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Consumption Growth and Inequality in a Heterogeneous Agent Model: Theory and Evidence from German Data

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Abstract

This paper studies the structure and dynamics of consumption and consumption growth inequality. The theoretical framework is a heterogeneous agent model with stochastic labor endowments, where the group mean consumption serves as consumption externality. The main finding is that households' preferences affect the within-group inequality through asset holding decisions: it decreases with groups' degree of patience, and increases with households' eagerness to keep up with the group average. The relationship between the group average consumption growth and within-group inequality predicted by the model is shown as slightly positive yet significant in a synthesis study combining the German Socio-Economic Panel (GSOEP) and the Einkommens- und Verbrauchsstichprobe (EVS). Age and household size are crucial for within-group inequality, as young and/or small households are more sensitive to income and consumption shocks. Large and well-educated households with unskilled jobs have undergone surprisingly inferior performance in consumption growth and growth variance. Moreover, the data also shows increases of within-group inequality directly after the reunification and the introduction of the euro.

Keywords: consumption inequality, consumption growth, German Socio-Economic Panel, EVS
JEL codes: E21, D91, D31, D64

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1 Introduction

The structure and evolution of income inequality has always been well documented for many countries, while studies on consumption inequality are less prevalent due to the availability of survey data. This is also the case for Germany where most inequality studies focus on wage income, disposable income or household wealth. For the purpose of examining the well-being of population, however, consumption is a more direct measure. The contribution of this paper is two-fold: providing a theoretical framework of heterogeneous agents in order to examine the link between within-group inequality and group average consumption growth, and finding empirical evidence on the aforementioned relationship using household data from the German Socio-Economic Panel (GSOEP) and the Einkommens- und Verbrauchsstichprobe (EVS).

The analysis of consumption inequality for Germany (West) in this paper complements a number of studies that use micro data to document the evolution of income or wage inequality in Germany in the last 25 years. As Biewen (2000) finds that the West German income distribution between 1984 and 1996 was stable, Ludsteck and Schönberg (2007), exploring the IABS 2% random sample of social security records for the years 1975 to 2004, conclude that German wage inequality has increased at the top of the distribution in the 1980s, while inequality at the bottom of the distribution started to rise in the 1990s. Apart from labor income, recent studies increasingly emphasize capital income and regard it as another source of inequality. Fräßdorf, Grabka and Schwarze (2008) compare Germany, the U.S. and the U.K., analyze the weight of capital income in disposable income, and find that a large part of the growing disparity of disposable income could be explained by the increasing capital income inequality.

As one of the first studies on the trend of consumption inequality in Germany, Fuchs-Schündeln, Krüger and Sommer (2008) look into both income and consumption inequality. Combining the GSOEP and EVS data, they document the inequality trends of wage income, consumption and wealth in West Germany and find upward trends in wage and market income inequality after the reunification. In contrast, disposable income and consumption inequality only display a modest rise over the same period. The pattern of consumption inequality in their findings can be confirmed in this paper, and what's more, by constructing yearly consumption data from the GSOEP, my study fills in the blanks between their observations

based on EVS, which is only available every five years.

The theoretical part of this paper is rooted in a rich body of literature on the connection between income shocks and consumption growth inequality. Incomplete risk-sharing/imperfect insurance are often considered as the explanations to the diverse evolution of income and consumption inequality. Examining U.S. panel data on income from the PSID (1978-1992) and cross-section Consumer Expenditure Survey (CEX) data on consumption (1980-1992), Blundell, Pistaferri and Preston (2008) confirm that consumption inequality follows closely the trends in permanent earnings inequality, as is shown earlier in Cutler and Katz (1991). They also find extremely strong evidence against full insurance for permanent income shocks but not for transitory income shocks, which is an extended result of a previous study by Blundell and Preston (1998).

Krüger and Perri (2005) notice in the CEX data that while between-group consumption inequality has tracked between-group income inequality quite closely, within-group consumption inequality has increased much less than within-group income inequality. Motivated by this observation, they propose a theoretical framework depicting the risk-sharing behavior within groups when idiosyncratic labor income shocks occur, where the market is imperfect due to the lack of contract enforceability. Their model with debt constrained markets shows that when income becomes more volatile, risk-sharing turns out to be more valuable for agents. As their incentive to default diminishes, within-group consumption inequality also decreases consequently. Parallel to limited enforcement of contracts, private information problem is another hypothesis on consumption smoothing and risk-sharing. Attanasio and Pavoni (2007) present such a setting, introducing moral hazard and hidden savings. Since agents in their model can insure more of their idiosyncratic risks under asymmetric information, consumption volatility is lower than in a single asset model.

Nonetheless, an incomplete market setup with one single asset and heterogeneous household preferences can offer partial but relatively good insurance against income shocks (Krusell and Smith, 1998), whereas under certain assumptions it can match the real-world wealth distribution relatively well: impatient agents have a high propensity to consume and can hardly accumulate wealth (under condition that social security exists), while more patient agents are much richer and have sufficient wealth to be well insured. As their marginal propensities to consume hardly depend on current wealth or income, more patient agents exhibit permanent

income savings behavior.

Accordingly, this paper uses an incomplete market setup with heterogeneous endowments, assuming preferences of the households, i.e. patience and the attitude towards the group average benchmark, are heterogeneous. I choose a comparatively easy model to capture the consumption evolution. In the spirit of Duesenberry's Relative Income Hypothesis (1949) and similar to the setup in Galí (1994), I apply a special type of consumption externalities, group average consumption, to a self insurance model. The attitude of households toward this externality is the key issue if the model produces "excessive smoothing" or vice versa. Acknowledging consumption growth inequality as a result of income uncertainties (permanent and transitory), I use this model to study the link between the group average consumption growth and within-group inequality. The main theoretical finding is that this consumption externality drives agents from the original consumption smoothing path. When restrictions on the time series properties of consumption growth are relaxed, the deviation can be even larger. Nevertheless, regardless of the extent of the deviation, the model most possibly predicts a positive correlation between the group average consumption growth and within-group inequality.

I further test this theoretical hypothesis using data from the German Socio-Economic Panel (GSOEP) and Einkommens- und Verbrauchsstichprobe (EVS). Both data sets have pros and cons and can relatively well complement each other. GSOEP tracks relatively good income flows of households in a panel setup but offers little information on consumption, while EVS contains detailed information on household consumption which is only available every five years. Since the households in both data sets can not be perfectly identified and matched, I combine the information in both data sets to serve the purpose of retrieving household consumption growth. In details, small cells can be built within EVS observations according to households' characteristics such as household size, household type, members' age, occupational background, number of kids, community size and etc., where households' consumption/income ratio (i.e. average propensity to consume) can be estimated. Interpolating this propensity for the years without EVS observations and applying the estimated propensities to those GSOEP households in the same cells, one can impute the consumption for GSOEP samples and further calculate the corresponding consumption growth rate. As a result, the imputed consumption growth rate would both reflect consumption information in

EVS and pick up the time structure in GSOEP.

There is, however, one drawback of this imputation. By construction consumption variance within cells would tightly follow income variances and the heterogeneity of the average propensity of the households within one cell would be similar to that of income. Thus, for the inequality analysis afterwards based on the pooled cells, within group variance is underestimated. Theoretically, this problem can be solved if cells are defined ultra small, and each cell contains only one household, while in reality it's almost impossible to achieve such a perfect match between the two data sets, since the available household characteristics are not totally the same.

Based on the imputed consumption growth rate, I define groups with available social demographic characteristics in GSOEP and find significant though only slightly positive correlation between the group average consumption growth rate and within group variance. Overall, the constructed consumption data does not show large changes in inequality during the sample period (1984-2005), except distinguishable increases directly after the reunification of Germany and the introduction of the euro. Compared to income inequality, the data shows a similar picture as what Krüger and Perri (2005) find in U.S. data, that between-group consumption inequality has tracked between-group income inequality much more closely than the within-group consumption inequality has followed the within-group income inequality.

The rest of the paper is organized as follows: Section Two presents the theoretical model and an extension based on the random walk hypothesis of consumption growth; Section Three introduces the data, discusses the grouping strategy and inequality trends in Germany in the sampling years; in Section Four tests are carried out on the relationship between consumption growth and inequality; Section Five concludes.

2 Consumption Growth and Inequality

In modern economics there are two major hypotheses connecting income shocks and consumption insurance. The complete market hypothesis assumes that consumption is fully insured against idiosyncratic income shocks (both permanent and transitory), which is soundly rejected in micro data (e.g. Attanasio and Davis, 1996). The other one, the permanent income hypothesis, assumes that personal savings serve as the only mechanism to smooth income

shocks, and exclusively against transitory shocks (Deaton, 1992). This hypothesis is also rejected by, for example, Attanasio and Pavoni (2007). Their empirical paper, as well as an earlier paper by Campbell and Deaton (1989), finds that consumption exhibits “excessive smoothness” by reacting too little to permanent income shocks; while some other studies find that consumption shows “excessive sensitivity” by reacting too much to transitory shocks (e.g. Hall and Mishkin, 1982). The truth seems to lie somewhere in between, and therefore partial insurance of consumption to income shocks becomes slowly the consensus.

These empirical studies question the theoretical framing of the household consumption and wealth distribution. Krüger and Perri (2006) have shown that, indeed, the complete market model with Arrow securities insure against heterogeneous income shocks to such a high degree that consumption and accordingly consumption inequality react too little to the shocks. In comparison, an incomplete market setup with one identical household preference turns to offer partial insurance and individual consumption fluctuations react too much to income shocks. As households’ “true” risk insurance may lie between a full set of contingent securities and a single asset, the latter setup together with heterogeneous preferences (such as in Krusell and Smith, 1998) can match real-world wealth distribution relatively well: impatient agents have a high propensity to consume and can hardly accumulate wealth (under the condition of social security), while more patient agents are much richer and have sufficient wealth to be well insured, whose marginal propensities to consume hardly depend on current wealth or income (exhibiting permanent income savings behavior). This is also the benchmark approach in the current paper.

Other extensions were added to the incomplete model setup in the asset pricing literature and one special “flavor” is to include relative consumption into household utility as a consumption externality. There is a refined difference if agents take the previous or current average consumption as benchmark. The former, which is a variation of the habit formation setup, is the case of “catching up with the Joneses” (Mehra and Prescott, 1985, Abel, 1990, Campbell and Cochrane, 1999) and the latter “keeping up with the Joneses” (Galí, 1994). While the former involves the interdependence between the agents’ past, present and future well-being, the latter setup emphasizes contemporaneous trade-offs and generates simpler results. In fact, Ljungqvist and Uhlig (2000) discusses optimal tax policies using these two differentiated cases and finds procyclical taxes for the former and a flat tax rate for the latter

to be optimal¹.

Following a similar route, this paper uses an incomplete market setup with heterogeneous endowments (income shocks), assuming heterogeneous preferences of the households (i.e. patience and the attitude towards group benchmark). Since the true task is to study the contemporaneous consumption distribution (cross-sectional) using a panel setting, the “keeping up with the Joneses” model is chosen so as to avoid more complex intertemporal considerations. To be specific, the model should help to discover the connection between consumption growth and inequality.

The paper starts with discussing two possible extensions of an incomplete market model with heterogeneous household preferences (henceforth IMH) where agents transfer resources intertemporally via trading one type of asset, which helps to store value and insure against income shocks. The first extension involves some external criteria for the households: the households cast their preference not only on their own consumption, but also on that of their neighbors (other households who are in the same social class). The relative standard of living becomes another important issue besides the absolute level. The consumption smoothing path of the household in the IMH model would be distorted, and variance of consumption growth would change. The direction of this change depends on household attitude toward their neighbors’ well-being (whether they are altruistic or meant to “keep up with the Joneses”). The result, however, is reached regardless of the assumption about the time series properties of the group average growth (the cross sectional moments such as means and variances). The relaxation of this assumption will be examined as the second extension.

2.1 A Heterogeneous Agent Model

In this economy there is a continuum of households of measure 1. Households belong to different groups $i \in \{1, ..M\}$, which capture the socio-economic heterogeneity in the population, such as age, education or job skill level. One can intuitively interpret a group as a highly similar neighborhood. p_i denotes the number of households in each group. Households within one group share the aforementioned features but are still subject to small idiosyncratic shocks, either from income or consumption innovation. Although households in a given group

¹Guo (2003) elaborates the latter case by adding capital accumulation and imperfect competition in the goods market and finds similar result.

do not observe the exact income of other group members, they can observe their consumption patterns. If they would like to compare with others in a similar socio-economic class, it is the case of “keeping up with the Joneses”. Otherwise, if they also benefit when others are doing well, we have “altruistic” households. I label the result of this additional externality a group effect on household consumption decisions.

The setup is similar to Galí (1994) and Abel (1990), only that in Abel’s case households regard agents’ own consumption habits and the group average consumption in the previous period as a benchmark for their current period consumption (“catching up with the Joneses”)². Besides, Galí (1994) and Abel (1990) both use the model to describe the (homogeneous) households in the whole economy, while this paper rather takes the perspective of each group, and the “keeping up” mechanism bounds the agents within one group only. The comparison of agents in different groups is not captured in this model, which does not deny the existence of such comparison, but is rather the result of model simplification and focus on the more likely comparison among people of similar socio-economic backgrounds.

There are several reasons why a heterogeneous agent model is considered here. Firstly, aging effects on consumption growth can vary for people with different education/occupation attainments, and heterogeneity in social economic characteristics are critical for within and between group income and consumption inequality. Secondly, using heterogeneous agents model enables the contemporaneous examination of consumption growth inequality within group, while still allowing for comparison in the time dimension and/or group-to-group dimension (more discussion of this issue can be found at the end of next subsection). In a word, such a setup allows mobility of the households on the total consumption distribution and enables their comparison with their contemporaneous benchmarks once they move from one group to another. Last but not least, the intensity of consumption externality tend to differ between groups, e.g. the urge of comparison with neighbors may depend on the size and consumption patterns of the neighborhood, which further depend on the social economic characteristics of the group.

²Abel (1990) introduces jointly the agent’s own consumption habit and past aggregate consumption into current utility: $u(C_t, v_t) = [C_t/v_t]^{1-\kappa} / (1-\kappa)$, where the preference parameter $v_t \equiv (C_{t-1}^D v_{t-1}^{1-D})^\gamma$. Let $D = 0$, then the current consumption only takes external habit (aggregate consumption) as benchmark, which is also the case here.

Similar to the setup of Krüger and Perri (2006), households are endowed with human capital every period and supply it inelastically to firms at unit price 1^3 . One household in group i has a stochastic labor endowment process $\{\alpha_{i,t}y_t\}$, where $\alpha_{i,t}$ is the deterministic, group specific endowment, and the idiosyncratic component $\{y_t\}$ holds economy wide. This implies that households within one group share a common deterministic group-specific trend while differing in being subject to idiosyncratic shocks in each period. Let y_t denote the current period labor endowment, and $\{y_t\}$ follow a Markov process with finite support Υ_t , initial probability distribution $\Pi_0(\cdot)$ and transition probabilities $\pi_t(y'|y)^4$. The distribution of $\{y_t\}$ relates to a business cycle concept in the sense that most households experience positive income shocks in boom, and negative shocks are prevalent in recession.

Even though a typical PIH model can also allow for endogenous labor supply and non-stationary income (Bewley, 1977), since the purpose of this paper is on the consumption dynamics, I reduce the household problem to consumption and asset holding decisions, assuming households supply endowed labor inelastically in each period. Intertemporally households transfer their resources by trading one single asset economy wide. The borrowing, however, is subject to a household-specific debt limit $A_{i,t}(y^t)$, the pre-specified credit line contingent on labor endowment histories up to period t and households' group.

For simplicity it is assumed that households have zero mobility across groups. The reason is two-fold. On the one hand, GSOEP data shows that mobility is not the dominant issue, since more than half of the households in the samples (56.8%) between 1984-2005 have never changed their groups, while among the group switchers over half of them (51.8%) have changed only once, among which over 52.7% happened due to aging. In a word, these het-

³This is merely a normalization of wage, which is not in the spotlight in the partial equilibrium discussed here.

⁴In a complete market setup, $y^t = (y_0, y_1 \dots y_t)$ captures the history of endowment shocks, such that the compound probability of a history y^t given an initial endowment y_0 is $\pi_t(y^t|y_0) = \pi_{t-1}(y_t|y_{t-1}) \pi_{t-2}(y_{t-1}|y_{t-2}) \dots \pi_0(y_1|y_0)$. Let Φ_0 be the initial distribution over types (i, a_0, y_0) . Total labor supply at period t is given by:

$$L_t = \sum_{y^t} \alpha_{i,t} y_t \pi_t(y^t|y_0) d\Phi_0$$

Let $\{q_t(y_{t+1}|y^t)\}_{t=0}^{\infty}$ be the economy-wide sequence of pricing kernels dependent on the history of the stochastic labor endowment and the realization of y_{t+1} at $t+1$, and let $\{a_{t+1}(a_0, y^t, y_{t+1}, x_i^t)\}_{t=0}^{\infty}$ be the claims on time $t+1$ consumption corresponding to each value of the time $t+1$ realization of y_{t+1} .

erogeneous households appeared to stay relatively persistently in their group. On the other hand, the later use of panel data is to examine consumption growth in sequential years where cross-sectional comparison is the final aim.

Define $c_t(\Delta_{i,t}, y_t, x_{i,t})$ as the time t consumption of a household in the i th group with group average endowment $\Delta_{i,t}$ and a stochastic labor endowment y_t , where group average consumption $x_{i,t} = \frac{\sum c_t(\Delta_{i,t}, y_t, x_{i,t})}{p_i}$. When p_i is sufficiently large, the mean of the stochastic component y_t approaches zero and plays no role for group mean consumption. γ is the risk aversion parameter and is usually larger than 1. With a group-specific discount factor β_i , which implies that groups are different in patience, a household from group i of type $(\Delta_{i,t}, y_t)$ chooses a consumption stream and asset holding plans for uncontingent claims to solve the following maximization problem⁵:

$$\max_{\{c_t(\Delta_{i,t}, y_t, x_{i,t})\}_{t=0}^{\infty}, \{\alpha_{t+1}(\Delta_{i,t}, y_t, x_{i,t})\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_i^t [u(c_t, x_{i,t})]$$

subject to

$$c_t(\Delta_{i,t}, y_t, x_{i,t}) + q_t * \alpha_{t+1}(\Delta_{i,t}, y_t, x_{i,t}) \leq \Delta_{i,t} y_t + \alpha_t(\Delta_{i,t-1}, y_{t-1}, x_{i,t-1}).$$

One unit asset is priced q_t in period t and pays one unit of consumption good in period $t + 1$. Initial asset holdings α_0 are given and the borrowing constraints hold in order to rule out Ponzi schemes:

$$-\alpha_{t+1}(\Delta_{i,t}, y_t, x_{i,t}) \leq A_{i,t}(y^t)$$

From now on, for simplicity I use $c_{ij,t} = c_t(\Delta_{i,t}, y_t, x_{i,t})$ to denote the consumption of the j th household in i th group, and the utility function has the following isoelastic form:

$$u(c_{ij,t}, x_{i,t}) = \frac{c_{ij,t}^{1-\gamma} x_{i,t}^{-(1-\gamma)\kappa_i} - 1}{1-\gamma}.$$

⁵Although a typical PIH model also allows for endogenous labor supply and non-stationary income (Bewley, 1977), since the purpose of this paper is on the consumption dynamics, I reduce the household problem to consumption and asset holding decisions, assuming households supply endowed labor inelastically in each period. To elaborate elastic labor supply, the utility function could take the form

$$u(c_t, x_{i,t}, l_t) = \frac{c_{ij,t}^{1-\gamma} x_{i,t}^{-(1-\gamma)\kappa} - 1}{1-\gamma} - \chi \frac{l_t^{1-\psi}}{1-\psi},$$

where $l_t = a_{i,t} y_t$.

Group consumption serves as an external benchmark, and $\kappa_i < \frac{\gamma}{\gamma-1}$ as the attitude of group i households towards this benchmark can be interpreted as “how important is my neighbors’ consumption for me”. Taking log of the core of the utility function yields:

$$(1 - \gamma) \ln c_{ij,t} - (1 - \gamma) \kappa_i \ln x_{i,t} = (1 - \gamma) \left[(1 - \kappa_i) \ln c_{ij,t} + \kappa_i \ln \frac{c_{ij,t}}{x_{i,t}} \right].$$

Scaled by parameter κ_i , the household’s consumption preference is a weighted average of the absolute and relative consumption (compared to group average). There is no restriction on κ_i to be positive or negative, which allows us to examine three cases considering the group effect in consumption:

1. When $0 < \kappa_i < \frac{\gamma}{\gamma-1}$, the household would like to “keep up with the Joneses”. Average consumption decreases the household’s utility level but increases household’s marginal utility of an additional unit of consumption. This reflects exactly the economic implication of “keeping up with the Joneses”, since “any given addition to his current level of consumption becomes more valuable”⁶. In the later part of the paper, it will become clear that such partial preferences, keeping up with the Joneses, could reduce contemporaneous consumption growth inequality but drive up consumption volatility over the business cycle further from a model without consumption externalities.

2. When $\kappa_i < 0$, households do not take the group mean as benchmark, but rather gain utility once the others in the group are doing well. For philanthropists this could be interpreted as altruism. However, a more economic intuition is that the group mean consumption acts as “substitute” for the household’s own consumption. This would be the feature of a public good. Here, a single household benefits from an increase in the group average.

3. When $\kappa_i = 0$, the utility function is reduced to a typical self insurance version, where agents are only concerned with their own consumption.

⁶Galí (1994).

The resulting Euler equation is⁷:

$$q_t = \beta_i E_t \left[\left(\frac{c_{ij,t+1}}{c_{ij,t}} \right)^{-\gamma} \left(\frac{x_{i,t+1}}{x_{i,t}} \right)^{-(1-\gamma)\kappa_i} \right].$$

Since all households in group i have the same optimization problem, through aggregation, it holds for group i :

$$q_t = \beta_i E_t \left[\left(\frac{x_{i,t+1}}{x_{i,t}} \right)^{-\gamma-(1-\gamma)\kappa_i} \right], \quad (1)$$

where q_t is determined by demand and supply in the financial market and is exogenous for single households. Equation (1) implies that the degree of risk aversion, and the group-specific discount factor as well as the attitude to neighbors' consumption determine the group consumption growth together. Group consumption growth is slow when households in the group are less patient (small β_i) and prefer current to future consumption, or when they put more value on their current relative position in the group (κ_i is positive and increases) and would rather “keep up” consumption than buying security (a similar effect to households' being “impatient”).

If for most households the idiosyncratic shock y_t turns out to be negative, implying a negative income shock in the aggregate, net borrowing demand (sales of the security) increases and ceteris paribus, the asset price q_t will decrease, and the return for those households purchasing the security increases. Needless to say, in a general equilibrium q_t is also subject to the distribution of β_i , κ_i and γ .

2.2 Implication on Consumption Dynamics

The permanent income hypothesis states that periodical consumption is subject to lifetime resources, instead of each period's income. Household wealth is thus a better candidate as a consumption constraint. However, while the change of household consumption is additionally

⁷In Abel's (1990) model households compare themselves with the previous consumption of the group members, so as to “catch up with the Joneses”. Households still buy one unit of risk-free bond at price q_t

$$q_t \left(\frac{x_{i,t}}{x_{i,t-1}} \right)^{-(1-\gamma)\kappa} = \beta E_t \left[\left(\frac{c_{ij,t+1}}{c_{ij,t}} \right)^{-\gamma} \right]$$

Taking logs gives the same result as above, since the growth rate of $x_{i,t}$ is time invariant. This picture, however, can be totally different if consumption growth is time-variant.

triggered by consumption innovations, the main shocks occurring to household consumption are often identified as contemporaneous income shocks in the related literature.

Following Meghir and Pistaferri (2004), I assume that per period labor income $Y_{ij,t}$ follows the following process:

$$y_{ij,t+1} = \ln Y_{ij,t} = \varphi Z_{ij,t} + P_{ij,t} + \varpi_{ij,t}$$

where $Z_{ij,t}$ is a set of observable characteristics, $P_{ij,t}$ is the permanent income component, and $\varpi_{ij,t}$ is the transitory component.⁸

The permanent component of income follows a martingale process (random walk):

$$P_{ij,t+1} = P_{ij,t} + \zeta_{ij,t+1}$$

where $\zeta_{ij,t}$ is the permanent shock and serially uncorrelated.

The log of income growth is therefore

$$\Delta y_{ij,t+1} = y_{ij,t+1} - y_{ij,t} = \varphi \Delta Z_{ij,t+1} + (\zeta_{ij,t+1} + \Delta \varpi_{ij,t+1}).$$

Once data are available on income and consumption, one can even identify the degrees to which permanent and transitory income shocks affect the change of consumption (see Blundell et al., 2008). Even though this is not the focus of my paper, it serves as the premise of my approach. Based on the strong correlation between consumption and income, a natural guess is that the change in log consumption is subject to part of the permanent income shock, transitory income shocks and consumption innovation shocks. The consumption growth rate of household j in group i is approximately the difference of log consumption and can be decomposed into $g_{i,t+1}$, the average growth rate of group i , and some household specific shock $v_{ij,t+1}$:

$$\begin{aligned} g_{ij,t+1} &= \ln c_{ij,t+1} - \ln c_{ij,t} = g_{i,t+1} + v_{ij,t+1} & (2) \\ \text{where } g_{i,t} &= \frac{1}{J_i} \sum_{j=1}^{J_i} g_{ij,t}, \quad j = 1, 2, \dots, J_i \\ \text{and } v_{ij,t+1} &\sim i.i.d.N(0, \sigma_{v_i}^2) \end{aligned}$$

⁸Meghir and Pistaferri (2004) use U.S. data to test the autocovariance of the unexplained earnings growth rate. Their result suggests that the transitory shock follows a moving average of degree 1.

As is mentioned, this unexplained consumption growth shock $v_{ij,t+1}$ contains information about income shocks (permanent and transitory), as well as some household specific consumption innovation (Blundell et al. 2008). The interaction of between-households comparison is captured in the group aggregate growth. For the simplest case, the group average consumption growth rate is assumed stationary and (conditionally and unconditionally) log-normally distributed $g_{i,t+1} \sim (g_i, \sigma_{g_i}^2)$ ⁹. With the help of a second order Taylor approximation, equation (1) turns out to be¹⁰:

$$q_t \approx \beta_i \exp \left[(-\gamma - (1 - \gamma) \kappa_i) g_i + \frac{(-\gamma - (1 - \gamma) \kappa_i)^2}{2} \sigma_{g_i}^2 \right] \quad (3)$$

The security price q_t is determined in the general equilibrium as a product of the aggregation of all groups' saving and borrowing decisions, which in turn depend on the group specific labor endowment and the distribution of the idiosyncratic income shocks. A none-zero κ_i leads to the deviation from the IMH case where the household optimization problem is independent of others' consumption behavior. This deviation could be one way to mitigate the equity premium puzzle in asset pricing. Rearranging equation (3) gives:

$$\sigma_{g_i}^2 = 2 \frac{[\gamma + (1 - \gamma) \kappa_i] g_i + \ln q_t - \ln \beta_i}{[\gamma + (1 - \gamma) \kappa_i]^2} \quad (4)$$

It yields a relationship between the within-group variance in consumption growth and group average consumption growth. Note that once the group average plays no role for single households ($\kappa_i = 0$), the equation is reduced to the IMH model:

$$\sigma_{g_i}^2 = 2 \frac{\gamma g_i + \ln q_t - \ln \beta_i}{\gamma^2} > 0 \quad (5)$$

Comparing these two equations can tell us the effect of minding neighbors' "business". For a reasonable value of risk aversion, i.e. $\gamma > 1$ ¹¹, there are the following propositions:

⁹Whereas define $G_{i,t+1} = 1 + g_{i,t+1} = \frac{x_{i,t+1}}{x_{i,t}}$, and $\ln \frac{x_{i,t+1}}{x_{i,t}} = \ln G_{i,t+1} \approx g_{i,t+1}$.

¹⁰See Appendix I for a detailed derivation.

¹¹Other than assuming the values of the key parameters, one can use maximum likelihood (MLE) to estimate them, which will be the next step of the research. The further task of the current paper is to examine the empirical relationship between group average consumption growth and within-group inequality.

Proposition 1 *At a given consumption growth rate, more patient groups have smaller within group variance.*

This proposition can be derived from taking partial derivatives of $\sigma_{g_i}^2$ according to group-specific discount factor β_i , where $\frac{\partial \sigma_{g_i}^2}{\partial \beta_i} < 0$. The implication is straightforward. Patient households tend to have a higher propensity to save, which insures the households against income shocks in next period to a higher degree. As result, the variance of growth is smaller within the group.

Proposition 2 *At a given consumption growth rate and in presence of precautionary saving, households' altruism / eagerness to "keep up" leads to a lower / higher within-group inequality in consumption growth compared with an externality-free model.*

Proof: Taking the partial derivative of $\sigma_{g_i}^2$ according to κ_i yields:

$$\frac{\partial \sigma_{g_i}^2}{\partial \kappa_i} = \frac{2(\gamma - 1)}{[\gamma + (1 - \gamma)\kappa_i]^2} \left[g_i + 2 \frac{(\ln q_t - \ln \beta_i)}{\gamma + (1 - \gamma)\kappa_i} \right]. \quad (6)$$

The return of the security is slightly below the discount rate of the most patient agents, because securities to some extent are used for precautionary savings. Accordingly, $\ln q_t > \ln \beta_i$. Meanwhile, because $\gamma + (1 - \gamma)\kappa_i > 0$, $\frac{\partial \sigma_{g_i}^2}{\partial \kappa_i} > 0$.

Households' preferences show a dislike of deviation from the group average. The faster the others in your group are upgrading than you are, the larger is the "punishment" of not being able to keep up with them. At a high degree of such dislike (the case of "keeping up with the Joneses, with a positive κ_i approaching 1), households prefer current consumption to security purchases, which leads to low insurance against future shock and higher within-group variance in consumption growth.

Following the contrary argument, group **inequality** is lower in the case when households are enthusiastic about group average well-being (regarding it as a public good) or lack the incentive to keep up. Higher volume of security sheds households against future uncertainty and reduce the within-group inequality $\sigma_{g_i}^2$.

Now recall the modeling strategy to exclude the mobility of the households across groups, then one can see that this result rather offers a snapshot of within-group variance and makes cross-sectional comparison possible, regardless of the mobility question. Apart from this contemporaneous effect, two further interpretations can be made from the result. Firstly there is a story on the evolution of inequality **over time**.

Proposition 3 *Households' preferences are sensitive to business cycle effects, and aggregate consumption growth turns to be more / less volatile in booms / recessions.*

Comparing to the first two propositions with a particular groups' perspective, the business cycle effects are general and apply to all groups. Let's get back to the partial derivative (6), which clearly increases with the security price q_t . In a business cycle context, most agents expect to receive positive income shocks in booms and are willing to lend out their resources (through buying more securities). Higher demand of securities drives up the unit price q_t in general equilibrium, which further intensifies the effect of households' preferences (degree of patience and households' attitude toward external benchmark). In contrast, when most agents are subject to negative income shocks in recessions, overwhelming borrowing wish leads to a decline of the security price and dampens the preference effect.

Proposition 4 *There is a positive relationship between group average consumption growth and within-group inequality, unless agents have extremely high desire in "keeping up with the Joneses" ($\kappa_i > \frac{\gamma}{\gamma-1}$).*

Under condition that κ_i is bounded by $\frac{\gamma}{\gamma-1}$, equation (4) reveals a positive relationship between $\sigma_{g_i}^2$ and g_i , suggesting that groups with higher consumption growth also have to pay the price of larger within-group inequality. Nonetheless, for a large κ_i , i.e. when it's extremely important for agents to keep up, they would short sell securities up to the liquidity constraints. By doing so, they indirectly insure their consumption next period, achieving a small within-group variance at a given consumption growth rate.

2.3 The Time Series Properties to Group Consumption Growth Rate

A further extension of the model includes relaxing the time-invariance assumption about the group average consumption growth. Note that the theoretical result (4) is achieved under condition that group-specific consumption growth is time-invariant or not. However, from asset pricing theory we know that assumption on the time series property of the consumption growth rate is very important for the theoretical model to reproduce the price volatilities of the risk-free securities.

Starting from the consumption level itself, there has been an intense debate on if it is a random walk process. Hall's famous random walk hypothesis was challenged by Campbell and

Mankiw (1989), who use aggregate data and test the predictability of consumption change from income change. Their result shows that about fifty percent of the change in consumption is predictable, implying a significant rejection of the random walk hypothesis. Will this also hold for the GSOEP data? A small exercise is carried out on group mean households' consumption, where 24 groups are defined by household size, household head's age, education and occupation. After the optimal lags of the group consumption are found through using Augmented Dickey-Fuller / Generalized Least Squares unit root tests, a confirmatory analysis jointly using a Dickey-Fuller test, a Phillips-Perron test and a stationarity KPSS test is applied to detect the stationarity of the group average consumption¹². The result, however, is not at all obvious: 2 out of 24 groups appear to follow a stationary process, also 2 out of 24 groups significantly show a nonstationary pattern, while the rest is unclear in stationarity. When a trend is assumed, 3 groups' consumption appears to be stationary and 3 other groups non-stationary, and the rest 18 groups bear unclear results.

Following the same procedure, I run the tests for the group-specific consumption growth¹³. The results here are much more clear compared to those for consumption level. Indeed at the 5% significance level, when a time trend is assumed, 19 out of 24 groups appear to be stationary processes; when no trend is included, for none of the 24 groups' consumption growth, the hypothesis of stationarity can be rejected.

Based on these observations, the group-wide consumption growth rate $g_{i,t+1}$ can be assumed following an $AR(1)$ process with a group-specific shock $u_{i,t+1}$:

$$g_{i,t+1} = (1 - \phi) g_i + \phi g_{i,t} + u_{i,t+1}; u_{i,t+1} \sim i.i.d.N(0, \sigma_u^2), \quad (7)$$

which suggests high persistence of the idiosyncratic shocks for $\phi \rightarrow 1$ (the diagonal elements of the transition probability $\pi_t(y'|y)$ are relatively big numbers). Rich households with a high labor endowment have a high probability to stay rich, while poor households turn to stay poor. Once this strong assumption holds, the mobility of households between groups is small

¹²This confirmatory analysis is used, because both Dickey-Fuller test and Phillips-Perron test have low power in rejecting the null hypothesis of a unit root, and therefore an opposite test for stationarity, which also has low rejecting power problem, is used for comparison.

¹³Alternatively one can do OLS regressions (alone, with fixed effect or with random effect) of group mean consumption growth on its lags, controlling for heteroskedasticity. The results of OLS with fixed effect on one lag and two lags are significant, confirming the result from the Augmented Dickey-Fuller / GLS unit root test.

and wealth distribution would be even more extreme, with a large stock of wealth in the rich groups, which implies that consumption growth is conditionally log-normally distributed:

$$E_t [g_{i,t+1}] = (1 - \phi) g_i + \phi g_{i,t}, \text{ and } \text{var}_t [g_{i,t+1}] = \sigma_u^2$$

The variance of this group-specific shock can be interpreted as one of the sources of between-group variance. Recall from (2) that the difference between individual consumption growth and the group mean is merely the idiosyncratic shock $v_{ij,t}$. Combining (7) with (2) yields $g_{ij,t+1} = (1 - \phi) g_i + \phi g_{i,t} + u_{i,t+1} + v_{ij,t+1}$, where the conditional mean and variance are

$$E_t [g_{ij,t+1}] = (1 - \phi) g_i + \phi g_{i,t}, \text{ and } \text{var}_t [g_{ij,t+1}] = \sigma_u^2 + \sigma_{v_i}^2.$$

Equation (3) would look different:

$$q_t \approx \beta_i \exp \left[-\gamma - (1 - \gamma) \kappa_i \left((1 - \phi) g_i + \phi g_{i,t} \right) + \frac{[-\gamma - (1 - \gamma) \kappa_i]^2 \sigma_u^2}{2} \right] \quad (8)$$

Now the uncertainty from group consumption growth drives up household demand for a secure transfer of their consumption between periods, which in general equilibrium increases the security price. Moreover, conditionally on a high current growth rate $g_{i,t}$, households expect high consumption growth tomorrow. Due to the smoothing motive, they would like to borrow against future growth, which drives down the security price.

Similar to (4), (8) also suggests that groups with faster consumption growth have also higher variance of growth. In reality, could they be groups with a lower consumption level and therefore especially sensitive to income shocks? Such “poor” households have a relatively high propensity to consume and a larger part of their income may be used to purchase basic goods with a very low demand elasticity. Because they have little capacity to buy security as insurance against future shocks, their future consumption and thus consumption growth are vulnerable towards the shocks. This hypothesis may be tested using the GSOEP data set where annual household consumption can be constructed in subsequent years between 1995 and 2005. The next section will describe the data and report the results.

3 Bringing the Model to the Data

3.1 GSOEP and EVS

For the purpose of testing this theoretical framework, panel data is needed to get the growth rate of household consumption, whereas the best choice is the German Socio-Economic Panel¹⁴. Starting from 1984, this panel data set is based on household interviews, and contains crucial questions on living and income. The sample used in this paper includes all West German and foreigner households from 1984, whereas immigrants households are added starting in 1995. However, since GSOEP does not offer much information on consumption, especially not on nondurable goods consumption, one solution is to construct consumption from the available information in GSOEP on financial inflows and outflows.

Besides the household monthly net income and savings, there are data on extra income: yearly rental income, capital/investment income, additional income from winnings and inheritance; and detailed expenditures: rent without heating costs, water and other expenses (noted as CR from here on), the cost of heating and water, the credit and interest repayment. Unfortunately, there are neither data on expenditure on durable goods (which would otherwise decrease the amount of nondurable consumption), nor data on total amount of consumption credit (which would otherwise increase the total amount of expenditure). I can only make the assumption that the two missing parts of the puzzle, the underestimation and overestimation, approximately cancel each other out, i.e. consumption credit is only used for durable goods. This, however, is a reasonable assumption due to the often high prices of durables.

I follow Cutler and Katz (1991) to construct consumption from the expenditure of house owners, where I need to impute the market-valued cold rent for all house owners. In case some house owners also report their own estimation of the rental value of the housing, I also impute the estimated CR for other house owners who did not report their estimation. The imputations are based on relevant house characteristics such as size of the apartment, family size, family monthly net income, area of the apartment (only available in 1985, 1994, 1999,

¹⁴Alternatively, the micro data from German Income and Expenditure Survey (Einkommens- und Verbrauchsstichprobe, EVS) can offer a deep and detailed view of the household consumption. However it is carried out only every 5 years, which, unfortunately, can not help constructing the growth rates.

2004). Year and federal state dummies are included¹⁵. Then I add the imputed CR/estimated CR to expenditure and deduct the mortgage payments and interest, the expenses to maintain the house, as well as the costs for water, garbage removal and street cleaning, in order to get an approximation of consumption for these house owners. Further I calculate consumption growth of each household. Regressions of within-group variances on the group average growth rate, using these two imputed values separately, yield slightly different results (see Figure 1 and 3).

Due to the lack of information on households' savings (stock), which is a crucial index for households' wealth and serves as the main insurance for households facing negative income shocks, the imputed consumption from the available GSOEP data is unavoidably underestimated and has a fat lower end. Alternatively, one can use information on household consumption-income ration (average propensity to consume) from the EVS data set, so as to attain a better picture of households' consumption.

EVS (Einkommens- und Verbrauchsstichprobe, or Income and Consumption Survey) is one of the major surveys containing personal and households' income and consumption distributions in Germany¹⁶. Federal Statistical Office delivers cross sectional survey every 5 years starting from early 1960s. Until 2008 there are surveys in 1962/1963, 1969, 1973, 1978, 1983 and 1988 for West Germany, and thereafter extended to East Germany in 1993, 1998 and 2003. Our research focus is mainly on the last two decades of 21st century, and therefore we choose the public-use micro data sets from 1978 until 2003. EVS is not a random sample with mandated response but a quota sample with voluntary participation. However, it takes as benchmark for recruiting participants the annual Current Population Survey of

¹⁵The two imputed results are highly correlated with a correlation coefficient of 0.915, which indicates that the estimated CR by the house owner does not differ too much from the market value. In 62.4% of the cases, the estimated rent is higher than the market value of the apartment/house.

¹⁶The survey is composed of three parts. As introduction there is an interview at the beginning of the year requesting the basic socio-demographic and socio-economic data of households and single persons, such as the housing situation and household facilities. The second part of the survey is recorded through books of household accounts, where the participating households note down all their income and expenditures for the whole survey period, which was one calendar year until 1993 and reduced to three months from 1998 on. After the reduction of the recording period, the households are equally divided into four groups, with each reporting for one quarter of the year. Last but not least, the third part of the survey is a heft containing a detailed record on food, beverage and tobacco products consumption.

Germany (Mikrozensus), which is a mandated random survey of large size. Consequently, the household net income brackets in the EVS are defined identically to those in the Mikrozensus.

The EVS data has several advantages. First, it covers a rather long time span. Second, it contains a large number of households (defined as consumer units), and even more observations when individuals are concerned. While individual samples are comparatively easy to be extracted from the household observations, they contain dependent employees, self-employed, unemployed as well as out of labor force. This large variety of occupational status enriches the objects of the study to the general population and makes it possible to examine the consumption and welfare effects over the time.

The idea is to form small cells according to households' characteristics which are common for EVS and GSOEP. They are household size, household type, household heads' age (5-year period), household heads' gender, occupational background, federal states, community size and etc., where households' consumption-income ratio can be estimated from the EVS from 1978-2003¹⁷. Interpolating this ratio for the between years using macroeconomic trend and applying the estimated propensities to those GSOEP households in the same cells, one can thus impute the consumption for GSOEP samples and further calculate the corresponding consumption growth rate. As a result, the imputed consumption growth rate would both reflect consumption information in EVS and pick up the time structure in GSOEP.

3.2 Grouping Strategy

Before entering the discussion about within-group and between-group effects, the crucial question would be, what criteria can be used to group the samples. For this purpose I carry out a two-step strategy to identify the criteria. Firstly, a factor analysis with principle component is used to distill the various household characteristics into most informative ones. With variables such as nationality, sex and federal land discarded, three blocks of variables are retained: age of household head and partner, their education level and their job types. Some variables overlap with each other, such as International Standard Classification of Education (ISCED-1997) and years of education, or Erikson Goldthorpe Classification (EGP) for job types and Occupational Position (Stellung im Beruf, coded by Statistisches Bundesamt).

¹⁷Some other household characteristics such as education level or years are available in either GSOEP or EVS, thus can not be used to construct the cells.

Therefore I run a simple OLS regression of log consumption on combinations of the crucial household characteristics mentioned above together with household size and year dummies, and look for the combination with the highest explanatory power. Since a Breusch-Pagan test shows that the residuals are heteroskedastic, I allow for heteroskedasticity in the regression. The regression yields an adjusted R-squares of 0.39¹⁸.

My result shows that in this data set the best explanatory variables for consumption are age and age-squared, ISCED for education ¹⁹, EGP for occupation ²⁰ of the household head, household size and its square, as well as the interaction term of education and occupation. Year dummies are included. As the fitted part counts for the between-group variance, the residuals (unexplained variables) are equivalent to the within-group inequality, and thus the data carries the information of the evolution of these inequalities over the sampling years. A further discussion of this important by-product can be found in next subsection.

Having extracted the most relevant attributes, I can start grouping the samples. Consequently, I use household size and household head's age, ISCED education level and EGP occupation to divide the sample into 24 groups. Particularly, a household is regarded as small once there are fewer than 3 members, otherwise it is large. Regarding age, suppose on average one person can work 40 years (between 25 and 65 years old), then the first 10 years (25-35) would be the phase of trying out and getting stabilized, and the last 10 years is the adjusting period before retirement, and the middle 20 years is the most stable period in the sense of income and social status. Therefore I consider the household head to be young if she or he is under 35, middle aged if between 35 and 55, and old if older than 55. For education levels, a household is counted as higher educated if one has at least post-secondary non-tertiary education, or lower educated otherwise. At last, I use the EGP to label the job as of higher level if the index is less or equal to 8 (including high/low level service, routine non-manual,

¹⁸For a robustness check, I randomize the observations and make 5000 random draws of half of the total samples. Similar OLS regressions controlling for heteroskedasticity generate very small R-squares and adjusted R-squares in comparison to 0.39, confirming the non-randomness of the grouping strategy.

¹⁹Dividing levels of education into: Pre-Primary Education, Primary Education or First Stage of Basic Education, Lower Secondary or Secondary Stage of Basic Education, (Upper) Secondary Education, Post-Secondary Non-Tertiary Education, First Stage of Tertiary Education, and Second Stage of Tertiary Education.

²⁰Dividing occupations into: High Service, Low Service, Routine Non Manual, Self-Employed With Employees, Self-Employed No Employees, Manual Supervise, Skilled Manual, Semi - Unskilled Manual, Farm Labor, Self-Employed Farm, Unemployed, and Pensioner.

self-employed, manual supervision, and skilled manual jobs), otherwise it is considered as lower level.

3.3 An Important By-Product: the Evolution of Inequalities

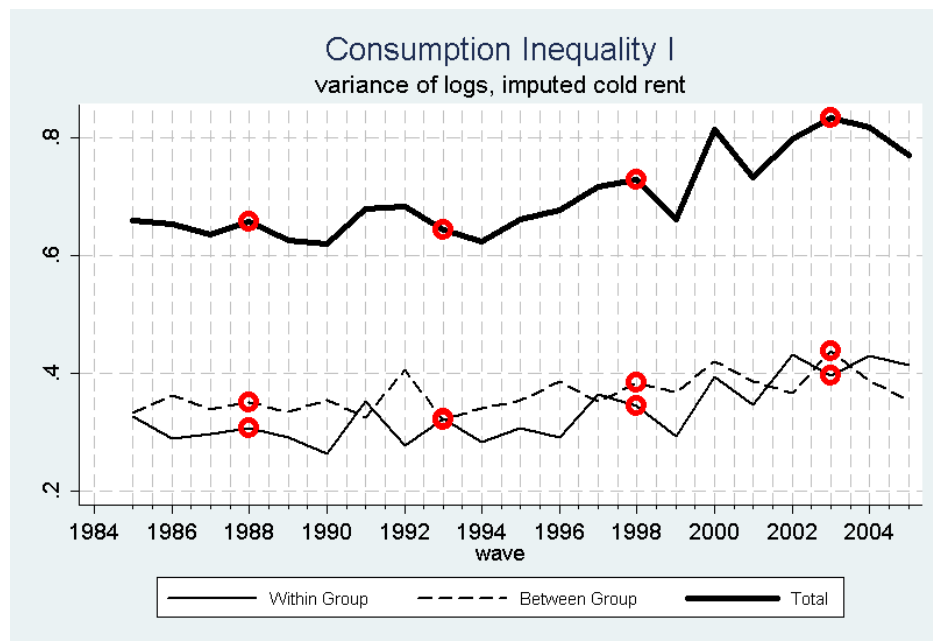


Figure 1: Consumption Inequality I

Figure 1 shows the evolution of total, between-group and within-group inequality. We can observe a notable increase in total inequality directly after the reunification in the year 1990. A closer look at the decomposed inequality (Figure 2) reveals that this rise was fueled by a 42.3% surge of within-group variance (emphasized by a shaded column). This may be a result of the influx of East German workers to West Germany. This is not a period when a trained doctor from the East could immediately get a job with the same payment as his West German colleague. Another time point of sharp increase in inequality comes in 2000 when the euro was introduced, and ascended further when Germany experienced a boom (2002/2003). High inflation followed the arrival of the new currency and joined the upturn of the economy, which may have distorted people’s usual consumption behavior. To be observed at first is an increase of within-group variance, which is more “nominal” and may result from differences in heterogeneous preferences and idiosyncratic shocks. Afterwards, the real economy may also

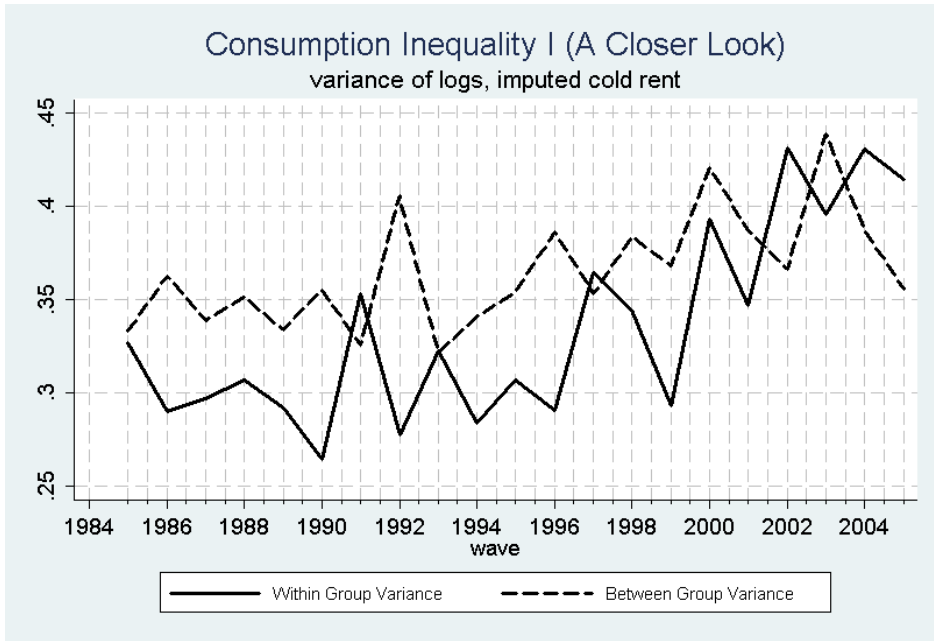


Figure 2: Consumption Inequality I (A Closer Look)

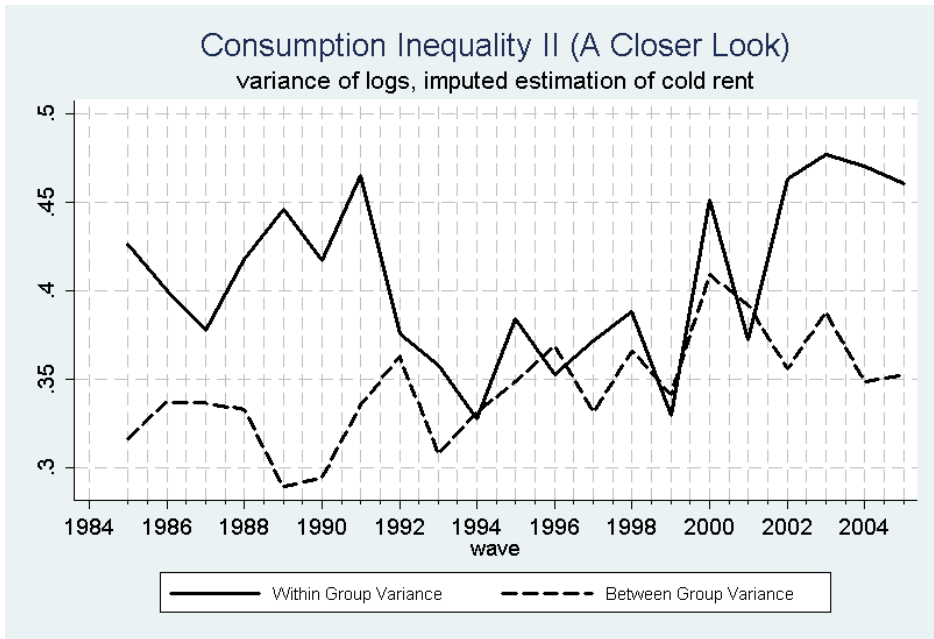


Figure 3: Consumption Inequality II (A Closer Look)

be affected and the structure of economic sectors and industries could potentially change, which may consequently raise the inequality between different groups.

Figure 3 shows a slightly different version of the evolution of inequality. Instead of imputed CR, the imputed estimation of cold rent is added to expenditure (and effectively to consumption). While the total and between-group variances do not change much, the within-group variance is higher in absolute value. Accordingly, in most of the years, within-group variance, which peaked in 1991 and 2003, overtakes between-group variance. This is a result from the overestimation of CR compared to the market value of the rent. The imputed expenses in consumption of house owners are thus higher, and so is the difference between them and the renters. Subsequently, consumption inequality within the same group increases, and the increase after the reunification is even more impressive.



Figure 4: Consumption Inequality II

Additionally, Figure 4 compares the inequality of income and consumption (calculated with the imputed CR) within groups and between groups. W_Income represents the within-group income inequality, W_Consumption the within-group consumption inequality, B_Income the between-group income inequality, and B_Consumption the between-group consumption

inequality. Similar to what is found by Krüger and Perri (2005) for U.S., between-group consumption inequality has tracked between-group income inequality more closely, while within-group consumption inequality has increased less than within-group income inequality.

4 Empirical Test for Correlation: Is Inequality the Price for Growth?

Although I try to include all legitimate explanatory variables, it may still happen that some important candidates are not available in the data set. An omitted-variable test suggests that the model does have omitted variables. Therefore the regression residual includes both the within-group variances and the error term $\varepsilon_{i,t}$ due to the lack of regressors.

$$g_{ij,t} = \kappa_i \Delta Z_{ij,t+1} + v_{ij,t} + \varepsilon_{i,t} \quad (9)$$

As the changes in group-specific characteristics are usually very limited across years, $\Delta Z_{ij,t+1}$ is close to zero. Averaging (9) over all j within each group i produces the group average growth rate, which is approximately equal to the measurement error $\varepsilon_{i,t}$.

As a result, a better estimation for the within-group variance is the variance of the demeaned growth rate

$$E(g_{ij,t} - g_{i,t})^2 = E(v_{ij,t}^2) = \sigma_v^2.$$

The regression equation is therefore,

$$g_{ij,t} = \alpha + \beta X_{ij,t} + \omega_{ij,t}$$

with X denoting the adjusted within-group variance and households' specific variables including household size, its square, household heads' gender, age, education level, job type, interaction term of age and education, interaction term of education and job type, as well as year dummies. Sample weights can be considered and the OLS regression is carried out for imputed consumption growth based on EVS and GSOEP separately.

The following table shows the OLS regression result of the group variance on average group growth and year dummies. Heteroskedasticity of the error terms is controlled for. At the 0.1% significance level, the regression coefficient of the group average growth rate is slightly positive (0.0124) when the EVS imputation is used, whereas if I use imputed

Regression of Within-Group Variance on Group Consumption Growth

VARIABLES	mean_growth (EVS imputation)	mean_growth (imputation from GSOEP)
within_variance (EVS imputation)	0.0124***	
within_variance (imputation from GSOEP)		0.0315***
household size	0.029***	0.026***
size square	-0.027**	-0.024**
head's gender	0.013*	0.014*
heads's age	0.006	0.007
Constant	0.38***	0.33***
	[0.00]	[0.00]
Observations	92391	90746
R-squared	0.109	0.127

Robust standard errors in parentheses

*** p<0.001, ** p<0.01, * p<0.05

Note: OLS regression. Heteroskedasticity-robust standard errors in brackets. All year dummies are highly significant but not reported.

estimation of CR from the GSOEP the coefficient increases to 0.0315 with a t-value of 13.47. This result points to a positive direction of the relationship, i.e., higher consumption growth is possibly accompanied by higher within-group consumption growth inequality. Admittedly this relation is relatively weak and one can not use it as a clear-cut evidence for the hypothesis posted at the equilibrium of theoretical model. It thus makes more sense to look into details of the groups.

The data shows the groups with highest consumption growth are young, small sized households with good education and skilled jobs (see later Figure 8), while the highest growth inequality appears in the group of young, small sized households with low education and unskilled jobs (see later Figure 17). It seems that young groups have at the meantime high growth and high equality. One possible reason can be that a large portion of the young population is still studying or under training, with very limited income. Besides, even among those who are working, people are subject to more shocks and changes at the beginning of their career, particularly due to different educational backgrounds and job types, which could also contribute to the high inequality within young population. A further deduction is, as they grow older, working experience can make up for the lack of education and the differences

in income and consumption growth between the group members should decrease.

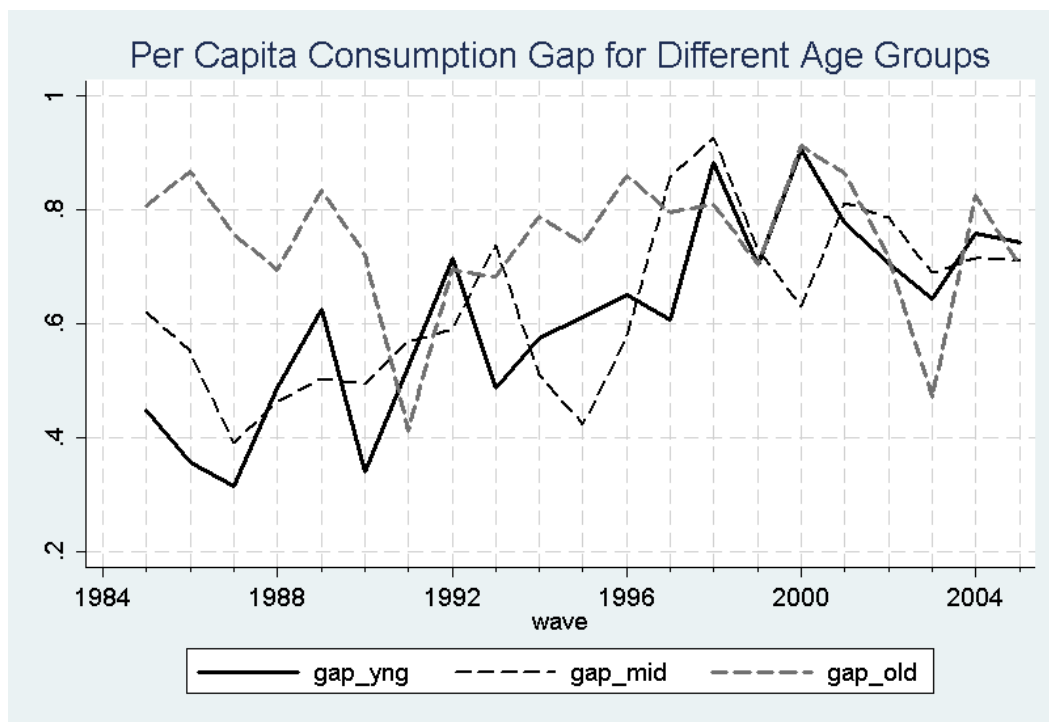


Figure 5: Per Capita Consumption Gap-Young v.s. Old

The data provides evidence in line with these arguments, which shows that the remarkable difference within each age group. Let's take a first look at the consumption gap, which is defined as the difference in per capita consumption of two totally different groups: large households with high education and skilled jobs versus small households with low education and unskilled jobs. Figure 5 plots the gaps for young-, middle- and old-aged households. One can see that the gaps are distinct for various age groups, with the largest gap almost always in the elderly households (except between 1997-1999). At the beginning of the sample period, the gap is relatively small for the young and big for the old population until the end 1980s. Within this starting period, the smallest gap occurred in 1987 for the young households, which is about 31.5%; the largest gap was among the old households and hits as high as 86.7 percentage points. In the first half of 1990s, the gap in old households keeps high, whereas between 1994-1996 the gap in the young surpasses that for the middle-aged. Since the end of 1990s the gaps have converged with an exceptionally low point for old in 2003, reaching 47.2%.

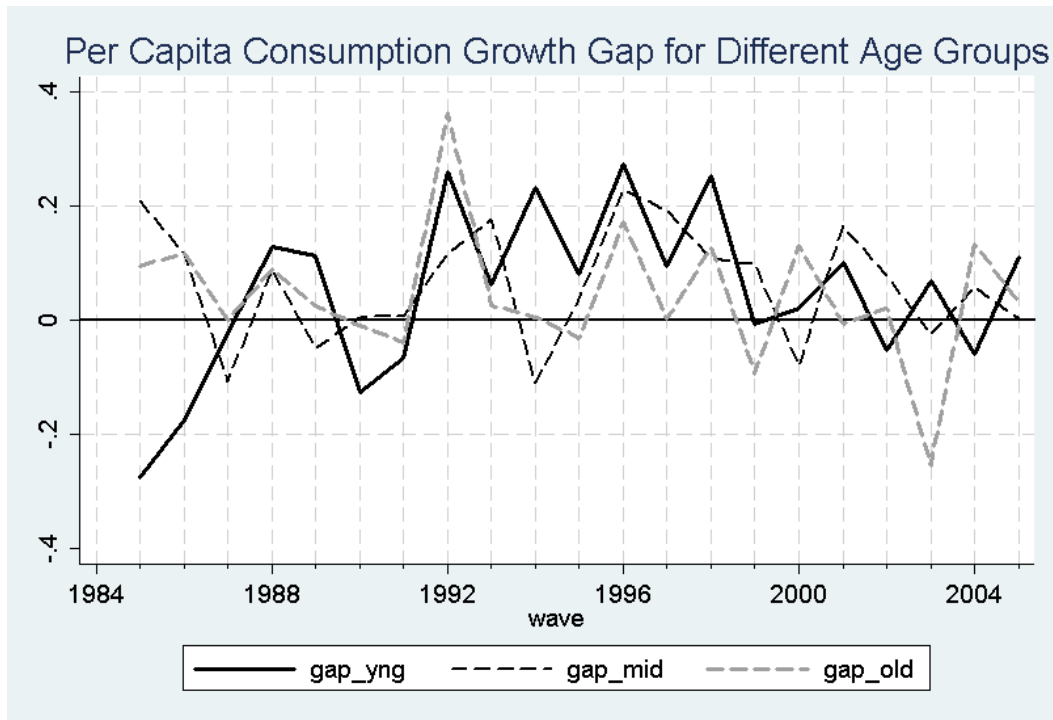


Figure 6: Per Capita Consumption Growth Gap-Young v.s. Old

The consumption growth also differs within different age groups, as is shown in Figure 6, with some similar trend between the young and old households from 1995 to 1999. Apart from the fact that the gaps in three age groups alternate in dominating the consumption growth during the whole sample period, it is also interesting to see that the gaps in consumption growth stay most time positive (a few exceptional negative points include: 1985-1987, 1990, 1991, 2002 and 2004 for gap in the young households; 1987, 1989, 1994, 2000 and 2003 for gap in the middle-aged households, 1991, 1995, 1999 and 2003 for gap in the old households). Recall in Figure 5 the absolute positiveness in the consumption gaps between these groups with different educational and occupational backgrounds. This result indicates that for each age group, the superior groups (groups with better education and jobs) not only have absolute advantage in consumption, but also undergo averagely higher consumption growth. As a result, we can observe in Figure 5 a slightly upward trend in the absolute consumption gaps.

The consumption gaps above capture the difference between the most and least advantaged groups, and the per capita consumption for each group are shown in Figure 7 and 8, where household head's age is controlled. It appears that average consumption in the house-

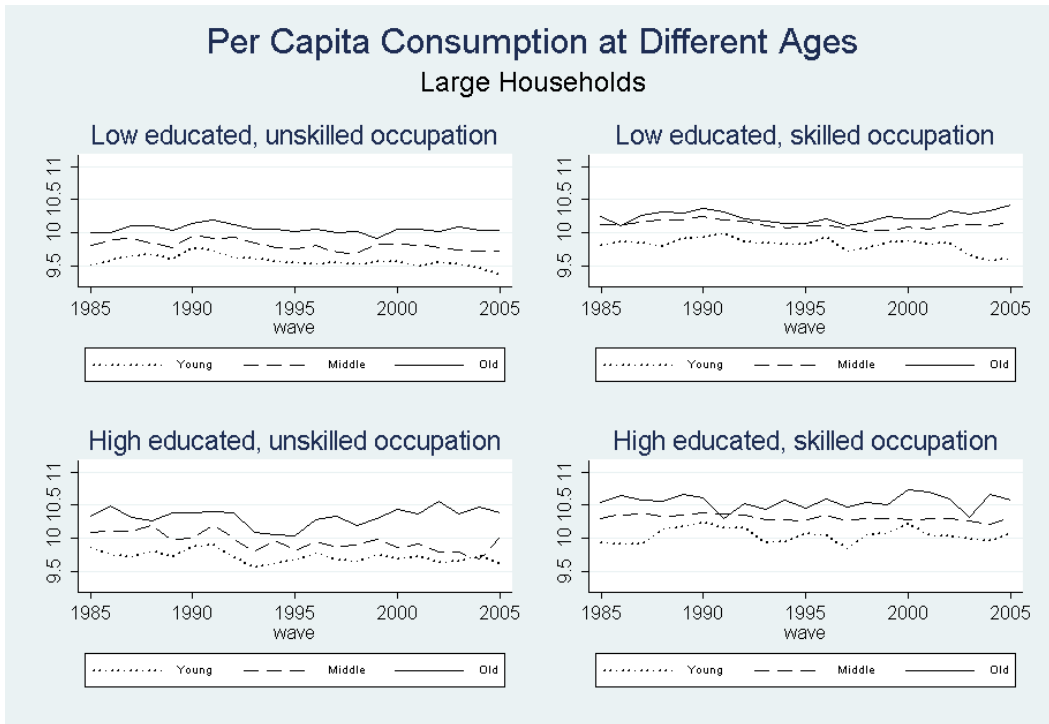


Figure 7: Per Capita Consumption-Large Households

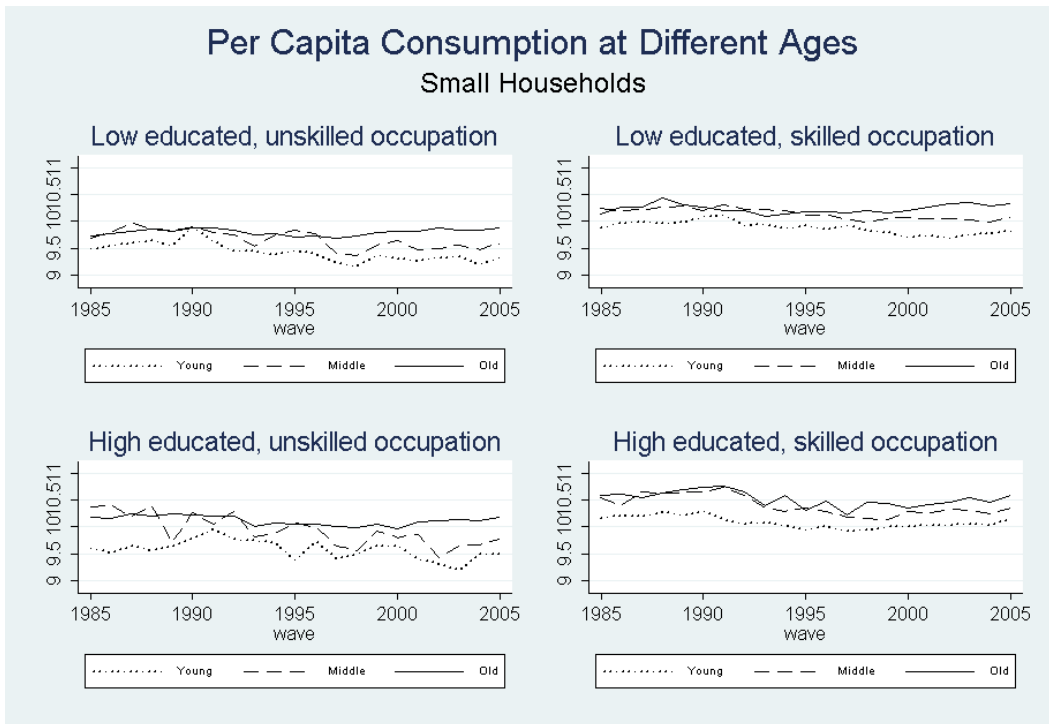


Figure 8: Per Capita Consumption-Small Households

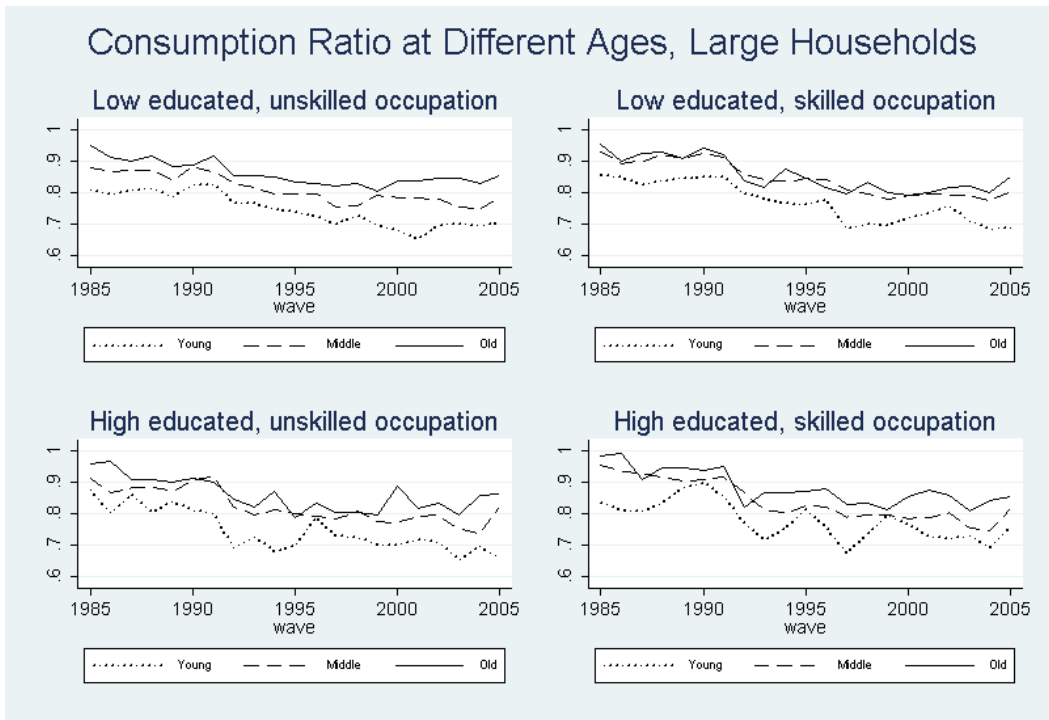


Figure 9: Consumption Ratio-Large Households

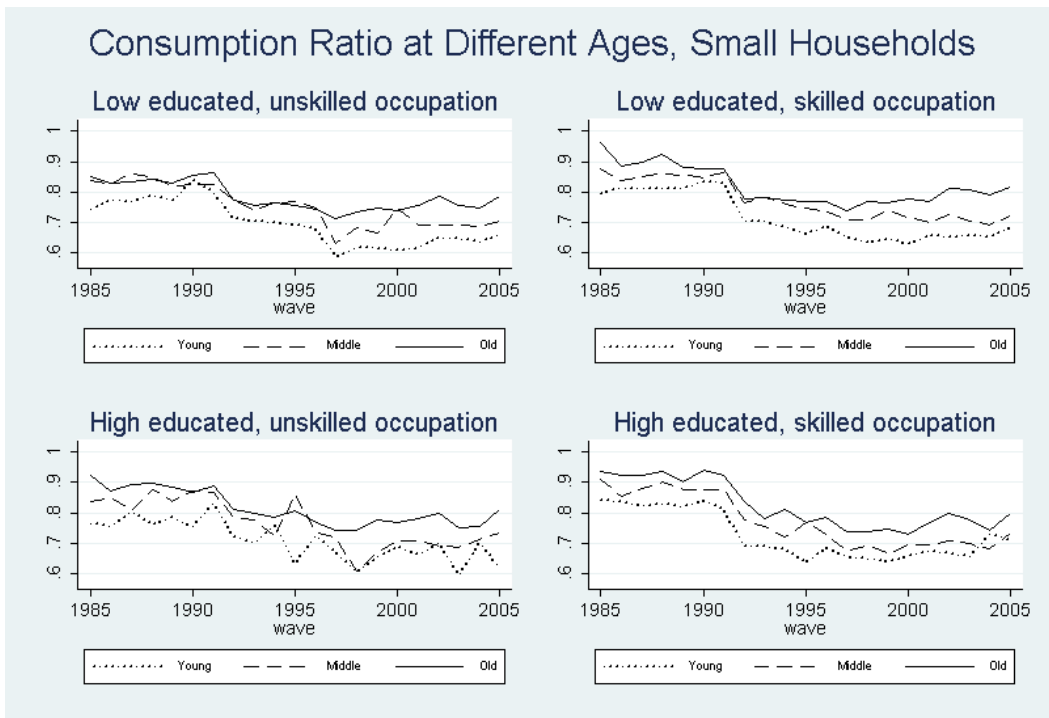


Figure 10: Consumption Ratio-Small Households

holds increases with household head's age, and per capita consumption of the old households almost always appears to be the highest, which is followed by middle-aged households. This ranking of consumption almost always holds, should households be large or small. Besides, old household consumption is also most stable over time, especially in the small household case.

Not only do households with elderly head have the highest absolute consumption, they also have the highest consumption ratio out of the total income (Figure 9 and 10). The ordering of the consumption ratio in different age groups is similar to that of absolute consumption, implying that the older the household head is, the higher consumption ratio the household has. Since we do not observe an obvious decline of household total income through aging, this high consumption ratio is more possibly attributed to the life-cycle effect, i.e. the elderly agents do not save so much as the younger ones for their future. Meanwhile, if the high consumption ratio corresponds to a high marginal propensity to consume, it may further imply that old households do not hold the highest level of wealth among all population.

Moreover, the consumption ratios experience a rather obvious downward sloping trend for each group. While such trend is not observed in the absolute per capita consumption in the previous Figure 7 and 8, it suggests a faster growth of total income compared to consumption and even an increasing living standard according to a broader interpretation of Engel's Law.

Groups with high consumption and high consumption ratio should also have high total income. This is shown in Figure 11, 12 and 13, which plot the net income, total income and consumption. Group 4 and 8, 12 and 16, 20 and 24 High educated and skilled households have the highest income and consumption levels in their respective age periods, and households in these groups share a common feature: they have high education and skilled jobs. We can also observe that the dotted lines are sometimes overtaken by the solid lines, showing that net income is sometimes not enough to cover the household consumption.

Do groups with high consumption level also have fast consumption growth? Or is it rather opposite that poor households experience high growth? Recall Figure ??, ?? and ?? in the introduction, which represent the young, middle-aged and old categories respectively and contain the group average consumption and income growth. There, only group 4, consisting of young, small households with high education and skilled job, has an obviously higher growth than others. Groups with particularly poor performance are 7, 15 and 23, which, different in

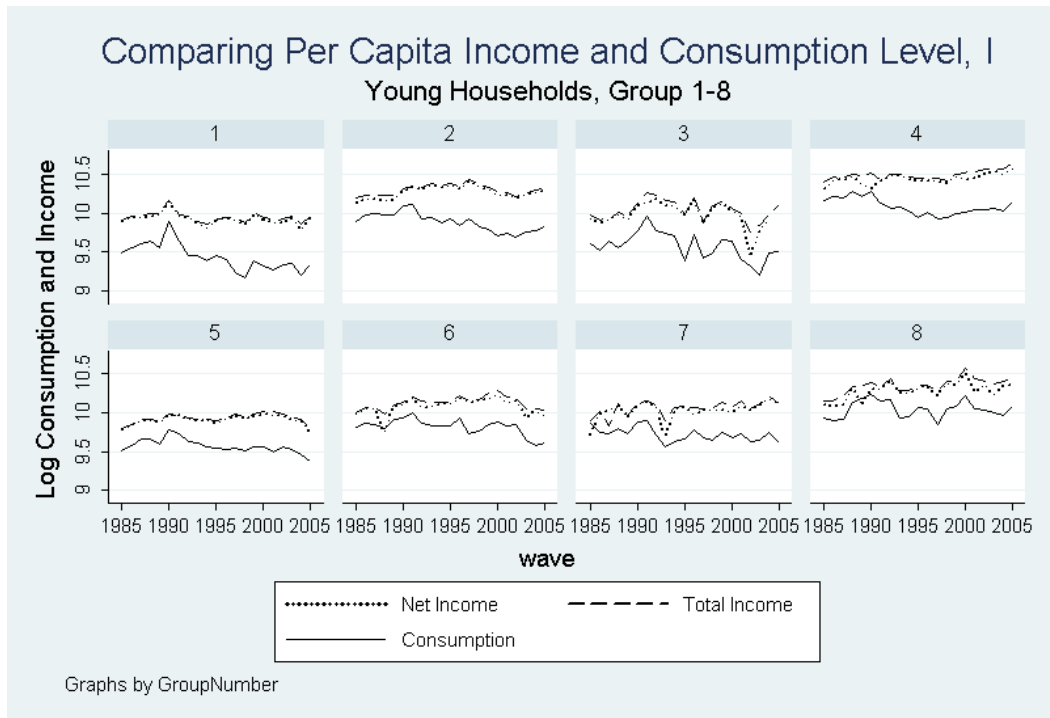


Figure 11: Per Capita Consumption and Income Level, Young

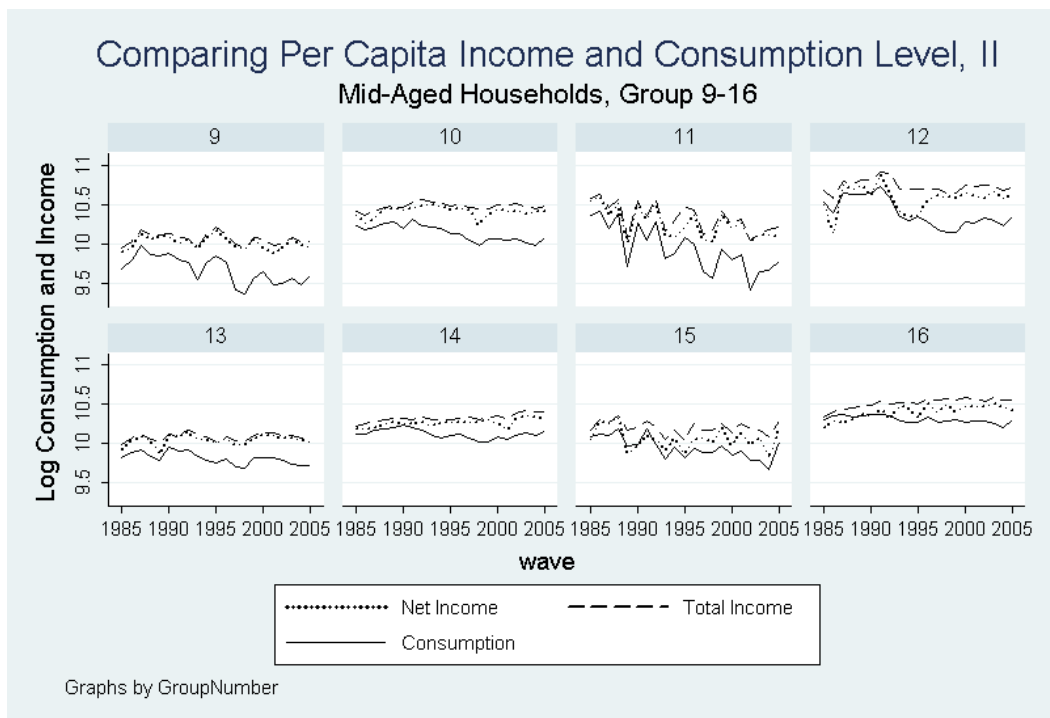


Figure 12: Per Capita Consumption and Income Level, Middle

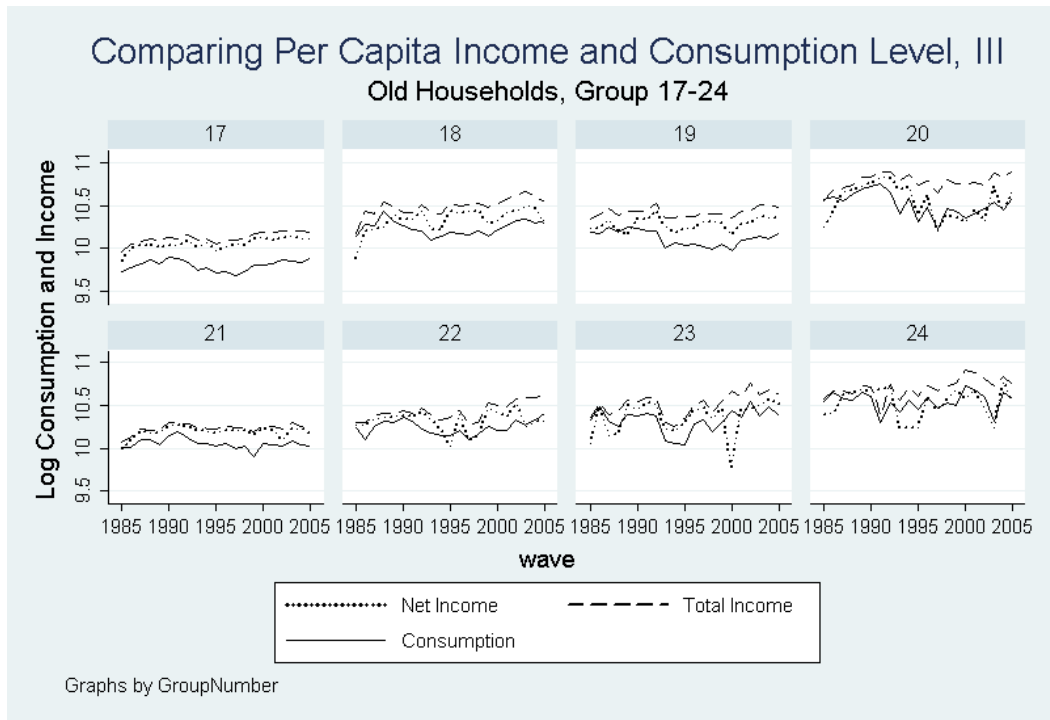


Figure 13: Per Capita Consumption and Income Level, Old

age, are large and well-educated households but with unskilled jobs. In fact most of the years these groups only have negative income and consumption growth, which could be a result of the unfortunate labor market mismatch of skills and jobs. Moreover, young households in Figure ?? appear to have higher fluctuation than older ones in income and consumption growth, regardless of other household features; middle-aged households in Figure ?? have the least volatile consumption growth, only with group 11 (small and well-educated but with unskilled jobs) as an exception.

Obviously the age of household head has very strong explanatory power for household consumption behavior. controlling for the household size, Figure 14 and 15 address the issue of age directly in plotting the per capita consumption growth for households in different ages. The figures show, unless they are small and have skilled jobs, young households do not necessarily have higher consumption growth than their older counterparts. As is mentioned before, for a well educated person, finishing study and finding a skilled job in the labor market change his/her income level and consequently consumption pattern dramatically, especially before the establishment of large family. Therefore we do not observe this consumption

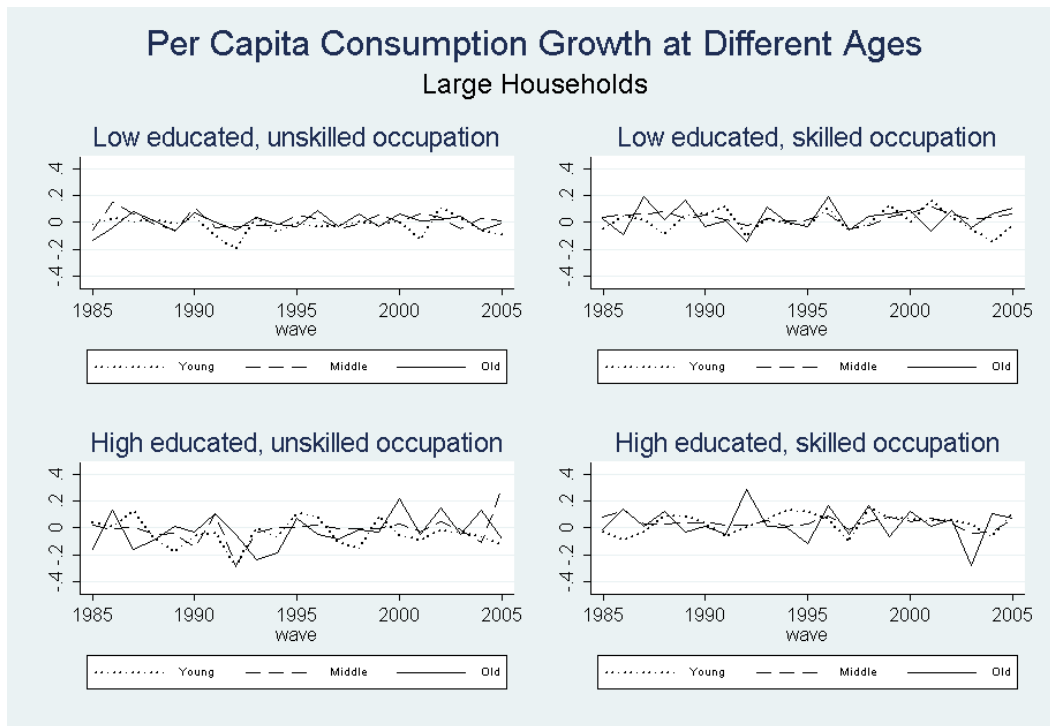


Figure 14: Per Capita Consumption Growth-Large Households

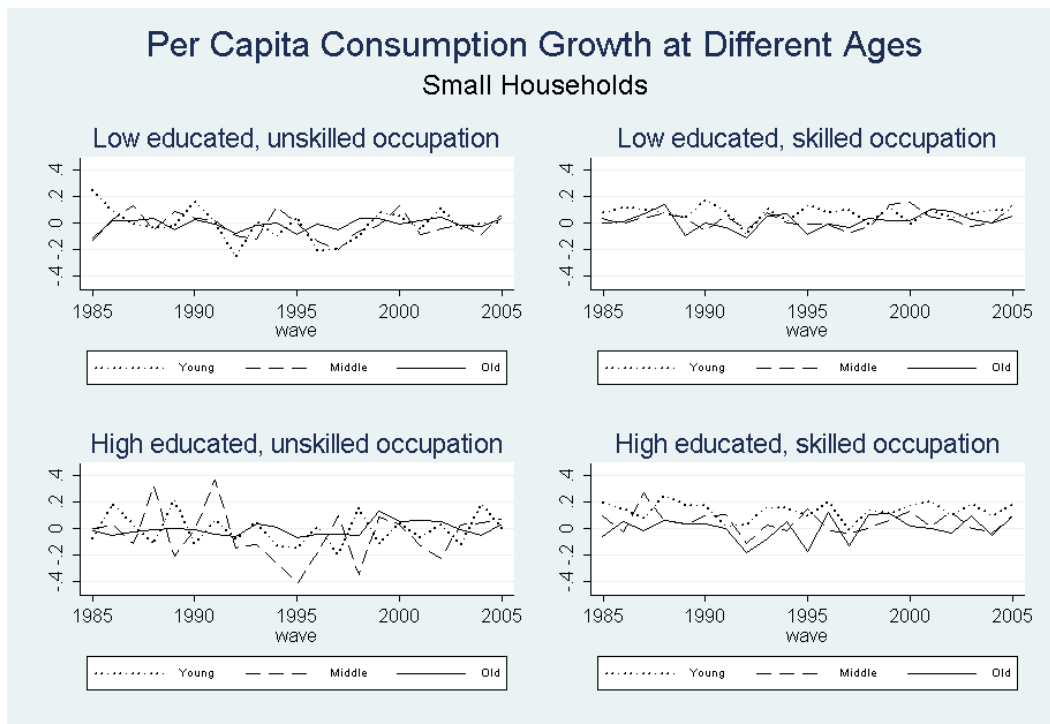


Figure 15: Per Capita Consumption Growth-Small Households

growth difference due to age in the groups of large households.

It is also confirmed that well-educated households with unskilled jobs have an inferior position, in the sense that their consumption growth are both irregular and often negative. In comparison, less educated households with unskilled jobs are even lower in consumption level (Figure 7 and 8); however, they have a rather smooth consumption growth, especially if they are large households.

Until now, the discussion has focused on the consumption level and growth. The intratemporal equilibrium of the model suggests a link between the consumption growth and consumption growth variance, with the latter shown in Figure 16 and 17. For the purpose of comparison, these two figures also look into large and small households respectively, where different age groups are shown for each combination of education and skills.

In the case of small households (Figure 17), consumption growth inequality is extremely and constantly high young households with low education and unskilled jobs (dotted line). It seems that experience for the unskilled (which grows with age) is important in stabilizing their consumption growth and bringing down the growth inequality. For the less educated households with skilled jobs, age does not show a noticeable influence on the inequality and we can observe a relative convergent pattern of the young, middle and old groups. In the large households (Figure 16) what is prominent is the almost constant inequality of low educated and unskilled workers, regardless of their age groups. Well-educated households with unskilled jobs are again at inferior position with relatively high variance in growth. In all, among large households (except low-educated with unskilled jobs), the old ones seem to have experienced high fluctuations in inequality in the past 20 years.

Another observation in Figure 18 is the effect of household size. For young and middle-aged groups, small households have a higher within-group inequality in consumption growth, but have a lower inequality in the old groups. It also becomes clear to draw a picture of the old households (four graphics in the last row): unless they have low education with unskilled jobs, old households have relatively high consumption growth inequality. The reasoning may relate to the capital income effect on consumption.

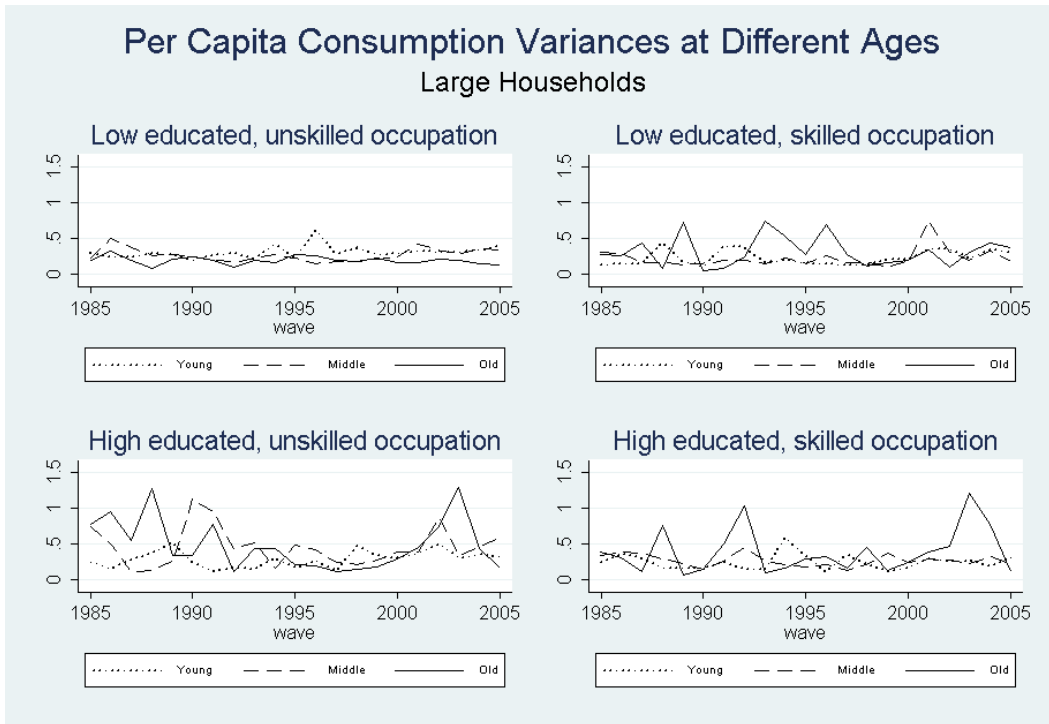


Figure 16: Variance of Per Capita Consumption Growth-Large Households

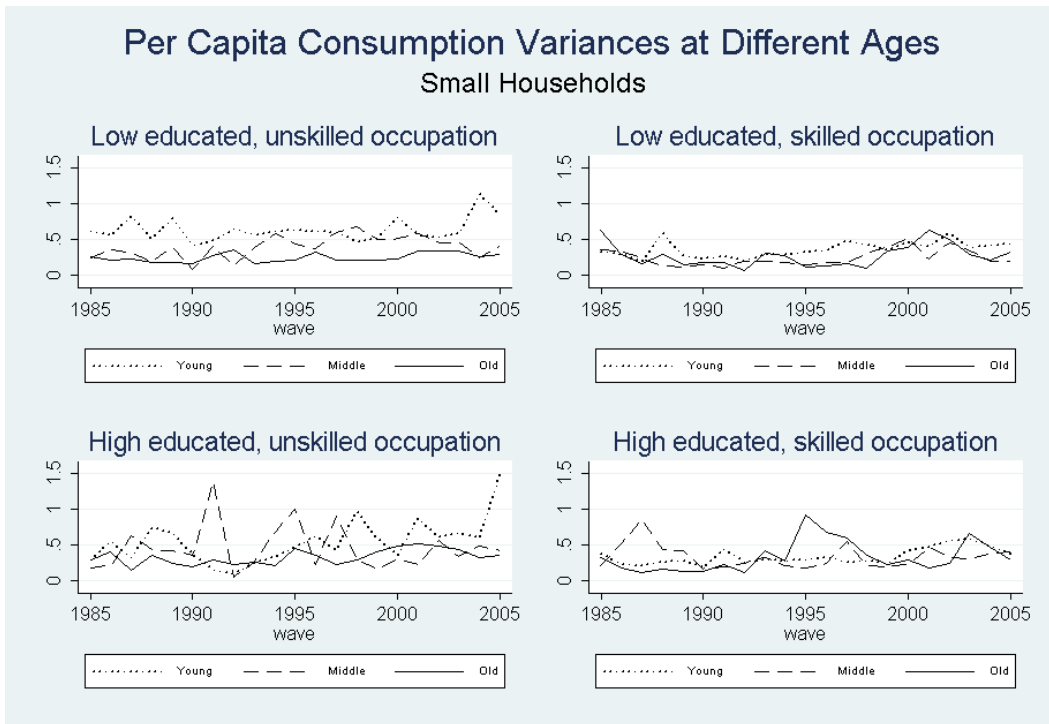


Figure 17: Variance of Per Capita Consumption Growth-Small Households

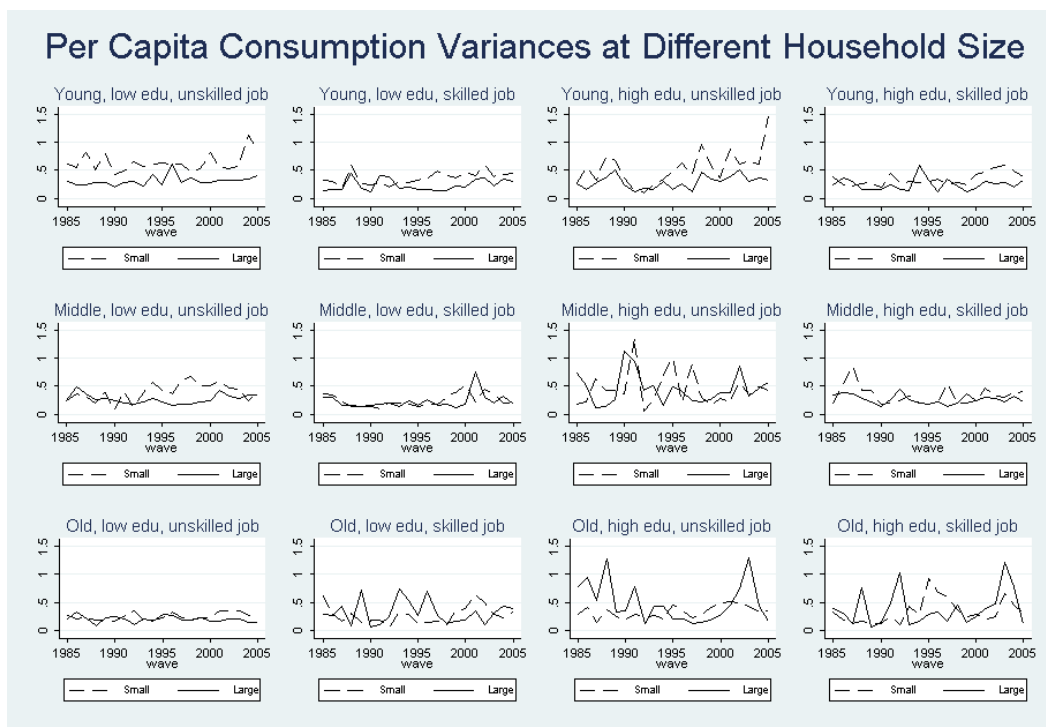


Figure 18: Variance of Per Capita Consumption Growth-Small v.s. Large

5 Summary

This paper offers an overview of consumption and consumption growth inequality in Germany between 1984-2005. A theoretical model borrowed from the asset pricing literature is used to examine the relationship between consumption growth and inequality.

Both complete market hypothesis and self insurance hypothesis are rejected in micro data. To get around this problem, the literature suggests settings of unenforceable contracts and private information. Alternatively, I adopt a simple extension from an incomplete market model, which, among other traits, can offer partial insurance against income shocks. I use it to examine the dynamics of consumption inequality. The basic idea is that even though households can not observe the income of other families with similar socio-economic status, they can observe the living standards and consumptions of others. How they evaluate others' consumptions affects the group consumption growth inequality. In particular, if they want to keep themselves in pace with their neighbors in consumption, their optimal security holding will change and consumption growth variance within the group becomes more than the

“altruism” case. The degree of (im)patience also plays a role for group variance, following the same argument of households’ decision on security holding against current consumption.

Concerning the time series properties of the group consumption growth rates, a test using the German Socio-Economic Panel suggests stationarity for the consumption growth whereas the deviation from the original consumption smoothing is even stronger. But however far the deviation is, the model would always predict a positive relationship between the group average growth rate and within-group variance given reasonable parameter values and under condition that households are not too keen in keeping up with group mean; i.e., the group with higher consumption growth should also observe higher inequality within the group. This theoretical result is tested using GSOEP survey data, where I divide the sample households into 24 groups according to household characteristics such as size, age, education, occupation of household head and so on. Two types of consumption are imputed using the limited information from GSOEP and the much more detailed EVS data separately.

The regression results shows a significant though only slightly positive relationship between group average consumption growth is and within-group consumption inequality. Furthermore, under my grouping strategy, age and household size are undoubtedly crucial for growth and growth variance. Since a large part of the young population is still out of the labor force and has limited income, consumption differences between them and young professionals are big. However, once they start working, the sudden relaxation of their financial constraint boosts up their consumption to such a degree that the consumption growth of the young groups is higher than the growth of the older groups.

In the sense of risk sharing, smaller households are more vulnerable to economic shocks. After economies of scale is ruled out and per capita consumption is examined, small households dominate the large ones, especially for young and middle-aged households. The data also shows that the otherwise relatively stable consumption growth inequality and within-group inequality in particular undergo increases immediately after the reunification of Germany and the introduction of the euro.

The study confirms the better income and consumption position of households with higher education and skilled jobs. Moreover, a rather surprising result is who is at the inferior position in the across group comparison. Taking consumption growth and growth variances as criteria, the group with the worst performance are not those with low education and

unskilled jobs, but rather the large and well-educated households who unfortunately possess unskilled jobs. It seems as if the mismatch in the labor market contributes to forming a twisted consumption pattern of the households.

These results offer an overall picture of consumption inequality in Germany in the last 20 years. Furthermore, business cycle effects are most likely also important for examining consumption inequality. Is within-group inequality generally procyclical, countercyclical or acyclical? Which groups are especially sensitive to booms and/or recessions? Concerning the theoretical model part, will the result be changed if mobility of households and between group comparison are allowed? These questions are left for future research.

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6 Appendix I: Taylor approximation of the group average Euler equation

According to second order Taylor approximation,

$$\ln G_{i,t+1} \approx \ln G_i + \frac{1}{G_i} (G_{i,t+1} - G_i) - \frac{1}{2G_i^2} (G_{i,t+1} - G_i)^2$$

Since $EG_{i,t+1} = G_i$, taking unconditional mean of both sides yields

$$E \ln G_{i,t+1} = \ln EG_{i,t+1} + \frac{1}{G_i} \underbrace{E (G_{i,t+1} - G_i)}_0 - \frac{1}{2G_i^2} E (G_{i,t+1} - G_i)^2.$$

Rearrange it, we have

$$\ln EG_{i,t+1} = E \ln G_{i,t+1} + \frac{1}{2} E \left[\left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right] \quad (10)$$

$$\text{or, } E[G_{i,t+1}] \approx \exp \left\{ E \ln G_{i,t+1} + \frac{1}{2} E \left[\left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right] \right\}.$$

Similarly according to Taylor approximation,

$$\begin{aligned} G_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} &\approx G_i^{(-\gamma-(1-\gamma)\kappa_i)} + (-\gamma - (1-\gamma)\kappa_i) G_i^{(-\gamma-(1-\gamma)\kappa_i-1)} (G_{i,t+1} - G_i) \\ &\quad + \frac{(-\gamma - (1-\gamma)\kappa_i)(-\gamma - (1-\gamma)\kappa_i - 1)}{2} G_i^{(-\gamma-(1-\gamma)\kappa_i-2)} (G_{i,t+1} - G_i)^2 \end{aligned}$$

and

$$\begin{aligned} EG_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} &\approx G_i^{(-\gamma-(1-\gamma)\kappa_i)} + (-\gamma - (1-\gamma)\kappa_i) G_i^{(-\gamma-(1-\gamma)\kappa_i-1)} \underbrace{E(G_{i,t+1} - G_i)}_0 \\ &\quad + \frac{(-\gamma - (1-\gamma)\kappa_i)(-\gamma - (1-\gamma)\kappa_i - 1)}{2} G_i^{(-\gamma-(1-\gamma)\kappa_i-2)} E(G_{i,t+1} - G_i)^2 \end{aligned}$$

$$EG_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} \approx G_i^{(-\gamma-(1-\gamma)\kappa_i)} \left[1 + \frac{(-\gamma - (1-\gamma)\kappa_i)(-\gamma - (1-\gamma)\kappa_i - 1)}{2} E \left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right]$$

Taking log of both sides yields

$$\begin{aligned} \ln EG_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} &\approx \ln G_i^{(-\gamma-(1-\gamma)\kappa_i)} \\ &\quad + \ln \left[1 + \frac{(-\gamma - (1-\gamma)\kappa_i)(-\gamma - (1-\gamma)\kappa_i - 1)}{2} E \left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right]. \end{aligned}$$

Since $E \left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2$ is very small,

$$\begin{aligned} &\ln \left[1 + \frac{(-\gamma - (1-\gamma)\kappa_i)(-\gamma - (1-\gamma)\kappa_i - 1)}{2} E \left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right] \\ &\approx \frac{(-\gamma - (1-\gamma)\kappa_i)(-\gamma - (1-\gamma)\kappa_i - 1)}{2} E \left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2. \end{aligned}$$

Using the result from (10),

$$\begin{aligned} \ln EG_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} &\approx (-\gamma - (1-\gamma)\kappa_i) \left[E \ln G_{i,t+1} + \frac{1}{2} E \left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right] \\ &\quad + \frac{(-\gamma - (1-\gamma)\kappa_i)(-\gamma - (1-\gamma)\kappa_i - 1)}{2} E \left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2 \\ &= (-\gamma - (1-\gamma)\kappa_i) E \ln G_{i,t+1} + \frac{(-\gamma - (1-\gamma)\kappa_i)^2}{2} E \left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2, \end{aligned}$$

where $E \ln G_{i,t+1} = E \ln (1 + g_{i,t+1}) \approx E g_{i,t+1} = g_i$, and

$$\begin{aligned}
& E \left[\left(\frac{G_{i,t+1} - G_i}{G_i} \right)^2 \right] \\
& \approx E \left[\ln \left(\frac{G_{i,t+1} - G_i}{G_i} + 1 \right) \right]^2 = E \left[\ln \left(\frac{G_{i,t+1}}{G_i} \right) \right]^2 = E [\ln G_{i,t+1} - \ln G_i]^2 \\
& = E [\ln (1 + g_{i,t+1}) - \ln (1 + g_i)]^2 \approx E [(g_{i,t+1} - g_i)^2] = \sigma_{g_i}^2.
\end{aligned}$$

Therefore

$$\begin{aligned}
\ln EG_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} &= (-\gamma - (1 - \gamma) \kappa_i) g_i + \frac{(-\gamma - (1 - \gamma) \kappa_i)^2}{2} \sigma_{g_i}^2 \\
\text{or } EG_{i,t+1}^{(-\gamma-(1-\gamma)\kappa_i)} &= \exp \left[(-\gamma - (1 - \gamma) \kappa_i) g_i + \frac{(-\gamma - (1 - \gamma) \kappa_i)^2}{2} \sigma_{g_i}^2 \right].
\end{aligned}$$