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## Conference Paper

# Inequality and Team Performance

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# Inequality and Team Performance

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## Abstract

We investigate the effects of wealth inequality on the incentives to contribute to a public good or a team output when agents are inequity averse. We show that inequality may increase total output when it favors the more able agents. Moreover, the more inequity averse the agents, the larger should be the inequality in wealth if total output is to be maximized.

**Key Words:** Public good, team incentives, inequity aversion, inequality, wage setting

**JEL Classification:** J31, D31, D63, M52, J41

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# 1 Introduction

There is a broad number of experimental and field studies indicating that many people tend to dislike inequity. Formal models of inequity aversion such as Fehr and Schmidt (1999) or Bolton and Ockenfels (2000) have been quite successful in explaining patterns of behavior observed in the lab and in the field.<sup>1</sup> In this paper we analyze the effect of ex-ante inequality in wealth on the motivation to contribute to a public good or a team output in an economic model. While it is sometimes argued that inequity aversion should lead to wage compression, we show that the optimal degree of inequality may actually increase with the importance of inequity aversion in the agents' preferences.

We consider a simple setting in which two agents which are inequity averse in the sense of Fehr and Schmidt (1999) simultaneously decide on their contributions. The team output is increasing in each agent's contribution but both agents may have different abilities which determine the marginal effect of their contributions. Furthermore, both agents benefit to the same extent from the public good but may differ in their ex-ante wealth. It turns out that ex-ante inequality is offset to some degree as the agents adapt their contributions to the public good according to their wealth. The agent with the lower wealth contributes less, and the agent with the higher wealth contributes more. But as people tend to suffer less from advantageous than from disadvantageous inequity, the effort reduction of the less wealthy agent is stronger than the effort increase of the more wealthy fellow agent. We show that nonetheless, if the difference in wealth is not too large the ex-ante inequality may be entirely compensated by the adaptation of contributions such that the agents will attain the same utility in equilibrium. Moreover,

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<sup>1</sup>For experimental evidence from bargaining games see for example Güth et al. (1982), Roth and Kagel (1995) and Camerer and Thaler (1995). Using a more general notion of fairness, field evidence is given by e.g. Bewley (1999), Campbell and Kamlani (1997) and Carpenter and Seki (2006). For a summary of the empirical evidence on social preferences see Fehr and Schmidt (2002) and Sobel (2005).

if the more wealthy agent is also the more able one, overall output can be higher when ex-ante wealth is distributed unequally.

In a next step we analyze the optimal degree of inequality for two simple settings. In the first setting, a social planner can decide on the allocation of a given amount of initial wealth. In the second setting, we consider a principal who can hire two agents to contribute to a team output but has to take into account their outside options. We show that in both settings, some ex-ante inequality in favor of the more able agent should be introduced. In the second setting, this is the case even when both agents have identical outside options. Moreover, the stronger the agents' inequity aversion, the greater should be the difference in the initial wealth distribution if total output is to be maximized.

In recent years, there has been a steadily growing number of theoretical papers investigating the effects of inequity aversion on incentives (see for instance Englmaier (2004) for a survey). Huck et al. (2003) and Neilson and Stowe (2003) examine the optimal linear incentive scheme when agents exhibit some form of other-regarding preferences. Demougin and Fluet (2003) and Grund and Sliwka (2005) study rank-order tournaments among inequity averse agents. Itoh (2004) shows in a two agents moral hazard context that wages decrease and the principal's expected utility increases the higher the agents' inequity aversion. Demougin et al. (2006) consider a scenario with two inequity averse agents working on two different verifiable tasks and find that greater inequity aversion decreases total output.

Similar to the second setting of our model, Rey-Biel (2004) examines how contracts can exploit inequity aversion of the agents to the principal's advantage. He shows that a selfish principal can indeed exploit agents' preferences for equity by offering them more equitable outcomes when her demands are met than when they are not. He highlights that inequity aversion itself can be a reason to form work teams of distributionally concerned agents, even when individual effort is contractible. Bartling and von Siemens (2004) analyze the

impact of inequity aversion on incentive provision in team production and find that inequity averse agents increase their effort as they suffer from the "shame from cheating" when shirking. Furthermore, if the agents are sufficiently inequity averse, efficient effort choices can be induced. Closely related to our approach is the study of Charness and Kuhn (2007). In their model, compared to the case with purely self-interested agents, a profit-maximizing principal should compress wages when agents have equity-concerned preferences. While they show that the optimal wage spread is decreasing in the degree of the agents' inequity aversion, our model predicts the opposite result. In particular, our model implies a larger wage spread in an equilibrium when agents are inequity averse.

The question whether equal wages are always the best wage policy for a profit-maximizing principal has also been studied both theoretically and experimentally. While it has often been argued that inequitable reward schemes provoke morale problems among co-workers leading to lower performances (e.g. Akerlof and Yellen (1990), Bewley (1999)), some other studies questioned whether equitable payment, realized by wage compression, does eliminate all these problems<sup>2</sup>. In this context, Winter (2004) shows that even in case of identical agents, it might be optimal to reward agents differently. This result is supported by Goerg et al. (2009) who present experimental evidence that unequal payments, depending on the production function, may indeed increase productivity by facilitating coordination. In another experiment, Abeler et al. (2009) find that paying equal wages after an unequal performance may lead to inequity and, in turn, to substantially lower efforts and a decline in efficiency over time. Similarly, our model predicts that the optimal wage spread is the higher the greater the difference in performances due to the agents' different abilities. However, we show that when agents are homogenous, i.e. have the same ability, no wage inequality should be

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<sup>2</sup>See e.g. Lazear (1989) who argues that " ... it is far from obvious that pay equality has these effects."

introduced in equilibrium.

The remainder of this paper is structured as follows. The model is described in Section 2. Section 3 presents the equilibrium analysis. In Section 4 we examine the optimal degree of inequity for two simple settings. Section 5 concludes.

## 2 The Model

Two Agents  $i$  and  $j$  can both contribute to a public good or a team output. An agent's contribution depends on her effort  $e_i$  and ability  $a_i$ . Individual effort costs are linear in the exerted effort and equal to  $c \cdot e_i$ ,  $c \in \mathbb{R}^+$ . The team output is given by

$$a_i\sqrt{e_i} + a_j\sqrt{e_j}.$$

The agents directly benefit from a higher team output. For simplicity we assume that each agent receives a share  $\eta$  of the team output. For instance  $\eta$  may be the degree of team identification or the individual valuation of the public good. We treat  $\eta$  as being exogenously given.

Each agent  $i$  has an initial endowment  $w_i$ . In a team production context within a firm  $w_i$  represents the wage. In a public goods context  $w_i$  represents the agent's initial wealth. Both agents are inequity averse with a Fehr and Schmidt (1999) type utility function. In contrast to previous studies, the disutility from inequity  $v(\Delta)$  in our model depends on the difference of the agents' net-wealth instead of the difference of their initial endowments. In a firm context, this allows a principal to reward unequal performance unequally without hurting the equity norm among agents. An agent's utility is

$$U_i = w_i - c \cdot e_i + \eta \cdot (a_i\sqrt{e_i} + a_j\sqrt{e_j}) - v(w_i - c \cdot e_i - w_j + c \cdot e_j)$$

with

$$v(\Delta) = \begin{cases} -\alpha \cdot \Delta & \text{if } \Delta < 0 \\ \beta \cdot \Delta & \text{if } \Delta > 0 \end{cases}$$

where  $\alpha$  measures the “psychological costs” of disadvantageous inequity and  $\beta$  that of advantageous inequity. Following Fehr and Schmidt (1999) we assume that  $\alpha > \beta > 0$ . Additionally, we assume that  $\beta \leq \frac{1}{2}$ .

### 3 Equilibrium Analysis

Each agent  $i$  maximizes

$$\max_{e_i} w_i - c \cdot e_i + \eta (a_i \sqrt{e_i} + a_j \sqrt{e_j}) - v(w_i - c \cdot e_i - w_j + c \cdot e_j).$$

The function is continuous but not continuously differentiable as it has a kink at  $e_i = \frac{w_i - w_j}{c} + e_j$ . Off the kink the second derivative with respect to  $e_i$  is  $-\frac{\eta a_i \sqrt{e_i}}{4e_i^2} < 0$ . As the right-sided derivative at the kink is strictly smaller than the left-sided derivative the function is strictly concave.

We have to consider two possible equilibrium types depending on whether there is inequity in equilibrium or whether both agents are equally well off. In an *inevitable equilibrium* one agent  $i$  is better off given the chosen effort levels  $w_i - ce_i > w_j - ce_j$ . Suppose that such an equilibrium exists. In that case both agents choose an effort level off the kink and

$$\begin{aligned} \frac{\partial U_i}{\partial e_i} &= -c + \frac{\eta a_i}{2\sqrt{e_i}} + \beta c = 0, \\ \frac{\partial U_j}{\partial e_j} &= -c + \frac{\eta a_j}{2\sqrt{e_j}} - \alpha c = 0. \end{aligned}$$

The respective equilibrium efforts are

$$e_i^* = \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} \text{ and } e_j^* = \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2}.$$

Such an equilibrium indeed exists if

$$w_i - c \cdot \left( \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} \right) > w_j - c \cdot \left( \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} \right).$$

This directly leads to the following result:

**Proposition 1** *If  $w_i - w_j > \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right)$  an inequitable equilibrium exists in which agent  $i$  is strictly better off than agent  $j$ . Then, the equilibrium effort levels are equal to*

$$e_i^* = \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} \text{ and } e_j^* = \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2}.$$

But there may also be *equitable equilibria* in which both agents have the same payoff. In that case  $w_i - ce_i = w_j - ce_j$  and both agents choose their effort levels at the kink of the respective utility function. An effort tuple  $(\bar{e}_i^*, \bar{e}_j^*)$  can be sustained in equilibrium if no agent has an incentive to deviate. As the function is strictly concave, necessary and sufficient conditions for the existence of the equilibrium are that for both agents the left hand side derivative of the utility function must be positive at  $(\bar{e}_i^*, \bar{e}_j^*)$ , the right hand side derivative negative and  $w_i - c\bar{e}_i^* = w_j - c\bar{e}_j^*$ . Let  $\Delta w_i = w_i - w_j$ . In an *equitable equilibrium*, the following five conditions must then be met:



$$\left. \frac{\partial_- U_i}{\partial e_i} \right|_{e_i = \bar{e}_i^*} = -c + \frac{\eta a_i}{2\sqrt{e_i}} + \beta c \geq 0 \Leftrightarrow \bar{e}_i^* \leq \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} \quad (1)$$

$$\left. \frac{\partial_+ U_i}{\partial e_i} \right|_{e_i = \bar{e}_i^*} = -c + \frac{\eta a_i}{2\sqrt{e_i}} - \alpha c \leq 0 \Leftrightarrow \bar{e}_i^* \geq \frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2} \quad (2)$$

$$\left. \frac{\partial_- U_j}{\partial e_j} \right|_{e_j = \bar{e}_j^*} = -c + \frac{\eta a_j}{2\sqrt{e_j}} + \beta c \geq 0 \Leftrightarrow \bar{e}_j^* \leq \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} \quad (3)$$

$$\left. \frac{\partial_+ U_j}{\partial e_j} \right|_{e_j = \bar{e}_j^*} = -c + \frac{\eta a_j}{2\sqrt{e_j}} - \alpha c \leq 0 \Leftrightarrow \bar{e}_j^* \geq \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} \quad (4)$$

$$\bar{e}_j^* = \bar{e}_i^* - \frac{\Delta w_i}{c} \quad (5)$$

From these conditions the following result can be derived:

**Proposition 2** *If  $\frac{\eta^2}{4c} \left( \frac{a_i^2}{(1+\alpha)^2} - \frac{a_j^2}{(1-\beta)^2} \right) \leq \Delta w_i \leq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right)$  there is a continuum of equitable equilibria with*

$$\max \left\{ \frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2}; \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} + \frac{\Delta w_i}{c} \right\} \leq \bar{e}_i^* \leq \min \left\{ \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}; \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c} \right\} \quad (6)$$

and  $\bar{e}_j^* = \bar{e}_i^* - \frac{\Delta w_i}{c}$ .

**Proof:**

Inserting the equity condition (5) in conditions (3) and (4) we can conclude that an effort level  $\bar{e}_i^*$  can be sustained if and only if

$$\max \left\{ \frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2}; \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} + \frac{\Delta w_i}{c} \right\} \leq \bar{e}_i^* \leq \min \left\{ \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}; \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c} \right\}.$$

Note that  $\frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2} < \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}$  and  $\frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} + \frac{\Delta w_i}{c} < \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c}$ . Hence, the set is non-empty for a certain value of  $\Delta w_i$  if

$$\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} \geq \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} + \frac{\Delta w_i}{c}$$

and

$$\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c} \geq \frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2}$$

which is the case when

$$\frac{\eta^2 a_i^2}{4(1+\alpha)^2 c} - \frac{\eta^2 a_j^2}{4(1-\beta)^2 c} \leq \Delta w_i \leq \frac{\eta^2 a_i^2}{4(1-\beta)^2 c} - \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c}. \quad (7)$$

■

The equilibrium effort levels are always bounded from below by  $\frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2}$  or  $\frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2}$  respectively. Hence, an agent never chooses an effort level of 0. Figures 1 and 2 show the sustainable equilibrium effort levels of both agents  $i$  and  $j$  depending on  $\Delta w_i$  for an example. There are two cut-off values for  $\Delta w_i$ . For small values of  $\Delta w_i$  ( $= -\Delta w_j$ ) below  $\frac{\eta^2}{4c} \left( \frac{a_i^2}{(1+\alpha)^2} - \frac{a_j^2}{(1-\beta)^2} \right)$  there is a unique inequitable equilibrium with  $e_i^* = \frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2}$  and  $e_j^* = \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2}$ . For large values of  $\Delta w_i$  above  $\frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right)$  there is a unique inequitable equilibrium with  $e_i^* = \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}$  and  $e_j^* = \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2}$ .

But when  $\Delta w_i$  is between these two cut-offs, there are multiple equitable equilibria. In particular, the set of equitable equilibria given by (7) is increasing in the agents' degree of inequity aversion. The reason for the multiplicity of equilibria is that when being inequity averse, agents have, to some extent, an interest to match their co-worker's effort. If an agent chooses a lower effort than her co-worker, she suffers from advantageous inequity. If she chooses a higher effort level than her co-worker, she suffers from disadvantageous inequity. On the one hand, the upper boundary of the equilibrium set for  $\bar{e}_i^*$  is increasing in  $\beta$  as  $\beta$  measures the degree of "compassion" and therefore determines the extent to which agents are willing to match a high effort by a co-worker. On the other hand, the lower boundary is decreasing in  $\alpha$  since the higher  $\alpha$ , the more an agent would be willing to match a low effort level exerted by a co-worker, even though her benefit from the team output

decreases.

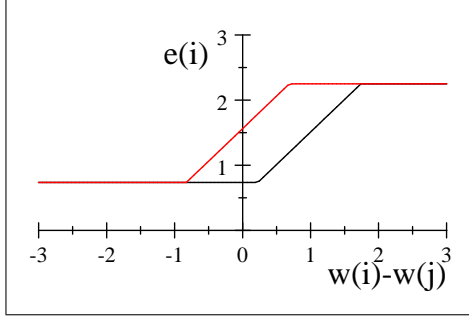


Figure 1

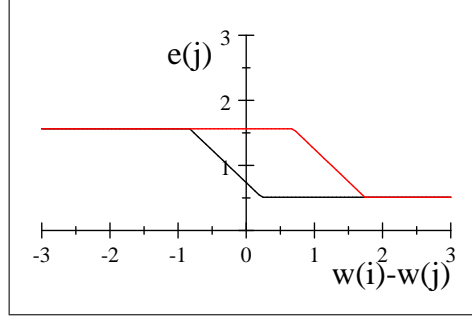


Figure 2

Note that both agents attain identical utility in an equitable equilibrium. Hence, among all equitable equilibria both agents prefer the same one. Consequently, it is important to compare the different feasible equitable equilibria with respect to the agents' utility. We can show the following result:

**Corollary 1** *As long as  $\beta \leq \frac{1}{2}$  the highest feasible equitable equilibrium is always pareto optimal.*

**Proof:**

The value of  $e_i^{\max}$  directly follows from the upper boundary given by (6). Let

$$v_i^E(e_i) = w_i - ce_i + \eta \left( a_i \sqrt{e_i} + a_j \sqrt{e_i - \frac{\Delta w_i}{c}} \right)$$

be agent  $i$ 's utility which is equal to agent  $j$ 's utility in any equitable equilibrium. To compare the equilibria in the set defined by (6) we have to check which value of  $e_i$  maximizes this utility. Note that

$$\begin{aligned} \frac{\partial v_i^E(e_i)}{\partial e_i} &= -c + \eta \left( \frac{a_i}{2\sqrt{e_i}} + \frac{a_j}{2\sqrt{e_i - \frac{\Delta w_i}{c}}} \right) \text{ and} \\ \frac{\partial^2 v_i^E(e_i)}{\partial e_i^2} &= \eta \left( -\frac{a_i}{4} e_i^{-\frac{3}{2}} - \frac{a_j}{4} \left( e_i - \frac{\Delta w_i}{c} \right)^{-\frac{3}{2}} \right) < 0. \end{aligned}$$

As  $v_i^E(e_i)$  is strictly concave,  $\left. \frac{\partial v_i^E(e_i)}{\partial e_i} \right|_{e_i=e_i^{\max}} \geq 0$  is a necessary and sufficient condition for  $e_i^{\max}$  to be pareto optimal. If  $\Delta w_i < \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1-\beta)^2} \right)$ ,  $e_i^{\max}$  is equal to  $\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c}$  and the condition is equivalent to

$$-c + \eta \left( a_i \frac{1}{2\sqrt{\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c}}} + a_j \frac{1}{2\sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c} - \frac{\Delta w_i}{c}}} \right) \geq 0 \Leftrightarrow$$

$$\Delta w_i \leq \frac{\eta^2}{4c} \left( \frac{a_i^2}{\beta^2} - \frac{a_j^2}{(1-\beta)^2} \right).$$

But  $\frac{\eta^2}{4c} \left( \frac{a_i^2}{\beta^2} - \frac{a_j^2}{(1-\beta)^2} \right) \geq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1-\beta)^2} \right)$  as long as  $\beta \leq \frac{1}{2}$ . Hence, both agent's utility is maximal at  $e_i^{\max}$  in this case. If, however,  $\Delta w_i \geq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1-\beta)^2} \right)$ ,  $e_i^{\max}$  is equal to  $\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}$  and the condition is equivalent to

$$-c + \eta \left( a_i \frac{1}{2\sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}}} + a_j \frac{1}{2\sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} - \frac{\Delta w_i}{c}}} \right) \geq 0 \Leftrightarrow$$

$$\Delta w_i \geq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{\beta^2} \right)$$

But  $\frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{\beta^2} \right) \leq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1-\beta)^2} \right)$  is again equivalent to  $\beta \leq \frac{1}{2}$ . ■

Hence, both agents benefit from playing the equitable equilibrium with the highest sustainable effort level. This effort level is strictly increasing in the degree of advantageous inequity aversion. Thus, the more “compassionate” the agents are, the higher the sustainable effort levels in equilibrium. This result is also obtained by Rey-Biel (2004) and Bartling and von Siemens (2004). However, in contrast to Huck and Rey-Biel (2003), we additionally show that also the team output in the equitable equilibrium is increasing in the agents' inequity aversion. When agents are inequity averse, they adjust

their effort relative to the efforts of the others in order to reduce the inequality in initial wealth. While the agent with the higher initial wealth increases her effort, the "disadvantaged" agent lowers it. In particular, the extent of the adjustment is increasing in the agents' degree of advantageous inequity aversion. All in all, the team output increases if the more able agent is also the more wealthy one.

## 4 The Optimal Distribution of Wealth

We now analyze the optimal distribution of wealth. In a first step we look at the case where a social planner can allocate a fixed budget  $W$  to both agents such that  $w_i + w_j = W$ . In a second step we consider the case of a principal, who wants to hire two agents and has to take into account their outside options.

### 4.1 (Re-)Allocating Wealth

We assume that a social planner wants to maximize total output and that the agent's choose the pareto-optimal equilibrium strategies. Hence, she solves the following problem

$$\begin{aligned}
& \max_{e_i, e_j, w_k, \Delta w_i} && (a_i \sqrt{e_i} + a_j \sqrt{e_j}) \\
\text{s.t. } (e_i, e_j) = & \begin{cases} \left( \frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2}, \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} \right) & \text{if } \Delta w_i < \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1+\alpha)^2} - \frac{a_j^2}{(1-\beta)^2} \right) \\ \left( \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c}, \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} \right) & \text{if } \Delta w_i \in \left[ \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1+\alpha)^2} - \frac{a_j^2}{(1-\beta)^2} \right), \frac{\eta^2}{4c} \left( \frac{a_i^2 - a_j^2}{(1-\beta)^2} \right) \right] \\ \left( \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}, \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} - \frac{\Delta w_i}{c} \right) & \text{if } \Delta w_i \in \left[ \frac{\eta^2}{4c} \left( \frac{a_i^2 - a_j^2}{(1-\beta)^2} \right), \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \right] \\ \left( \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}, \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} \right) & \text{if } \Delta w_i > \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \end{cases} \\
& 2w_j + \Delta w_i = W.
\end{aligned}$$

The solution of this program leads to the following result:

**Proposition 3** *The social planner chooses a wealth distribution in which the more able agent receives a higher initial wealth than the less able one. The difference in initial wealth is  $\Delta w_i^* = \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2}$ , which is strictly increasing in  $\beta$ . The optimal initial wealth levels are given by*

$$\begin{aligned} w_i^* &= \frac{W}{2} + \frac{\eta^2}{8c} \frac{a_i^2 - a_j^2}{(1-\beta)^2}, \\ w_j^* &= \frac{W}{2} - \frac{\eta^2}{8c} \frac{a_i^2 - a_j^2}{(1-\beta)^2}. \end{aligned}$$

*Under the optimal wealth distribution, both agents attain the same utility. The total output is strictly increasing in the degree of advantageous inequity aversion  $\beta$ .*

**Proof:**

By substituting the incentive conditions and simplifying we obtain that the social planner maximizes

$$\max_{\Delta w_i} \begin{cases} \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4(1+\alpha)^2 c^2}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2}} \right) & \text{if } \Delta w_i < \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1+\alpha)^2} - \frac{a_j^2}{(1-\beta)^2} \right) \\ \left( a_i \sqrt{\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2}} \right) & \text{if } \Delta w_i \in \left[ \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1+\alpha)^2} - \frac{a_j^2}{(1-\beta)^2} \right), \frac{\eta^2}{4c} \left( \frac{a_i^2 - a_j^2}{(1-\beta)^2} \right) \right] \\ \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} + a_j \sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} - \frac{\Delta w_i}{c}} \right) & \text{if } \Delta w_i \in \left[ \frac{\eta^2}{4c} \left( \frac{a_i^2 - a_j^2}{(1-\beta)^2} \right), \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \right] \\ \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2}} \right) & \text{if } \Delta w_i > \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \end{cases}$$

under the budget constraint  $2w_j + \Delta w_i = W$ .

Note for the first and the last term the total output is independent of  $\Delta w_i$  to the extent that  $2w_j + \Delta w_i = W$  still has to be met. The total outputs are given by  $\frac{\eta a_i^2}{2(1+\alpha)c} + \frac{\eta a_j^2}{2(1-\beta)c}$  and  $\frac{\eta a_i^2}{2(1-\beta)c} + \frac{\eta a_j^2}{2(1+\alpha)c}$ .

Since the second term is increasing and the third term is decreasing in  $\Delta w_i$ , the total output is maximized at  $\Delta w_i = \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2}$ . In both cases, the

total output is equal to  $\frac{\eta a_i^2}{2(1-\beta)c} + \frac{\eta a_j^2}{2(1-\beta)c}$  which is higher than the total output in the first and the last case for  $\alpha > \beta > 0$ . In particular,  $\Delta w_i$  is strictly positive if agent i is the more able one. The optimal values for the initial wealth follow directly from the social planner's budget constraint.

Note that under the optimal wealth distribution the total output is equal to

$$\frac{\eta (a_i^2 + a_j^2)}{2(1-\beta)c}$$

which is increasing in  $\beta$ . ■

Figure 3 shows the total output as a function of  $\Delta w$  for an example. The solid line shows the upper boundary of the equilibrium set. The maximum of the function is attained at the optimal wage differential  $\Delta w_i^* = \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2}$ .

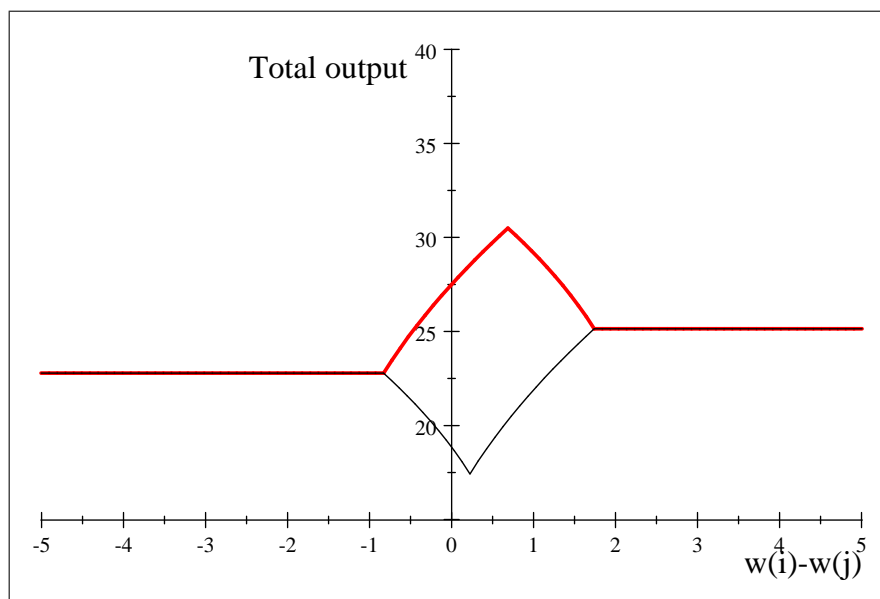


Figure 3

Hence, the total output is maximized with an unequal wealth distribution favoring the more able agent. Moreover, the optimal difference in initial wealth is *increasing* in the agents' degree of advantageous inequity aversion  $\beta$ . The intuition behind this result is as follows: As we have shown above,

there are multiple equilibria when wealth levels do not differ too strongly. In addition, we showed that, compared to the case of inequitable equilibria, both agents are better off in the highest feasible equilibrium. Moreover, the higher  $\beta$ , the higher the effort level that can be sustained in the highest feasible equilibrium. The reason is that the upper boundary of the equilibrium set is defined by the highest attainable pair of efforts at which no agent has an incentive to deviate by unilaterally reducing her own effort level. But the marginal gain from an effort reduction is the smaller the higher  $\beta$  as such an unilateral effort reduction causes advantageous inequity which agents with a higher  $\beta$  dislike more.

Furthermore, it is straightforward to see that an output-maximizing social planner should choose a wealth distribution which benefits the more able agent. In particular, the optimal difference in initial wealth is increasing in the difference of the agents' abilities. Finally, the ex-ante inequality may be entirely compensated by the adaption of agents' effort choices such that they will receive the same utility in equilibrium.

## 4.2 The Optimal Wage Policy

We now consider the case of a principal who wants to hire two agents and has to take into account their outside options. We want to check whether and under which conditions the principal voluntarily introduces wage inequality. As it is straightforward that wages are unequal when agents have different outside options we focus on the case in which both agents have identical outside options. The principal's profit is given by

$$(a_i\sqrt{e_i} + a_j\sqrt{e_j}) - \Delta w_i - 2w_j.$$



The agents' participation constraints are

$$\begin{aligned} w_j + \Delta w_i - c \cdot e_i + \eta \cdot (a_i \sqrt{e_i} + a_j \sqrt{e_j}) - v(\Delta w_i - c \cdot e_i + c \cdot e_j) &\geq u_0, \\ w_j - c \cdot e_j + \eta \cdot (a_i \sqrt{e_i} + a_j \sqrt{e_j}) - v(-\Delta w_i - c \cdot e_j + c \cdot e_i) &\geq u_0. \end{aligned}$$

We proceed by first determining the optimal contract which induces an equitable and inequitable equilibrium respectively. We then compare the principal's profits under these contracts.

**Proposition 4** *The principal chooses a contract in which the more able agent receives a higher wage than the less able one. The wage spread is*

$$\Delta w_i^* = \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1 - \beta)^2}.$$

which is strictly increasing in  $\beta$ . The wages are given by

$$\begin{aligned} w_i^* &= u_0 - \frac{\eta^2 \left( (1 - 2\beta)a_i^2 + 2(1 - \beta)a_j^2 \right)}{4(1 - \beta)^2 c}, \\ w_j^* &= u_0 - \frac{\eta^2 \left( (1 - 2\beta)a_j^2 + 2(1 - \beta)a_i^2 \right)}{4(1 - \beta)^2 c}. \end{aligned}$$

Under the optimal contract both agents receive the same utility.

**Proof:** See the appendix.

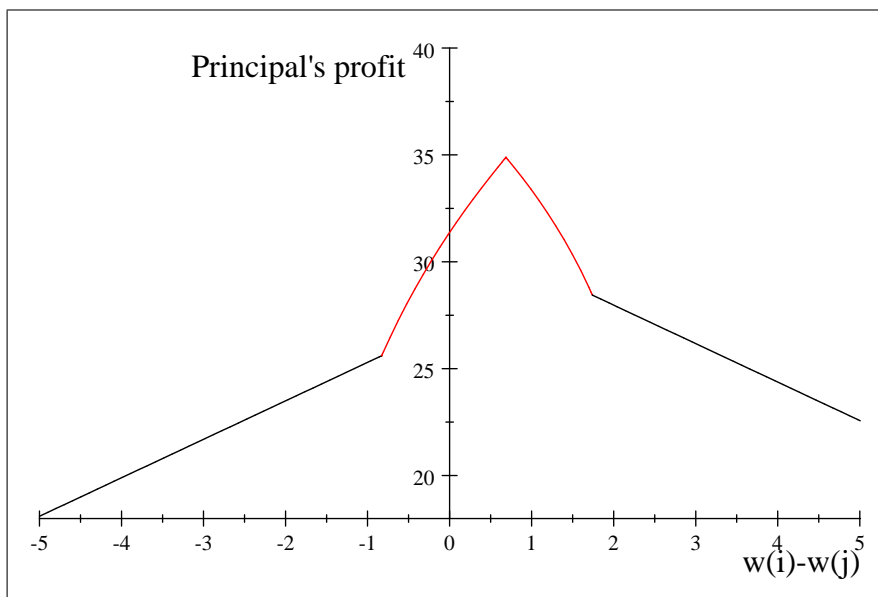


Figure 4

Hence, a principal who has to take the agents' participation constraints into account, chooses the same wage differential as a social planner allocating a fixed budget. As illustrated by Figure 4 showing the principal's profit in equilibrium as a function of the wage spread, inequitable equilibria impose additional costs for the principal as higher degrees of inequity have to be compensated with higher wages. However, in equitable equilibria agents do not suffer from inequity at all and therefore, the principal chooses the same optimal wage spread as in the case in which these additional "inequity costs" had not been taken into account. Furthermore, it is straightforward to see that a profit-maximizing principal should pay a higher wage to the more able agent. In particular, the optimal wage spread is the higher the larger the difference in the agents' abilities. Finally, the ex-ante inequality may be entirely compensated by the adaption of agents' effort choices such that they will receive the same utility in equilibrium.

### 4.3 Benchmark Case with selfish agents

We finally compare the optimal wage policy for inequity averse agents to the one for selfish agents. By setting  $\beta = 0$  in the above results we directly obtain the following corollary:

**Corollary 2** *When agents are only selfish, profits are smaller than with inequity averse agents and wage compression is stronger.*

**Proof:** See the appendix.

Profits are smaller with selfish agents because efforts increase with  $\beta$  as inequity aversion helps to reduce free-riding. The reason for the smaller wage spread with selfish agents is that when agents are inequity averse, a higher wage spread is beneficial as the favored agent increases her effort level to a stronger extent than the disadvantaged agent decreases her effort level. If the favored agent is also the more able one, this has a positive effect on the team output.

## 5 Conclusion

In this paper, we analyze the effects of wealth inequality on the incentives to contribute to a public good or team output when agents are inequity averse. While inequality may decrease the total contributions, we show that nevertheless, total output can be increased when the more able agent is favored. In this case, the initial wealth inequality induces her to increase her contribution to the common team output such that the utility gap can be reduced in equilibrium. Although the disadvantaged agent decreases her effort levels, the former effect dominates as long as the degree of inequality does not become too large, because the marginal productivity of the favored agent's effort is higher. Applying our model to the context of wage policy, we show that it is profitable for a principal to voluntarily introduce wage inequality when the agents are inequity averse. In particular, the higher the

agents' inequity aversion, the larger should be the wage spread if profit is to be maximized. Furthermore, compared to the case when agents are only self-interested, the principal's profit is higher when agents also care about the utility of their co-workers as inequity aversion helps to overcome the free-rider problem to some extent, especially when wage inequality is optimally adapted to the difference in the agents' abilities.

## 6 Appendix

### Proof of Proposition 4:

First, suppose that the principal wants to induce an equitable equilibrium and the agents play the pareto dominant one. In this case we can substitute  $e_j = e_i - \frac{\Delta w_i}{c}$ . The participation constraints then coincide and are equal to

$$w_j + \Delta w_i - c \cdot e_i + \eta \cdot \left( a_i \sqrt{e_i} + a_j \sqrt{e_i - \frac{\Delta w_i}{c}} \right) \geq u_0.$$

The principal maximizes

$$\begin{aligned} \max_{e_i, w_j, \Delta w_i} & \quad \left( a_i \sqrt{e_i} + a_j \sqrt{e_i - \frac{\Delta w_i}{c}} \right) - 2w_j - \Delta w_i \\ \text{s.t.} & \quad w_j + \Delta w_i - c \cdot e_i + \eta \cdot \left( a_i \sqrt{e_i} + a_j \sqrt{e_i - \frac{\Delta w_i}{c}} \right) \geq u_0 \\ & \quad \Delta w_i \geq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1+\alpha)^2} - \frac{a_j^2}{(1-\beta)^2} \right) \\ & \quad \Delta w_i \leq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \\ & \quad e_i = \begin{cases} \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c} & \text{if } \Delta w_i < \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1-\beta)^2} \right) \\ \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} & \text{if } \Delta w_i \geq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1-\beta)^2} \right) \end{cases} \end{aligned}$$

Note that  $w_j$  will be set such that the participation constraint will always be binding as it does not affect the incentive constraint. The value of  $w_j$  can be substituted into the objective function which becomes

$$\begin{aligned} & \left( a_i \sqrt{e_i} + a_j \sqrt{e_i - \frac{\Delta w_i}{c}} \right) - \Delta w_i \\ & - 2 \left( u_0 - \eta \cdot \left( a_i \sqrt{e_i} + a_j \sqrt{e_i - \frac{\Delta w_i}{c}} \right) + c \cdot e_i - \Delta w_i \right) \\ = & (1 + 2\eta) \left( a_i \sqrt{e_i} + a_j \sqrt{e_i - \frac{\Delta w_i}{c}} \right) + \Delta w_i - 2u_0 - 2c \cdot e_i. \end{aligned}$$

By substituting the incentive conditions and simplifying we obtain that the principal maximizes

$$\max_{\Delta w_i} \begin{cases} (1+2\eta) \left( a_i \sqrt{\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c}} + a_j \sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}} \right) & \text{if } \Delta w_i < \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2} \\ -\Delta w_i - 2u_0 - \frac{\eta^2 a_j^2}{2(1-\beta)^2 c} & \\ (1+2\eta) \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} + a_j \sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} - \frac{\Delta w_i}{c}} \right) & \text{if } \Delta w_i \geq \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2} \\ +\Delta w_i - 2u_0 - \frac{\eta^2 a_i^2}{2(1-\beta)^2 c} & \end{cases}$$

under the constraints that  $\frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \geq \Delta w_i \geq \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1+\alpha)^2} - \frac{a_j^2}{(1-\beta)^2} \right)$ . The first derivative of this function with respect to  $\Delta w_i$  is equal to

$$\begin{cases} \frac{(1+2\eta)a_i}{2c} \left( \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c} \right)^{-\frac{1}{2}} - 1 & \text{if } \Delta w_i < \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2} \\ -\frac{(1+2\eta)a_j}{2c} \left( \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} - \frac{\Delta w_i}{c} \right)^{-\frac{1}{2}} + 1 & \text{if } \Delta w_i \geq \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2} \end{cases},$$

and the second derivative is

$$\begin{cases} -\frac{(1+2\eta)a_i}{4c^2} \left( \frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i}{c} \right)^{-\frac{3}{2}} & \text{if } \Delta w_i < \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2} \\ -\frac{(1+2\eta)a_j}{4c^2} \left( \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} - \frac{\Delta w_i}{c} \right)^{-\frac{3}{2}} & \text{if } \Delta w_i \geq \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2} \end{cases}.$$

Note that the left first derivative at  $\Delta w_i = \frac{\eta^2}{4c} \frac{a_i^2 - a_j^2}{(1-\beta)^2}$  is equal to

$$\frac{\frac{(1+2\eta)a_i}{2c}}{\frac{\eta}{2c} \frac{a_i}{(1-\beta)}} - 1 = \frac{(1-\beta)(1+2\eta)}{\eta} - 1 = \frac{(1-\beta) + (1-2\beta)\eta}{\eta} > 0$$

and the right first derivative at this point is

$$-\frac{\frac{(1+2\eta)a_j}{2c}}{\frac{\eta}{2c} \frac{a_j}{(1-\beta)}} + 1 = -\frac{(1-\beta)(1+2\eta)}{\eta} + 1 = -\frac{(1-\beta) + (1-2\beta)\eta}{\eta} < 0.$$

Hence, the function is strictly concave and attains a maximum at  $\Delta w_i^* = \frac{\eta^2}{4c} \left( \frac{a_i^2 - a_j^2}{(1-\beta)^2} \right)$ . For this value of  $\Delta w_i$  indeed an equitable equilibrium is attained. The optimal wages can be computed from the binding participation constraint.

Second, suppose that the principal wants to induce an inequitable equilibrium and the agents play the pareto dominant one. Let agent i be the favored agent. The agents' participation constraints are

$$\begin{aligned} \widehat{w}_j + \Delta \widehat{w}_i - c \cdot \widehat{e}_i + \eta \cdot \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) - \beta (\Delta \widehat{w}_i - c \cdot \widehat{e}_i + c \cdot \widehat{e}_j) &\geq u_0, \\ \widehat{w}_j - c \cdot \widehat{e}_j + \eta \cdot \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) + \alpha (\Delta \widehat{w}_i - c \cdot \widehat{e}_i + c \cdot \widehat{e}_j) &\geq u_0. \end{aligned}$$

The principal maximizes

$$\begin{aligned} \max_{\widehat{e}_i, \widehat{e}_j, \widehat{w}_j, \Delta \widehat{w}_i} \quad & \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) - 2\widehat{w}_j - \Delta \widehat{w}_i \\ \text{s.t.} \quad & \widehat{w}_j + \Delta \widehat{w}_i - c \cdot \widehat{e}_i + \eta \cdot \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) - \beta (\Delta \widehat{w}_i - c \cdot \widehat{e}_i + c \cdot \widehat{e}_j) > u_0 \\ & \widehat{w}_j - c \cdot \widehat{e}_j + \eta \cdot \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) - \alpha (\Delta \widehat{w}_i - c \cdot \widehat{e}_i + c \cdot \widehat{e}_j) = u_0 \\ & \widehat{e}_i = \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} \quad \text{if} \quad \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \leq \Delta \widehat{w}_i \\ & \widehat{e}_j = \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} \quad \text{if} \quad \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \leq \Delta \widehat{w}_i \end{aligned}$$

Note that  $\widehat{w}_j$  will be set such that agent j's participation constraint will always be binding as it does not affect the incentive constraint. Hence,  $\widehat{w}_j = u_0 + c \cdot \widehat{e}_j - \eta \cdot \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) + \alpha (\Delta \widehat{w}_i - c \cdot \widehat{e}_i + c \cdot \widehat{e}_j)$  and agent i's

participation constraint will always be fulfilled. The program becomes

$$\begin{aligned}
& \max_{\widehat{e}_i, \widehat{e}_j, \widehat{w}_j, \Delta \widehat{w}_i} && \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) \\
& && -2 \left( u_0 + c \cdot \widehat{e}_j - \eta \cdot \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) + \alpha \left( \Delta \widehat{w}_i - c \cdot \widehat{e}_i + c \cdot \widehat{e}_j \right) \right) - \Delta \widehat{w}_i \\
& \text{s.t.} && \widehat{w}_j + \Delta \widehat{w}_i - c \cdot \widehat{e}_i + \eta \cdot \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) - \beta \left( \Delta \widehat{w}_i - c \cdot \widehat{e}_i + c \cdot \widehat{e}_j \right) > u_0 \\
& && \widehat{e}_i = \frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2} \quad \text{if} \quad \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \leq \Delta \widehat{w}_i \\
& && \widehat{e}_j = \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c^2} \quad \text{if} \quad \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \leq \Delta \widehat{w}_i
\end{aligned}$$

By substituting the incentive conditions and simplifying we obtain that the principal maximizes

$$\begin{aligned}
& \max_{\widehat{w}_j, \Delta \widehat{w}_i} && (1 + 2\eta) \left( \frac{\eta a_i^2}{2(1-\beta)c} + \frac{\eta a_j^2}{2(1+\alpha)c} \right) \\
& && -2 \left( u_0 + \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c} + \alpha \left( \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c} - \frac{\eta^2 a_i^2}{4(1-\beta)^2 c} \right) \right) - 2\alpha \Delta \widehat{w}_i - \Delta \widehat{w}_i \\
& \text{s.t.} && \widehat{w}_j + \Delta \widehat{w}_i - c \cdot \widehat{e}_i + \eta \cdot \left( a_i \sqrt{\widehat{e}_i} + a_j \sqrt{\widehat{e}_j} \right) - \beta \left( \Delta \widehat{w}_i - c \cdot \widehat{e}_i + c \cdot \widehat{e}_j \right) > u_0
\end{aligned}$$

under the constraints that  $\frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \leq \Delta \widehat{w}_i$ .

Note that the principal's profit is strictly decreasing in  $\Delta \widehat{w}_i$ . Hence, the optimal wage spread takes the minimum value at  $\Delta \widehat{w}_i^* = \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right)$  such that the equilibrium constraint will be binding. For this value of  $\Delta \widehat{w}_i$  indeed an inequitable equilibrium is attained. The optimal wages can be computed from the binding participation constraint.

Now, we compare the principal's profit in both equilibria. Note that in an



equitable equilibrium the principal's profit is given by

$$\begin{aligned}
U^* &= (1 + 2\eta) \left( a_i \sqrt{\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{\Delta w_i^*}{c}} + a_j \sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}} \right) \\
&\quad - \Delta w_i^* - 2u_0 - \frac{\eta^2 a_j^2}{2(1-\beta)^2 c} \\
&= (1 + 2\eta) \left( a_i \sqrt{\frac{\eta^2 a_j^2}{4(1-\beta)^2 c^2} + \frac{1}{c} \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1-\beta)^2} \right)} + a_j \sqrt{\frac{\eta^2 a_i^2}{4(1-\beta)^2 c^2}} \right) \\
&\quad - \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1-\beta)^2} \right) - 2u_0 - \frac{\eta^2 a_j^2}{2(1-\beta)^2 c} \\
&= (1 + 2\eta) \left( \frac{\eta a_i^2}{2(1-\beta)c} + \frac{\eta a_j^2}{2(1-\beta)c} \right) - \left( \frac{\eta^2 a_i^2}{4(1-\beta)^2 c} - \frac{\eta^2 a_j^2}{4(1-\beta)^2 c} \right) \\
&\quad - 2u_0 - \frac{\eta^2 a_j^2}{2(1-\beta)^2 c} \\
&= \frac{(2 - 2\beta + 3\eta - 4\beta\eta)\eta}{4(1 - \beta)^2 c} (a_i^2 + a_j^2) - 2u_0
\end{aligned}$$

and in an inequitable equilibrium it is equal to

$$\begin{aligned}
\widehat{U}^* &= (1 + 2\eta) \left( \frac{\eta a_i^2}{2(1-\beta)c} + \frac{\eta a_j^2}{2(1+\alpha)c} \right) \\
&\quad - 2 \left( u_0 + \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c} + \alpha \left( \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c} - \frac{\eta^2 a_i^2}{4(1-\beta)^2 c} \right) \right) \\
&\quad - 2\alpha \Delta \widehat{w}_i^* - \Delta \widehat{w}_i^* \\
&= (1 + 2\eta) \left( \frac{\eta a_i^2}{2(1-\beta)c} + \frac{\eta a_j^2}{2(1+\alpha)c} \right) \\
&\quad - 2 \left( u_0 + \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c} + \alpha \left( \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c} - \frac{\eta^2 a_i^2}{4(1-\beta)^2 c} \right) \right) \\
&\quad + 2\alpha \left( \frac{\eta^2 a_j^2}{4(1+\alpha)^2 c} - \frac{\eta^2 a_i^2}{4(1-\beta)^2 c} \right) - \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1-\beta)^2} - \frac{a_j^2}{(1+\alpha)^2} \right) \\
&= \frac{(2 - 2\beta + 3\eta - 4\beta\eta)\eta a_i^2}{4(1 - \beta)^2 c} + \frac{(2 + 2\alpha + 3\eta + 4\alpha\eta)\eta a_j^2}{4(1 + \alpha)^2 c} - 2u_0
\end{aligned}$$

Subtracting  $\widehat{U}^*$  from  $U^*$  results to

$$\begin{aligned}
U^* - \widehat{U}^* &= \frac{(2 - 2\beta + 3\eta - 4\beta\eta)\eta}{4(1 - \beta)^2c} (a_i^2 + a_j^2) - 2u_0 \\
&\quad - \left( \frac{(2 - 2\beta + 3\eta - 4\beta\eta)\eta a_i^2}{4(1 - \beta)^2c} + \frac{(2 + 2\alpha + 3\eta + 4\alpha\eta)\eta a_j^2}{4(1 + \alpha)^2c} - 2u_0 \right) \\
&= \frac{(2 - 2\beta + 3\eta - 4\beta\eta)\eta a_j^2}{4(1 - \beta)^2c} - \frac{(2 + 2\alpha + 3\eta + 4\alpha\eta)\eta a_j^2}{4(1 + \alpha)^2c} \\
&= \frac{((1 + \alpha)^2(2 - 2\beta + 3\eta - 4\beta\eta) - (1 - \beta)^2(2 + 2\alpha + 3\eta + 4\alpha\eta))\eta a_j^2}{4(1 + \alpha)^2(1 - \beta)^2c} \\
&= \frac{((2(1 - \alpha\beta) + 2\eta(1 - 2\alpha\beta))(\alpha + \beta) + (2 + 3\eta)(\alpha + \beta)(\alpha - \beta))\eta a_j^2}{4(1 + \alpha)^2(1 - \beta)^2c} > 0
\end{aligned}$$

Note that the difference in utilities is strictly positive for  $\alpha < 1$ . Furthermore, it is also strictly increasing in  $\alpha$ .

$$\frac{\partial}{\partial \alpha} \left( -\frac{(2 + 2\alpha + 3\eta + 4\alpha\eta)\eta a_j^2}{4(1 + \alpha)^2c} \right) = \frac{(2 + 2\alpha + 2\eta + 4\alpha\eta)\eta a_j^2}{4(1 + \alpha)^3c} > 0$$

Hence, the principal always receives the highest profit in an equitable equilibrium.

Furthermore, the optimal wages in an equitable equilibrium are given by

$$\begin{aligned}
w_j^* &= u_0 - \eta \cdot \left( a_i \sqrt{e_i^*} + a_j \sqrt{e_i^* - \frac{\Delta w_i^*}{c}} \right) + c \cdot e_i^* - \Delta w_i^* \\
&= u_0 - \eta \cdot \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4(1 - \beta)^2 c^2}} + a_j \sqrt{\frac{\eta^2 a_i^2}{4(1 - \beta)^2 c^2} - \frac{1}{c} \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1 - \beta)^2} - \frac{a_j^2}{(1 - \beta)^2} \right)} \right) \\
&\quad + c \cdot \frac{\eta^2 a_i^2}{4(1 - \beta)^2 c^2} - \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1 - \beta)^2} - \frac{a_j^2}{(1 - \beta)^2} \right) \\
&= u_0 - \frac{\eta^2 \left( (2 - 2\beta)a_i^2 + (1 - 2\beta)a_j^2 \right)}{4(1 - \beta)^2c}
\end{aligned}$$

$$\begin{aligned}
w_i^* &= w_j^* + \Delta w_i^* \\
&= u_0 - \frac{\eta^2 \left( (2 - 2\beta)a_i^2 + (1 - 2\beta)a_j^2 \right)}{4(1 - \beta)^2 c} + \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1 - \beta)^2} - \frac{a_j^2}{(1 - \beta)^2} \right) \\
&= u_0 - \frac{\eta^2 \left( (1 - 2\beta)a_i^2 + (2 - 2\beta)a_j^2 \right)}{4(1 - \beta)^2 c}
\end{aligned}$$

In an inequitable equilibrium, they are equal to

$$\begin{aligned}
\widehat{w}_j^* &= u_0 + c \cdot \widehat{e}_j^* - \eta \cdot \left( a_i \sqrt{\widehat{e}_i^*} + a_j \sqrt{\widehat{e}_j^*} \right) + \alpha \left( \Delta \widehat{w}_i^* - c \cdot \widehat{e}_i^* + c \cdot \widehat{e}_j^* \right) \\
&= u_0 + c \cdot \frac{\eta^2 a_j^2}{4(1 + \alpha)^2 c^2} - \eta \cdot \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4(1 - \beta)^2 c^2}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4(1 + \alpha)^2 c^2}} \right) \\
&\quad + \alpha \left( \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1 - \beta)^2} - \frac{a_j^2}{(1 + \alpha)^2} \right) - c \cdot \frac{\eta^2 a_i^2}{4(1 - \beta)^2 c^2} + c \cdot \frac{\eta^2 a_j^2}{4(1 + \alpha)^2 c^2} \right) \\
&= u_0 - \frac{\eta^2 a_i^2}{2(1 - \beta)c} - \frac{(1 + 2\alpha)\eta^2 a_j^2}{4(1 + \alpha)^2 c}
\end{aligned}$$

$$\begin{aligned}
\widehat{w}_i^* &= \widehat{w}_j^* + \Delta \widehat{w}_i^* \\
&= u_0 - \frac{\eta^2 a_i^2}{2(1 - \beta)c} - \frac{(1 + 2\alpha)\eta^2 a_j^2}{4(1 + \alpha)^2 c} + \frac{\eta^2}{4c} \left( \frac{a_i^2}{(1 - \beta)^2} - \frac{a_j^2}{(1 + \alpha)^2} \right) \\
&= u_0 - \frac{\eta^2 a_j^2}{2(1 + \alpha)c} - \frac{(1 - 2\beta)\eta^2 a_i^2}{4(1 - \beta)^2 c}
\end{aligned}$$

■

### Proof of Corollary 2:

When the agents are only self-interested the principal's maximization prob-

lem becomes

$$\begin{aligned}
& \max_{\tilde{e}_i, \tilde{e}_j, \tilde{w}_i, \tilde{w}_j} && \left( a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j} \right) - \tilde{w}_i - \tilde{w}_j \\
& \text{s.t.} && \tilde{w}_i - c \cdot \tilde{e}_i + \eta \cdot \left( a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j} \right) \geq u_0 \\
& && \tilde{w}_j - c \cdot \tilde{e}_j + \eta \cdot \left( a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j} \right) \geq u_0 \\
& && \tilde{e}_i = \frac{\eta^2 a_i^2}{4c^2} \\
& && \tilde{e}_j = \frac{\eta^2 a_j^2}{4c^2}
\end{aligned}$$

Note that  $\tilde{w}_i$  and  $\tilde{w}_j$  will be set such that the participation constraints will always be binding as it does not affect the incentive constraint and as the principal's profit is strictly decreasing in  $\tilde{w}_i$  and  $\tilde{w}_j$ . Hence,  $\tilde{w}_i^* = u_0 + c \cdot \tilde{e}_i - \eta \cdot (a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j})$  and  $\tilde{w}_j^* = u_0 + c \cdot \tilde{e}_j - \eta \cdot (a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j})$ . The program becomes

$$\begin{aligned}
& \max_{\tilde{e}_i, \tilde{e}_j} && \left( a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j} \right) - \left( u_0 + c \cdot \tilde{e}_i - \eta \cdot \left( a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j} \right) \right) \\
& && - \left( u_0 + c \cdot \tilde{e}_j - \eta \cdot \left( a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j} \right) \right) \\
& \text{s.t.} && \tilde{e}_i = \frac{\eta^2 a_i^2}{4c^2} \\
& && \tilde{e}_j = \frac{\eta^2 a_j^2}{4c^2}
\end{aligned}$$

The principal's profit in the equilibrium with selfish agents is given by

$$\begin{aligned}
\tilde{U}^* &= \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4c^2}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4c^2}} \right) - \left( u_0 + c \cdot \frac{\eta^2 a_i^2}{4c^2} - \eta \cdot \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4c^2}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4c^2}} \right) \right) \\
&\quad - \left( u_0 + c \cdot \frac{\eta^2 a_j^2}{4c^2} - \eta \cdot \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4c^2}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4c^2}} \right) \right) \\
&= \frac{(2 + 3\eta)\eta}{4c} (a_i^2 + a_j^2) - 2u_0
\end{aligned}$$

Subtracting  $\tilde{U}^*$  from  $U^*$  results to

$$\begin{aligned}
U^* - \tilde{U}^* &= \frac{(2-2\beta+3\eta-4\beta\eta)\eta}{4(1-\beta)^2c} (a_i^2 + a_j^2) - 2u_0 - \left( \frac{(2+3\eta)\eta}{4c} (a_i^2 + a_j^2) - 2u_0 \right) \\
&= \left( \frac{(2-2\beta+3\eta-4\beta\eta)\eta}{4(1-\beta)^2c} - \frac{(2+3\eta)\eta}{4c} \right) (a_i^2 + a_j^2) \\
&= \frac{\eta}{4c} \left( \frac{(2-2\beta+3\eta-4\beta\eta)}{(1-\beta)^2} - (2+3\eta) \right) (a_i^2 + a_j^2) \\
&= \frac{\eta}{4(1-\beta)^2c} \left( (1-\beta)2\beta(1+\eta) - \beta^2\eta \right) (a_i^2 + a_j^2) \\
&= \frac{\eta}{4(1-\beta)^2c} \underbrace{\left( (1-\beta) \cdot 2\beta \cdot (1+\eta) - \beta \cdot \beta \cdot \eta \right)}_{>0} (a_i^2 + a_j^2) > 0
\end{aligned}$$

Hence, the Principal's profit is always higher in the case when both agents are inequity averse.

Furthermore, the optimal wages are given by

$$\begin{aligned}
\tilde{w}_j^* &= u_0 + c \cdot \tilde{e}_j - \eta \cdot (a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j}) \\
&= u_0 + c \cdot \frac{\eta^2 a_j^2}{4c^2} - \eta \cdot \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4c^2}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4c^2}} \right) \\
&= u_0 - \frac{\eta^2 a_j^2}{4c} - \frac{\eta^2 a_i^2}{2c}
\end{aligned}$$

$$\begin{aligned}
\tilde{w}_i^* &= u_0 + c \cdot \tilde{e}_i - \eta \cdot (a_i \sqrt{\tilde{e}_i} + a_j \sqrt{\tilde{e}_j}) \\
&= u_0 + c \cdot \frac{\eta^2 a_i^2}{4c^2} - \eta \cdot \left( a_i \sqrt{\frac{\eta^2 a_i^2}{4c^2}} + a_j \sqrt{\frac{\eta^2 a_j^2}{4c^2}} \right) \\
&= u_0 - \frac{\eta^2 a_i^2}{4c} - \frac{\eta^2 a_j^2}{2c}
\end{aligned}$$

Note that in this case a profit-maximizing principal will pay a higher fixed wage to the more able agent, who in turn supplies greater effort than the less

able agent. Furthermore, the optimal wage spread is given by

$$\Delta \tilde{w}_i^* = \frac{\eta^2}{4c} (a_i^2 - a_j^2)$$

Compared to the equitable equilibrium with inequity averse agents, a profit-maximizing principal compresses wages when agents are purely self-interested

$$\Delta \tilde{w}_i^* = \frac{\eta^2}{4c} (a_i^2 - a_j^2) < \frac{\eta^2}{4(1-\beta)^2c} (a_i^2 - a_j^2) = \Delta w_i^*.$$

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