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# The strategic advantage of negatively interdependent preferences in action-monotonic games

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## Abstract

Investigating the strategic advantage of negatively interdependent preferences in action monotonic games, we derive characterizing conditions both for general action monotonic games and for the subclass of action monotonic games with spillovers. Examples demonstrate the generality of our findings, in particular that the strategic advantage prevails beyond the classes of super- and submodular games. The application of two-player rent-seeking contests illustrates how our criteria simplify analyzing the strategic advantage.

Keywords: strategic advantage, interdependent preferences, spillovers, action monotonicity.

JEL-Class.-No.: C72, D72, C79.

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# 1 Introduction

Imagine a situation where individuals differing in their attitude towards others interact playing a game with symmetric material payoff. Some individuals have individualistic preferences, solely caring about their own material payoff, while others have interdependent preferences, displaying altruism or spite. The latter not only have a concern for their own payoff, but additionally about their own payoff relative to that of other individuals. Altruism translates into a positive interdependence, whereas status-seeking or spite represents a negative one. Focussing on the interaction of individualistic and negatively interdependent preferences, our central question is which of the two preference types realizes higher material payoff in equilibrium. Earning higher material payoff, then defines the *strategic advantage* of the corresponding preference type (Koçkesen et al, 2000a,b).

Obviously, the issue at stake requires two levels at which evaluations take place. First, there is the level of preferences, which evaluate alternative material outcomes of the game, thus guiding a player's course of action. And second, there is the level of material payoff, determining the material success of a certain preference type (e.g. the strategic advantage). Implicitly we associate a feedback process: Preferences govern equilibrium actions, which in turn pin down material payoff. The material success then causes the success or failure of preferences. Through this channel, the ultimate success of a preference type depends on the material success of the behavior that is induced by the preference type in the first place.

## RELEVANCE

Such a feedback process lies at the heart of many applications. We confine ourselves to two examples. First, the evolution of preferences, such as analyzed under the indirect evolutionary approach investigates the evolutionary stability of preference types (Güth & Yaari, 1992; Bester & Güth, 1998; Possajennikov, 2000; Bolle 2000; Güth & Peleg, 2001). Here, a (not explicitly modelled) process of evolutionary selection connects the survival of preferences with the material success of the induced behavior. A special issue of the "Journal of Economic Theory" complements the earlier contributions to the indirect evolutionary approach (see Samuelson 2001a,b; Ok & Vega-Redondo, Ely & Yilankaya, Sethi & Somanathan, Bisin & Verdier, all 2001). The particular relevance to identify the strategic advantage of interdependent preferences originates from the implied instability of individualistic preferences.

Second, the literature on strategic delegation examines whether a player can profit from delegating his position and which incentives he should provide the delegate with. This framework was originally proposed and developed by Vickers (1984) and Fershtman & Judd (1987) in the context of oligopoly. Later on, similar issues were raised in the context of rent-seeking

contests (see e.g. Baik & Kim, 1997; Kräkel & Sliwka, 2002; or Baik, 2003; all of these contributions restrict to the special case of a constant returns-to-scale technology). Regarding this literature, a strategic advantage of negatively interdependent preferences in a certain game indicates that delegation would be profitable and that the incentives should incorporate a relative performance component (see also Koçkesen et al, 2000a).

Interestingly, Dufwenberg & Güth (1999) relate these two strands of literature to each other, underscoring both technical similarities and conceptual differences between the two.

#### RELATED LITERATURE

To our knowledge, two papers have investigated the strategic advantage of negatively interdependent preferences (NIPs) in games. Koçkesen et al (2000a) examine super- and submodular games that are symmetric with respect to material payoff. Two groups of players interact – one group with individualistic (or independent) preferences, the other with negatively interdependent ones. Koçkesen et al (2000a) identify sufficient conditions such that players with NIPs earn higher material payoff at all intragroup symmetric equilibria of a given super- or submodular game. In Koçkesen et al (2000b), the authors adopt a similar approach to study the strategic advantage of NIPs in differentiable aggregative games.

Among the conditions identified in Koçkesen et al (2000a), action monotonicity turns out to play a crucial role in that it is sufficient for the strategic advantage to be independent of the relative size of the two groups. Action monotonicity requires a tight connection between payoffs and actions. In positively (negatively) action monotonic games, the player with the larger (smaller) action earns higher *material* payoff. While Koçkesen et al (2000a) assume this property to hold true for all action profiles, we relax action monotonicity, requiring it to apply only at Nash equilibrium profiles of the preference-induced game.

#### APPROACH

Unlike Koçkesen et al (2000a), we take action-monotonicity as our starting-point and characterize the strategic advantage of NIPs. That is, we derive equivalent criteria, representing both sufficient and necessary conditions for the strategic advantage. To underscore the relevance of our undertaking, we provide examples showing that the strategic advantage prevails beyond the identified subclasses of super- and submodular games. Additionally, we adduce the two-player model of rent-seeking with general winning probabilities (like in Dixit, 1987) to illustrate how our criteria simplify analyzing the strategic advantage. Notice that the rent-seeking model is not covered by the results of the two mentioned papers (Koçkesen et al, 2000a,b), first, since the underlying game is neither super- nor submodular; and second, as payoff is not monotonic in a player's own action.

## 2 The model

We investigate a model where two groups of players, one with independent preferences – the other with interdependent preferences, interact with each other, playing a *symmetric* material  $n$ -person game  $\Gamma$ .

### THE MATERIAL GAME

Let  $X$  and  $\pi_h : X^n \rightarrow \mathbf{R}$  denote the action space and the material payoff function of player  $h$ , respectively, that is  $\Gamma = (X, \{\pi_h\}_{h=1,\dots,n})$ . Symmetry implies  $\pi_h(x) = \pi_g(x')$ , where  $x'$  is obtained from  $x$  by exchanging  $x_g$  and  $x_h$ . We endow the action space  $X$  with a linear order  $\succsim$  to obtain a chain. The corresponding class of games is denoted by  $\mathcal{G}$ . Let  $BR(\cdot)$  denote the best-reply correspondence of the material game  $\Gamma$ .

### THE PERCEIVED GAME

The set of players being divided into two groups, let  $k$  represent the number of players in the first group. We index members of this group by  $i \in I_k := \{1, \dots, k\}$ . Each of them chooses an action  $x_i$  in order to maximize his own material payoff  $\pi_i(x_1, \dots, x_n)$  such as given by the material game  $\Gamma \in \mathcal{G}$ . Members of group  $I_k$  have *independent preferences*. We call them *individualists*.

The remaining  $n - k$  players form the second group and are indexed  $j \in J_k := \{k + 1, \dots, n\}$ . These players additionally care about their own performance relative to that of the other players. They have (*negatively*) *interdependent preferences*. We call them *status-seekers*.

**Assumption A** Any status-seeker  $j$ 's relative payoff function,  $\rho_j : \mathbf{R}^n \rightarrow \mathbf{R}$ , is assumed to satisfy the following three properties.

- (i) (Monotonicity) Relative payoff  $\rho_j(\pi_1, \dots, \pi_n)$  is strictly increasing in own material payoff,  $\pi_j$ , and strictly decreasing in each other player's material payoff,  $\pi_h$  ( $h \neq j$ ).
- (ii) (Normalization) Whenever all players earn identical material payoff, relative payoff coincides, i.e.

$$\hat{\rho} := \rho_j(\pi', \dots, \pi') = \rho_j(\pi'', \dots, \pi''), \quad (1)$$

for all  $\pi', \pi'' \in \mathbf{R}$ .

- (iii) (Symmetry) Relative payoff is symmetric between players, i.e.

$$\rho_h(\pi_1, \dots, \pi_n) = \rho_g(\pi'_1, \dots, \pi'_n),$$

where  $(\pi'_1, \dots, \pi'_n)$  is obtained from  $(\pi_1, \dots, \pi_n)$  by exchanging  $\pi_h$  and  $\pi_g$ .

With slight abuse of notation, we write  $\rho_j(x) := \rho_j(\pi_1(x), \dots, \pi_n(x))$  to represent the relative payoff for some given effort profile  $x$ .

This axiomatic approach to define relative payoff encompasses many different forms of relative payoff functions that have been used in the literature. For instance, Konrad & Lommerud (1993) and Guse & Hehenkamp (2003) use differences between own and average payoff to represent relative payoff, while Koçkesen et al (2000a,b) employ ratios. Konrad & Lommerud relate an individual's consumption to the average consumption of *other* individuals, whereas in Guse & Hehenkamp a player's payoff is related to the average payoff of all players, including his own payoff.

The function  $F$  below represents a status-seeker's *negatively interdependent preferences*. Correspondingly, the *utility* of status-seekers depends both on absolute and on relative material payoff. Status-seekers choose their action  $x_j$  in order to maximize

$$F_j(x) = F(\pi_j(x), \rho_j(\pi(x))), \quad (2)$$

where  $F$  is assumed strictly increasing in both arguments and where  $\pi(x) := (\pi_1(x), \dots, \pi_n(x))$  collects the material payoffs of all players. The set of all strictly increasing preference functions  $F : \mathbf{R}^2 \rightarrow \mathbf{R}$  is denoted by  $\mathcal{F}$ .

Notice that it is important to distinguish between *material payoff*, which determines the material success of players (or their strategies, respectively), and *utility*, which represents the players' independent or interdependent preferences. While for individualists the notions of material payoff and utility coincide, they differ for status-seekers.

Given any material game  $\Gamma \in \mathcal{G}$  and any number of individualists,  $k \in \{1, \dots, n-1\}$ , the perceived game is then

$$\Gamma_F(k) = (X, \{\pi_i\}_{i \in I_k} \cup \{F_j\}_{j \in J_k}).$$

While the material success of players (or preference types) depends on the payoffs given by  $\Gamma$ , the players are engaged in playing the perceived game  $\Gamma_F(k)$ . Let  $BR_F(\cdot)$  denote the best-reply correspondence of a status-seeker in the perceived game  $\Gamma_F(k)$ .

#### SOLUTION CONCEPT

In this paper, we examine Nash equilibria of the perceived game  $\Gamma_F(k)$ . Like Koçkesen et al (2000a), we focus on intragroup symmetric (IGS) equilibria, where players from the same group choose the same action. We write  $x = ([a]_k, [b]_{n-k})$ , where  $x_i = x_1 =: a, \forall i \in I_k = \{1, \dots, k\}$  and  $x_j = x_n =: b, \forall j \in J_k = \{k+1, \dots, n\}$  and where  $[t]_l$  denotes the  $l$ -replication of object  $t$ . Restricting attention to IGS equilibria is not as innocent as it might appear. However, many applications give rise to a unique equilibrium, which then necessarily is intragroup symmetric.

The set of intragroup symmetric equilibria,  $x = ([a]_k, [b]_{n-k})$ , corresponding to some given  $k \in \{1, \dots, n-1\}$  and  $F \in \mathcal{F}$ , is denoted  $N_{sym}(\Gamma_F(k))$ .

Accordingly,  $N_{sym}(\Gamma_{\mathcal{F}}) \equiv \bigcup_{k=1}^{n-1} \{x \in N_{sym}(\Gamma_F(k)) : F \in \mathcal{F}\}$  represents the set of IGS Nash equilibria that might arise for any  $k \in \{1, \dots, n-1\}$  and any  $F \in \mathcal{F}$ .

#### STRATEGIC ADVANTAGE AND ACTION MONOTONICITY

The following two definitions introduce the concepts of strategic advantage and action monotonicity.

**Definition 1** *Let  $\Gamma \in \mathcal{G}$ . Then (negatively) interdependent preferences yield a strategic advantage over independent preferences if and only if*

$$\pi_n(\hat{x}) > \pi_1(\hat{x}) \quad \forall \hat{x} \in N_{sym}(\Gamma_{\mathcal{F}}) \text{ s.t. } \hat{x}_1 \neq \hat{x}_n.$$

According to Definition 1, interdependent preferences yield a strategic advantage over independent preferences for some given game  $\Gamma \in \mathcal{G}$  if they earn higher *material* payoff in equilibrium independently of the number of individualists,  $k \in \{1, \dots, n-1\}$ , and independently of the particular representation of interdependent preferences,  $F \in \mathcal{F}$ .

**Definition 2** (i) *We call an  $n$ -person normal form game  $\Gamma \in \mathcal{G}$  positively (negatively) action monotonic at equilibrium profiles if*

$$\hat{x}_h \succ (\prec) \hat{x}_{h'} \quad \implies \quad \pi_h(\hat{x}) > \pi_{h'}(\hat{x}), \quad (3)$$

for all  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$  and all players  $h, h' \in I_k \cup J_k$ .

(ii) *We call an  $n$ -person normal form game  $\Gamma \in \mathcal{G}$  positively (negatively) action monotonic if*

$$x_h \succ (\prec) x_{h'} \quad \implies \quad \pi_h(x) > \pi_{h'}(x), \quad (4)$$

for all  $x \in X$  and all players  $h, h' \in I_k \cup J_k$ .

Positive (negative) action monotonicity *at equilibrium profiles* requires the player with the larger (smaller) action to have higher material payoff in any IGS equilibrium that might arise for the class of games  $\Gamma_{\mathcal{F}}$ . This notion of *local* action monotonicity is weaker and satisfied for a larger class of games than the notion of (*global*) action monotonicity, which assumes the player with larger (smaller) action to earn higher payoff at *all* action profiles. Koçkesen et al (2000a) study games that are action monotonic in the latter sense.

Thinking of action monotonicity at equilibrium profiles as a prerequisite for our first theorem below, it is sufficient and typically easier to establish action monotonicity at the best-reply curve of the material game. To this end, it suffices to show that implication (3) holds true for all  $x = ([a]_k, [b]_{n-k}) \in X$  such that  $a \in BR([a]_{k-1}, [b]_{n-k})$  and  $\{x_h, x_{h'}\} = \{a, b\}$ . Notice that this criterion allows to establish action monotonicity at equilibrium profiles in terms of the material game  $\Gamma$  (i.e. independently of the interdependent utility function  $F \in \mathcal{F}$ ).

### 3 Locally action-monotonic games with spillovers

We start with investigating the subclass of action-monotonic games that display spillovers. Spillovers characterize the impact of other players' action on a certain player's payoff.

**Definition 3** *An  $n$ -person normal form game  $\Gamma \in \mathcal{G}$  is said to have negative spillovers if, for any  $x \in N_{sym}(\Gamma_{\mathcal{F}})$ ,*

$$t' \succ x_r \succ t'' \quad \implies \quad \pi_q(x_{-r}, t') < \pi_q(x) < \pi_q(x_{-r}, t'')$$

for all  $r$  and  $q \neq r$ . Games with positive spillovers are defined dually.

Accordingly, in games with negative spillovers, in any equilibrium profile of the perceived game  $\Gamma_{\mathcal{F}}$ , another player's action has a negative impact on each other players' payoff, whereas in games with positive spillovers the opposite holds true.

Similar to action monotonicity, we think of spillovers as a prerequisite for Theorem 1 below. Again, it is sufficient to establish the property at the best-reply curve, spillovers then being directly related to the material game.

Lemma 1 below extends Lemma 1 from Koçkesen et al (2000a) to the class of relative payoff functions encompassed by Assumption A. It shows that a larger action by status-seekers is necessary for the strategic advantage of interdependent preferences in games with negative (positive) spillovers. Moreover, if  $k = n - 1$  then also positive (negative) action monotonicity at equilibrium profiles is necessary for the strategic advantage of interdependent preferences.

**Lemma 1** *Fix  $k \in \{1, \dots, n - 1\}$ ,  $F \in \mathcal{F}$  and let  $\Gamma \in \mathcal{G}$  have negative spillovers.*

(i) *For any  $\hat{x} \in N_{sym}(\Gamma_F(k))$ , we have:*

$$\pi_j(\hat{x}) \geq (>) \pi_i(\hat{x}) \quad \implies \quad \hat{x}_j \succsim (>) \hat{x}_i$$

for all  $(i, j) \in I_k \times J_k$ .

(ii) *If  $k = n - 1$ , then also the reverse implication holds true.*

*(Dual results apply to the case of positive spillovers.)*

**Proof.** See Appendix A. ■

For games with negative spillovers, any increase in one's own action will reduce others' material payoff. If increasing one's own action additionally increases own material payoff, then any status-seeker doing so gains in terms of his interdependent utility. Accordingly, no action profile where a status-seeker can raise his material payoff by increasing his action can ever represent a Nash equilibrium of  $\Gamma_F$ , given any interdependent preference  $F \in \mathcal{F}$ .



For positively action monotonic games, interdependent preferences yield a strategic advantage over independent preferences whenever interdependent preferences induce a larger action. Combining negative spillovers with positive action monotonicity at equilibrium profiles, a sufficient condition for the strategic advantage of NIPs would then be that status-seekers can increase their material payoff by playing a larger action in any candidate profile of an IGS equilibrium where they choose the lower action.

This provides the rationale behind the following definition and the subsequent theorem.

**Definition 4** *We say an  $n$ -person normal form game  $\Gamma \in \mathcal{G}$  exhibits profitable increases at the bottom (PIB) (resp. decreases at the top) if, for any  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$ ,*

$$\hat{x}_1 \succ (\prec) \hat{x}_n \quad \implies \quad \exists t \succ (\prec) \hat{x}_n : \pi_n(\hat{x}_{-n}, t) \geq \pi_n(\hat{x}). \quad (5)$$

Notice that, in contrast to the notions of spillovers and action monotonicity, the notion of PIB explicitly relates to the roles of independent and interdependent players. Similar to the preceding definitions, to establish profitable increases at the bottom it is sufficient to show that  $\Gamma \in \mathcal{G}$  possesses profitable increases along that part of the best-reply curve where  $\hat{x}_1 \succ \hat{x}_n$  (and  $\hat{x} = ([a]_k, [b]_{n-k})$ ).

Theorem 1 shows that PIB represent a characterizing criterion for the strategic advantage of NIPs.

**Theorem 1** *Let  $\Gamma \in \mathcal{G}$  be a game with negative (positive) spillovers that is positively (negatively) action monotonic at equilibrium profiles. Then  $\Gamma$  exhibits profitable increases at the bottom (decreases at the top) if and only if  $\pi_n(\hat{x}) > \pi_1(\hat{x})$  for all  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$  with  $\hat{x}_1 \neq \hat{x}_n$ .*

**Proof.** We establish the claim for the case of positive action monotonicity at equilibrium profiles and negative spillovers. The dual case of negative action monotonicity at equilibrium profiles and positive spillovers can be dealt with similarly.

First, consider any  $\Gamma \in \mathcal{G}$  exhibiting profitable increases at the bottom. Let  $\hat{x} = ([a]_k, [b]_{n-k}) \in N_{sym}(\Gamma_{\mathcal{F}})$  be arbitrary such that  $a \neq b$ ,  $k \in \{1, \dots, n-1\}$ ,  $F \in \mathcal{F}$  and suppose to the contrary that  $\pi_n(\hat{x}) \leq \pi_1(\hat{x})$ . By positive action monotonicity at equilibrium profiles and because of  $a \neq b$ , we obtain  $a \succ b$ .

On the one hand, since  $\Gamma$  exhibits PIB, there exists  $b' \succ b$  such that  $\pi_n(\hat{x}_{-n}, b') \geq \pi_n(\hat{x})$ . On the other, negative spillovers imply that  $\pi_h(\hat{x}_{-n}, b') < \pi_h(\hat{x})$  for all  $h \neq n$ . Combining both implications, it follows from monotonicity of relative payoff that  $\rho_n(\hat{x}_{-n}, b') > \rho_n(\hat{x})$ . Since this contradicts  $b \in BR_F([a]_k, [b]_{n-k-1})$ , we must have  $\pi_n(\hat{x}) > \pi_1(\hat{x})$ .

Second, suppose that  $\pi_n(\hat{x}) > \pi_1(\hat{x})$  for all  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$  with  $\hat{x}_1 \neq \hat{x}_n$ . By positive action monotonicity at equilibrium profiles we have  $\hat{x}_n \succ \hat{x}_1$ . Thus,  $\Gamma \in \mathcal{G}$  is increasing at the bottom as the premise of (5) is not satisfied.

■

Observe that the notion of PIB does not rely on compactness or convexity of the action space  $X$ , but is solely based on order relations. Accordingly, Theorem 1 characterizes the strategic advantage of NIPs within the boundaries of the lattice-theoretic framework.

The following example serves to illustrate that Theorem 1 applies to games beyond the classes of super- and submodular games. The example presents a game that is positively action monotonic game at equilibrium profiles, has negative spillovers, and a non-monotonic best-reply function. Moreover, the action space cannot be reordered so as to make the best-reply function monotonic, without losing symmetry of the game.

**Example 1** Consider the symmetric two-person material game  $\Gamma$  given by the following bimatrix

$1/2$	$\alpha$	$\beta$	$\gamma$
$\alpha$	8,8	6,7	2,9
$\beta$	7,6	4,4	3,5
$\gamma$	9,2	5,3	1,1

Let the action space  $X = \{\alpha, \beta, \gamma\}$  be equipped with the linear order  $\alpha \prec \beta \prec \gamma$  and  $\alpha \prec \gamma$ . Obviously,  $\Gamma$  is positively action monotonic, has negative spillovers, and a non-monotonic best-reply function. Moreover, the best-reply structure is cyclic so that the action space cannot be reordered to make the best-reply function monotonic.

## 4 Action-monotonic games

In this section, we give up the assumption of spillovers. This relaxation comes at a price. First, we have to replace positive action monotonicity at equilibrium profiles by (global) positive action monotonicity. Moreover, we restrict profitable increases at the bottom to profitable switches where an interdependent player materially gains from switching to an independent player's action whenever the latter is playing the larger action. More formally, we define:

**Definition 5** An  $n$ -person normal form game  $\Gamma \in \mathcal{G}$  exhibits profitable switches at the bottom (PSB) (resp. at the top) if, for any  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$ ,

$$\hat{x}_1 \succ (\prec) \hat{x}_n \quad \implies \quad \pi_n(\hat{x}_{-n}, \hat{x}_1) \geq \pi_n(\hat{x}). \quad (6)$$

By definition, profitable switches at the bottom (at the top) imply profitable increases at the bottom (decreases at the top), while the opposite inclusion does not apply. Nevertheless, the notion of PSB incorporates most properties of PIB. First, PSB explicitly relates to the roles of independent and interdependent players. Second, to establish PSB it is sufficient to show that (6) holds along that part of the best-reply curve where  $\hat{x}_1 \succ \hat{x}_n$ . Finally, PSB only relates to order relations and hence connects to the lattice-theoretic framework.

Observe that either of the following conditions would be sufficient for PSB (and hence for PIB): (i) strict supermodularity of the material payoff function  $\pi$ ; or (ii) weak supermodularity of  $\pi$  and unique best-replies (so that the best-reply correspondence reduces to a function).

Theorem 2 shows that the notion of PSB fully characterizes the strategic advantage of NIPs in action-monotonic games without spillovers.

**Theorem 2** *Let  $\Gamma \in \mathcal{G}$  be positively (negatively) action monotonic. Then  $\Gamma$  exhibits profitable switches at the bottom (at the top) if and only if  $\pi_n(\hat{x}) > \pi_1(\hat{x})$  for all  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$  with  $\hat{x}_1 \neq \hat{x}_n$ . For  $k = n-1$ , action monotonicity at equilibrium profiles is sufficient for this equivalence.*

**Proof.** Again, we content ourselves with establishing the claim for the case of positive action monotonicity.

First, consider any  $\Gamma \in \mathcal{G}$  exhibiting profitable switches at the bottom. Let  $\hat{x} = ([a]_k, [b]_{n-k}) \in N_{sym}(\Gamma_{\mathcal{F}})$  be arbitrary such that  $a \neq b$ ,  $k \in \{1, \dots, n-1\}$ , and  $F \in \mathcal{F}$  and suppose to the contrary that  $\pi_n(\hat{x}) \leq \pi_1(\hat{x})$ . By positive action monotonicity (at equilibrium profiles) and because of  $a \neq b$ , we have that  $a \succ b$  and hence  $\pi_1(\hat{x}) > \pi_n(\hat{x})$ . It follows that

$$\begin{aligned} \rho_n(\hat{x}) &= \rho_n([\pi_1(\hat{x})]_k, [\pi_n(\hat{x})]_{n-k}) \\ &< \rho_n([\pi_n(\hat{x})]_n) = \hat{\rho}, \end{aligned}$$

where the inequality follows from Assumption A(i) and A(iii) and the last equality from normalization A(ii).

On the one hand, since  $\Gamma$  exhibits profitable switches at the bottom, it follows that  $\pi_n(\hat{x}_{-n}, a) \geq \pi_n(\hat{x})$ . On the other hand, positive action monotonicity implies  $\pi_n(\hat{x}_{-n}, a) > \pi_j(\hat{x}_{-n}, a)$  for all  $j \in J_k \setminus \{n\}$  (this step is only relevant for  $k < n-1$ ). By symmetry of material payoff, we obtain  $\pi_n(\hat{x}_{-n}, a) = \pi_i(\hat{x}_{-n}, a)$  for all  $i \in I_k$ . From Assumption A it hence follows that  $\rho_n(\hat{x}_{-n}, a) \geq \hat{\rho}$ . Because of  $\rho_n(\hat{x}) < \hat{\rho}$ , this contradicts  $b \in BR_F([a]_k, [b]_{n-k-1})$ . Thus, we must have  $\pi_n(\hat{x}) > \pi_1(\hat{x})$ .

Second, suppose that  $\pi_n(\hat{x}) > \pi_1(\hat{x})$  for all  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$  with  $\hat{x}_1 \neq \hat{x}_n$ . By positive action monotonicity (at equilibrium profiles) we have  $\hat{x}_n \succ \hat{x}_1$ . Thus,  $\Gamma \in \mathcal{G}$  exhibits profitable switches at the bottom as the premise of (6)

cannot be satisfied.

■

The basic intuition underlying Theorem 2 can be described looking at the two-player case. Suppose  $\hat{x} = (a, b)$ ,  $a \neq b$ , were an equilibrium of  $\Gamma_F$  such that no strategic advantage of interdependent preferences were present, i.e.  $\pi_1(\hat{x}) \geq \pi_2(\hat{x})$ . By action monotonicity this can only occur when  $a \succ (\prec)b$  and hence  $\pi_1(\hat{x}) > (<)\pi_2(\hat{x})$ . Since  $\Gamma_F$  displays profitable switches at the bottom (at the top), player 2 can increase his material payoff, switching to  $x_2 = a$  instead. By symmetry of  $\Gamma$  and Assumption A, we moreover obtain that  $\rho_2(\hat{x}_1, a) = \hat{\rho} > \rho_2(\hat{x})$ , which yields a contradiction to  $b \in BR_F(a)$ . Thus, it must be that  $\pi_1(\hat{x}) < \pi_2(\hat{x})$ . Theorem 2 then shows that this intuition, slightly modified, remains valid for the case of more than two players.

**Corollary 1** *Let  $\Gamma \in \mathcal{G}$  be a positively (negatively) action monotonic game with negative (positive) spillovers. Then  $\Gamma$  exhibits profitable increases at the bottom (decreases at the top) if and only if  $\Gamma$  exhibits profitable switches at the bottom (at the top).*

The following example illustrates two aspects. First, like Theorem 1, Theorem 2 covers games with non-monotonic best-reply functions. Therefore, the strategic advantage of negatively interdependent preferences prevails beyond the classes of supermodular and submodular games such as identified in Koçkesen et al (2000a). Second, Theorem 2 extends to games that are not covered by Theorem 1. For, the game in Example 2 displays the properties of positive action monotonicity at equilibrium profiles and profitable switches at the bottom, whereas it has neither positive nor negative spillovers.

**Example 2** *Consider the symmetric two-person normal form game  $\Gamma$  represented by the bimatrix*

$1/2$	$\alpha$	$\beta$	$\gamma$
$\alpha$	$7,7$	$5,6$	$1,8$
$\beta$	$6,5$	$3,3$	$3,4$
$\gamma$	$8,1$	$4,3$	$2,2$

*Equipping the action space  $X = \{\alpha, \beta, \gamma\}$  with the order from Example 1, the game is positively action monotonic, has a non-monotonic best-reply function and displays profitable switches at the bottom. Moreover, it displays neither positive nor negative spillovers at  $(\beta, \gamma)$  and  $(\alpha, \beta)$ . Then, Theorem 2 tells us that the action profile  $(\gamma, \alpha)$  cannot represent a Nash equilibrium in game  $\Gamma_F$ , for any  $F \in \mathcal{F}$ . Since  $(\gamma, \alpha)$  is the only action profile with  $x_1 \succ x_3$  and  $x_1 \in BR(x_2)$ , it follows that, whenever  $\hat{x}_1 \neq \hat{x}_n$ , interdependent prefer-*

ences have a strategic advantage over independent preferences.

Notice that the assumptions of Theorem 1 and 2 cover different classes of games, respectively: While we can examine Example 2 by means of Theorem 2, but not with Theorem 1, the aforementioned Example 1 presents the opposite instance. There, Theorem 1 applies, but not so Theorem 2. For, game  $\Gamma$  in Example 1 is increasing at the bottom, but does not exhibit profitable switches at the bottom.

We conclude this section providing an example which shows that global action monotonicity is indispensable for establishing Theorem 2 if  $k < n - 1$ .

**Example 3** Consider the symmetric three-person material game  $\Gamma$  represented below.

3: $\alpha$			3: $\beta$		
1/2	$\alpha$	$\beta$	1/2	$\alpha$	$\beta$
$\alpha$	1, 1, 1	1, 4, 1	$\alpha$	1, 1, 4	40, 4, 4
$\beta$	4, 1, 1	4, 4, 40	$\beta$	4, 40, 4	5, 5, 5

Let  $I = \{1\}$ ,  $J = \{2, 3\}$  and equip the state space  $X = \{\alpha, \beta\}$  with the order  $\alpha \prec \beta$ . Moreover, set  $\rho_j = \pi_j - (\sum_{h=1}^n \pi_h) / n$  and  $F(\pi_j, \rho_j) := \rho_j + \pi_j$ .<sup>1</sup> Then the perceived game is given by

3: $\alpha$			3: $\beta$		
1/2	$\alpha$	$\beta$	1/2	$\alpha$	$\beta$
$\alpha$	1, 1, 1	1, 6, 0	$\alpha$	1, 0, 6	40, -8, -8
$\beta$	4, 0, 0	4, -8, 64	$\beta$	4, 64, -8	5, 5, 5

(recall that player 1's concern is material payoff). Clearly, game  $\Gamma$  is positively action-monotonic at equilibrium profiles, but not at profile  $(\beta, \beta, \alpha)$ . Moreover, game  $\Gamma$  exhibits PSB. However, the action profile  $(\beta, \alpha, \alpha)$  constitutes a unique IGS equilibrium of the perceived game, where the individualist earns higher material payoff than the two status-seekers do.

## 5 An illustrative application

In this section we illustrate how our findings can help identifying conditions that characterize the strategic advantage even in the absence of sub- or supermodularity. To this end, we investigate the class of general two-player rent-seeking contests.

Two players,  $h = 1, 2$ , compete for a rent with common value  $V$ . To this end, each player  $h$  chooses an level of effort  $x_h \in X$ . Given his own effort

<sup>1</sup>The table of status-seekers' relative payoff is provided in Appendix B.

level  $x_h$  and the effort  $x_g$  of the other player, player  $h$  wins the rent with probability  $p(x_h, x_g)$ . A player's winning probability increases in his own effort and decreases in the other player's effort. Material payoff of player  $h$  is then given by

$$\pi_h(x_1, x_2) = p(x_h, x_g)V - x_h. \quad (7)$$

This general formulation of rent-seeking contests was proposed and analyzed by Dixit (1987). It covers the most important variants of rent-seeking models that have been used in the literature, e.g. the logit and the probit model (see e.g. Tullock, 1980; Nalebuff & Stiglitz, 1983; Hirshleifer, 1989; Skaperdas, 1996; or Lockard & Tullock, 2000).

Notice first that the above material game  $\Gamma$  has negative spillovers, by virtue of a players' winning probability being decreasing in the other player's effort. Second, without putting additional restrictions on  $p(\cdot, \cdot)$ , the material game  $\Gamma$  will in general be neither super- nor submodular. E.g., for Tullock's (1980) omnipresent example of logit winning probabilities  $p^T(x_h, x_g) := x_h^r / (x_h^r + x_g^r)$  with  $r > 0$ , it turns out that best reply correspondence associated with the material game is increasing for  $x_h > x_g$  and decreasing for  $x_h < x_g$  (see Pérez-Castrillo and Verdier, 1992). Third, game  $\Gamma$  is positively action monotonic at equilibrium profiles. This follows directly from Lemma 1(ii), since  $\Gamma$  displays negative spillovers.

We focus on the differentiable case. The action space  $X$  is given by some interval  $X = [0, \bar{X}]$  (where  $\bar{X} \geq V$ ), equipped with the natural ordering relation  $\geq$ . The material payoff function and the winning probability function are assumed differentiable on  $X^2$ .

#### THE CRITERION

The following proposition identifies a condition characterizing the strategic advantage in two-player rent-seeking contests.

**Proposition 1** *Let  $\Gamma \in \mathcal{G}$  be a two-player rent-seeking contest with material payoff function (7) and winning probability function  $p(x_h, x_g)$ . Then negatively interdependent preferences yield a strategic advantage over independent preferences if and only if, for any  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$ ,*

$$\hat{x}_1 > \hat{x}_2 \quad \implies \quad p_1(\hat{x}_2, \hat{x}_1) > p_1(\hat{x}_1, \hat{x}_2), \quad (8)$$

where  $p_1(x_h, x_g) := \partial p(x_h, x_g) / \partial x_h$  represents the derivative with respect to the first argument.

**Proof.** Since  $\Gamma$  is positively action monotonic at equilibrium profiles and since it displays negative spillovers, it follows from Theorem 1 that the strategic advantage is equivalent to profitable increases at the bottom. Therefore, it is sufficient to establish (i) that property (8) implies profitable increases at the bottom and (ii) that the strategic advantage implies (8).

Ad (i): Let  $\hat{x} \in (\hat{x}_1, \hat{x}_2) \in N_{sym}(\Gamma_{\mathcal{F}})$  be arbitrary. From the first order condition of player 1, we obtain  $p_1(\hat{x}_1, \hat{x}_2)V = 1$ . Inserting this into the marginal material payoff of player 2, condition (8) implies

$$\frac{\partial \pi_2}{\partial x_2} = p_1(\hat{x}_2, \hat{x}_1)V - 1 = [p_1(\hat{x}_2, \hat{x}_1) - p_1(\hat{x}_1, \hat{x}_2)]V > 0,$$

Hence, there exists  $t \in (\hat{x}_2, \hat{x}_2 + \varepsilon)$  such that  $\pi_2(\hat{x}_1, t) > \pi_2(\hat{x}_1, \hat{x}_2)$ , for some  $\varepsilon > 0$  sufficiently small. That is, game  $\Gamma$  exhibits profitable increases at the bottom.

Ad (ii): Let  $\hat{x} \in (\hat{x}_1, \hat{x}_2) \in N_{sym}(\Gamma_{\mathcal{F}})$  be arbitrary such that  $\pi_2(\hat{x}) > \pi_1(\hat{x})$ . Because of action monotonicity at equilibrium profiles, we have  $\hat{x}_2 > \hat{x}_1$ . Thus, the premise of (8) is not satisfied, which completes the proof. ■

Condition (8) says that, at any Nash equilibrium  $\hat{x} \in N_{sym}(\Gamma_{\mathcal{F}})$  with  $\hat{x}_1 > \hat{x}_2$ , the marginal winning probability of player 2 should exceed that of player 1. A sufficient condition for (8) would be that  $p_1(x_2, x_1) > p_1(x_1, x_2)$  holds for all  $x_1, x_2 \in X$  with  $x_1 > x_2$ .

#### TULLOCK'S EXAMPLE OF LOGIT PROBABILITY FUNCTIONS

The above condition is e.g. satisfied for Tullock's aforementioned example of logit probability functions.

**Lemma 2** *Let  $p(x_h, x_g) = p^T(x_h, x_g)$ . Then, we have  $p_1(x_2, x_1) > p_1(x_1, x_2)$  if and only if  $x_1 > x_2$ .*

**Proof.** Notice that  $p_1^T(x_h, x_g) = r x_h^{r-1} x_g^r / (x_h^r + x_g^r)^2$ . The claim then follows from

$$\frac{p_1^T(x_2, x_1)}{p_1^T(x_1, x_2)} = \frac{x_1}{x_2}.$$

■

We thus obtain as a corollary to Proposition 1:

**Corollary 2** *Let  $p(x_h, x_g) = p^T(x_h, x_g)$ . Then, negatively interdependent preferences yield a strategic advantage over independent preferences.*

Adopting a simpler and more elegant approach, Corollary 2 restates Theorem 1 in Guse & Hehenkamp (2003).

#### GENERAL LOGIT PROBABILITY FUNCTIONS

Let us now consider the case of general logit probability functions, where  $p(x_h, x_g) = p^{\text{logit}}(x_h, x_g) := f(x_h) / (f(x_1) + f(x_2))$  and  $f : X \rightarrow \mathbf{R}$  is assumed increasing. For this class of logit probability functions, condition (8) enables us to derive a direct characterization of the strategic advantage.

**Proposition 2** *Let  $p(x_h, x_g) = p^{\text{logit}}(x_h, x_g)$ . Then, negatively interdependent preferences have a strategic advantage over independent preferences if and only if, for any  $\hat{x} \in N_{\text{sym}}(\Gamma_{\mathcal{F}})$ ,*

$$\hat{x}_1 > \hat{x}_2 \quad \implies \quad f'(x_2)f(x_1) > f'(x_1)f(x_2).$$

**Proof.** The claim follows from  $p_1^{\text{logit}}(x_h, x_g) = f'(x_h)f(x_g)/[f(x_1) + f(x_2)]^2$ .

■

Obviously, it is sufficient for the strategic advantage of NIPs that  $\varphi(z) := f'(z)/f(z)$  is decreasing in  $z$ . Sufficient for this latter condition in turn would be that  $f'(z)$  is weakly decreasing. In contrast, for convex contest success functions  $f(z)$  the strategic advantage does not apply whenever  $\varphi(z)$  is weakly increasing between at least two  $\hat{x}_1 > \hat{x}_2$  so that  $(\hat{x}_1, \hat{x}_2) \in N_{\text{sym}}(\Gamma_{\mathcal{F}})$  and  $\varphi(\hat{x}_2) \leq \varphi(\hat{x}_1)$ . In this case, individualistic preferences earn higher material payoff in equilibrium.

Notice that our analysis refers to equilibria in pure actions. While for weakly concave contest success functions  $f(z)$  existence and uniqueness of a pure action equilibrium can be established, this is not necessarily the case if  $f(z)$  is convex. In that case, Proposition 2 characterizes the strategic advantage of *all* equilibria in pure actions.

## 6 Conclusion

In this paper, we have characterized the strategic advantage of negatively interdependent preferences in action monotonic games. To this end we derived conditions such that the strategic advantage was independent (i) of the number of individualists in the population of players, (ii) independent of the particular representation of relative payoff, and (iii) independent of the representation of negatively interdependent preferences.

For games that are positively action monotonic at equilibrium profiles and that have negative spillovers, it turned out that profitable increases at the bottom (PIB) are equivalent to the strategic advantage. For positively action monotonic games, the related but stronger criterion of profitable switches at the bottom (PSB) has been identified to characterize the strategic advantage. For negatively action monotonic games dual results apply. Examples have illustrated the novelty and applicability of our findings. In particular, it turns out that the strategic advantage is not restricted to the classes of super- and submodular games.

Our characterizations remain with the lattice-theoretic framework and do not rely on topological or vector space properties such as compactness or convexity, respectively. This is remarkable in that some of the earlier results on the strategic advantage rely on topological or vector space properties such as compactness or convexity, respectively.



Finally, we established our results adopting an axiomatic formalization of relative payoff, thus incorporating different representations of relative payoff that have been used in the literature.

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## Appendix A

**Proof of Lemma 1.** We only show (ii). Part (i) can be shown as in Koçkesen et al (2000a).

Let  $k = n - 1$ ,  $\hat{x} = ([a]_{n-1}, b) \in N_{sym}(\Gamma_{\mathcal{F}})$  and  $F \in \mathcal{F}$ . We have to show that  $b \succ a$  implies  $\pi_n(\hat{x}) > \pi_1(\hat{x})$ . Suppose to the end of contradiction that  $b \succ a$  and  $\pi_n(\hat{x}) \leq \pi_1(\hat{x})$ . Observe that, by monotonicity and normalization of relative payoff,  $\pi_n(\hat{x}) \leq \pi_1(\hat{x})$  implies

$$\rho_n(\hat{x}) = \rho_n([\pi_1(\hat{x})]_{n-1}, \pi_n(\hat{x})) \leq \rho_n([\pi_1(\hat{x})]_n) = \hat{\rho}.$$

Consider  $\check{x} := ([a]_n)$ . By symmetry of material and relative payoff, respectively, we have  $\rho_n(\check{x}) = \hat{\rho}$  and hence  $\rho_n(\check{x}) \geq \rho_n(\hat{x})$ . Because of  $b \in BR_F([a]_{n-1})$ , it follows that  $\pi_n(\check{x}) \leq \pi_n(\hat{x})$ . Thus, negative spillovers and symmetry of the material game imply

$$\pi_1(\hat{x}) < \pi_1(\check{x}) = \pi_n(\check{x}) \leq \pi_n(\hat{x}),$$

in contradiction to  $\pi_n(\hat{x}) \leq \pi_1(\hat{x})$ . ■

## Appendix B

**Relative payoff to status-seekers in Example 3.**

3: $\alpha$			3: $\beta$		
1/2	$\alpha$	$\beta$	1/2	$\alpha$	$\beta$
$\alpha$	·, 0, 0	·, 2, -1	$\alpha$	·, -1, 2	·, -12, -12
$\beta$	·, -1, -1	·, -12, 24	$\beta$	·, 24, -12	·, 5, 5

■