawn that it incorporates for each level of employment-dependent nominal consumption demand (which is equal to the value added in *Department II*,  $P_c C_r$ , in a situation of ‘completely successful reproduction’) the corresponding level of investment demand (equal to the value added in *Department I* under ‘completely successful reproduction’) given by the adapted multiplier formula.

**Figure 2:** Demand under completely successful reproduction



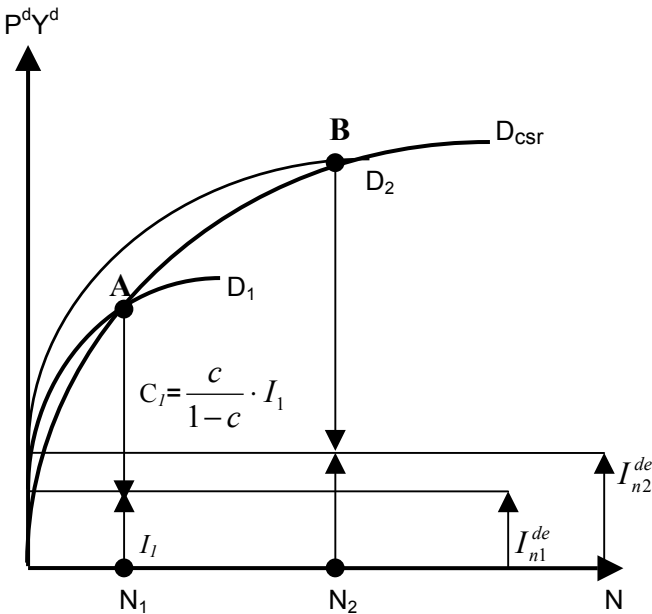
To repeat, the curve is in nominal terms. Along the Y-axis, proceeds are measured as the product of the aggregate demand price ( $P^d$ ) and real output in both departments ( $Y$ ). Note that the demand price  $P^d$  has replaced the value (or price of production). If  $P$  is the value (or price

of production) of a unit of output in Marx’s sense, then  $P^d$ , the price that is effectively paid for this unit of output, can deviate from it due to peculiarities of the distribution sphere. But that means that the *effective* rate of surplus-value is different from the relation between ‘surplus labor’ and ‘necessary labor’. The net value added (or income) represented by each unit of output is the sum of variable capital (wage) and the *effective* surplus-value (profit) inherent in it. Since income is the magnitude that influences demand as well as the equilibrium proportion of departments, we have to concentrate on  $P^d$  instead of  $P$ .

The  $D_{csr}$ -curve reflects what NELL (1998, p. 350) calls a ‘core’ relationship of capitalist economies. It changes its shape only if changes in productivity or the marginal propensity to consume should occur. As will be shown shortly, it does *not* reflect what actually happens because of the possibility of expectational errors. But note that, since completely successful reproduction implies maximization of realized profit,  $D_{csr}$  is the curve that entrepreneurs would want to know *ex ante*.

The next step will be to combine  $D_{csr}$  with the  $D/Z$ -diagram (which has been interpreted as the aggregate result of a thought experiment of each entrepreneur aiming to estimate *ex ante* which output and employment level will realize maximum profit). Let us concentrate on  $D$  first.

**Figure 3:** Aggregate demand and macroeconomic imbalance



The difference between the  $D_{csr}$ -curve and the  $D$ -curves in Figure 3 is that, for the latter, a specific level of estimated investment expenditure ( $I_n^{de}$ ) has to be taken for granted, whereas for  $D_{csr}$  a different level of investment is implicit at each  $N$ . Let us assume for simplicity that

*Department I* produces on order. Then the whole burden of expectation-building lies with the entrepreneurs of the consumption goods department.  $I_n^{de}$  is the value of investment that the entrepreneurs of *Department II* expect. The two D-curves show alternative expectations as to the consequences of different investment levels for the nominal demand for consumption goods. Let  $D_1$  depict a situation where the entrepreneurs of *Department II* hold correct expectations about the marginal propensity to consume as well as the volume of nominal investment demand and the demand price level (that means:  $c_1^e = c$ ,  $I_{n1}^{de} = I_1$ ,  $P_1^{de} = P^d$ ), then  $D_1$  intersects  $D_{csr}$  at Point *A*, and the economy is in macroeconomic balance at the end of the production period. The two curves have to intersect at Point *A*, because, given the premises mentioned,  $C_1$  will be the expected total ‘emanation’ (in value terms) of the expected investment expenditure  $I_{n1}^{de} = I_1$  into the consumption goods department. This ‘emanation’ includes the total consumption demand of those producing consumption goods (and receiving profits therefrom), and it can be calculated *ex ante* using the modified multiplier formula. (Of course, there is no reason why  $N_1$  should imply full employment of labor.)

If the entrepreneurs of *Department II* do not forecast  $c$  and  $I_n$  and  $P^d$  correctly, then the economy will only find itself in a position of balance at the end of the production period if the various deviations should by chance exactly cancel out. In Figure 3,  $D_2$  depicts a situation where the expected demand-price level  $P^{de}$  is higher than the *ex post* demand-price level (at the end of the production period)  $P^d$ . The higher the expected demand price level is, the more towards the top of the diagram will the D-curve be situated, because not only nominal investment demand is overestimated, but also the ‘emanation’ of demand into *Department II*.<sup>11</sup> But if price expectations turn out to be overly optimistic, the condition for macroeconomic balance will be violated. We have to be careful in interpreting what that means, though. Prices are set by firms. So, if entrepreneurs expect *ex ante* that  $P_2^{de}$  will be enforceable, they will set prices at that level, and  $P_2^{de}$  will rule in the market initially. But if buyers are reluctant to accept such a high price level, proceeds will fall short of  $P_2^{de} \cdot Y(N_2)$  (Point *B*). If entrepre-

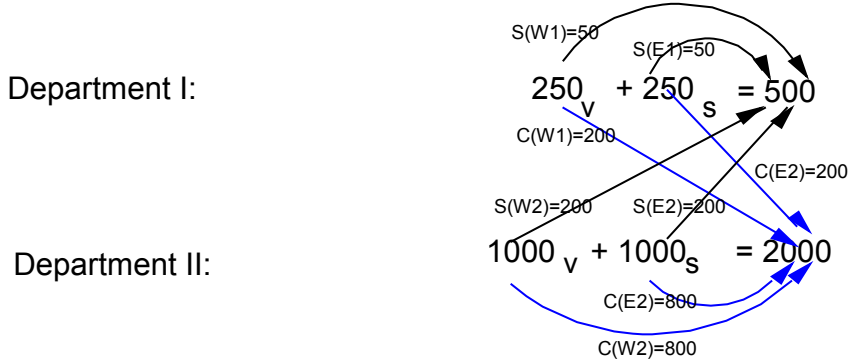
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<sup>11</sup> Note that, in the absence of productivity growth and cuts in the wage unit, entrepreneurs will only expand employment in reaction to a higher (expected) aggregate demand if they expect that the higher marginal costs can be covered by a higher enforceable demand price per unit of output. We can conclude that the D-curve can only shift to the top if there is (also) a shift in the price component inherent in it (cf. KOENIG 1980, p. 445). Keynes seems to have believed that the higher supply price necessary to accommodate higher marginal costs would always be enforceable if the economy was to expand. He wrote: “(T)here cannot be rising output without rising prices” (KEYNES 1965, p. 33).

neurs react by cutting prices, and it is assumed here that they do so to some extent – the alternative being accumulation of inventories –, the *ex post* demand-price level  $P^d$  will fall short of  $P_2^{de}$ , and the condition for completely successful reproduction will be violated. The same result will emerge if the entrepreneurs of *Department II* overestimate *ex ante* real investment demand or the marginal propensity to consume.

It has been said that the expectation of higher demand prices shifts the D-curve to the top. A crucial issue to note at this juncture is that this can only happen if price increases are expected in both departments. To see what this implies consider Figure 4.

**Figure 4:**Marx-Keynes reproduction scheme I - Completely successful reproduction



In the reproduction scheme of Figure 4, it is assumed that the rate of surplus-value is unity in both departments. This implies that the amount of variable capital (subscript ‘v’) and of surplus-value (subscript ‘s’) are equal in each department. What the entrepreneurs (E) spend as variable capital constitutes income for the workers (W). The rest of the value added, the surplus-value, belongs to the entrepreneurs and constitutes their income. Of course, the income of the entrepreneurs has to be realized in the process of reproduction by selling their products to other entrepreneurs, and to the workers.

Keynes does not distinguish between workers and entrepreneurs as far as the disposal of their income is concerned. As ‘households’ they dispose of their income according to the ‘fundamental psychological law’ (KEYNES, 1973a, p. 96). That is why the marginal propensities to consume are assumed to be the same for workers and entrepreneurs in the scheme. The main rationale for sticking to Keynes at this point is that it keeps the formula for the equilibrium proportion of departments as simple as possible.<sup>12</sup>

<sup>12</sup> HARTWIG (2004b) relaxes this simplifying assumption, but here we will keep it. Further, we assume away the government and foreign sectors for expositional ease.



The workers of *Department I* want to consume the equivalent of 80% of their income of \$250, which gives a value of desired consumption of \$200 (indicated by the arrow annotated by  $C_{(W1)} = 200$ ). The entrepreneurs of *Department I* also want to spend \$200 on consumption whereas both the workers and the entrepreneurs of *Department II* want to consume the equivalent of \$800. All the consumption plans add up to \$2000 which happens to be the value added of *Department II*. In the same way the savings sum up to the value added of *Department I*.<sup>13</sup> At the end of the production period, the whole output has been sold for its value. The reproduction has been ‘completely successful’. The reason, of course, is that the two departments stand in equilibrium proportion: *Department I* : *Department II* as  $1 : c/(1-c) = 1 : 4$ .

A further remark is apposite. Since constant capital is absent from the scheme, it may not be concluded that we have a situation of ‘simple reproduction’ in Marx’s sense. It is not possible to tell whether the net value added of *Department I* constitutes an addition to the capital stock or not, since it is unknown how much constant capital has to be replaced. Constant capital has to be absent from the scheme (in other words: it is necessary to deal with net value added instead of the total value product of the two departments) to account for Keynes’s treatment of the transfer of the value of constant capital onto the output. National accounts distinguish between gross and net income. Keynes’s “indubitable [proposition] ... that the income derived in the aggregate by all elements in the community concerned in a productive activity has a value exactly equal to the value of the output” (KEYNES, 1973a, p. 20) is correct for gross income, but it is only (net) ‘national income’ that households have at their disposal for consumption or saving and that is relevant for the reproduction scheme (as well as for the D-curve). In other words, the value of constant capital transferred onto the output during a production period has to be paid for by the purchasers and constitutes gross income for the entrepreneurs. But the marginal propensity to save with regard to this part of gross income is unity (cf. KEYNES, 1973a pp. 57-58). So if we are applying Keynes’s ‘fundamental psychological law’ ( $0 < c < 1 \Rightarrow 0 < s < 1$ ), it is convenient to concentrate on those

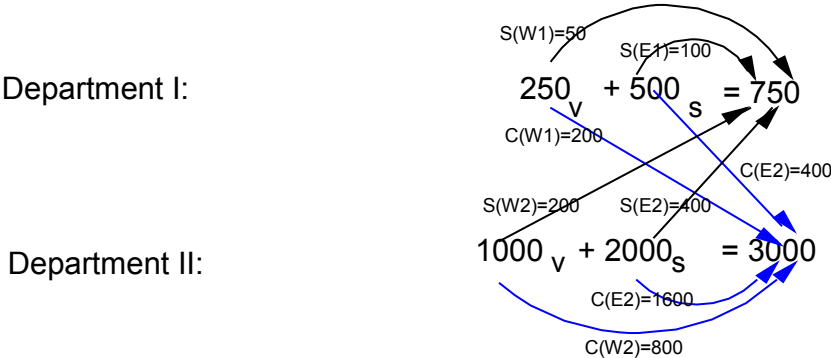
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<sup>13</sup> A remark is apposite about workers’ savings. In the reproduction scheme it looks as if their savings would constitute a direct demand for investment goods. Of course, this is not the case. The transfer of workers’ savings to the entrepreneurs is effected by what Keynes has once called the ‘financial machine’ (KEYNES, 1973d, p. 352). It is assumed here that no part of workers’ savings is hoarded. (Indeed, this is a precondition for ‘completely successful reproduction’, cf. also ROCHON, 1999, p. 35.) If workers save part of their income (and do not hoard) they are, in effect, granting loans to the entrepreneurs. Then they can participate in the distribution of the surplus-value (which is divided between profit and interest on debt). It follows that it is an oversimplification not to distinguish between the functional and personal income distribution in the scheme. On the other hand, that will not be harmful as long as the marginal propensities to consume are assumed to be identical.

parts of gross income for which this ‘law’ can possibly hold – and these constitute net income or value added (cf. also BHADURI, 1986, Chapters 1-2).<sup>14</sup>

Now, let us assume that, at the beginning of production period  $t+1$ , the entrepreneurs of *Department II* expect a demand price level that is 50% higher in both departments. Assume further that they do not expect the variable capital parts to alter so that all the increase in proceeds would constitute surplus value. (Keynes might have called such a situation ‘true inflation’, cf. KEYNES, 1973a, pp. 118-119.) The result is shown in Figure 5.

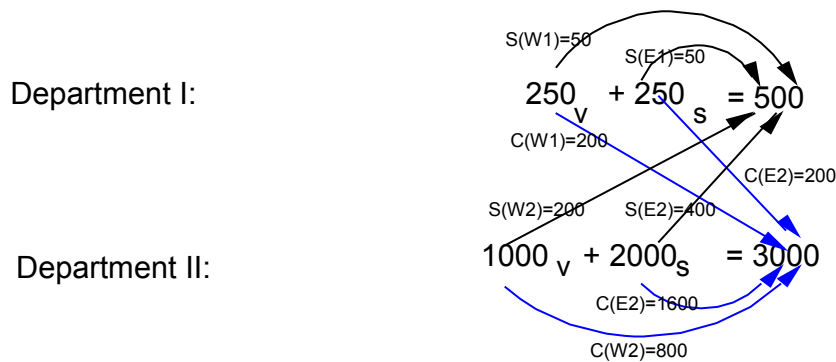
**Figure 5:**Marx-Keynes reproduction scheme II - Completely successful reproduction



Reproduction is still ‘completely successful’ because the two departments have conserved their equilibrium proportion. But this will not be the case if the entrepreneurs of *Department II* expect the demand-price increase to happen only in their own department. Then, as is shown in Figure 6, the value added in *Department II* would be \$3000, but the aggregate consumption demand would only add up to  $\$800_{C(W2)} + \$1600_{C(E2)} + \$200_{C(W1)} + \$200_{C(E1)} = \$2800$ . The output of *Department II* could only be sold for \$200 below value. On the other hand, the demand for the output of *Department I* would be higher than the value added there.

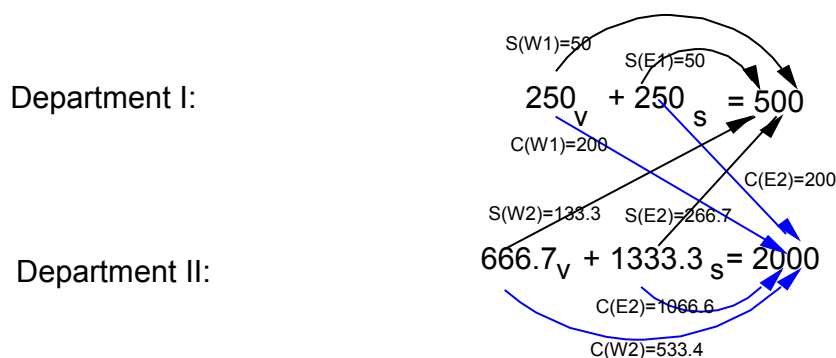
<sup>14</sup> The value of constant capital transferred onto the output (i.e. total depreciation) equals (in terms of Chapter 6 of the *General Theory*) the sum of user cost and supplementary cost minus what modern production accounts (not Keynes) call ‘intermediate transactions’. MILLAR (1972) has translated Keynes’s Chapter 6-definitions into the terms of modern national accounts.

**Figure 6:**Marx-Keynes reproduction scheme III -  
Incomplete reproduction



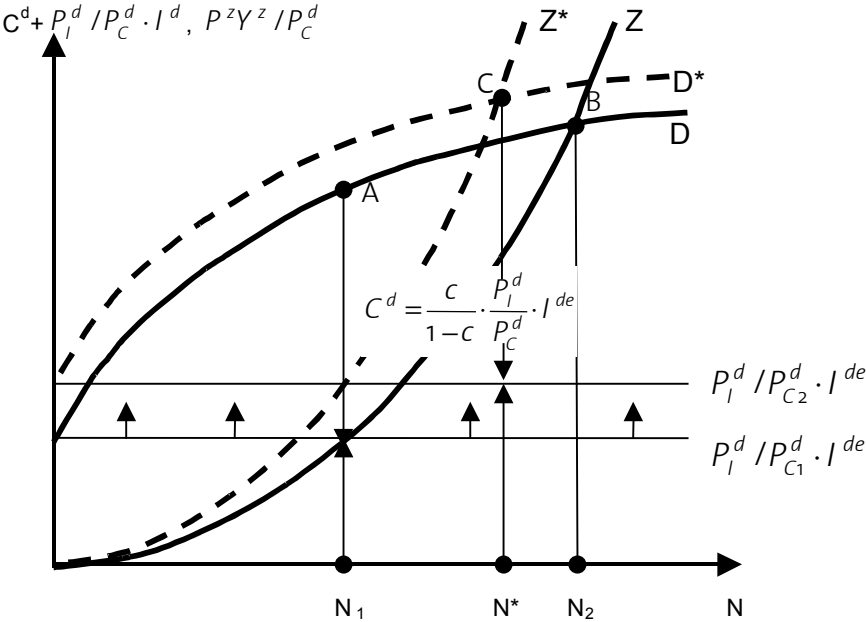
If the entrepreneurs of *Department II* are reluctant to cut prices, they could also accumulate inventories; but regardless of what the entrepreneurs do, the condition for completely successful reproduction is violated: the output cannot be sold for its value, and not all plans are being realized. Let us assume that the net value added of *Department II* (of \$3000) incorporates 1000 units of the aggregate consumption good, and that the expected unit demand price is \$3. If the entrepreneurs of the consumption goods department do not expect the price level of *Department I* to ‘advance *pari passu*’ they know *ex ante* that the rate of surplus-value ( $s/v$ ) will fall short of the projected value of 2 if they produced 1000 units at cost of \$1000. Instead, the rate would only be 1.8 (=\$1800/\$1000). If they insist on a rate of 2, they will produce less than in the production period before the expected demand price rose. By cutting back production to 666.7 units (this implies a cut-back in employment which may even be greater than 33.3% if we assume a decreasing marginal productivity of labor), and by selling each unit for the expected enforceable price of \$3, the entrepreneurs of *Department II* can realize a rate of surplus-value of 2. They can realize it because, by doing so, they will restore the equilibrium proportion of departments (cf. Figure 7).

**Figure 7:**Marx-Keynes reproduction scheme IV -  
Completely successful reproduction



The last task is to bring the aggregate supply curve back into the picture. From what has been said so far it follows that entrepreneurs (especially of *Department II*) try to estimate which level of aggregate demand will imply macroeconomic balance so that they can sell their output “without more disturbance to the price of consumption goods than consequential, in conditions of decreasing returns, on an increase in the quantity which is produced” (KEYNES, 1973a, p. 122). Let that estimation have led them to Point A in Figure 8. If the aggregate supply function is given by Z, then the Point of Effective Demand will be Point B (the point of intersection of D and Z). If the entrepreneurs supplied only  $Y(N_1)$ , the aggregate demand price (A) would be higher than the aggregate supply price, and they would have (in Keynes’s words) “an incentive ... to increase employment beyond N” (KEYNES, 1973a, p. 25). However, when they do supply  $Y(N_2)$  at Point B, they will find out during the production period that they cannot sell the whole output because the departments are out of balance. At the price level  $P^d = P^z(N_2)$  the consumption goods department is ‘too big’: as in the reproduction scheme of Figure 6 it cannot sell its entire output at the price level expected to rule in the market.

**Figure 8:** The point of effective demand and macroeconomic balance



At first sight, the entrepreneurs of *Department II* seem to have two alternatives. Supplying at Point A means that they can expect to sell the whole output at the expected demand price level but to forego additional profits. Supplying at Point B means that the expected unit demand price exactly equals the marginal cost of the last-produced unit, but it also means that part of the output cannot be sold – and entrepreneurs know that *ex ante*. This gives the hint how to

reconcile the two alternatives. If entrepreneurs of *Department II* know *ex ante* that part of the output cannot be sold if they are supplying at the Point of Effective Demand, then they must expect price cuts in the consumption goods department during the period, while investment goods prices can be expected to stay the same or even rise. (We will assume that they stay the same, but the results in no way hinge on this assumption.) If the prices for consumption goods must be expected to rise, this means that the demand price level inherent in *D* is not yet the correct one. Expectations concerning the enforceable price level will have to be revised downward; and this ‘profit deflation’ in *Department II* will move the *D*-curve upward. We now have the opposite case to the one discussed in the context of Figures 6 and 7: a downward revision in the expected demand price level only in *Department II* disturbs the equilibrium proportion of departments. *Department II* becomes ‘too small’ in terms of its value added in relation to *Department I*, and it has to expand its output to grow into the correct proportion. What has been lost in nominal terms (reduction in the expected unit demand price) has to be gained in real terms to achieve what entrepreneurs want: completely successful reproduction.

What is implied here is that the proportion of departments corresponding to Point *A* is not yet the equilibrium proportion because it has been calculated for the wrong relation of demand price levels of *Departments I* and *II*. To show that, in Figure 8 aggregate nominal consumption demand, aggregate nominal investment demand, and the aggregate supply price are deflated by the demand price level of consumption goods. (Since this price level is not dependent on employment, the deflation does not change the shapes of *D* and *Z*.) If expectations concerning the demand price level in the consumption goods department have to be revised downward – as in our example – the horizontal line:  $P_I^d / P_C^d \cdot I^{de}$  will move upward. This reflects the fact that, given a certain nominal investment demand and marginal propensity to consume, in macroeconomic balance the consumption goods department will be the bigger in real terms the lower the price level of consumption goods is, while, of course, the equilibrium proportion of departments in nominal or value terms does not change with changes in the price level of *Department II*. The *D*-curve shifts upwards to *D\**. A certain point on the new *D*-curve (let that be Point *C*) implies macroeconomic balance. This means that the entrepreneurs of the consumption goods department now expect a real demand of

$$C^{de} = \frac{c}{1-c} \cdot \frac{P_I^d}{P_C^d} \cdot I^{de} \quad (\text{which is bigger than before}).$$

The *Z*-curve, also being deflated by  $P_C^d$ ,

shifts to the top, too. The new Point of Effective Demand does not necessarily coincide with Point *C*, but if it doesn’t, a new round of revision in demand price expectations in *Department*

*H* sets in, resulting in shifts of *D* and *Z*, until the Point of Effective Demand implies macroeconomic balance. Only in this case, demand price expectations are consistent. In all other situations, such as Point *A* in Figure 8, they have to change before entrepreneurs can be satisfied with their thought experiment aiming to estimate *ex ante* which output and employment level will realize their maximum profit. There is only one (expected) aggregate demand price level – brought about in our example by variations in consumer prices with prices for investment goods unchanged – for which the Point of Effective Demand coincides with macroeconomic balance. Hence, the Principle of Effective Demand produces a precise theoretical forecast for the demand price level.

### 3 Concluding Remarks

The ‘received view’ of what Keynes’s Principle of Effective Demand is about can be described as follows: quantity reactions of real income equilibrate supply and demand. But this interpretation contradicts Keynes’s own presentation of the Principle as was shown in this paper. Keynes presents the Principle of Effective Demand as the aggregate result of a thought experiment of each entrepreneur aiming to estimate *ex ante* which output and employment level will realize her maximum profit. The Post Keynesian *D/Z*-model is designed to capture this idea, even though not all Post Keynesians would admit it.

Apart from the distinction between the supply price level and the demand price level, it is the adherence to the *D/Z*-model that distinguishes this paper from the – in other respects similar – work of NELL (1978, 1998) and LAVOIE (2003). Nell rejects diminishing marginal returns for the analysis of mass production economies and thus discards the *D/Z*-diagram which rests on that assumption (cf. NELL, 1998, pp. 618–621). Also, he denies the significance of ‘psychological propensities of households’ in the context of multiplier analysis (cf. NELL, 1998, p. 563) and thus prefers the ‘classical savings hypothesis’. But his perception of the multiplier (cf. NELL, 1998, pp. 560–564) is very similar to the one presented in section 2 of this paper.

LAVOIE (2003, p. 167) believes that the *D/Z*-framework “should be abandoned and that more efficient and heuristic means should be adopted”. He reshapes his analysis within the space of employment and real wages rather than sticking to the employment and proceeds space. But in accordance with the present paper, Lavoie respects Keynes’s ‘neoclassical vestiges’, e.g. the production function with decreasing returns and the assumption of profit maximization; and he thus reaches similar conclusions. A ‘notional labor demand curve’ is contrasted with an ‘expected labor demand curve’ dependent on entrepreneurs’ expectations

concerning aggregate demand. These two curves essentially have the same properties as the  $Z$ - and  $D$ -curves presented here. According to LAVOIE's (2003, p. 172) re-interpretation of Keynes, disequilibria on the goods market are cured by price reactions in the short term. Again, this conforms to the analysis put forward here.

The present paper has linked the  $D/Z$ -model to a Marxian reproduction scheme. This link-up has permitted the conclusion that the point of effective demand implies an *ex ante* expectation of macroeconomic balance between the two departments of the economy – balance being defined as the proportionality: *Department I* : *Department II* as  $1 : c/(1-c)$ . Since imbalance leads to (demand-) price reactions that can be anticipated by entrepreneurs, we should stop seeing the Principle of Effective Demand as a theory of quantity reactions. Instead, we should interpret it as a theory to explain not only the level of real output and its split into consumption goods and investment goods, but also the aggregate price level.

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