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# The Doping Threshold in Sport Contests: An Auction-based Approach\*

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September, 2010

## Abstract

We analyze the influence of the number of competitors, the costs of doping and the distribution of talents on doping behavior. In an n-player strategic game modeled as an all-pay auction, the players have private information about their talent and the amount of doping. The main finding of the analysis is the existence of a doping threshold. In the leading case only athletes with a talent above this threshold dope. The value of the doping threshold is increasing in the number of participating athletes and the size of the relative doping costs. The amount of doping the players take increases with the talent. In addition, the more the distribution of talents is skewed to the left the smaller is the doping threshold.

Keywords: Auctions, Contests, Doping.  
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# 1 Introduction

In sport contests a multitude of different behaviors and strategies of athletes can be observed. The major objectives of elite athletes are victories and positive results at important sport competitions, the World Champion title in their specific discipline, or even a gold medal at the Olympic Games. To achieve such goals athletes have to push their physical and mental limits far and farther out. Past and present incidents show that for achieving their dreams some athletes even cross the border and take performance enhancing drugs. It has to be mentioned that doping is by far not a recent phenomenon. Written sources show that already ancient Greek athletes doped by using stimulants (Verroken, 2005).

The ongoing commercialization of elite sports has led to a radical increase in the sums of money paid out in prizes to athletes and the possible contracts to be gained, including sponsoring contracts. This has probably increased the incentive to resort to doping substances and caused the doping problem to spread. A typical example of this development is found in competitive cycling.<sup>1</sup> So far the anti-doping regulations of the IOC and the efforts of the national anti-doping organizations have been unable to establish a doping-free environment for sports competitions. For a decade now sports events like the Tour de France have suffered from negative advertising primarily associated with doping cases and have only marginally benefited from the suspense of the sport and the prowess of its athletes. This has caused a significant loss in spectator interest and the withdrawal of sponsor funding. From a game-theoretical perspective an analysis of the doping issue presents a typical example of the prisoner's dilemma.

Many researchers have investigated the doping problem and have explored actual or promising anti-doping regulations (e.g., Kräkel (2007)). A representative example of such an enquiry is the paper by Berentsen (2002), which analyzes the anti-doping regulations of the IOC in a strategic two-player game. It gives a fundamental overview of the doping problem and presents a ranking-based sanction scheme for attaining a no-doping equilibrium that is incentive compatible and less costly than the actual sanction scheme. However, before embarking on a consideration of anti-doping measures, a better understanding of the various incentives that motivate athletes and the mechanisms operating in competition should first be gained. Most sports involve competition between more than two athletes, very often between teams. An athlete's performance in a competition will depend on his form on that day, his talent, the standard of training he has, and other unpredictable factors. Each athlete has his own strengths and weaknesses. Aspects such as these have only partially been considered in previous studies.

In this paper we want to investigate the rationale of doping in an  $n$ -player game with heterogeneous players. Moreover, we take step backwards and abstract from an anti-doping agency

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<sup>1</sup> Ever since the Festina scandal in 1998, the Tour de France has appeared repeatedly in the headlines because of new doping scandals. In addition to doped favorites also large-scale police raids seizing needles and doping substances covered the media. See Dilger et al. (2007).

that punishes positively tested athletes.<sup>2</sup> We analyze the influence of the number of competitors on optimal doping behavior. A further aspect is that nowadays better and stronger doping substances are being developed. The pharmaceutical industry facilitates this by continually releasing new medications onto the market that extend the limits of performance. Then, the relation between the cost of the doping substances and the winners' bonuses is decisive. Moreover, the opponents' performances are a determining factor in the optimal choice of behavior that an athlete will make. The aim of the paper is to gain a more comprehensive understanding of doping behavior where the number of competitors is larger than two and the players possess different talents.

We show that even in an environment where doping is not prohibited not all players take performance-enhancing drugs. It can be shown that the prisoner's dilemma in two-player doping games (e.g., Daumann (2003)) only partly applies in an n-player game. The main finding of the paper is that for the majority of underlying parameter values a doping threshold exists, which depends on the number of players, the costs of doping and the distribution of players' talents. In the standard case there are two categories of behavior. Strong players dope, and players beneath the doping threshold have no incentive to dope. Furthermore, in our model the optimal level of doping increases with the athlete's talent. The most talented athlete will dope the most because the optimal doping behavior of his competitors forces him to dope more to defend his advantage.

Another finding is that the most talented athlete will win the competition. Doping does not change the rank order of the competition. At the same time, however, doping imposes huge costs on athletes above the doping threshold. This would speak in favor of the introduction of a control mechanism that would inhibit athletes from doping. A doping free environment would increase an athlete's utility, and not simply incur expenses in monitoring costs.

Our model and the results shed new light on the behavior of athletes. An athlete will not abstain from doping on purely ethical grounds, but rather because this behavior maximizes his expected return. The results of our model should be taken into account when agencies like the WADA<sup>3</sup> implement new anti-doping regulations. Designing optimal control mechanisms is not a simple task since controls and punishments change the incentives of the athletes.

The paper is structured as follows: In Section 2 we develop the doping model and formally derive the doping threshold, which describes the level of talent at which an athlete resorts to doping. In Section 3 we discuss the results of the doping model. Here, we show in a first step how the doping threshold is dependent on the number of athletes and on the costs of doping. Finally, the influence of the distribution of talents is analyzed. Section 4 summarizes and concludes.

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<sup>2</sup>This is comparable to a situation with toothless anti-doping regulations. For example, in situations where athletes can easily manipulate their test results.

<sup>3</sup>WADA is the abbreviation of World Anti-Doping Agency. For further informations about the anti-doping program see <http://www.wada-ama.org>.

## 2 The model

In a competition  $n$  athletes compete against each another. The winner receives prize money  $v$ . All athletes are risk-neutral and maximize the expected utility of the payoff. Players have private information about their talent  $t_i$ , for  $i = 1, \dots, n$ . The talent  $t_i$  is drawn from a given distribution  $F$ . The talents  $t_i$  are independent and identically distributed (i.i.d.). Where there is no doping, the most talented athlete wins the competition. The distribution of the talents is normalized to lie between 0 and 1. We assume a power function distribution  $F(t) = t^\alpha$ , where  $t \in [0, 1]$ . If  $\alpha = 1$  is chosen, then talent  $t_i$  is uniformly distributed between 0 and 1. For  $\alpha < 1$  the density of the talent distribution is the largest for small  $t$ 's. Where on the other hand for  $\alpha > 1$  there are relatively more talented players than untalented.

In the competition athletes have the possibility of improving their performance by doping. In contrast to other contributions, this paper investigates athletes' doping behavior when doping is permitted. Every athlete can dope to improve his performance capability in the competition. The athlete's performance  $p_i$  is thus a linear combination of his talent  $t_i$  and the amount of doping  $d_i$  he chooses to take:  $p_i = t_i + d_i$ . Doping substances are not free of charge and the athlete has to pay for his doping substances before the game begins. Since the amount of doping  $d$  is arbitrary, it is possible for a weaker athlete to beat a more talented athlete. Each player thus faces a trade-off between gaining a higher likelihood of winning through doping and the cost that this incurs. Each athlete is free to choose his personal doping amount and is not limited to the two discrete options of 'doping' or 'not doping'. This general assumption of the model is superior to a model with only two discrete decision possibilities because it also determines the optimal doping amount.<sup>4</sup>

The doping amount is equal to his performance  $p_i$  minus his talent  $t_i$ . If his performance corresponds to his talent, then the costs of doping are zero. This is reasonable, as the athlete in this case does not dope. Every athlete's doping cost function will therefore be a function of the difference  $p_i - t_i$ . The cost function is  $k(p, t_i) = c(p - t_i)$ , with marginal costs  $k'(p, t_i) = c$ . The linearity of the function ensures a closed-form solution. We assume that the costs of doping substances are the same for all athletes.

Since we are interested in  $n$  player contests, it is convenient to refer to the tools of auction theory. The doping game is similar to an all-pay auction where every player has to pay for their personal doping amounts. But in contrast to this kind of auction, it is the talent that the players possess that is variable, while the prize that they receive for winning (e.g., the personal valuation of winning) is constant for all players. The athlete's probability of winning depends on his chosen performance  $p$  and on the performance of the other athletes. The player only wins the competition if his  $p$  is greater than the  $p$  of all other players. We gain the probability of winning

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<sup>4</sup>The model can readily be adapted to apply to a binary decision between 'doping' and 'not doping'. This simplification does not change the qualitative results.

$G(x)$  from the first order statistic  $Y_{(1:n-1)}$  of the remaining players.<sup>5</sup> Given the assumed power function distribution the probability of having the largest talent is  $G(t) = F(t)^{n-1} = t^{\alpha(n-1)}$ .

## 2.1 The symmetric Nash equilibrium

In order to find the symmetric Nash equilibrium in the doping game, we apply the usual approach used in auction theory. First, we assume that in a symmetric Nash equilibrium a player with talent  $t$  chooses the performance  $p(t)$  and then formulate the expected return for this player (using the first order statistic). Every player can deviate from this equilibrium strategy by choosing another performance. Moreover, we assume that the equilibrium performance function is increasing in  $t$ . If this is the case, then it does not make sense to choose a performance lower than  $p(0)$  or higher than  $p(1)$ . In the first instance, one would never win, and, in the second instance, one would always win but have to pay too much. For this reason, deviations from the equilibrium performance function can be modeled as follows: An athlete with talent  $t$  who pretends to have a different talent  $x$  chooses the associated performance  $p(x)$  in the competition.

The utility function of an athlete with talent  $t$  who pretends to have talent  $x$  is the product of the prize  $v$  multiplied by the probability of winning of a player with talent  $x$ , minus the cost of doping in order to achieve the performance  $p(x)$ . His utility function is thus:  $u(t, x) = vG(x) - k(p(x), t)$ . The athlete will choose the  $x$ , which maximizes his utility. To receive the optimum we differentiate the utility function with respect to  $x$  and set this equal to zero. We obtain the FOC:  $vG'(x) - k'_p(p(x), t)p'(x) \stackrel{!}{=} 0$ . The optimal behavior of an athlete with talent  $t_i$  is to imitate the strategy of an athlete with talent  $x$  so that the FOC holds.

In the Nash equilibrium, the incentive compatibility (IC) constraint has to be satisfied for every possible  $t$ . There must be no gain in deviating from the equilibrium strategy. The IC constraint is satisfied if  $u(t, t) \geq u(t, x)$  for all  $t, x$ . We assume that when the athlete is indifferent between  $u(t, t)$  and  $u(t, x)$ , the athlete will undertake the equilibrium strategy  $t$ . In a symmetric equilibrium, the optimal  $x$  corresponds to the athlete's true talent  $t$ . Therefore, we insert  $x = t$  and  $p(x) = p(t)$  into the FOC and get  $vG'(t) - k'_p(p(t), t)p'(t) = 0$ . This condition states that the expected marginal return ( $vG'(t)$ ) has to be equal to the marginal costs of increasing the winning probability ( $k'_p(p(t), t)p'(t)$ ). Our assumption of a linear doping cost function implies that an athlete with talent  $t_i$  is indifferent between his equilibrium performance  $p(t_i)$  and all other  $p(t)$ 's for which the nonnegativity constraint of doping is not binding.<sup>6</sup>

<sup>5</sup>For further information on order statistics see David and Nagaraja (2003).

<sup>6</sup>This are particular circumstances and have to be kept in mind when investigating the equilibrium. If we would instead use a doping cost function with slightly decreasing marginal costs or even a quadratic doping cost function then the single crossing property is guaranteed. For example, assume that the marginal cost function is:  $c(t) = c \exp(-at)$ , where  $a$  is very small. As  $a$  goes to zero the marginal costs go to  $c$  ( $\lim_{a \rightarrow 0} c(t) = c$ ). The eventual performance function is a continuous function of  $c(t)$ . This implies that the limit of the equilibrium performance function is equal to the performance function under constant marginal costs using the continuous mapping theorem.

Solving for  $p'(t)$  gives us Equation (1) for the assumed linear cost function  $k(\cdot)$ .

$$p'(t) = \frac{vG'(t)}{k'_p(p(t), t)} = \frac{vG'(t)}{c}. \quad (1)$$

From Equation (1) we see that only the ratio of the prize money and marginal costs matters and not the absolute values. Therefore, we define  $w$  as the ratio ( $w \equiv v/c$ ), and we get  $p'(t) = wG'(t)$ .

So far we have not taken into account the nonnegativity constraint on the amount of doping an athlete can choose. In equilibrium the constraint  $p(t) \geq t$  has to hold for every possible  $t$ . Solving Equation (1) with respect to the performance function  $p(t)$  without taking account of the constraint can result in a performance function that runs partly below the underlying talent.

Figure 1 depicts the performance function when the constraint would be ignored ( $\alpha = 1, n = 10, w = 2$ ).<sup>7</sup> For athletes with a talent less than 0.917 the performance function (red) is smaller than the underlying talent. Hence, the constraint is violated. One could argue that augmenting the performance function of the red part to the talent solves the problem. It is important to understand why the proposed solution (thick dark blue line) in the figure is not an equilibrium.

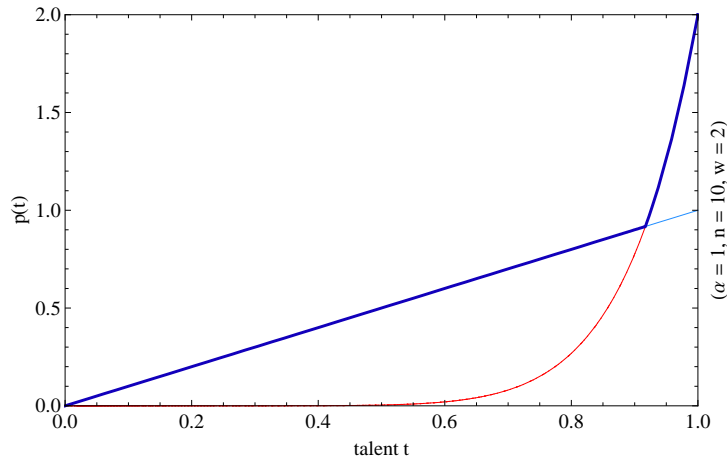


Figure 1: The nonnegative doping constraint

The reason is that all athletes optimize their behavior given the behavior of all other athletes. Augmenting the performance function to the 45-degree line increases competition for athletes just to the left of the former intersection. This athletes will increase their performance by doping so that it is no more profitable for athletes with lower talents to imitate their performance. This will also change the behavior of the athletes on the right side of the intersection. Hence, the nonnegativity constraint plays an important role in the doping game.

<sup>7</sup>The formula of the performance function not taking into account the nonnegativity constraint is:  $p(t) = wt^{\alpha(n-1)}$ .

To ensure that a player's performance is equal to or greater than his talent, we borrow the concept of the *reserve price*, which is used in auctions.<sup>8</sup> In our model we refer to the *reserve price* as the *reserve talent*. The introduction of a reserve talent ensures that the athletes with  $t < t^*$ , show their true talent, such that  $p(t)$  is no longer smaller than  $t$ . In contrast to an auction with a reserve price<sup>9</sup>, weaker athletes can still win, since their bid is the sum of their chosen doping quantity and their given talent. The introduction of the reserve talent thus ensures that weak athletes demonstrate their true talent in competition; that is, that their performance is not lower than their talent. The reserve talent must not become effective, where performance would outstrip talent.

In order to gain the performance function  $p(t)$  of an athlete with talent  $t$ , Equation (1) is then integrated from a not yet specified reserve talent  $t^*$  to  $t$ .

$$\int_{t^*}^t p'(z)dz = w \int_{t^*}^t G'(z)dz = w[F(t)^{n-1} - F(t^*)^{n-1}].$$

We solve the integral on the left-hand side of the last equation and take  $p(t^*)$  over to the right-hand side. Finally, we substitute the power distribution function and replace  $p(t^*)$  by  $t^*$ . Since the performance function at position  $t^*$  must exactly correspond to  $t^*$  ( $p(t^*) = t^*$ ). We thus gain the performance function of the athletes, given the underlying distribution of talents and there exists a reserve talent  $t^*$ . Additionally, we state the derivative with respect to  $t$ .

$$p(t) = t^* + w[F(t)^{n-1} - F(t^*)^{n-1}] = t^* + w[t^{\alpha(n-1)} - t^{*\alpha(n-1)}] \quad \text{for } t^* \leq t \leq 1, \quad (2a)$$

$$p'_t(t) = \alpha(n-1)wt^{\alpha(n-1)-1}. \quad (2b)$$

The next step is to determine whether there exists a positive reserve talent or not. If there exists a reserve talent we have to compute the exact value to obtain the doping behavior of the athletes. Therefore, the characteristic of the derivative (2b) of Equation (2a) is of importance. The derivative is the expected marginal gain divided by the marginal costs of doping. Taking the second derivative of Equation (2a) shows that the first derivative is increasing if  $\alpha(n-1) > 1$  and decreasing if  $\alpha(n-1) < 1$  for every  $t \in (0, 1]$ . This critical value is important to distinguish between different doping behaviors. The following cases exist.

<sup>8</sup>For a good overview on auctions and the concept of the reserve price see Krishna (2002).

<sup>9</sup>The reserve price is used in auction theory in order to close out bid valuations that are too low. The bids have to at least meet the reserve price, which prevents bidders with valuations lower than  $r$  from placing a bid. Since they have to bid  $r$  or more and therefore would suffer a loss in case that they win the auction. The existence of the reserve price has the consequence of shifting the bid function upwards, because the bid function for the value  $r$  must at least equate to  $r$  ( $\beta(r) = r$ ).



### The four doping cases:

- (i) *Strong athletes dope*
- (ii) *Weak athletes dope*
- (iii) *Nobody dopes*
- (iv) *Everybody dopes*

In which situation we actually are depends on the underlying parameter values of  $\alpha, n, w$ . In section A of the appendix the inequalities which have to hold under each of the four cases are displayed. Case (i) is the standard case which arises in the majority of possible underlying parameter values. The other three cases are special cases that emerge only under exceptional circumstances like very high/low costs of doping or an extreme skewed distribution of talents. Therefore, we will first center our investigations on case (i) and derive the proper reserve talent. At the end of Section 2 we will refer to the other cases.

We will show in Claim 1 for what parameter values athletes behave like in case (i). Moreover, we will show that for this values weak athletes have no incentive to dope in equilibrium. This means that the nonnegativity constraint of the doping behavior is binding for weak talents. Under the circumstances that the derivative (2b) is increasing in  $t$  the approach with the reserve talent can be applied. This is the case if  $\alpha(n-1) > 1$ . To make clear how to determine the optimal reserve talent we give the intuition of the proper reserve talent. The reserve talent  $t^*$  has to be chosen such that the performance function  $p(t)$  of athletes with talent above the reserve talent does not sink below the 45-degree line ( $p(t) \geq t$  for  $t \in [t^*, 1]$ ). Moreover,  $t^*$  must not be chosen too large, so that  $p(t)$  does not have a kink at position  $t^*$ . Therefore, the performance function must have the same slope like the 45-degree line at position  $t^*$ . This is the case if the slope of the performance function is equal to 1 at the proper reserve talent, which will be the doping threshold. The proof that this is a Nash equilibrium follows after we have derived the performance function in equilibrium.

To determine the optimal reserve talent  $t^*$  we set the derivative (2b) of the performance function equal to one. By solving the equation for  $t$  we gain the formula for the doping threshold  $t_{(\alpha, n, w)}^*$ . In Claim 1 we show under what parameter values the doping threshold exists. The resulting doping threshold is:

$$t_{(\alpha, n, w)}^* = [\alpha w(n-1)]^{\frac{1}{1-\alpha(n-1)}}. \quad (3)$$

**Lemma 1.** *The **doping threshold**  $t_{(\alpha, n, w)}^*$  defines the talent where the athlete is indifferent between doping or not doping. Athletes with talent beneath the threshold do not dope, and athletes with talent above this value take performance-enhancing drugs. The doping threshold depends on the distribution of talents, the number of competing athletes and the ratio of the prize money to the marginal cost of doping.*

*Claim 1.* We can differentiate between two cases if  $\alpha(n-1) > 1$ . The first one is case (i) where only strong athletes dope. If  $\alpha(n-1)w > 1$  then there exists a unique doping threshold  $t^* \in (0, 1)$  and athletes behave like in case (i).

The second one is case (iii) where nobody dopes. If  $\alpha(n-1)w < 1$  then there exists no doping threshold over the support  $t \in [0, 1]$  and athletes behave like in case (iii).

The proof of Claim 1:

When  $\alpha(n-1) > 1$  then  $p(t)$  is strictly increasing in  $t$ . Moreover, the derivative (2b) is continuous and strictly increasing. The derivative at  $t = 0$  is zero and if the derivative at  $t = 1$   $p'_t(1) = \alpha(n-1)w > 1$ . Then, it follows that there must exist a unique solution of the doping threshold  $t^*$  over the support  $\in (0, 1)$ . Using the same reasoning, there exists no doping threshold over the support  $t \in [0, 1]$  if  $p'_t(1) = \alpha(n-1)w < 1$ .

The next step is to derive the performance function and the doping function of the athletes. The athlete's performance function is gained by inserting Equation (3) in Equation (2a). The performance function is

$$p(t) = \begin{cases} t & \text{if } t < t^* \\ t^* + w[t^{\alpha(n-1)} - t^{*\alpha(n-1)}] & \text{if } t \geq t^*, \end{cases} \quad (4)$$

with the doping threshold:  $t_{(\alpha, n, w)}^* = [\alpha w(n-1)]^{\frac{1}{1-\alpha(n-1)}}$ .

The applied approach with the reserve talent to ensure the nonnegativity constraint is non-standard in the auction literature. Therefore, it remains to be shown that the performance function is a Nash equilibrium over the whole support of  $t$ . The equilibrium performance function with the doping threshold derived in Equation (3) is a Nash equilibrium if no athlete with talent  $t_i$  for all  $t_i \in [0, 1]$  is better off by unilaterally deviating from the equilibrium strategy. Note that the performance function is strictly increasing in  $t$ . It follows that if an athlete imitates the strategy of an athlete with talent  $t_j$  his winning probability is the first order statistic  $G(t_j)$ . Moreover, note that an athlete with talent  $t_i$  can only imitate athletes with a performance greater than his talent ( $p(t_j) \geq t_i$ ). Hence, we can limit the verification on the range  $p(t_j) \geq t_i$ , for all  $t_i \in [0, 1]$ .

For the proof we introduce the auxiliary function  $\rho(G(t), t_i)$ . To derive the auxiliary function we take advantage of the indifference property of Equation (2a) resulting from the assumed linear doping cost function. We can derive  $\rho(G(t), t_i)$  by substituting  $t_i$  for  $t^*$  in Equation (2a).

**Definition 1.** The function  $\rho(G(t), t_i)$  indicates the performance level at which a player with talent  $t_i$  and winning probability  $G(t)$  would be indifferent to his equilibrium strategy  $p(t_i)$ . Function  $\rho(G(t), t_i)$  is defined for  $t_i < t^*$  and  $t \in [t_i, 1]$ . It intersects with the performance function at  $t = t_i$ . More formally the function is

$$\rho(G(t), t_i) = [t_i - wt_i^{\alpha(n-1)}] + wG(t) = [t_i - wt_i^{\alpha(n-1)}] + wt^{\alpha(n-1)} \quad \text{for } t \geq t_i.$$

Function  $\rho(G(t), t_i)$  allows to compare the expected revenue of an athlete at his equilibrium performance directly with the performance function  $p(t)$ . If the performance function  $p(t)$  runs below (above)  $\rho(G(t), t_i)$  then the athlete is better off (worse off) by deviating. Comparing the functional form of  $\rho(G(t), t_i)$  and  $p(t)$  unveils that both have the same slope within the range of  $t \in [t^*, 1]$ .

*Proof.* Remember the implication of the linear doping cost function. An athlete with talent  $t_i$  above the doping threshold is indifferent between his equilibrium performance and all other  $p(t)$ 's in the range where the nonnegativity constraint is not binding.

(1) From this indifference property it follows directly that for all  $t_i \in [t^*, 1]$  no athlete can be better off by imitating another performance ( $p(t_j) \geq t_i$ ).

For athletes with talents  $t_i < t^*$  we make use of the function  $\rho(G(t), t_i)$ . Note that the derivative with respect to  $t$  of  $\rho(G(t), t_i)$  is equal to Equation (2b). Given that case (i) applies, then it follows that the derivative  $\rho_t(G(t), t_i)$  at  $t = t_i$  is strictly increasing in  $t_i$ . Note that by construction  $\rho(G(t), t_i)$  at  $t_i = t^*$  is identical to the performance function. Therefore, the derivative of  $\rho(G(t), t_i)$  at  $t = t^*$  is equal to 1.

(2) It follows that  $\rho(G(t), t_i)$  runs below the performance function  $p(t)$  in the range of  $t \in [t_i, 1]$  for all  $t_i \in [0, t^*)$ . Hence, an athlete with talent  $t_i$  is worse off by imitating the strategy of a talent  $t > t_i$ .

It rests to prove that the doping threshold  $t^*$  is optimally chosen. Suppose that the optimal threshold  $\tilde{t}^*$  is smaller than  $t^*$ . Then, the performance function would violate the nonnegativity constraint since the derivative (2b) of the performance function is smaller than 1 for all  $\tilde{t}^* \in [0, t^*)$ . Now suppose that the optimal threshold  $\tilde{t}^*$  is greater than  $t^*$ . Then,  $\rho(G(t), \tilde{t}^* - \epsilon)$  of an athlete with talent slightly smaller than  $\tilde{t}^*$  would run above the performance function since the derivative of  $\rho(G(t), \tilde{t}^* - \epsilon)$  for  $t \in (t^*, 1]$  is greater than 1. Hence, the athlete with talent  $\tilde{t}^* - \epsilon$  would be better off by deviating from his equilibrium strategy. It follows that the doping threshold  $t^*$  is optimally chosen and therefore the performance function is a Nash equilibrium.  $\square$

Beside the performance function the formula for the doping behavior of the athlete is certainly important. As the performance function  $p(t)$  is the sum of talent  $t$  and the doping quantity  $d$ , the doping function can be simply derived from the performance function. The doping quantity is an athlete's performance minus his talent. From this, we get the equation for the doping function  $d(t)$ :

$$d(t) = \begin{cases} 0 & \text{if } t < t^* \\ t^* + w[t^{\alpha(n-1)} - t^{*\alpha(n-1)}] - t & \text{if } t \geq t^*, \end{cases} \quad (5)$$

with the doping threshold:  $t_{(\alpha,n,w)}^* = [\alpha w(n-1)]^{\frac{1}{1-\alpha(n-1)}}$ .

Having gained the doping threshold, the performance function and the doping function, we now have the necessary equations needed for describing the equilibrium behavior of the athletes in the doping model. Section 3 will go into the doping behavior of the athletes in case (i) in more detail. With comparative statics and figures we will show how different values of  $\alpha$ ,  $n$  and  $w$  influence the doping threshold, the performance function and the doping function.

Beforehand, we will address the doping behavior of athletes in case (ii) and case (iv). This behavior arises when the distribution of talents is highly skewed to the left. The derivative (2b) of the performance function is negative. Moreover,  $p'(0)$  is not defined. It is the case that  $\lim_{t \rightarrow 0} p'(t) = \infty$ . Therefore, the nonnegativity constraint is not binding for small  $t$ 's and we do not apply the approach with the reserve talent. To derive the performance function we integrate Equation (1) from 0 to  $t$ . Note that the performance function  $p(t)$  has to be equal or greater than  $t$  over the whole support  $t \in [0, 1]$ . The performance function is

$$p(t) = \max\{wt^{\alpha(n-1)}, t\}. \quad (6)$$

Like before, the doping quantity is an athlete's performance minus his talent. Then, the equation for the doping function  $d(t)$  is:

$$d(t) = \max\{wt^{\alpha(n-1)} - t, 0\}. \quad (7)$$

Figure A.1 in the appendix gives an example of the doping behavior in case (ii). Only weak athletes takes performance enhancing drugs. The reason that strong athletes do not dope is that the probability to encounter a stronger athlete and the ratio of the prize money and the doping costs are so small that it is not optimal to increase once performance over the talent level. In Claim 2 we show under what parameter values case (ii) and (iv) exist. The formula of the doping threshold in case (ii) is stated in Equation (A.1) in the appendix.

*Claim 2.* We can differentiate between two cases if  $\alpha(n-1) < 1$ . The first one is case (ii) where only weak athletes dope. If  $w < 1$  then there exists a unique doping threshold  $t^* \in (0, 1)$  and athletes behave like in case (ii).

The second one is case (iv) where everybody dopes. If  $w > 1$  then there exists no doping threshold over the support  $t \in [0, 1]$  and athletes behaves like in case (iv).

The proof of Claim 2:

When  $\alpha(n-1) < 1$  then the derivative (2b) of the performance function is strictly decreasing in  $t$ . Furthermore the derivative  $p'(0)$  is not defined. It can be shown that  $\lim_{t \rightarrow 0} p'(t) = \infty$ . The performance function in case (ii) for  $t = 1$  not considering the doping nonnegativity constraint would be  $p(1) = w1^{\alpha(n-1)} = w < 1$ . Therefore, there must exist a unique intersection point with the 45-degree line. Hence, there exists a doping threshold  $t^*$  over the support  $t \in (0, 1)$ . Using the same reasoning, there exists no doping threshold over the support  $t \in [0, 1]$  if  $w > 1$ .

### 3 Implications

A competition is held between  $n$  athletes, where each athlete may make use of performance-enhancing drugs. Each athlete knows how many competitors are participating in the game and his own talent. The distribution of talents is common knowledge. In such a game, it is not always an optimal strategy for all athletes to dope. As each athlete maximizes his utility, all competitors have to choose an amount of doping for which the marginal cost of doping is exactly equal to the marginal expected increase in prize revenue. Given the assumed distribution of talents in case (i), for low  $t$ 's the expected marginal profit to be gained by doping is smaller than the marginal cost of doping. The reason is that the probability of confronting a stronger adversary is more likely. Therefore, there is no advantage in doping for weak athletes. The winning probability function has a convex form owing to the first order statistic. It becomes more convex, the more competitors there are. This is why there is a threshold value for almost all parameter values of  $\alpha$ ,  $n$  and  $w$ , beyond which it makes sense to dope.

#### 3.1 The number of athletes and the costs of doping

Athletes with a talent below the doping threshold value do not dope. While athletes with a talent above this threshold do dope. In discussing the results, we shall first investigate the effects of  $n$  and  $w$ . For this, we will assume that  $\alpha = 1$ . The talents are uniformly distributed between 0 and 1. In Section 3.1 we will investigate the effects of different talent distributions ( $\alpha \neq 1$ ) on doping behavior. Now, we will consider the doping threshold. With an uniform distribution of talents, Equation (3) becomes

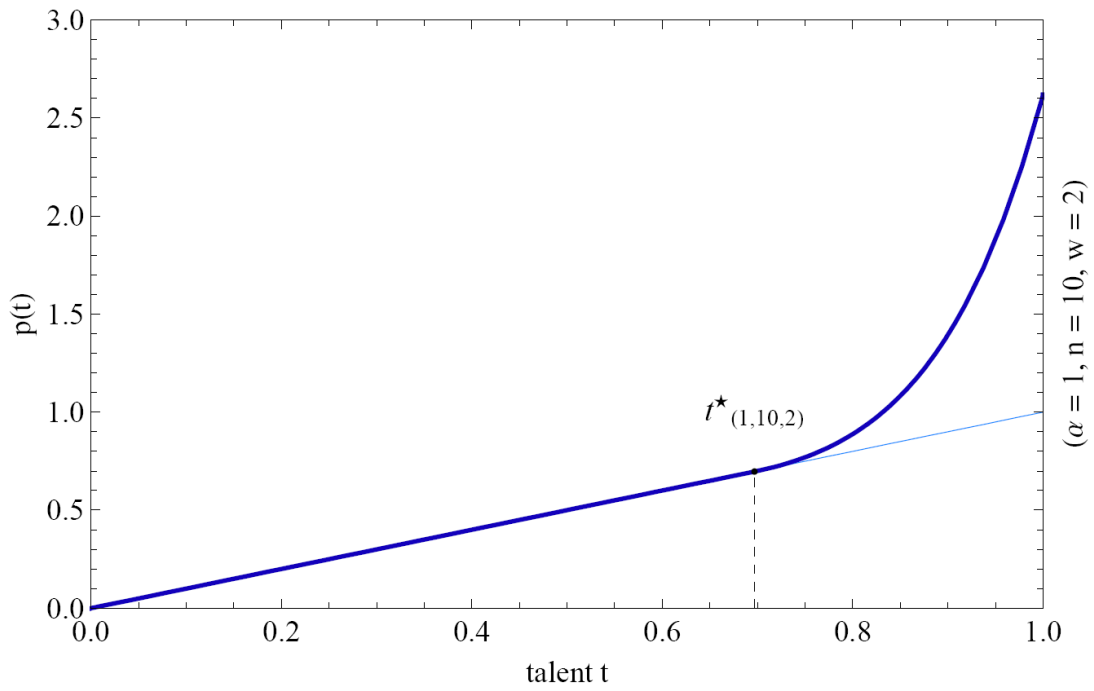
$$t_{(\alpha=1,n,w)}^* = [w(n-1)]^{\frac{1}{2-n}}. \quad (8)$$

The number of competitors is decisive for the size of the doping threshold. If only two athletes compete, then the formula of the doping threshold is not defined. The performance function consequently has the following appearance:  $p(t) = wt$  for  $w > 1$ . The doping quantity  $p(t) = wt$  for  $w > 1$  increases linearly in line with talent. If  $w \leq 1$ , the two athletes do not dope because doping is too expensive. Hence, either all athletes dope or none dopes. If the number of athletes competing is larger than two, then two classes of athletes form, one dopes and the other not. The indifferent talent is situated exactly on the doping threshold. The more athletes participate, the larger the doping threshold becomes. A larger field of participants makes it increasingly unattractive for weak athletes to dope.

Furthermore, the relationship between the prize money and the marginal cost  $w$  determines the level of the doping threshold. If  $w$  is large, then the participants can win a large sum of prize money and the doping costs are relatively low. If  $w$  becomes smaller, that is, the relative doping costs increase, then the doping threshold rises. For the case that doping is very expensive (tiny  $w$ ) and only a few athletes participate, then the doping threshold formula would produce values that are greater than 1. But since the assumption is that an athlete's talent can take a maximum value of 1, this entails that we are in case (iii) where nobody will dope.

Figure 1 shows the performance function and the doping function for a competition between 10 athletes and a ratio of prize money to marginal cost of 2 to 1. The performance function is shown in the upper chart (the dark blue line) of Figure 1. It represents an athlete's doping talent up to the threshold. Beyond this point, the athlete's performance is greater than his talent. The doping threshold lies at approximately 0.7. On the lower graph, one sees the athlete's doping function  $d(t)$ . The calculated doping function indicates the amount of doping that a player with talent  $t$  will buy. Up to the doping threshold, the doping function is equal to zero, as it is not worth to dope in this array, and the athletes let their true talent determine the outcome. For talents above the doping threshold, the doping function has a positive value. From the threshold on, athletes will dope, and dope all the more, the more talented they are. The player with the greatest talent dopes the most. This result is in line with evidence from positive doping cases.

### Performance function $p(t)$



### Doping function $d(t)$

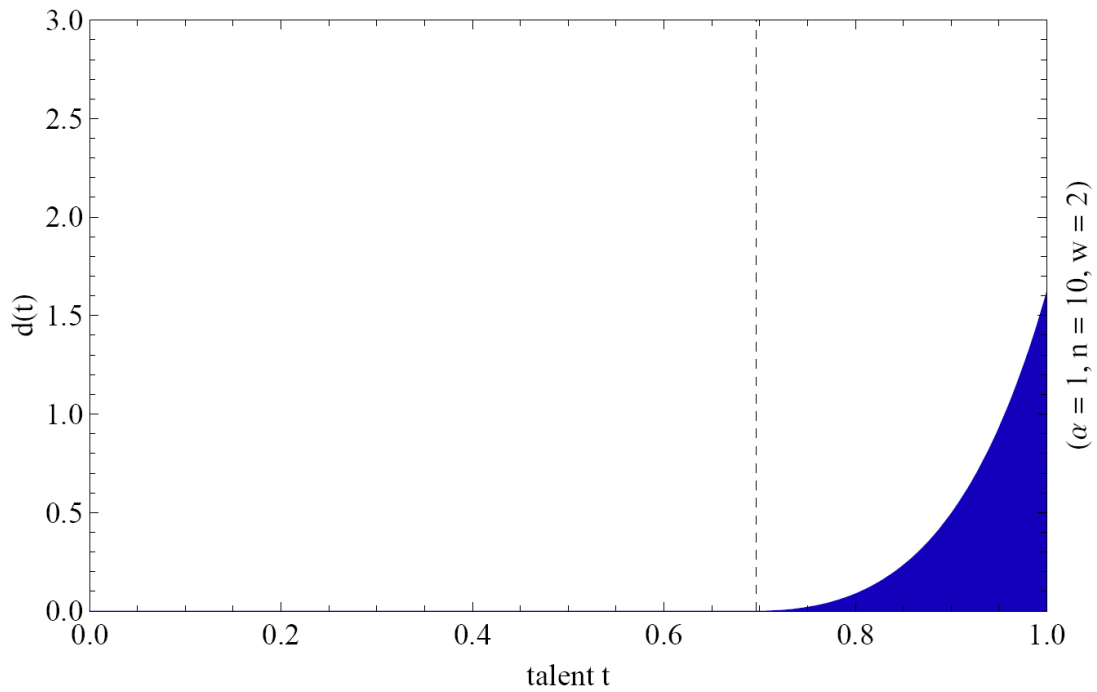


Figure 2: The doping threshold

The extreme difference between an athlete's performance and talent in this example is striking. The performance function of an athlete with the talent  $t = 1$  is 2.5 times greater than his talent. The amount of doping substances  $d(t)$  chosen is a much more crucial determinant of the outcome of the competition than the natural differences in talent for this underlying parameter values. The larger the ratio of prize money to doping costs  $w$  becomes, the more discrepant is the difference between them. If one follows the developments in competition sports, one sees how marketing has catapulted prize money in certain sports and how successful athletes' public images are used for corporate sponsoring events. Thus the ratio  $w$  has been seen to increase rather than decrease. A larger  $w$  leads to a smaller doping threshold and raises the amount of doping substances used.

An athlete will thus always have difficulty winning without resorting to doping. In order to illustrate this, we will use the example of a player with talent  $t = 1$ . This athlete always wins in a world where no athletes resort to doping. However, where he is the only athlete not to dope, his chance of winning falls dramatically. In order to calculate his chance of winning in this competition, we need talent  $t'$ , for which the performance function  $p(t')$  is exactly 1.<sup>10</sup> Our best athlete, who is also honest, is only able to win against athletes whose  $t < t'$ . His likelihood of winning corresponds to the first order statistic of  $t'$ .  $G(t = 1_{fair}) = t'^{n-1} = 0.19$ . In a doping environment, his chance of winning thus falls from 100% to just 20%. This low winning probability is due to the fact that he has to compete against not simply one competitor, whose talent should be smaller than his ( $t_i < t'$ ). No, he has to win against all nine other competitors.

### 3.2 The distribution of talents

The nature of the distribution has a direct influence on the doping threshold and the optimal amount of doping substances used. The power distribution with an arbitrary  $\alpha$  can illustrate different distributions of talents and possesses a closed-form solution for the doping threshold. With such a distribution the variations in the doping threshold can also be studied when applied to competitions of numerous talented athletes or numerous untalented athletes.

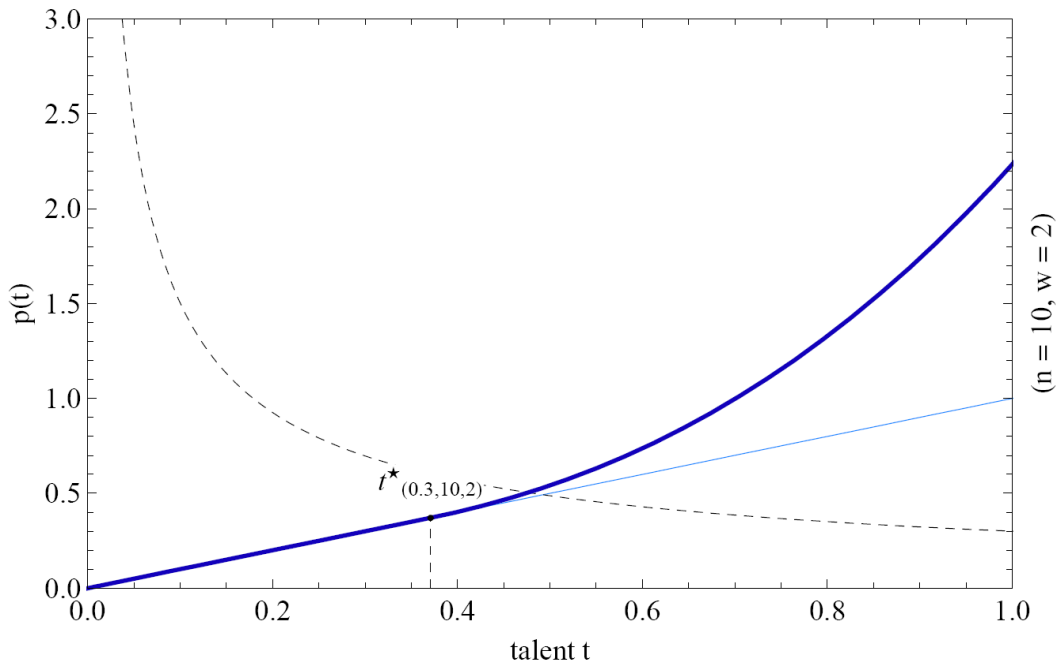
Figure 2 compares two different distributions of talents. In both cases, 10 athletes compete against each other, and the relationship between prize money and the marginal cost of doping is equal to 2. The dashed line is the density of athletes' talents. In the upper graph, there are a large number of untalented athletes and only a few talented athletes. The density of the power function distribution has a parameter value of  $\alpha = 0.3$ . The light-blue line is the performance of the athletes when they do not dope. The athlete's optimal behavior, when doping takes place, is illustrated by the dark-blue performance function.

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<sup>10</sup>We set the performance function equal to 1 and then solve for  $t$ . For the competition with  $n = 10$  and  $w = 2$  we get:  $t' = 0.832$ .



**Many untalented contrahents:  $\alpha \rightarrow 0.3$**



**Many talented contrahents:  $\alpha \rightarrow 3$**

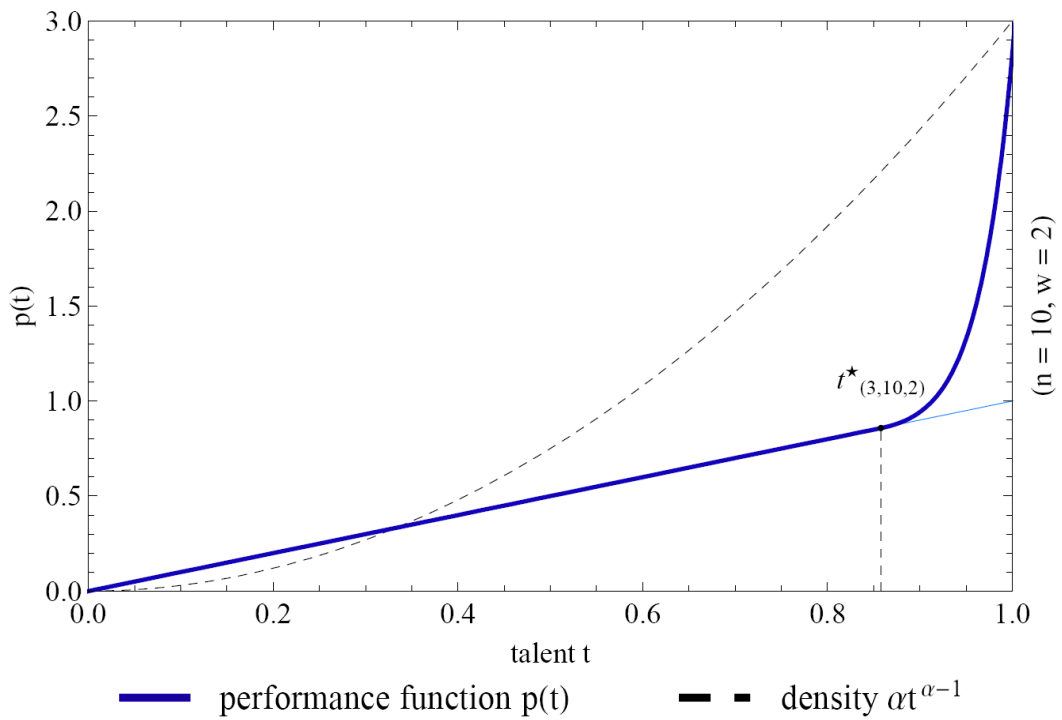


Figure 3: The distribution of talents

This comparison of athletes' doping behavior illustrates how important the distribution of talents is. For the upper case, with a large number of untalented athletes, the doping threshold is very low. As there are only a few talented athletes, even relatively weak athletes make use of doping, as the probability of coming up against a stronger athlete in a match is relatively small. The performance function of the athletes increases gradually.

The second case is quite different. The doping threshold is higher. This is because a weaker athlete can expect to come up against a stronger competitor, given the probability of the talent distribution. The performance function of the athletes increases much more sharply than it did in the upper case. Because there are many more talented athletes, competition pressure increases between the strong athletes. This leads the strongest athlete to take more doping substances than he would have in the upper case. Although the doping threshold is further to the right, the actual performance of an athlete with  $t = 1$  is higher than it would be in a competition with numerous weak athletes. In the upper graph, the performance of the strongest athlete lies at 2.2, while it lies above 2.6 in the lower graph.

In addition to the number of athletes that compete and the ratio of prize money and the marginal cost of doping, the distribution of talents also plays an important role. A high doping threshold does not always imply that the doping function increases less strongly. Competitive pressure plays a decisive role in an athlete's choice of the optimal doping quantity.

## 4 Conclusions

Performance enhancing drug scandals resulting in the conviction of the Festina cycling team in the nineties or the Fuentes affair in the year 2006 are just the tip of the iceberg. Doping is a phenomenon with a long history. During antiquity, Greek athletes already used dubious mixtures to enhance their strength and endurance. However, due to the commercialization of sports within the last century the incentive to use such substances increased drastically. But doping is not limited to sport competitions only. A form of doping is also prevalent in modern economics, where some companies, institutions and even countries revert to illegal acts. For instance companies cook their books with the intention to receive better ratings. In March 2009, Bernard Madoff, a renown public figure at Wall Street, was pleaded guilty of several felonies, including securities fraud. Based on a large Ponzi scheme he deceived his investors, thus gaining access to monetary assets of about 65 billion dollars (FinancialTimes (2009)).

Another example is Greece which falsified their statistics for years. In environments where the costs to camouflage such acts are smaller, the incentives to cheat are higher. In October 2009 the newly elected socialist government revealed that the country's deficit was far larger than the previous government disclosed. As a result rating agencies downgraded Greece's debt and

increased the risk premiums on Greek bonds. The acknowledgment of Greece's prime minister that "corruption was a national disease" shows that doping is a social problem (FinancialTimes (2010)).

The goal of this paper is to highlight how the doping behavior depends on the general conditions and regulations that prevail. The cultural environment and the existing framework are crucial determinants for the unfair behavior of athletes in a sport competition or companies in the economic system. Therefore we developed a doping model to shed more light on the doping problematic.

The model can be applied to other areas where the competitor with the best composed bid receives a prize. All-pay auctions are often used to model bribery (Milgrom, 2004). A direct application of the model would be to model public building contract requests in rather corrupt governments. Building contractors have to submit project designs and cost estimates. The best project will receive the contract. The contractors can improve their chance of winning with illegal gifts and bribery. This would be a straightforward application to our model. Moreover, the method with the reserve talent could be used in other theoretical applications. Especially, in situations with a constraint.

Sport competitions are most often held between more than two athletes. For example, every year approximately 200 cyclists participate in the well-known cycling event, the Tour de France. The participating cyclists differ much in strength, talent, technical or financial support. However, all have the same dream to win for their team and to 'sport' the winner's coveted jersey. Models that investigated the doping behavior of athletes in a two-player setting have led to interesting findings, but in some ways they abstract away too many details from reality. That was our stimulus to develop a doping model with heterogeneous agents in an n-player setting. We asked ourselves what are the important determinants for an athlete to use performance enhancing drugs? Obviously the size of the prize money and the costs of doping are crucial. But other determinants like the number of competitors and their strengths are important, too. In order to develop a monitoring mechanism, it is important to first analyze the optimal behavior of an athlete where doping is allowed.

In particular, the paper investigated how the number of athletes competing and the distribution of the talents affect doping behavior. In the standard case this led to a very interesting result. If more than two athletes compete, doping is only profitable above a certain talent level. In a competition, there will be a group of athletes who do not dope. Only when the athlete's talent is above this doping threshold he resorts to doping substances. Athletes can therefore be divided into two groups: Weak athletes playing fairly and strong athletes resorting to stronger and stronger amounts of doping.<sup>11</sup>

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<sup>11</sup>If doping is modeled as a binary choice between 'doping' and 'not doping', then a doping threshold will also exist.

Our doping model, where every player is free to use performance-enhancing drugs, produced interesting results. Below we recap our main findings. The symmetric Nash equilibrium of the competition shows that it is not always optimal for every player to dope. We distinguish between four cases. In the standard case the expected marginal revenue from taking drugs for weak players is smaller than the marginal costs of doping. We receive a doping threshold, which divides between doped and non-doped athletes. The height of the threshold in our model depends on three things. First, it depends on the number of competing athletes. The more athletes participate, the higher is the doping threshold. If there are more competitors the probability that there is a stronger competitor increases. Therefore, it only makes sense to dope if the athlete belongs to the superior part of the field. Due to the fact that the chances to win are higher and the expected profit of doping is bigger than its cost.

Secondly, the threshold depends on the prize-cost ratio of the competition. By prize-cost ratio we are referring to the relation of prize money and the marginal doping costs. If the relative doping costs increase, the doping threshold does, too. Recent developments in competitive sports affected the prize-cost ratio. Prize money and sponsoring contracts are more lucrative nowadays due to the growing media interest in some widely known sports. At the same time, more effective substances are available on the market for more or less the same price. If offenders are not penalized appropriately the doping threshold will decrease.

And last, the threshold depends on the distribution of talents. In our doping model we chose a power function distribution for the talents. This allows us to take a closer look at the influence of the distribution of talents on doping behavior by varying the factor  $\alpha$  of the probability density function. If this factor is small, then there are many untalented players. In such a game the doping threshold is low. For high values of this factor the game has many talented players. Here the doping threshold is higher than in the former case. Even though fewer athletes take performance-enhancing drugs, the doping amount of the most talented participant may be higher than in the former case.

The results of the n-player doping game have shown that a doping threshold exists for the majority of cases. Hence, in most circumstances talented players take performance-enhancing drugs. The size of the doping threshold depends positively on the number of players, and negatively on the prize-cost ratio of doping. Moreover, the distribution of talents influences the value of the threshold. The most talented player is also the player who takes the largest doping amounts. On the one hand he already has a higher chance to win and therefore can invest more money into doping. On the other hand he needs to defend his role as a leader. Hence, he is forced to dope more than the others. In the end doping has no effect on the rankings of the game. The same person who would win in a doping-free environment also wins the doping game. These findings are not only of theoretical interest but should also be considered in the context of establishing optimal anti-doping regulations. Future extensions of the model could include such regulations.

Despite many differences between a standard auction and our doping model, there are some major similarities. Doping does not influence the ranking aspect of the game; the most talented athlete will win nonetheless. The performance function is increasing in the talent. Hence, the ranking position of the athletes stays the same, except that some costs for illegal substances accrue. Altogether the prevention of these practices would not harm the players since the additional expenses would vanish.

Unfortunately the banishment of doping may not be free of costs for competitive sport events. The detection of drug consumption can be quite difficult or even impossible. A good mechanism to fight doping consists of rigorous, independent control bodies and appropriate penalties laid upon offenders. One result of our model indicates that not every athlete has the same incentives to enhance his performance that way. Usually, athletes being weak compared to their competitors, do not profit of using such products. The anti-doping agencies need to pay attention to this. Otherwise penalties or inaccurate tests could actually lead to unintentional outcomes. Nonetheless, it makes sense to focus those tests on the top athletes. An extension of our model would consist of the inclusion of a doping controller, who checks the pool of players and tries to keep doping out of the sport completely. A doping model with heterogeneous athletes that can cope with flawed test results and takes windfall-profit effects<sup>12</sup> into account is demanding. Possible disqualifications imply that the winning probability of the runner-up is greater than zero. To account for these possibilities in the optimization problem of the athlete further order statistics need to be included. Reasonable extensions of our doping model wait for implementation to give a better understanding of the doping problem.

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<sup>12</sup>We speak of windfall-profit effects when the alleged winner is tested positive and the prize goes to the runner-up. Windfall-profit effects have been investigated by Kräkel (2005) beforehand.

## Appendix

### A The four cases of doping behavior

The following distinction of cases can be made:

#### (A) Existence of the doping threshold

(i) **Strong athletes dope**

$$\alpha \geq 1/(n-1) \cap w\alpha(n-1) > 1.$$

(ii) **Weak athletes dope**

$$\alpha < 1/(n-1) \cap w < 1.$$

#### (B) Nonexistence of the doping threshold

(iii) **Nobody dopes**

$$\alpha \geq 1/(n-1) \cap w\alpha(n-1) \leq 1.$$

(iv) **Everybody dopes**

$$w \geq 1 \cap \alpha < 1/(n-1).$$

## A.1 The doping behavior in case (ii)

In case (ii) only weak athletes dope. The formula of the doping threshold differs from Equation (3) of the doping threshold in case (i). If only weak athletes dope the equation of the doping threshold is the following one:

$$t_{\alpha,n,w}^* = w^{\frac{1}{1-\alpha(n-1)}}. \quad (\text{A.1})$$

Figure A.1 displays the performance function  $p(t)$  in case (ii). At the doping threshold the performance function has a kink. Having argued in case (i) that there can be no kink at the doping threshold, in case (ii) this is different. Given that all other athletes play the symmetric equilibrium of the performance function in Equation (6) it can be shown that an athlete with a talent  $t_i$  below the doping threshold is indifferent to imitating the performance of an athlete with a talent  $t_j < t^*$  and would decrease his expected return by imitating an athlete with a talent  $t_j > t^*$ . For athletes with a talent above the doping threshold the nonnegativity constraint is binding. That implies that the marginal return of increasing the performance is lower than the marginal costs of doping.

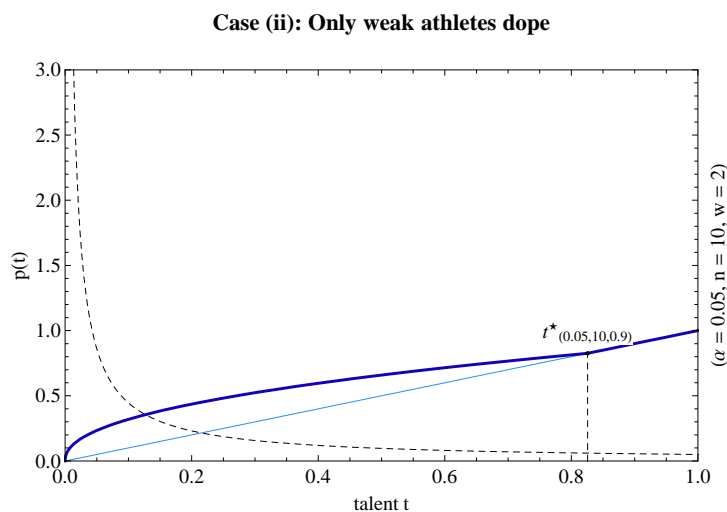


Figure A.1: The weak athletes dope

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