

Ludwig, Alexander; Zimper, Alexander

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# A decision-theoretic model of asset price fluctuations\*

Alexander Ludwig<sup>†</sup>      Alexander Zimper<sup>‡</sup>

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## Abstract

Based on the psychological interpretation of conditional non-additive probability measures arising in Choquet expected utility theory we introduce a behavioral model of asset price fluctuations. In this model naïve agents are prone to a confirmatory bias in the interpretation of new information about economic fundamentals. As a conceptual advantage over rational expectations asset pricing models our formal approach gives rise to model-endogenous concepts of “overpricing” and “underpricing”. As another interesting feature our model generates equilibrium price patterns that reflect underreaction of asset prices with respect to one period good news as well as overreaction with respect to several periods of good news. These empirical phenomena have received significant attention in the behavioral finance literature. In contrast to our approach, however, previous formal explanations have not been derived from decision-theoretic first principles but were rather ad hoc.

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<sup>†</sup>CMR, University of Cologne; Albertus-Magnus-Platz; 50923 Köln; Germany; Email: ludwig@wiso.uni-koeln.de.

<sup>‡</sup>School of Economic and Business Sciences, University of the Witwatersrand, Johannesburg, South Africa. E-mail: zimper@bigfoot.com

# 1 Introduction

## 1.1 Motivation

Rational asset pricing models derive prices from the portfolio optimization problems of agents who fully understand the behavior of other economic agents as well as the stochastic processes that drive the economic fundamentals. The common theme of rational models is the efficient market hypothesis (=EMH) which implies that price fluctuations should only reflect changes in the agents' information about economic fundamentals. Several empirical price patterns, suggest, however, that asset price dynamics cannot be satisfactorily explained by rational models alone. For example, the persistent empirical phenomenon that prices tend to underreact to one period of good news whereas they tend to overreact to several periods of good news is difficult to reconcile with the EMH.

As a formal attempt to describe the occurrence of price fluctuations that are not based on economic fundamentals, we develop in this paper an asset pricing model based on sound decision-theoretic foundations. Our approach makes thereby two behavioral assumptions. Firstly, we assume that the agents' belief generation process is governed by a confirmatory bias heuristic according to which the interpretation of new information depends on the asset's past performance in terms of dividend payments. Secondly, we assume that the agents are naïve in the sense that they do not fully understand this biased belief generation process. As a specific feature of our approach, we formalize these behavioral assumptions within the axiomatic framework of Choquet expected utility (CEU) theory. In contrast to standard expected utility theory, the beliefs of CEU agents are described by non-additive rather than additive probability measures (cf. Schmeidler 1986, 1989; Gilboa 1987). In order to focus our analysis, we further restrict attention to neo-additive capacities in the sense of Chateauneuf, Eichberger and Grant (2007) according to which an agent's non-additive belief about the likelihood of an event is a weighted average of an ambiguous part and an additive part.

Our formal concept of a confirmatory bias heuristic exploits the fact that there exist several perceivable Bayesian update rules for non-additive probability measures expressing different psychological attitudes towards the interpretation of new informa-

tion (Gilboa and Schmeidler 1993; Sarin and Wakker 1998).<sup>1</sup> In particular, we consider the so-called *optimistic* and *pessimistic* (Gilboa and Schmeidler 1993; Sarin and Wakker 1998) as well as the *full Bayesian* update rule (Pires 2002; Eichberger, Grant, and Kelsey 2006; Siniscalchi 2006). Depending on the agent’s information about the asset’s past performance in terms of dividend payments, three corresponding asset pricing regimes—the optimistic, the pessimistic, as well as the default pricing scheme—emerge. Our formal approach thus captures the intuitive notion that good news result in an optimistic whereas bad news result in a pessimistic bias in the agent’s belief about the asset’s future prospects. These according biases then give rise to natural definitions of over- and underpricing, respectively, in our model of asset-price equilibrium.

Whenever the beliefs of our approach reduce to additive probability measures, these three different pricing regimes collapse into the familiar asset pricing model of expected utility theory with additively time-separable preferences. However, whenever the non-additive beliefs of our model express ambiguity attitudes, the existence of the three different pricing schemes results in a higher fluctuation of equilibrium prices than in the standard expected utility model. We further illustrate the explanatory power of our behavioral model of asset-price fluctuations by demonstrating that our equilibrium price process quite naturally generates the so-called phenomena of “underreaction” and “overreaction”.

## 1.2 Underreaction and overreaction

There is by now a rich literature in behavioral finance that addresses the empirical phenomena of underreaction and overreaction. On the one hand, stock prices tend to underreact in the short run in the sense that one period of goods news is not fully reflected in the asset’s price. On the other hand, however, stock prices tend to overreact if there is a prolonged sequence of good news. Recent empirical evidence on over- and underreaction is presented, e.g., in Chan (2003) and Antweiler and Frank (2006).

For the purpose of this paper we adapt the formal definitions of underreaction and

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<sup>1</sup>As explained below, this “indeterminacy” of update rules is a direct consequence of the violation of Savage’s (1954) sure thing principle as elicited in paradoxes of the Ellsberg (1961) type.

overreaction of Barberis et al. (1998). Consider the adapted process of equilibrium returns for a given asset  $(R_t)_{t \geq 1}$  such that

$$R_t = \frac{p_t^* + Y_t}{p_{t-1}^*} \text{ for all } t \geq 1 \quad (1)$$

whereby  $p_t^*$  denotes period  $t$ 's ex-dividend equilibrium price and  $Y_t$  period  $t$ 's dividend payment, respectively. Furthermore, let  $G_t$ , respectively  $B_t$ , denote the event that there is “good”, respectively “bad”, news in period  $t$ .

### Definitions.

**(i) Underreaction to one period of good news:**

$$E [R_{t+1}, \pi (\cdot | G_t)] \geq E [R_{t+1}, \pi (\cdot | B_t)]. \quad (2)$$

**(ii) Overreaction to a sequence of good news:** *For some  $j \geq 2$ ,*

$$E [R_{t+j}, \pi (\cdot | G_t, \dots, G_{t+j-1})] \leq E [R_{t+j}, \pi (\cdot | B_t, \dots, B_{t+j-1})]. \quad (3)$$

The interpretation of these definitions is straightforward. In the case of one period good news the asset is—compared to the case of one-period bad news—underpriced in period  $t$  whereas this underpricing is corrected later-on in period  $t + 1$ . A rather low price  $p_t^*$  and a rather high price  $p_{t+1}^*$  imply then a rather high return in the case of good news in period  $t + 1$  as formally expressed by (2). In the case of a sequence of  $j \geq 2$  periods of good news the opposite effect happens in that the price of the good-news asset in period  $t + j - 1$ ,  $p_{t+j-1}^*$ , is rather too high whereas the corrected—rather low—price in period  $t + j$ ,  $p_{t+j}^*$ , implies then a rather low return in period  $t + j$  as formally expressed by (3).

To explain these phenomena, three different scenarios with respect to the nature and reception of information about a company's earnings have been considered in the literature. Daniel et al. (1998) differentiate between public and private information whereby

they make the behavioral assumption that investors believe too strongly in their own information and discount public signals. Consequently, there is underreaction to public information but overreaction to private information. Hong and Stein (1999) base their theory on the assumption that there exist heterogeneous groups of investors which differ in their reactions to public news. In particular, one of these groups ignores news but reacts to prices only. As a consequence, there is initial underreaction followed by subsequent overreaction. Finally, Barberis et al. (1998) consider a representative investor—so that public and private information coincide—who believes that either one of two—actually incorrect regimes—correctly describes the firm’s earning process. Barberis et al. proceed to assume that, firstly, the investor changes his beliefs about the likelihood of either regime in the light of new information about past dividend payments and, secondly, that one regime is mean-reverting whereas the other regimes follows a trend. Under these assumptions, Barberis et al. can generate over-, respectively underreaction whenever the dividend payment process follows a random walk.

As in Barberis et al. (1998) our approach generates underreaction and overreaction within a representative agent economy. In contrast to Barberis et al., however, our model of asset-pricing has been developed from decision-theoretic first principles without explicitly aiming at any ad hoc explanation of the under- and overreaction phenomena. We thereby address an important self-criticism of Barberis et al. (1998) who remark: “The idea that the investor believes that the world is governed by one of the two incorrect models is a crude way of capturing the psychological phenomena of the previous section.” (p. 318). The fact that agents may not learn the correct earnings process in the long run results in our model as a formal consequence of our description of agents’ beliefs by non-additive probability measures arising in CEU theory whereas Barberis et al. introduce it by assumption. As a consequence, our approach is therefore not prone to the standard criticism against the relevance of incorrect beliefs stating that additive beliefs will converge in the long run to correct probabilities in standard models of Bayesian learning (cf. Zimmer (2009) and Zimmer and Ludwig (2009)).

The remainder of our analysis is structured as follows. Section 2 presents the decision

theoretic framework on which our model is based. In section 3 we then describe our behavioral assumptions, namely that agents are prone to confirmatory biases and naïve in the sense that they ignore these biases. Next, section 4 merges our assumptions on expectations and behavior with a standard Lucas (1978) type asset pricing model and thereby describes the equilibrium price processes emanating from our model. Based on these insights, section 5 describes and illustrates how our model gives rise to mispricing relative to the EMH hypothesis and consequently overpricing, respectively underpricing, of assets. That our model gives rise to over- and underreaction in correspondence to the formal definitions given above is shown in section 6. Finally, section 7 concludes. All proofs are relegated to the appendix.

## **2 Decision-theoretic preliminaries**

The formalism of our behavioral model is not ad hoc but rather based on Choquet decision theory which we review in this section. In the first subsection we recall basic elements of CEU (=Choquet expected utility) theory and we introduce neo-additive capacities as a specific class of non-additive probability measures. In the second subsection we present different Bayesian update rules for non-additive probability measures and their psychological interpretation whereby we demonstrate that the existence of different Bayesian update rules is an immediate consequence of the failure of Savage's sure thing principle as demonstrated by experiments of the Ellsberg-type. (The reader who is familiar with Choquet decision theory may wish to skip this section and proceed directly to our behavioral model.)

### **2.1 Choquet decision theory and neo-additive capacities**

CEU (=Choquet expected utility) theory was first axiomatized by Schmeidler (1986, 1989) within the Anscombe and Aumann (1963) framework, which assumes preferences over objective probability distributions. Subsequently, Gilboa (1987) as well as Sarin and Wakker (1992) have presented CEU axiomizations within the Savage (1954) framework, assuming a purely subjective notion of likelihood. When restricted to the domain

of gains, CEU theory is formally equivalent to *cumulative prospect theory* (Tversky and Kahneman, 1992; Wakker and Tversky, 1993) which generalizes the celebrated *prospect theory* of Kahneman and Tversky (1979). Moreover, as a representation of preferences over lotteries, CEU theory coincides with *rank dependent utility theory* as introduced by Quiggin (1981, 1982). Within the context of CEU theory, properties of such capacities are used in the literature for formal definitions of, e.g., *ambiguity* and *uncertainty attitudes* (Schmeidler 1989; Epstein 1999; Ghirardato and Marinacchi 2002), *pessimism* and *optimism* (Eichberger and Kelsey 1999; Wakker 2001), as well as *sensitivity to changes in likelihood* (Wakker 2004).

Let us consider a measurable space  $(\Omega, \mathcal{F})$  with  $\mathcal{F}$  denoting a  $\sigma$ -algebra on the state space  $\Omega$  and a non-additive probability measure (=capacity)  $\nu : \mathcal{F} \rightarrow [0, 1]$  satisfying

- (i)  $\nu(\emptyset) = 0, \nu(\Omega) = 1$
- (ii)  $A \subset B \Rightarrow \nu(A) \leq \nu(B)$  for all  $A, B \in \mathcal{F}$ .

The Choquet integral of a bounded function  $f : \Omega \rightarrow \mathbb{R}$  with respect to capacity  $\nu$  is defined as the following Riemann integral extended to domain  $\Omega$  (Schmeidler 1986):

$$E[f, \nu(d\omega)] = \int_{-\infty}^0 (\nu(\{\omega \in \Omega \mid f(\omega) \geq z\}) - 1) dz + \int_0^{+\infty} \nu(\{\omega \in \Omega \mid f(\omega) \geq z\}) dz \quad (4)$$

whereby we will simply write  $E[f, \nu]$  for  $E[f, \nu(d\omega)]$ . For example, assume that  $f$  takes on  $m$  different values such that  $A_1, \dots, A_m$  is the unique partition of  $\Omega$  with  $f(\omega_1) > \dots > f(\omega_m)$  for  $\omega_i \in A_i$ . Then the Choquet expectation (4) becomes

$$E[f, \nu] = \sum_{i=1}^m f(\omega_i) \cdot [\nu(A_1 \cup \dots \cup A_i) - \nu(A_1 \cup \dots \cup A_{i-1})]. \quad (5)$$

Our own approach focuses on non-additive beliefs that are defined as *neo-additive capacities* in the sense of Chateauneuf, Eichberger and Grant (2007).

**Definition.** For a given measurable space  $(\Omega, \mathcal{F})$  the neo-additive capacity,  $\nu$ , is defined, for some  $\delta \in (0, 1), \lambda \in [0, 1]$  by

$$\nu(A) = \delta \cdot \lambda + (1 - \delta) \cdot \pi(A) \quad (6)$$



for all  $A \in \mathcal{F}$  such that  $A \notin \{\emptyset, \Omega\}$  whereby  $\pi$  is some additive probability measure on  $(\Omega, \mathcal{F})$ .

Neo-additive capacities can thus be interpreted as non-additive beliefs that stand in for deviations from additive beliefs such that a parameter  $\delta$  (*degree of ambiguity*) measures the lack of confidence the decision maker has in some subjective additive probability distribution  $\pi$ . The following proposition extends a result (Lemma 3.1) of Chateauneuf, Eichberger and Grant (2007) for finite random variables to the more general case of random variables with a closed and bounded range.

**Observation 1.** *Let  $f$  be real-valued function with closed and bounded range. The Choquet expected value (4) of  $f$  with respect to a neo-additive capacity (6) is given by*

$$E[f, \nu] = \delta \left( \lambda \max_{\omega \in \Omega} f(\omega) + (1 - \lambda) \min_{\omega \in \Omega} f(\omega) \right) + (1 - \delta) E[f, \pi]. \quad (7)$$

According to observation 1, the Choquet expected value of a random variable  $f$  with respect to a neo-additive capacity is a convex combination of the expected value of  $f$  with respect to some additive probability measure  $\pi$  and an ambiguity part. If there is no ambiguity, i.e.,  $\delta = 0$ , then the Choquet expected value (7) reduces to the standard expected value of a random variable with respect to an additive probability measure. In case there is some ambiguity, however, the second parameter  $\lambda$  measures how much weight the decision maker puts on the best possible outcome of  $f$  when resolving his ambiguity. Conversely,  $(1 - \lambda)$  is the weight he puts on the worst possible outcome of  $f$ . As a consequence, we interpret  $\lambda$  as an “optimism under ambiguity” parameter whereby  $\lambda = 1$ , resp.  $\lambda = 0$ , corresponds to extreme optimism, resp. extreme pessimism, with respect to resolving ambiguity in the decision maker’s belief.

## 2.2 Bayesian updating of neo-additive capacities

CEU theory has been developed in order to accommodate paradoxes of the Ellsberg type which show that real-life decision-makers violate Savage's *sure-thing principle* according to which preferences over acts shall be unaffected by consequences in states in which the two acts have the same outcome. In this section we demonstrate that abandoning the sure-thing principle bears two important implications for conditional CEU preferences over Savage-acts. First, in contrast to Bayesian updating of additive probability measures, there exist several perceivable Bayesian update rules for non-additive probability measures (cf. Gilboa and Schmeidler 1993, Sarin and Wakker 1998, Pires 2002, Eichberger, Grant and Kelsey 2006, Siniscalchi 2006). Second, any preferences that (strictly) violate the sure-thing principle cannot be updated in a dynamically consistent way. That is, there does not exist any updating rule for capacities such that ex-ante CEU preferences that (strictly) violate the sure-thing principle are updated in a dynamically consistent manner to ex-post CEU preferences.

To see this define the Savage-act  $f_B h : \Omega \rightarrow X$  such that

$$f_B h(\omega) = \begin{cases} f(\omega) & \text{for } \omega \in B \\ h(\omega) & \text{for } \omega \in \neg B \end{cases}$$

where  $B$  is some non-empty event. That is, the act  $f_B h$  gives the same consequences as the act  $f$  in all states belonging to event  $B$  and it gives the same consequences as the act  $h$  in all states outside of event  $B$ . Recall that Savage's sure-thing principle states that, for all acts  $f, g, h, h'$  and all events  $B \in \mathcal{F}$ ,

$$f_B h \succeq g_B h \text{ implies } f_B h' \succeq g_B h'. \quad (8)$$

That is, preferences over Savage-acts  $f$  and  $g$  should be unaffected by any states in which these acts give the same consequences. Let us now interpret event  $B$  as new information received by the agent. The sure-thing principle then implies a straightforward way for deriving ex-post preferences  $\succeq_B$ , conditional on the new information  $B$ , from the agent's original preferences  $\succeq$  over Savage-acts. Namely, we have

$$f \succeq_B g \text{ if and only if } f_B h \succeq g_B h \text{ for any } h, \quad (9)$$

so that an agent's ex-post preferences over two acts in light of new information  $B$  are given as the agent's ex-ante preferences over these acts whenever both acts give arbitrary but identical consequences in states of the world that do not belong to  $B$  (i.e., in states that will be declared *impossible* by the new information  $B$ ). Equation (9) implies for a subjective EU decision-maker

$$f \succeq_B g \Leftrightarrow E[u(f), \pi(\cdot | B)] \geq E[u(g), \pi(\cdot | B)]$$

where  $u : X \rightarrow \mathbb{R}$  is a von Neumann-Morgenstern utility function and  $\pi(\cdot | B)$  is a conditional additive probability measure defined, for all  $A, B \in \mathcal{F}$  such that  $\pi(B) > 0$ , by

$$\pi(A | B) = \frac{\pi(A \cap B)}{\pi(B)}.$$

In case the sure-thing principle does not hold, the specification of act  $h$  in (9) is no longer arbitrary. For CEU preferences there therefore exist several possibilities of deriving ex post preferences from ex ante preferences. That is, in a CEU framework there exist several perceivable ways of defining a conditional capacity  $\nu(\cdot | B)$  such that

$$f \succeq_B g \Leftrightarrow E[u(f), \nu(\cdot | B)] \geq E[u(g), \nu(\cdot | B)].$$

Let us at first consider so-called *h-Bayesian update rules* for preferences  $\succeq$  over Savage acts as introduced by Gilboa and Schmeidler (1993). That is, we consider some collection of conditional preference orderings,  $\{\succeq_B^h\}$  for all events  $B$ , such that for all acts  $f, g$

$$f \succeq_B^h g \text{ if and only if } f_B h \succeq g_B h \tag{10}$$

where

$$h = (x^*, E; x_*, \neg E), \tag{11}$$

with  $x^*$  denoting the best and  $x_*$  denoting the worst consequence possible and  $E \in \mathcal{F}$ . For the so-called *optimistic* update rule  $h$  is the constant act where  $E = \emptyset$ . That is, under the optimistic update rule the null-event,  $\neg B$ , becomes associated with the worst consequence possible. Gilboa and Schmeidler (1993) offer the following psychological motivation for this update rule:

“[...] when comparing two actions given a certain event  $B$ , the decision maker implicitly assumes that had  $B$  not occurred, the worst possible outcome [...] would have resulted. In other words, the behavior given  $B$  [...] exhibits ‘happiness’ that  $B$  has occurred; the decisions are made as if we are always in ‘the best of all possible worlds’.”

As corresponding optimistic Bayesian update rule for conditional beliefs of CEU decision makers we obtain

$$\nu_{opt}(A | B) = \frac{\nu(A \cap B)}{\nu(B)}. \quad (12)$$

**Observation 2.** *An application of the optimistic update rule (12) to a neo-additive capacity (6) results in the conditional neo-additive capacity*

$$\nu_{opt}(A | B) = \delta_{opt}(B) + (1 - \delta_{opt}(B)) \cdot \pi(A | B) \quad (13)$$

with

$$\delta_{opt}(B) = \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1 - \delta) \cdot \pi(B)}.$$

For the *pessimistic* (or Dempster-Shafer) update rule  $h$  is the constant act where  $E = \Omega$ , associating with the null-event,  $\neg B$ , the best consequence possible. The psychological interpretation for this update rule according to Gilboa and Schmeidler (1993) is as follows:

“[...] we consider a ‘pessimistic’ decision maker, whose choices reveal the hidden assumption that all the impossible worlds are the best conceivable ones.”

The corresponding pessimistic Bayesian update rule for CEU decision makers is

$$\nu_{pess}(A | B) = \frac{\nu(A \cup \neg B) - \nu(\neg B)}{1 - \nu(\neg B)}. \quad (14)$$

**Observation 3.** *An application of the pessimistic update rule (14) to a neo-additive capacity (6) results in the conditional neo-additive capacity*

$$\nu_{pess}(A | B) = (1 - \delta_{pess}(B)) \cdot \pi(A | B) \quad (15)$$

with

$$\delta_{pess}(B) = \frac{\delta \cdot (1 - \lambda)}{\delta \cdot (1 - \lambda) + (1 - \delta) \cdot \pi(B)}.$$

In addition to the two  $h$ -Bayesian update rules described above, we also consider conditional CEU preferences satisfying, for all acts  $f, g$ ,

$$f \succeq_B g \text{ if and only if } f_B h \succeq g_B h$$

where  $h$  is the so-called conditional certainty equivalent of  $g$ , i.e., given information  $B$  the agent is indifferent between the act  $g$  and the act  $h$  that gives in every state of  $B$  the same consequence. The corresponding Bayesian update rule for the non-additive beliefs of a CEU decision maker is the so-called full Bayesian update rule which is given by (Eichberger, Grant, and Kelsey 2006)

$$\nu_{FB}(A | B) = \frac{\nu(A \cap B)}{\nu(A \cap B) + 1 - \nu(A \cup \neg B)}, \quad (16)$$

where  $\nu^{FB}(A | B)$  denotes the conditional capacity for event  $A \in \mathcal{F}$  given information  $B \in \mathcal{F}$ .

**Observation 4.** *An application of the full Bayesian update rule (16) to a neo-additive capacity (6) results in the conditional neo-additive capacity*

$$\nu_{FB}(A | B) = \delta_{FB}(B) \cdot \lambda + (1 - \delta_{FB}(B)) \cdot \pi(A | B) \quad (17)$$

whereby

$$\delta_{FB}(B) = \frac{\delta}{\delta + (1 - \delta) \cdot \pi(B)}. \quad (18)$$

### 3 The behavioral model

Our behavioral model is based on two main assumptions. Firstly, the agents' belief generation process is prone to a confirmatory bias heuristic in case the asset's performance in terms of dividend-payments, i.e., underlying profits, is regarded as good, respectively bad. The according heuristic of a performance-driven confirmatory bias will be introduced in subsection 3.2. Secondly, the agents do not fully understand this biased belief generation process in the sense that they do not correctly anticipate the beliefs of their future selves. The agent's naïve beliefs will be introduced in subsection 3.3. As a consequence of these behavioral assumptions, our model violates the rational beliefs paradigm in two different ways. On the one hand, the agents of our model have an incorrect world view because—due to their confirmatory bias—their subjective beliefs about economic fundamentals do not coincide with the objective distribution of these fundamentals regardless of how much information the agents receive. On the other hand, while solving their portfolio optimization problem the agents incorrectly assume that their future selves interpret new information in the same way a they do.

#### 3.1 Information structure

Let  $(Y_t)_{t \geq 0}$  denote a sequence of random variables whereby the range of  $Y_t$  is given by some finite set  $\mathbf{Y}_t \subset \mathbb{R}_+$  containing at least two elements. We interpret  $Y_t$  as the asset's random period  $t$  dividend payments which we identify—by convention—with the underlying firm's period  $t$  profits. For our purpose it is convenient to consider  $Y_t$  as coordinate variable on the state space

$$\Omega = \times_{t=0}^{\infty} \mathbf{Y}_t, \quad (19)$$

i.e., for all  $t \geq 0$ ,  $Y_t : \Omega \rightarrow \mathbf{Y}_t$  such that

$$Y_t(\omega) = y_t \text{ for } \omega = (y_0, y_1, \dots). \quad (20)$$

As period  $t$ 's agent information partition we define

$$\mathcal{P}_t = \left\{ \{(y_0, \dots, y_t)\} \times \left( \times_{s=t+1}^{\infty} \mathbf{Y}_s \right) \right\}_{(y_0, \dots, y_t) \in \times_{s=0}^t \mathbf{Y}_s}. \quad (21)$$

Observe that the agents' information partitions are getting finer with increasing  $t$ , implying that the period  $t$  agent can observe any realized stream of past dividend payments  $y_0, \dots, y_t$ . Further denote by  $\mathcal{F}_t$  the Borel  $\sigma$ -algebra generated by  $\mathcal{P}_t$ ; that is,  $\mathcal{F}_t$  contains all sets

$$A \times \left( \times_{s=t+1}^{\infty} \mathbf{Y}_s \right) \quad (22)$$

in  $\Omega$  such that  $A$  is any subset of  $\times_{s=0}^t \mathbf{Y}_s$ . As in standard asset pricing models we thus assume perfect memory since the  $\sigma$ -algebras  $\mathcal{F}_t$  generated by the  $\mathcal{P}_t$  constitute a *filtration*, i.e.,  $\mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \dots \subseteq \mathcal{F}$  whereby  $\mathcal{F}$  denotes the standard product algebra generated by  $\mathcal{F}_1, \mathcal{F}_2, \dots$

### 3.2 Performance-driven confirmatory bias heuristic

Given a measurable space  $(\Omega, \mathcal{F})$  we now describe the agents' belief generation process by some conditional neo-additive probability measure  $\nu(\cdot | \cdot)$ . Recall that the information received by a period  $t$  agent is given as the stream of realizations of past profits  $y_0, \dots, y_t$ . Let us formally define this information as the following event in  $\mathcal{F}_t$

$$I_t = (y_0, \dots, y_t) \times \left( \times_{s=t+1}^{\infty} \mathbf{Y}_s \right)$$

whereby we will henceforth speak of "agent  $I_t$ " whenever a period  $t$  agent has received information  $I_t$ .

The key to our behavioral approach is a heuristic assumption according to which the period  $t$  agent might have an optimistic, respectively pessimistic, confirmatory bias in the evaluation of an asset's future performance which results from the agent's observation of the asset's past "performance". As a simplifying but plausible assumption we suppose that an agent evaluates the asset's performance in terms of profits, respectively dividend payments. The following assumptions on the conditional neo-additive probability measure  $\nu(\cdot | \cdot)$  naturally link our heuristic concept of "good" versus "bad" asset performance to the optimistic versus pessimistic Bayesian update rules for neo-additive beliefs of subsection 2.2.

**Confirmatory bias heuristic.** Fix some number  $n \geq 1$  and let

$$I_t = (y_0, \dots, y_{t-n}, \dots, y_t) \times \left( \times_{s=t+1}^{\infty} \mathbf{Y}_s \right) \in \mathcal{F}.$$

Then  $\nu(\cdot | \cdot)$  must satisfy the following conditions.

- (i) If  $y_{t-n} < \dots < y_t$ , then agent  $I_t$  is optimistically biased in the sense that, for all  $A \in \mathcal{F}$ ,

$$\nu(A | I_t) = \nu_{opt}(A | I_t) \quad (23)$$

with  $\nu_{opt}(A | I_t)$  given by (13).

- (ii) If  $y_{t-n} > \dots > y_t$ , then agent  $I_t$  is pessimistically biased in the sense that, for all  $A \in \mathcal{F}$ ,

$$\nu(A | I_t) = \nu_{pess}(A | I_t) \quad (24)$$

with  $\nu_{pess}(A | I_t)$  given by (15).

- (iii) If neither  $y_{t-n} < \dots < y_t$  nor  $y_{t-n} > \dots > y_t$ , then, for all  $A \in \mathcal{F}$ ,

$$\nu(A | I_t) = \nu_{FB}(A | I_t) \quad (25)$$

with  $\nu_{FB}(A | I_t)$  given by (17).

In words: If the period  $t$  agent has observed strictly increasing past profits  $y_{t-n}, \dots, y_t$  for a fixed number  $n$  of periods, he interprets the prospect of future profits in an optimistic way. If, in contrast, he has observed strictly decreasing past profits  $y_{t-n}, \dots, y_t$ , he interprets the prospect of future profits in a rather pessimistic way.

These heuristic assumptions on  $\nu(\cdot | \cdot)$  conclude the construction of  $(\nu(\cdot | \cdot), \Omega, \mathcal{F})$  as the conditional neo-additive probability space that henceforth governs the agents' "true" belief process about dividend payments.

### 3.3 Naïve agents

Whereas  $(\nu(\cdot | \cdot), \Omega, \mathcal{F})$  describes—by definition—the belief generation process for every agent  $I_t$  about next period's profits  $Y_{t+1}$ , the agents themselves will have an incorrect



view on this belief generation process. More precisely, we assume that every agent  $I_t$  is convinced that his and his future-selves' beliefs are governed by some process  $(\nu^{I_t}(\cdot | \cdot), \Omega, \mathcal{F})$  whereby  $\nu(\cdot | I_s)$  and  $\nu^{I_t}(\cdot | I_s)$  coincide whenever  $s = t$  but not on future periods  $s > t$  (cf. the remark at the end of this subsection). That is, our agents are “naïve” in the sense that they do not correctly anticipate that their future selves' beliefs are governed by  $\nu(\cdot | \cdot)$  rather than by  $\nu^{I_t}(\cdot | \cdot)$ .

In what follows we impose formal properties on  $\nu^{I_t}(\cdot | \cdot)$  whereby we slightly misuse notation and simply write  $y_{s+1}$  for the event

$$(Y_{s+1} = y_{s+1}) \in \mathcal{F}_{s+1}$$

and  $y_0, \dots, y_s$  for the event

$$(Y_0 = y_0, \dots, Y_s = y_s) \in \mathcal{F}_s.$$

**Agent  $I_t$ 's “naïve” belief  $\nu^{I_t}(\cdot | \cdot)$ .**

- (i) *Suppose that  $I_t$  is optimistically biased. Then, for all  $y_{s+1} \in \mathcal{F}_{s+1}$  and all  $y_0, \dots, y_s \in \mathcal{F}_s$  with  $s \geq t$ ,*

$$\nu_{opt}^{I_t}(y_{s+1} | y_0, \dots, y_s) = \delta_{opt}^{I_t} + (1 - \delta_{opt}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s)$$

whereby

$$\delta_{opt}^{I_t} = \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1 - \delta) \cdot \pi(I_t)}.$$

- (ii) *Suppose that  $I_t$  is pessimistically biased. Then, for all  $y_{s+1} \in \mathcal{F}_{s+1}$  and all  $y_0, \dots, y_s \in \mathcal{F}_s$  with  $s \geq t$ ,*

$$\nu_{pess}^{I_t}(y_{s+1} | y_0, \dots, y_s) = (1 - \delta_{pess}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s) \quad (26)$$

whereby

$$\delta_{pess}^{I_t} = \frac{\delta \cdot (1 - \lambda)}{\delta \cdot (1 - \lambda) + (1 - \delta) \cdot \pi(I_t)}.$$

(iii) Suppose that  $I_t$  is neither optimistically nor pessimistically biased. Then, for all  $y_{s+1} \in \mathcal{F}_{s+1}$  and all  $y_0, \dots, y_s \in \mathcal{F}_s$  with  $s \geq t$ ,

$$\nu_{FB}^{I_t}(y_{s+1} \mid y_0, \dots, y_s) = \delta_{FB}^{I_t} \cdot \lambda + (1 - \delta_{FB}^{I_t}) \cdot \pi(y_{s+1} \mid y_0, \dots, y_s) \quad (27)$$

whereby

$$\delta_{FB}^{I_t} = \frac{\delta}{\delta + (1 - \delta) \cdot \pi(I_t)}.$$

**Remark.** Since the naïve agents of our model are not aware of the true nature of their confirmatory bias, they are not *sophisticated* in the sense of Strotz (1956) and Pollak (1968) nor *behavioral consistent* in the sense of Karni and Safra (1990) nor *sequentially rational* in the sense of Kreps and Wilson (1982). To see this consider, e.g., two optimistically biased agents  $I_t$  and  $I_s$  such that  $s > t$ . Then

$$\begin{aligned} \nu^{I_t}(y_{s+1} \mid y_0, \dots, y_s) &= \delta_{opt}^{I_t} + (1 - \delta_{opt}^{I_t}) \cdot \pi(y_{s+1} \mid y_0, \dots, y_s) \\ &\neq \delta_{opt}^{I_s} + (1 - \delta_{opt}^{I_s}) \cdot \pi(y_{s+1} \mid y_0, \dots, y_s) \\ &= \nu^{I_s}(y_{s+1} \mid y_0, \dots, y_s) \\ &= \nu(y_{s+1} \mid I_s) \end{aligned}$$

since

$$\begin{aligned} \delta_{opt}^{I_t} &= \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1 - \delta) \cdot \pi(I_t)} \\ &\neq \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1 - \delta) \cdot \pi(I_s)} \\ &= \delta_{opt}^{I_s} \\ &= \delta_{opt}(I_s) \end{aligned}$$

because of  $\pi(I_t) > \pi(I_s)$ . That is, agent  $I_t$ 's belief about dividend payments in period  $s + 1$  conditional on the dividend payments  $y_0, \dots, y_s$  does not coincide with the belief of agent  $I_s$  who has actually observed dividend payments  $y_0, \dots, y_s$ . It is in this specific sense that the agents of our model hold incorrect opinions about the beliefs of their future-selves.

## 4 The equilibrium price process

In the previous section we have developed our behavioral model according to which agents are prone to a confirmatory bias if they have observed a good, respectively bad, asset performance in terms of the stochastic dividend payment process  $(Y_t)_{t \geq 0}$ . In this section we derive the corresponding equilibrium prices of an representative agent economy as an adapted stochastic process  $(p_t^*)_{t \geq 0}$ .

Consider the decision situation of a representative agent  $I_t$  who maximizes his Choquet expected utility from an infinite consumption stream with respect to his belief  $\nu^{I_t}(\cdot | \cdot)$  by deciding about his period  $t, t+1, \dots$  asset holdings. In line with most of the literature we assume that the agent's vNM utility of an infinite consumption stream is additively time-separable. The agent's maximization problem is then given as

$$\max_{(z_t, z_{t+1}, \dots)} u(c_t) + E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \nu^{I_t}(\cdot | \cdot) \right]$$

subject to

$$c_s = e_s + Y_s z_s + p_s (z_s - z_{s+1}) \text{ for all } s \geq t.$$

$\beta \in (0, 1)$  denotes the time-discount factor,  $u : \mathbb{R} \rightarrow \mathbb{R}$  is a strictly concave, differentiable function,  $e_s \in \mathbb{R}$  stands for the agent's additional income in period  $s$ , and  $p_s$  denotes the period  $s$  asset price. We further assume  $z_s \in Z$ ,  $s > 1$ , for some open, bounded and convex  $Z \subset \mathbb{R}_+$  with  $\{1\} \in Z$  whereby we set for the initial asset endowment  $z_0 = 1$ . In our representative agent economy it must hold for any period  $t$  equilibrium allocation that  $z_t^* = 1$ . We further assume that the Choquet expected utility

$$E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \nu^{I_t}(\cdot | \cdot) \right]$$

is finite for all  $(z_t, z_{t+1}, \dots)$ . Define for  $s > t$  the following random variable (the so-called *stochastic discount factor*)

$$M_{t,s} = \beta^{s-t} \cdot \frac{u'(c_s)}{u'(c_t)}.$$

For the corresponding equilibrium price process of our model,  $(p_t^*)_{t \geq 0}$  we have to consider three different pricing schemes reflecting the three possibilities that agent  $I_t$  either has

an optimistic, or a pessimistic or no confirmatory bias. That is, for a fixed  $n \geq 1$ , we have, for all  $t$ ,

$$p_t^* = \begin{cases} p_t^{opt} & \text{if } y_{t-n} < \dots < y_t \\ p_t^{pess} & \text{if } y_{t-n} > \dots > y_t \\ p_t^{FB} & \text{else.} \end{cases}$$

The following propositions establish for all three pricing regimes the existence of unique period  $t$  equilibrium prices  $p_t^{opt}$ ,  $p_t^{pess}$ , and  $p_t^{FB}$  conditional on received information  $I_t$ .

**Proposition 1 “Optimistically biased pricing regime”.** *Suppose that agent  $I_t$  is optimistically biased. Then there exists a unique period  $t$  equilibrium asset price such that*

$$p_t^{opt} = \delta_{opt}^{I_t} \cdot \sum_{s=t+1}^{\infty} \min M_{t,s} \cdot \max Y_s + (1 - \delta_{opt}^{I_t}) \cdot E \left[ \sum_{s=t+1}^{\infty} M_{t,s} \cdot Y_s, \pi(y_s | y_0, \dots, y_{s-1}) \right]$$

such that

$$\delta_{opt}^{I_t} = \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1 - \delta) \cdot \pi(I_t)}$$

whenever the transversality condition

$$\lim_{s \rightarrow \infty} (M_{t,s} \cdot p_s^{opt}) = 0$$

holds.

**Proposition 2 “Pessimistically biased pricing regime”.** *Suppose that agent  $I_t$  is pessimistically biased. Then there exists a unique period  $t$  equilibrium asset price such that*

$$p_t^{pess} = \delta_{pess}^{I_t} \cdot \sum_{s=t+1}^{\infty} \max M_{t,s} \cdot \min Y_s + (1 - \delta_{pess}^{I_t}) \cdot E \left[ \sum_{s=t+1}^{\infty} M_{t,s} \cdot Y_s, \pi(y_{s+1} | y_0, \dots, y_{s-1}) \right]$$

such that

$$\delta_{pess}^{I_t} = \frac{\delta \cdot (1 - \lambda)}{\delta \cdot (1 - \lambda) + (1 - \delta) \cdot \pi(I_t)}.$$

whenever the transversality condition

$$\lim_{s \rightarrow \infty} (M_{t,s} \cdot p_s^{pess}) = 0$$

holds.

**Proposition 3 “Default pricing regime”.** *Suppose that agent  $I_t$  is neither optimistically nor pessimistically biased. Then there exists a unique period  $t$  equilibrium asset price such that*

$$\begin{aligned} p_t^{FB} = & \delta_{FB}^{I_t} \cdot \left( \lambda \sum_{s=t+1}^{\infty} \min M_{t,s} \cdot \max Y_s + (1 - \lambda) \sum_{s=t+1}^{\infty} \max M_{t,s} \cdot \min Y_s \right) \\ & + (1 - \delta_{FB}^{I_t}) \cdot E_t \left[ \sum_{s=t+1}^{\infty} M_{t,s} \cdot Y_s, \pi(y_{s+1} \mid y_0, \dots, y_{s-1}) \right] \end{aligned}$$

such that

$$\delta_{FB}^{I_t} = \frac{\delta}{\delta + (1 - \delta) \cdot \pi(I_t)}.$$

whenever the transversality condition

$$\lim_{s \rightarrow \infty} (M_{t,s} \cdot p_s^{FB}) = 0$$

holds.

**Remark 1.** Key to the formal proofs of the above propositions (relegated to the appendix) is the fact that our assumptions allow us to transform the Choquet expected utility optimization problems into equivalent expected utility optimization problems for which standard arguments such as sufficient characterization of global optima by first order conditions as well as the law of iterated expectations go through. The reader

should be aware, however, that this formal equivalence would break down if we had considered a portfolio choice problem with several assets that do not have a comonotonic payoff-structure. Formally, the corresponding portfolio optimization problem would then exhibit kinks so that first order conditions are no longer sufficient criteria for global optima.

**Remark 2.** While the assumption of naïve agents has—in our opinion—a lot of realistic appeal, it is also technically very convenient. By this assumption, we can solve for equilibrium prices as a standard single-agent problem with fixed additive probability measures  $\pi_{opt}^{I_t}$ ,  $\pi_{pess}^{I_t}$  and  $\pi_{FB}^{I_t}$ , respectively, to the effect that the law of iterated expectations can be used. In the case of sequentially rational agents who are able to correctly anticipate their future agents beliefs we would instead have to solve for equilibrium prices by attaching modified additive probability measures to every future agent whereby these changing measures would be determined by the dynamics of the belief generation process  $\nu(\cdot | \cdot)$ . As a consequence, the law of iterated expectations, being formulated for a unique additive probability measure on  $(\Omega, \mathcal{F})$ , were not be applicable to sequentially rational agents.

## 5 Underpricing and overpricing

### 5.1 Definitions

Let us interpret the additive part  $\pi$  of the neo-additive capacity  $\nu$  as the “true” probability measure that governs the objective dividend-payment process. In that case, we can define the resulting expected utility price of the asset, i.e.,

$$p_t^{EU} = E \left[ \sum_{s=t+1}^{\infty} M_{t,s} \cdot Y_s, \pi(\cdot | I_t) \right], \quad (28)$$

as the “correct” price which properly takes into account the stochastic process of economic fundamentals as well as the representative agent’s “tastes” for intertemporal consumption as expressed by the stochastic discount factor. Consequently, the different

pricing regimes of our model's equilibrium price process  $(p_t^*)_{t \geq 0}$  can thus be expressed as follows

$$\begin{aligned}
p_t^{opt} &= \delta_{opt}^{I_t} \cdot \sum_{s=t+1}^{\infty} \min M_{t,s} \cdot \max Y_s + (1 - \delta_{opt}^{I_t}) \cdot p_t^{EU} \\
p_t^{pess} &= \delta_{pess}^{I_t} \cdot \sum_{s=t+1}^{\infty} \max M_{t,s} \cdot \min Y_s + (1 - \delta_{pess}^{I_t}) \cdot p_t^{EU} \\
p_t^{FB} &= \delta_{FB}^{I_t} \cdot \left( \lambda \sum_{s=t+1}^{\infty} \min M_{t,s} \cdot \max Y_s + (1 - \lambda) \sum_{s=t+1}^{\infty} \max M_{t,s} \cdot \min Y_s \right) \\
&\quad + (1 - \delta_{FB}^{I_t}) \cdot p_t^{EU}.
\end{aligned}$$

Observe that all different pricing schemes collapse into the correct pricing scheme (28) whenever the neo-additive beliefs reduce to additive probability measures since  $\delta = 0$  implies

$$\delta_{opt}^{I_t} = \delta_{pess}^{I_t} = \delta_{FB}^{I_t} = 0.$$

However, if there is some ambiguity in the agents' beliefs, i.e.,  $\delta > 0$ , prices in our model are given as a convex combination between the asset's correct price and "something else" whereby this something else depends on the respective pricing regime. Our formal approach gives therefore rise to the following natural definitions of underpricing and overpricing.

**Definition.** *We say that the equilibrium price process  $(p_t^*)_{t \geq 0}$  overprices, respectively underprices the asset, iff  $p_t^* \geq p_t^{EU}$ , respectively iff  $p_t^* \leq p_t^{EU}$ .*

For almost risk-neutral agents, whose stochastic discount factor,  $M_{t,s}$ , is approximately given by the time-discount factor  $\beta^{s-t}$ , we readily obtain the following result.

**Observation 5.** *Consider (almost) risk-neutral agents.*

- (i) *The optimistic pricing regime always overprices the asset.*

(ii) *The pessimistic pricing regime always underprices the asset.*

(iii) *The default pricing regime overprices the asset iff*

$$\sum_{s=t+1}^{\infty} \beta^{s-t} \cdot (\lambda \max Y_s + (1 - \lambda) \cdot \min Y_s) \geq \sum_{s=t+1}^{\infty} \beta^{s-t} \cdot E_t [Y_s, \pi] \quad (29)$$

According to our behavioral model, overpricing occurs if and only if  $y_{j-n} < \dots < y_{j+k}$  for a fixed  $n$ . On the one hand, this determination of price deviations from economic fundamentals in terms of previous performance offers a straightforward link between the movement of economic fundamentals and price movements via the agent's belief generation process. On the other hand, however, the question whether this model-specific link is sensible or not, is ultimately an empirical question and beyond the scope of the present paper's highly stylized analysis.

## 5.2 An illustrative example

We now consider a calibrated version of our model in order to illustrate the emergence of overpricing and underpricing. As the model is highly stylized, our exercise is for illustrative purposes only and we do not attempt to provide a realistic parametrization.

### Calibration

We consider a situation with i.i.d. dividends that are distributed as normal with mean  $EY$  and variance  $\sigma_Y^2$ , whereby we take discrete realizations of the dividend process by drawing from a normal distribution with Gaussian quadrature methods. We set  $EY = 1.5$ ,  $\sigma_Y = 0.1$ , and take  $m = 5$  discrete realizations (nodes). Notice that  $\sigma_Y$  is only a scaling parameter. Table 1 shows the respective probabilities and nodes for our discretization.

By the i.i.d. assumption the equilibrium price under rational expectations is constant and given by

$$p_t^{EU} = \sum_{s=t+1}^{\infty} \beta^{t-s} EY = \frac{\beta}{1 - \beta} EY. \quad (30)$$



Table 1: Dividends: Probabilities and Nodes

$\pi_i$	$Y_i$
0.0113	1.2143
0.2221	1.3644
0.5333	1.5
0.2221	1.6356
0.0113	1.7857

Consequently, the expected return under rational expectations is also constant and given by

$$ER_{t+1}^{EU} = 1 + Er_{t+1}^{EU} = E \frac{p_{t+1}^{EU} + Y_{t+1}}{p_t^{EU}} = \frac{1}{\beta}. \quad (31)$$

We interpret the frequency of our model as annual and therefore consider as a realistic value for the real return on the risky asset under rational expectations  $Er_{t+1}^{EU} = 0.075$ . With this target value, the implied value for the annual discount factor is

$$\beta = \frac{1}{ER_{t+1}^{EU}} = 0.93023$$

and we therefore have that

$$p_t^{EU} = \frac{\beta}{1 - \beta} EY = \frac{1}{Er_{t+1}^{EU}} EY = 20.$$

By the i.i.d. assumption asset prices in the three pricing regimes are history (or state) dependent and given by

$$p_t^{opt} = \delta_{opt}^t \cdot \frac{\beta}{1 - \beta} \cdot \max Y + (1 - \delta_{opt}^t) \cdot p_t^{EU}$$

with

$$\delta_{opt}^t = \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1 - \delta) \cdot \prod_{s=0}^t \pi(y_s)},$$

as well as

$$p_t^{pess} = \delta_{pess}^t \cdot \frac{\beta}{1 - \beta} \cdot \min Y + (1 - \delta_{pess}^t) \cdot p_t^{EU}$$

with

$$\delta_{pess}^t = \frac{\delta \cdot (1 - \lambda)}{\delta \cdot (1 - \lambda) + (1 - \delta) \cdot \prod_{s=0}^t \pi(y_s)}$$

and

$$p_t^{FB} = \delta_{FB}^t \cdot \frac{\beta}{1 - \beta} \cdot (\lambda \cdot \max Y + (1 - \lambda) \cdot \min Y) + (1 - \delta_{FB}^t) \cdot p_t^{EU}$$

with

$$\delta_{FB}^t = \frac{\delta}{\delta + (1 - \delta) \cdot \prod_{s=0}^t \pi(y_s)}.$$

As  $\prod_{s=0}^t \pi(y_s) \rightarrow 0$ , for  $t \rightarrow \infty$ , the limiting asset prices where  $\delta^{FB} = \delta^{opt} = \delta^{pess} = 1$  in the three regimes are given by

$$\begin{aligned} p_t^{opt} &= \frac{\beta}{1 - \beta} \cdot \max Y \\ p_t^{pess} &= \frac{\beta}{1 - \beta} \cdot \min Y \\ p_t^{FB} &= \frac{\beta}{1 - \beta} \cdot (\lambda \cdot \max Y + (1 - \lambda) \cdot \min Y). \end{aligned}$$

As we shall see below, convergence of our model to this limiting case is fast—even for our chosen extremely low degree of initial ambiguity of  $\delta = 1.0E^{-10}$ —so that these limits play a crucial role in our analysis.

We determine the value of  $\lambda$  such that in all periods  $t$  the price under rational expectations is the same as in the full Bayesian pricing regime. This means that we rule out overpricing, respectively underpricing, in the default pricing regime, cf. observation 5. We accordingly solve

$$p_t^{FB} = \delta_{FB}^t \cdot \frac{\beta}{1 - \beta} \cdot (\min(Y) + \lambda(\max(Y) - \min(Y))) + (1 - \delta_{FB}^t) p_t^{EU} = p_t^{EU}$$

for  $\lambda$  which, given that the distribution of dividends is symmetric, cf. table 1, results in  $\lambda = 0.5$ .

Finally, we choose the parameter that determines a switch of pricing regimes according to our heuristic as described in subsection 3.2 to be  $n = 3$ . We take the simulation period to be  $T = 100$  years. All calibration parameters of our model are summarized in table 2.

Table 2: Calibration

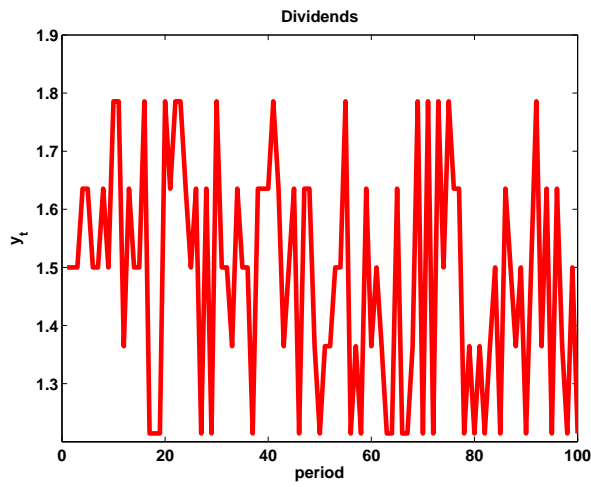
Expected value of dividends, $EY$	1.5
Standard Deviation of dividends, $\sigma_Y$	0.1
Number of nodes, $m$	5
Expected rate of return, $Er_{t+1}^{EU}$	0.075
Discount factor, $\beta$	0.93023
Degree of pessimism, $\lambda$	0.5
Degree of ambiguity, $\delta$	$1.0E - 10$
Sequence of good (bad) news, $n$	3
Time Periods, $T$	100

## Results

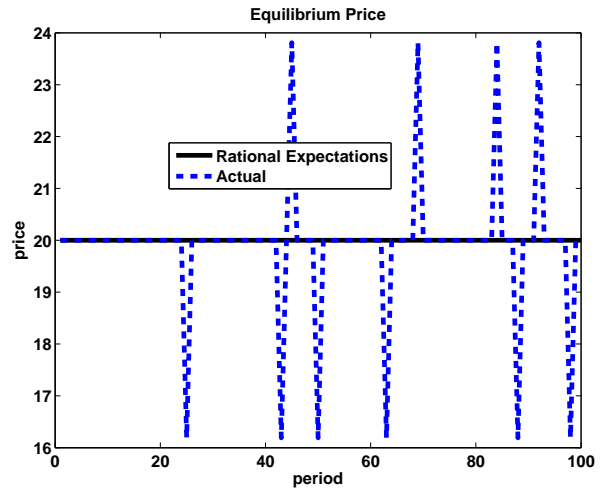
Figure 1 summarizes the results of our simulation for  $T=100$  model periods. Panel (a) shows a random time path of dividends drawn from the distribution displayed in table 1. Panel (c) of the figure displays the degree of ambiguity for this simulation. Even when the initial degree of ambiguity is as low as in our case, cf. table 2, convergence to the limiting case where  $\delta_t^{FB} = \delta_t^{opt} = \delta_t^{pess} = 1$  occurs within less than 20 model periods. Convergence of the corresponding equilibrium prices in the three pricing regimes is shown in panel (d). Given the choice of variance for the dividend process,  $\sigma_Y^2 = 0.01$ , the resulting equilibrium prices in the optimistic (pessimistic) pricing regime are about 20% above (below) the respective level of 20. Recall that this is the equilibrium price under rational expectations as well as, by our choice of  $\lambda$ , under the full Bayesian pricing regime. Finally, panel (b) of the figure shows the resulting equilibrium path of prices. The rational expectations equilibrium is the constant line at an equilibrium price of 20. The actual equilibrium prices emanating in our model are shown as the dashed blue line. For the simulation period we observe 6 events of underpricing, i.e., switches to the pessimistic pricing regime, and 4 event of overpricing, i.e., switches to the optimistic pricing regime.

Figure 1: Dividends and Prices

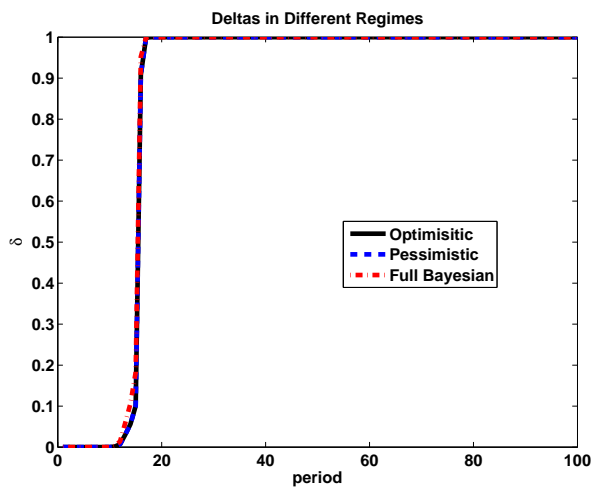
(a) Dividends



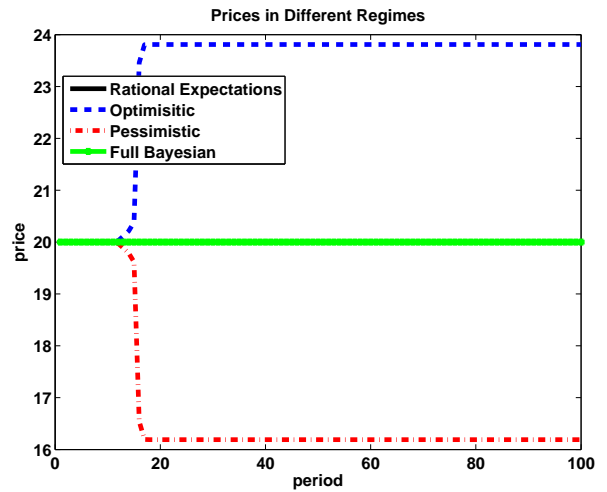
(b) Equilibrium Price



(c) Degree of Ambiguity



(d) Prices in Regimes



## 6 Underreaction and overreaction revisited

We demonstrate in this section that our model naturally generates a pattern of asset prices that have been discussed in the literature as underreaction and overreaction. To be more specific let us assume that—as in the previous section—the additive part  $\pi$  of the agents’ neo-additive beliefs governs the objective dividend-payment process. Interpreting “good news in period  $t$ ” as the event of a dividend-payment increase  $y_{t-1} < y_t$  and “bad news in period  $t$ ” as a dividend-payment decrease  $y_{t-1} > y_t$ , we can then re-formulate the formal (and empirically tested) definitions of underreaction (2) and overreaction (3) by Barberis et al. (1998) for our model as follows.

### Definitions.

(i) **Underreaction to one period of good news:**

$$E [R_{t+1}, \pi (\cdot \mid y_{t-1} < y_t)] \geq E [R_{t+1}, \pi (\cdot \mid y_{t-1} > y_t)]. \quad (32)$$

(ii) **Overreaction to a sequence of good news:** *For some  $j \geq 2$ ,*

$$E [R_{t+j}, \pi (\cdot \mid y_{t-1} < \dots < y_{t+j-1})] \leq E [R_{t+j}, \pi (\cdot \mid y_{t-1} > \dots > y_{t+j-1})]. \quad (33)$$

The following proposition—which is proved in detail in the appendix—establishes sufficient conditions under which our behavioral model of asset-pricing generates underreaction and overreaction as defined above.

**Proposition 4.** *Consider a dividend-payment process  $(Y_t)_{t \geq 0}$  such that the  $Y_t \in \{\min Y, \dots, \max Y\}$  are i.i.d. and assume that the agents are (almost) risk-neutral. Then we have for the equilibrium return process  $(R_t^*)_{t \geq 0}$  of our behavioral model:*

(i) *Condition (32), i.e., underreaction to one period of good news, is always satisfied.*

- (ii) Condition (33), i.e., overreaction to a sequence of good news, is satisfied for sufficiently large  $t$  whereby our model determines  $j \geq n + 1$  with  $n$  denoting the parameter of our confirmatory bias heuristic.

**Sketch of the proof of proposition 4(i).** The proof of proposition 4 proceeds by reformulating the inequalities (32) and (33) in terms of equilibrium prices. Under our confirmatory bias heuristic and the independence assumption (32) is satisfied if the sufficient condition

$$E [p_{t+1}^*, \pi(\cdot | y_{t-1} < y_t)] \geq E [p_{t+1}^*, \pi(\cdot | y_{t-1} > y_t)]$$

holds, i.e., if average future prices are greater for good than for bad news. A simple stochastic dominance argument—again based on our confirmatory bias heuristic and independence—gives readily the desired result of underreaction to one period good news. Intuitively, average future returns are higher for good than for bad news in our model because good news tend to induce an optimistic pricing regime whereas bad news tend to induce a pessimistic pricing regime.

**Sketch of the proof of proposition 4(ii).** The formal argument in the case of overreaction is more difficult and a long-run argument is required. At first notice that (33) is satisfied if the sufficient condition

$$\frac{E [p_{t+j}^*, \pi(\cdot | y_{t-1} < \dots < y_{t+j-1})]}{p_{t+j-1}^{opt}} \leq \frac{E [p_{t+j}^*, \pi(\cdot | y_{t-1} > \dots > y_{t+j-1})]}{p_{t+j-1}^{pess}} \quad (34)$$

holds. To see this let  $j \geq n + 1$  so that, by our confirmatory bias heuristic, the price  $p_{t+j-1}^*$  following a sequence of  $n$  good news is  $p_{t+j-1}^{opt}$  whereas it is  $p_{t+j-1}^{pess}$  for a sequence of  $n$  bad news. Combined with the independence assumption we thus obtain the following period  $t + j - 1$  expectations of period  $t + j$  returns:

$$\begin{aligned} & E [R_{t+j}, \pi(\cdot | y_{t-1} < \dots < y_{t+j-1})] \\ = & \frac{E [p_{t+j}^*, \pi(\cdot | y_{t-1} < \dots < y_{t+j-1})] + E [Y_{t+j}, \pi]}{p_{t+j-1}^{opt}} \end{aligned}$$

and

$$\begin{aligned} & E [R_{t+j}, \pi(\cdot \mid y_{t-1} > \dots > y_{t+j-1})] \\ = & \frac{E [p_{t+j}^*, \pi(\cdot \mid y_{t-1} > \dots > y_{t+j-1})] + E [Y_{t+j}, \pi]}{p_{t+j-1}^{pess}}. \end{aligned}$$

Since  $p_{t+j-1}^{opt} \geq p_{t+j-1}^{pess}$ , (33) is thus satisfied whenever (34) holds.

On the one hand, we have that the return following a sequence of good news is discounted by a higher price, i.e.,  $p_{t+j-1}^{opt}$ , than under bad news, i.e.,  $p_{t+j-1}^{pess}$ . On the other hand, however, expected future prices that follow a sequence of good news are higher than after a sequence of bad news, i.e.,

$$E [p_{t+j}^*, \pi(\cdot \mid y_{t-1} < \dots < y_{t+j-1})] > E [p_{t+j}^*, \pi(\cdot \mid y_{t-1} > \dots > y_{t+j-1})]. \quad (35)$$

The non-trivial part in establishing overreaction is therefore to demonstrate that (34) holds in spite of (35). We conclude this section by illustrating the corresponding formal argument by a simple example.

**Example.** Consider an economy with (almost) risk-neutral agents and i.i.d. dividend-payment process such that any value of  $Y_t$  is equally likely, i.e.,

$$\pi(Y_t = y) = \frac{1}{m} \text{ for all } y \in \mathbf{Y}$$

with  $m = \#\mathbf{Y}$ . By independence of dividend payments, the correct period  $t$  price is constant over all states of the world, i.e.,

$$\begin{aligned} p_t^{EU} &= E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} \cdot Y_s, \pi(\cdot \mid I_t) \right] \\ &= \frac{\beta}{1-\beta} \cdot \sum_{y \in \mathbf{Y}} \frac{y}{m}. \end{aligned}$$

Observe that for all  $I_t$ ,  $\pi(I_t) = \frac{1}{m^t}$  so that—within a given particular pricing regime—period  $t$  equilibrium prices are constant over all states of the world. In particular, we have

$$p_t^{opt} = \delta_{opt}^t \cdot \frac{\beta}{1-\beta} \cdot \max Y + (1 - \delta_{opt}^t) \cdot p_t^{EU}$$

with

$$\delta_{opt}^t = \frac{\delta \cdot \lambda}{\delta \cdot \lambda + (1 - \delta) \cdot \frac{1}{m^t}},$$

as well as

$$p_t^{pess} = \delta_{pess}^t \cdot \frac{\beta}{1 - \beta} \cdot \min Y + (1 - \delta_{pess}^t) \cdot p_t^{EU}$$

with

$$\delta_{pess}^t = \frac{\delta \cdot (1 - \lambda)}{\delta \cdot (1 - \lambda) + (1 - \delta) \cdot \frac{1}{m^t}},$$

and

$$p_t^{FB} = \delta_{FB}^t \cdot \frac{\beta}{1 - \beta} \cdot (\lambda \cdot \max Y + (1 - \lambda) \cdot \min Y) + (1 - \delta_{FB}^t) \cdot p_t^{EU}$$

with

$$\delta_{FB}^t = \frac{\delta}{\delta + (1 - \delta) \cdot \frac{1}{m^t}}.$$

Since the prices are constant within each pricing scheme, we obtain as average period  $t + j$  equilibrium prices for the respective pricing schemes

$$E [p_{t+j}^{opt}, \pi(\cdot | \cdot)] = p_{t+j}^{opt} \quad (36)$$

$$E [p_{t+j}^{pess}, \pi(\cdot | \cdot)] = p_{t+j}^{pess} \quad (37)$$

$$E [p_{t+j}^{FB}, \pi(\cdot | \cdot)] = p_{t+j}^{FB} \quad (38)$$

Observe now that the sufficient condition for overreaction (34) holds if

$$\begin{aligned} & \frac{E [p_{t+j}^{opt}, \pi(\cdot | \cdot)] \cdot \pi(y_{t+j-1} < Y_{t+j}) + E [p_{t+j}^{FB}, \pi(\cdot | \cdot)] \cdot \pi(y_{t+j-1} \geq Y_{t+j})}{p_{t+j-1}^{opt}} \\ & \leq \frac{E [p_{t+j}^{pess}, \pi(\cdot | \cdot)] \cdot \pi(y_{t+j-1} > Y_{t+j}) + E [p_{t+j}^{FB}, \pi(\cdot | \cdot)] \cdot \pi(y_{t+j-1} \leq Y_{t+j})}{p_{t+j-1}^{pess}} \end{aligned}$$

which is, after substituting the expected values (36)–(38), equivalent to

$$\begin{aligned} & \left( \frac{p_{t+j}^{opt}}{p_{t+j-1}^{opt}} - \frac{p_{t+j}^{FB}}{p_{t+j-1}^{pess}} \right) \cdot \pi(y_{t+j-1} < Y_{t+j}) \quad (39) \\ & \leq \left( \frac{p_{t+j}^{pess}}{p_{t+j-1}^{pess}} - \frac{p_{t+j}^{FB}}{p_{t+j-1}^{opt}} \right) \cdot \pi(y_{t+j-1} > Y_{t+j}) + \left( \frac{p_{t+j}^{FB}}{p_{t+j-1}^{pess}} - \frac{p_{t+j}^{FB}}{p_{t+j-1}^{opt}} \right) \cdot \pi(y_{t+j-1} = Y_{t+j}) \quad (40) \end{aligned}$$



Now observe that

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{p_{t+j}^{opt}}{p_{t+j-1}^{opt}} &= \lim_{t \rightarrow \infty} \frac{p_{t+j}^{pess}}{p_{t+j-1}^{pess}} = 1 \\ \lim_{t \rightarrow \infty} \frac{p_{t+j}^{FB}}{p_{t+j-1}^{pess}} &= \frac{\lambda \cdot \max Y + (1 - \lambda) \cdot \min Y}{\min Y} \geq 1 \\ \lim_{t \rightarrow \infty} \frac{p_{t+j}^{FB}}{p_{t+j-1}^{opt}} &= \frac{\lambda \cdot \max Y + (1 - \lambda) \cdot \min Y}{\max Y} \leq 1 \end{aligned}$$

so that, by continuity, there exist sufficiently large  $t$  such that the lhs (39) is smaller equal zero and the rhs (40) is greater equal zero. This establishes overreaction in the sense of (3) in the long run for this economy.  $\square$

## 7 Concluding remarks and outlook

We introduce a behavioral model of asset price fluctuations. In this model naïve agents are prone to a confirmatory bias in the interpretation of new information about economic fundamentals. Our formal approach gives rise to model-endogenous concepts of “overpricing”, “underpricing”. These pricing scenarios are induced by different pricing regimes—an optimistic, a pessimistic, and an “in between” default pricing regime—and endogenous switches between there regimes. As another interesting feature our model generates equilibrium price patterns that reflect underreaction of asset prices with respect to one period good news as well as overreaction with respect to several periods of good news. These empirical phenomena have received substantial attention in the behavioral finance literature. In contrast to our approach, however, previous formal explanations for these phenomena were rather ad hoc and have not been derived from decision-theoretic first principles.

The key message of this paper is to illustrate how such pricing phenomena emerge from a non-standard dynamic Bayesian learning model with a sound decision theoretic basis. On purpose, we develop a parsimonious model and keep the analysis extremely stylized. Our current work extends this framework to develop a joint model for asset pricing anomalies such as the equity premium and excess volatility puzzles (Mehra and

Prescott 1985; Shiller 1981). As such, our current research focuses on a joint theory for normal times, i.e., periods with standard asset price fluctuations, and exceptional times, i.e., periods with significant deviations from asset pricing according to fundamentals.

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## Appendix

### Proof of propositions 1-3.

Observe at first that—independent of the specific choice of  $(z_t, z_{t+1}, \dots)$ —the maximal, respectively minimal, economic outcome occurs for every portfolio-choice in the same state  $(\max Y_1, \max Y_2, \dots) \in \Omega$  resp.  $(\min Y_1, \min Y_2, \dots) \in \Omega$ . Because of this fact and because of our assumption that every agent  $I_t$  assumes that  $\nu^{I_t}(\cdot | \cdot)$  governs the beliefs of his future-selves, we can transform the neo-additive Choquet expected utility maximization problem into an equivalent (standard) expected utility maximization problem for conveniently constructed additive probability measures  $\pi_{opt}^{I_t}$ ,  $\pi_{pess}^{I_t}$  and  $\pi_{FB}^{I_t}$ . More specifically, we obtain the following equivalent maximization problems for the respective pricing regimes.

Optimistic pricing regime:

$$\begin{aligned} & \max_{(z_t, z_{t+1}, \dots)} u(c_t) + E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \nu^{I_t}(\cdot | \cdot) \right] \\ &= \max_{(z_t, z_{t+1}, \dots)} u(c_t) + E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \pi_{opt}^{I_t}(y_{s+1} | y_0, \dots, y_s) \right] \end{aligned}$$

such that

$$\pi_{opt}^{I_t}(y_{s+1} | y_0, \dots, y_s) = \begin{cases} \delta_{opt}^{I_t} + (1 - \delta_{opt}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s) & \text{for } y_{s+1} = \max Y_{s+1} \\ (1 - \delta_{opt}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s) & \text{for } y_{s+1} < \max Y_{s+1} \end{cases}$$

Pessimistic pricing regime:

$$\begin{aligned} & \max_{(z_t, z_{t+1}, \dots)} u(c_t) + E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \nu^{I_t}(\cdot | \cdot) \right] \\ &= \max_{(z_t, z_{t+1}, \dots)} u(c_t) + E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \pi_{pess}^{I_t}(y_{s+1} | y_0, \dots, y_s) \right] \end{aligned}$$

such that

$$\pi_{pess}^{I_t}(y_{s+1} | y_0, \dots, y_s) = \begin{cases} \delta_{pess}^{I_t} + (1 - \delta_{pess}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s) & \text{if } y_{s+1} = \min Y_{s+1} \\ (1 - \delta_{pess}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s) & \text{if } y_{s+1} > \min Y_{s+1} \end{cases}$$

Default pricing regime:

$$\begin{aligned} & \max_{(z_t, z_{t+1}, \dots)} u(c_t) + E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \nu^{I_t}(\cdot | \cdot) \right] \\ &= \max_{(z_t, z_{t+1}, \dots)} u(c_t) + E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \pi_{FB}^{I_t}(y_{s+1} | y_0, \dots, y_s) \right] \end{aligned}$$

such that

$$\begin{aligned} & \pi_{FB}^{I_t}(y_{s+1} | y_0, \dots, y_s) \\ &= \begin{cases} \delta_{FB}^{I_t} \cdot \lambda + (1 - \delta_{FB}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s) & \text{if } y_{s+1} = \max Y_{s+1} \\ (1 - \delta_{FB}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s) & \text{if } \min Y_{s+1} < y_{s+1} < \max Y_{s+1} \\ \delta_{FB}^{I_t} \cdot (1 - \lambda) + (1 - \delta_{FB}^{I_t}) \cdot \pi(y_{s+1} | y_0, \dots, y_s) & \text{if } y_{s+1} = \min Y_{s+1} \end{cases} \end{aligned}$$

Given this transformation of the CEU into an EU maximization problem the above results follow readily from standard arguments. For the sake of completeness we demonstrate this for the optimistically biased pricing regime. Consider an optimistically biased agent so that

$$\begin{aligned} & E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \nu^{I_t}(\cdot | \cdot) \right] \\ &= \delta_{opt}^{I_t} \cdot \max_{\omega \in I_t} \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s) + (1 - \delta_{opt}^{I_t}) \cdot E \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s), \pi(y_{s+1} | y_0, \dots, y_s) \right]. \end{aligned}$$

The corresponding period  $s$  first order conditions, evaluated at the equilibrium allocation  $z_t^* = 1$  for all  $t$ , imply

$$\begin{aligned} p_s^* &= \delta_{opt}^{I_t} \cdot \beta \cdot \frac{u'(e_{s+1} + \max Y_{s+1})}{u'(e_s + y_s)} \cdot (\max Y_{s+1} + p_{s+1}^*) \\ &\quad + (1 - \delta_{opt}^{I_t}) \cdot E \left[ \beta \cdot \frac{u'(e_{s+1} + Y_{s+1})}{u'(e_s + y_s)} \cdot (Y_{s+1} + p_{s+1}^*), \pi(y_{s+1} | y_0, \dots, y_s) \right] \\ &= \delta_{opt}^{I_t} \cdot \min M_{s,s+1} \cdot (\max Y_{s+1} + p_{s+1}^*) \\ &\quad + (1 - \delta_{opt}^{I_t}) \cdot E [M_{s,s+1} \cdot (Y_{s+1} + p_{s+1}^*), \pi(y_{s+1} | y_0, \dots, y_s)] \end{aligned}$$

for all  $s \geq t$ . Notice that

$$\min M_{s,s+1} = \beta \cdot \frac{u'(e_{s+1} + \max Y_{s+1})}{u'(e_s + y_s)}$$

follows from the concavity of  $u$ . As a consequence, any period  $t$  equilibrium asset price  $p_t^{opt}$  is characterized by the following system of equations

$$\begin{aligned} p_s^{opt} &= E \left[ M_{s,s+1} \cdot (Y_{s+1} + p_{s+1}^{opt}), \pi_{opt}^{I_t}(y_{s+1} \mid y_0, \dots, y_s) \right] \\ &\equiv E_s^{opt} \left[ M_{s,s+1} \cdot (Y_{s+1} + p_{s+1}^{opt}) \right] \end{aligned}$$

for all  $s \geq t$  with  $\pi_{opt}^{I_t}(y_{s+1} \mid y_0, \dots, y_s)$  defined above. Substitute

$$p_{t+1}^{opt} = E_{t+1}^{opt} \left[ m_{t+1,t+2} \cdot (Y_{t+2} + p_{t+2}^{opt}) \right]$$

in

$$p_t^{opt} = E_t^{opt} \left[ M_{t,t+1} \cdot (Y_{t+1} + p_{t+1}^{opt}) \right]$$

and observe that

$$\begin{aligned} p_t^{opt} &= E_t^{opt} \left[ M_{t,t+1} \cdot (Y_{t+1} + E_{t+1}^{opt} \left[ M_{t+1,t+2} \cdot (Y_{t+2} + p_{t+2}^{opt}) \right]) \right] \\ &= E_t^{opt} \left[ M_{t,t+1} \cdot Y_{t+1} \right] + E_t^{opt} \left[ M_{t,t+1} \cdot E_{t+1}^{opt} \left[ M_{t+1,t+2} \cdot (Y_{t+2} + p_{t+2}^{opt}) \right] \right] \\ &= E_t^{opt} \left[ M_{t,t+1} \cdot Y_{t+1} \right] + E_t^{opt} \left[ E_{t+1}^{opt} \left[ M_{t,t+1} \cdot M_{t+1,t+2} \cdot (Y_{t+2} + p_{t+2}^{opt}) \right] \right] \\ &= E_t^{opt} \left[ M_{t,t+1} \cdot Y_{t+1} \right] + E_t^{opt} \left[ E_{t+1}^{opt} \left[ M_{t,t+2} \cdot (Y_{t+2} + p_{t+2}^{opt}) \right] \right] \\ &= E_t^{opt} \left[ M_{t,t+1} \cdot Y_{t+1} \right] + E_t^{opt} \left[ M_{t,t+2} \cdot Y_{t+2} \right] + E_t^{opt} \left[ M_{t,t+2} \cdot p_{t+2}^{opt} \right]. \end{aligned}$$

The third line thereby results from the fact that the random variable  $M_{t,t+1}$  is a constant with respect to any given  $E_{t+1}^{opt}$  and the fifth line is implied by the law of iterative expectations for additive probability measures. Applying the same reasoning to  $p_{t+2}^{opt}, p_{t+3}^{opt}, \dots$  gives the desired result whenever the transversality condition is satisfied.  $\square$

### Proof of proposition 4(i).

Observe at first that

$$\begin{aligned} &E \left[ \frac{p_{t+1}^* + Y_{t+1}}{p_t^*}, \pi(\cdot \mid y_{t-1} < y_t) \right] \\ &= \frac{E \left[ p_{t+1}^*, \pi(\cdot \mid y_{t-1} < y_t) \right] + E \left[ Y_{t+1}, \pi(\cdot \mid y_{t-1} < y_t) \right]}{p_t^*} \\ &\geq \frac{E \left[ p_{t+1}^*, \pi(\cdot \mid y_{t-1} < y_t) \right] + E \left[ Y_{t+1}, \pi(\cdot \mid y_{t-1} < y_t) \right]}{p_t^{FB}} \end{aligned}$$



since  $y_{t-1} < y_t$  implies either  $p_t^* = p_t^{FB}$  or  $p_t^* = p_t^{opt}$ .

Now observe that

$$\begin{aligned} & E \left[ \frac{p_{t+1}^* + Y_{t+1}}{p_t^*}, \pi(\cdot \mid y_{t-1} < y_t) \right] \\ = & \frac{E [p_{t+1}^*, \pi(\cdot \mid y_{t-1} > y_t)] + E [Y_{t+1}, \pi(\cdot \mid y_{t-1} > y_t)]}{p_t^*} \\ \leq & \frac{E [p_{t+1}^*, \pi(\cdot \mid y_{t-1} > y_t)] + E [Y_{t+1}, \pi(\cdot \mid y_{t-1} > y_t)]}{p_t^{FB}} \end{aligned}$$

since  $y_{t-1} > y_t$  implies either  $p_t^* = p_t^{FB}$  or  $p_t^* = p_t^{pess}$ .

Consequently,

$$E [R_{t+1}, \pi(\cdot \mid y_{t-1} < y_t)] \geq E [R_{t+1}, \pi(\cdot \mid y_{t-1} > y_t)]$$

is satisfied if

$$E [p_{t+1}^*, \pi(\cdot \mid y_{t-1} < y_t)] \geq E [p_{t+1}^*, \pi(\cdot \mid y_{t-1} > y_t)]. \quad (41)$$

In accordance with our confirmatory bias heuristic, for good news  $p_{t+1}^*$  is given either by the optimistic or the default pricing scheme, implying

$$\begin{aligned} E [p_{t+1}^*, \pi(\cdot \mid y_{t-1} < y_t)] &= E [p_{t+1}^{opt}, \pi(\cdot \mid y_{t-(n-1)} < \dots < y_t)] \cdot \pi(y_{t-(n-1)} < \dots < y_t) \\ &\quad + E [p_{t+1}^{FB}, \pi(\cdot \mid \overline{y_{t-(n-1)} < \dots < y_t})] \cdot [1 - \pi(y_{t-(n-1)} < \dots < y_t)] \end{aligned}$$

For bad news,  $p_{t+1}^*$  follows either the pessimistic or the default pricing scheme, implying

$$\begin{aligned} E [p_{t+1}^*, \pi(\cdot \mid y_{t-1} > y_t)] &= E [p_{t+1}^{pess}, \pi(\cdot \mid y_{t-(n-1)} > \dots > y_t)] \cdot \pi(y_{t-(n-1)} > \dots > y_t) \\ &\quad + E [p_{t+1}^{FB}, \pi(\cdot \mid \overline{y_{t-(n-1)} > \dots > y_t})] \cdot [1 - \pi(y_{t-(n-1)} > \dots > y_t)] \end{aligned}$$

Furthermore, by independence

$$\begin{aligned} E [p_{t+1}^{opt}, \pi(\cdot \mid y_{t-(n-1)} < \dots < y_t)] &= E [p_{t+1}^{opt}, \pi(y_{t+1})] \\ E [p_{t+1}^{FB}, \pi(\cdot \mid \overline{y_{t-(n-1)} < \dots < y_t})] &= E [p_{t+1}^{FB}, \pi(\cdot \mid \overline{y_{t-(n-1)} > \dots > y_t})] \\ &= E [p_{t+1}^{FB}, \pi(y_{t+1})] \\ E [p_{t+1}^{pess}, \pi(\cdot \mid y_{t-(n-1)} > \dots > y_t)] &= E [p_{t+1}^{pess}, \pi(y_{t+1})] \end{aligned}$$

whereby risk-neutrality implies, by observation 5,

$$E [p_{t+1}^{opt}, \pi (y_{t+1})] \geq E [p_{t+1}^{FB}, \pi (y_{t+1})] \geq E [p_{t+1}^{pess}, \pi (y_{t+1})].$$

As a consequence the lhs in (41) first-order stochastically dominates the lhs in (41) so that we obtain the desired result.  $\square$

**Proof of proposition 4(ii).**

Let  $j \geq n + 1$  so that  $p_{t+j-1}^*$ , conditional on  $y_{t-1} < \dots < y_{t+j-1}$ , is given as  $p_{t+j-1}^{opt}$  whereas it becomes, conditional on  $y_{t-1} > \dots > y_{t+j-1}$ ,  $p_{t+j-1}^{pess}$ . By independence,

$$\begin{aligned} E [Y_{t+j}, \pi (\cdot \mid y_{t-1} < \dots < y_{t+j-1})] &= E [Y_{t+j}, \pi (\cdot \mid y_{t-1} > \dots > y_{t+j-1})] \\ &= E [Y_{t+j}, \pi], \end{aligned}$$

implying

$$\begin{aligned} &E [R_{t+j}, \pi (\cdot \mid y_{t-1} < \dots < y_{t+j-1})] \\ &= \frac{E [p_{t+j}^*, \pi (\cdot \mid y_{t-1} < \dots < y_{t+j-1})] + E [Y_{t+j}, \pi]}{p_{t+j-1}^{opt}} \end{aligned}$$

and

$$\begin{aligned} &E [R_{t+j}, \pi (\cdot \mid y_{t-1} > \dots > y_{t+j-1})] \\ &= \frac{E [p_{t+j}^*, \pi (\cdot \mid y_{t-1} > \dots > y_{t+j-1})] + E [Y_{t+j}, \pi]}{p_{t+j-1}^{pess}} \end{aligned}$$

As a consequence, our model generates overreaction (3) whenever the sufficient condition

$$\frac{E [p_{t+j}^*, \pi (\cdot \mid y_{t-1} < \dots < y_{t+j-1})]}{p_{t+j-1}^{opt}} \leq \frac{E [p_{t+j}^*, \pi (\cdot \mid y_{t-1} > \dots > y_{t+j-1})]}{p_{t+j-1}^{pess}} \quad (42)$$

holds. In accordance with our confirmatory bias heuristic,  $p_{t+j}^*$  is given for good news either by the optimistic or the default pricing scheme, implying

$$\begin{aligned} E [p_{t+j}^*, \pi (\cdot \mid y_{t-1} < \dots < y_{t+j-1})] &\leq E [p_{t+j}^{opt}, \pi (\cdot \mid y_{t-1} < \dots < y_{t+j-1} < y_{t+j})] \\ &\quad \cdot \pi (y_{t+j-1} < Y_{t+j}) \\ &\quad + E [p_{t+j}^{FB}, \pi (\cdot \mid y_{t-1} < \dots < y_{t+j-1} \geq y_{t+j})] \\ &\quad \cdot \pi (y_{t+j-1} \geq Y_{t+j}) \end{aligned}$$

Accordingly,  $p_{t+j}^*$  results for bad news either from the pessimistic or from the default pricing scheme, implying

$$\begin{aligned}
E [p_{t+j}^*, \pi(\cdot | y_{t-1} > \dots > y_{t+j-1})] &\geq E [p_{t+j}^{pess}, \pi(\cdot | y_{t-1} > \dots > y_{t+j-1} > y_{t+j})] \\
&\quad \cdot \pi(y_{t+j-1} > Y_{t+j}) \\
&\quad + E [p_{t+j}^{FB}, \pi(\cdot | y_{t-1} > \dots > y_{t+j-1} \leq y_{t+j})] \\
&\quad \cdot \pi(y_{t+j-1} \leq Y_{t+j})
\end{aligned}$$

Substituting in inequality (42) and rearranging gives

$$\begin{aligned}
&\left( \frac{E [p_{t+j}^{opt}, \pi(\cdot | y_{t-1} < \dots < y_{t+j-1} < y_{t+j})]}{p_{t+j-1}^{opt}} - \frac{E [p_{t+j}^{FB}, \pi(\cdot | y_{t-1} > \dots > y_{t+j-1} < y_{t+j})]}{p_{t+j-1}^{pess}} \right) \quad (43) \\
&\quad \cdot \pi(y_{t+j-1} < Y_{t+j}) \quad (44) \\
\leq &\left( \frac{E [p_{t+j}^{pess}, \pi(\cdot | y_{t-1} > \dots > y_{t+j-1} > y_{t+j})]}{p_{t+j-1}^{pess}} - \frac{E [p_{t+j}^{FB}, \pi(\cdot | y_{t-1} < \dots < y_{t+j-1} < y_{t+j})]}{p_{t+j-1}^{opt}} \right) \\
&\quad \cdot \pi(y_{t+j-1} > Y_{t+j}) \quad (45) \\
&+ \left( \frac{E [p_{t+j}^{FB}, \pi(\cdot | y_{t-1} > \dots > y_{t+j-1} = y_{t+j})]}{p_{t+j-1}^{pess}} - \frac{E [p_{t+j}^{FB}, \pi(\cdot | y_{t-1} < \dots < y_{t+j-1} = y_{t+j})]}{p_{t+j-1}^{opt}} \right) \\
&\quad \cdot \pi(y_{t+j-1} = Y_{t+j}) \quad (46)
\end{aligned}$$

Observe that, by our iid assumption,  $\lim_{t \rightarrow \infty} \pi(I_t) = 0$  for all  $I_t$ , which implies

$$\lim_{t \rightarrow \infty} \delta_{opt}^{I_t} = \lim_{t \rightarrow \infty} \delta_{pess}^{I_t} = \lim_{t \rightarrow \infty} \delta_{FB}^{I_t} = 1.$$

As a consequence, the respective equilibrium prices of a given regime converge to some constant, i.e.,

$$\begin{aligned}
\lim_{t \rightarrow \infty} p_t^{opt} &= \frac{\beta}{1 - \beta} \cdot \max Y, \\
\lim_{t \rightarrow \infty} p_t^{pess} &= \frac{\beta}{1 - \beta} \cdot \min Y, \\
\lim_{t \rightarrow \infty} p_t^{FB} &= \frac{\beta}{1 - \beta} \cdot (\lambda \cdot \max Y + (1 - \lambda) \cdot \min Y).
\end{aligned}$$

By continuity, the expected prices in (43) must therefore approximately satisfy

$$\begin{aligned} E [p_{t+j}^{opt}, \pi(\cdot | \cdot)] &= p_{t+j-1}^{opt} \\ E [p_{t+j}^{FB}, \pi(\cdot | \cdot)] &= p_{t+j-1}^{FB} \\ E [p_{t+j}^{pess}, \pi(\cdot | \cdot)] &= p_{t+j-1}^{pess} \end{aligned}$$

for sufficiently large  $t$ . Substituting in (43) gives, for sufficiently large  $t$ ,

$$\begin{aligned} &\left(1 - \frac{p_{t+j-1}^{FB}}{p_{t+j-1}^{pess}}\right) \cdot \pi(y_{t+j-1} < y_{t+j}) \leq \left(1 - \frac{p_{t+j-1}^{FB}}{p_{t+j-1}^{opt}}\right) \cdot \pi(y_{t+j-1} > y_{t+j}) \\ &+ \left(\frac{p_{t+j-1}^{FB}}{p_{t+j-1}^{pess}} - \frac{p_{t+j-1}^{FB}}{p_{t+j-1}^{opt}}\right) \cdot \pi(y_{t+j-1} = y_{t+j}), \end{aligned}$$

which is always satisfied because the lhs is negative and the rhs is positive because of  $p_{t+j-1}^{opt} \geq p_{t+j-1}^{FB} \geq p_{t+j-1}^{pess}$ .  $\square\square$

## Supplementary appendix

**Proof of observation 1:** By an argument in Schmeidler (1986), it suffices to restrict attention to a non-negative valued random variable  $f$  so that

$$E[f, \nu] = \int_0^{+\infty} \nu(\{\omega \in \Omega \mid f(\omega) \geq z\}) dz,$$

which is equivalent to

$$E[f, \nu] = \int_{\min f}^{\max f} \nu(\{\omega \in \Omega \mid f(\omega) \geq z\}) dz$$

since the range of  $f$  is closed and bounded. We consider a partition  $P_n$ ,  $n = 1, 2, \dots$ , of  $\Omega$  with members

$$A_n^k = \{\omega \in \Omega \mid a_{k,n} < f(\omega) \leq b_{k,n}\} \text{ for } k = 1, \dots, 2^n$$

such that

$$\begin{aligned} a_{k,n} &= [\max f - \min f] \cdot \frac{(k-1)}{2^n} + \min f \\ b_{k,n} &= [\max f - \min f] \cdot \frac{k}{2^n} + \min f. \end{aligned}$$

Define the step functions  $a_n : \Omega \rightarrow \mathbb{R}$  and  $b_n : \Omega \rightarrow \mathbb{R}$  such that, for  $\omega \in A_n^k$ ,  $k = 1, \dots, 2^n$ ,

$$\begin{aligned} a_n(\omega) &= a_{k,n} \\ b_n(\omega) &= b_{k,n}. \end{aligned}$$

Obviously,

$$E[a_n, \nu] \leq E[f, \nu] \leq E[b_n, \nu]$$

for all  $n$  and

$$\lim_{n \rightarrow \infty} E[b_n, \nu] - E[a_n, \nu] = 0.$$

That is,  $E[a_n, \nu]$  and  $E[b_n, \nu]$  converge to  $E[f, \nu]$  for  $n \rightarrow \infty$ . Furthermore, observe that

$$\begin{aligned} \min a_n &= \min f \text{ for all } n, \text{ and} \\ \max b_n &= \max f \text{ for all } n. \end{aligned}$$

Since  $\lim_{n \rightarrow \infty} \min b_n = \lim_{n \rightarrow \infty} \min a_n$  and  $E[b_n, \pi]$  is continuous in  $n$ , we have

$$\begin{aligned} \lim_{n \rightarrow \infty} E[b_n, \nu] &= \delta \left( \lambda \lim_{n \rightarrow \infty} \max b_n + (1 - \lambda) \lim_{n \rightarrow \infty} \min b_n \right) + (1 - \delta) \lim_{n \rightarrow \infty} E[b_n, \pi] \\ &= \delta (\lambda \max f + (1 - \lambda) \min f) + (1 - \delta) E[f, \pi]. \end{aligned}$$

In order to prove proposition 3, it therefore remains to be shown that, for all  $n$ ,

$$E[b_n, \nu] = \delta (\lambda \max b_n + (1 - \lambda) \min b_n) + (1 - \delta) E[b_n, \pi].$$

Since  $b_n$  is a step function, the Choquet expectation of  $b_n$  with respect to  $\nu$  becomes

$$\begin{aligned} E[b_n, \nu] &= \sum_{A_n^k \in P_n} \nu(A_n^{2^n} \cup \dots \cup A_n^k) \cdot (a_{k,n} - a_{k-1,n}) \\ &= \sum_{A_n^k \in P_n} a_{k,n} \cdot [\nu(A_n^{2^n} \cup \dots \cup A_n^k) - \nu(A_n^{2^n} \cup \dots \cup A_n^{k-1})], \end{aligned}$$

implying for a neo-additive capacity

$$\begin{aligned} E[b_n, \nu] &= \max b_n [\delta \lambda + (1 - \delta) \pi(A_n^{2^n})] + \sum_{k=2}^{2^n-1} a_{k,n} (1 - \delta) \pi(A_n^k) \\ &\quad + \min b_n \left[ 1 - \delta \lambda - (1 - \delta) \sum_{k=2}^{2^n} \pi(A_n^k) \right] \\ &= \delta \lambda \max b_n + (1 - \delta) \sum_{k=1}^{2^n} a_{k,n} \pi(A_n^k) + \min b_n [\delta - \delta \lambda] \\ &= \delta (\lambda \max b_n + (1 - \lambda) \min b_n) + (1 - \delta) E[b_n, \pi]. \end{aligned}$$

□

**Proof of observation 2:** An application of the full Bayesian update rule to a neo-additive capacity gives

$$\begin{aligned}
\nu^{FB}(A|B) &= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \pi(A \cap B)}{\delta \cdot \lambda + (1 - \delta) \cdot \pi(A \cap B) + 1 - (\delta \cdot \lambda + (1 - \delta) \cdot \pi(A \cup \neg B))} \\
&= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \pi(A \cap B)}{1 + (1 - \delta) \cdot (\pi(A \cap B) - \pi(A \cup \neg B))} \\
&= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \pi(A \cap B)}{1 + (1 - \delta) \cdot (\pi(A \cap B) - \pi(A) - \pi(\neg B) + \pi(A \cap \neg B))} \\
&= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \pi(A \cap B)}{1 + (1 - \delta) \cdot (-\pi(\neg B))} \\
&= \frac{\delta \cdot \lambda + (1 - \delta) \cdot \pi(A \cap B)}{\delta + (1 - \delta) \cdot \pi(B)} \\
&= \frac{\delta \cdot \lambda}{\delta + (1 - \delta) \cdot \pi(B)} + \frac{(1 - \delta) \cdot \pi(B)}{\delta + (1 - \delta) \cdot \pi(B)} \pi(A|B) \\
&= \delta_B^{FB} \cdot \lambda + (1 - \delta_B^{FB}) \cdot \pi(A|B)
\end{aligned}$$

with  $\delta_B^{FB}$  given by (18).  $\square$