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The perception of the income tax: Evidence from Germany

Denis Beninger*

January 2010 !!PRELIMINARY!!

Abstract

The paper explores whether the tax rule defined by the tax law or the perception of the tax system by the single household matters in explaining household behaviour. In particular, Germany applies joint taxation with marital splitting to married couples. However, the couple has the choice between two formulas when paying the tax. In both cases, the *perceived* marginal tax rate, i.e. the rate which practically applies, and the *real* one, i.e. the one calculated from the tax function, differ systematically.

Using the 2004 wave of the German Socio-Economic Panel, we find that (i) the perceived marginal tax rate is a better indicator than the real one when explaining the couples' behaviour, (ii) there is a correlation between income pooling and the system of tax collection chosen by the couple, and (iii) the intra-household bargaining rule depends on the specificities of the tax system.

Key Words: Collective model, taxation, intra household allocations. **JEL Classification:** D11, D12, J22

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1 Introduction

Progressive income tax applies in most countries with the aim to achieve a certain vertical equity in the society. O'Donoghue/Sutherland (1998) show indeed that income tax reduces the spread of income distribution between families. In the literature, there are many papers on the effect of taxation on *inter*-household equity. Unfortunately, *intra*-household aspects are usually neglected, although income taxation affects also the time and money arrangements inside the families, beyond the sole impact on the household revenue. Indeed, if the individual incomes are not pooled within the family, as shown for example by Thomas (1990), Browning et al. (1994) and Lundberg/Pollack/Wales (1997), the redistribution of the resources inside the household depends on the income structure of the family. Since income taxation changes the marginal contribution of each individual to the family income, taxation affects the intra-household bargaining and may have very different impacts on the individual welfare of the family members (Beninger et al. 2006).

Only few studies are concerned with the impact of taxation on intrahousehold issues. Apps/Rees (1988) and Brett (1998) introduced the discussion in highlighting that these should be considered as well when evaluating policy measures. Beninger/Laisney/Beblo (2007), Vermeulen (2006) and Beninger (2009) use different discrete choice approaches in collective models to show e.g. that a move from joint to individual taxation favours women.

Donni (2003) extends the seminal paper by Chiappori (1988) to the case of progressive income taxation. To identify the model, i.e. in order to recover sharing rule and individual preferences, the budget set is linearized using "implicit prices". These are defined as the wages deflated by the marginal tax rate which applies at the optimum. Moreau/Donni (2002) use this method for an application on French data. However, they prespecify how the tax burden is shared between the spouses. In France, joint taxation with family splitting is applied to the family income. Thus, using the "implicit prices" as defined in Donni (2003) means that each spouse pays a part of the income tax which is proportional to his contribution to the household income. There is no reason that the couple does not choose a different rule as for the intra-household share of the tax burden though. In particular, this may depend on the own wish of equity inside the families and on their own perception of the tax system.

In order to test for this hypothesis, we use the specificities of the way in which the income tax is collected in Germany. Basically, joint taxation with marital splitting applies to German married couples. However, when paying the tax, the household can choose between two systems which lead to very different perceived marginal tax rates for the spouses. These perceived marginal tax rates are also different from the real marginal tax rate calculated from the tax function. We find that (i) the perceived marginal tax rate is a better indicator than the real one when explaining the couples' behaviour, (ii) there is a high correlation between income pooling and the system of tax collection chosen by the couple, and (iii) the intra-household bargaining rule depends on the specificities of the tax system.

The paper is organised as follows. The second section presents the implications of introducing progressive income tax into a collective model. This section includes an example to illustrate the theoretical framework. In the third section, we do the empirical application using the 2004 wave of the German Socio-Economic Panel (SOEP). The fourth section concludes.

2 Progressive income tax and collective model: Theory and illustration

When progressive income tax is introduced, the household no longer faces a linear budget as assumed in the simple collective model. The marginal price of leisure is thus no longer constant along the budget constraint since it depends on the labour supply choices of the family members. If the budget set is nonetheless convex, the budget constraint can be linearized at the optimum in using "implicit prices", i.e. the wages deflated by the marginal tax rate which applies at this point.

However, the sole implicit prices may not completely explain how income tax affects the intra-household distribution of resources. Indeed, in the collective framework, the bargaining process within the family depends on the structure of the income. In using the implicit prices, we implicitly specify the impact of income tax on the intra-household bargaining process. However, this effect may be a very different one, since the household may perceive the income tax in a different way than implicitly supposed by the model. In this section, we first introduce progressive income tax into the collective model. We then discuss how income tax affects the intra-household bargaining process. We give an example in order to illustrate the main points of the discussion.

2.1 Theory

We first introduce income tax into the general collective proposed by Browning/Chiappori (1998). We then concentrate on the model introduced by Chiappori (1988).

2.1.1 The general model

Here, we follow Browning/Chiappori (1998). Let us define the budget constraint as follows:

$$g\left(\mathbf{p},\mathbf{q},B\right) \le 0,\tag{1}$$

where \mathbf{q} is a vector of goods. \mathbf{p} represents the price of goods \mathbf{q} . B is the household disposable income. Function g is continuous, continuously differentiable in each argument and represents a convex budget bundle. Function g is supposed to be completely specified. This assumption is essential in deriving testable restrictions on demand functions. Vector \mathbf{q} may include leisure. The corresponding price is then the gross wage rate. \mathbf{q} is composed of private goods \mathbf{q}_i , and public goods \mathbf{Q} typically not assignable:

$$\mathbf{q} = \mathbf{q}_f + \mathbf{q}_m + \mathbf{Q},\tag{2}$$

where f (resp. m) is the subscript for the wife (resp. the husband) in a two-person household composed of a married couple. The spouses are supposed to know each other perfectly. They are thus assumed to reach Pareto-efficient outcomes. The family optimizes:

$$\max_{\mathbf{q}} U^{f}(\mathbf{q})$$
(3)
s.t.
$$\begin{cases} U^{m}(\mathbf{q}) \geq \overline{U}^{m} \\ g(\mathbf{p}, \mathbf{q}, B) \leq 0, \end{cases}$$

or equivalently:

$$\max_{\mathbf{q}} U^{f}(\mathbf{q}) + \lambda(\mathbf{p}, \mathbf{q}, B) U^{m}(\mathbf{q})$$
s.t. $g(\mathbf{p}, \mathbf{q}, B) \leq 0,$
(4)

where $(U^i)_{i=f,m}$ are the individual utility functions. The utility set is supposed to be convex. In this first step, the preferences are very general; in particular we allow for altruism between the spouses. \overline{U}^m is the husband threat point in the intra-family negotiation process. \overline{U}^m is a measure for the allocation of the resources inside the household. λ (.) is the male Paretoweight. It represents his power in the intra-household negotiation process, as \overline{U}^m . We can show (see Appendix) that there is a matrix *PS*, defined as the sum of a symmetric and negative semi definite matrix *S* plus a matrix of rank at most one \mathbf{uv}' . The rank of matrix \mathbf{uv}' is zero if the household pools its income, i.e. if the collective model collapses into a unitary framework. Matrix *PS* depends on the implicit prices $\widetilde{\mathbf{p}}$ and the income *B*, where $\widetilde{\mathbf{p}}$ is defined as

$$\widetilde{\mathbf{p}} = \left. \frac{\partial g}{\partial \mathbf{q}} \right|_{\mathbf{q}^*},\tag{5}$$

i.e. $\tilde{\mathbf{p}}$ the marginal price of goods \mathbf{q} at the optimum. This property is similar to the SR1 condition in Browning/Chiappori (1998). This means, a tax reform has an impact

1. on the household indirect utility. The Pareto-frontier moves through effects due to variation in the implicit prices $\tilde{\mathbf{p}}$ and in the disposable income B, keeping λ constant,

2. on income distribution within the household. μ also varies, since λ depends on $\tilde{\mathbf{p}}$ and B. There is an additional move along the (new) Pareto-frontier.

Similarly to the linear case, the SR1 condition can be tested if there are at least five commodities in a parametric specification (Browning/Chiappori 1998) or three goods in a non-parametric context (Cherchye/De Rock/Vermeulen 2007). It can be straightforwardly shown that all other extensions (more than two person households, model with distribution factors) proposed by Browning/Chiappori (1998) can be extended to the case of a convex budget set.

2.1.2 Model à la Chiappori (1988)

Many data sets do not provide enough information on commodities to test property SR1. In particular, in cross section data, there is not sufficient price variation. Only the Hicksian aggregate, which is typically not assignable, is included. Thus, without further restrictions on the assumptions of the model, the collective framework does not have any implication for price responses. Chiappori (1988) shows, in the linear case, that with additional assumptions on preferences (the individuals are supposed to be egoistic or caring), testable restrictions on the preferences can be derived as soon as observations on three goods are available. The preferences and the sharing rule can then be identified up to an additive constant. This section extends Chiappori's results to the collective model with convex budget constraint.

In this case, the family optimizes:

$$\max_{c_f, c_m, l_f, l_m} U^f(c_f, l_f)$$

$$s.t. \begin{cases} U^m(c_m, l_m) \ge \overline{U}_m \\ c_f + c_m \le g(l_f, l_m, w_f, w_m, y), \end{cases}$$
(6)

where c_i , l_i and w_i are the consumption level, the leisure demand and the gross wage of spouse *i*. *y* is the household unearned income. It can be shown that the following restrictions have to be fulfilled:

1. $\alpha_{\tilde{y}}A + aA_{\tilde{y}} - \alpha_{\tilde{w}_m} = 0,$ 2. $\beta_{\tilde{y}}B + \beta B_{\tilde{y}} - \beta_{\tilde{w}_f} = 0,$ 3. $\alpha A \left(\mathcal{L}^f_{\tilde{w}_f} \alpha B + \mathcal{L}^f_{\tilde{w}_m} \left(\beta B - T + \mathcal{L}_f \right) \right) \leq 0,$ 4. $\beta B \left(\mathcal{L}^m_{\tilde{w}_m} \beta A + \mathcal{L}^m_{\tilde{w}_f} \left(\alpha A - T + \mathcal{L}_m \right) \right) \leq 0,$ where $\alpha = \left(1 - \frac{BA_{\tilde{y}} - A_{\tilde{w}_f}}{AB_{\tilde{y}} - B_{\tilde{w}_m}}\right)^{-1}$ and $\beta = \left(1 - \frac{AB_{\tilde{y}} - B_{\tilde{w}_m}}{BA_{\tilde{y}} - A_{\tilde{w}_f}}\right)^{-1}$. $A = \frac{h_{\tilde{w}_m}^f}{h_{\tilde{y}}^f}$ and $B = \frac{h_{\tilde{w}_f}^m}{h_{\tilde{y}}^m}$. $h^i = h^i \left(\tilde{w}_i, \rho\left(\tilde{w}_f, \tilde{w}_m, \tilde{y}\right)\right)$ represent the labour supply

function, and $\rho(\tilde{w}_f, \tilde{w}_m, \tilde{y})$ is the intra-household income sharing rule. \tilde{w}^f and \tilde{w}^m are the implicit wages:

$$\widetilde{w}_{f}(w_{f}, w_{m}, y) = g_{l_{m}}(l_{f}^{*}, l_{m}^{*}, w_{f}, w_{m}, y),$$

$$\widetilde{w}_{m}(w_{f}, w_{m}, y) = g_{l_{f}}(l_{f}^{*}, l_{m}^{*}, w_{f}, w_{m}, y),$$
(7)

where l_f^* and l_m^* are the leisure demand functions at the optimum, and $g_{l_i} = \frac{\delta g}{\delta l_i}$. The implicit uncarned income \tilde{y} at the optimum is then

$$\tilde{y}(w_f, w_m, y) = g(l_f^*, l_m^*, w_f, w_m, y) - \sum_{i=f,m} l_i^*.\tilde{w}_i.$$
 (8)

The original proof is given by Chiappori (1988). Donni (2003) extends to the case of a convex budget constraint.

The following discussion essentially outlines the potential effects of income taxation on the allocation process. Indeed the redistribution of resources inside the family may differ substantially when income tax is introduced. The intuition is that the negotiation of the income sharing rule depends on the characteristics of the tax system. A reason is that the income tax may have diverging impacts on the welfare of both spouses.

2.2 Impact of the income tax on the intra-household allocations

2.2.1 Discussion

The most interesting aspect of the collective model is that it allows to consider the income redistribution within the household. The bargaining power of the spouses is indeed revenue dependent. Thus a change in the budget constraint, due for example to the introduction of income taxation, affects the household behaviour through both a pure income and a redistribution effect.

However, introducing a progressive taxation implies that the marginal wages is varying along the budget constraint. In particular the individual budget sets are potentially affected by the spouse's decisions, implying that the income sharing rule may be renegotiated. This means that the income tax has a double effect on the redistribution of the resources within the family: (i) there is a *price effect* due to the variation of the implicit wages and of the unearned income; (ii) a *taxation effect* applies when the spouses renegotiate the way that they share the tax. In this case the income sharing rule is not *tax neutral*.

Define by $\Psi(\tilde{w}_f, \tilde{w}_m, \tilde{y}, \pi)$ the tax sharing rule, i.e. the rule defining how the spouses share the tax burden. π is a vector of characteristics of the tax system. Ψ has the usual properties. The tax sharing may have a different rule that the income sharing rule ϱ . Define by $\Theta(\tilde{w}_f, \tilde{w}_m, \tilde{y}, \pi)$ the product of the income and tax sharing rules:

$$\Theta\left(\tilde{w}_{f}, \tilde{w}_{m}, \tilde{y}, \boldsymbol{\pi}\right) = \varrho\left(\tilde{w}_{f}, \tilde{w}_{m}, \tilde{y}\right) \times \Psi\left(\tilde{w}_{f}, \tilde{w}_{m}, \tilde{y}, \boldsymbol{\pi}\right).$$
(9)

 Θ can be defined as the generalized income sharing rule. Θ is not observable, but can be recovered up to an additive constant by the usual techniques. Note that Donni (2003) stipulates that the income sharing rule may depend on π : $\varrho = \varrho (\tilde{\omega}_f, \tilde{\omega}_m, \tilde{y}, \pi)$, but does not distinguish between income and tax sharing. However introducing or reforming taxation may have an impact not only on the argument of the sharing rule, but on the rule itself. Donni (2003) constrains the parameters of the income sharing rule to remain unchanged by a tax reform. Our model extends in this way the one proposed by Donni (2003). From the optimality condition, we have following restrictions:

$$\frac{\partial \Theta}{\partial \pi} = \frac{h_{\pi}^f}{h_{\tilde{w}_m}^f} \Theta_{\tilde{w}_m} = \frac{h_{\pi}^m}{h_{\tilde{w}_f}^m} \Theta_{\tilde{w}_f}.$$
 (10)

The tax neutrality is ensured by the restrictions:

$$\frac{\partial \Theta}{\partial \pi} = 0$$
(11)
$$\frac{\partial \Theta}{\partial \Omega'} = \frac{\partial \varrho}{\partial \Omega'},$$

where $\Omega = (\tilde{w}_f, \tilde{w}_m, \tilde{y})'$. These can be tested in using the variability in the tax parameter.

The remaining part of this section illustrates the potential effects of income taxation on the budget constraint and on the intra-household allocations, through an example assuming joint taxation of the spouses' incomes.

2.2.2 Illustration

In case of joint taxation, the tax is applied on the pooled household income. The tax law possibly distinguishes between different types of incomes (earned income, benefits, transfers, revenue from capital, etc.), but the tax liability does not depend on who is the recipient of the income. Implicitly, both spouses have the same marginal tax rate. The budget constraint of the couple is:

$$g(l_f, l_m, w_f, w_m, y) = g^J(w_f(T - l_f) + w_m(T - l_m) + y),$$

where g^J represents the budget set of the household when joint or *common* taxation applies. g^J is a positive, continuous, twice continuously differentiable increasing and convex function: $\frac{\partial g^J}{\partial X} > 0$ and $\frac{\partial^2 g^J}{\partial X^2} < 0$, where $X = w_f (T - l_f) + w_m (T - l_m) + y$.

Example 1 For illustrative purpose, we consider that the household has a fixed income y. The spouses do not work. The family utility function is: , where a_f and a_m are of the wife and the husband respectively):

$$U(c_f, c_m) = \lambda a_f \ln c_f + (1 - \lambda) a_m \ln c_m,$$

where $a_i \in [0, 1[$ is the propensity for consumption of spouse *i*. λ represents the female bargaining power. Note that λ (~ the income sharing rule) may be revenue dependent: $\lambda = \lambda(y)$. The budget constraint is:

$$c_f + c_m \le y,$$

The first order conditions are:

(1)
$$\lambda a_f c_f^{-1} - \mu = 0,$$

(2) $(1 - \lambda) a_m c_m^{-1} - \mu = 0$
(3) $c_f + c_m = y,$

where μ is the Kuhn-Tucker coefficient associated to the budget constraint. So the consumption depends positively of the income. From conditions (1) and (2) above we obtain:

$$c_f = \frac{\lambda}{1-\lambda}c_m$$
 if $a_f = a_m = a_f$

Substituting in condition (3), the optimal spouses' consumption $(c_i^*)_{i=f,m}$ are:

$$c_f^* = \lambda y$$
 and $c_m^* = (1 - \lambda) y$.

Consider now that joint taxation with marital splitting applies. The budget constraint is:

$$c_f + c_m \le g^c(y) = 2\left(\frac{y}{2}\right)^{\alpha} = y_J.$$

The spousal optimal consumption $(c_{iJ}^*)_{i=f,m}$ in this case are: $c_{fJ}^* = \lambda_J y_J$ and $c_{mJ}^* = (1 - \lambda_J) y_J$, where λ_J is the new value of the female bargaining power resulting from the impact of the vitation in the household revenue on the internal distribution of resources.

Define by g_i the net income of a single individual:

$$g_i\left(y\right) = y_i^\alpha,$$

where y_i represents the individual unearned income. Suppose that, in case of divorce, each spouse receives the part of the gross unearned income y corresponding to the sharing rule, i.e. the consumptions $(c_{iD}^*)_{i=f,m}$ and the household disposable income y_D are:

$$c_{fD}^* = (\lambda y)^{\alpha} \quad and \quad c_{mD}^* = ((1-\lambda)y)^{\alpha},$$

$$y_D = (\lambda y)^{\alpha} + ((1-\lambda)y)^{\alpha}.$$

Therefore, if the reallocation of resources is not too sensitive to the level of income y ($\lambda_J \simeq \lambda$), the poorest spouse (the one with the lowest relative weight, in this particular case) is penalized by the joint taxation with regard of 'divorced'. Note that since g is a convex function, we have: $y_J > y_D$. Define by 'joint no threat' the following situation: the richest spouse transfers to his poorest partner a part of his income, so that the poorest's revenue is equal to that he would obtain if he divorced. The budget constraint of the household remains unchanged: $\tilde{y}_T = \tilde{y}_J$. The outcomes for 'joint' and 'joint no threat' draw therefore the same Pareto-frontier. In transferring resources within the household, the optimal solutions moves on the Paretofrontier. Define by $(c_{iT}^*)_{i=f,m}$ the resulting optimal consumption. It follows:

 $c_{iT}^* = c_{iJ}^* - (c_{jD}^* - c_{jJ}^*)$ if *i* is the spouse with the highest relative weight, $c_{iT}^* = c_{iD}^*$ otherwise.

Therefore, we have

 $c_{iJ}^* > c_{iT}^* > c_{iD}^*$ if *i* is the spouse with the highest relative weight, $c_{iJ}^* < c_{iT}^* = c_{iD}^*$ otherwise.

The above inequalities result from the convexity of the tax function.

In the linear case, the spouses share the unearned income following a negotiated rule. The impact of a external shock on the budget constraint is twofold. Firstly, there is an income effect due to the variation of the income: the Pareto-frontier moves. Secondly, there is a redistribution effect due to the change of the bargaining position; there is a move on the Pareto-frontier.

The Example illustrates which is the impact on the household behaviour of the introduction of progressive taxation. An interesting result highlighted by Figure 1 is:¹ the poorest spouse has to support a part of the tax mainly due to his richer partner. This means that he would be better off as a single person as he would indeed have a lower marginal tax rate. Thus, the weakest spouse will claim to have a higher share of the unearned income, otherwise he divorces. The threat is believable: the welfare of his partner would be lower again in case of divorce. In other words, marriage dominates divorce,²

¹female's bargaining power $\mu = 1/(1 - \lambda)$; for the illustration, we set: a = .6, $\alpha = .5$. λ is not revenue dependent.

 $^{^{2}}$ McElroy and Horney (1981) assume that the intra-household allocation is determined by a Nash-solution of a cooperative game where the threat points are the utility level obtained in case of divorce. This section shows that assuming only Pareto-efficiency of the intra-household decisions is sufficient to obtain for each spouse a utility level at least equals to what he would reach as a single individual.



Figure 1: Difference in male's utility by female's bargaining power

This graph illustrates the Example. The reference is 'divorced'.



Figure 2: Difference in household's utility by female's bargaining power

This graph illustrates the Example. The reference is 'divorced'.



Figure 3: Household's utility by female's bargaining power

This graph illustrates the Example.

i.e. there is an Pareto-improving allocation, such that, in redistributing the resources within the household, both spouses in a married couple are better off than two single individuals with identical characteristics. Therefore, to save the marriage, from which both spouses gain, the richest spouse has to accept to make transfers to his partner. Thus introducing taxation may lead to redefine the intra-household sharing rule, beyond the impact that the variation of the income has on the redistribution of the family income. In case of a convex budget set, a third effect might to be introduced: the *taxation effect*.

Globally, the household gains to keep married, as depicted in Figure 2. Interesting is the difference in the household utility between 'joint' and 'joint no threat'. In these two situations the household faces the same budget constraint. After transfers from the rich to the poor spouse, the new allocation remains Pareto-optimal, even if it does not correspond to the social optimum of the household, given the female's bargaining power. The loss in welfare of the richest spouse is not completely compensated by the welfare gain of his partner. This loss in household welfare is the vertical distance (i.e. for a given female's bargaining power) between the two utility curves in Figure 3. The horizontal difference gives the variation in female's bargaining power so that, given the allocation, the household stays on the same Pareto-frontier.

In a more general pattern, people work and may have unearned income. Assume that joint splitting taxation applies and that the individuals are strictly egoistic. A rich person ideally marries a poor one, in order to minimize the tax liability. However the poor partner negotiates a more important share of the income to compensate the higher marginal tax rate. The richer partner accepts: in case of divorce, his welfare would be lower. In empirical applications, one can guess that the convexity has a positive impact on the female bargaining power: women are often the worse off inside the household. Since the threat to divorce is believable, they might obtain a higher share of the income of the family. However that does not mean that the intra-household discrepancies between spouses is reduced: the reallocation of resources in favour of the women is a compensation for the high tax rate, mainly due to the wealth of their husband.

3 The empirical study

In this section, we aim to evaluate if the way how income tax is collected influences the perception of the tax system by the single households. For this purpose, we estimate on the 2004 wave of the German Socio-Economic Panel (SOEP) a collective model of household labour supply for double-earner couples using (i) the real marginal tax rate computed from the German income tax function, and (ii) the perceived marginal tax rates for both spouses as they are defined when paying the income tax. Using these estimates, we then calculate the imputed labour supplies and we test by best fit which of the real or perceived marginal tax rates best explains the couples' behaviour. In the following, we first describe the German tax system. We then present the estimation model. In the following subsection, we present the econometric specification of the model we estimate. The data is described in the fourth sub-section. Finally, we give the estimation and the tests results.

3.1 The German tax system

Germany has a personal income tax system administered at the federal level and regulated by the Personal Income Law ("Einkommensteuergesetz"). The German tax system is characterized by a comprehensive tax which covers labour earnings as well as income from other sources such as capital, investment etc. and by joint taxation for married couples. The function applied to the tax base is progressive, but in contrast with the systems of most other countries, the tax function is smooth and not piece-wise linear (see Table 1). Gross taxable income forms the base from which all further allowances are deducted. In particular the tax relief for each working spouse is 920 euros. In 2004 the top rate applied was 45 percent for yearly earnings in excess of 52,152 euros. Earnings below the basic personal allowance of 7,664 euros are tax free. Germans pay an additional 5.5 percent tax termed "solidarity supplement for the reconstruction of East Germany" ("Solidaritätszuschlag"). This tax is based on the individual amount of income tax using a specific taxable income measure which, among other items, includes

Table 1: 2004 German income tax function

Income (X) bracket	Income tax liability
0-7,664	0
7,665-12,739	$(793.10 \cdot Y + 1,600) \cdot Y$
12,740-52,151	$(265.78 \cdot Z + 2,405) \cdot Z + 1,016$
>52,152	$0.45 \cdot X - 8,845,$

where X=rounded taxable income, Y=(X-7,664)/10,000, Z=(X-12,739/10,000)

the "Kinderfreibetrag", whether the parents opt for this allowance or the perception of children benefit (154 euros monthly per child).

German employees pay compulsory social contributions (for health insurance, etc.) which are about 21 percent of the gross income. The tax law considers the social contributions as an income. These are, for their main part, not deductible from the taxable income.

Since we concentrate on double earner couples, we do not take into account potential social subsidies for low earning families. Double-working couples are typically not eligible for revenue dependent social benefits. Of course, we take account for all means-tested benefits the family is eligible for (for example child benefit). Therefore, in our paper, the households face a convex budget constraint.

For couples, taxes are paid and benefits are received independently of the intra-household allocation. The tax schedule used is the same for singles and for couples, but for couples, the "splitting method" is applied: the tax rate is applied to one-half of the joint taxable income, and the outcome is doubled to obtain the total income tax liability of the spouses. A specificity of the German income tax law is in the way how the tax is collected. For the employees, the income tax is deducted from the monthly salary. Since the tax burden due by each spouse in a double-earner couple can not be calculated in a non-ambiguous way, the tax law defines how much is paid by each spouse. Two systems cohabit:

- In the first system ("Steuerklasse IV/IV"), both spouses pay taxes as if they were a single person. Since the couple pays too much income tax due to the convexity of the tax function, the tax authority refunds the surplus at the end of the year. In this system, the perceived marginal tax rate of each spouse depends on his/her personal labour income. It is higher than the real tax rate for the main earner. It is usually lower than the real tax rate for the second earner.
- In the second system ("Steuerklasse III/V"), the main earner pays income tax as if he/she was the only earner in the couple. The second earner has then to pay the difference between the total couple's tax burden and the amount paid by the main earner. In this system, the second earner is therefore systematically overtaxed. The perceived

marginal tax rate of the main earner is moderate. On the contrary, the one of the second earner is usually far higher than the real tax rate.

3.2 The model

For the purpose of the paper, we use a household model which takes account of each individual in the family. We additionally introduce the German tax system as described above. The model is a collective framework. The household optimizes its utility under budget restriction as follows:

$$\max_{c_{f}, c_{m}, l_{f}, l_{m}} U^{f}(c_{f}, l_{f}) + \lambda \left(w_{f}, w_{m}, y\right) U^{m}(c_{m}, l_{m})$$
(12)
s.t. $c_{f} + c_{m} \leq g_{R} \left(l_{f}, l_{m}, w_{f}, w_{m}, y\right),$

where λ is the male relative weight in the household. This model is equivalent to the model 6 presented in Section 2.1.2.

 $g_R(.)$ represents the household net income. The latter is calculated as follows:

$$g_R(l_f, l_m, w_f, w_m, y) = I_H - f_R(w_f h_f + w_m h_m, y),$$

where h_i is the labour supply of spouse *i*. $h_i = T - l_i - \bar{l}$. *T* and \bar{l} are the total disposable and the regeneration time. I_H is the household gross income: $I_H = w_f h_f + w_m h_m + y$. f_R (.) is the German tax system as described in the former section. However, if the couple chooses the first system for paying the income tax ("Steuerklasse IV/IV"), its budget constraint can be rewritten as follows:

$$g_{IV/IV}(l_f, l_m, w_f, w_m, y) = I_H - f_{IV}(w_f h_f) - f_{IV}(w_m h_m) - y_{IV}^*,$$

where $f_{IV}(w_ih_i)$ is the tax paid by spouse *i* when the German tax function applies on his own income as if he/she was a single individual. $y_{IV}^* = f_{IV}(w_fh_f^*) + f_{IV}(w_mh_m^*) - f_R(w_fh_f^* + w_mh_m^*, y)$ is the surplus of income tax paid. This amount is refunded by the tax authority to the couple at the end of each year. In the model, y_{IV}^* is considered as an uncarned income. If the couple chooses the second system for paying the income tax ("Steuerklasse III/V"), its budget constraint is:

$$g_{III/V}(l_f, l_m, w_f, w_m, y) = I_H - f_{III}(w_i h_i) - f_V(w_j h_j) - y^*_{III/V}.$$

Spouse *i* is the main earner, i.e. the tax burden deducted from his salary, $f_{III}(w_ih_i)$, corresponds to the one of a married person whose spouse does not work. The income tax deduction on the salary of spouse *j*, who is the

second earner, is $f_V(w_jh_j) = f_R(w_fh_f + w_mh_m) - f_{III}(w_ih_i)$. $y_{III/V}^* = f_{III}(w_ih_i^*) + f_V(w_jh_j^*) - f_R(w_fh_f^* + w_mh_m^*, y)$. We therefore have: $\widetilde{w}_{III} > \widetilde{w}_R > \widetilde{w}_{IV} > \widetilde{w}_V$,

where $\tilde{w}_x = \frac{\partial g_x}{\partial l_i} = w_i - \frac{\partial f_x}{\partial h_i} \Big|_{i=f,m;x=III,R,IV,V}$. \tilde{w}_x are the different implicit wages. These depend on the way the income tax is paid, and not only on the income tax function. We use these different implicit wages to perform differentiated estimations for the labour supply equations and intra-household income sharing rule, and to test whether the *perceived* marginal tax rate \tilde{w}_{III} , \tilde{w}_{IV} or \tilde{w}_V , or the *real* marginal tax rate \tilde{w}_R better explains the household behaviour.

3.3 The econometric specification

Since the sample is composed exclusively of double-earner couples, the estimation procedure proposed by Chiappori/Fortin/Lacroix (2002) is most convenient. We thus use a semi-logarithmic specification for the labour supply functions. However, since we introduce a progressive income tax function, we use the implicit wages as defined in the former section, instead of the gross wages and add the vector of parameter $\boldsymbol{\pi}$, which represents the caracteristics of the tax system. The labour supply functions are therefore:

$$h_{i} = \alpha_{0}^{i} + \alpha_{1}^{i} \ln \widetilde{w}_{i} + \alpha_{2}^{i} \ln \widetilde{w}_{j} + \alpha_{3}^{i} \widetilde{y} + \alpha_{4}^{i} \ln \widetilde{w}_{i} \ln \widetilde{w}_{j} + \alpha_{5}^{i} \mathbf{s} + \alpha_{6}^{i} \mathbf{z} + \alpha_{7}^{i} \boldsymbol{\pi} + \boldsymbol{\varepsilon}_{i} \qquad i = f, m,$$
(13)

where **s** and **z** are vectors of distribution factors and socio-demographic characteristics. ε_i are normal distributed error terms. The distribution factor we use in this study is the ratio in body mass index (BMI) between the female and male. The BMI is surely related to the consumption pattern, but it is doubtful that it is correlated with the preference for consumption. One can argue that the BMI is partly determined by genetics, and that people with high BMI do not consume more, but consume differently. The usual distribution factor used in similar studies, the sex ratio, is not a meaningful variable for Germany. Indeed, the sex ratio is highly unbalanced between East and West-Germany due to the higher migration of young women from Eastern to the Western part of the country. This migration is mainly related to better job opportunities in the West. Therefore people remaining in the East may have a lower preference for labour. As for π , we choose following variable:

i.e. when estimating the labour supply equations on the base of the real tax function, parameter π is the ration between the household marginal tax rate if the female and the male were earning twice as much as they do. When estimation the labour supply equations on the base of the perceived wages, parameter π is the ration between the female and the male marginal tax rates.

The restrictions which apply to this model are (these can be computed from restrictions 1. and 2. in the system of restrictions given in Section 2.1.2):

$$\frac{\alpha_4^f}{\alpha_4^m} = \frac{\alpha_5^f}{\alpha_5^m}.$$
(14)

Further restrictions are the Slutsky conditions (restriction 3. and 4. in Section 2.1.2):

$$\frac{\alpha_1^f \alpha_4^m - \alpha_4^f \alpha_1^m}{\alpha_4^m \widetilde{w}_f} - \frac{\alpha_3^f \alpha_4^m - \alpha_4^f \alpha_3^m}{\alpha_4^m} h_f \geq 0$$

$$\frac{\alpha_4^f \alpha_1^m - \alpha_1^f \alpha_4^m}{\alpha_4^f \widetilde{w}_m} - \frac{\alpha_4^f \alpha_3^m - \alpha_3^f \alpha_4^m}{\alpha_4^f} h_m \geq 0.$$
(15)

The income sharing rule is given by (see Chiappori/Fortin/Lacroix 2002):

$$\varrho = \beta_0 + \beta_1 \ln \widetilde{w}_i + \beta_2 \ln \widetilde{w}_j + \beta_3 \widetilde{y} + \beta_4 \ln \widetilde{w}_i \ln \widetilde{w}_j + \beta_5 \mathbf{s}, \qquad (16)$$

where

$$\beta_{1} = \frac{\alpha_{4}^{f} \alpha_{1}^{m}}{\alpha_{3}^{f} \alpha_{4}^{m} - \alpha_{4}^{f} \alpha_{3}^{m}}$$

$$\beta_{2} = \frac{\alpha_{2}^{f} \alpha_{4}^{m}}{\alpha_{3}^{f} \alpha_{4}^{m} - \alpha_{4}^{f} \alpha_{3}^{m}}$$

$$\beta_{3} = \frac{\alpha_{3}^{f} \alpha_{4}^{m}}{\alpha_{3}^{f} \alpha_{4}^{m} - \alpha_{4}^{f} \alpha_{3}^{m}}$$

$$\beta_{4} = \frac{\alpha_{4}^{f} \alpha_{4}^{m}}{\alpha_{3}^{f} \alpha_{4}^{m} - \alpha_{4}^{f} \alpha_{3}^{m}}$$

$$\beta_{5} = \frac{\alpha_{5}^{f} \alpha_{4}^{m}}{\alpha_{3}^{f} \alpha_{4}^{m} - \alpha_{4}^{f} \alpha_{3}^{m}}$$

$$(17)$$

 β_0 is not determined: the sharing rule is only identified up to an additive constant (see Chiappori 1988). We can therefore use these equations to test for income pooling in the household, depending on the different tax collecting.

3.4 The data

We use the 2004 wave from the German Socio-Economic Panel (GSOEP), a representative panel data sample of households and individuals living in Germany. The panel gives a wealth of information on the labour market status of individuals and on the various income sources of families.

For the purpose of the paper, we selected double-earner couples. The individuals are German nationals aged between 25 and 55 years. All are employees with a contractual labour supply of at least five hours per week. The restriction on hours is introduced to avoid extraordinary high wage rates as the ratio of earnings over hours for people with less than five hours. We excluded self-employed, recipients of pension payments as well as individuals in parental leave or in education or registered as unemployed. The couples may have dependent children living in or outside the household. The sample consists of 1,627 couples.

The originality of the 2004 wave of the German Socio-Economic Panel is that it contains the information on which system is chosen by the couples as for the payment of the income tax. From Table 2, we see that 786 couples choose to pay income tax as if the spouses were single individuals. Approximately half of the couples declare the husband as the main earner. On the contrary, the wife is the main earner only for less than two percent of the couples (these households are not shown in the statistics table).

Interestingly, over 60 percent of married women choosing to pay taxes as if they were single individuals work full-time. This is the case of less than 15

	Real		IV/IV		III/V	
variable	mean	st. dev.	mean.	st. dev.	mean.	st. dev.
female hourly gross wage	13.2	5.1	14.2	5.2	12.3	4.7
male hourly wage gross	20.1	8.7	21.0	9.3	19.3	8.0
household monthly unearned income	1,456	$6,\!443$	1,586	$5,\!245$	1,332	$7,\!423$
female labour supply	26.2	11.1	31.9	9.8	20.7	9.3
female works full-time (dummy)	.369	.483	.609	.489	.134	.340
male labour supply	40.0	6.7	40.5	7.3	39.7	5.9
m rBMI	.953	.213	.954	.199	.948	.217
child (dummy)	.800	.400	.793	.405	.806	.395
country side (dummy)	.214	.411	.209	.407	.221	.414
East (dummy)	.163	.369	.260	.439	.070	.255
female age	42.7	7.18	43.2	7.44	42.3	6.90
male age	43.2	7.10	43.8	6.92	42.8	7.18
high educ. female (dummy)	.495	.500	.578	.494	.417	.493
high educ. male (dummy)	.497	.500	.513	.500	.479	.500
# obs.	1,627		786		816	

Table 2: Summary statistics

percent of the couples in which the female is the second earner. It is therefore surprising that the choice how the couple chooses to pay income tax does not depend on the presence of children in the household. An explanation is that mothers continue to work part-time, even once the children have left the household. Remarkable is that the choice of "Steuerklasse III/V" is far more common in Western Germany. Only seven percent of the East German couples choose this way of payment whereas they represent more than 16 percent of the sample.

The female body mass index is in average five percent lower than the male's. Note that the distribution of ratio between female and male body mass index (rBMI) is spread. rBMI is thus technically a good distribution factor.

3.5 The estimation results

We perform three different estimations by the general method of moments (GMM) of equation (13) using (i) the real marginal tax rate (implicit wages are \tilde{w}_R), (ii) perceived marginal tax rate when the couple choose to paid the income tax following the first system (implicit wages are \tilde{w}_{IV}), and (iii) perceived marginal tax rate when the husband is the main earner (implicit wages are \tilde{w}_{III}^m and \tilde{w}_V^f). We have only very few observations with the female as the main earner. We do therefore not do an estimation for this case.

Tables 3 and 4 show that the labour supplies are far more sensitive to the marginal wages for those couples choosing to pay the income tax as if

	Real		IV/IV		III/V	
h_f	coef.	st. err.	coef.	st. err.	coef.	st. err.
$\ln \widetilde{w}_f$	12.1	4.8	17.2	6.7	4.6	3.7
$\ln \widetilde{w}_m$	7.3	4.1	10.5	5.8	5.6	3.0
$\widetilde{y}/1000$.3	.4	1.2	1.3	.8	1.0
$\ln \widetilde{w}_f \ln \widetilde{w}_m$	-2.4	1.6	-3.7	2.0	-1.3	1.0
rBMI	2.7	1.2	2.8	1.1	3.8	1.1
child (dummy)	-1.4	.6	-1.8	1.2	-1.7	.9
country (dummy)	.7	.6	.8	.9	.7	.4
East (dummy)	7.7	.7	4.3	1.9	8.0	2.1
age/10	.4	.3	.5	.6	.5	.3
high educ. (dummy)	2.5	.6	2.0	1.0	4.3	1.4
π	.9	.4	.7	.4	.7	.3
cons.	-14.2	12.2	-14.9	10.7	-8.9	7.7
Objective function	91.02		57.01		58.08	

 Table 3: Estimation results - female labour supply

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 Table 4: Estimation results - male labour supply

	Real		IV/IV		III/V	
h_m	coef .	st. err.	coef.	st. err.	coef.	st. err.
$\ln \widetilde{w}_f$	3.2	3.1	2.6	3.4	2.0	3.4
$\ln \widetilde{w}_m$	3.0	2.6	8.1	3.5	2.5	3.0
$\widetilde{y}/1000$	6.7	4.0	9.7	5.4	6.0	4.1
$\ln \widetilde{w}_f \ln \widetilde{w}_m$	-1.2	1.0	-2.3	1.1	-1.5	1.1
rBMI	2	.8	3	.9	4	.8
child (dummy)	.8	.4	1.0	.6	.8	.5
country (dummy)	1	.4	1	.4	1	.4
East (dummy)	.4	.4	1.2	.6	.6	.5
age/10	.4	.2	.5	.2	.5	.2
high educ. (dummy)	1.5	.4	1.9	.9	1.9	.8
π	.5	.3	.4	.3	.4	.2
cons.	28.9	8.0	24.6	8.1	30.1	8.7
Objective function	87.14		53.84		54.85	

	Real		IV	/IV	III/V	
ϱ	coef.	st. err.	coef.	st. err.	coef.	st. err.
$\ln \widetilde{w}_f$	8.345	3.920	5.385	2.068	3.852	2.101
$\ln \widetilde{w}_m$	9.747	$3,\!941$	5.289	1.999	4.021	2.124
$\widetilde{y}/1000$	4,520	1,224	2.307	1.028	2.380	994
$\ln \widetilde{w}_f \ln \widetilde{w}_m$	15.726	7.250	10.079	4.088	7.141	4.840
rBMI	7.585	2.451	3.254	1.474	4.891	1.546

Note: The standard errors have been computed using the delta method.

Table 5: Mean absolute error in imputed labour supply

	Real	IV/IV	III/V
Female	6.42	4.11	3.82
Male	2.41	2.37	2.28

Note: Mean absolute error between real labour supply, and labour supply imputed using the estimates.

the spouses were two single individuals. On the contrary, marginal wages seem to be less determinant as for the labour supply when the second system is used. Note that for the males, the labour supply barely depends on the wages. This is explained by the concentrated distribution of the male labour supply.

As for the socio-economic variables, the labour supply increases with the level of education, in particular for women. Labour supply is also significantly higher for Eastern German women. On the contrary, the presence of children does not seem to have any impact on the labour supply, confirming therefore the observation from the statistics.

Table ?? shows the parameters of the income sharing rule (equation (16)). The impact of the distribution factor is as expected. An increase of ratio of the body mass index, due for example to an increase of the female BMI, increases the bargaining power of the male. BMI can be seen as a proxy for beauty. This result supports then the idea that in the negotiation process as for the intra-household bargaining rule, beauty is an argument in order to negotiate a higher power. An attractive person has indeed better outside options. Interestingly, the income related variables (the wages and the unearned income) are more or less significative, depending on the formula the couple chooses for the income tax payment. In particular, among those couples choosing equal taxation ("Steuerklasse IV/IV"), the income structure explains in a significant way the intra-household share of the resources. On the contrary, for the couples choosing "Steuerklasse III/V", the income related variables are barely significant in the income sharing rule equation. This leads us to conclude that income pooling seems to be less common among couples choosing to pay the income tax as if they were single individuals.

Table 5 indicates that the perceived wages are a better indicator for the household behaviour than the real marginal tax rate, as a better fit is obtained when imputing the labour supply decision from the estimates from the estimation of the equation with the perceived implicit wages.

Finally, we perform different tests. The first is on the collective rationality (equation (14)). Here we use an LR type test introduced by Newey/West (1987). Aim is to test if the objective functions of the model estimated with unrestricted parameters (see Tables 3 and 4) and with restricted distribution factor proportionality are significantly different.³ For all three estimations, we accept empirically the null hypothesis, since the test statistics is comprised between .79 and 1.03, which are far under the critical value $\chi^2(1) = 3.84$. Couples in the different subsamples are therefore collective rational. Second, we verify the Slutsky conditions (equations (15)). These are accepted for almost all individuals in the samples. For less than two percent of the males with low wages, the condition is violated. The last test is on the tax neutrality (equation (11)). Here we want to test whether the different marginal tax rates influence the income share. We reject the hypothesis of tax neutrality. We therefore conclude that the way how the couple perceives the tax system matters when the couple allocates the resources between the spouses.

4 Conclusion

The main result of the paper is that the redistribution of the resources inside the household depends on how the household perceives the tax system, rather than on the tax rule as defined in the tax law. In particular, the labour supplies are better predicted when using the estimates resulting from the estimation of the equation with the perceived implicit wage. The test on tax neutrality confirms that the perception of the tax system matters when evaluating the impact of the tax system on the household decisions. Another result is that the non-income pooling test is less robust for couples declaring the male as the main earner in comparison to couples paying taxes as if the spouses were two single individuals. It seems there is a correlation between income pooling and the way in which the couples decide to pay the income tax.

 $^{^{3}}$ In the model with restricted distribution factor proportionality, equation (14) is imposed. The estimates of this model are not shown in the paper.

References

- Apps P.F. and R. Rees (1988), Taxation and the Household, *Journal of Public Economics* 35, 355-369.
- [2] Beninger D. (2009), A discrete choice estimation of a collective model of household labour supply: An application for Germany, mimeo.
- [3] Beninger D., F. Laisney and M. Beblo (2007), Welfare Analysis of Fiscal Reforms: Does the Representation of the Family Decision Process Matter? Evidence for Germany, *Journal of Population Economics* 20(4), 869-893.
- [4] Beninger D., O. Bargain, M. Beblo, R. Blundell, R. Carrasco, M.C. Chiuri, F. Laisney, V. Lechene, E. Longobardi, N. Moreau, M. Myck, J. Ruiz-Castillo and F. Vermeulen (2006), Tax Reform Analysis: The Choice of the Representation of Household Decision Processes Does Matter, *Review of Economics of the Household* 4, 159-180.
- [5] Brett C. (1998), Tax Reform and Collective Family Decision-Making, Journal of Public Economics 70, 425-440.
- [6] Browning M. and P.-A. Chiappori (1998), Efficient Intra Household Allocations: a General Characterization and Empirical Tests, *Econometrica* 66, 1241-1278.
- [7] Browning M., F. Bourguignon, P.-A. Chiappori and V. Lechene (1994), Incomes and Outcomes: a Structural Model of Intra Household Allocations, *Journal of Political Economy* 102, 1067-1096.
- [8] Cherchye L., B. De Rock and F. Vermeulen (2007), The collective model of household consumption: a nonparametric characterization, *Econometrica* 75, 553-574.
- [9] Chiappori, P.-A. (1988), Rational Household Labor Supply, *Econometrica* 56, 63-89.
- [10] Chiappori, P.-A., B. Fortin and G. Lacroix G. (2002), Household labour Supply, Sharing Rule and the Marriage Market, *Journal of Political Economy*, 110, 37-72.
- [11] Donni O. (2003), Collective Household Labour Supply: Nonparticipation and Income Taxation, *Journal of Public Economics* 87, 5-6, 1179-1198.
- [12] Lundberg S.J., R.A. Pollak and T.J. Wales (1997), Do Husbands and Wives pool their Resources? Evidence from the United Kingdom Child Benefit, *The Journal of Human Resources* 32, 463-480.

- [13] Moreau N. and O. Donni (2002), Une estimation d'un modèle collectif d'offre de travail avec taxation, Annales d'économie et de statistiques 65, 55-83.
- [14] Newey W and K West (1987), Hypothesis testing with efficient mothod of moment estimators, *International Economic Review* 28, 777-787:
- [15] O'Donoghue C. and H. Sutherland (1998), Accounting for the Family: the Treatment of Marriage and Children in European Income Tax System, Innocenti - UNICEF International Child Development Center Publications.
- [16] Thomas D. (1990), Intra-Household Resource Allocation: An Inferential Approach, *Journal of Human Resources* 25, 635-664.
- [17] Vermeulen F. (2006), A collective model for female labour supply with nonparticipation and taxation, *Journal of Population Economics* 19, 99-118.

5 Appendix

Here, we follow Browning/Chiappori (1998). Let us define the budget constraint as follows:

$$g\left(\mathbf{p},\mathbf{q},B\right) \le 0,\tag{18}$$

where \mathbf{q} is a vector of goods. \mathbf{p} represents the price of goods \mathbf{q} . B is the household disposable income. Function g is continuous, continuously differentiable in each argument and represents a convex budget bundle. Function g is supposed to be completely specified. This assumption is essential in deriving testable restrictions on demand functions. Vector \mathbf{q} may include leisure. The corresponding price is then the wage rate. \mathbf{q} is composed of private goods \mathbf{q}_i and public goods \mathbf{Q} typically not assignable

$$\mathbf{q} = \mathbf{q}_f + \mathbf{q}_m + \mathbf{Q},\tag{19}$$

where f (resp. m) is the subscript for the wife (resp. the husband) in a two-person household composed of a married couple. The spouses are supposed to know each other perfectly. They are thus assumed to take efficient outcomes. The family optimizes:

$$\max_{\mathbf{q}} U^{f}(\mathbf{q}) \tag{20}$$
s.t.
$$\begin{cases} U^{m}(\mathbf{q}) \geq \overline{U}^{m} \\ g(\mathbf{p}, \mathbf{q}, B) \leq 0, \end{cases}$$

where $(U^i)_{i=f,m}$ are the individual utility functions. The utility set is supposed to be convex. In this first step, the preferences are very general; in particular we allow for altruism between the spouses. \overline{U}^m is the husband threat point in the intra-family negotiation process. \overline{U}^m is a measure for the allocation of the resources inside the household. It is supposed to depend on the prices **p** and income *B*. Using the second welfare theorem, Problem (3) can be written

$$V(\mathbf{p}, B, \mu) = \max_{\mathbf{q}} \mu U^{f}(\mathbf{q}) + (1 - \mu) U^{m}(\mathbf{q}) \qquad (21)$$

s.t. $g(\mathbf{p}, \mathbf{q}, B) \leq 0$,

where $V(\mathbf{p}, B, \mu)$ is the household indirect utility function for any μ . μ represents the wife's relative weight in the household (or her bargaining power). μ is an equivalent measure to \overline{U}^m . μ is therefore a function of prices and income, among other arguments. This means that in case of a variation of prices \mathbf{p} or income B, the bargaining power of the spouses changes. The total effect can be decomposed in a *price* or *income* effect (the Pareto-frontier moves due to the change in the budget constraint) and in a *redistribution* effect (move *on* the Pareto-frontier due to the change in the budget of the change in the bargaining power).

Let denote by $\tilde{\mathbf{p}}$ the vector of implicit prices

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$$\widetilde{\mathbf{p}} = \left. \frac{\partial g}{\partial \mathbf{q}} \right|_{\mathbf{q}^*},\tag{22}$$

i.e. the marginal price of goods \mathbf{q} at the optimum.⁴ Define by $\mathbf{f}(\mathbf{\tilde{p}}, B, \mu)$ the Marshallian demand function, (unique) solution of Program (??). The Roy's identity yields for good q_n

$$-\frac{\partial V}{\partial \tilde{p}_n} \bigg/ \frac{\partial V}{\partial B} = f_n \left(\tilde{\mathbf{p}}, B, \mu \right) = q_n.$$
(23)

From the dual program, i.e. from the corresponding cost function, we have:

$$E\left(\widetilde{\mathbf{p}}, u, \mu\right) = \min_{\mathbf{q}} \widetilde{\mathbf{p}}' \mathbf{q} \qquad (24)$$

s.t. $\mu U^{f}(\mathbf{q}) + (1-\mu) U^{m}(\mathbf{q}) \ge u$,

The expenditure function E is linear homogenous, non-decreasing, continuous and concave in $\tilde{\mathbf{p}}$. From the envelope theorem, we have

$$\frac{\partial E}{\partial \tilde{p}_n} = h_n \left(\widetilde{\mathbf{p}}, u, \mu \right), \tag{25}$$

where $\mathbf{h}(\widetilde{\mathbf{p}}, u, \mu)$ is the Hicksian demand function, solution of Problem (24). Problems (24) and (??) are dual. Therefore

$$\mathbf{f}\left(\widetilde{\mathbf{p}}, E\left(\widetilde{\mathbf{p}}, u, \mu\right), \mu\right) = \mathbf{h}\left(\widetilde{\mathbf{p}}, u, \mu\right).$$
(26)

It follows that

$$\frac{\partial f_n}{\partial \tilde{p}_k} + \frac{\partial f_n}{\partial B} f_n = \frac{\partial h_n}{\partial \tilde{p}_k}.$$
(27)

Holding μ constant, equation (27) is equivalent to $\sigma_{kn} = \frac{\partial f_n}{\partial \tilde{p}_k} + \frac{\partial f_n}{\partial B} q_n$. $S = (\sigma_{kn})$ is the matrix of the compensated price responses on the demand functions, i.e. for any value μ , a variation $\Delta \tilde{p}_n$ of the price is compensated by a change in income $\Delta B = q_n \Delta \tilde{p}_n$, so the household indirect $V(\mathbf{p}, B, \mu)$ utility remains unchanged. However the individual utilities may vary. Matrix S corresponds to the Slutsky matrix in the unitary case.

$$\begin{array}{cccc} \mathbb{R}^n_{++} & \leftrightarrow & \mathbb{R}^n_{++} \\ \mathbf{p} & \leftrightarrow & \widetilde{\mathbf{p}}. \end{array}$$

 $^{{}^{4}\}widetilde{\mathbf{p}} = \widetilde{\mathbf{p}}(\mathbf{p})$. We assume that $\left|\frac{\partial \widetilde{\mathbf{p}}}{\partial \mathbf{p}'}\right| \neq 0$. This assumption ensures the derivability relatively to $\widetilde{\mathbf{p}}$ of the functions depending initially on \mathbf{p} . A bijection between both price vector can then be defined:

The Marshallian demand function $\mathbf{f}(\mathbf{\tilde{p}}, B, \mu)$ is not observable. For a specific $\mu = \mu(\mathbf{\tilde{p}}, B)$, only the demand function $\boldsymbol{\zeta}(\mathbf{\tilde{p}}, B)$ are observed. The latter is defined by

$$\boldsymbol{\zeta}\left(\widetilde{\mathbf{p}},B\right) = \mathbf{f}\left(\widetilde{\mathbf{p}},B,\mu\left(\widetilde{\mathbf{p}},B\right)\right).$$
(28)

 $\boldsymbol{\zeta}$ is zero-degree-homogenous, continuously differentiable in $\tilde{\mathbf{p}}$ and B, and, by definition, adds-up: $\tilde{\mathbf{p}}'\boldsymbol{\zeta} = B$. From definition (28), we obtain

$$PS = \frac{\partial \mathbf{f}}{\partial \mathbf{\tilde{p}}'} + \frac{\partial \mathbf{f}}{\partial B} \boldsymbol{\zeta}' + \frac{\partial \mathbf{f}}{\partial \mu} \left(\frac{\partial \mu}{\partial \mathbf{\tilde{p}}'} + \frac{\partial \mu}{\partial B} \boldsymbol{\zeta} \right)', \qquad (29)$$

where PS is the *pseudo-Slutsky* matrix. Therefore each term in PS is the sum of a substitution term for given μ (*price / income effect*) plus a term corresponding to the change of μ holding V unchanged (*distribution effect*). It follows

$$PS = S + \mathbf{uv}',\tag{30}$$

where $\mathbf{u} = \frac{\partial \mathbf{f}}{\partial \mu}$ and $\mathbf{v} = \frac{\partial \mu}{\partial \tilde{\mathbf{p}}'} + \frac{\partial \mu}{\partial B} \boldsymbol{\zeta}$. *PS* is the sum of a symmetric and negative semi definite matrix *S* plus a matrix of rank at most one \mathbf{uv}' , which rank is zero in the unitary case. This property is called *SR*1 condition by Browning and Chiappori (1998).