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# Insurance Market with Self-Protection Activities and Risk Interdependencies: Profits, Market Structure, and Risk Exposure\*

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## Abstract

Common economic models of insurance theory assume that insurers play no role in modifying the loss potential. Individual loss prevention decisions tend to reduce risk and even often affect risk faced by others. Incorporating these important features into an insurance market, we argue that insurers may alter the loss potential via their pricing strategy. This is because the (market equilibrium) price of insurance affects the individuals' prevention decisions. Individuals weigh the cost associated with prevention against the price of insurance coverage. In particular, when the insurance market is monopolistic, the insurer will take advantage of this effect and determine loss propensities in a sense optimal for him. This is because different loss propensities affect the individuals' willingness to pay for insurance and thus the potential extent of expected profit per policy. Premiums are then optimally chosen to reflect the trade-off between the number of purchasers and expected profit per policy. Such a strategic behavior of insurers may be a reason why they tend to invest in understanding the loss process: Insurers may indeed have an interest in understanding and eventually affecting underlying loss propensities in order to increase expected profits. Consistent with empirical evidence, we show that consumers are exposed to higher overall risk in a competitive insurance market compared with a monopolistic market.

**JEL Classification:** D40 · D62 · G22.

**Keywords:** self-protection, interdependent risks, insurance monopoly

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# 1 Introduction

It is common knowledge that risk-averse individuals have an incentive to invest in costly self-protection (self-insurance) to reduce or eliminate the probability (the magnitude) of potential loss.<sup>1</sup> Common economic models of insurance theory assume that, while individuals are assumed to be able to influence the loss potential via self-protection or self-insurance investments, the insurer plays no role in modifying the loss potential. Seminal studies by Rothschild and Stiglitz (1976) and Stiglitz (1977) as well as subsequent studies treat loss distributions of insurance customers as exogenously given. In these models, insurers know the proportions of high and low risks in the population of potential customers, but not necessarily individual risk type.

This paper addresses the question why insurers tend to invest in understanding the loss process. For instance, auto insurers often study how crashes occur and the consequences. Property insurers study fire exposures and fire safety mechanisms. We argue that insurers may indeed have an interest in understanding and eventually affecting underlying loss propensities of economic agents in order to increase expected profits. The rationale is as follows. Since the (market equilibrium) price for insurance coverage affects the individuals' prevention decisions, different prices will lead to different prevention levels in society. Different prevention levels tend to result in different overall risk, which again affects the individuals' willingness to pay for insurance and thus the potential extent of insurers' expected profits.

The profit maximizing strategy of an insurer – who is able to somehow alter loss distributions via its pricing strategy – may take advantage of this ability and determine proportions of high and low risks in a way optimal for him. In this paper, we argue that the insurer may influence the loss potential via its profit maximizing pricing strategy and in this way determine the overall loss distribution in a sense optimal for him. We consider an insurance market with self-protection activities where risks are interdependent, i.e. individual self-protection affects risks of others. Although risk interdependencies have not been paid major attention in the insurance literature so far, they seem to be present in many individual decision problems. Indeed, in many cases of individual investment decisions, the ultimate risk of each decision-maker depends in some way on the aggregate actions of others. For instance, the mounting of a sprinkler system in a company decreases the risk of fire for the neighboring company. A farming business that utilizes certain hazardous substances also influences the risks of surrounding farms (i.e. through changes in genetic pools, etc.). Risks of traffic accidents are interdependent. Interestingly, many health risks are imposed by others. A common example is smokers creating cancer risks not only for themselves, but also for non-smokers via passive

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<sup>1</sup> See Ehrlich and Becker (1972).

smoking (known as the second-hand smoking problem). Any individual investment to stop smoking benefits non-smokers as well, and hence such investment generates an externality. Further examples include catastrophe, terrorism, computer security, and liability risks. To keep our analysis simple at this point, we focus on cases where preventive investments can eliminate individual risk. Examples where loss prevention can eliminate individual risk include the following discrete individual choices: vaccination against an infectious disease (risk of infection depends upon number of vaccinated people), not dining in smoking area in restaurants (risk of second-hand smoking related disease depends upon number of smokers), using tram instead of automobile (risk of being involved in an automobile accident depends upon number of cars on the street), not going on holiday in a terror-prone country (number of foreigners spending their holiday in a terror-prone country influences (perceived) terrorism risk), and many others.<sup>2</sup>

The rest of the article is structured as follows. The next section explains some preliminaries. Section 3 introduces our model. We study market equilibria under risk interdependencies (1) when there is no insurance market, (2) in case of a competitive insurance market, and (3) when the insurance market is monopolistic. Section 4 extends our model to imperfect loss prevention, i.e. when loss prevention cannot perfectly eliminate risk. We also analyze the impact of underwriting cost on the insurer's optimal pricing strategy. We then discuss how risk exposure is affected in the different market settings and compare our results with seminal empirical evidence in Europe. Our results contribute to explain these findings. The last section concludes.

## 2 Preliminaries

Assume a representative risk-averse individual and a risk-neutral monopolistic insurer with no transaction costs. Both the insurer and the individual have perfect information concerning the loss distribution and the preferences of the consumer. What would be the loss distribution that is potentially the most profitable for the insurer? Consider a simple two-state model in which the representative individual with given initial wealth  $W$  faces a potential loss  $L < W$  occurring with probability  $p$ . The individual's von Neumann-Morgenstern utility  $U(\cdot)$  is a function of final wealth, twice continuously differentiable, increasing and strictly concave indicating risk aversion.<sup>3</sup> Following Pratt (1964), the individual is willing to incur a non-random cost that exceeds the expected loss by an amount not greater than the risk premium

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<sup>2</sup> See, for instance, Soenmez and Graefe (1998). For more examples, see Avery, Heymann, and Zeckhauser (1995) and Kunreuther and Heal (2003).

<sup>3</sup> See Friedman and Savage (1948).

$\pi$  in order to eliminate all riskiness. The risk premium  $\pi(p)$  is defined via the equation

$$U(W - pL - \pi(p)) = (1 - p)U(W) + pU(W - L), \quad (1)$$

which gives

$$\pi(p) = W - pL - U^{-1}\{(1 - p)U(W) + pU(W - L)\}. \quad (2)$$

For a given probability  $p$ , if there is no transaction cost involved for the insurer in offering insurance coverage, the monopolist's expected profit is maximized by selling a full-coverage policy at a premium that equals the sum of actuarial value of the policy and the risk premium. Expected profit is then equal to the risk premium:

$$E\Pi(\pi(p), p) = pL + \pi(p) - pL = \pi(p), \quad (3)$$

where the premium  $P = pL + \pi(p)$  corresponds to the individual's reservation price (given  $p$ ).

It is well-established in the insurance literature that if an insurer offers any desired level of coverage at a price equal to actuarial value of the policy plus a fixed fee, then the individual will always desire full coverage as long as the fixed loading is not higher than the risk premium.<sup>4</sup> Thus with the fixed loading set equal to the individual's risk premium, the level of coverage desired is always full coverage. In case of positive transaction cost, full coverage yields highest expected profit so long as marginal transaction cost is not an increasing function of the level of insurance coverage. As a result, we consider only full-coverage insurance. For simplicity reasons, we first assume zero transaction cost so that the excess of the insurance premium over the actuarial value of the policy can be interpreted as expected profit per policy.

Since  $\pi(p)$  is a concave function and  $\pi(0) = \pi(1) = 0$ , we may determine the value of  $p$ ,  $0 < p^* < 1$ , that maximizes  $\pi(p)$  and thus the insurer's expected profit from offering a full-insurance contract at price  $P(p, \pi(\cdot)) = pL + \pi(p)$ . Following Schlesinger and Venezian (1986), the first order condition for the maximum of  $\pi(p)$  is

$$\frac{\partial U^{-1}\{(1 - p)U(W) + pU(W - L)\}}{\partial EU} = \frac{L}{U(W) - U(W - L)}, \quad (4)$$

which can be rewritten to

$$U'(C(p^*)) = \frac{U(W) - U(W - L)}{L}, \quad (5)$$

where the certainty equivalent  $C(p^*)$  determines the probability  $p^*$  that maximizes  $\pi(p)$  via

$$p^* = \frac{U(W) - U(C(p^*))}{U(W) - U(W - L)}. \quad (6)$$

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<sup>4</sup> See, for instance, Doherty (1975).

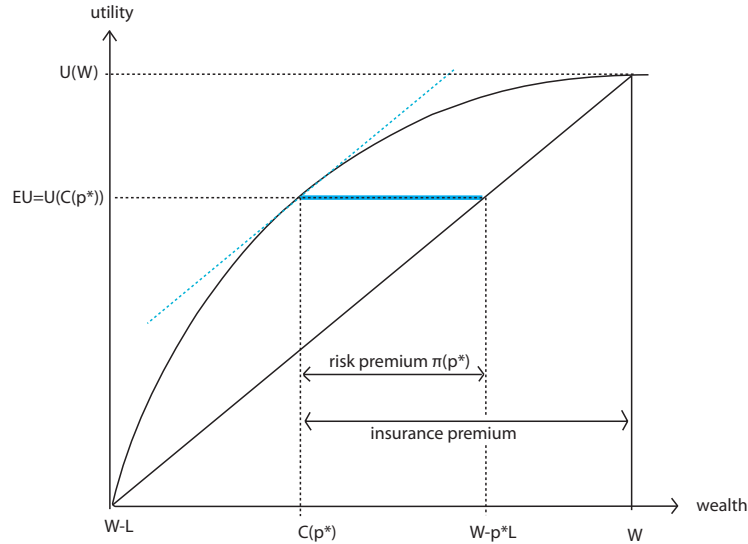


Figure 1: Maximum risk premium.

The insurer will thus offer a full-coverage insurance policy at a premium equal to  $W - C(p^*)$  that yields the maximum expected profit of  $\pi(p^*) = W - p^*L - C(p^*)$ . As a result, if the insurer could somehow influence the individual's loss distribution, he may want to increase or decrease the probability of loss and optimally shift the probability to  $p^*$ .<sup>5</sup>

Individuals generally consider investing in loss prevention in order to reduce or eliminate risk of loss. As discussed above, risks are also often found to be interdependent, i.e. prevention decisions tend to affect risks faced by others. In a model with many heterogeneous individuals who can influence their individual loss potential via preventive investments, an insurer's pricing strategy may have an impact on the individuals' prevention decisions, which, in turn, determine their overall loss potential. This is because individuals will weigh the cost associated with prevention against the price they would have to spend in order to obtain insurance coverage for their risk. Therefore, a profit-maximizing insurer may have an incentive to influence the loss distribution faced by individuals in a way optimal for him. In the next section, we propose a model exhibiting these characteristics. To illustrate the mechanism, we focus on a very simple model: individuals can either invest in prevention or not, i.e. an investment in loss prevention is a discrete choice. If an investment in prevention is undertaken, the

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<sup>5</sup> This result is somewhat general: considering not only a two-state model but allowing for some continuous loss distribution bounded on  $[0, L]$ , second-order stochastic dominance yields the two-state case as the most risky from among all distributions with equivalent expected value. As a consequence, the representative individual's risk premium and the insurer's expected profit are always higher for an actuarially equivalent distribution with just a two-point support. Therefore, the best two-state loss distribution for the insurer, the one with loss probability  $p^*$ , is also the best among all bounded continuous distributions. See Schlesinger and Venezian (1986), p. 231 and Rothschild and Stiglitz (1970).

probability of loss is reduced to zero. Then, depending upon individual characteristics, each individual will decide whether to invest in prevention or not.

### 3 The Model

Society is viewed as a continuum of risk-averse individuals (or firms) each being endowed with initial wealth  $W$  that is subject to potential loss  $L < W$ . The individuals' utility  $U(y)$  is increasing and strictly concave in final wealth  $y$ . Individuals differ in prevention cost or prevention technology given by  $\theta$ . Then individuals can be listed in ascending order according to their individual cost  $\theta$ . The total number of individuals in the economy is normalized to unity. Prevention costs are distributed with the (non-degenerate) distribution function  $F(\theta)$  and density function  $f(\theta)$ , defined over the support  $[0, L]$ . All individuals are free to choose whether or not they want to invest in prevention. All individuals maximize their expected utility of final wealth.

Individuals have to decide whether or not to invest in loss prevention. This decision is a discrete choice: an individual either invests or not. Those individuals with "low" cost will tend to invest in prevention, while those with "high" cost will not. There are risk interdependencies, that is, a loss is indirectly occurring via the actions of others. In particular, investing in prevention generates an externality: a preventive investment benefits others.<sup>6</sup> We assume that a loss can be avoided with certainty by an investment in prevention at cost  $\theta$ . Three possible states of final wealth of an individual result: without prevention, the final wealth of an individual is  $y_1 = W$  in case of no loss, and  $y_2 = W - L$  in case of loss. If an individual invests in prevention, his final wealth is  $y_3 = W - \theta$  in both cases.

The probability of a loss is indirectly determined via the actions of others: it is  $p(x) \in [0, 1]$ , where  $x$  denotes the proportion of individuals in the economy *without* loss prevention. Generally, the higher the proportion of individuals without prevention in the economy, the higher will be the risk of loss, i.e.  $p'(x) > 0$ , and we have  $p(1) = 1$ ,  $p(0) = 0$ . We also assume  $p'(0) = 0$  and  $p'(1) = +\infty$ . These assumptions may be interpreted as follows. If nobody invests in prevention ( $x = 1$ ), then a loss occurs with certainty.<sup>7</sup> In contrast, if everybody invests in prevention ( $x = 0$ ), then the risk of loss in the economy is zero.

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<sup>6</sup> One may easily think of the mounting of a sprinkler system in a company that decreases the risk of fire for the neighboring company.

<sup>7</sup> Note that we might limit  $p(1)$  to a maximum without losing generality.

### 3.1 No Insurance

The expected utility of an individual who does not invest in prevention is

$$EU_0(x) = p(x)U(W - L) + (1 - p(x))U(W), \quad (7)$$

while the expected utility of an individual who invests in prevention at individual cost  $\theta$  is given by

$$EU_1(\theta) = U(W - \theta). \quad (8)$$

Generally, an individual will invest in loss prevention if the *excess* of expected utility with a preventive investment over the expected utility without such an investment is non-negative:  $\Delta(x, \theta) = EU_1(x) - EU_0(\theta) \geq 0$ .<sup>8</sup> To find the proportions of high and low risks in the population, define the excess of expected utility to the marginal individual with prevention cost  $c$  as  $\Delta(x(c), c)$  so that

$$\Delta(x(c), c) = EU_1(x(c)) - EU_0(c) \quad (9)$$

with

$$EU_0(c) = p(x(c))U(W - L) + (1 - p(x(c)))U(W) \quad (10)$$

and

$$EU_1(c) = U(W - c), \quad (11)$$

where  $x(c)$  denotes the proportion of individuals *without prevention* in the economy, given that the marginal individual has a cost  $c$ :

$$x(c) = \int_c^L f(\theta) d\theta = 1 - F(c). \quad (12)$$

$dx(c)/dc = -f(c) < 0$ , i.e. the proportion of individuals without prevention is strictly decreasing in  $c$ . Along with  $0 \leq p(x(c)) \leq 1$ , it follows that at  $c = 0$  we have (1)  $\Delta(x(0), 0) > 0$  and at  $c = L$  we have (2)  $\Delta(x(L), L) < 0$ , so that (1) if nobody invests in prevention and therefore the risk of loss is very high, it is worth to undertake preventive measures to reduce expected loss when prevention is costless, while (2) if everybody invests in prevention and therefore the risk is zero, then an investment in prevention is not worth being undertaken. The difference in expected utility is decreasing in  $c$ , i.e. we also have

$$\frac{d\Delta(\cdot)}{dc} = \frac{dp(x(c))}{dc} \cdot [U(W) - U(W - L)] - U'(W - c) < 0, \quad (13)$$

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<sup>8</sup> Without loss of generality, we assume that an individual invests in prevention when he is indifferent between investing and not investing.



which ensures that there exists a unique interior solution  $\bar{c}$ , where  $0 < \bar{c} < L$ . To simplify notation, we write  $p(x(c))$  in the following as  $p(c)$  and similarly for other expressions. The interior solution  $\bar{c}$  satisfies  $\Delta(\bar{c}) = 0$ . It indicates the proportions of high and low risks in the population via the equation

$$U(W - \bar{c}) = (1 - p(\bar{c}))U(W) + p(\bar{c})U(W - L). \quad (14)$$

The RHS of (14) can be rewritten, using the definition of risk premium by Pratt (1964), and we obtain

$$U(W - \bar{c}) = U(W - p(\bar{c})L - \pi(p(\bar{c}))). \quad (15)$$

Since  $U(\cdot)$  is a strictly increasing function and thus  $U(a) = U(b)$  necessarily implies that  $a = b$ , we obtain

$$\bar{c} = p(\bar{c})L + \pi(p(\bar{c})). \quad (16)$$

The equilibrium cutoff  $\bar{c}$  divides all individuals into two groups: those with  $\theta \leq \bar{c}$  who do invest in prevention and those with  $\theta > \bar{c}$  who don't.

### 3.2 Competitive Insurance Market

Assume now a competitive insurance market. As price competition drives profits down to zero, insurance premiums are actuarially fair. Then, those individuals with prevention cost smaller than the fair premium would invest in prevention and all other individuals would prefer to purchase insurance.

Again, the difference in expected utility is positive at  $c = 0$ , negative at  $c = L$  and decreasing in  $c$ . The equilibrium, say  $\underline{c}$ , is then determined by

$$U(W - \underline{c}) = U(W - p(\underline{c})L) \quad (17)$$

which gives

$$\underline{c} = p(\underline{c})L. \quad (18)$$

In a competitive insurance market, insurers make zero expected profits and all individuals with  $\theta \leq \underline{c}$  invest in prevention while all others ask for full insurance at the fair price  $\underline{P} = p(\underline{c})L$ . Note that the equilibrium does not include any risk premium. This is because all individuals, being now fully protected from risk of loss, behave as if they were risk neutral.

In order to compare market equilibria given by the cutoff cost levels  $\underline{c}$  and  $\bar{c}$ , define  $\lambda(c) \equiv c - p(c)L$ . Since  $\lambda'(c) = 1 - p'(c)L > 0$ , a higher equilibrium cutoff cost level means a higher  $\lambda(\cdot)$ . Note that  $\lambda(\underline{c}) = 0$  while  $\lambda(\bar{c}) = \pi(p(\bar{c})) > 0$ . As a result,  $\lambda(\bar{c}) > \lambda(\underline{c})$  and thus  $\bar{c} > \underline{c}$ ,

i.e. in a situation without insurance there is a higher proportion of individuals investing in prevention. Hence, a competitive insurance market leads to a lower overall prevention level compared to a situation without insurance. This result is very plausible since it does not make economic sense for individuals to bear excessive cost of loss prevention activities if there is ultimately a cheaper way of cancelling out risk: a fair insurance contract.

### 3.3 Insurance Monopoly

When we introduce a monopolistic insurer into this market, note that by setting a premium for (full) insurance that differs from the cutoff price  $\bar{c} = p(\bar{c})L + \pi(p(\bar{c}))$  the insurer may influence the proportions of high and low risks in the population because the availability of insurance at a given price influences investments in prevention. Of course, at a price of

$$\bar{P} = p(\bar{c})L + \pi(p(\bar{c})) \quad (19)$$

the insurer would gain all individuals who did not invest in prevention in a situation without insurance. The proportions of high and low risk types would not change. His expected profit would then be

$$E\Pi(\pi(\cdot), p(\bar{c})) = \pi(p(\bar{c})) \cdot \int_{\bar{c}}^L f(\theta) d\theta, \quad (20)$$

where the first term is expected profit per policy (which is also the individuals' reservation price given the cutoff level  $\bar{c}$ ) and the second is the number of policyholders. But this premium might not be optimal. The insurer might offer a smaller price and then gain more policyholders (which is, of course, associated with less expected profit per policy). If he offered a smaller price, say  $\hat{c} < \bar{c}$ , less people would invest in prevention and the policyholders' risk of loss would increase, i.e.  $p(\hat{c}) > p(\bar{c})$ . This, in turn, affects the risk premium the individuals are prepared to pay to transfer their risk so that  $\pi(p(\hat{c})) \neq \pi(p(\bar{c}))$ .<sup>9</sup> Indeed, the problem of the insurer is to find the optimal trade-off between the number of policyholders and expected profit per policy. The problem of the insurer is to maximize expected profit

$$\max_c E\Pi(\pi(\cdot), p(c), \lambda(c)) = \lambda(c) \cdot \int_c^L f(\theta) d\theta, \quad c \in [\underline{c}, \bar{c}] \quad (21)$$

where the first factor is the premium loading, i.e. net profit per policy  $\lambda(c) \equiv c - p(c)L$ , and the second is the proportion of high risks demanding insurance coverage.<sup>10</sup> The optimal

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<sup>9</sup> Note that reducing the premium means reducing the loading but increasing probability of loss for policyholders.

<sup>10</sup> The optimal cutoff  $c$  lies in the interval  $[\underline{c}, \bar{c}]$  because (1) offering a lower premium than the actuarially fair premium would imply negative expected profits for the insurer and so the fair premium  $\underline{c}$  is somewhat

premium  $P(\cdot)$  is the premium that satisfies

$$P(c^*) = p(c^*)L + \lambda(c^*), \quad \text{where } c^* \equiv \arg \max_{c \in [\underline{c}, \bar{c}]} E\Pi(\pi(\cdot), p(c), \lambda(c)), \quad (22)$$

where the first part of the premium is the actuarial value of the policy given the cutoff  $c^* \in [\underline{c}, \bar{c}]$  and the second is the loading  $\lambda(c^*) \in [0, \pi(p(\bar{c}))]$  representing net expected profit per policy.<sup>11</sup> The first order condition is<sup>12</sup>

$$\frac{\lambda'(c)}{\lambda(c)} = \frac{f(c)}{1 - F(c)}. \quad (23)$$

In order to interpret the f.o.c., we may rewrite it in terms of elasticity. Then the optimal pricing rule is

$$\varepsilon_{\lambda(c), x(c)} = \frac{\frac{\lambda'(c)}{\lambda(c)}}{-\frac{x'(c)}{x(c)}} = 1. \quad (24)$$

In the profit maximum, the elasticity of the net price of insurance with respect to the number of purchasers equals unity. This is a well-known microeconomic result whereby the optimal markup-over-cost pricing strategy for a monopolist is to set a price at which elasticity of demand with respect to the markup is unitary. The optimal premium  $P(c^*) \in [\underline{c}, \bar{c}]$  takes into account the trade-off between the number of purchasers determined by the group of individuals without prevention and the expected profit per policy. Since the premium consists of the actuarial value of the policy plus some fee not higher than the individuals' risk premium, all individuals fully insure themselves.

As in the standard one-insurer-one-representative-customer model, our results imply a positive relationship between the insurer's expected profit and the consumers' degree of risk aversion. However, in our setting the insurer's expected profit also depends on the distribution of prevention cost.

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the lowest potential price implying zero expected profits, and (2) offering a higher premium than the one corresponding to the cutoff in a situation without insurance would imply no demand. This is because the premium would be higher than the individuals' reservation price given the risk of loss.

<sup>11</sup> Note that due to  $\lambda(\underline{c}) = 0$ ,  $\lambda(\bar{c}) = \pi(p(\bar{c}))$  and  $\lambda'(c) > 0$ , we must have  $\lambda(c) \in [0, \pi(p(\bar{c}))] \forall c \in [\underline{c}, \bar{c}]$ . Note also that  $\pi(p(c)) \geq \lambda(c) \forall c \in [\underline{c}, \bar{c}]$ . This is because the difference  $(\pi(\cdot) - \lambda(\cdot))$  is positive at  $\underline{c}$ , zero at  $\bar{c}$  and, since the risk premium  $\pi(\cdot)$  is a concave function in  $p$ , it is either decreasing in  $c$  or first increasing and then decreasing. As a result, the difference is never negative for any  $c \in [\underline{c}, \bar{c}]$ .

<sup>12</sup> The first derivative of the expected profit function is positive at  $c = \underline{c}$  but not necessarily negative at  $c = \bar{c}$ . Therefore, the second order condition is fulfilled or a corner solution results depending on the distribution of prevention cost  $F(\theta)$ .

## 4 Imperfect Prevention, Underwriting Cost, and Risk Exposure

This section studies two important extensions of our framework and discusses our results. First, we allow for prevention to be imperfect; second, we introduce underwriting cost into our model and show how this affects the first-order conditions. We then compare a competitive to a monopolistic insurance market from a risk exposure viewpoint. Interestingly, our results are consistent with controversially discussed empirical evidence on risk exposure.

### 4.1 Imperfect Prevention

In many individual decision problems, a possible loss cannot be perfectly avoided by a preventive investment. A part of the risk, call it the direct risk part, is eliminated via prevention but a residual (indirect) risk remains. This risk depends on the behavior of other individuals. These effects result, for example, with investments in prevention relating to certain catastrophe, health and liability risks. For instance, the mounting of a sprinkler system in a company decreases the risk of fire for the neighboring company. Many health risks are imposed by others. Smokers create cancer risks not only for themselves, but also for non-smokers via passive smoking. Individual investment to stop smoking benefits non-smokers, and therefore such an investment generates an externality. Risk interdependencies are especially pronounced in networks. Internet communities and social networks as well as traffic networks are prone to such externalities.<sup>13</sup> In a traffic network, the direct risk is the risk that a careless driver causes an accident himself, the indirect risk is the risk that the driver is involved in an accident caused by somebody else. He can only reduce the first risk by driving more cautiously, but he can do nothing about the second one. However, careful driving behavior diminishes the indirect risk for others.<sup>14</sup>

Following 3.1, let again  $p(c)$  be the indirect risk in the population. Let  $q \in (0, 1)$  be the direct risk. We assume that both risks are independent and that only the direct risk part is eliminated via preventive effort (i.e.  $q$  is reduced to  $q = 0$ ). However, the indirect part, given by  $p(c)$  remains. Hence, an individual without prevention faces the overall risk  $q + (1 - q)p(c)$  given that  $p(\cdot)$  and  $q$  are independent.

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<sup>13</sup> See Kearns (2005).

<sup>14</sup> For more examples, see Avery, Heymann, and Zeckhauser (1995).

#### 4.1.1 No Insurance

Following the same argument as in 3.1 above, let us first consider the case without insurance. Then, to find the equilibrium, we can write expected utility with an investment in prevention as

$$EU_2 = p(c)U(W - c - L) + (1 - p(c))U(W - c) \quad (25)$$

while expected utility without prevention is

$$EU_3 = qU(W - L) + (1 - q)\{p(c)U(W - L) + (1 - p(c))U(W)\}, \quad (26)$$

where  $p(c)$  is defined as in 3.1 above. We may rewrite (25) and (26) so that

$$EU_2 = U(W - c - p(c)L - \pi(p(c))) \quad (27)$$

and

$$EU_3 = U(W - qL - (1 - q)p(c)L - \pi(q + (1 - q)p(c))). \quad (28)$$

Following section 3.1, in equilibrium, excess of expected utility is zero, so that the equilibrium  $\bar{c}_q$  is given by

$$\bar{c}_q = q(1 - p(\bar{c}_q))L + \pi(q + (1 - q)p(\bar{c}_q)) - \pi(p(c)) \quad (29)$$

so that individuals with  $\theta \leq \bar{c}_q$  are low risk types investing in prevention while those with  $\theta > \bar{c}_q$  are high risk types who don't.

#### 4.1.2 Competitive Insurance Market

Assume a competitive insurance market where insurers offer fair premiums and make zero expected profits. Since prevention does not fully protect individuals from risk of loss (the indirect risk remains), there are different insurance contracts for different risk types (we assume perfect information). Given a cutoff  $c$ , the fair price in case an individual does not invest in prevention is given by

$$P_{np} = \{q + (1 - q)p(c)\}L \quad (30)$$

while the fair price for covering the direct risk  $q$  under loss prevention is, of course,

$$P_p = p(c)L. \quad (31)$$

As the price for insurance is actuarially fair, all individuals ask for insurance. Again, the excess of expected utility function is positive at  $c = 0$ , negative at  $c = L$  and decreasing. The equilibrium is where the excess is zero, i.e. at

$$U(W - c - p(c)L) = U(W - qL + (1 - q)p(c)L) \quad (32)$$

which gives

$$\underline{c}_q = q(1 - p(\underline{c}_q))L. \quad (33)$$

The equilibrium is easily interpreted. All individuals with prevention cost less than the difference in insurance premia,  $P_{np} - P_p = q(1 - p(\cdot))L$ , invest in prevention while the others don't.

### 4.1.3 Insurance Monopoly

A monopoly insurer may again influence the shares of high and low risks in the population via his pricing strategy. Of course, individuals with prevention cost higher than the insurance premium would demand insurance coverage. However, those individuals with prevention cost lower than the premium face the problem whether to invest in prevention (and reduce only part of the risk) or to buy insurance (and eliminate the risk completely). As there is perfect information, the insurer designs different premiums for different risk types. Given that prevention is imperfect but insurance covers the full risk of loss, what is the insurer's optimal pricing strategy given the density  $f(\theta)$  in the potential customer pool?

The insurer faces the profit maximization problem

$$\max_c E\Pi_q(\pi(\cdot), p(c)) = [c - p(c)L] \cdot \int_0^c f(\theta)d\theta + [c - qL - (1 - q)p(c)L] \cdot \int_c^L f(\theta)d\theta. \quad (34)$$

The first derivative of the expected profit function is

$$E\Pi'_q(c) = [1 - p'(c)L](1 - x(c)) + [1 - (1 - q)p'(c)L] \cdot x(c) + [q(1 - p(c))L] \cdot f(c) \quad (35)$$

which is positive for all  $c \in (0, L)$ . Therefore the insurer's expected profit is maximized at the corner solution  $c^* = \bar{c}_q$  and the insurer offers each risk group its reservation price given the cutoff. Therefore, the premia for low and high risks are

$$P_p^* = p(\bar{c}_q)L + \pi(p(\bar{c}_q)) \quad (36)$$

and

$$P_{np}^* = [q + (1 - q)p(\bar{c}_q)]L + \pi(q + (1 - q)p(\bar{c}_q)). \quad (37)$$

As a result, the prevention level in the population is unchanged compared to a situation without insurance. The only difference is that rent is transferred from consumers to the insurer. In this way, the insurer extracts maximum consumer rent. Note that this result holds for any given  $q$  and any distribution of prevention cost in the population of consumers.

## 4.2 The Model with Underwriting Cost

In this section, we allow for transaction cost of underwriting insurance. Underwriting cost may be either proportional to the insurance premium or it may only be a fixed cost per policy. The first type of underwriting cost reflects the fact that certain expenses of the insurer are proportional to expected loss or the premium. These expenses include sales commissions for insurance agents, premium taxes, or cost of claims adjustment. A fixed underwriting cost per policy reflects costs independent of the insurance premium as for instance in case of administrative and other fixed cost.

Note first the case when underwriting cost consists of a fixed cost  $\gamma > 0$  per policy. Provided that the fixed cost is not too high (which may imply underwriting loss), the optimal  $P(c^*)$  reflects the fixed cost. The problem of the insurer is to maximize expected profit

$$\max_c E\Pi(p(c), \gamma) = [c - \gamma - p(c)L] \cdot \int_c^L f(\theta) d\theta, \quad (38)$$

and the f.o.c. becomes

$$\frac{1 - p'(c)L}{c - \gamma - p(c)L} = \frac{f(c)}{1 - F(c)}. \quad (39)$$

As a result, taking underwriting cost into account, the profit maximum is again attained when the elasticity of the net price of insurance with respect to the number of policyholders equals unity.

If, in contrast, transaction cost  $\lambda$  of underwriting insurance is a cost proportional to the policy premium, we have underwriting cost

$$UC = \lambda \cdot Premium, \quad (40)$$

where  $0 \leq \lambda < 1$ . The insurer then wishes to maximize expected profit

$$E\Pi(p(c), \lambda) = [(1 - \lambda)c - p(c)L] \cdot \int_c^L f(\theta) d\theta, \quad (41)$$

The f.o.c. can then be written as

$$\frac{1 - \lambda - p'(c)L}{(1 - \lambda)c - p(c)L} = \frac{f(c)}{1 - F(c)}. \quad (42)$$

When prevention is imperfect, as in section 4.1.3 above, the insurer's expected profit is as follows. In case of a fixed cost  $\gamma$  per policy

$$E\pi_q^*(\gamma) = [\pi(p(\bar{c}_q)) - \gamma] \cdot \int_0^{\bar{c}_q} f(\theta) d\theta + [\pi(q + (1 - q)p(\bar{c}_q) - \gamma)] \cdot \int_{\bar{c}_q}^L f(\theta) d\theta, \quad (43)$$

where we assume that  $\gamma < \pi(p(\bar{c}_q))$  for simplicity. Note that expected profit is maximal since (34) does not change when we introduce a fixed cost  $\gamma$  as long as  $\gamma$  is not 'too high'. In case of a cost proportional to the premium, we obtain

$$E\pi_q^*(\lambda) = \left[ (1-\lambda)\bar{c}_q - p(\bar{c}_q)L \right] \cdot \int_0^{\bar{c}_q} f(\theta)d\theta + \left[ (1-\lambda)\bar{c}_q - qL - (1-q)p(\bar{c}_q)L \right] \cdot \int_{\bar{c}_q}^L f(\theta)d\theta, \quad (44)$$

which results because (34) is positive again for all  $c \in [\underline{c}, \bar{c}]$ .

### 4.3 Risk Exposure

In our model, the insurer's pricing strategy affects the overall risk exposure in the population. When we look at consumer welfare  $CW$ , i.e. the sum of all consumers' utility, in a situation without insurance

$$CW(c) = \int_0^c U(W - \theta)f(\theta)d\theta + EU_0(c) \cdot x(c), \quad (45)$$

we can evaluate its slope at the equilibrium prevention level  $c = \bar{c}$ . This gives

$$CW'(c)|_{c=\bar{c}} = f(\bar{c}) \cdot \underbrace{\Delta(\bar{c})}_{=0} - x(\bar{c}) \cdot p'(\bar{c}) \cdot [U(W) - U(W - L)] > 0 \quad (46)$$

which tells us that the welfare function has a positive slope at  $c = \bar{c}$ .<sup>15</sup> Hence, consumer welfare is not maximized in the no insurance market equilibrium with the marginal consumer's prevention cost  $\bar{c}$ . The prevention level is 'too low' from a social welfare viewpoint, that is, the overall risk exposure is actually 'too high' compared to the social optimum. This result is intuitive, since the equilibrium outcome happens to be inefficient due to prevention externalities.

By introducing insurance, the risk allocation is improved as risk is transferred from risk-averse consumers to some risk-neutral party. Comparing market structures, however, we see that in a competitive insurance market under interdependent risks, overall prevention is less than in a situation without insurance (i.e.  $\underline{c} < \bar{c}$ ). Therefore, consumers are exposed to higher risk in a competitive market setting compared to a situation without insurance or with a monopolistic insurer (i.e.  $p'(c) < 0$  implies  $p(\underline{c}) > p(\bar{c})$ ). This implies that a monopolistic insurance market is superior to a competitive market from an overall risk exposure viewpoint. Interestingly, seminal empirical studies in Europe find that insurance monopolies indeed tend to affectively reduce risk exposures and thus claims levels are also found to be lower. All these studies suggest that a market with a state-monopoly for insurance leads to a remarkably

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<sup>15</sup> Note that the welfare function is concave since  $CW'(c)$  is positive at  $c = 0$  and negative at  $c = L$ .



higher overall investment in loss prevention, and accordingly, a lower overall risk exposure of consumers.<sup>16</sup> Our model suggests a theoretical explanation to these empirical findings.<sup>17</sup>

## 5 Concluding Remarks

Common economic models of insurance theory assume that insurers cannot modify the loss distribution. Seminal studies treat loss distributions of individuals as exogenously given. In these models, insurers know the proportions of high and low risks but not individual risk type. Insurers then offer a self-selection design which makes customers reveal their risk type. However, individual self-protection often affects the loss probabilities faced by others.

We make two contributions to the insurance literature. Firstly, we argue that individual self-protection may depend upon insurance prices and address the question why insurers tend to invest in understanding the loss process. For instance, auto insurers often study how crashes occur and the consequences, and property insurers study fire exposures and fire safety mechanisms. We argue that insurers may indeed have an interest in understanding and eventually affecting underlying loss propensities of economic agents in order to increase expected profits. The rationale is as follows. Since the (market equilibrium) price for insurance coverage affects the individuals' prevention decisions, different prices will lead to different prevention levels in society. Different prevention levels tend to result in different overall risk, which again affects the individuals' willingness to pay for insurance and thus the potential extent of insurers' expected profits. Secondly, we focus on a monopolistic insurance market and show how the insurer can take advantage of risk interdependencies in order to determine loss propensities in a sense optimal for him. Premiums are optimally chosen to reflect the trade-off between the number of policyholders and expected profit per policy.

From our theoretical model, we may derive the following hypotheses: (1) the higher the policyholders' risk aversion, the higher will tend to be a monopolistic insurer's expected overall profit, (2) the lower the price for insurance (in an interdependent risk market), the less people tend to invest in loss prevention and the higher the risk of loss faced by policyholders will be, and (3) policyholders are exposed to higher risk in a competitive insurance market compared to a monopoly. While there is commonly no doubt about the first hypothesis being generally fulfilled, the second is less obvious. Empirical evidence from Danzon and

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<sup>16</sup> See Epple and Schäfer (1996) and Felder (1996) for Germany as well as von Ungern-Sternberg (1996), and Jametti and von Ungern-Sternberg (2005) for Switzerland and other European countries.

<sup>17</sup> Note that a monopolistic insurer might even implement the socially optimal prevention level by using premium discrimination in a market where insurance is compulsory. Hence, introducing compulsory insurance may make the effect even stronger. See Hofmann (2007).

Harrington (2001) provides support for the second hypothesis. Danzon and Harrington (2001) develop and test the hypothesis that, in the long run, rate suppression in the U.S. workers' compensation insurance market exacerbates loss growth, leading to higher overall losses. They find a clear tendency for rate suppression to increase injury rates.<sup>18</sup> Although other factors we have not discussed here might have an impact of this observed tendency, our model provides a tentative theoretical explanation of this effect. Finally, as discussed above in the risk exposure section, our third hypothesis is also found to be consistent with empirical evidence in Europe suggesting that policyholders face higher overall risk in a competitive insurance market compared with a monopolistic market.

It is worth noting that our analysis exhibiting negative externalities presents no restriction. We could in the same way consider positive externalities in our model. Remember that there was a negative externality associated with *not* investing in prevention, i.e. the increased risk to others.<sup>19</sup> Of course, this externality might also be reversed leading to reduced risk to others. In other words, a positive externality would imply that an individual investment in prevention might actually *increase* risks of others. An illustrative example is a house owner living in a residential where most families have installed costly burglar alarm systems in their houses, but he has not. Then, the risk of robbery in his less well-protected home may become more likely. In the case of terrorism insurance, for instance, this kind of scenario involving positive externalities seems to be especially important since terrorists commonly respond to increased protection of one target by turning to more vulnerable ones.<sup>20</sup> Our results would partly be reversed. For instance, in a setting without insurance, the prevention level in the population would be 'too high' from a social welfare viewpoint. The basic mechanism, however, remains and insurers' pricing strategies affect prevention and therefore will have an impact on loss propensities in the population.

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<sup>18</sup> See Danzon and Harrington (2001), p. 21.

<sup>19</sup> It is a question of definition how we treat the externality. Alternatively, we may say that in our model externalities associated with an investment in prevention are positive since such investment actually reduces risk to others.

<sup>20</sup> See, for instance, Enders and Sandler (2004) or Keohane and Zeckhauser (2003). A more detailed discussion about self-protection externalities referring to terrorism is provided by Lakdawalla and Zanjani (2005) who investigate the rationale for public intervention into the terrorism insurance market.

## References

- AVERY, C., J. HEYMANN, AND R. ZECKHAUSER (1995): "Risks to Selves, Risks to Others," *American Economic Review*, 85, 61–66.
- DANZON, P., AND S. HARRINGTON (2001): "Workers' Compensation Rate Regulation: How Price Controls Increase Costs," *Journal of Law and Economics*, 44(1), 1–36.
- DOHERTY, N. A. (1975): "Some Fundamental Theorems of Risk Management," *Journal of Risk and Insurance*, 42, 447–460.
- EHRlich, I., AND G. S. BECKER (1972): "Market Insurance, Self-Insurance, and Self-Protection," *Journal of Political Economy*, 80, 623–648.
- ENDERS, W., AND T. SANDLER (2004): "What do we know about the Substitution Effect in Transnational Terrorism?," in *Researching Terrorism: Trends, Achievements, Failures*. Frank Cass.
- EPPLE, K., AND R. SCHÄFER (1996): "The Transition from Monopoly to Competition: The Case of Housing Insurance in Baden-Württemberg," *European Economic Review*, 40(3-5), 1123–1131.
- FELDER, S. (1996): "Fire Insurance in Germany: A Comparison of Price-Performance between State Monopolies and Competitive Regions," *European Economic Review*, 40(3-5), 1133–1141.
- FRIEDMAN, M., AND L. SAVAGE (1948): "The Utility Analysis of Choices involving Risk," *Journal of Political Economy*, 56(4), 279.
- HOFMANN, A. (2007): "Internalizing Externalities of Loss-Prevention through Insurance Monopoly: An Analysis of Interdependent Risks," *Geneva Risk and Insurance Review*, 32, 91–111.
- JAMETTI, M., AND T. VON UNGERN-STERNBERG (2005): "Assessing the Efficiency of an Insurance Provider - A Measurement Error Approach," *Geneva Risk and Insurance Review*, 30, 15–34.
- KEARNS, M. (2005): "Economics, Computer Science, and Policy," *Issues in Science and Technology*, 21, 37–47.
- KEOHANE, N. O., AND R. J. ZECKHAUSER (2003): "The Ecology of Terror Defense," *Journal of Risk and Uncertainty*, 26, 231–249.
- KUNREUTHER, H., AND G. HEAL (2003): "Interdependent Security," *Journal of Risk and Uncertainty*, 26, 231–249.

- LAKDAWALLA, D., AND G. ZANJANI (2005): “Insurance, Self-Protection and the Economics of Terrorism,” *Journal of Public Economics*, 89, 1891–1905.
- PRATT, J. W. (1964): “Risk Aversion in the Small and in the Large,” *Econometrica*, 32, 122–136.
- ROTHSCHILD, M., AND J. STIGLITZ (1970): “Increasing Risk I. A Definition,” *Journal of Economic Theory*, 2, 225–243.
- (1976): “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information,” *Quarterly Journal of Economics*, 90, 629–650.
- SCHLESINGER, H., AND E. VENEZIAN (1986): “Insurance Market with Loss-Prevention Activity: Profits, Market Structure, and Consumer Welfare,” *RAND Journal of Economics*, 17, 227–238.
- SOENMEZ, S., AND A. GRAEFE (1998): “Influence of Terrorism Risk on Foreign Tourism Decisions,” *Annals of Tourism Research*, 25(1), 112–144.
- STIGLITZ, J. (1977): “Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market,” *Review of Economic Studies*, 44, 407–430.
- VON UNGERN-STERNBERG, T. (1996): “The Limits of Competition: Housing Insurance in Switzerland,” *European Economic Review*, 40(3-5), 1111–1121.