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The sunk cost bias and managerial pricing practices

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Working Paper #0076

The Sunk Cost Bias and Managerial Pricing Practices [*]

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Abstract

This paper provides an explanation for why the sunk cost bias persists among firms in a competitive environment in which rich learning possibilities are allowed. We envision firms that experiment with cost methodologies that are consistent with real-world accounting practices, including ones that confuse the relevance of variable, fixed, and sunk costs to pricing decisions. Firms follow "naive" adaptive learning to adjust prices and reinforcement learning to modify their costing methodologies. Costing and pricing practices that increase profits are reinforced. We show that all firms eventually display the sunk cost bias in their pricing behavior.

Contents

1 Introduction **1**

2 Price Setting and Costing Methodologies in Practice **5**
- 2.1 Full-cost Pricing . . . 5
- 2.2 Absence of Common Standards and the Importance of Flexibility . . . 6
- 2.3 Representational Faithfulness . . . 7
- 2.4 Budgets and Variances . . . 8
- 2.5 Summary . . . 8

3 The Model **9**
- 3.1 Demand and Cost Fundamentals . . . 9
- 3.2 Economic vs. Accounting Profits . . . 10
- 3.3 The Static Price Competition Game . . . 11

4 An Illustrative Example **12**

5 Main Results **14**
- 5.1 Adaptive Learning and Comparative Statics . . . 14
- 5.2 Distortion Result . . . 15
- 5.3 Adjustment of Costing Methodologies . . . 16
- 5.4 No-distortion Results . . . 18

6 Special Case: Symmetric Linear Demand **19**

7 Discussion **26**
- 7.1 Experimental Results . . . 26
- 7.2 Cournot Competition and Behavioral Biases . . . 28
- 7.3 Alternative Explanations: Noisy Observation of Cost, Delegation, and Evolution . . . 29

8 Concluding Remarks **30**

9 Appendix **32**

1 Introduction

Economic theory offers the unambiguous prescription that only marginal cost is relevant for profit-maximizing pricing decisions. On-going fixed costs or previously incurred sunk costs, although relevant for entry and exit decisions, are irrelevant for pricing. This theoretical prescription stands in stark contrast with evidence that pricing decisions of real world firms display a *sunk cost bias*. In surveys of pricing practices of U.S. companies, Govindarajan and Anthony (1995), Shim (1993), and Shim and Sudit (1995) find that most firms price their products based on costing methodologies that treat fixed and sunk costs as relevant for pricing decisions. Leading textbooks on managerial and cost accounting paint a similar picture. Maher, Stickney, and Weil (2004, p. 258) assert that, when it comes to pricing practices, "[o]verwhelmingly, companies around the globe use full costs rather than variable costs." They cite surveys of U.S. industries "showing that full-cost pricing dominated pricing practices (69.5 percent), while only 12.1 percent of the respondents used a variable-cost based approach." Horngren, Foster, and Datar (2000, pp. 427-40), another leading accounting textbook, report other surveys in which a majority of managers in the United States, the United Kingdom and Australia take fixed and sunk costs in to account in pricing.

While suggestive, surveys of pricing practices do not answer the question whether the sunk cost bias has a *material effect* on observed prices. Managers may be influenced by a combination of biases that "cancel out," leading them to price "as if" they understood the economic reasoning of marginal revenue and marginal cost. One way to resolve this issue is through controlled experimental settings where decision makers face environments that differ only in the presence of sunk cost. Offerman and Potters (2006) conduct such an experiment in a Bertrand duopoly context. In their baseline treatment with no fixed or sunk cost, Offerman and Potters find that prices converge to the Bertrand equilibrium. In the sunk cost treatment, subjects face the same demands and marginal costs as in the baseline treatment, but they must pay a sunk entry fee to play the game. In this treatment, Offerman and Potters find that prices are significantly higher: once the sunk entry fee is paid, the average markup over marginal cost is 30 percent higher than the markup in the baseline treatment.

A standard reaction to this evidence is that initial behavioral biases eventually disappear in response to learning and competition. Pursuit of profit is a powerful incentive for firms to learn to price optimally. Also, competition among managers for promotion and advancement within a firm should favor those who price optimally. Alchian (1950)'s classic argument that learning and imitation would propagate good practices suggests that inter-firm competition would only reinforce the case for optimal pricing: "[W]henever successful enterprises are observed, the elements common to these observable successes will also be associated with success and copied by others in their pursuit of profits or success. 'Nothing succeeds like success.' "

Yet, important mechanisms for propagating best accounting practices lend little if any support for the use of economics-based pricing principles. For example, textbooks in managerial accounting often list marginal-cost based pricing as just one of several acceptable methodologies, alongside others that incorporate sunk costs into pricing. Some managerial accounting texts even argue *against* basing prices on marginal costs.[1]

Our goal in this paper is to understand why the sunk cost bias persists in real-world pricing practices despite the forces of learning and competition. In fact, we will argue that the sunk bias is *reinforced* by these forces. We study a Bertrand oligopoly with product differentiation where firms occasionally experiment with new "costing methodologies" and reinforce the ones that yield higher profits. Firms are "naive" in the sense that they adjust their prices adaptively, and may confound fixed, sunk and variable costs. The choice of costing methodologies is subject to inertia: firms adjust prices much more frequently than they experiment with new costing methodologies.[2]

[1] To cite one example, Shank and Govindarajan (1989) admonish managers to avoid committing the "Braniff fallacy," the practice (named after the now-defunct Braniff Airlines) of being content to sell seats at prices that merely cover the incremental cost of the seat. They state that "Looking at *incremental* business on an *incremental* cost basis will, at best, *incrementally* enhance overall performance. It cannot be done on a *big* enough scale to make a *big* impact. If the scale is that large, then an incremental look is not appropriate!" (p. 29, emphasis in original) Shank and Govindarajan make it clear that they believe that managers should incorporate sunk costs into pricing decisions arguing that "Business history reveals as many sins by taking an incremental view as by taking the full cost view.."

[2] An indirect indication of the inertia involved in the selection of costing systems comes from the empirical literature on the adoption of activity-based costing (ABC). ABC is a set of practices for assigning costs to products in a multi-product firm that is widely believed to be superior to traditional methodologies for allocating common costs. Beginning in the mid-1980s, academics and consultants began touting the virtues of ABC, but available evidence suggests that the adoption of ABC in the

Our main result is that all firms eventually display the sunk cost bias.[3] We reach this conclusion by analyzing two learning processes. Theorem 1 shows that if a firm suffers from a sunk cost bias, adaptive learning eventually leads to higher equilibrium profits. We then model the process through which firms experiment with new costing practices and reinforce successful ones. Theorem 2 shows that this process of experimentation and reinforcement leads all firms to eventually display the sunk cost bias almost surely. This can explain why the sunk cost bias has been so hard to eradicate in managerial pricing practices.

A key assumption in our analysis is that the pricing game is supermodular, so the adaptive learning results of Vives (1990) and Milgrom and Roberts (1990) are applicable. We especially rely on Echenique (2002), who shows that adaptive learning leads to the "right" comparative statics when the pricing game has multiple equilibria. Modeling the process through which firms reinforce successful costing methodologies is more problematic. First, it is implausible that firms know their rivals' distorted costing practices. Second, even if these practices are known, computing payoff functions requires calculating the limit of the adaptive adjustment in prices. Firms that are naive enough to confuse cost concepts are unlikely to have the requisite sophistication and understanding of the game to carry out such computations. Thus, our firms face a learning problem where neither opponents' strategies nor the payoff functions are known.[4] Despite these difficulties, we display a reinforcement learning process in which all firms eventually distort their relevant costs by incorporating sunk costs in their pricing decisions (Theorem 2).

Our analysis does more than rationalize the pervasiveness of the sunk cost bias. It also makes predictions about which industry structures reinforce this bias, and which tend to eradicate it. There is no reason to suspect that managers' innate predispositions to behavioral biases is correlated with industry structure. Yet Section 5.4 shows that

mid-to late 1990s was not widespread (see for example, Brown, Booth, and Giacobbe (2004), Roztocki and Schultz (2003)). Among the factors that have slowed the adoption of ABC systems include the need to make significant new investments in information technology in order to implement an ABC system and the need to obtain acceptance from key managers to support the transition to an ABC system.

[3] Our analysis also covers, without any modifications, unsunk flow fixed costs.

[4] In the pricing game we assume that each firm knows its own demand and the recent history of its rivals' prices.

the sunk cost bias should *not* persist in a monopoly, perfectly competitive markets and undifferentiated Bertrand competition. These predictions can, in principle, be tested. In fact, our prediction that monopolists would eventually rid themselves of the sunk cost bias is consistent with the experimental findings of Offerman and Potters (2006).

The sunk cost bias in our model acts as if it is a commitment to raise prices. But it is a spurious commitment in the sense that it can be easily eliminated through learning. Firms learning to price correctly is allowed in our model, and is indeed what prevails under some market structures. This is to be contrasted with the classical analysis of strategic commitment, where sophisticated players make binding decisions to limit their flexibility and foresee the impact of these decisions on their rivals' future behavior. Here, we envision real-world managers whose behavior is shaped by myopic incentives and refined by naive adaptive learning. The driving force in our model is confusion about which costs are relevant for pricing decisions. Since such confusion is unlikely to be observable, it is quite different from the familiar foundation of commitment, namely credible, transparent public actions designed to shape equilibrium outcomes in subsequent stages of the game. See Section 7.3 for further discussion.

The sunk cost bias in human decision-making manifests itself in a myriad of ways. One pattern, extensively supported by experimental evidence (see, Arkes and Ayton (1999)), is for individuals to persist in an activity "to get their money's worth."[5] Under this pattern, individuals deflate the true cost of the activity. In this paper we focus on how the sunk cost bias manifests itself in pricing decisions. The evidence reported above, as well as a wealth of anecdotal examples, suggests that firms inflate relevant cost by including a sunk cost component.[6] We allow firms to be rational, inflate or deflate relevant cost. Our analysis identifies circumstances under which a systematic

[5] For example, in a well-known experiment (see Arkes and Ayton (1999)) season tickets were sold at three randomly selected prices. Those charged the lower price attended fewer events during the season. Apparently, those who had sunk the most money into the season tickets were most motivated to use them.

[6] A particularly vivid example is Edgar Bronfman, former owner of Universal Studios, who criticized the movie industry's pricing model because ticket prices do not reflect differences in the sunk production costs of different movies. "He ... observed that consumers paid the same amounts to see a movie that costs $2 million to make as they do for films that cost $200 million to produce. 'This is a pricing model that makes no sense, and I believe the entire industry should and must revisit it.' " *Wall Street Journal*, April 1, 1998.

bias towards inflating cost appears.

The paper proceeds as follows. In Section 2 we provide an interpretation cost accounting practices as they relate to pricing decisions. Sections 3-5 introduces the model and our main results. In Section 6, we specialize our general model to the canonical case of symmetric linear demand. We obtain a closed-form solution that shows how the distortion of relevant cost depends on the degree of product differentiation and the number of firms. Finally, Section 7 compares the implications of our model with available experimental evidence and discusses related literature.

2 Price Setting and Costing Methodologies in Practice

An economist reading managerial accounting textbooks may be surprised to discover that they mainly consist of a compilation of common company practices. In contrast to the traditional economic treatment of optimal pricing, managerial accounting offers no rigid guidelines as to how various costs should factor into firms' pricing practices.

In this section, we present our understanding and interpretation of the accounting principles used as a guide in day-to-day costing practices.

2.1 Full-cost Pricing

The most common set of real-world pricing practices fall under the rubrics *cost-based pricing*, *cost-plus pricing,* or *full-cost pricing*. Although they come in a wide range of variations, they all base price on a calculation of an average or unit cost that includes variable, fixed, and sunk costs.[7]

Firms justify full-costing using a variety of arguments, with typically little or no foundation in economic theory:

- *Simplicity.* Full-cost formulas are thought to be relatively straightforward to implement because they do not require detailed analysis of cost behavior in order to separate the fixed and variable components of various cost items. A related

[7] See Kaplan and Atkinson (1989) for a detailed discussion of full-cost pricing.

argument is that estimates of marginal cost are often imprecise and indeed even-misleading in large, multi-product firms (Kaplan and Atkinson (1989)).

- *Promote full recovery of all costs of the product.* Pricing based on a full-cost formulas is sometimes justified because it provides a clear indicator of the minimum price needed to ensure the long-run survival of the business (Horngren, Foster, and Datar (2000, p. 437, henceforth HFD)). The idea is that without full-cost pricing, a firm's managers would not be as aware of the " pricing hurdle" that the firm would need to clear in order to generate economic profits for the firm.[8]

- *Competitive discipline.* Some managers believe that full-costing methodologies promote pricing stability by limiting "the ability of salespersons to cut prices" and reducing the "temptation to engage in excessive long-run price cutting." (HFD, p. 437). Further, full-costing may allow firms to better coordinate on price increases: "At a time when all firms in the industry face similar cost increases due to industry-wide labor contracts or material price increases, firms will implement similar price increases even with no communication or collusion among individual firms." (Kaplan and Atkinson (1989)).

2.2 Absence of Common Standards and the Importance of Flexibility

An underlying message of the accounting literature is that firms should adapt their costing methodologies to fit their particular competitive environment and product line idiosyncrasies. That is, an emphasis is placed on *flexibility*. The typical theme is that the appropriate methodology depends on the industry environment. For example, full-cost pricing is considered to be well suited for firms that operate in differentiated product industries (HFD, p. 427), while companies that operate in highly competitive commodity industries are encouraged to set prices based on competitors' prices.[9]

[8] This argument may seem strange to economists who might wonder why it is not possible for the firm's managers to price to maximize profits based on standard marginal reasoning and to simply do a "side calculation" to determine whether that profit-maximizing price allows the firm to recover its total costs, including the rental rate on capital.

[9] This emphasis on flexibility and "sticking with what what works" is consistent with theoretical models of reinforcement learning.

Managerial practice mirrors the textbook emphasis on flexibility. For example, the cost of shared assets is often allocated to individual product lines according to fixed, and more or less arbitrary, percentages. In computing unit costs, there is no universally accepted benchmark for what quantity should go in the "denominator" : some firms calculate unit cost at full capacity (in one of its various forms), while others use historical output levels, while still others base unit costs on forecasts of future sales. Some companies compute prices by applying a single markup to a chosen cost base, while others apply different markup percentages to different cost categories.

2.3 Representational Faithfulness

Given the arbitrariness and flexibility in pricing methodologies, there is no presumption that a firm includes all of its fixed and sunk costs in its computation of unit costs for price-setting purposes. Still, it is reasonable to believe that *"firms do not create costs out of thin air!!"* In fact, we shall assume that firms' distortions display a minimal degree of *coherence*, requiring that they only allocate existing fixed and sunk costs.

This can be justified by the accounting principle of *representational faithfulness*, which is one of the two major principles (the other being verifiability) that defines the standard to which external financial statements are expected to adhere (Financial Accounting Standards Board (1980, p. 28)). This principle requires the "correspondence or agreement between a measure or description and the phenomenon it purports to represent." Although internal financial information is not required to meet external standards of reliability, external reporting systems often have an important impact on accounting information that is used for internal purposes. It therefore seems plausible that a principle designed to keep external accounting information grounded in the economic fundamentals of transactions would also keep internal accounting information similarly grounded.

Another justification may be based on psychological evidence that the sunk cost bias results from decision makers' taste for taking actions that rationalize past choices. See Eyster (2002). Clearly, if no sunk or fixed costs are committed, there is nothing to rationalize and, under this theory, the sunk cost bias should not appear.

2.4 Budgets and Variances

For pricing and other operating decisions, firms typically utilize a budget of forecasts of costs and quantities for a given accounting period. Budgets aid in decision making and also serve as a benchmark against which decisions can be evaluated *ex post*. Budgeted amounts may be based on historical performance, engineering studies of how various cost items might behave, and hypotheses about competitors' actions.[10] For example, a budget might specify a firm's per-unit cost based on the quantity of output produced in the most recent time period.

The budget process is subject to inertia. A firm may change its price and other operating decisions many times before it would consider changing the methodology used to determine its budget.

At the end of each accounting period, firms typically compute a *variance*, the difference between the budget amount and the actual result.[11] Variances provide a mechanism by which anticipated and actual performance can be reconciled. For example, suppose a firm has computed its unit costs based on a target output, and the actual output in the accounting period differs from this target output. A variance would then reconcile the unit cost the firm assumed would prevail and the unit cost that actually prevailed. Variance analysis assures that managers ultimately are held accountable for the actual performance of the firm.

2.5 Summary

The discussion above highlights key features of managerial practice which we will incorporate in our formal model:

1. Firms choose "costing methodologies" that might confound sunk, fixed, and variable costs. Firms do not necessary allocate all of fixed and sunk costs to the cost base, but this allocation cannot exceed actual fixed and sunk costs.

[10] Sophisticated firms often build flexibility into their budgeting. This flexibility allows the firm to adjust the values of key variables to reflect unexpected shocks (*e.g.,* abrupt increase in the cost of a raw material, labor strikes, and so on).

[11] See Maher, Stickney, and Weil (2004) for a through discussion of variances.

2. Firms compute the per-unit sunk cost based on a budgeted output level that remains constant during an accounting period.

3. Day-to-day management of the firm's pricing strategy is guided by the firm's budget and costing methodology. Although firms can adjust prices quickly, budgets and costing methodologies are less flexible and change less often than prices.

4. Firms periodically reconcile actual and budgeted profits by adding back variances ensuring that, at the end of each accounting period, firms observe their actual economic profit.

3 The Model

We consider an oligopoly in which boundedly rational firms may adopt costing methodologies which cause their pricing decisions to depart from those prescribed by textbook economic theory. This section sets up the basic model.

3.1 Demand and Cost Fundamentals

The industry consists of N single-product firms engaged in (Bertrand) price competition with differentiated products. (By a slight abuse of notation N also denotes the set of firms.) Firm n's quantity, denoted q_n, is determined by a demand function $q_n = D_n(p)$, where $p = (p_1, \ldots, p_N)$ denotes the industry's vector of prices.[12] We assume that firm n knows its own demand function, D_n, but not the demand functions of the other firms.

We assume that D_n is differentiable and, wherever $D_n > 0$, satisfies $\frac{\partial D_n}{\partial p_n} < 0$ and $\frac{\partial D_n}{\partial p_m} > 0$ for $m \neq n$. Further, we assume that for any pair of firms $n,\ m$, demand is strictly log-supermodular, i.e., $\dfrac{\partial^2 \log D_n(p_n, p_{-n})}{\partial p_n \partial p_m} > 0$. Log-supermodularity is equivalent to the intuitive condition that a firm's demand becomes less price elastic as a rival's price goes

[12] We use the following standard vector notation. For any vector $x \in R^l$, $x = (x_1, \ldots, x_l)$ and $x_{-n} \in R^{l-1}$ denotes the vector $(x_1, \ldots, x_{n-1}, x_{n+1}, \ldots, x_l)$. Also, $x \geq 0$ means that $x_j \geq 0$ for all j; $x > 0$ means that $x_j \geq 0$ for all j and $x \neq 0$; and $x >> 0$ means that $x_j > 0$ for all j.

up, a property that is satisfied by many common demand systems, including linear, logit, and CES.

Each firm has a constant marginal cost c_n, with $c = (c_1, \ldots, c_N) \geq 0$ denoting the vector of these costs. In addition, each firm has a per-period sunk cost, denoted $F_n \geq 0$. Our analysis also covers, without any modifications, unsunk flow fixed costs. However, we will use the term sunk cost to avoid redundancy. For example, F_n may represent the per-period capital charge for an asset that has no redeployment value. The calculation of F_n will depend on the specific accounting standards used by the firm (for instance, how depreciation is computed). We abstract from these issues and take F_n as given. In our multi-period analysis, we also assume that F_n is constant over time.

Finally, we shall assume that the set of Nash equilibria is compact and is contained in the interior of a cube $P = [0, p^+] \subset R^N$.

3.2 Economic vs. Accounting Profits

Firm n's objective measure of success is its *economic profit*, defined as

$$\pi_n^e(p) \equiv (p_n - c_n)D_n(p) - F_n.$$

The sunk cost bias distorts the firm's perception of relevant costs. We model this in a way that is both simple and consistent with managerial accounting practices discussed in Section 2. We assume that a firm may act as if its true marginal cost is inflated by a constant distortion component, $s_n \geq 0$, representing the part of sunk cost (mis-) allocated as a per unit variable cost.

To motivate this, imagine that firm n bases its cost allocation on a budgeted quantity $\hat{q}_n$ (see Section 2.4). In practice, $\hat{q}_n$ is based on past realized quantities, and is therefore independent of the firm's current decisions. For now we take $\hat{q}_n > 0$ as exogenously given. In footnote 22, we explain how its choice can be made endogenous after the full learning model is introduced.

Given $\hat{q}_n$, the firm's *accounting profit* is:

$$\pi_n(p, s_n) \equiv (p_n - c_n - s_n)D_n(p) - (F_n - s_n\hat{q}_n), \tag{1}$$

where $(F_n - s_n\hat{q}_n)$ denote the unallocated sunk cost. The accounting profit function π_n represents the manager's *perception* of the relevant payoff function within a given round of short-term price adjustments. Note that the difference between economic and accounting profits, $s_n(D_n(p)-\hat{q}_n)$, disappears if the firm is rational, $s_n = 0$, or if budgeted and actual quantities coincide. Footnote 22 elaborates on how discrepancies between budgeted and actual quantities are reconciled at the end of the accounting period.

Actual economic profit is not observed within a round of short-run price adjustments. However, at the end of the accounting period, budgeted and actual quantities are reconciled using variances, ensuring that firms ultimately observe their economic profit.

We shall require that the unallocated sunk cost be non-negative, and that firms do not "make-up" sunk costs when there are none. Formally, we require

$$s_n \leq \frac{F_n}{\hat{q}_n}.$$

In particular, $F_n = 0$ implies $s_n = 0$, so a firm with no sunk cost would be unable to inflate its true marginal cost. Note that we do not require that firms allocate all of their sunk cost. This may be justified in terms of the arbitrariness and flexibility in costing methodologies discussed in Section 2.

3.3 The Static Price Competition Game

Formally, a price competition game $\Gamma(s)$ with cost distortions s is an N-player game, each with strategy set P and payoffs function given by the accounting profit function defined in Equation (1).

In $\Gamma(s)$ firm n maximizes $\pi_n(\cdot, s_n)$ given a forecast p_{-n} of its competitors's prices, yielding a first-order condition:

$$(p_n - c_n - s_n)\frac{\partial D_n(p)}{\partial p_n} + D_n(p) = 0, \tag{2}$$

provided $D_n(p) > 0$. This gives rise to firm n's reaction function[13]

$$r_n(p, s_n) = \arg\max_{p'_n} \pi_n(p'_n, p_{-n}, s_n).$$

[13] Our assumptions on demand imply that this is well defined and single valued.

Our assumptions on demand imply that $r_n(s_n) : P^N \to P$ is differentiable on the interior of P^{N-1} and strictly increasing:

$$\forall n \neq m, \ \frac{\partial r_n}{p_m} > 0.$$

Let $r(s) : P^N \to P^N$ denote the vector of reaction functions. An equilibrium for $\Gamma(s)$ is a price vector $\bar{p}$ such that $\bar{p} = r(\bar{p}, s)$. Let $\mathcal{E}(s)$ denote the set of equilibria for $\Gamma(s)$. Given the assumption that demand is log-supermodular, the price-setting game $\Gamma(s)$ will be supermodular, which implies that a pure strategy equilibrium exists (Milgrom and Roberts (1990, Theorem 5)). Our assumptions do not, however, rule out the possibility of multiple equilibria. Note that any equilibrium $\bar{p}$ such that all firms are active (*i.e.,* $\bar{q} >> 0$) must satisfy $\bar{p} >> c + s$.

4 An Illustrative Example

To motivate our formal analysis, we consider a simple example. There are two firms facing common marginal cost $c = 10$, sunk cost $F > 0$, and demand curves[14]

$$D_n(p_n, p_m) = 124 - 2p_n + 1.6p_m, \quad m \neq n, \quad m, n \in \{1, 2\}.$$

Initially, firms are at the (unique, symmetric) Bertrand equilibrium of this game.

Suppose that this game is repeatedly played by myopic players. Unbeknownst to its opponent, firm 1 "experiments" with a new costing methodology which inflates its relevant costs by $s_1 = 10$ per unit, and that firms adjust their prices *adaptively*, say, by following a simple procedure like the Cournot dynamic.[15] Then firm 1's experiment triggers a process of pricing adjustments that leads to higher limiting prices and economic profits for both firms. In fact, firm 1's equilibrium price increases from 60 to about 65 and its economic profits increase from 5,000 to about 5,149. Suffering from a sunk cost bias yields higher profits to firm 1!

[14] This is the cost and demand system used in Offerman and Potters (2006). See Section 7.1 for comparison with their experimental findings.

[15] Under the Cournot dynamic firms best respond to their opponents' last period price (see, for instance, Vives (1999)). Our analysis applies more generally, as described in the next section.

This informal story has a flavor similar to the familiar logic of strategic commitment. There are crucial differences, however: Strategic commitments matter only to the extent that they are observable and irreversible. By contrast, firms' internal accounting methodologies and the confusions about relevant costs that these methodologies entail are neither observable nor irreversible. The intuition underlying our analysis is based instead on firms' predisposition to commit the sunk cost bias, their use of naive adaptive rules to set prices, and the stickiness of their costing methodologies. Our formal model shows that firms' initial predisposition is reinforced under some market structures, while it disappears under others.

Learning plays two crucial roles in our analysis. First, it is implausible that firms that succumb to the sunk cost fallacy would, at the same time, have the sophistication necessary to carry out the complex reasoning justifying Nash equilibrium play. The alternative view, which we take here, is that a Nash equilibrium in the pricing game is the result of a naive adaptive process carried out by boundedly rational players. We use the theory of learning for supermodular games to model myopic firms who adjust prices adaptively, unaware of the demand and costing practices of their opponents. Second, although a positive cost distortion will shift the equilibrium *set* upward,[16] equilibrium analysis is consistent with the selection of a lower equilibrium price within the new set. In this case, arbitrary equilibrium selection, rather than fundamentals, end up driving the persistence of the sunk cost bias. Our first main result, Theorem 1, shows that learning can serve as a foundation for meaningful comparative statics when multiple equilibria may be present.

In the above informal discussion, we focused on the consequences of firms' experiments, while taking the experiments themselves as exogenously given. Our second main result, Theorem 2, endogenizes the process of experimentation in a model where costing methodologies are reinforced depending on how well they have done in the past on average.

[16] In the sense that the smallest (largest) equilibrium price vector of $\Gamma(s')$ is larger than the smallest (largest) equilibrium price vector of $\Gamma(s)$. See Milgrom and Roberts (1990).

5 Main Results

5.1 Adaptive Learning and Comparative Statics

We first define adaptive pricing processes in the game $\Gamma(s)$ repeatedly played by myopic players.

Definition 1 *An* adaptive pricing adjustment *for* $\Gamma(s)$ *starting at* $\bar{p}$ *is a sequence of prices* $\{p_t\}$ *such that* $p_0 = \bar{p}$ *and there is positive integer* γ *such that for every* t, p_t *satisfies*[17]

$$r(\inf\{p_{t-\gamma}, \ldots, p_{t-1}\}, s) \;\leq\; p_t \;\leq\; r(\sup\{p_{t-\gamma}, \ldots, p_{t-1}\}, s).$$

That is, at time t, each firm best responds to some probability distribution on the recent past history of play of their opponents. The class of adaptive pricing adjustment process is quite broad, and includes, as special cases, the Cournot dynamic where $p_t = r(p_{t-1})$, a version of fictitious play in which only the past γ rounds of play are taken into account, and sequential best response.

The key comparative statics result needed for our analysis is:

Proposition 1 *Suppose that* $\bar{p} \leq r(\bar{p}, s)$. *Then*

$$\lim_{t\to\infty} p_t = \inf\{z \in \mathcal{E}(s); z \geq \bar{p}\}$$

for any adaptive pricing adjustment sequence $\{p_t\}$ *for* $\Gamma(s)$ *starting at* $\bar{p}$.

The proof is essentially that of Theorem 3 in Echenique (2002) specialized to the case where best replies are single-valued.[18] The comparative statics implications of this result is that starting with an equilibrium price $\bar{p}$, if one firm distorts its cost, then any adaptive pricing adjustment must converge to the *lowest* price vector that is higher than $\bar{p}$. Note that this result yields unambiguous prediction on the direction of price changes,

[17] For $t < \gamma$ we set $p_{t-\gamma} = p_0$.

[18] As r is a strictly increasing reaction function, the lowest and highest selections from r are equal. Also, the lowest and highest selections from iterated applications of r are also equal. The proof of part 2 of Theorem 3 in Echenique (2002) shows that these selections define the lower and upper bounds of the limit points of any adaptive dynamic. Hence, as r is a best-reply function, this limit is unique. Finally, part 1 of Theorem 3 shows that this limit must be equal to the lowest equilibrium higher than $\bar{p}$.

but is silent on the effect on economic profits, which is our primary concern. This is what our first theorem deals with.

5.2 Distortion Result

Theorem 1 *There exists $\Delta > 0$ such that given any firm n, vector of distortions $\bar{s}_{-n} \geq 0$, equilibrium price vector $\bar{p}$ of $\Gamma(\bar{s}_{-n}, s_n = 0)$ with corresponding quantity vector $\bar{q} >> 0$, if firm n chooses $0 < \hat{s}_n < \Delta$ then for any adaptive pricing adjustment $\{p_t\}$ for $\Gamma(\bar{s}_{-n}, \hat{s}_n)$ starting at $\bar{p}$ there is a time T such that:*

$$\pi_n^e(p_t) > \pi_n^e(\bar{p}) \text{ for every } t \geq T \ .$$

Proofs of this and subsequent results are in the Appendix.

Theorem 1 says that no matter what costing methodologies are employed by rival firms, there is always a cost distortion $\hat{s}_n$ that eventually results in higher economic profits for firm n. To provide an intuition for the theorem, suppose we are initially at any equilibrium $\bar{p}$ of the game $\Gamma(\bar{s}_{-n}, s_n = 0)$. A cost distortion by firm n to $\hat{s}_n > 0$ causes firm n to raise its price. This triggers an adaptive adjustment process that eventually settles at a strictly higher equilibrium price. The question is, does this yield higher economic profits for firm n? Note, first, that for any sequence of distortions converging to 0, the corresponding equilibrium prices converge to an equilibrium $\hat{p}$ of $\Gamma(\bar{s}_{-n}, s_n = 0)$. There are two cases. First, if $\hat{p} >> \bar{p}$ then it is easy to see that economic profits of firm n must strictly increase. Roughly, this happens if $\bar{p}$ is not locally stable, so a slight perturbation leads to a large jump in prices. The second case is that $\hat{p} = \bar{p}$. Here, roughly, the net effect on the economic profits of firm n consists of a negative direct effect (due to the distortion of its price, holding the prices of its rivals fixed) and a positive strategic effect (due to its rivals raising their prices). We show that for a small enough distortion of relevant costs, the latter dominates the former.

We note that no assumptions are made about firm n's knowledge about the demand functions, costs, or costing methodologies of its rivals. Nor does any firm need to know, specifically, what new equilibrium would be reached if it distorted its relevant cost.

5.3 Adjustment of Costing Methodologies

We now turn to the process through which firms choose costing methodologies. As suggested in the introduction, new complications arise, namely that firms do not know their rivals' strategies or their own payoff functions. Since firms must learn about their payoffs as well as resolve the strategic uncertainty about their opponents' behavior, the learning model here is of necessity different from the *adaptive* model used in the pricing game.[19]

We introduce a model of *reinforcement* learning through trial and error. In this model, motivated by Milgrom and Roberts (1991), firms know their strategy set and observe their realized payoffs. But they do not know their payoff functions or the strategies used by their opponents. Firms experiment with different costing methodologies from time to time and reinforce those that have done well on average in the past.

5.3.1 A Dynamic Game of Experimentation and Cost Adjustments

We assume firms can adjust prices much more frequently than they can adjust their costing methodologies. To model this, we first restrict firms to choose from a finite grid of relevant cost distortions $S_n = \{0, \Delta, 2\Delta, \ldots, K\Delta\}$, where K is a positive integer and $\Delta > 0$ (S_n is the same for all firms). Firm n chooses $s_n(\tau) \in S_n$ at times $\tau = 1, 2, \ldots$. Within each "interval" $[\tau, \tau + 1]$, we have a sequence of *price adjustment sub-periods* $t = 1, 2, \ldots$ during which firms may adjust prices, but not their distortion of relevant costs.[20]

To define the dynamic game of cost adjustments, fix $s(0)$ arbitrarily and let $p(0)$ be

[19] Most adaptive learning models assume that firms know their own payoff function and the actions taken by their opponents. We are aware of only a few models that can handle the case of unobserved opponents' actions and unknown own-payoff function, all of which rely on players' randomly experimenting with different strategies. Milgrom and Roberts (1991) and Hart and Mas-Colell (2001) describe adaptive procedures that eventually leads to the play of only serially undominated strategies and correlated equilibrium respectively. Friedman and Mezzetti (2001) propose a procedure that leads to the play of a Nash equilibrium in supermodular (as well as other classes of games with a property called weak Finite Improvement Property). The problem is that supermodularity of the the costing methodology game requires stringent, difficult to interpret conditions on the underlying model. In particular, it does not necessarily follow from the supermodularity of the pricing game.

[20] Thus, formally, we have a double index $\{\tau_t\}_{\tau,t=1}^{\infty}$ and where indices are ordered lexicographically: $\tau'_{t'} > \tau_t$ if and only if $\tau' > \tau$ or $[\tau' = \tau$ and $t' > t]$.

any equilibrium of the game $\Gamma(s(0))$.[21] At time τ, firm n is chosen with probability $\frac{1}{N}$, at which point this firm is allowed to re-evaluate its costing methodology. With probability $\epsilon > 0$, this firm experiments by picking a distortion $s_n(\tau)$ uniformly from S_n. We call these periods, "firm n's experimental periods." With probability $1-\epsilon$, firm n picks the distortion that generated the highest average payoff in its past experimental' periods. Firms experiment independently from each other and over time. All other firms $m \neq n$ set $s_m(\tau) = s_m(\tau-1)$.

Firm n's payoff at τ is:

$$\Pi_n(s(\tau), p(\tau-1)) \equiv \pi_n^e(p(\tau))$$

where $p(\tau) = \lim_{t\to\infty} p_t$ and $\{p_t\}$ is any adaptive pricing adjustment sequence for $\Gamma(s(\tau))$ starting at $p(\tau-1)$.[22] To motivate this definition, suppose that $\tau > 0$ is an experimental period for firm n in which firm n chooses $s_n(\tau) \neq s_n(\tau-1)$. This triggers a process of price adjustments starting from $p(\tau-1)$ and converging to a new equilibrium price $p(\tau)$. Our assumption that prices adjust more rapidly than costing methodologies implies that the limiting price $p(\tau)$ obtains "before" period $\tau+1$ arrives. The payoff function above says that firm n uses the profit from this final price, $\pi_n^e(p(\tau))$, in judging its experiment at τ.

Our model is motivated by Milgrom and Roberts (1991)'s study of learning and experimentation in repeated, normal form games. Their analysis is inapplicable in our setting, however, because we are dealing with a *dynamic* game with history dependent payoffs. In our model, the payoff structure at time τ is a reduced form for an underlying pricing dynamic whose outcome may depend on the initial condition $p(\tau-1)$.

5.3.2 Learning result

Theorem 2 *There is $\Delta > 0$ such that, with probability one, there is $\bar{\tau}$ such that for every firm n, in all non-experimental periods $\tau \geq \bar{\tau}$, $s_n(\tau) > 0$.*

[21] The assumption that $p(0)$ is an equilibrium of $\Gamma(s(0))$ can be relaxed, but not entirely dropped. See Echenique (2002).

[22] With this notation, we can now endogenize the choice of the reference quantity $\hat{q}_n$ used to determine the per unit sunk cost $\frac{F_n}{\hat{q}_n}$ that provides an upper bound on s_n. Formally, we set $\hat{q}_n(\tau) = q_n(\tau-1)$ for $\tau > 1$ and set $\hat{q}_n(0) > 0$ arbitrarily.

That is, on almost all paths all firms eventually distort their relevant costs. The strength of the results is in how weak the assumptions are. Firms only know their past distortion choices and, in each case, how well they have done in price competition. This rather coarse information does not allow firms to correlate changes in their payoffs with changes in their rivals actions, or conduct counterfactuals like "I would do better by choosing s_n' given the actions of my rivals." Nevertheless, this coarse information is sufficient for firms to eventually reject "rational" costing.

5.4 No-distortion Results

Our analysis makes sharp predictions about which environments are unlikely to reinforce the sunk cost bias, and which environments tend to eliminate it.

5.4.1 Monopoly Market

The intuition underlying our analysis is that the benefit from distorting relevant costs stems from its favorable strategic effects. This strategic effect is absent in monopoly, so our model implies that no distortion should persist in a monopoly market. Even though a monopolist is just as likely to be predisposed to the sunk cost bias initially, (non-strategic) learning will eventually lead the firm to price optimally. This conclusion is consistent with the experimental results of Offerman and Potters (2006) who find that the effects of the sunk cost bias disappear in the monopoly treatment of their experiments.[23]

5.4.2 Perfect Competition

The sunk cost bias would also not persist in a market with price-taking firms. Let p^* be the equilibrium price. Since an individual firm's output decision has no effect on the price and hence on other firms output and pricing decisions, a firm that distorts its relevant costs moves its own output away from the profit-maximizing quantity and hence reduces its economic profits. There is indirect evidence in support of this conclusion: Maher, Stickney, and Weil (2004, p. 258) note that "cost-based pricing is far less prevalent in

[23] See Section 7.1 for a more detailed discussion.

Japanese process-type industries (for example, chemicals, oil, and steel)", all of which are products with very weak differentiation.

5.4.3 Price Competition with Homogenous Product Oligopolies

Finally, no distortion should persist in a Bertrand oligopoly with homogenous products. To establish this, suppose there are N firms with constant marginal costs ordered so that $c_1 \leq c_2 \ldots \leq c_N$. Let $D(p)$ be a demand curve which is continuous and strictly decreasing at all p where $D(p) > 0$ and that there is a "choke price" $\bar{p} < \infty$ such that $D(p) = 0$ for all $p \geq \bar{p}$ and we assume $\bar{p} > c_1$. The demand for firm n is

$$D_n(p) = \begin{cases} D(p_n) \text{ if } p_n \text{ is the lowest price} \\ \frac{1}{J} D(p_n) \text{ if } p_n \text{ and } J-1 \text{ other firms set the lowest price} \\ 0 \text{ if } p_n \text{ is not the lowest price.} \end{cases}$$

We also assume that the price firm 1 sets if it is a monopolist is greater than c_2. Otherwise, firm 1 is effectively a monopolist even in the presence of competition and hence the analysis is very similar to the monopoly case above.

Proposition 2 *There is no incentive to distort relevant costs in a homogeneous product oligopoly.*

The intuition underlying this result is straightforward. In a homogenous product Bertrand oligopoly, each firm has such a strong incentive to undercut its competitor to capture the entire market that it is impossible to soften price competition via cost distortion.

6 Special Case: Symmetric Linear Demand

In this section we examine the important special case of symmetric linear demand. This additional structure enables us to generate comparative statics predictions about how the distortion changes with the number of firms, the degree of product differentiation, and so on. Theorems 1 and 2 illustrated that our main points hold generally. Once we move the special structure of symmetric linear demand, the analysis becomes much simpler: there is a unique equilibrium in the pricing game for any profile of distortions.

The cost distortion game is supermodular, dominance solvable, and its Nash equilibrium is also the unique correlated equilibrium (Milgrom and Roberts (1990)). A broad class of learning models, including our own, converge to the unique equilibrium in the distortion game.

We assume that the N firms in the industry have a common marginal cost c and face a symmetric system of firm-level demand curves that is consistent with maximization by a representative consumer who has a quadratic net benefit function. The representative consumer chooses quantities $q = (q_1, \ldots q_N)$ to maximize:

$$U(q) = aq - \frac{1}{2}bq\Theta q - pq,$$

where a and b are positive constants and

$$\Theta = \begin{pmatrix} 1 & \theta & \ldots & \theta \\ \theta & 1 & \ldots & \theta \\ \vdots & & \ddots & \vdots \\ \theta & \theta & \ldots & 1 \end{pmatrix},$$

where $\theta \in (0, 1)$ parameterizes the extent of horizontal differentiation among the goods. As $\theta \to 0$, the goods become independent, and as $\theta \to 1$, the goods become perfect substitutes. Throughout, we assume that $a > c$.

The system of demand functions implied by the solution to the representative consumer's utility maximization problem is given by:

$$q_n = D_n(p) = \alpha - \beta p_n + \gamma \sum_{m \neq n} p_m, \; n \in \{1, \ldots, N\},$$

where

$$\begin{aligned} \alpha &\equiv \frac{a(1-\theta)}{b(1-\theta)\left[1+(N-1)\theta\right]} > 0, \\ \beta &\equiv \frac{1+(N-2)\theta}{b(1-\theta)\left[1+(N-1)\theta\right]} > 0, \\ \gamma &\equiv \frac{\theta}{b(1-\theta)\left[1+(N-1)\theta\right]} \in (0, \beta). \end{aligned}$$

For later use, note that

$$2\beta - (N-1)\gamma = \frac{2(1-\theta)+(N-1)\theta}{b(1-\theta)\left[1+(N-1)\theta\right]} > 0.$$

The (unique, symmetric) Nash equilibrium in prices *without* distortions is

$$p^0 = \lambda a + (1-\lambda)c,$$

where

$$\lambda = \frac{(1-\theta)}{2(1-\theta)+(N-1)\theta} \in (0, \frac{1}{2}).$$

Now, suppose that firm n chooses costing methodology $s_n \in [0, s^+]$, where $s^+ \equiv \frac{F}{\hat{q}}$ is the maximum distortion consistent with a firm's sunk cost and is assumed to be common across all firms. The first-order condition for a firm is:

$$\frac{\partial \pi_n(p, s_n)}{\partial p_n} = -\beta(p_n - c) + \alpha - \beta p_n + \gamma \sum_{m \neq n} p_m + \beta s_n = 0, \ n \in \{1, \ldots, N\}. \quad (3)$$

Given a vector s of distortions, firm n's second-stage equilibrium price $\overline{p}_n(s)$ is found by solving this system of first-order conditions:

$$\overline{p}_n(s) = p^0 + \frac{\beta}{2\beta+\gamma} s_n + \frac{\beta}{2\beta+\gamma}\left(\frac{\gamma}{2\beta-(N-1)\gamma}\right)\sum_{m=1}^{N} s_m. \quad (4)$$

This implies

$$\frac{\partial \overline{p}_n}{\partial s_n} = \frac{\beta}{2\beta+\gamma}\left[1 + \frac{\gamma}{2\beta-(N-1)\gamma}\right] > 0. \quad (5)$$

$$\frac{\partial \overline{p}_n}{\partial s_m} = \frac{\beta}{2\beta+\gamma}\frac{\gamma}{2\beta-(N-1)\gamma} > 0. \quad (6)$$

Finally, firm n's economic profit is

$$\overline{\pi}_n^e(s) \equiv \overline{\pi}_n^e(\overline{p}_n(s)) = (\overline{p}_n(s) - c)\left[\alpha - \beta \overline{p}_n(s) + \gamma \sum_{m \neq n} \overline{p}_m(s)\right]. \quad (7)$$

We call the game where the firms choose distortions and payoffs are determined by (7) and (4), the *distortion game.*

Firm n's problem in the distortion game is

$$\max_{s_n \in [0, s^+]} \overline{\pi}_n^e(s) = (\overline{p}_n(s) - c)\left[\alpha - \beta \overline{p}_n(s) + \gamma \sum_{m \neq n} \overline{p}_m(s)\right].$$

We begin by establishing some basic properties of the distortion game:

Proposition 3 *The distortion game is supermodular and has a unique, symmetric equilibrium distortion s^*.*

Throughout, we assume that the symmetric equilibrium in the first-stage is less than the upper bound s^+, and below we show that this distortion is generally positive for $\theta \in (0,1)$. Given this, the equilibrium distortion s^*satisfies

$$\frac{\partial \overline{\pi}_n^e(s^*)}{\partial s_n} = 0, \ n \in \{1, \ldots, N\},$$

with the induced equilibrium price $p^* = \overline{p}_n(s^*)$, $n \in \{1, \ldots, N\}$. These conditions can be shown to imply:

$$s^* = \frac{(p^* - c)(N-1)\xi\gamma}{\beta} \tag{8}$$

$$p^* = p^0 + \frac{\beta s^*}{2\beta - (N-1)\gamma}, \tag{9}$$

where $\xi \equiv \frac{\frac{\partial \overline{p}_n}{\partial s_m}}{\frac{\partial \overline{p}_n}{\partial s_n}} = \frac{\frac{\gamma}{2\beta-(N-1)\gamma}}{1+\frac{\gamma}{2\beta-(N-1)\gamma}} \in (0,1)$.

With this derivation in hand, we can establish a baseline set of no-distortion results, that mirror those in Section 5.4:

Proposition 4 *(a) As the goods become independent, the equilibrium distortion goes to zero, i.e.,* $\lim_{\theta \to 0} s^* = 0$*; (b) As the goods become perfect substitutes, the equilibrium distortion goes to zero, i.e.,* $\lim_{\theta \to 1} s^* = 0$*; (c) As the number of firms becomes infinitely large, the equilibrium distortion goes to zero, i.e.,* $\lim_{N \to \infty} s^* = 0$

We now turn our attention to circumstances under which the equilibrium distortion is positive. To do so, we solve (8) and (9) in terms of the primitives of the model: a, b, c, θ,and N.

$$p^* - c = \frac{\left(p^0 - c\right)\left[(2-\theta) + (N-1)\theta\right]}{(2-\theta) + (N-1)\theta(1-\theta)} \tag{10}$$

$$s^* = \frac{\left(p^0 - c\right)(N-1)\theta^2}{\left[(2-\theta) + (N-1)\theta(1-\theta)\right]\left[(1-\theta) + (N-1)\theta\right]}. \tag{11}$$

An immediate implication of these expression is that when there is some degree of (imperfect) product differentiation, firms distort their relevant costs in equilibrium, which in turn elevates the equilibrium price relative to the single-stage Nash equilibrium:

Proposition 5 *If $\theta \in (0,1)$ the equilibrium distortion is positive, i.e., $s^* > 0$. Moreover, the induced equilibrium price exceeds the Nash equilibrium price without distortions, i.e., $p^* > p^0$.*

Table 1 shows calculations of the Nash price, the equilibrium distortion s^*, and the induced equilibrium price p^* for different numbers N of firms and values of the product differentiation parameter, θ.[24] As one would expect given Proposition 4, the equilibrium distortion is not monotone in θ. For any N, it initially increases in θ but eventually begins to decrease, going to zero as the products become perfect substitutes. The equilibrium distortion is not monotone in N either. For low values of θ, the distortion initially increases in N, but eventually decreases to 0 as N becomes sufficiently large. When the number of firms is large, the equilibrium distortion is extremely small. However, in an industry with a small number of firms and an intermediate degree of product differentiation, the equilibrium distortion can be significant. For instance, when $N = 2$ and $\theta = 0.80$, the equilibrium distortion is \$21.82, about 36 percent of the undistorted Nash price of \$60.

[24] The parameters for these calculations are as follows: $a = 310, b = \frac{350}{252}$, and $c = 10$.

θ		$N=2$	$N=3$	$N=5$	$N=10$	$N=100$
$\to 0$	p^0	160.00	160.00	160.00	160.00	160.00
	s^*	0	0	0	0	0
	p^*	160.00	160.00	160.00	160.00	160.00
0.10	p^0	152.11	145.00	132.73	110.00	33.08
	s^*	0.71	1.17	1.65	1.81	0.18
	p^*	152.45	145.65	133.71	111.20	33.24
0.20	p^0	143.33	130.00	110.00	80.59	21.21
	s^*	2.70	3.70	3.95	2.80	0.10
	p^*	144.83	132.22	112.63	82.73	21.31
0.40	p^0	122.50	100.00	74.29	47.50	14.41
	s^*	9.47	9.18	6.29	2.62	0.04
	p^*	128.42	106.43	79.23	49.80	14.45
0.60	p^0	95.71	70.00	47.50	29.35	11.99
	s^*	17.70	12.05	5.76	1.72	0.02
	p^*	108.36	79.64	52.54	30.97	12.01
0.80	p^0	60.00	40.00	26.67	17.89	10.75
	s^*	21.82	9.88	3.40	0.80	0.01
	p^*	78.18	48.89	29.88	18.68	10.76
0.90	p^0	37.27	25.00	17.89	13.61	10.34
	s^*	17.48	6.12	1.80	0.39	≈ 0
	p^*	53.17	30.81	19.64	14.00	10.34
$\to 1$	p^0	10.00	10.00	10.00	10.00	10.00
	s^*	0	0	0	0	0
	p^*	10.00	10.00	10.00	10.00	10.00

Table 1 suggests that even though the distortion is non-monotonic in the number of firms, the induced equilibrium price is. The next proposition confirms this:

Proposition 6 *The induced equilibrium price p^* is strictly decreasing in the number of firms.*

Since firms do not internalize the beneficial impact of their distortion of relevant costs on their rivals' profits, they end up with a price lower than the monopoly price:

Proposition 7 *When $\theta \in (0,1)$, the induced equilibrium price is less than the monopoly price.*

Let us now turn to a comparative statics analysis of the equilibrium distortions with respect to the other parameters of the model, a and c. To avoid conflating the impact of a and c on the distortion with their effect on the overall price level, we derive the comparative statics result for the percentage distortion above the undistorted Nash equilibrium price, $\frac{s^*}{p^0}$

Proposition 8 *The percentage distortion $\frac{s^*}{p^0}$ is increasing in a and decreasing in c. Hence an increase in demand, as measured by a larger value of a, will result in a greater percentage distortion, while an increase in marginal cost will result in a lower percentage distortion.*

This result makes sense. The difference $a - c$ measures the intrinsic value of the industry profit opportunity. The incremental benefit to a firm from suppressing price competition will be greater the more intrinsically profitable the market is, and as a result, the degree of distortion is greater the greater is a or the lower is c.

We conclude by considering the implications of this analysis for entry. To do so, we imagine that each firm faces a sunk cost of entry F. With no distortion, the equilibrium number of firms N^0 is given by the condition:

$$(p^0 - c)(\alpha - \left[\beta - (N^0 - 1)\gamma\right] p^0 = F,$$

which can be rewritten

$$\left(\frac{a}{b} - \frac{p^0}{b}\right) \frac{\left(p^0 - c\right)}{1 + (N^0 - 1)\theta} = F. \tag{12}$$

Similarly, with distortions, the equilibrium number of firms N^* is given by:

$$\left(\frac{a}{b} - \frac{p^*}{b}\right) \frac{(p^* - c)}{1 + (N^* - 1)\theta} = F. \tag{13}$$

Proposition 9 *When $\theta \in (0, 1)$, the equilibrium number of firms is greater than the equilibrium number of firms in the game where no firm distorts, i.e., $N^* > N^0$.*

This is because distortions increase equilibrium margins, making entry more desirable.

7 Discussion

7.1 Experimental Results

As noted in the introduction, there is strong evidence from the psychology literature that individual decision makers commit the sunk cost fallacy. A well-cited example is a study by Arkes and Blumer (1991) which found that the sunk costs incurred by season ticket holders for a university theater company apparently influenced attendance decisions.

Experimental economists have also studied sunk cost biases, but the evidence from this work is more mixed. Economists' experiments on the sunk cost fallacy differ from those in the psychology literature in that the latter typically do not focus on decision making in market settings in which competition might be expected to create strong pressures for decision makers to "get it right." By contrast, economist's experiments on sunk costs generally place subjects in the context of some sort of market rivalry. For example, Phillip, Battalio, and Kogut (1991) study bidding in first-price sealed bid auctions in which subjects paid a sunk admissions fee in order to participate. They found that 95 percent of the subjects correctly ignored sunk costs in their bidding behavior. Summarizing a variety of experiments in competitive market settings in which sunk costs might effect market outcomes Smith (2000) writes: "These results showed no evidence of market failure due to the sunk cost fallacy."

The finding that subjects in competitive market experiments do not succumb to the sunk cost fallacy is consistent with the prediction of our theory that it would run counter to the self-interest of a price-taking firm, or a price-setting firm in a homogeneous product oligopoly, to base decisions on sunk costs. However, a more direct test of the implications of our theory would be to explore pricing behavior in a differentiated-product oligopoly market in which firms incur sunk costs. Unfortunately, most of the existing experimental investigations of price competition in a differentiated product oligopoly (e.g., Dolbear, Lave, Bowman, Lieberman, Prescott, Reuter, and Sherman (1968)) are not relevant to our theory. This is because these experiments typically provided subjects with a table showing profit as a function of own price and the prices of rivals. Participants were simply asked to choose a price, and they did not have to figure out for themselves which costs might be relevant for pricing decisions.

An important exception to this approach is a recent paper by Offerman and Potters (2006). In this study, subjects assumed the role of decision makers in a differentiated product duopoly, and they were given information on firms' demand curves and marginal costs. Subjects were placed in one of three treatments: one in which rights to operate in the market were auctioned to the two highest bidders; one in which entry rights were allocated randomly to two subjects who were required to pay an up-front fee;[25] and one in which entry rights were allocated randomly without payment of a fee. Offerman and Potters find that when an entry fee is charged (either via a fixed fee or by means of an auction) prices are significantly higher than the undistorted (symmetric) Nash equilibrium price, but when no entry fee is charged, prices were close to the undistorted equilibrium price. By contrast, in a monopoly market, prices tend to equal the profit-maximizing monopoly price, irrespective of the level or form of the sunk entry fee.

Our theoretical model gives exactly the same predictions. Our theory implies that a monopolist would have no incentive to succumb to the sunk cost fallacy. In a duopoly with differentiated products but with no sunk costs, firms might ultimately benefit from distorting their relevant costs, but because they lack a compelling justification for distortionary behavior, our theory would predict that they would end up charging the undistorted Nash equilibrium price. When sunk costs are present, firms have the opportunity to distort relevant costs upwards, and given the demand curves and marginal costs in the Offerman and Potters' environment, it makes sense to do so. Indeed, in this case, the fit between our model and Offerman and Potters' quantitative results is remarkably tight. In the calculations in Table 1 in the previous section, we chose parameters so that when $\theta = 0.80$ and $N = 2$, we have an economic environment that is identical to that in Offerman and Potters' duopoly experiments.[26] As shown in Table 1, the equilibrium price in the two-stage game predicted by our model is 78.18, about 30 percent higher than the undistorted Nash equilibrium price of 60. In Offerman and Potters's fixed entry

[25] Unbeknownst to the subjects, the fixed entry fees were set equal to the entry fees generated in the auction treatment.

[26] In particular, recall that we set $a = 310, b = \frac{350}{252}$, and $c = 10$. When $\theta = 0.80$ and $N = 2$, the firms' demand curves are given by $D_1(p_1, p_2) = 124 - 2p_1 + 1.6p_2$ and $D_2(p_1, p_2) = 124 - 2p_2 + 1.6p_1$, exactly as in the Offerman and Potters experiment.

fee and auction treatments, the average prices were 75.65 and 72.73, respectively.[27]

Offerman and Potters interpret their results as suggesting that entry fees generate tacit collusion. In fact, their explanation itself relies on a bounded rationality assumption as they study finite repetitions of an oligopoly game which has a unique subgame perfect equilibrium. Moreover, their explanation predicts that tacit collusion should also occur in a Bertrand oligopoly with homogenous products. This does not coincide with our predictions and hence provides a way of testing our theory against theirs.

In conclusion, we note that Offerman and Potters's experiment is not a formal test of our model. Our model requires separate pricing and costing decisions and a specific timing structure in which adjustments in these decisions occur at different rates. Nevertheless, models should ideally provide robust insights that hold beyond the narrow formalism on which they are based. It is in this sense that we view their results as suggestive of the plausibility of the core intuition of our paper.

7.2 Cournot Competition and Behavioral Biases

Suppose firms engage in Cournot quantity competition, rather than Bertrand price competition. In the Bertrand model, a firm that suffers from the sunk cost fallacy charges a higher price for any given prices of its rivals. That is, it becomes a less aggressive competitor and this triggers a movement to higher prices and profits. In the Cournot model, a firm benefits from producing a higher quantity for any given quantities of its rivals and by becoming a *more* aggressive competitor. This occurs when the firm acts as if its marginal cost is *lower* than it actually is.

One bias that might lead to this behavior is a tendency to give explicit weight to market share in the firm's objective function. Alternatively, managers may overlook relevant opportunity costs.[28] For example, a manager whose firm has purchased an input under a long-term contract may treat the cost of that input as sunk when, in fact, it is variable if the input can be resold in the marketplace. A similar bias can arise if the contract price of the input is less than the current market price, and the manager

[27] These calculations are based on the data reported in Table 1 in Offerman and Potters.

[28] Phillip, Battalio, and Kogut (1991) found evidence that subjects treat opportunity costs as different from direct monetary outlays.

computes marginal cost based on the historical, rather than current, market price.

But the presence of sunk costs should not further exacerbate these biases. This, then, implies that the difference between Cournot quantity setting and Bertrand price setting provides a refutable implication of our theory. Under Bertrand price setting, prices should be higher in an experimental treatment in which firms face sunk costs than in one in which sunk costs are zero.[29] By contrast, under Cournot quantity setting, the equilibrium price should not vary across experimental treatments that differ only in the degree to which firms have sunk costs.

7.3 Alternative Explanations: Noisy Observation of Cost, Delegation, and Evolution

Cost and price distortions may arise as a result of firms observing their total cost, but not its split between fixed, sunk and variable components. In this case rational firms' estimates of relevant costs are likely to include non-zero errors similar to the distortions introduced in this paper. In this case a plausible scenario is that firms' errors have zero mean, in which case no systematic distortion in relevant costs, prices, or profits would appear.

It also is instructive to compare our model with models of strategic delegation, as in the seminal work of Fershtman and Judd (1987). In contrast to delegation models, we study firms with myopic incentives who arrive at their decisions through a process of naive adaptive learning. Our firms do not actively fine-tune their incentive schemes in anticipation of more favorable second-stage equilibrium outcomes. Our model also has testable implications that separates it from the delegation framework. First, the delegation model would predict higher prices regardless of the presence of sunk cost, a prediction that can be experimentally tested. Second, the logic of delegation has no bite when prices are set by the firms' owners, as in experiments where each firm is a single subject (so there is literally no delegation). In our model, whether the owner of the firm is the same as the agent setting prices is irrelevant.

[29] Recall, this is what Offerman and Potters (2006) found.

Related ideas appear in the literature on the evolution of preferences. Samuelson (2001) offers an excellent overview of this literature.[30] Closer to our model is the recent work of Heifetz, Shannon, and Spiegel (2004). They offer an interesting model of the evolution of optimism, pessimism and interdependent preferences in dominance-solvable games. Like strategic delegation, the evolution of preferences literature underscores the value of commitment, but this time using distortions of players' perceptions of their payoffs as a commitment device. The unobservability of firms' costing practices again separates our model from evolution of preferences approach. For instance, Dekel, Ely, and Yilankaya (1998) point out, this approach has little bite when players cannot observe their opponents' preferences. And as in the delegation paradigm, sunk cost plays no role in evolutionary arguments; these arguments work equally well in settings where no sunk cost is present.

8 Concluding Remarks

There is extensive evidence that real-world decision makers violate the predictions of standard economic theory. Among these violations, the sunk bias in pricing decisions stands out on a number of grounds. First, its impact is not limited to small-stakes decisions: pricing is one of the most critical decisions a company can make. Second, unlike other cognitive biases that disappear once the underlying fallacy is explained, the sunk cost bias in pricing seems to thrive despite the relentless efforts of economics and business educators to stamp it out. Third, while many cognitive biases disappear through learning and training, the survey evidence we report provide no indication that this bias is disappearing over time.

In this paper we provided a theory of why the sunk cost bias persists in pricing practice.[31] We showed that under conditions rooted in actual cost accounting practices,

[30] The special issue of *The Journal of Economics Theory*, 2001, vol. 97 which is devoted for this line of research

[31] The only other model we are aware of that studies the effect of the sunk cost bias in differentiated Bertrand competition is by Parayre (1995). He considers a two stage game in a linear environment similar to the special case of symmetric linear demand we study in Section 6 (in his model, no naive learning justification of behavior is considered). Parayre assumes that the sunk cost bias has the implication that

price competition with product differentiation reinforces managers' innate predisposition to succumb to the sunk cost bias. And although there is no reason to suspect this predisposition to systematically vary across industries, no similar forces appear in monopoly, perfect competition, or price competition with homogenous products. Thus, our analysis makes predictions that can be empirically and experimentally tested.

Our theory builds on recent advances in learning in games played by naive players who know little about their environments. As such, this paper illustrates how learning theory can be a valuable tool in understanding the nature of behavioral biases in economics.

firms produce in order to use up whatever capacity is available to them. Compared to the standard Bertrand equilibrium, Parayre's model leads to *lower* prices and profits. Our model sheds light on the form of distortion that is likely to persist in a competitive environment. While the form of distortion we propose will be perpetuated in the long run, our model predicts that Parayre's form of distortion must disappear.

9 Appendix

The proof of Theorem 1 relies on Proposition 1 and the following result:

Proposition 10 *There exists $\Delta > 0$ such that for any firm n, any vector of distortions $s_{-n} \geq 0$, any $s_n \leq \Delta$, and any equilibrium price vector p of $\Gamma(s_{-n}, 0)$ with corresponding quantity vector $q >> 0$, if p^+ is an equilibrium of $\Gamma(s_{-n}, s_n)$ with $p^+ > p$,*

$$\pi_n^e(p^+) > \pi_n^e(p).$$

Proof of Theorem 1: Let Δ be as in Proposition 10 and choose any $0 < \hat{s}_n \leq \Delta$. Fix any $\bar{s}_{-n}$ and $\bar{p}$ as in the statement of the theorem. By Proposition 1, any adaptive adaptive price adjustment $\{p_t\}$ starting from $\bar{p}$ for $\Gamma(\bar{s}_{-n}, \hat{s}_n)$ converges to $p = \inf\{z \in \mathcal{E}(\bar{s}_{-n}, \hat{s}_n), z \geq \bar{p}\}$. It is easy to see that in fact $p >> \bar{p}$. The result now follows from Proposition 10. ■

Proof of Proposition 10: Suppose, by way of contradiction, that for every positive integer k, there is $s_n < 1/k$, a vector of distortions s_{-n}^k, an equilibrium $\bar{p}^k$ of $\Gamma(\bar{s}_{-n}^k, 0)$ and an equilibrium $\hat{p}^k$ of $\Gamma(s_{-n}^k, s_n)$ with $\hat{p}^k > \bar{p}^k$ yet $\pi_n^e(\hat{p}^k) \leq \pi_n^e(\bar{p}^k)$. Passing to subsequences if necessary, we may assume that $s_{-n}^k \to s_{-n}$, $\bar{p}^k \to \bar{p}$, and $\hat{p}^k \to \hat{p}$. Clearly, $\hat{p} \geq \bar{p}$ and $\pi_n^e(\hat{p}) \leq \pi_n^e(\bar{p})$. We note that, by Lemma 2 below, $\hat{p}$ is an equilibrium of $\Gamma(s_{-n}, 0)$. We consider two cases:

Case 1: $\hat{p} > \bar{p}$ In this case, the fact that best responses are strictly increasing implies $\hat{p} >> \bar{p}$. Next we show that $\pi_n^e(\hat{p}) > \pi_n^e(\bar{p})$. From our assumptions on demand and the fact that $q >> 0$:

$$\pi_n(\hat{p}_{-n}, \bar{p}_n, s_n = 0) > \pi_n(\bar{p}, s_n = 0) = \pi_n^e(\bar{p}). \tag{14}$$

That is, relative to $\bar{p}$, firm n achieves higher profit if his opponents strictly increase their prices while all other prices remain unchanged. Since $\hat{p}$ is an equilibrium, we must also have

$$\pi_n^e(\hat{p}) = \pi_n(\hat{p}, s_n = 0) \geq \pi_n(\hat{p}_{-n}, \bar{p}_n, s_n = 0),$$

which combining with (14) implies $\pi_n^e(\hat{p}) > \pi_n^e(\bar{p})$. By the continuity of π_n^e, for k large enough, $\pi_n^e(\hat{p}^k) > \pi_n^e(\bar{p}^k)$, a contradiction.

Case 2: $\hat{p} = \bar{p}$ For each k, define

$$z^k = \frac{\hat{p}^k - \bar{p}^k}{\| \hat{p}^k - \bar{p}^k \|}.$$

Let S^N denote the unit sphere and, given $\alpha > 0$, define $B_\alpha = S^N \cap \{x^N \in R_+^N, x_n \leq 1-\alpha\}$. We first need the following lemma:

Lemma 1 *There is $\alpha > 0$ such that for all large enough k, $z^k \in B_\alpha$.*

From the definition of derivatives, for every $\epsilon > 0$ there is $\bar{h}$ such that for any $0 < h < \bar{h}$ and any $z \in S^N$

$$\left| \frac{\pi_n^e(\bar{p}^k + hz) - \pi_n^e(\bar{p}^k)}{h} - [\partial \pi_n^e(\bar{p}^k)]z \right| < \epsilon$$

where $\partial \pi_n^e(\bar{p}^k)$ is a row vector of partial derivatives of π_n^e evaluated at $\bar{p}^k$. Notice that $\frac{\partial \pi_n^e}{\partial p_n}(\bar{p}^k) = 0$ as $\bar{p}^k$ is an equilibrium and $\frac{\partial \pi_n^e}{\partial p_m}(\bar{p}^k) > 0$ as $\frac{\partial D_n}{\partial p_m} > 0$ for $m \neq n$ and $q_m^k > 0$ for all k large enough. Note that $[\partial \pi_n^e(\bar{p})]z > 0$ for all $z \in B_\alpha$.

Next we note that

$$\liminf_{k \to \infty} \min_{\substack{z \in B_\alpha \\ \bar{p}^k \in \mathcal{E}(s_{-n}^k, 0)}} [\partial \pi_n^e(\bar{p}^k)]z \geq \min_{\substack{z \in B_\alpha \\ \bar{p} \in \mathcal{E}(s_{-n}, 0)}} [\partial \pi_n^e(\bar{p})]z.$$

To see this, write

$$\min_{\substack{z \in B_\alpha \\ \bar{p}^k \in \mathcal{E}(s_{-n}^k, 0)}} [\partial \pi_n^e(\bar{p}^k)]z = [\partial \pi_n^e(\check{p}^k)]\check{z}^k.$$[32]

For any pair of subsequences $\{\check{p}^l\}$, $\{\check{z}^l\}$ converging to $\check{p}$ and $\check{z}$, respectively, we have $\check{z} \in B_\alpha$ and $\check{p} \in \mathcal{E}(s_{-n}, 0)$ (the latter holds by Lemma 2). From the continuity of π_n^e we have

$$\min_{\substack{z \in B_\alpha \\ \bar{p}^l \in \mathcal{E}(s_{-n}^l, 0)}} [\partial \pi_n^e(\bar{p}^l)]z = [\partial \pi_n^e(\check{p}^l)]\check{z} \to [\partial \pi_n^e(\check{p})]\check{z} \geq \min_{\substack{z \in B_\alpha \\ \bar{p} \in \mathcal{E}(s_{-n}, 0)}} [\partial \pi_n^e(\bar{p})]z.$$

[32] The minimum is achieved by the compactness of B_α and $\mathcal{E}(s_{-n}^k, 0)$.

Fix $\alpha > 0$ and set $\epsilon = 0.25 \min_{\substack{z \in B_\alpha \\ \bar{p} \in \mathcal{E}(s_{-n},0)}} [\partial \pi_n^e(\bar{p})]z$. Then, given any $z \in B_\alpha$ and $\bar{p}^k \in \mathcal{E}(s_{-n}^k, 0)$, for all k large enough we have

$$\begin{aligned}
\frac{\pi_n^e(\hat{p}^k) - \pi_n^e(\bar{p}^k)}{\| \hat{p}^k - \bar{p}^k \|} &\equiv \frac{\pi_n^e(\bar{p}^k + \| \hat{p}^k - \bar{p}^k \| z^k) - \pi_n^e(\bar{p}^k)}{\| \hat{p}^k - \bar{p}^k \|} \\
&> [\partial \pi_n^e(\bar{p}^k)]z^k - \epsilon \\
&\geq \min_{\substack{z \in B_\alpha \\ \bar{p}^k \in \mathcal{E}(s_{-n}^k,0)}} [\partial \pi_n^e(\bar{p}^k)]z - \epsilon \\
&\geq 0.5 \min_{\substack{z \in B_\alpha \\ \bar{p} \in \mathcal{E}(s_{-n},0)}} [\partial \pi_n^e(\bar{p})]z - \epsilon \\
&\geq 0.25 \min_{\substack{z \in B_\alpha \\ \bar{p} \in \mathcal{E}(s_{-n},0)}} [\partial \pi_n^e(\bar{p})]z > 0.
\end{aligned}$$

Hence $\pi_n^e(\hat{p}^k) - \pi_n^e(\bar{p}^k) > 0$ for all k large enough. A contradiction. ■

Proof of Lemma 1: Given k, for $m \neq n$

$$\lim_{h \to 0} \frac{r_m(\bar{p}_{-n}, \bar{p}_n + h, s_m) - \bar{p}_m}{h} = \frac{\partial r_m}{\partial p_n}(\bar{p}, s_m)$$

As $\hat{p}^k \to \bar{p}$, $\bar{p}^k \to \bar{p}$ and $s_{-n}^k \to s_{-n}$, since $\frac{\partial r_m}{\partial p_n}$ is continuous, for every $\epsilon > 0$, there is a $\bar{k}$ such that for all $k > \bar{k}$:

$$\frac{r_m(\hat{p}_{-n}^k, \bar{p}_n^k + \left(\hat{p}_n^k - \bar{p}_n^k\right), s_m^k) - \bar{p}_m^k}{(\hat{p}_n^k - \bar{p}_n^k)} > \frac{\partial r_m}{\partial p_n}(\bar{p}, s_m) - \epsilon \tag{15}$$

for all $m \neq n$.

Note further that

$$\hat{p}_m^k = r_m(\hat{p}_{-n}^k, \hat{p}_n^k, s_m^k) = r_m(\hat{p}_{-n}^k, \bar{p}_n^k + \left(\hat{p}_n^k - \bar{p}_n^k\right), s_m^k)$$

and hence from (15), for every $\varepsilon > 0$, for all k large enough and all $m \neq n$,

$$\frac{\hat{p}_m^k - \bar{p}_m^k}{\hat{p}_n^k - \bar{p}_n^k} > \frac{\partial r_m}{\partial p_n}(\bar{p}, s_m) - \epsilon > 0$$

if ϵ is small enough.

To prove the lemma, suppose, by way of contradiction, that $\{z^k\}$ has as an accumulation point the unit vector z' with $z'_n = 1$ and all other entries equal to zero. Passing to

subsequences if necessary, we may assume that $z^k \to z'$. This implies that $\frac{\hat{p}_m^k - \bar{p}_m^k}{||\hat{p}^k - \bar{p}^k||} \to 0$ for every $m \neq n$, and $\frac{\hat{p}_n^k - \bar{p}_n^k}{||\hat{p}^k - \bar{p}^k||} \to 1$. Thus, given any $\tau > 0$, for ϵ small and for all k large enough, $\left| \frac{\hat{p}_n^k - \bar{p}_n^k}{||\hat{p}^k - \bar{p}^k||} - 1 \right| < \tau$, hence $||\hat{p}^k - \bar{p}^k|| < \frac{\hat{p}_n^k - \bar{p}_n^k}{1-\tau}$. Then for $m \neq n$,

$$\| z^k - z' \| \geq \frac{\hat{p}_m^k - \bar{p}_m^k}{\| \hat{p}^k - \bar{p}^k \|} \geq \frac{\hat{p}_m^k - \bar{p}_m^k}{\frac{p_n^k - \bar{p}_n}{1-\tau}} > (1-\tau) \left(\frac{\partial r_m}{\partial p_n}(\bar{p}, s_m) - \epsilon \right) > 0,$$

contradicting the assumption that $z^k \to z'$. ■

Lemma 2 *Suppose that $s^k \to s$, and $p^k \to p$ where for each k, p^k is an equilibrium of $\Gamma(s^k)$. Then p is an equilibrium of $\Gamma(s)$.*

Proof: If p were not an equilibrium of $\Gamma(s)$, then there would be a firm m and a strategy p'_m such that

$$\pi_m(p_{-m}, p'_m, s_m) > \pi_m(p_{-m}, p_m, s_m).$$

Since π_m is continuous in p and s_m, for all large enough k,

$$\pi_m(p_{-m}^k, p'_m, s_m^k) > \pi_m(p_{-m}^k, p_m^k, s_m^k).$$

This contradicts the fact that p^k is an equilibrium of $\Gamma(s^k)$ for all k. ■

Lemma 3 *For any s_{-n}, $s_n \geq \Delta$, and $p \in \mathcal{E}(s)$,*

$$p_0 \equiv \sup\{z \in \mathcal{E}(s_{-n}, 0), z \leq p\} << \inf\{z \in \mathcal{E}(s_{-n}, \Delta), z \geq p_0\} \leq \sup\{z \in \mathcal{E}(s_{-n}, \Delta), z \leq p\}.$$

Proof: First, note that, since best responses are strictly increasing in opponents' prices and cost, $p_o << p$. Define a sequence of price vectors $\{p_t\}_{t=0}^{\infty}$ such that $p_0 = \sup\{z \in \mathcal{E}(s_{-n}, 0), z \leq p\}$, and for $t > 0$,

$$p_t = (r_m(p_{t-1}, s_m), m \neq n, r_n(p_{t-1}, \Delta)).$$

We show by induction that $p_t << p$ for all t. The claim holds by definition for $t = 0$. Assume that it holds for $t \geq 0$, then since $p_{t,-m} << p_{-m}$ for all m, and best responses

are strictly increasing in opponents' prices, $r_m(p_t, s_m) < r_m(p, s_m)$. A similar argument shows that $p_t >> p_0$ for all $t > 0$. Since $[p_0, p]$ is compact, passing to subsequences if necessary, we may assume that $\mathrm{p}_t \to p_\Delta \in [p_0, p]$. Since $\{p_t\}$ is an adaptive pricing adjustment starting at p_0, by Proposition 1, $p_\Delta = \inf\{z \in \mathcal{E}(s_{-n}, \Delta), z \geq p_0\}$. Clearly, $p_\Delta \leq \sup\{z \in \mathcal{E}(s_{-n}, \Delta), z \leq p\}$. ■

Lemma 4 *There is $\phi > 0$ such that for any s and $p \in \mathcal{E}(s)$,*

$$\Pi_n(s_{-n}, \Delta, p) - \Pi_n(s_{-n}, 0, p) > \phi.$$

Proof: Consider any s', $p' \in \mathcal{E}(s')$. Assume first that $s'_n > \Delta$. By Proposition 1,

$$\Pi_n(s'_{-n}, \Delta, p') - \Pi_n(s'_{-n}, 0, p') = \pi^e_n(\sup\{z \in \mathcal{E}(s'_{-n}, \Delta), z \leq p'\}) - \pi^e_n(\sup\{z \in \mathcal{E}(s'_{-n}, 0), z \leq p'\}).$$

Combining Lemma 3 and Proposition 10, this expression is strictly positive for every s', $p' \in \mathcal{E}(s')$. The conclusion now follows from the facts that s' ranges over a finite set, $\mathcal{E}(s')$ is compact for every s', and π^e_n is continuous. The remaining cases $s'_n = 0, \Delta$ can be dealt with similarly. ■

Proof of Theorem 2:

Choose Δ as in Proposition 10. Fix an arbitrary firm n and define $L_\tau(s)$ to be the sum of payoffs in all periods $\tau' < \tau$ such that τ' was an experimental period and the distortion s was chosen. Define

$$X_\tau = L_\tau(\Delta) - L_\tau(0).$$

Since experiments are independent across periods and all distortions are picked with equality probability, the strong law of large numbers implies that, almost surely, for all τ large enough, all distortions are played with equal frequencies. Thus, for all large non-experimental periods τ, $X_\tau > 0$ implies that firm n does not choose $s_n = 0$. Thus, to prove the theorem, it suffices to show that $X_\tau \to \infty$ almost surely.

Define

$$x_\tau = [L_\tau(\Delta) - L_\tau(0)] - [L_{\tau-1}(\Delta) - L_{\tau-1}(0)]$$

$$y_\tau = x_\tau - E(x_\tau|h_{\tau-1}), \qquad z_\tau = E(x_\tau|h_{\tau-1})$$
$$Y_\tau = \sum_{s=1}^{\tau} y_s, \qquad Z_\tau = \sum_{s=1}^{\tau} z_s.$$

Clearly,

$$X_\tau = Y_\tau + Z_\tau,^{33} \tag{16}$$

and $\{Y_\tau\}$ is a martingale.[34] Since each Y_τ has bounded support, by Theorem 6.8.5 in Ash and Doleans-Dade (2000, p. 280) the random variables $Y_\tau - Y_{\tau-1} = y_\tau$ are orthogonal, and by Theorem 5.1.2 in Chung (2001, p. 108), $\frac{Y_\tau}{\tau} = \frac{\sum_{s=1}^{\infty} y_s}{\tau} \to 0$ almost surely (*i.e.,* the conclusion of the law of large numbers holds). From this and equation (16) it follows that

$$\frac{X_\tau - Z_\tau}{\tau} \to 0 \quad a.s. \tag{17}$$

On the other hand,

$$\begin{aligned} z_\tau &= E([L_\tau(\Delta) - L_\tau(0)] - [L_{\tau-1}(\Delta) - L_{\tau-1}(0)]|h_{\tau-1}) \\ &= E(\Pi_n(s_{-n}(\tau), \Delta, p(\tau-1)) - \Pi_n(s_{-n}(\tau), 0, p(\tau-1))|h_{\tau-1}) \\ &\geq \frac{1}{N}\frac{\epsilon}{K}\phi > 0, \end{aligned}$$

where the last two inequalities follow from Lemma 4, and K is the number of distortions in the grid. This implies that $\frac{Z_\tau}{\tau} \geq \frac{1}{N}\frac{\epsilon}{K}\phi$. This fact and equation (17) imply $X_\tau \to \infty$ almost surely. ■

Proof of Proposition 2: In any equilibrium of the pricing game, the lowest price offered must be at or just below than the second highest cost including any distortion. Suppose this is not the case and let p^* be the lowest price. Then, of the two lowest cost firms, at least one is not capturing the entire market. But that firm can price just under p^* and capture the entire market and increase its accounting profit, a contradiction. Also, in any equilibrium, the firm with the lowest cost including distortions must be setting the lowest price. Otherwise, it is making zero profits and can deviate and make positive accounting profit by matching the lowest price. If its cost including distortion

[33] This is the well-known Doob decomposition. See, for instance, Chung (2001, Theorem 9.3.2, p. 337).
[34] $E(Y_{\tau+1}|Y_\tau) = E(y_{\tau+1} + Y_\tau|Y_\tau) = E(x_{\tau+1}|Y_\tau) - E(E(x_{\tau+1}|Y_\tau)|Y_\tau) + Y_\tau = Y_\tau$.

is lower than the second-highest cost, it will price slightly below the second-highest cost to capture the entire market. This observation implies that in any equilibrium with no distortions, the lowest price must be at or just below c_2. Also, firm 1 must be setting the lowest price and either capturing the entire market, if $c_2 > c_1$, or sharing it, if $c_1 = c_2$. In either case, firms 2 and above make zero profits and firm 1 makes positive profits only if $c_2 > c_1$.

Suppose firm 2 or higher distorts its relevant costs upwards by s. Then, let c^* be the lowest cost including the distortion among all the firms apart from firm 1. By our argument above, any equilibrium of the pricing game must have the price set at or just below c^*. Therefore, firms 2 and higher makes zero profits in equilibrium and hence have no incentive to distort their relevant costs upwards. Now, suppose firm 1 distorts upwards by s. If $c_1+s < c_2$, then there is no change in the equilibrium in the pricing game and hence no incentive to distort. If $c_1 + s > c_2$, in any equilibrium in the pricing game, firm 1 must make zero profits as firm 2 has the lowest cost and will always undercut the price set by the firm with the second-highest cost including any distortion. Hence, firm 1 has no incentive to distort relevant costs upwards. Moreover, there is no incentive for firm 1 to distort downwards as it does not alter the pricing equilibrium. Similarly, if firm $n > 1$ distorts downwards by s it still makes zero profits, when $c_n - s \geq c_1$, or makes a loss, when $c_n - s < c_1$ and it sets a price just under c_1 which is less than its true cost c_n. ■

Proof of Proposition 3

We establish existence by showing that $\overline{\pi}_n^e(s)$ is supermodular in s (Milgrom and Roberts 1990, Theorem 5). Differentiating $\overline{\pi}_n^e(s)$with respect to s_n we have:

$$\frac{\partial \overline{\pi}_n^e(s)}{\partial s_n} = \left[-\beta \left(\overline{p}_n(s) - c \right) + \alpha - \beta \overline{p}_n(s) + \gamma \sum_{m \neq n} \overline{p}_m(s) \right] \frac{\partial \overline{p}_n}{\partial s_n} + \left(\overline{p}_n(s) - c \right) \gamma \sum_{m \neq n} \frac{\partial \overline{p}_m}{\partial s_n}.$$

Using (3), we can write this as:

$$\frac{\partial \overline{\pi}_n^e(s)}{\partial s_n} = -\beta \frac{\partial \overline{p}_n}{\partial s_n} s_n + (N-1)\gamma \frac{\partial \overline{p}_m}{\partial s_n} \left(\overline{p}_n(s) - c \right), \tag{18}$$

where $\frac{\partial \overline{p}_n}{\partial s_n}$ and $\frac{\partial \overline{p}_n}{\partial s_m}$ are given by (5) and (6), respectively. This implies:

$$\frac{\partial^2 \overline{\pi}_n^e(s)}{\partial s_n \partial s_m} = (N-1)\gamma \left(\frac{\partial \overline{p}_n}{\partial s_m}\right)\left(\frac{\partial \overline{p}_m}{\partial s_n}\right) > 0.$$

Thus, the first-stage game is supermodular in s. To establish uniqueness we establish the contraction condition $\Lambda \equiv \frac{\partial^2 \overline{\pi}_n^e(s)}{\partial s_n^2} + \sum_{n \neq m} \frac{\partial^2 \overline{\pi}_n^e(s)}{\partial s_n \partial s_m} < 0$ for all $s \geq 0$. We first note that

$$\frac{\partial^2 \overline{\pi}_n^e(s)}{\partial s_n^2} = \frac{\partial \overline{p}_n}{\partial s_n}\left[-\beta + (N-1)\gamma \frac{\partial \overline{p}_m}{\partial s_n}\right].$$

Thus, we can write

$$\Lambda = \frac{\partial \overline{p}_n}{\partial s_n}\left\{-\beta + \left[(N-1)\gamma + (N-1)^2\gamma\xi\right]\frac{\partial \overline{p}_m}{\partial s_n}\right\},$$

where $\xi \equiv \frac{\frac{\partial \overline{p}_n}{\partial s_m}}{\frac{\partial \overline{p}_n}{\partial s_n}} = \frac{\frac{\gamma}{2\beta-(N-1)\gamma}}{1+\frac{\gamma}{2\beta-(N-1)\gamma}} \in (0,1)$. Since $\frac{\partial \overline{p}_n}{\partial s_n} > 0$ and $\frac{\partial \overline{p}_m}{\partial s_n} = \frac{\beta}{2\beta+\gamma}\frac{\gamma}{2\beta-(N-1)\gamma}$, $\Lambda < 0$ if and only if

$$\left[(N-1) + (N-1)^2\xi\right]\frac{\gamma^2}{(2\beta+\gamma)(2\beta-(N-1)\gamma)} < 1. \tag{19}$$

Now, straightforward algebra establishes that:

$$\frac{\gamma^2}{(2\beta+\gamma)(2\beta-(N-1)\gamma)} = \left(\frac{\theta}{(N-1)\theta+2(1-\theta)}\right)\left(\frac{\theta}{(2N-1)\theta+2(1-\theta)}\right).$$

Furthermore $\xi < 1$, so $(N-1)+(N-1)^2\xi < (N-1)+(N-1)^2 = N(N-1)$. Thus

$$\begin{aligned}&\left[(N-1) + (N-1)^2\xi\right]\frac{\gamma^2}{(2\beta+\gamma)(2\beta-(N-1)\gamma)}\\ <\ &\left(\frac{(N-1)\theta}{(N-1)\theta+2(1-\theta)}\right)\left(\frac{N\theta}{(2N-1)\theta+2(1-\theta)}\right).\end{aligned}$$

But $N < 2N-1$, since $N > 1$. Thus

$$\begin{aligned}&\left(\frac{(N-1)\theta}{(N-1)\theta+2(1-\theta)}\right)\left(\frac{N\theta}{(2N-1)\theta+2(1-\theta)}\right)\\ <\ &\left(\frac{(N-1)\theta}{(N-1)\theta+2(1-\theta)}\right)\left(\frac{(2N-1)\,\theta}{(2N-1)\theta+2(1-\theta)}\right).\end{aligned}$$

Since each term in the above expression is positive but less than one, we immediately have

$$\left(\frac{(N-1)\theta}{(N-1)\theta+2(1-\theta)}\right)\left(\frac{(2N-1)\,\theta}{(2N-1)\theta+2(1-\theta)}\right) < 1,$$

which establishes that $\Lambda < 0$. The equilibrium in s_n is thus unique, and because firms are symmetric, the equilibrium distortion will be symmetric across firms. ■

Proof of Proposition 4

Proof (a): We begin by noting that $\theta = 0 \Rightarrow \gamma = 0$ and $\beta = \frac{1}{b}$. Using this, it is straightforward to verify that $\lim_{\theta \to 0} \frac{\partial \overline{\pi}_n^e}{\partial s_n} = -\frac{s_n}{2b} < 0$. Hence, no firm has an incentive to choose a positive distortion.

Proof (b): It is straightforward to show that

$$\begin{aligned} \lim_{\theta \to 1} p^0 &= c. \\ \lim_{\theta \to 1} \frac{\gamma}{\beta} &= \frac{1}{N-1}. \\ \lim_{\theta \to 1} \frac{\beta}{2\beta - (N-1)\gamma} &= 1. \\ \lim_{\theta \to 1} \xi &= \frac{1}{N}. \end{aligned}$$

Now let $s^*(1) = \lim_{\theta \to 1} s^*$ and $p^*(1) = \lim_{\theta \to 1} p^*$. Taking limits of each side of (8) and (9), and using the expressions above we have

$$\begin{aligned} s^*(1) &= \frac{(p^*(1) - c)}{N}. \\ p^*(1) &= c + s^*(1). \end{aligned}$$

These can only be satisfied if $s^*(1) = 0$ and $p^*(1) = c$.

Proof (c): It is straightforward to show that

$$\begin{aligned} \lim_{N \to \infty} p^0 &= c. \\ \lim_{N \to \infty} (N-1)\xi &= 1. \\ \lim_{N \to \infty} \frac{\gamma}{\beta} &= 0. \end{aligned}$$

Now let $s^*(\infty) = \lim_{N \to \infty} s^*$ and $p^*(\infty) = \lim_{N \to \infty} p^*$. Taking limits of (8) and using the above expressions implies that $s^*(\infty) = 0$. ■

Proof of Proposition 5

If $\theta \in (0, 1)$, then $p^0 > c$. Furthermore, it is straightforward to show that

$$\frac{(N-1)\theta^2}{[(2-\theta) + (N-1)\theta(1-\theta)]\,[(1-\theta) + (N-1)\theta]} > 0,$$

which implies that $s^* > 0$. When $\theta \in (0,1)$,

$$\frac{(2-\theta)+(N-1)\theta}{(2-\theta)+(N-1)\theta(1-\theta)} > 1.$$

Thus, $p^* - c > p^0 - c$, or $p^* > p^0$. ■

Proof of Proposition 6

The expression for $p^* - c$ can be written as follows:

$$p^* - c = (a-c)(1-\theta)y(N)z(N),$$

where

$$\begin{aligned} y(N) &= \frac{1}{2-\theta+(N-1)\theta(1-\theta)} \\ z(N) &= \frac{2-\theta+(N-1)\theta}{2(1-\theta)+(N-1)\theta}. \end{aligned}$$

Clearly $y(N)$ is strictly decreasing in N and so too is $z(N)$:

$$z'(N) = -\frac{\theta^2}{[2(1-\theta)+(N-1)\theta]^2} < 0.$$

Hence $p^* - c$ is strictly decreasing in N. ■

Proof of Proposition 7

The monopoly price p^M satisfies

$$p^M - c = \frac{a-c}{2}.$$

From (10), $p^* - c$ can be written as

$$p^* - c = \frac{(a-c)(1-\theta)\left[2-\theta+(N-1)\theta\right]}{\left[2-\theta+(N-1)\theta(1-\theta)\right]\left[2(1-\theta)+(N-1)\theta\right]}.$$

$p^* < p^M$ if and only if

$$\frac{(1-\theta)\left[2-\theta+(N-1)\theta\right]}{\left[2-\theta+(N-1)\theta(1-\theta)\right]\left[2(1-\theta)+(N-1)\theta\right]} < \frac{1}{2}. \tag{20}$$

This can be re-written as

$$[2-\theta+(N-1)\theta(1-\theta)]\,[2(1-\theta)+(N-1)\theta] > 2(1-\theta)\,[2-\theta+(N-1)\theta)]\,.$$

Now, for $N \geq 3, [2(1-\theta)+(N-1)\theta] \geq 2$ since $\theta \in (0,1)$ and $N-1 \geq 2$. Hence, we have:

$$\begin{aligned}[2-\theta+(N-1)\theta(1-\theta)]\,[2(1-\theta)+(N-1)\theta] &> 2\,[2-\theta+(N-1)\theta(1-\theta)] \\ &> 2(1-\theta)[2-\theta+(N-1)\theta].\end{aligned}$$

For $N=2$, the inequality (20) boils down to:

$$\frac{1-\theta}{2-\theta} < \frac{2-\theta^2}{4},$$

which can be shown to hold for $\theta \in (0,1)$. Hence, (20) holds for all $N \geq 2$, establishing $p^* < p^M$. ■

Proof of Proposition 8

These results follows directly from the expression (11). ■

Proof of Proposition 9

We know from the above results that

$$p^M > p^* > p^0.$$

Since p^M maximizes $(p-c)\left(\frac{a}{b}-\frac{p}{b}\right)$, it therefore follows that

$$\left(\frac{a}{b}-\frac{p^*}{b}\right)(p^*-c) > \left(\frac{a}{b}-\frac{p^0}{b}\right)(p^0-c).$$

Given (12) and (13), it immediately follows that

$$1+(N^*-1)\theta > 1+(N^0-1)\theta,$$

or $N^* > N^0$. ■

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