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## Conference Paper

# The Green Paradox and the Choice of Capacity

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# The Green Paradox

## and the Choice of Capacity

January 31, 2010

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### Abstract

A number of recent papers extend traditional Hotelling frameworks by the topical issue of climate change. They study the effects of different environmental taxes on the resource extraction paths and derive important and far-reaching policy implications. In particular the most recent strain of literature devoted to the so called “Green Paradox” shows how over various economic channels a steady increase in environmental taxation rather accelerates resource extraction instead of resulting in the desired slow down. The cause of this paradox lies in the inability to tax the supply side of the resource and thus limits the policy instruments to the demand. Thus, e.g. an oil-sheikh has an interest to sell his oil while it is still relatively low taxed and he can still generate higher profits. This effect could be seen as a kind of intertemporal carbon leakage transferring future extraction to the present. The Green Paradox literature generally concludes that a binding global certificate system covering all CO<sup>2</sup> sources is the only solution and that attempts of implementing greener policies in the transition process are counterproductive. However, the underlying implicit assumption is that extraction capacities are sufficient and that capacity adjustments are costless. Our paper accounts for an endogenous capacity building decision under convex

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adjustment costs by extending the original Hotelling type formulation of the Green Paradox. The analysis shows that for typical assumptions about the cost structure greener policies stay a useable instrument in the transition process. We are able to define the necessary conditions for the policy measures to be effective and show that the evaluation of the Green Paradox differs for pre and post oil-peak regimes.

**Keywords:** Global warming, carbon taxation, optimal taxation

**JEL Classification:** Q38, Q54, H21

# 1 Introduction

A vast literature models the extraction of a non-renewable resource using the framework put forward by Hotelling (1931). Hotelling suggested that, in optimum, the price of a non-renewable resource grows at the rate of interest. Various extensions of this framework have been considered in subsequent research efforts. The focus of this literature, however, undertook an important change. A larger number of papers that emerged during the 1970s and 1980s focus on the scarcity problem. Long (1975), for instance, investigates the impact of insecure property rights, Long and Sinn (1985) the consequences of sudden price shifts on resource extraction. Dasgupta and Heal (1979) provide a comprehensive treatment of the resource extraction problem. It is obvious that the literature in those days found its motivation in concerns regarding the future availability of oil. Motivated by the two oil crises the dominant question in these days was the optimal allocation of a scarce resource neglecting the negative externalities like e.g. global warming. A recent sprout therefore extends this traditional framework by the topical issue of climate change. Early examples include Ulph and Ulph (1994), Withagen (1994) as well as Hoel and Kverndokk (1996). These days fighting climate change reached the top of the political agenda. In consequence, research in this area is has never been as important as today. Examples of recent papers include Chakravorty et al. (2006, 2008), who consider pollution ceilings as well as the order of extraction of different carbon resources. Of particular importance, however, is Sinn's (2008) formulation of the Green Paradox. The paper assumes that resource owners are confronted with carbon taxes that increase over time - a plausible assumption given the increasing immediacy of the climate change problem. It is shown that this eventually results in a faster rather than a slower extraction of the carbon resource. The cause of this paradox lies in the inability to tax the supply side of the resource and thus limits the policy instruments to the demand. Thus, e.g. an oil-sheikh has an interest to sell his oil while it is still relatively low taxed and he can still generate higher profits. This effect could be seen as a kind of intertemporal carbon leakage transferring future extraction to the present. It is obvious that this result has important and far-reaching policy implications. The Green Paradox literature generally concludes that a binding global certificate system covering all CO<sup>2</sup> sources is the only solution and that attempts of implementing greener policies in the transition process are counterproductive. Additionally, other recent papers including Eichner and Pethig (2009), Smulders et al. (2009) and van der Werf (2009) explicitly seize on the idea of the Green Paradox.

A prerequisite for this bringing forward of extraction is, obviously, that the production capacity is sufficiently large. Neither this specific literature nor other papers that emerged from these research efforts discuss this issue. The papers by Campbell (1980), Holland (2003) as well as Ghoddusi (2009) form notable exceptions in this regard. Therefore, the focus of this paper is to further develop the resource extraction decision modelled by Sinn (2008) by adding an endogenous capacity choice under the assumption of convex adjustment costs. Generally, the inclusion of this decision leads to a two-part extraction path.

Initially, there is a phase in which the extraction capacity is build up and where extraction quantities are rising. This pre-peak oil phase is followed by the post peak oil phase in which extraction declines and the (costly) overcapacities are reduced accordingly. The analysis shows that for typical assumptions about the cost structure greener policies stay a useable instrument in the transition process. We are able to define the necessary conditions for the policy measures to be effective and show that the evaluation of the Green Paradox differs for pre and post oil-peak regimes. The remainder of the paper is organized as follows. Section 2 presents the model, Section 3 the results. Section 4 offers some concluding remarks.

## 2 Model

Sinn (2008) shows that for an exhaustible resource the extension of a standard Hotelling model with an increasing tax leads to a faster extraction of the resource. This paper employs the extended Hotelling resource extraction model used by Sinn (2008) and makes the following essential additional assumption: The resource owner faces a binding capacity constraint  $R$  which is always fully utilized but can but adjusted (increased or decreased) by  $V$  for the cost  $F(V)$ . Capacity is either increased or decreased, but never both at the same time. The variable extraction cost are denoted  $C(S(t))$  rather than  $g(S(t))$ . This means that Sinn's (2008) model is the special case of our generalized model for  $F(V) = 0$ . The full utilization of the extraction capacity is a useful simplification. Relaxing that assumption wouldn't change the result in a deterministic analysis.  $R$  can either be interpreted as capacity or as a simple form of a production function where  $f(R) = R$ . Most likely the capacities we think of in this context are not the extraction capacities but the loop-hole limiting the production are the refinery capacities. Therefore also the extraction costs are only a negligible fraction of the production cost which are dominantly caused by the installed refining capacities. The costs are assumed to increase linear in the installed capacity but we allow the marginal costs  $C(S)$  to increase as resources get scarce and thus e.g. longer pipelines have to be installed to supply the refinery with crude oil.

Following Sinn (2008),  $\hat{\Theta}$  is the growth rate of the cash flow tax,  $i + \pi$  is the interest rate and the expropriation probability,  $\Theta$  is the cash flow tax,  $P$  is the consumer price for a unit of the resource,  $R$  is the production capacity,  $V$  is an adjustment of capacity,  $F(V)$  is the adjustment cost of adding (or subtracting)  $V$  units of capacity and  $S$  is the stock of the resource.

$$\max_V \int_0^{\infty} \Theta(0) \cdot [\Theta \cdot P(t) - C(S)] \cdot R - F(V) \cdot e^{-[i+\pi-\hat{\Theta}] \cdot t} dt \quad (1)$$

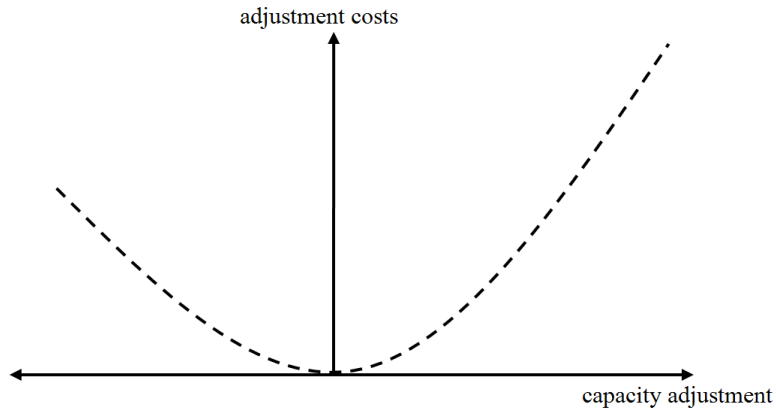
$$s.t. \quad \dot{S} = -R(t) \quad (2)$$

$$\dot{R}(t) = V \quad (3)$$

$$S(0) = S_0 \quad (4)$$

It is worth noticing that the tax function is defined as  $\Theta(t) = \Theta(0) \cdot e^{\hat{\Theta} \cdot t}$  where  $\Theta(0) = 1 - \tau(0)$  and  $\tau(0)$  is the tax rate in percent. Thus  $\Theta(0)$  is share of the of the consumer price  $P(t)$  that the producer receives. An increasing tax would therefore be expressed by  $\hat{\Theta} < 0$ . The adjustment cost function shall be convex, contineous and differentiable with its minimum at  $V = 0$ . Further we assume the adjustment cost function to have the following properties:

Figure 1: adjustment costs



Source: own illustration

$$\begin{aligned}
 & F(V) > 0 \\
 & F'(V) = 0 \quad \text{for} \quad V = 0 \\
 & F'(V) > 0 \quad \text{for} \quad V > 0 \\
 & F'(V) < 0 \quad \text{for} \quad V < 0 \\
 & F''(V) > 0 \\
 & F'''(V) \text{ is monotonous} \tag{5}
 \end{aligned}$$

These assumptions will result in a plausible adjustment cost function as the stylised illustration in figure 1 shows. As long as the general assumptions hold we do explicitly not impose any symmetry restriction for positive vs. negative capacity adjustments. Thus the slope for capacity decreases can vary from that for capacity increases.

### 3 Results

The following section only presents the main findings. The complete analytical solution can be found in the appendix. The optimal control problem stated

above can be formulated in the following current value Hamiltonian:

$$H = \Theta(0) \cdot [[\Theta \cdot P(t) - C(S)] \cdot R(t) - F(V)] - \lambda_1(t) \cdot R(t) + \lambda_2(t) \cdot V \quad (6)$$

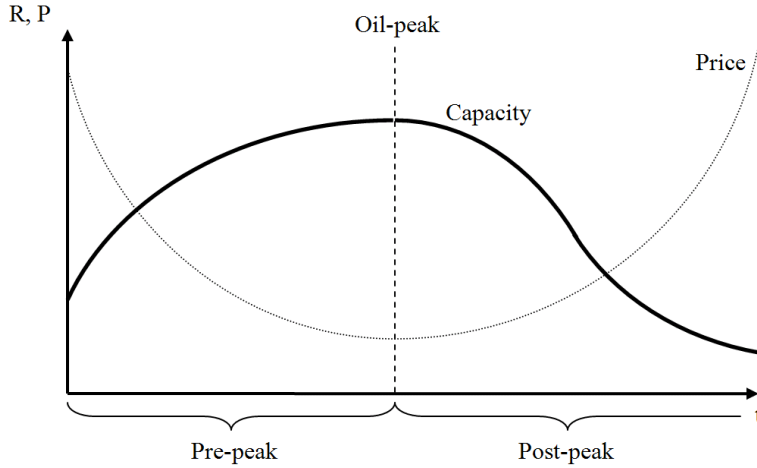
Solving the optimal control problem analytical and displaying it in the conventional form yields the resulting equivalent to the Hotelling rule:

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot \dot{P}(t)}{\Theta \cdot P(t) - C(S) - [i + \pi - \hat{\Theta}] \cdot F_V(V)} \quad (7)$$

First we can observe that this rule deviates from the original Green Paradox only in the last term of the denominator on the right side of the equation. As was stated above the original analysis is a special case of our analysis for which  $F(V) = 0$  in which case obviously  $F_V(V) = 0$  and thus equation (7) reduces to the original Green Paradox:

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot \dot{P}(t)}{\Theta \cdot P(t) - C(S)} \quad (8)$$

Figure 2: regime type



Source: own illustration

The growth rate of marginal profits (right hand side of the equation) has to be equal to the sum of the time preference rate (which is in itself the sum of the interest rate  $i$  and additional discount  $\pi$  accounting for the expropriation probability) and the tax rate growth  $\hat{\Theta}$ . As shown in Sinn (2008) this implies that an increasing tax rate (expressed by  $\hat{\Theta} < 0$ ) will increase the left hand side of the equation and thus the extracted amount of the resource has to be larger

for any given stock of the resource (since the growth rate of marginal profits has to rise). Thus the resource will be extracted quicker which will exacerbate global warming. Going back to equation (7) we observe that it is more difficult to derive a clear cut result since the additional term in the denominator depends on the growth of the tax rate as well. This effect might oppose the Green Paradox. It stays true that the increasing taxes give an incentive to bring forward resource extraction. At the same time the faster increase of the taxes reduces the present value of the whole project (this is even true in welfare terms since the sum of the present value of the project the tax revenues is by the tax burden smaller than the undistorted extraction disregarding the environmental externality). This will in turn reduce the incentive to invest in capacities for the extraction and the processing of the resource. The resource owners will require the marginal capacities to have a higher return on investment which implies lower investments in capacities. In the following analysis we will determine the conditions under which the latter effect dominates the Green Paradox. There exist two regimes (capacity increase and capacity decrease) which have a natural order given that we start from zero capacity. – Pre-oil-peak regime (capacity increase, existence conditional on increasing marginal adjustment costs). – Post-oil-peak regime (capacity decrease, once regime is reached, it is optimal not to switch back) (Feichtinger and Hartl (1986)). Thus, both the optimal extraction and the price path the model generates are in line with important streams of literature. The peak-oil literature can be traced back to Hubbert (1956), who correctly predicted the peak in U.S. crude oil production in the 1970s. This issue has been addressed in a large number of papers. Holland’s (2008) contribution is one of the most interesting ones as it shows that Hotelling models and peak-oil are reconcilable with each other. Also the U-shaped price path has been addressed in a vast literature. Slade’s (1983) seminal paper attempts to empirically test this type of price path assumption. Figure 3 displays historical oil prices and suggests that a U-shaped price path appears to be present. Dvir and Rogoff (2008) argue that in the periods prior to 1900 and after 1970 uncertainty regarding the availability of sufficient amounts of oil are present which results in the strong volatility in the corresponding periods. It is plausible to argue that the scarcity in the first period was attributable to insufficient capacities, while for the present period the limited future availability of oil appears to be the culprit. For our further analysis we will distinguish when necessary between the two regimes identified by positive or negative capacity adjustments, respectively (it is never optimal to build up and reduce capacity at the same time because it’s costly and just a waste of money). In our deterministic framework the resource owner will strictly start with building up the capacity in the first regime and then switch once and for all to the second regime as can be seen in the illustration in figure 2:

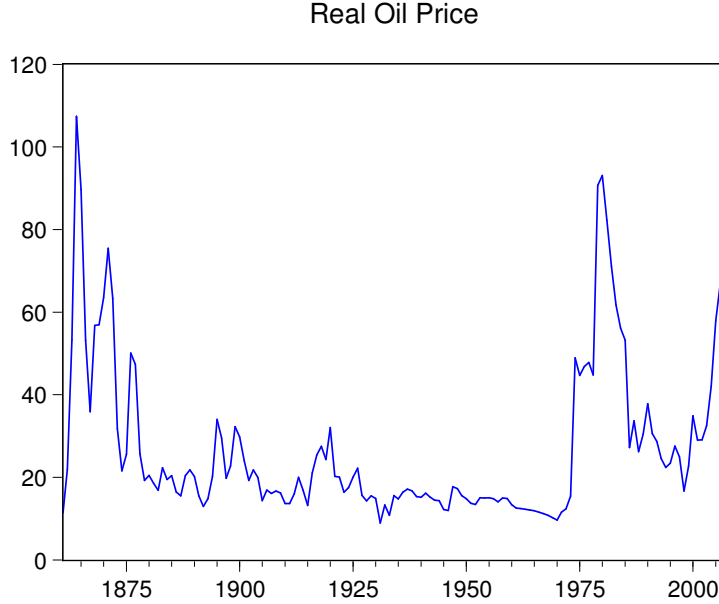
- regime 1  $V \geq 0$
- regime 2  $V < 0$

Comparing our theoretical predictions illustrated in figure 2 with the historical development of the oil price it doesn’t seem odd to assume such a time structure



in the processing capacities. To access the validity of the Green Paradox we start by using the condition that the markets have to clear and thus supply has to equal demand. Thus, substituting the inverse demand function  $P = P(R(t))$  in the optimal extraction rule yields equation (9).

Figure 3: historic oil price



Source: BP

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot P_R(R(t)) \cdot V}{\Theta \cdot P(R(t)) - C(S) - [i + \pi - \hat{\Theta}] \cdot F_V(V)} \quad (9)$$

Equation (9) could obviously be solved for a specific demand function in order to determine the optimal capacity buildup  $V$  depending only on the current capacity  $R(t)$  and exogenous factors, thus  $V^* = V(R(t), [i + \pi - \hat{\Theta}], \Theta, C(S))$ . The obvious question for the climate effect (faster extraction is bad for climate) is now how a change in  $\hat{\Theta}$  will change  $V$  for any given level of  $R(t)$  (higher  $R(t)$  faster extraction). The answer to that question delivers the sign of the derivative of  $V^*$  with respect to  $\hat{\Theta}$  which we derive by totally differentiating equation (9) we get:

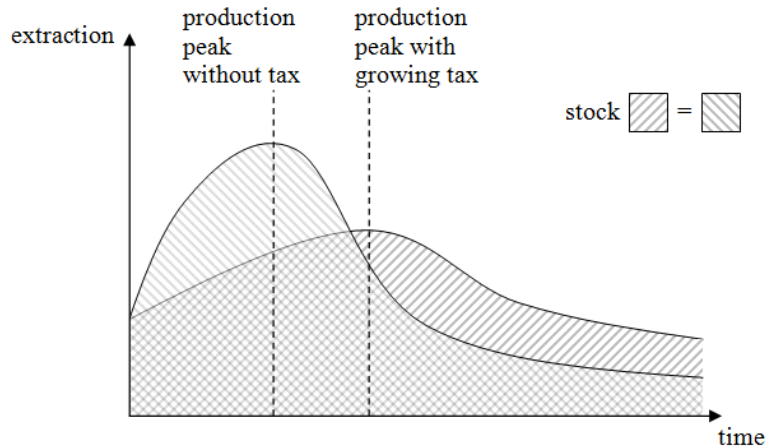
$$\frac{dV}{d\hat{\Theta}} = \frac{\Omega(V, R(t))}{V \cdot \left[ [i + \pi - \hat{\Theta}] \cdot F_{VV}(V) + \frac{\Theta \cdot P_R(R(t))}{i + \pi - \hat{\Theta}} \right]} \quad (10)$$

Where  $\Omega(V, R(t)) > 0$ . We can rearrange the Hotelling rule from equation (7) to

$$\Theta \cdot P(R(t)) - C(S) = V \cdot \left[ [i + \pi - \hat{\Theta}] \cdot \frac{F_V(V)}{V} + \frac{\Theta \cdot P_R(R(t))}{i + \pi - \hat{\Theta}} \right] \quad (11)$$

which shows that - given the assumptions about the cost function - that the denominator of equation (10) is positive if  $F_{VV}(V) = \frac{F_V(V)}{V}$  which is the case if the cost function is quadratic. An increasing tax rate is present if  $\hat{\Theta} < 0$  and therefore a further increase in the tax growth requires a decrease in  $\hat{\Theta}$ . Since for a quadratic cost function the derivative in equation (10) is positive for an increase  $\hat{\Theta}$  it will be negative for a further increase of the tax growth. Thus for an increase in the tax growth, the capacity adjustment  $V$  will decrease. That is good news for the mitigation of climate change since in the first regime where the capacity increase will be slowed down (the positive value of  $V$  will be reduced for any given Value of  $R$ ) and capacity reduction will be speeded up (the negative value of  $V$  will be further reduced for any given Value of  $R$ ). The effect of an introduction of a growing tax rate on the extraction path given quadratic adjustment costs is illustrated in figure 4. Imagine that in both scenarios we start from some arbitrary small but positive initial value of  $R$ . In both scenarios the total stock will be extracted eventually. Therefore the area under the shown capacity curve has to be the same in both scenarios. Since the capacity will increase slower and decrease faster in the tax scenario for any given level of  $R$ , the production peak has to be delayed in order to allow for the total extraction to be the same.

Figure 4: path comparison before and after the introduction of increasing taxes with quadratic adjustment costs



Source: own illustration

If the cost function is not quadratic we can derive sufficient conditions for the validity of the green paradox from the relation of  $F_{VV}(V)$  vs.  $\frac{F_V(V)}{V}$ . First it is helpful to note that for  $F_{VV}(V) \geq 0$  the marginal cost function is convex and given the assumptions about the cost function it holds true that  $F_{VV}(V) \geq \frac{F_V(V)}{V}$ . Consequently for  $F_{VV}(V) \leq 0$  the marginal cost function is concave and thus  $F_{VV}(V) \leq \frac{F_V(V)}{V}$ . Therefore we can record that a convex marginal cost function is sufficient condition for:

$$\frac{dV}{d\hat{\Theta}} \geq 0 \quad \text{in regime 1} \quad (12)$$

Meaning that for any given capacity  $R$  the increase in capacity will be lower after the increase of the tax rate growth  $\hat{\Theta} < 0$ . This is equivalent to a slower extraction in that regime and thus the Green Paradox is not valid. Likewise a concave marginal cost function is a sufficient condition for

$$\frac{dV}{d\hat{\Theta}} \geq 0 \quad \text{in regime 2} \quad (13)$$

Meaning that for any given capacity  $R$  the decrease in capacity will be higher after the increase of the tax rate growth  $\hat{\Theta} < 0$ . This is also equivalent to a slower extraction in that regime and thus the Green Paradox is not valid. To derive the necessary condition for the validity of the Green Paradox we can directly analyze what is the condition for the denominator to be positive. From rearranging the Hotelling rule

$$\Theta \cdot P(R(t)) - C(S) = [i + \pi - \hat{\Theta}^*] \cdot F_V(V) + \frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}^*} \quad (14)$$

And defining

$$\Upsilon \equiv [i + \pi - \hat{\Theta}^*] \cdot [F_V(V) - F_{VV}(V) \cdot V] \quad (15)$$

then it is true for the for the denominator above that

$$\Theta \cdot P(R(t)) - C(S) - \Upsilon = [i + \pi - \hat{\Theta}^*] \cdot F_{VV}(V) \cdot V + \frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}^*} \quad (16)$$

Then the Green Paradox is not valid (the denominator is positive) if:

$$\Theta \cdot P(R(t)) - C(S) > \Upsilon \quad (17)$$

which is equivalent to

$$\Theta \cdot P(R(t)) - C(S) > [i + \pi - \hat{\Theta}^*] \cdot [F_V(V) - F_{VV}(V) \cdot V] \quad (18)$$

or

$$\frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}^*} > [i + \pi - \hat{\Theta}^*] \cdot [-F_{VV}(V) \cdot V] \quad (19)$$

which we can interpret for the two regimes separately:

- Regime 1 ( $V > 0$ )

$$\frac{\Theta}{i + \pi - \hat{\Theta}^*} \cdot \underbrace{|P_R(R(t))|}_{\text{slope of inverse demand curve}} < [i + \pi - \hat{\Theta}^*] \cdot \underbrace{F_{VV}(V)}_{\text{slope of marginal cost curve}} \quad (20)$$

- Regime 2 ( $V < 0$ )

$$\frac{\Theta}{i + \pi - \hat{\Theta}^*} \cdot |P_R(R(t))| > [i + \pi - \hat{\Theta}^*] \cdot F_{VV}(V) \quad (21)$$

## 4 Conclusions

The resource extraction framework put forward by Hotelling (1931) has been embraced by generations of researchers. Its flexibility allows one to consider issues that range from uncertain property rights to effects of backstop technologies. In various recent extension the topical issue of climate change is investigated. Sinn's (2008) discovery of the Green Paradox is of particular importance because it, firstly, has far-reaching policy implications, and, second, sparked considerable research efforts. The heart of the Green Paradox is that resource owners who face increasing environmental taxes bring forward rather than postpone the extraction of carbon resources. Although sufficient extraction capacities or costless capacity adjustments are a prerequisite for this, this issue is neglected by Sinn (2008). This paper closes this gap and includes a capacity building decision in Sinn's (2008) model. The analysis shows that this extension clearly affects the results obtained by Sinn (2008). For plausible assumptions regarding the cost function for capacity adjustments the Green Paradox does not emerge.

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## 6 Appendix

$$\max_V \int_0^\infty \Theta(0) \cdot [[\Theta \cdot P(t) - C(S)] \cdot R - F(V)] \cdot e^{-[i+\pi-\hat{\Theta}] \cdot t} dt \quad (22)$$

$$s.t. \quad \dot{S} = -R(t) \quad (23)$$

$$\dot{R}(t) = V \quad (24)$$

$$S(0) = S_0 \quad (25)$$

Assumptions about the adjustment cost function:

$$\begin{aligned} F(V) &> 0 \\ F'(V) &= 0 \quad \text{for} \quad V = 0 \\ F'(V) &> 0 \quad \text{for} \quad V > 0 \\ F'(V) &< 0 \quad \text{for} \quad V < 0 \\ F''(V) &> 0 \\ F'''(V) &\text{ is monotoneous} \end{aligned} \quad (26)$$

Adjustment costs:

$$H = \Theta(0) \cdot [[\Theta \cdot P(t) - C(S)] \cdot R(t) - F(V)] - \lambda_1(t) \cdot R(t) + \lambda_2(t) \cdot V \quad (27)$$

Partial derivate of equation (27) with respect to  $V$ :

$$\lambda_2(t) = \Theta(0) \cdot F_V(V) \quad (28)$$

Differentiate equation (28) with respect to time:

$$\dot{\lambda}_2 = 0 \quad (29)$$

$$\dot{\lambda}_1 = [i + \pi - \hat{\Theta}] \cdot \lambda_1(t) - H_S \quad (30)$$

$$\dot{\lambda}_1 = [i + \pi - \hat{\Theta}] \cdot \lambda_1(t) + \Theta(0) \cdot C_S(S) \cdot R(t) \quad (31)$$

$$\dot{\lambda}_2 = [i + \pi - \hat{\Theta}] \cdot \lambda_2(t) - H_R \quad (32)$$

with

$$-H_R = -\Theta(0) \cdot [[\Theta \cdot P(t) - C(S)]] + \lambda_1(t) \quad (33)$$

$$\dot{\lambda}_2 = [i + \pi - \hat{\Theta}] \cdot \lambda_2(t) - \Theta(0) \cdot [\Theta \cdot P(t) - C(S)] + \lambda_1(t) \quad (34)$$

Use equation (28) and equation (29) in equation (34):

$$0 = [i + \pi - \hat{\Theta}] \cdot \Theta(0) \cdot F_V(V) - \Theta(0) \cdot [\Theta \cdot P(t) - C(S)] + \lambda_1(t) \quad (35)$$

$$\lambda_1(t) = \Theta(0) \cdot [\Theta \cdot P(t) - C(S)] - [i + \pi - \hat{\Theta}] \cdot \Theta(0) \cdot F_V(V) \quad (36)$$

Differentiate equation (35) with respect to time:

$$\dot{\lambda}_1 = \Theta(0) \cdot [\Theta \cdot \dot{P}(t) - C_S(S) \cdot \dot{S}] \quad (37)$$

$$\dot{\lambda}_1 = \Theta(0) \cdot [\Theta \cdot \dot{P}(t) + C_S(S) \cdot R(t)] \quad (38)$$

Use equation (35) and equation (37) in equation (31):

$$\begin{aligned} \dot{\lambda}_1 &= [i + \pi - \hat{\Theta}] \cdot \lambda_1(t) + \Theta(0) \cdot C_S(S) \cdot R(t) \\ &\quad \Theta(0) \cdot [\Theta \cdot \dot{P}(t) + C_S(S) \cdot R(t)] \\ &= [i + \pi - \hat{\Theta}] \cdot [\Theta(0) \cdot [\Theta \cdot P(t) - C(S)] - [i + \pi - \hat{\Theta}] \cdot \Theta(0) \cdot F_V(V)] + \Theta(0) \cdot C_S(S) \cdot R(t) \end{aligned} \quad (40)$$

simplify

$$\Theta \cdot \dot{P}(t) = [i + \pi - \hat{\Theta}] \cdot [\Theta \cdot P(t) - C(S) - [i + \pi - \hat{\Theta}] \cdot F_V(V)] \quad (41)$$

rearrange

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot \dot{P}(t)}{\Theta \cdot P(t) - C(S) - [i + \pi - \hat{\Theta}] \cdot F_V(V)} \quad (42)$$

Use the inverse demand function  $P = P(R(t))$  in equation (42):

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot P_R(R(t)) \cdot \dot{R}}{\Theta \cdot P(R(t)) - C(S) - [i + \pi - \hat{\Theta}] \cdot F_V(V)} \quad (43)$$

Use equation (24) in equation (43):

$$i + \pi - \hat{\Theta} = \frac{\Theta \cdot P_R(R(t)) \cdot V}{\Theta \cdot P(R(t)) - C(S) - [i + \pi - \hat{\Theta}] \cdot F_V(V)} \quad (44)$$

Which can obviously be solved to determine the optimal capacity buildup  $V$  depending only on the current capacity  $R(t)$  and exogenous factors, thus  $V^* = V(R(t), [i + \pi - \hat{\Theta}], \Theta, C(S))$ . The obvious question for the climate effect (faster extraction bad for climate) is now how a change in  $\hat{\Theta}$  will change  $V$  for any given level of  $R(t)$  (higher  $V$  faster extraction). The answer to that

question delivers the sign of the derivative of  $V^*$  with respect to  $\hat{\Theta}$  which we derive by totally differentiating equation (44):

$$\left[ i + \pi - \hat{\Theta} \right] \cdot F_V(V) = \Theta \cdot P(R(t)) - C(S) - \frac{\Theta \cdot P_R(R(t)) \cdot V}{\left[ i + \pi - \hat{\Theta} \right]} \quad (45)$$

$$\begin{aligned} & F_V(V) \cdot d\hat{\Theta} - \left[ i + \pi - \hat{\Theta} \right] \cdot F_{VV}(V) \cdot dV \\ &= \frac{\Theta \cdot P_R(R(t)) \cdot V}{\left[ i + \pi - \hat{\Theta} \right]^2} \cdot d\hat{\Theta} + \frac{\Theta \cdot P_R(R(t))}{\left[ i + \pi - \hat{\Theta} \right]} \cdot dV \end{aligned} \quad (46)$$

$$\begin{aligned} & \left[ \left[ i + \pi - \hat{\Theta} \right] \cdot F_{VV}(V) + \frac{\Theta \cdot P_R(R(t))}{\left[ i + \pi - \hat{\Theta} \right]} \right] \cdot dV \\ &= \left[ F_V(V) - \frac{\Theta \cdot P_R(R(t)) \cdot V}{\left[ i + \pi - \hat{\Theta} \right]^2} \right] \cdot d\hat{\Theta} \end{aligned} \quad (47)$$

$$\frac{dV}{d\hat{\Theta}} = \frac{\overbrace{V \cdot \left[ \left[ i + \pi - \hat{\Theta} \right] \cdot F_V(V) - \frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}} \right]}^{\text{positive}}}{V \cdot \left[ \left[ i + \pi - \hat{\Theta} \right] \cdot F_{VV}(V) + \frac{\Theta \cdot P_R(R(t))}{i + \pi - \hat{\Theta}} \right]} \quad (48)$$

with

$$\Omega(V, R(t)) = V \cdot \left[ \left[ i + \pi - \hat{\Theta} \right] \cdot F_V(V) - \frac{\Theta \cdot P_R(R(t)) \cdot V}{i + \pi - \hat{\Theta}} \right] \quad (49)$$

$$\frac{dV}{d\hat{\Theta}} = \frac{\Omega(V, R(t))}{V \cdot \left[ \left[ i + \pi - \hat{\Theta} \right] \cdot F_{VV}(V) + \frac{\Theta \cdot P_R(R(t))}{i + \pi - \hat{\Theta}} \right]} \quad (50)$$

Where  $\Omega(V, R(t)) > 0$  and we can rearrange the Hotelling rule from equation (44) to:

$$\Theta \cdot P(R(t)) - C(S) = V \cdot \left[ \left[ i + \pi - \hat{\Theta}^* \right] \cdot \frac{F_V(V)}{V} + \frac{\Theta \cdot P_R(R(t))}{i + \pi - \hat{\Theta}^*} \right] \quad (51)$$