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**Working Paper**

## A behavioral portfolio analysis of retirement portfolios

Thünen-Series of Applied Economic Theory - Working Paper, No. 104

**Provided in Cooperation with:**

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*Suggested Citation:* Singer, Nico (2009) : A behavioral portfolio analysis of retirement portfolios, Thünen-Series of Applied Economic Theory - Working Paper, No. 104, Universität Rostock, Institut für Volkswirtschaftslehre, Rostock

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**Thünen-Series of Applied Economic Theory**  
**Thünen-Reihe Angewandter Volkswirtschaftstheorie**

Working Paper No. 104

**A Behavioral Portfolio Analysis of Retirement Portfolios**

by

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Wirtschafts- und Sozialwissenschaftliche Fakultät  
Institut für Volkswirtschaftslehre

2009

# A Behavioral Portfolio Analysis of Retirement Portfolios\*

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February 23, 2009

## Abstract

With regard to retirement savings individual investors tend to hold large positions of their wealth in riskless assets, although equity products offer higher returns. In this article we study a behavioral portfolio model which captures this phenomenon by considering two behavioral aspects: fear and hope. In detail, we extend Shefrin's and Statman's stochastic behavioral portfolio model [SS00] and provide an equivalent deterministic model, which can be solved numerically. This allows us to apply this behavioral portfolio model even to a large amount of return data of retirement assets. When we assume fear, we find an optimal retirement portfolio with large positions in riskless assets. In this case, the proportion invested in equity is very small up to zero, while it is large when we assume hope. In short, a fear-driven behavior results in a smaller expected portfolio return and a shifting of wealth from risky to riskless assets; a hope-driven behavior results in a larger expected portfolio return and a shifting of wealth from riskless to risky assets.

*Keywords:* behavioral portfolio choice, decision making under risk, retirement portfolios

*JEL Classification:* G11

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\*I would like to thank Alexander Conrad for helpful interaction and Benjamin Strohner for analyzing results.

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# 1 Introduction

Because of its practical significance and its conceptual simplicity portfolio models based on risk and return have been common methods to evaluate individual and institutional portfolios. But, in reality investment decisions were not only made by risk and return, but also by behavioral aspects like personality, risk attitude, gender and even emotions. These and other behavioral aspects might be reasons for some asset allocation phenomena such as the observation that long-term oriented individual investors hold large positions in riskless assets (e.g. savings accounts, property used by owners, and endowment insurance), even though equity products offer higher returns (see for example DeBondt [DeB98] and Agell and Edin [AE90]). Classical portfolio theory such as the mean-variance model [Mar52] can not explain this phenomenon, because behavioral aspects were neglected and portfolios were only determined with regard to risk and return. Behavioral portfolio theory includes, in addition to risk and return, behavioral arguments and may explain this phenomenon. Shefrin and Statman [SS00] were the first who suggested a behavioral portfolio model by allowing for emotions such as fear and hope. Their model is a result of combining *SP/A theory* [Lop87, LO99], an alternative approach of decision making under risk, and Telser's safety-first rule [Tel55]. As they studied their model only from a theoretical point of view, there is little information available on the applicability of this model to real return data. Furthermore, it is an open question whether this model can explain the phenomenon mentioned.

The aim of the present article is twofold. First, we extend Shefrin's and Statman's model such that it can be applied even to a large amount of data. In detail, we reformulate the stochastic model, restrict portfolio weights to be non-negative, i.e. short selling is prohibited and transform it to a deterministic model which can be solved numerically. Second, we apply return data of retirement assets and calculate efficient portfolios for both fear-driven and hope-driven investors. Based on our calculation, we provide an explanation of the phenomenon mentioned, namely individuals are driven by fear when constructing retirement portfolios.

The sequel of this article is organized as follows: In section two we present the underlying model, a mixed-binary linear optimization program, which combines Lopes' SP/A theory and Telser's safety-first rule. In section three we report our return data on which our calculations are based on. Section four contains the main results and section five offers our conclusions.

## 2 Model Description

This section is divided into three parts: First, we review the basics of SP/A theory [Lop87, LO99], second we provide a modified version of Shefrin's and Statman's stochastic behavioral portfolio model [SS00] and, third, we suggest an equivalent deterministic model.

We consider an investment horizon of  $n$  assets, each asset generates a random return  $R_j$ ,  $j = 1, \dots, n$ . As SP/A theory is a discrete decision model under risk, we assume

## 2 Model Description

discrete distributed asset returns. In the sequel of this article we always refer to realization tableau 1, where  $\hat{R}_{i,j}$  is the realized return of asset  $j$  at state  $i$  and  $\sum_{i=1}^m p_i = 1$ .

	Asset 1	Asset 2	...	Asset $n$
$p_1$	$\hat{R}_{1,1}$	$\hat{R}_{1,2}$	$\cdots$	$\hat{R}_{1,n}$
$p_2$	$\hat{R}_{2,1}$	$\hat{R}_{2,2}$	$\cdots$	$\hat{R}_{2,n}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$p_m$	$\hat{R}_{m,1}$	$\hat{R}_{m,2}$	$\cdots$	$\hat{R}_{m,n}$

Table 1: Realization tableau

### 2.1 SP/A Theory

The SP/A theory, established by Lopes [Lop87, LO99], is an alternative approach of decision making under risk, including two criteria: The *SP-criterion*, where  $S$  stands for security and  $P$  for potential, describes two contrasting behavioral elements: On the one hand individuals are looking for a general desire for security and, on the other hand, they want to participate on some upward potential. In this portfolio context the terms security and potential are understood as financial certainty and return seeking, respectively. This criterion emerged from the Friedman-Savage-paradox [FS48], that is the observation that individuals who buy insurance policies often buy lottery tickets as well. According to Lopes, the *SP-criterion* is equivalent to two emotions that drive financial decisions: fear and hope<sup>1</sup>. The *A-criterion*, where  $A$  stands for aspiration, defines a target value or, in terms of portfolio theory, a return, which shall be achieved. In the upcoming subsection 2.2 we will describe in detail how the aspiration level  $A$  is included in the model.

In the sequel of this subsection, we demonstrate how the *SP-criterion* can be modeled mathematically: Assume that the realizations of the random asset return  $R_j$  are ranked such that  $\hat{R}_{1,j} \leq \dots \leq \hat{R}_{m,j}$ . Given a utility function  $u$  we calculate the expected utility of  $R_j$  according to *rank-dependent expected utility*<sup>2</sup> (see Quiggin [Qui82, Qui93]) as

$$\mathbb{E}(u(R_j)) = \sum_{i=1}^m u(\hat{R}_{i,j}) w_i(\mathbf{p}), \quad (1)$$

---

<sup>1</sup>In the literature the terms pessimism and optimism are sometimes used instead.

<sup>2</sup>Rank-dependent expected utility theory is also known as anticipated utility theory or generalized expected utility theory and is the natural extension of von Neumann and Morgenstern expected utility theory.

where

$$w_i(\mathbf{p}) = h\left(\sum_{k=1}^i p_k\right) - h\left(\sum_{k=1}^{i-1} p_k\right)$$

and  $h : [0, 1] \rightarrow [0, 1]$  is a probability transformation function. Remark that equation (1) collapses to *von Neumann and Morgenstern expected utility* when  $h$  is assumed to be the identity function. Let  $D_{i,j} = P(R_j \geq \hat{R}_{i,j})$  be the decumulative distribution function and assuming a linear utility function  $u$ , equation (1) can equivalently be written as<sup>3</sup>

$$\mathbb{E}(R_j) = \sum_{i=1}^m h(D_{i,j})(\hat{R}_{i,j} - \hat{R}_{i-1,j}), \quad (2)$$

with  $\hat{R}_{0,j} = 0 \forall j$ . (For a formal prove of equation (2) see the appendix.) Equation (2) is the functional form of SP/A theory where fear and hope can be modeled by transforming decumulative probabilities. We call the case where neither fear nor hope is assumed risk neutral behavior. If we assume fear-driven behavior, individuals overweight probabilities attached to unfavorable outcomes, which can be expressed by the transformation function<sup>4</sup>

$$h_S(D) = D^{1+q_S}, \quad q_S > 0. \quad (3)$$

Hope-driven behavior, which operates as an overweighting of probabilities attached to favorable outcomes, can be expressed by

$$h_P(D) = 1 - (1 - D)^{1+q_P}, \quad q_P > 0. \quad (4)$$

Because fear and hope reside within individuals simultaneously, the final shape of the transformation function is a convex combination of  $h_S$  and  $h_P$ :

$$h(D) = \lambda h_S(D) + (1 - \lambda) h_P(D), \quad \lambda \in [0, 1]. \quad (5)$$

Remark that an individual is (a) risk averse if the transformation function is everywhere convex, i.e.  $\lambda = 1$ , and (b) risk seeking if the transformation function is everywhere concave, i.e.  $\lambda = 0$ .

## 2.2 A Stochastic Behavioral Portfolio Model

Shefrin and Statman provided a single mental account version of their behavioral portfolio theory which is based on a safety-first rule, established by Telser [Tel55]. The safety-first rule allows the investor to define a target return which shall be achieved by a given probability. In our portfolio context, the target return can exactly be viewed as the aspiration level  $A$  from SP/A theory. The aspiration level serves as reference point as portfolio returns which do not exceed  $A$  are neglected and those exceeding  $A$  are accepted.

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<sup>3</sup>Rank-dependent expected utility theory with linear utility function  $u$  is exactly the case of Yaari's *dual theory of choice under risk* [Yaa87]

<sup>4</sup>For sake of simplicity we omit subscripts of  $D$  at this point.

In contrast to Shefrin and Statman, who developed their model by defining initial wealth and asset prices, we normalize our model by a portfolio vector  $\mathbf{x} \in [0, 1]^n$ ,  $x_j$  is the proportion invested in asset  $j$ . Given the vector  $\mathbf{R} = (R_1, \dots, R_n)^T$  of random asset returns, we define the random portfolio return as the product  $Z := \mathbf{x}^T \mathbf{R}$ . Also given the expected returns  $\mathbb{E}(R_j)$  of each asset, calculated by equation (2), and a chosen target return  $A$ , we can formulate the stochastic linear optimization problem as

**Problem 1.**

$$\begin{aligned} \max \mathbb{E}(Z) = \max \sum_{j=1}^n x_j \mathbb{E}(R_j) \quad & s.t. \\ P(Z < A) & \leq \alpha, & (6) \\ \mathbf{1}^T \mathbf{x} & \leq 1, & (7) \\ x_j & \geq 0 \quad j = 1, \dots, n, & (8) \end{aligned}$$

where the objective function is the expected portfolio return that should be maximized. With the safety-first constraint (6) we allow the portfolio return  $Z$  to fall below the target return  $A$  by at most a small probability  $\alpha$ . Inequality (7) arises from our portfolio definition<sup>5</sup> and, with (8), short selling is prohibited.

### 2.3 An Equivalent Deterministic Behavioral Portfolio Model

As constraint (6) in problem 1 is stochastic, it is cumbersome for practical applications. For this reason we need to transform problem 1 to a deterministic optimization problem, which allows us to apply it even to a large amount of data. Using methods of stochastic linear programming (for an overview see Kall [KM05]), there are several ways to do this, e.g. assuming the existence of the second moments of the asset return distributions, we can approximate the shortfall probability  $P(Z < A)$  by employing Tchebyshev's inequality and, as a result, constraint (6) collapses to a quadratic inequality<sup>6</sup>.

In this paper we assume, as is also done in the SP/A theory, that the random asset returns have a finite discrete distribution according to realization tableau 1. We employ Raike's transformation method (see Raike [Rai70] and Kall [KM05]) and formulate the deterministic optimization problem as

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<sup>5</sup>Without loss of generality, constraint (7) can be written as equality constraint. Nevertheless, in many applications it is useful to attach lower bounds to portfolio weights to avoid very small portfolio weights. Applying lower bounds, an inequality constraint (7) is necessary.

<sup>6</sup>See Breuer et al. [BGS06] and Elton and Gruber [EG95] for a detailed analysis.

**Problem 2.**

$$\max \mathbb{E}(Z) \quad s.t. \quad \mathbf{x}^T \hat{\mathbf{R}}_i + M(1 - d_i) \geq A, \quad i = 1, \dots, m, \quad (9)$$

$$\sum_{i=1}^m p_i d_i \geq 1 - \alpha, \quad (10)$$

$$d_i \in \{0, 1\}, \quad i = 1, \dots, m, \quad (11)$$

$$\mathbf{1}^T \mathbf{x} \leq 1, \quad (12)$$

$$x_j \geq 0 \quad j = 1, \dots, n, \quad (13)$$

where  $\hat{\mathbf{R}}_i$  is the  $i$ th row of realization tableau 1, i.e. the  $i$ th realization of the random return vector  $\mathbf{R}$ , and,  $M$  is a real number that is chosen large enough<sup>7</sup>. Note that the decision variables are the portfolio weights  $x_j$ ,  $j = 1, \dots, n$  and the binary variables  $d_i$ ,  $i = 1, \dots, m$ , where the  $d_i$ 's serve as auxiliary variables.

The stochastic constraint (6) in problem 1 is equivalently substituted by the deterministic constraints (9)-(11) in problem 2. The deterministic behavioral portfolio problem 2 is a mixed-binary linear optimization problem, which can be solved numerically by using the SCIP-solver (**S**olving **C**onstraint **I**nteger **P**rograms) developed by Achterberg [Ach07], currently the fastest non-commercial mixed-integer programming solver.

### 3 Data

In this section we provide return data of several retirement assets. We considered assets which were most frequently chosen by individuals according to survey data<sup>8</sup> and we additionally considered assets which were recommended by the investment community, but overlooked by individuals. Table 2 summarizes our asset basket.

For all additional products, displayed in the second column of table 2, we can provide exact return data, but, for those preferred by individuals, we can not. Thus, we approximated these returns as follows:<sup>9</sup>

*Savings deposits:* The interest rates for savings deposits are composed of deposit rates of banks in Germany with minimum rates of return and with agreed notice of three months. The average rates are calculated as unweighted arithmetic means from the interest rates reported to be within the spread. The spread is ascertained by eliminating

<sup>7</sup>In the appendix we provide a precise declaration of  $M$ .

<sup>8</sup>We analyzed data from the Debt and Credit survey 2004, for which more than 10,000 individual investors in Germany were asked about their asset allocation with regard to retirement savings, among other areas surveyed.

<sup>9</sup>We employ Westerheide's [Wes05] approximation method. Return data was extracted from the Bundesbank, the Statistisches Bundesamt (Federal Office of Statistics) and the Deutsche Boerse (German Bourse).



Products preferred by individuals	Additional products
<ul style="list-style-type: none"> <li>• savings deposits</li> <li>• fixed-term deposits</li> <li>• savings plan with periodic payments</li> <li>• property used by owners</li> <li>• endowment insurance (EnIn)</li> <li>• building society savings contract (BSSC)</li> </ul>	<ul style="list-style-type: none"> <li>• equity products: DAX Performance Index (DAXP); Siemens stock (Siem); Munich Re stock (MuRe); equity fund, Europe (EFEu); equity fund, Germany (EFGe)</li> <li>• pension funds: REX Performance Index (REXP)</li> <li>• real estate fund, Germany (REFG)</li> <li>• gold</li> </ul>

Table 2: Asset basket

the reports in the top 5% and the bottom 5% of the interest rate range. Including rates for savings deposits, we consider the differences between the Bundesbank’s interest rate statistics and the new European Central Bank’s statistics, with the latter starting at January 2003.

*Other investments with banks:* We consider savings bonds with fixed maturity of four years, overnight money, savings with/without a contract period for varying investment volumes, and fixed-term deposits. We obtain a representative time series by calculating the arithmetic mean of the mentioned time series.

*Endowment insurance:* To approximate the return of endowment insurances, we observe investment products that insurance companies are likely to purchase for their deposits, which are loans for public households, Federal securities, fixed-income securities, mortgages and public “Pfandbriefe.” To achieve one time series, we calculate the arithmetic mean of the products mentioned.

*Building society savings contract:* We approximated the return of building society savings contracts by yields on debt securities outstanding issued by residents.

*Property used by owners:* We approximated the return of property used by owners using the index for housing calculated by the Bundesbank, based on data provided by BulwienGesa AG.

All together, our raw data contains continuously compounded monthly returns of 13 assets from July 1987 to May 2008, which implies a  $252 \times 13$  realization tableau. A choice of  $m = 252$  in Problem 2 implies 252 inequalities in constraint (9), which associates a high complexity concerning computer time. To solve optimization problem

2 in an appropriate computer time, we reduced the number of states from 252 to 120, so that the reduced return history contains monthly returns of ten years, from January 1998 to December 2007. We also found that savings deposits, property used by owners and other investments with banks are dominated by the endowment insurance, i.e. in each state the return on savings account, property used by owners and other investments with banks, respectively, is smaller than the return of the endowment insurance. Hence, we omit the return histories of these assets. Having employed these two reductions we obtained a  $120 \times 10$  realization tableau.

## 4 Results

The  $120 \times 10$  return matrix and the following parameters are included in the optimization problem:

$A$  : aspiration level, measures the monthly return in percent,

$\alpha$  : shortfall probability, step size=0.05,

$q_S$  : measures the strength of fear,  $q_S > 0$ ,

$q_P$  : measures the strength of hope,  $q_P > 0$ , and

$\lambda$  : determines the strength of fear relative to hope.

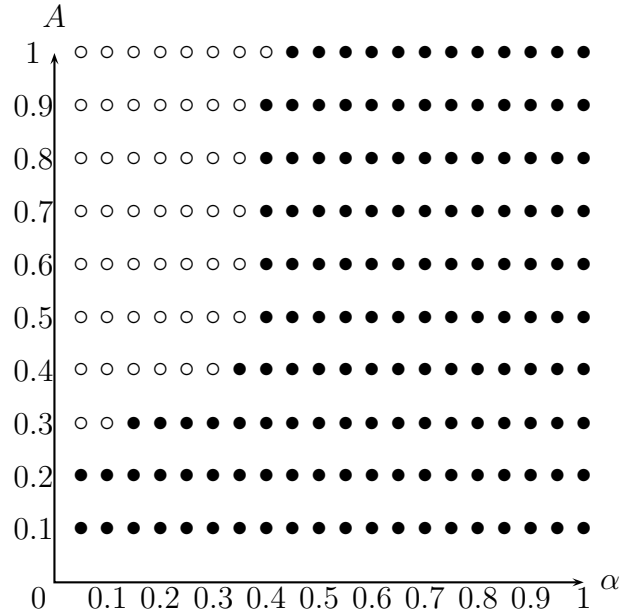
Note, in the special case where  $q_S = q_P = 0$  the model (problem 1 and 2) collapses to Telser's safety-first model, where the expected returns of each asset is determined by the arithmetic mean.

Because hope and/or fear does only affect the objective function in problem 1 and problem 2, respectively, the feasible domain does not change when assuming hope and/or fear or nothing of both. Hence, the existence of a feasible solution of problem 1 and problem 2, respectively, depends solely on the choice of  $A$  and  $\alpha$ . A small  $A$ , e.g.  $A = 0.1$ , can be achieved even by small  $\alpha$ , whereas a large  $A$ , e.g.  $A = 0.8$ , can be achieved by only a large  $\alpha$ . Figure 1 contains the feasible  $(A, \alpha)$ -combinations. Note that the feasible domain of problem 1 and 2 is infinite rather than finite as displayed in figure 1.

We use the concept of the *SP/A-efficient frontier*<sup>10</sup> for presenting our main findings. The SP/A-efficient frontier displays a criterion used to evaluate alternative risky assets, plotted in  $(\mathbb{E}(Z), \alpha)$ -space. Shefrin [She05] defines the SP/A-efficient frontier as monotone non-decreasing, because investors prefer higher  $\mathbb{E}(Z)$ , but lower  $\alpha$ . But, this definition contains some imprecision, because constant segments are allowed (see figure 2). This implies that all portfolios on a constant segment have the same expected return ( $\mathbb{E}(Z)$ ) but different risk levels ( $\alpha$ ). Thus, the furthest left portfolio on a constant segment dominates all others with the same expected return, because it has the lowest risk level. In short, this definition does not exclude all inefficient portfolios.

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<sup>10</sup>Shefrin [She05] first introduced the term SP/A-efficient frontier to refer to SP/A theory.

Figure 1: Feasible  $(A, \alpha)$ -combinations ( $\bullet$ )

Nevertheless, this concept is a powerful tool to demonstrate efficient behavioral portfolios. Figure 2 displays SP/A-efficient frontiers for different aspiration levels, where risk neutral behavior is assumed, i.e.  $q_S = q_P = 0$ . Note that for  $A = 0.3$ , there exist solutions only for  $\alpha \geq 0.15$  and for  $A = 0.5$  only for  $\alpha \geq 0.4$ .

#### 4.1 Fear-driven Portfolios

Choosing  $q_S > 0$  and  $\lambda = 1$  we assume fear-driven behavior, that is, the investor is more security-oriented. Figure 3 shows the difference between the risk neutral and the fear-driven investor with different aspiration levels. We observed that a fear-driven investor's expected portfolio return is always smaller than the risk neutral investor's expected portfolio return.

Table 3 and 4 present the corresponding portfolio weights for  $A = 0.2$  and  $A = 0.3$ , respectively. For small  $\alpha$  almost the entire wealth is invested in the endowment insurance. However, there is a difference between the risk neutral and the fear-driven portfolio. For example, for  $A = 0.2$  and  $\alpha = 0.15$ , the risk neutral investor holds about 80% in the endowment insurance and the rest in risky assets, while the fear-driven investor holds more than 90% in the endowment insurance. An increasing  $\alpha$  is associated with an increasing proportion invested in risky assets. For example, for  $A = 0.2$  and  $\alpha = 0.3$ , the risk neutral investor holds his entire wealth in risky assets, while the fear-driven investor holds 63% in the endowment insurance and only 37% is invested in risky assets. All our calculations exhibit a shift of wealth from risky to

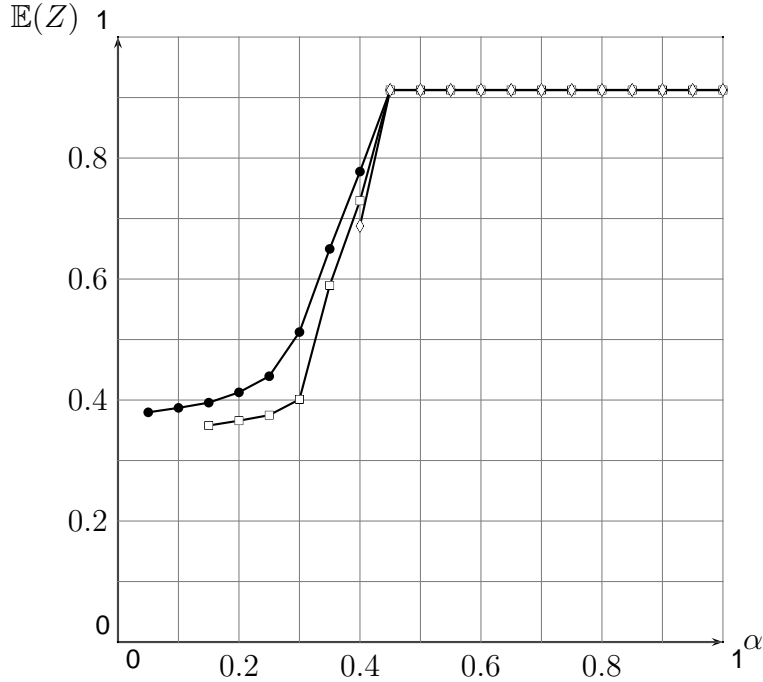


Figure 2: SP/A-efficient frontiers for  $A = 0.1$  ( $\bullet$ ),  $A = 0.3$  ( $\square$ ) and  $A = 0.5$  ( $\diamond$ ).

riskless assets, while switching from risk neutral to fear-driven behavior. In short, a fear-driven behavior is accompanied by smaller expected portfolio returns and larger proportions invested in riskless assets.

## 4.2 Hope-driven Portfolios

Choosing  $q_P > 0$  and  $\lambda = 0$  we assume a hope-driven behavior, that is, the investor is more potential-oriented. Figure 4 shows SP/A-efficient frontiers for risk neutral and hope-driven investors with different aspiration levels. In contrast to fear-driven portfolios, we observed an opposite result: The hope-driven investor's expected portfolio return is always larger than the risk neutral investor's expected portfolio return.

Table 5 and 6 present the corresponding portfolio weights for  $A = 0.2$  and  $A = 0.3$ , respectively. Concerning portfolio weights, we observed a portfolio switching from riskless to risky assets.

## 5 Discussion

The first aim of this paper was to extend Shefrin's and Statman's [SS00] stochastic behavioral portfolio model to a deterministic behavioral portfolio model. For this purpose we modified the original model by introducing a portfolio vector instead of

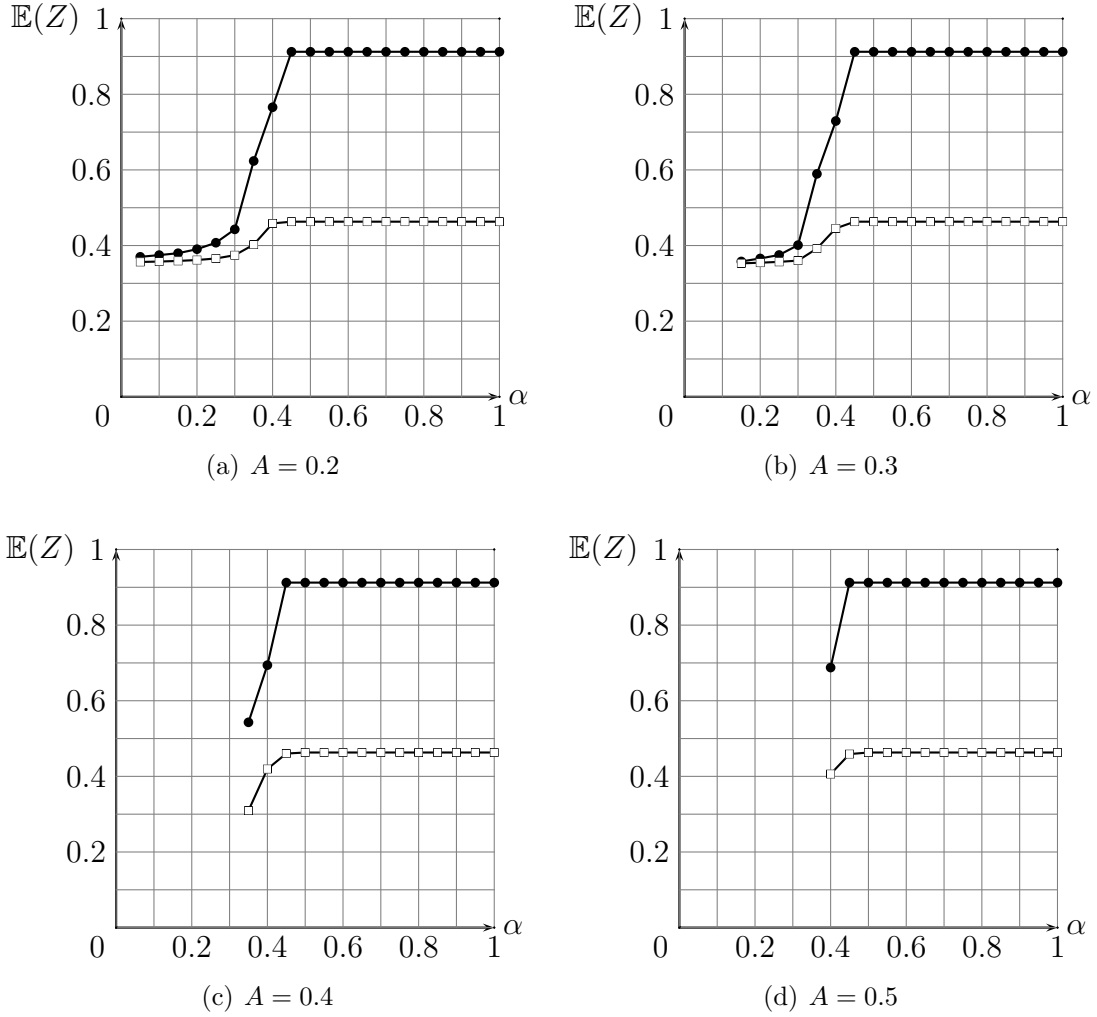


Figure 3: SP/A-efficient frontiers for risk neutral ( $\bullet$ ),  $q_S = q_P = 0$ , and fear-driven ( $\square$ ) behavior,  $q_S = 0.05$ ;  $\lambda = 1$ . Note that in some cases there exist no solution for small  $\alpha$ .

5 Discussion

$\alpha$	DAXP	REXP	Siem	MuRe	UBSD	Gold	EnIn
0.3	0.026	0.751	0.015	0.006	0	0.202	0
	0	0.141	0.038	0	0.009	0.183	0.629
0.25	0.033	0.113	0.019	0.001	0	0.128	0.706
	0.022	0.032	0.02	0	0	0.126	0.8
0.2	0.039	0.191	0.007	0.001	0.002	0.072	0.688
	0	0.012	0.003	0	0	0.085	0.9
0.15	0.026	0.106	0.002	0	0.017	0.052	0.797
	0	0.022	0	0	0	0.066	0.912
0.1	0.001	0.105	0.009	0	0.001	0.044	0.841
	0	0.099	0.009	0	0	0.044	0.849
0.05	0.001	0.056	0.008	0	0	0.035	0.9
	0	0.05	0.008	0	0	0.035	0.907

Table 3: Portfolio weights for  $A = 0.2$ , for every  $\alpha$  the first row displays the risk neutral and the second row displays the fear-driven behavior.

$\alpha$	DAXP	REXP	Siem	MuRe	UBSD	Gold	EnIn
0.35	0.327	0.081	0.195	0.046	0	0.351	0
	0.243	0.2	0.014	0	0	0.543	0
0.3	0.021	0.467	0.014	0	0	0.072	0.425
	0.016	0.122	0.014	0	0	0.075	0.773
0.25	0.016	0.01	0.005	0	0.004	0.055	0.911
	0.016	0.01	0.005	0	0.001	0.053	0.914
0.2	0.004	0.115	0.003	0	0	0.019	0.859
	0	0.035	0.003	0	0	0.021	0.94
0.15	0.003	0.03	0	0	0.002	0.005	0.959
	0.002	0.005	0	0	0	0.01	0.983

Table 4: Portfolio weights for  $A = 0.3$ , for every  $\alpha$  the first row displays the risk neutral and the second row displays the fear-driven behavior.

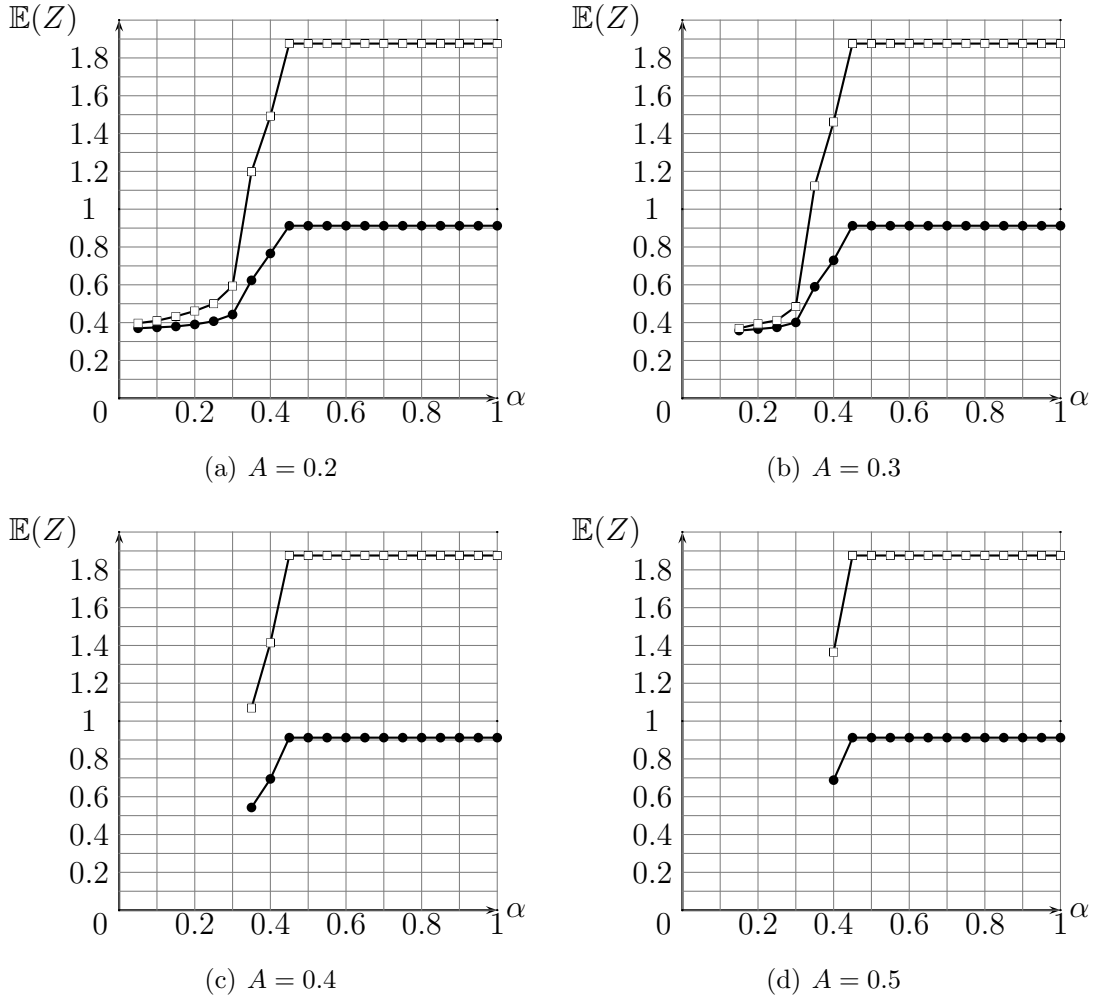


Figure 4: SP/A-efficient frontiers for risk-neutral ( $\bullet$ ),  $q_S = q_P = 0$ , and hope-driven ( $\square$ ) behavior,  $q_P = 0.1$ ;  $\lambda = 1$ ,  $A = 0.2, \dots, 0.5$ . Note that in some cases there exist no solution for small  $\alpha$ .

5 Discussion

$\alpha$	DAXP	REXP	Siem	MuRe	UBSD	Gold	EnIn
0.3	0.026	0.751	0.015	0.006	0	0.202	0
	0.012	0.783	0.026	0	0.049	0.13	0
0.25	0.033	0.113	0.019	0.001	0	0.128	0.706
	0.067	0.251	0.01	0.008	0.005	0.077	0.582
0.2	0.039	0.191	0.007	0.001	0.002	0.072	0.688
	0.04	0.191	0.007	0.001	0.002	0.072	0.687
0.15	0.026	0.106	0.002	0	0.017	0.052	0.797
	0.023	0.11	0.003	0.001	0.017	0.052	0.794
0.1	0.001	0.105	0.009	0	0.001	0.044	0.841
	0.009	0.072	0.008	0.002	0	0.044	0.865
0.05	0.001	0.056	0.008	0	0	0.035	0.9
	0	0.061	0.009	0	0	0.034	0.896

Table 5: Portfolio weights for  $A = 0.2$ , for every  $\alpha$  the first row displays the risk neutral and the second row displays the hope-driven behavior.

$\alpha$	DAXP	REXP	Siem	MuRe	UBSD	Gold	EnIn	BSSC
0.4	0	0	0.406	0	0.011	0.019	0.564	0
	0.128	0	0.587	0.227	0	0	0.058	0
0.35	0.327	0.081	0.195	0.046	0	0	0.351	0
	0.327	0.081	0.195	0.046	0	0	0.351	0
0.3	0.021	0.467	0.014	0	0	0	0.072	0.425
	0.021	0.467	0.014	0	0	0	0.072	0.425
0.25	0.016	0.010	0.005	0	0	0.004	0.055	0.911
	0.001	0.201	0.009	0	0.001	0	0.031	0.756
0.2	0.004	0.115	0.003	0	0	0	0.019	0.859
	0.014	0.104	0.002	0.001	0	0.003	0.012	0.863
0.15	0.003	0.03	0	0	0	0.002	0.005	0.959
	0.003	0.03	0.001	0	0	0.002	0.005	0.959

Table 6: Portfolio weights for  $A = 0.3$ , for every  $\alpha$  the first row displays the risk neutral and the second row displays the hope-driven behavior.



initial wealth and asset prices, and, we prohibit short sales. Problem 1 is the result of our modifications. By assuming finite discrete distributed asset returns we transformed problem 1 to an equivalent deterministic portfolio model (problem 2), which can be solved numerically by mixed-binary linear programming. The second aim was to demonstrate the model's applicability to real return data, which we have obtained by analyzing individual behavior with respect to retirement savings. For this purpose we constructed an investment basket including assets that individual investors are likely to hold, and, assets that were recommended by the investment community; especially equity assets.

We achieved the following results: When we assume an investor acting under fear, the optimal expected portfolio return decreases, and, we observed a portfolio switching from risky to riskless assets. When we assume an investor acting under hope, the optimal expected portfolio return increases, and, we observed a portfolio switching from riskless to risky assets. We also stated that the role of equity for long-term investments is important, because in the hope-driven case as well as in the risk neutral case the proportion of equity is large. But, our analysis contains two basic shortcomings: First as we could not provide real return data for some riskless assets, e.g. the endowment insurance, we had to approximate those by return time series of related assets. Second, solving mixed-binary linear optimization programs, such as problem 2, is very complex with regard to computer time. This led us to reduce our data set from more than 20 years to 10 years, which might distort our results.

Nevertheless, our model can explain why individual investors choose conservative portfolios, such as observed by DeBondt [DeB98] and Agell and Edin [AE90]; they act under pessimism or, in terms of SP/A theory, under fear. This behavior is no surprise and it reflects exactly previous research on this topic, but the way how we achieved this result, namely by using a behavioral portfolio model, is new. Because of its deterministic nature, this model is well-suited for practical purposes, especially for asset management and financial risk management. With the help of this model asset managers could act on different behavioral patterns and may obtain a better understanding of their client's behavior. Furthermore, they could state how their clients behave, i.e. search for a feasible portfolio, and how they should behave, i.e. suggest a portfolio on the efficient frontier.

For further research on this topic several extensions are possible: The most obvious extensions is the generalization from discrete SP/A theory to continuous SP/A theory, which was also recognized by Shefrin and Statman [SS00]. This would allow the application to continuous distributed asset returns, which affects the behavioral portfolio model such that the probability constraint (6) collapses to the associated continuous distribution function. The equivalent deterministic portfolio model based on continuous distributed asset returns is, in general, less complex with regard to processing time than the behavioral portfolio model based on discrete distributed asset returns. Moving from the discrete model to the continuous model does not only raise generality, but it also reduces complexity. A second extension concerns the aspiration level  $A$ , which was assumed to be fixed. But, especially for long-term investments the investor's aspiration level may vary or he wants to adjust his aspiration according to a

benchmark, which can be, for instance, an index like the DAX or the S&P 500. Thus, we suggest a stochastic aspiration level, which was also stimulated by Kall [KM05].

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## A Appendix

### A.1 A Proof of Equation (2)

**Proposition 1.** *If  $\hat{R}_1 \leq \dots \leq \hat{R}_m$  are realizations of the random asset return  $R$  and  $D_i = P(R \geq \hat{R}_i)$  is the decumulative distribution function. Then the expected asset return can be expressed by*

$$\mathbb{E}(R) \stackrel{Def.}{=} \sum_{i=1}^m p_i \hat{R}_i \stackrel{Prop.}{=} \sum_{i=1}^m D_i (\hat{R}_i - \hat{R}_{i-1}),$$

where  $\hat{R}_0 = 0$ .

**Proof:** Because the return realizations are ranked from worst to best the decumulative distribution function can be written as  $D_i = \sum_{j=i}^m p_j$ . We prove the proposition by induction. The base case ( $m = 1$ ) is trivial. To prove the general case, we assume that the proposition is true for  $m$  and show that it is true for  $m + 1$ :

$$\begin{aligned} \sum_{i=1}^{m+1} D_i (\hat{R}_i - \hat{R}_{i-1}) &= \sum_{i=1}^{m+1} \sum_{j=i}^{m+1} p_j (\hat{R}_i - \hat{R}_{i-1}) \\ &= \underbrace{p_{m+1} \sum_{i=1}^{m+1} (\hat{R}_i - \hat{R}_{i-1})}_{=p_{m+1} \hat{R}_{m+1}} + \underbrace{\sum_{i=1}^m \sum_{j=i}^m p_j (\hat{R}_i - \hat{R}_{i-1})}_{=\sum_{i=1}^m p_i \hat{R}_i, \text{ by assumption}} \\ &= p_{m+1} \hat{R}_{m+1} + \sum_{i=1}^m p_i \hat{R}_i \\ &= \sum_{i=1}^{m+1} p_i \hat{R}_i \end{aligned}$$

□

## A.2 A Necessary Condition for the Existence of a Feasible Solution of Problem 2

We define the index set  $I := \{i \in \{1, \dots, m\} : \hat{R}_{i,j} < A \ \forall j = 1, \dots, n\}$  of critical events, i.e.,  $I$  includes the indices of those states, which can not achieve the aspiration level  $A$  by any portfolio vector.

**Proposition 2.** *The feasible domain of problem 2 is not empty implies*

$$\sum_{i \in I} p_i \leq \alpha. \quad (14)$$

**Proof:** We show that  $\sum_{i \in I} p_i > \alpha$  implies, that the feasible domain is the empty set. We have the following decomposition (recall that  $d_i$  is a binary variable)

$$\sum_{i=1}^m d_i p_i = \sum_{i \in I} d_i p_i + \sum_{i \notin I} d_i p_i, \quad (15)$$

where the first addend is zero, because constraint (9) of problem 2 holds only, if  $d_i = 0 \ \forall i \in I$ . Equation (15) collapses to

$$\sum_{i=1}^m d_i p_i = \sum_{i \notin I} d_i p_i \leq \sum_{i \notin I} p_i < 1 - \alpha,$$

which contradicts constraint (11) of problem 2. □

## A.3 A Precise Expression for $M$ in Problem 2

Consider again optimization problem 2. If  $d_i = 1$ ,  $M$  vanishes and we do not have to care about  $M$ . If  $d_i = 0$ , we have to choose  $M$  such that constraint (9) always holds. The worst case which may occur in one optimization step is when everything is invested in the smallest return realization, e.i.

$$\hat{R}_{\min} := \min_{\substack{i=1, \dots, m \\ j=1, \dots, n}} \hat{R}_{i,j}.$$

Thus, when we choose  $M$  as  $A - \hat{R}_{\min}$  than constraint (9) always holds. In our data set the smallest return is the September 2002 return of the Munich Re stock with  $\hat{R}_{\min} = -52.65$ . When we choose  $M = A + 52.65$  constraint (9) always holds.