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**Working Paper**

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IZA Discussion Papers, No. 2947

**Provided in Cooperation with:**

IZA – Institute of Labor Economics

*Suggested Citation:* Engström, Per; Holmlund, Bertil (2007) : Worker absenteeism in search equilibrium, IZA Discussion Papers, No. 2947, Institute for the Study of Labor (IZA), Bonn

This Version is available at:

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IZA DP No. 2947

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July 2007

# Worker Absenteeism in Search Equilibrium

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Discussion Paper No. 2947  
July 2007

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## ABSTRACT

### **Worker Absenteeism in Search Equilibrium\***

The paper presents a general equilibrium model of search unemployment that incorporates absence from work as a distinct labor force state. Absenteeism is driven by random shocks to the value of leisure that are private information to the workers. Firms maximize profits while recognizing that the compensation package may affect the queue of job applicants and the absence rate. The analysis provides results concerning the effects of social insurance benefits and other determinants of workers' and firms' behavior. The normative analysis identifies externalities associated with firm-provided sick pay and examines the welfare implications of alternative policies.

JEL Classification: J21, J64, J65

Keywords: absenteeism, search, unemployment, social insurance

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\* We gratefully acknowledge useful comments from anonymous referees as well as from Peter Fredriksson, Kevin Lang, Oskar Nordström Skans, Ann-Sofie Kolm and Åsa Rosén.

# 1 Introduction

Worker absenteeism is a pervasive feature of employment relationships. In many countries, sickness absence represents an underutilization of the labor force of a magnitude comparable to unemployment. Data from Sweden and Norway over the period 2000-2004 show that sickness absence amounted to 7-8 percent of contractual work hours.<sup>1</sup>

This paper presents a general equilibrium analysis of employment and nonemployment where sickness absence is incorporated as a distinct labor force state. Absenteeism is triggered by random shocks to the worker's utility function that are private information to the worker. Although there are institutions in place whereby employers and insurance providers try to verify health conditions, perfect monitoring is bound to be prohibitively costly.

Previous research on sickness absence has almost exclusively been empirical and typically focused on how the individual worker responds to changes in sick pay or other plausible determinants of absence. Brown and Sessions (1996) provide a survey of the literature. There is by now considerable evidence that increased generosity of sickness benefits tends to increase absence rates; see, for example, Allen (1981), Johansson and Palme (1996, 2002) and Henrekson and Persson (2004). Time series data from some countries, notably Norway and Sweden, reveal markedly pro-cyclical absence rates. Arai and Skogman Thoursie (2005) as well as Askildsen et al (2005) provide evidence and interpretations of pro-cyclical absenteeism in those countries. There is not much evidence on the prevalence and determinants of sickness reporting among unemployed individuals, however. The scanty evidence there is indicates higher prevalence of reported sickness among unemployed individuals than among employed workers; see Larsson (2004).

Theoretical work on sickness absence is rare and has typically elaborated on the static neoclassical model of labor supply. Ehrenberg (1970) is a seminal paper where labor demand considerations are taken into account. The paper by Barmby et al (1994) proposes an efficiency wage model. Other theoretical contributions include Coles and Treble (1996) and Chatterji and Tilley (2002), who emphasize the interactions between absenteeism and productivity.

Our analysis of absenteeism in a search equilibrium framework is new

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<sup>1</sup>Nyman et al (2002) contain international comparisons of sickness absence based on data from the labor force surveys. See also Barmby et al (2002).

in the literature. It derives from the notion that absence from work in a frictional labor market is fundamentally different from being “absent” from the labor force. A worker on sick leave can return to work more or less instantaneously without having to engage in costly search. By contrast, a nonparticipant is restricted by labor market frictions and must compare the benefits of entry to the costs of search, recognizing the randomness of job offers.

The supply side of our model relates to some existing multistate models of labor force dynamics. Toikka (1976) is a seminal paper and other contributions include Flinn and Heckman (1982) as well as Burdett et al (1984). Those papers provide partial equilibrium analyses in the sense that wages are taken as given. Individual search and labor supply decisions are examined in stochastic environments, allowing for nonparticipation as a distinct state in addition to employment and unemployment. The more recent contribution by Garibaldi and Wasmer (2005) takes this approach into a general equilibrium setting by incorporating endogenous wage determination.<sup>2</sup>

The framework we propose can shed light on a number of issues. For example, we can show how changes in sickness benefits affect employed workers’ absence decisions as well as nonemployed workers’ search decisions. We can also illuminate how those changes impact on firms’ wage and recruitment decisions. Analogously, we can show how nonemployment benefits affect not only behavior among the nonemployed but also absence behavior among employees. Welfare policy interdependencies can thus be analyzed in a coherent general equilibrium framework. As noted by Krueger and Meyer (2002), not much research has been devoted to interactions between social insurance programs.<sup>3</sup>

Our model also relates to recent empirical work on how absenteeism responds to employment protection rules. Ichino and Riphahn (2004, 2005) report that transitions from (insecure) temporary jobs to (secure) “permanent” jobs are associated with an increase in absenteeism. This pattern is consistent with the hypothesis that workers perceive the risk of job loss as

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<sup>2</sup>The model by Garibaldi and Wasmer (GW) has some similarities with our model but there are also important differences. GW focus on risk neutral workers; we assume risk aversion which implies a motive for income insurance. GW consider bargaining over wages whereas we have wage posting by firms. Finally, GW do not treat absence as a distinct labor force state.

<sup>3</sup>See Larsson (2006) for a recent empirical study of interactions between unemployment insurance and sickness insurance.

positively correlated with absence from work, an hypothesis supported also by the analysis in Hesselius (2002). Our framework allows for the possibility that the probability of job loss depends on whether the employee is present or absent from work.

The analysis offers results regarding the impact of social insurance benefits and other determinants of workers' and firms' behavior. For example, higher statutory sick pay is shown to increase absenteeism but also to reduce wage costs; the effect on employment is ambiguous. Higher nonemployment benefits lead to lower employment but also higher absenteeism among employed workers. Our normative analysis identifies externalities associated with firm-provided sick pay and examines the welfare implications of alternative policies.

The model is presented in section 2 of the paper. Section 3 presents comparative statics results, section 4 turns to welfare analyses and section 5 concludes.

## 2 The Model

### 2.1 Workers' Behavior<sup>4</sup>

The number of individuals is normalized to unity. Individuals are homogeneous ex ante, i.e., before they have been hit by shocks to their utility functions. The four states are as follows:  $p$  is present at work (or simply work),  $s$  is sickness absence (sick leave),  $u$  is unemployment, and  $n$  is nonparticipation. Work and sickness absence represent employment ( $e$ ), whereas unemployment and nonparticipation are referred to as nonemployment ( $o$ ). We think of nonparticipants as "inactive" nonemployed individuals who belong to the potential labor force but do not actively pursue job search. Let  $j$  indicate the labor force state,  $j \in \{p, s, u, n\}$ .

The worker's utility is taken to be quasi-linear of the form

$$v^j = \ln C^j - a^j \xi \tag{1}$$

where  $C^j$  is consumption,  $a^j$  a positive parameter, and  $\xi$  a utility shifter that is increasing in sickness. Successive  $\xi$  are independently and identically distributed random variables drawn from a known distribution  $F(\xi)$  with

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<sup>4</sup>The supply side of the model is described in more detail in Appendix A.

support  $(0, \infty)$  and density  $f(\xi)$ . Consumption is equal to after-tax income in every period.<sup>5</sup> Consumption while at work (wage income) is given as  $C^p = w$ , and work-hours are taken as fixed. The individual is entitled to non-work benefits when he does not work; the levels of these benefits may differ across the three states of non-work. An employed worker who is absent from work receives sickness benefits (sick pay),  $C^s = \rho^s w$ , where  $\rho^s$  is the replacement rate that applies to the *wage in the firm* where the worker is employed. The replacement rate may be exogenously given by law or determined by the firm as part of its optimal compensation package.

An unemployed person receives unemployment benefits,  $C^u = b^u$ , and nonparticipants receive what is referred to as sickness assistance,  $C^n = b^n$ . Each firm takes  $b^u$  and  $b^n$  as exogenous to its wage decisions, although these benefit levels are in fact indexed to the *average wage* in the economy through exogenously given replacement rates,  $\rho^u$  and  $\rho^n$ .<sup>6</sup> The general equilibrium features a common economy-wide wage. Benefit differentiation between the unemployed and the nonparticipants is feasible only if search effort can be monitored by the labor market authorities. If monitoring is impossible, there is only room for a uniform replacement rate for nonemployed workers, i.e.,  $\rho^u = \rho^n \equiv \rho^o$ .

We set  $a^j = 1$  as normalization for  $j \in \{s, n\}$ , i.e., for “inactive” individuals who don’t work or don’t search. We set  $a^j > 1$  for  $j \in \{p, u\}$ , i.e., for “active” individuals who are present at work or who are unemployed. Superscript  $j$  is dropped in the subsequent exposition, thus assuming  $a^p = a^u = a > 1$ . The assumptions concerning  $a^j$  capture the idea that the disutility of work or search is increasing in sickness.

The probability of job loss may differ between workers who are present at work and workers on sick leave. Let  $\phi^p$  denote the job loss probability for a person at work and  $\phi^s$  the corresponding probability for a person on sick leave. Assume that work can never be more risky than sick leave, i.e.,  $\phi^p \leq \phi^s$ .<sup>7</sup> Equal separation risks, i.e.,  $\phi^p = \phi^s = \phi$ , may correspond to a

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<sup>5</sup>We thus rule out the possibility of smoothing consumption through borrowing and saving. Analytical treatments of equilibrium search models with risk aversion and precautionary savings have proved to be highly complex and few results are available. See Costain (1997) for a numerical analysis.

<sup>6</sup>The indexation of benefits to wages is done for tractability as well as for realism. Our assumptions have the realistic implication that unemployment remains unaffected by trend growth in productivity.

<sup>7</sup>A similar idea is contained in Wang and Williamson (1996), who assume that a



stringent employment protection legislation.<sup>8</sup>

We assume that job finding requires active search and let  $\alpha$  denote the probability of job finding when searching. Job finding depends on labor market conditions and we make the usual assumption that it is increasing in the ratio between vacancies,  $v$ , and unemployment,  $u$ , i.e.,  $\alpha = \alpha(v/u) = \alpha(\theta)$  where  $\theta \equiv v/u$ . The probability that a firm with a vacancy finds a worker is given as  $q(\theta) = \alpha(\theta)/\theta$  by virtue of a constant returns to scale matching function.

Individual optimization involves choosing reservation values of sickness. Let  $Q$  denote the value of  $\xi$  that equalizes the employed worker's expected present values of being present at work and absent from work. Analogously, let  $R$  denote the value of  $\xi$  that equalizes the values to the nonemployed worker of being an unemployed job seeker and a nonparticipant. The probability of being present at work is given by  $F(Q)$  whereas the probability of being searching when nonemployed is  $F(R)$ .

Let  $M^e$  and  $M^o$  denote the expected present values of employment ( $e$ ) and nonemployment ( $o$ ) given that optimal decision rules are adhered to. These values can be written as asset equations of the form:

$$rM^e = \tilde{v}^e + \tilde{\phi}(M^o - M^e) \quad (2)$$

$$rM^o = \tilde{v}^o + \tilde{\alpha}(M^e - M^o) \quad (3)$$

where  $r$  is the discount rate,  $\tilde{\phi} \equiv \phi^p F(Q) + \phi^s [1 - F(Q)] = \tilde{\phi}(Q)$  the average firing rate,  $\tilde{\alpha} \equiv F(R)\alpha(\theta) = \tilde{\alpha}(R, \theta)$  the average hiring rate, and  $\tilde{v}^e$  ( $\tilde{v}^o$ ) the expected per-period utility if employed (nonemployed). Per-period utilities are given as

$$\begin{aligned} \tilde{v}^e &\equiv F(Q) [\ln w - aE(\xi | \xi \leq Q)] + [1 - F(Q)] [\ln \rho^s w - E(\xi | \xi > Q)] \\ \tilde{v}^o &\equiv F(R) [\ln \rho^u w - aE(\xi | \xi \leq R)] + [1 - F(R)] [\ln \rho^n w - E(\xi | \xi > R)] \end{aligned}$$

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worker's probability of job retention depends on effort supplied on the job.

<sup>8</sup>By allowing  $\phi^s \geq \phi^p$  we wish to capture that absence may respond to labor market conditions, a relationship that has empirical support. A formal rationalization of the assumption may require a paper of its own. One scenario is when worker redundancies require that many workers are laid off. Being on sick leave is likely to increase the risk of being laid off as soon as there is some cost to the firm – in addition to lost production – of having workers on sick leave. In our model, the presence of experience rating is such a cost, but other costs are plausible as well.

where  $E$  stands for the expectations operator and where we have imposed the replacement rate conditions also for the nonemployed workers, i.e.,  $b^u = \rho^u w$  and  $b^n = \rho^n w$ ; recall that the latter replacement rates apply to the common aggregate wage. The difference in present values is

$$M^e - M^o = \frac{\tilde{v}^e(\rho^s) - \tilde{v}^o(\rho^u, \rho^n)}{\tilde{\alpha}(R, \theta) + \tilde{\phi}(Q) + r} \quad (4)$$

As shown in Appendix A, the equations for the reservation values of sickness can be written as:

$$(a - 1)Q = -\ln \rho^s + (\phi^s - \phi^p) \left[ \frac{\tilde{v}^e(\rho^s) - \tilde{v}^o(\rho^u, \rho^n)}{\tilde{\alpha}(R, \theta) + \tilde{\phi}(Q) + r} \right] \quad (5)$$

$$(a - 1)R = \ln \rho^u - \ln \rho^n + \alpha(\theta) \left[ \frac{\tilde{v}^e(\rho^s) - \tilde{v}^o(\rho^u, \rho^n)}{\tilde{\alpha}(R, \theta) + \tilde{\phi}(Q) + r} \right] \quad (6)$$

Individual optimization implies that  $M^e$  and  $M^o$  are locally independent of the relevant reservation values of sickness. Note also that  $M^e - M^o$  is independent of the wage, an implication of constant replacement rates.

Eqs. (5) and (6) imply that both  $Q$  and  $R$  depend on compensation while nonemployed as well as labor market conditions. Non-work benefits have direct effects, holding  $M^e - M^o$  constant, as well as indirect “wealth effects” through  $M^e - M^o$ . The indirect effects may offset or reinforce the direct effects. For example, a rise in  $\rho^s$  has a direct negative effect on  $Q$ , whereas the indirect effects work in the opposite direction (since higher sick pay increases the value of employment relative to the value of nonemployment). The direct effect dominates the indirect effect.<sup>9</sup> A rise in  $\rho^u$  has a direct positive effect on  $R$ , whereas the indirect effect tends to reduce  $R$  (since the value of employment falls relative to the value of nonemployment). The direct effect dominates the indirect effect also in this case.<sup>10</sup> The following results hold in partial equilibrium with tightness taken as constant:

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<sup>9</sup>The employed worker spends only a fraction,  $1 - F(Q)$ , of his time as absent from work, a fact that attenuates the wealth effect. The wealth effect is also smaller, the smaller the excess firing risk associated with absence. We have  $\text{sign}(\partial Q / \partial \rho^s) = \text{sign} \left[ -1 + (\tilde{\phi} - \phi^p) / (\tilde{\alpha} + \tilde{\phi} + r) \right] < 0$ .

<sup>10</sup>The worker spends a only fraction,  $F(R)$ , of his time as nonemployed as active job searcher; this attenuates the wealth effect of higher unemployment benefits. We have  $\text{sign}(\partial R / \partial \rho^u) = \text{sign} \left[ 1 - \tilde{\alpha} / (\tilde{\alpha} + \tilde{\phi} + r) \right] > 0$ .

**Lemma 1:** *Higher replacement rates affect  $Q$  and  $R$  as given by*

$$\begin{aligned} \frac{\partial Q}{\partial \rho^s} &< 0, & \frac{\partial Q}{\partial \rho^u} &\leq 0, & \frac{\partial Q}{\partial \rho^n} &\leq 0, & \frac{\partial Q}{\partial \rho^o} &\leq 0 \\ \frac{\partial R}{\partial \rho^s} &> 0, & \frac{\partial R}{\partial \rho^u} &> 0, & \frac{\partial R}{\partial \rho^n} &< 0, & \frac{\partial R}{\partial \rho^o} &< 0 \end{aligned}$$

where  $\partial Q/\partial \rho^o$  and  $\partial R/\partial \rho^o$  denote the effects of simultaneous increases of  $\rho^u$  and  $\rho^n$ . The weak inequalities hold as equalities when  $\phi^p = \phi^s$ .

**Lemma 2:** *An increase in labor market tightness affect  $Q$  and  $R$  as given by  $\partial Q/\partial \theta \leq 0$  for  $\phi^p \leq \phi^s$ , and  $\partial R/\partial \theta > 0$ .*

It is also clear from (5) that  $Q$  is increasing in  $(\phi^s - \phi^p)$ ; the larger the excess firing risk associated with absence, the larger is  $Q$ , i.e., the lower is the probability of being absent from work. Empirical evidence suggests that absenteeism increases in labor market tightness, an observation consistent with  $\partial Q/\partial \theta < 0$  (implied by  $\phi^s > \phi^p$ ). Pro-cyclical labor force participation, for which there is ample evidence, is consistent with  $\partial R/\partial \theta > 0$ .

This completes the description of the supply side where labor market tightness is taken as given. We now turn to the behavior of firms and the determination of wages and tightness.

## 2.2 Zero Profits

From now on we focus on the limiting case where the discount rate approaches zero ( $r \rightarrow 0$ ) and consider a firm with potentially many workers.<sup>11</sup> The fraction of workers present at work is given by  $F(Q)$  whereas the fraction absent is  $1 - F(Q)$ . The firm is operating under constant returns to labor and  $y$  denotes the constant marginal product. The wage cost per employee at work, inclusive of the payroll tax  $t$ , is  $w_c = w(1 + t)$ . There is a cost  $\kappa y$  of holding a vacancy open.<sup>12</sup> The firm's profits per employed worker can be

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<sup>11</sup>The workers' present values of employment and nonemployment go to infinity as the discount rate approaches zero. However, the flow values,  $rM^e$  and  $rM^o$ , as well as the difference  $M^e - M^o$ , are well defined for arbitrarily small discount rates. Cf. eqs. (2), (3) and (4). All other key relationships of the model are therefore also well defined for arbitrarily small discount rates.

<sup>12</sup>This relationship between the vacancy cost and output can be rationalized from a model of a firm that allocates its labor force between production and recruitment activities (see Holmlund, 2002).

written as

$$\pi = F(Q)(y - w_c) - \tau [1 - F(Q)] \rho^s w_c - \frac{\kappa y \tilde{\phi}}{q(\theta)} \quad (7)$$

where  $k\tilde{\phi}/q(\theta)$  is the vacancy cost; note that  $\tilde{\phi}/q(\theta)$  is the vacancy/employment ratio in the firm in a steady state with constant employment. The parameter  $\tau$  is referred to as the degree of experience rating of sickness insurance. With  $\tau = 1$ , the firm fully finances sick pay for its workers; with  $\tau = 0$ , the firm finances nothing (directly) of its workers' sick pay. It is assumed that firm-provided sick pay is taxed at the same rate as wages.

The firm maximizes profit per employed worker and free entry drives profits to zero, i.e.,

$$Z(w_c, \rho^s, \theta) \equiv F(Q)(y - w_c) - \tau [1 - F(Q)] \rho^s w_c - \frac{\kappa y \tilde{\phi}}{q(\theta)} = 0 \quad (8)$$

We will typically focus on two alternative versions of this zero profit condition. (i) Sick pay ( $\rho^s$ ) is provided by the government and exogenous to the firm. The natural benchmark regarding financing is  $\tau = 0$ , i.e., financing exclusively by taxes. We allow absence-dependent firing risks, i.e.,  $\phi^s \geq \phi^p$  and thus  $\tilde{\phi} = \tilde{\phi}(Q)$ . (ii) Sick pay is provided exclusively by firms and financed by them directly, i.e.,  $\tau = 1$ . To simplify this analysis, we assume  $\phi^p = \phi^s = \phi$  and thus  $\tilde{\phi} = \phi$ . Absence from work has thus no consequences for the risk of job loss in this case. By making use of (5) and differentiate the zero profit condition we can state a useful lemma:

**Lemma 3:** *If  $\rho^s$  is exogenous to the firm, the zero profit condition implies a negative relationship between the wage cost,  $w_c$ , and labor market tightness,  $\theta$ , i.e.,  $\partial w_c / \partial \theta < 0$ .*

A tighter labor market is associated with higher vacancy costs which has to be offset by lower wage costs so as to maintain zero profits. Absence appears in this relationship because it affects the surplus associated with more employed workers, and possibly the direct costs of having more workers absent from work (if  $\tau > 0$ ) as well as the effective cost of holding a vacancy open (if  $\phi^p < \phi^s$ ).

## 2.3 Wage Posting

Firms post wages so as to attract job applicants, recognizing that higher wages attract a longer “queue” of applicants. The inverse of tightness, i.e.,

$\theta^{-1} = u/v$ , can be thought of as the length of the queue. Unemployed workers allocate themselves to firms, recognizing wage offers as well as job offer probabilities. Worker mobility across job queues equalizes the expected values across those queues and firms take as given the common expected value of being unemployed. The model is thus of the competitive search variety; see Moen (1997) for a seminal contribution and Rogerson et al (2005) for a recent survey.<sup>13</sup>

The worker's choice of job queue depends on the value of the  $w, \theta$ -pair on offer, as given by  $r\tilde{U} \equiv \alpha(\theta)rM^e(w) + [1 - \alpha(\theta)]rM^o$ . This is the flow value of being unemployed before the veil of ignorance regarding  $\xi$  has been lifted.<sup>14</sup> A firm must offer its workers a  $w, \theta$ -pair that is no less attractive than the best alternative available in the market. Let  $r\bar{U}$  denote the value of the most attractive offer. A competitive  $w, \theta$ -pair must respect the inequality  $r\tilde{U} \geq r\bar{U}$ . Note that this market restriction, written as an equality, is equivalent to  $\bar{U} = M^o + \tilde{\alpha}(\theta) [\tilde{v}^e(w) - rM^o] / \tilde{\phi}$ . The Lagrangian for the firm's problem can then be written as

$$L = \pi(\cdot) + \mu \left[ \bar{U} - M^o - \frac{\tilde{\alpha}(\theta) [\tilde{v}^e(w) - rM^o]}{\tilde{\phi}} \right]$$

where  $\mu$  is the Lagrange multiplier. The firm takes  $\bar{U} - M^o$  as exogenous and maximizes with respect to  $w$  and  $\theta$ . The first-order conditions take the form:

$$\pi_w = \mu \tilde{\alpha}(\theta) \tilde{v}_w^e / \tilde{\phi} \tag{9}$$

$$\pi_\theta = \mu [\tilde{v}^e(w) - rM^o] \tilde{\alpha}_\theta / \tilde{\phi} \tag{10}$$

which imply that the marginal rates of substitution between  $w$  and  $\theta$  are equal for firms and workers. The two first-order conditions can be combined to yield

$$\tilde{v}^e - rM^o = \left( \frac{\eta}{1 - \eta} \right) \frac{\kappa y \tilde{\phi}(Q) / q(\theta)}{[F(Q) + \tau(1 - F(Q)) \rho^s] w_c}$$

where  $\eta \equiv -\theta q'(\theta) / q(\theta)$  is the elasticity of matching with respect to unemployment,  $\eta \in (0, 1)$ . We note that  $\kappa y \tilde{\phi}(Q) / q(\theta)$  is the vacancy cost per worker. In an equilibrium with free entry, the firm's gross profit per worker

<sup>13</sup>For ease of exposition, we suppress submarket subscripts on  $w$  and  $\theta$ .

<sup>14</sup>It can be shown that a worker's sickness status, as captured by  $\xi$ , does not affect preferences over job queues.

will be equal to the vacancy cost. Write gross profits as  $\pi^g \equiv \pi + \kappa y \tilde{\phi}/q(\theta)$  and note that  $\pi^g = \kappa y \tilde{\phi}/q(\theta)$  when  $\pi = 0$ . Using this fact, we obtain

$$\tilde{v}^e - rM^o = \left( \frac{\eta}{1-\eta} \right) \left( -\frac{\partial \ln \pi^g}{\partial \ln w_c} \right)^{-1} \quad (11)$$

since

$$-\frac{\partial \ln \pi^g}{\partial \ln w_c} = \frac{F(Q) + \tau [1 - F(Q)] \rho^s}{F(Q) \left( \frac{y}{w_c} - 1 \right) - \tau [1 - F(Q)] \rho^s} \quad (12)$$

The more elastic profits are with respect to the wage, the lower compensation given to the worker and the lower the worker's utility while employed,  $\tilde{v}^e$ . To arrive at a general equilibrium relationship we must recognize that  $rM^o$  is endogenous and influenced by sick pay and labor market conditions. We use eqs. (2) and (3) to derive

$$\tilde{v}^e - rM^o = \left[ \frac{\tilde{v}^e - \tilde{v}^o}{\tilde{\phi}(Q) + \tilde{\alpha}(\theta, R)} \right] \tilde{\phi}(Q) \quad (13)$$

and obtain:

$$W(w_c, \rho^s, R, \theta) \equiv \frac{\tilde{v}^e(\rho^s) - \tilde{v}^o(\rho^u, \rho^n)}{\tilde{\phi}(Q) + \tilde{\alpha}(\theta, R)} - \left( \frac{\eta}{1-\eta} \right) \left( -\frac{\partial \ln \pi^g}{\partial \ln w_c} \right)^{-1} \frac{1}{\tilde{\phi}(Q)} = 0 \quad (14)$$

The equation involves four endogenous variables, viz. the wage cost, tightness, and the reservation values of sickness while employed and unemployed:  $w_c$ ,  $\theta$ ,  $Q$  and  $R$ ; and possibly sick pay,  $\rho^s$ , as well. We can use (5) and (6) to substitute out  $Q$  and  $R$ , recognizing  $Q = Q(\theta)$  and  $R = R(\theta)$ , with  $Q'(\theta) \leq 0$  and  $R'(\theta) > 0$ . By differentiating (14), we obtain a positive relationship between  $w_c$  and  $\theta$ , for given  $\rho^s$ . Eq. (14) can be thought of as a positively sloped "wage curve" in the  $w_c, \theta$ -space. The general equilibrium is illustrated in Figure 1.

A useful representation of the equilibrium is obtained by invoking (8) to substitute out  $w_c$  from (14):

$$\hat{W}(\rho^s, \theta) \equiv \frac{\tilde{v}^e(\rho^s) - \tilde{v}^o(\rho^u, \rho^n)}{\tilde{\phi}(Q) + \tilde{\alpha}(\theta, R)} - \left( \frac{\eta}{1-\eta} \right) \left[ \frac{\kappa}{F(Q)q(\theta) - \kappa \tilde{\phi}(Q)} \right] = 0 \quad (15)$$

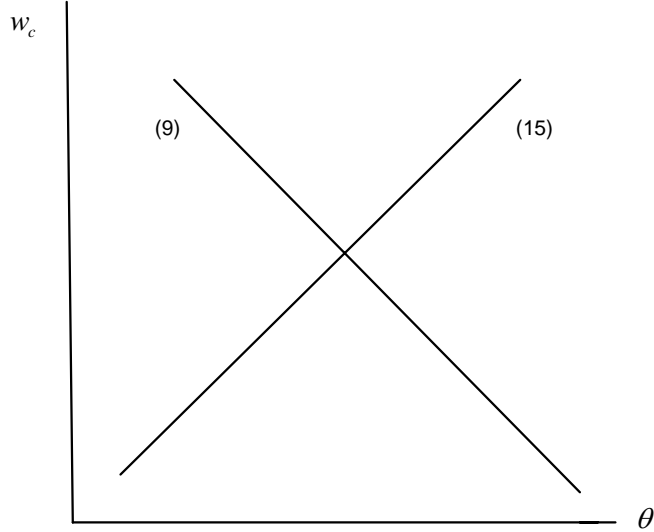


Figure 1: General equilibrium

where  $F(Q)q(\theta) - \kappa\tilde{\phi}(Q) > 0$  from (8). This equation determines  $\theta$  as a function of parameters when  $\rho^s$  is exogenous; recall  $Q'(\theta) \leq 0$  and  $R'(\theta) > 0$ . Existence requires  $\tilde{v}^e - \tilde{v}^o > 0$ , i.e.,  $M^e > M^o$ ; this restriction puts some restrictions on the parameters that are assumed to be fulfilled. Uniqueness is guaranteed by the fact that  $\hat{W}_\theta < 0$ .<sup>15</sup> The wage cost is obtained by invoking the zero profit condition, i.e., (8). In summary:

**Lemma 4:** *Free entry and zero profits together with wage posting yield – for a given  $\rho^s$  – a vertical  $w_c, \theta$ -locus that determines tightness independently of the wage cost, i.e., eq. (15). The wage cost is obtained recursively from the zero profit condition, i.e., eq. (8).*

We note that the tax rate and the degree of experience rating do not appear in (15) and thus do not affect the equilibrium level of tightness.

<sup>15</sup>A sufficient condition for  $\hat{W}_\theta < 0$  is that the elasticity of matching is non-decreasing in  $\theta$ , i.e.,  $\eta'(\theta) \geq 0$ . The empirical literature has largely focused on Cobb Douglas matching functions with constant  $\eta$ . One exception is Blanchard and Diamond (1989), who estimate the elasticity of substitution between vacancies and unemployment to be smaller than one, implying  $\eta'(\theta) > 0$ . Our analysis presumes that  $\eta$  is constant.

### 2.3.1 Firm-Provided Sick Pay

Consider now the case where sickness benefits are provided by the firm as part of its optimal compensation package. As noted above, we simplify this analysis by assuming  $\phi^p = \phi^s$  and thus have  $(a - 1)Q = -\ln \rho^s$  from (6). Maximization of the Lagrangian with respect to  $\rho^s$  yields a first-order condition of the form

$$\pi_{\rho^s} = \mu \tilde{\alpha}(\theta) \tilde{v}_{\rho^s}^e / \tilde{\phi} \quad (16)$$

and the optimal compensation package involves the efficiency condition

$$\frac{\pi_w}{\pi_{\rho^s}} = \frac{\tilde{v}_w^e}{\tilde{v}_{\rho^s}^e} \quad (17)$$

which equalizes the marginal rates of substitution between wages and sick pay for the firm and the worker. The former rate is affected by the degree of experience rating; the latter is not. The efficiency condition given by (17) traces out a positively sloped “contract curve” in the  $w, \rho^s$ -space that can be written as:<sup>16</sup>

$$\Gamma(\rho^s, w_c) \equiv \tau \rho^s - \frac{F(Q) - \left(\frac{y}{w_c} - 1\right) \varepsilon^s}{F(Q) + \varepsilon^s} = 0 \quad (18)$$

where  $Q = Q(\rho^s)$  and

$$\varepsilon^s \equiv \frac{d \ln s^r}{d \ln \rho^s} = \frac{d \ln [1 - F(Q)]}{d \ln \rho^s} \quad (19)$$

is the elasticity of the absence rate with respect to the replacement rate. Absent moral hazard we have  $\varepsilon^s = 0$  and full insurance, i.e.,  $\rho^s = 1$ , under full experience rating, i.e.,  $\tau = 1$ ; this is of course a natural outcome with risk averse workers and risk neutral firms. We assume that the elasticity  $\varepsilon^s$  is locally constant.<sup>17</sup>

To determine the general equilibrium configuration of  $\theta, w_c$  and  $\rho^s$ , we need to invoke (18) together with (8) and (15), i.e.,  $Z(w_c, \rho^s, \theta) = 0$  and  $\hat{W}(\rho^s, \theta) = 0$ . Existence can be shown to hold for some parameter configurations but we have not been able to derive simple and transparent conditions

<sup>16</sup>To derive (18), use (7) and the definition of  $\tilde{v}^e$ .

<sup>17</sup>Note that  $\varepsilon^s$  is everywhere constant,  $\varepsilon^s = \lambda/(a - 1)$ , if  $f(\xi)$  is exponential, i.e.,  $f(\xi) = \lambda \exp(-\lambda \xi)$ ,  $\lambda > 0$ .



for existence. However, provided that an equilibrium exists, it will be unique. Hence:

**Lemma 5:** *The general equilibrium of an economy where firms post both wages and sick pay will be unique and determine  $\theta$ ,  $w_c$  and  $\rho^s$ .*

**Proof:** (Sketch - see Appendix B for more details.) Combine (18) and (8) to obtain a relationship  $\tilde{\Gamma}(\theta, \rho^s) = 0$ , where  $(\partial\theta/\partial\rho^s)_{\tilde{\Gamma}} < 0$ . Combine also  $\tilde{\Gamma}(\theta, \rho^s) = 0$  and (15) to obtain a relationship  $\tilde{W}(\theta, \rho^s) = 0$ , where  $(\partial\theta/\partial\rho^s)_{\tilde{W}} \geq 0$ . It can be shown that  $(\partial\theta/\partial\rho^s)_{\tilde{\Gamma}} < (\partial\theta/\partial\rho^s)_{\tilde{W}}$ . The two relationships between  $\theta$  and  $\rho^s$  can cross only once and an equilibrium that determines  $\theta$  and  $\rho^s$  is therefore unique. The wage cost,  $w_c$ , follows from (8) once  $\theta$  and  $\rho^s$  are determined. ■

## 2.4 Flow Equilibrium and Balanced Budget

Having determined  $w_c$ ,  $\theta$ ,  $Q$  and  $R$ , and possibly  $\rho^s$  as well, it is straightforward to determine average hiring and firing rates, i.e.,  $\tilde{\alpha}(R, \theta)$  and  $\tilde{\phi}(Q)$ , as well as the sickness absence rate,  $s^r(Q)$ . The stocks of employed, absentees, unemployed and nonparticipants are obtained by imposing flow equilibrium in the labor market which implies the following relationships:

$$\begin{aligned} e &= \tilde{\alpha} / (\tilde{\alpha} + \tilde{\phi}) \\ p &= \tilde{\alpha}F(Q) / (\tilde{\alpha} + \tilde{\phi}) = F(Q)e \\ s &= \tilde{\alpha}[1 - F(Q)] / (\tilde{\alpha} + \tilde{\phi}) = [1 - F(Q)]e \\ u &= \tilde{\phi}F(R) / (\tilde{\alpha} + \tilde{\phi}) = F(R)(1 - e) \\ n &= \tilde{\phi}[1 - F(R)] / (\tilde{\alpha} + \tilde{\phi}) = [1 - F(R)](1 - e) \end{aligned}$$

The unemployment rate, as conventionally measured, is obtained as  $u^r = u/(u + e)$ . The tax rate,  $t$ , is “neutral” in the sense that it does not affect the real outcomes in the economy (except for the real wage which fully absorbs any tax changes). The tax rate can thus be determined recursively to balance the government’s budget period by period, assuming a given degree of experience rating,  $\tau$ :

$$t(p + \tau\rho^s s) = (1 - \tau)\rho^s s + \rho^u u + \rho^n n \quad (21)$$

### 3 Comparative Statics

The parameters of the model have their origins in preferences and shocks to preferences, vacancy costs, matching technologies, job separation rates and benefit policies. Changes in parameters impact on the economy by influencing workers' decisions on absence and search and by influencing firms' decisions on wages and recruitments. We confine the discussion to a subset of parameters, beginning with benefit policies when sick pay is exogenous. To provide some feel for how the model works, the discussion of the effects of higher sick pay is relatively detailed. Some effects can only be established numerically based on the calibrated version of the model as presented in Appendix C.

#### 3.1 Exogenous Sick Pay

**Higher Sick Pay** Consider how the economy responds to an exogenous increase in sick pay,  $\rho^s$ , that is financed by taxes. Holding labor market tightness constant, the effect on the reservation value of sickness while employed,  $Q$ , and thus on absence, is given by (5). The direct effect on  $Q$  is obviously negative. However, there is also a “wealth effect” involved since higher sick pay increases the value of employment relative to nonemployment,  $M^e - M^o$ . The direct effect dominates the indirect effect so  $Q$  does indeed fall; cf. Lemma 1. The increase in absence triggers an increase in firings (if  $\phi^s > \phi^p$ ) and thus contributes to a decline in employment.

As is clear from (6), the fact that higher sick pay increases the value of employment relative to nonemployment strengthens the incentives for job search among nonemployed individuals. They substitute job search as unemployed for inactivity, i.e., they raise the reservation value of sickness,  $R$ . This leads to a rise in hirings as well as in employment. The net effect on employment is generally unclear but positive when  $\phi^s = \phi^p$ ; in this case there is no increase in firings associated with higher sick pay.

So far we have taken labor market tightness as given. As is clear from (5) and (6), both absence and search decisions depend in general on tightness. Higher tightness reduces the value of employment relative to nonemployment and makes the employed worker less reluctant to call in sick ( $Q$  falls). For the nonemployed worker, a stronger labor market makes it more attractive to engage in search and the reservation value of sickness while nonemployed,  $R$ ,

thus increases. To determine the effect on labor market tightness, we need to consider firms' wage and recruitment decisions.

A rise in sick pay that increases absence reduces the firm's surplus per worker; cf. (8). To maintain zero profits, firms offer fewer jobs thereby reducing expected vacancy costs. The process can be illustrated as a shift to the left of the zero profit condition in the  $w_c, \theta$ -space. However, the rise in sick pay also makes employment more attractive to workers which induces firms to increase the workers' job finding probabilities. Wage offers are reduced so as to allow this shortening of queue lengths. The wage curve as given by (14) shifts to the right. The wage cost falls unambiguously and the worker's real consumer wage also falls as long as the tax rate does not decrease.

The effect on tightness is ambiguous. The wage moderation effect is counteracted by the adverse labor demand effect as higher absence reduces the firm's surplus per worker. Recall that we obtained an unambiguously positive employment response to higher sick pay when  $\phi^s = \phi^p$  and tightness was taken as given. This prediction does not carry over the case with endogenous tightness: higher sick pay may reduce tightness which weakens search incentives and reduces the average hiring rate,  $\tilde{\alpha}(R, \theta)$ . The numerical analysis suggests that tightness as well as employment would decline.

How does then sickness absence respond to higher sick pay? From Lemma 1 we have  $\partial Q/\partial \rho^s < 0$ , holding tightness constant. This effect, implying higher absenteeism, may conceivably be offset by changes in tightness; recall  $\partial Q/\partial \theta \leq 0$  from Lemma 2. However, by invoking (5) and (15), we can establish an unambiguous increase in absenteeism, regardless of how tightness is affected. In summary:

**Proposition 1** *Higher sick pay increases absenteeism,  $\partial s^r/\partial \rho^s > 0$ , and reduces the wage cost,  $\partial w_c/\partial \rho^s < 0$ .*

**Higher Nonemployment Benefits** A uniform increase in nonemployment benefits,  $\rho^o$ , can be illustrated as a shift to the left of the wage curve (14), and possibly as a shift of the zero profit condition as well. There will be a negative effect on tightness, an implication that follows immediately from (15). Search activity among nonemployed workers falls unambiguously since eq. (6) implies  $\partial R/\partial \rho^o < 0$  and  $\partial R/\partial \theta > 0$ . Sickness absence among employees is not affected as long as  $\phi^p = \phi^s$ , but will unambiguously increase when  $\phi^s > \phi^p$ . Note from (5) that the rise in nonemployment benefits tends

to increase absenteeism (reduce the reservation value of sickness during employment,  $Q$ ) as the per-period utility difference between employment and nonemployment increases. However, the fall in tightness reduces job finding which makes nonemployment less attractive, which in turn makes absenteeism more costly. By using (5) and (15), it is straightforward to establish that absenteeism does indeed increase ( $Q$  falls when  $\phi^s > \phi^p$ ); the indirect effect through adjustment in tightness can never offset the direct effect.

The fall in tightness reduces job finding and the fact that  $Q$  can never increase imply that firings can never fall. It is clear, then, that employment must fall. To obtain the effect on the wage cost, we need to invoke the zero profit condition, i.e., eq. (8). As long as  $\phi^p = \phi^s$  holds, there is no effect on this relationship; the shift to the left of the wage curve given by (14) implies an unambiguous increase in the wage cost. In general, the zero profit condition may shift in either direction and the effect on the wage cost is thus ambiguous. The numerical analysis suggests a positive effect on the wage cost. In summary:

**Proposition 2** *A rise in nonemployment benefits reduces tightness, job finding and employment:  $\partial\theta/\partial\rho^o < 0$ ,  $\partial\tilde{\alpha}/\partial\rho^o < 0$  and  $\partial e/\partial\rho^o < 0$ . Sickness absence and the average firing rate increase if  $\phi^s > \phi^p$  but remain unaffected when  $\phi^s = \phi^p$ :  $\partial s^r/\partial\rho^o > 0$  and  $\partial\tilde{\phi}/\partial\rho^o > 0$  if  $\phi^s > \phi^p$ , and  $\partial s^r/\partial\rho^o = \partial\tilde{\phi}/\partial\rho^o = 0$  if  $\phi^s = \phi^p$ . The wage cost increases as long as  $\phi^s = \phi^p$ :  $\partial w_c/\partial\rho^o > 0$  if  $\phi^s = \phi^p$ .*

**Higher Experience Rating** It is clear from (15) that experience rating,  $\tau$ , does not affect tightness, i.e.,  $\partial\theta/\partial\tau = 0$ . It is therefore also neutral with respect to employment. Experience rating is in this respect similar to general taxes,  $t$ ; recall that  $\partial\theta/\partial t = 0$ . The wage cost per labor input, i.e.,  $w_c = w(1+t)$ , falls as  $\tau$  increases but there is no effect on the expected total labor cost per employee, i.e.,  $w_c^T \equiv [F(Q) + \tau(1 - F(Q))\rho^s]w_c$ . The real consumer wage,  $w$ , remains unchanged; this result is obtained by using the zero profit condition and the government's budget restriction to obtain an equation that determines  $w$  independently of  $\tau$ . Hence:

**Proposition 3** *The degree of experience rating has no effect on tightness, employment and the real consumer wage:  $\partial\theta/\partial\tau = \partial e/\partial\tau = \partial w/\partial\tau = 0$ .*

**Higher Vacancy Costs** A rise in vacancy costs,  $\kappa$ , causes an unambiguous fall in tightness, an implication that is clear from eq. (15). Job findings therefore fall and employment falls as long as  $\phi^p = \phi^s$ . When  $\phi^p < \phi^s$ , the fall in tightness will reduce worker absenteeism and therefore reduce firings as well as hirings; the net effect on employment is then ambiguous. The numerical analysis suggests that employment would fall.

The effect on the wage cost depends also on how the zero profit condition is affected. The “direct” effect can be represented as a shift to the left in the  $w_c, \theta$ -space; zero profits require lower wage costs at a given level of tightness when vacancy costs have increased. However, this effect is potentially counteracted by lower absenteeism among workers. However, it can be shown that the direct effect dominates the indirect effect.

**Proposition 4** *Higher vacancy costs reduce tightness, job finding and the wage cost:  $\partial\theta/\partial\kappa < 0$ ,  $\partial\tilde{\alpha}/\partial\kappa < 0$ , and  $\partial w_c/\partial\kappa < 0$ . Absenteeism and firings fall if  $\phi^s > \phi^p$ :  $\partial s^r/\partial\kappa \leq 0$  and  $\partial\tilde{\phi}/\partial\kappa \leq 0$  as  $\phi^s \geq \phi^p$ . Employment falls as long as  $\phi^p = \phi^s$ :  $\partial e/\partial\kappa < 0$  if  $\phi^p = \phi^s$ .*

**Cyclical Effects** Time series data from several countries indicate procyclical absenteeism. Our analysis involves comparisons of steady states and the model as it stands is not directly suitable for studies of business cycle effects. However, we have noted a positive partial equilibrium association between sickness absence and labor market tightness (Lemma 2), an association that it is tempting to interpret as a cyclical effect. But absenteeism and tightness are both endogenous variables and the covariations between the two generally depend on the origins of the exogenous disturbances. Higher vacancy costs lead to lower tightness as well as lower absenteeism, i.e., a positive covariation between tightness and absence. However, higher non-employment benefits lead to lower tightness but higher absenteeism, i.e., a negative covariation between tightness and absence.

We have indexed vacancy costs to productivity, a relationship that may not hold in the short run. If we allow productivity to move without affecting vacancy costs, the model implies a positive covariation between output, tightness and absenteeism. Productivity changes are presumably more important than changes in nonemployment benefits for business cycle fluctuations. Procyclical absenteeism would therefore seem to be the most plausible prediction from a suitably extended version of the model.

### 3.2 Firm-Provided Sick Pay

The general equilibrium now features three key endogenous variables, viz. tightness, wage cost and sick pay ( $\theta$ ,  $w_c$  and  $\rho^s$ ) and we need to invoke eq. (18) in addition to the other relationships. We examine the labor market responses to exogenous changes in nonemployment benefits and in experience rating. Since the firing probability is taken as independent of absence behavior in this case, the absence rate is exclusively determined by  $\rho^s$ .

**Nonemployment Benefits** Higher nonemployment benefits reduce tightness, job finding and employment. The level of sick pay increases, which in turn leads to an increase in absenteeism. The mechanism at work is that higher nonemployment benefits make nonemployed workers more willing to accept an increase in queue length. Firms respond by increasing worker compensation – wages and sick pay – while allowing a decline in tightness, which in turn leads to a fall in employment.

**Proposition 5** *A rise in nonemployment benefits reduces tightness, job finding and employment:  $\partial\theta/\partial\rho^o < 0$ ,  $\partial\tilde{\alpha}/\partial\rho^o < 0$  and  $\partial e/\partial\rho^o < 0$ . The level of sick pay provided by firms increases,  $\partial\rho^s/\partial\rho^o > 0$ , which triggers an increase in absenteeism,  $\partial s^r/\partial\rho^o > 0$ . The wage cost also increases:  $\partial w_c/\partial\rho^o > 0$ .*

**Proof** (Sketch.) Combine (18) and (8) to obtain a relationship  $\tilde{\Gamma}(\theta, \rho^s) = 0$ , where  $(\partial\theta/\partial\rho^s)_{\tilde{\Gamma}} < 0$ ; combine also  $\tilde{\Gamma}(\theta, \rho^s) = 0$  and (15) to obtain a second relationship  $\tilde{W}(\theta, \rho^s) = 0$ , where  $(\partial\theta/\partial\rho^s)_{\tilde{W}} \gtrless 0$ . Note that  $(\partial\theta/\partial\rho^s)_{\tilde{\Gamma}} < (\partial\theta/\partial\rho^s)_{\tilde{W}}$  (see Appendix B) and that  $\rho^o$  features only in the second relationship. Differentiate with respect to  $\rho^o$  and obtain  $\partial\rho^s/\partial\rho^o > 0$  and  $\partial\theta/\partial\rho^o < 0$ . The other results follow by noting that job finding as well as employment depends on tightness, that absence depends on sick pay, and that (18) implies a positive relationship between  $\rho^s$  and  $w_c$ . ■

It is noteworthy that nonemployment benefits have the same qualitative effects on tightness, employment and the wage cost irrespectively of whether sick pay is provided by the government or the firm. The effects on absenteeism are potentially different, however. When sick pay is determined by the government, higher nonemployment benefits increases absenteeism only to the extent that  $\phi^s > \phi^p$  holds; higher nonemployment benefits reduces the cost of job loss and makes the worker less reluctant to risk a job loss. When sick pay is provided by firms, higher nonemployment benefits increases absenteeism through the induced increase in the level of sick pay.

**Experience Rating** Consider next how the firm's choice of sick pay responds to higher experience rating,  $\tau$ . Obviously, a reduction of  $\tau$  below one would imply a subsidy to firm-provided sick pay. The following result is obtained by using a proof analogous to the proof of Proposition 5:

**Proposition 6** *Higher experience rating reduces sick pay and sickness absence, i.e.,  $\partial \rho^s / \partial \tau < 0$  and  $\partial s^r / \partial \tau < 0$*

Since experience rating from the firm's perspective is a tax on sick pay, it is not surprising that firms respond by reducing sick pay which in turn reduces absence. The effects on tightness and employment work through the induced effects on sick pay; the direction of these effects are generally ambiguous. Recall that the effects on tightness and employment from exogenously imposed increases in sick pay were also ambiguous. However, the numerical analysis suggests that tightness as well as employment would fall. Analogously, the numerical analysis suggests that higher experience rating, via the induced fall in sick pay, leads to an increase in tightness and employment.

## 4 Welfare Analysis

Competitive search equilibria have been shown to be socially optimal under some conditions; see Moen (1997) and Rogerson et al (2005). These results do not obtain in our case where individuals are risk averse, credit markets are imperfect and the government finances social insurance benefits by means of taxes. The presence of a social insurance system creates externalities that operate through the government's budget constraint. We analyze these externalities and provide a quantitative comparison of the welfare implications of alternative social insurance policies.

### 4.1 Private versus Public Provision of Sick Pay

Does firm-provided sick pay yield higher or lower compensation compared to what a benevolent government would choose? We approach this issue by asking whether a social planner can improve welfare by implementing a small (infinitesimal) change of sick pay at the privately optimal solution. To facilitate this exercise, it is useful to reformulate the private solution so that it mimics the decision problem facing the planner. This is accomplished

by means of a “dual” approach where the private equilibrium is viewed as the result of a maximization of the worker’s expected utility against a zero profit constraint. This approach yields outcomes that are identical to those obtained from the “primal” approach where profits are maximized against an expected utility constraint for the worker. We assume complete experience rating, i.e.,  $\tau = 1$ . Discounting is ignored ( $r \rightarrow 0$ ) so the relevant welfare objective can be stated as the worker’s expected utility, i.e.,  $\Lambda = e\tilde{v}^e + (1 - e)\tilde{v}^o$ .

It is useful to begin by invoking the zero profit constraint, eq. (8), and obtain tightness as a function of the wage cost and sick pay:

$$\theta = \theta(w_c, \rho^s), \quad \theta_{w_c} < 0, \quad \theta_{\rho^s} < 0 \quad (22)$$

which can be substituted into the worker’s objective function:

$$\Lambda = e[\theta(w_c, \rho^s), R][\tilde{v}^e(w, \rho^s) - \tilde{v}^o(w, \rho^o)] + \tilde{v}^o(w, \rho^o) \quad (23)$$

where  $e[\cdot] = \tilde{\alpha}(\theta)/[\tilde{\alpha}(\theta) + \phi]$ . Employment generally depends on search effort on the extensive margin, i.e.,  $R$ , which in turn depends on tightness,  $R = R(\theta)$ . However, this relationship can be ignored as long as we focus on small (infinitesimal) deviations from the privately optimal solution. The worker’s expected utility is invariant to derivative changes of  $R$ , by the envelope theorem.

The first-order conditions pertaining to the private solution can now be written as:

$$\Lambda_w \equiv A = (\tilde{v}^e - \tilde{v}^o)e_\theta\theta_w + e\tilde{v}_w^e = 0 \quad (24)$$

$$\Lambda_{\rho^s} \equiv B = (\tilde{v}^e - \tilde{v}^o)e_\theta\theta_{\rho^s} + e\tilde{v}_{\rho^s}^e = 0 \quad (25)$$

which imply equality between the firm’s and the worker’s marginal rate of substitution between wages and sick pay. The social planner contemplates a small increase in  $\rho^s$  at the prevailing equilibrium and recognizes the government’s budget constraint,  $t = t[\rho^s, Q(\rho^s), e(\theta, R), \rho^o]$ , in addition to the zero profit constraint. The relevant expression is

$$\begin{aligned} \frac{d\Lambda}{d\rho^s} &= (\tilde{v}^e - \tilde{v}^o)e_\theta \left( \theta_{w_c} \frac{dw_c}{d\rho^s} + \theta_{\rho^s} \right) \\ &+ e \left( \tilde{v}_{\rho^s}^e + \tilde{v}_w^e \frac{dw}{d\rho^s} \right) + (1 - e)\tilde{v}_w^o \frac{dw}{d\rho^s} \end{aligned} \quad (26)$$



where  $\theta_{w_c} = \theta_w/(1+t)$ , and

$$\frac{dw_c}{d\rho^s} = (1+t)\frac{dw}{d\rho^s} + w\frac{dt}{d\rho^s} < 0 \quad (27)$$

where  $dt/d\rho^s$  is the total derivative of the government's budget restriction. Now evaluate the welfare derivative at the privately optimal  $\rho^s$  and obtain:

$$\left(\frac{d\Lambda}{d\rho^s}\right)_{A=B=0} = (\tilde{v}^e - \tilde{v}^o) e_\theta \theta_{w_c} \left(w\frac{dt}{d\rho^s}\right) + (1-e)\tilde{v}_w^o \frac{dw}{d\rho^s} \quad (28)$$

The two terms on the right-hand side of (28) capture externalities associated with the tax system and the unemployment insurance system. The first term involves the tax externality: private agents do not internalize the fact that their decisions affect tax rates via the government's budget constraint. The second term captures what may be referred to as a "wage externality": private agents do not internalize the linkage between their wage decisions, the aggregate wage, and the level of consumption during nonemployment. Nonemployment benefits are indexed to the aggregate wage,  $\tilde{v}_w^o > 0$ ; absent this linkage, there would be no wage externality.

Is the privately chosen  $\rho^s$  too low or too high? Clearly,  $d\Lambda/d\rho^s > 0$  would imply that the private system yields too low sick pay, and vice versa. There is a presumption that  $dt/d\rho^s > 0$ , although this cannot be analytically verified; this would pull in the direction of  $d\Lambda/d\rho^s < 0$ . This may be offset by the wage externality in so far as  $dw/d\rho^s > 0$ . However, we find  $dw/d\rho^s < 0$  in our calibrated model. We obtain  $d\Lambda/d\rho^s < 0$  when we evaluate the derivative at our calibrated equilibrium with firm-provided sick pay. This would suggest, then, that firm-provided sick pay would be too generous relative to what a social planner would prefer.

Instead of asking whether firm-provided sick pay is set too low or too high, we could ask if there are incentives for firms to offer sick pay in addition to the statutory benefits, if the latter were optimally chosen by the government. To answer this question, we evaluate the derivative of the firm's objective function at the socially optimal level of government-provided sick pay. The sign of this derivative is the negative of (28): if  $d\Lambda/d\rho^s < 0$ , the firm could thus increase its profit by topping up the statutory sick pay. Indeed, in countries where statutory sick pay involves relatively low replacement rates, as in the United Kingdom, many employers have their own schemes which top up statutory sick pay.

The exercises so far involve “local” changes in sick pay, taking other instruments as given. We proceed to an analysis of optimal policies that makes use of all available instruments.

## 4.2 Optimal Policies

The problem facing the social planner is to maximize social welfare, which involves maximization of the worker’s expected utility subject to a zero profit constraint and the government’s budget constraint. Appendix C describes the numerical calibration of the model. We use as benchmark a policy with publicly provided sick pay and uniform replacement rates, i.e.,  $\rho^s = \rho^u = \rho^n$ . We also set  $\tau = 1$ , recalling that the degree of experience rating is neutral (Proposition 3). Column (1) in Table 1 shows the outcomes of the welfare maximizations. The optimal uniform replacement rate is 0.31. The other policies, displayed in columns (2) through (6), are as follows: (2) Publicly provided sick pay with optimal differentiation of  $\rho^s$  and  $\rho^o$ ; (3) Publicly provided sick pay with optimal differentiation of  $\rho^s$ ,  $\rho^u$  and  $\rho^n$ ; (4) Privately provided sick pay with optimal  $\rho^o$  and  $\tau = 1$ ; (5) Privately provided sick pay with optimal  $\rho^o$  as well as optimal  $\tau$ ; (6) Privately provided sick pay with optimal  $\rho^u$ ,  $\rho^n$  and  $\tau$ .

Table 1. Welfare comparisons, private and public sick pay.

	Publicly provided			Privately provided		
	Optimal uniform $\rho^j$	Optimal $\rho^s, \rho^o$	Optimal $\rho^s, \rho^u, \rho^n$	Optimal $\rho^o$ $\tau = 1$	Optimal $\rho^o, \tau$	Optimal $\rho^u, \rho^n, \tau$
	(1)	(2)	(3)	(4)	(5)	(6)
$\rho^s$	.307	.228	.225	.262	.228	.225
$\rho^o$	.307	.398		.397	.398	
$\rho^u$	.307	.398	.534	.397	.398	.534
$\rho^n$	.307	.398	.170	.397	.398	.170
$\tau$	1	1	1	1	1.181	1.226
$w_c$	.931	.943	.951	.924	.929	.937
$\theta$	.501	.377	.285	.376	.377	.285
$e$	.930	.911	.915	.911	.911	.915
$p$	.842	.863	.869	.849	.863	.869
$s$	.088	.048	.047	.063	.048	.047
$u$	.062	.070	.081	.071	.070	.081
$n$	.008	.019	.003	.018	.019	.003
$u^r$	.063	.072	.082	.072	.072	.082
$s^r$	.094	.052	.051	.069	.052	.051
$t$	.058	.054	.063	.041	.038	.047
$w$	.881	.895	.895	.888	.895	.895
$\Delta\Lambda$ (%)		0.84	1.56	0.79	0.84	1.56

Notes:  $\phi^p = \phi^s$  in all simulations. See Appendix C for details regarding the parameters. The welfare changes are measured relative to the optimal uniform public system.

The welfare effect of a specific policy is measured relative to case (1). It is expressed as the equivalent of a consumption tax that equalizes welfare across policy regimes. Let  $\Lambda^U$  represent welfare associated with reference case (1) with uniform benefits and  $\Lambda^A$  welfare associated with an alternative policy. The measure of the welfare gain of policy  $A$  relative to policy  $U$  is given by the value of the tax rate  $z$  that solves  $\Lambda^A [(1-z)w; \cdot] = \Lambda^U$ . With logarithmic utility functions we have  $\Delta\Lambda \equiv \Lambda^A - \Lambda^U = -\ln(1-z) \approx z$ .

Table 1 immediately reveals that privately and publicly provided sick pay are equivalent provided that the planner makes optimal use of experience rating; cf. the outcomes in columns (2) and (5), and those in columns (3) and (6). The planner can choose  $\rho^s$  directly, as in the public system; or  $\rho^s$  can

be controlled indirectly by means of  $\tau$ , as in the private system. The welfare implications are identical. The only variables that depend on private/public regime are  $w_c$  and  $t$ ; however, these effects are uninteresting since they do not affect consumption possibilities (lower  $t$  is offset by higher  $\tau$  so there will be no effect on firms' total labor costs).

The equivalence result can be stated formally as follows:

**Proposition 7** *Publicly provided optimal sick pay is welfare equivalent to privately provided sick pay provided that the degree of experience rating is chosen optimally.*

**Proof** Let  $X(\rho^s, \tau)$  denote the solution vector of endogenous variables in the system with public provision of sick pay. The key variables are determined by eqs. (8) and (15). We have  $\partial X/\partial \tau = 0$  by Proposition (3). Consider next a system with firm-provided sick pay, where the key variables are determined by eqs. (8), (15) and (18). We obtain  $\rho^s = \rho^s(\tau)$ , with  $\partial \rho^s/\partial \tau < 0$  by Proposition (5). Substitution of  $\rho^s = \rho^s(\tau)$  into  $X(\rho^s, \tau)$  yields the vector  $X(\rho^s(\tau), \tau)$ . The social planner maximizes  $\Lambda = \Lambda[X(\rho^s, \tau)]$  by choosing  $\rho^s$  directly, or indirectly via  $\tau$ . The relevant first-order condition for the first case (public provision) is  $\Lambda_X(\partial X/\partial \rho^s) = 0$ ; note that  $\Lambda_X(\partial X/\partial \tau) = 0$  is always satisfied. The first-order condition for the second case (private provision) is  $\Lambda_X(\partial X/\partial \rho^s)(\partial \rho^s/\partial \tau) + \Lambda_X(\partial X/\partial \tau) = 0$ , which can be written as  $\Lambda_X(\partial X/\partial \rho^s)(\partial \rho^s/\partial \tau) = 0$  since  $(\partial X/\partial \tau) = 0$ . It follows that the solution vector for the two systems is identical. ■

The government can thus “delegate” the decision on sick pay to firms, provided that it exercises appropriate control over the degree of experience rating. By choosing experience rating optimally, the government induces the private agents to internalize all relevant externalities. As our numerical results in Table 1 indicate, the optimal degree of experience rating involves  $\tau > 1$ ; firms should be charged with *more* than the full amount of their expenditure on sick pay.

In the simulations reported in Table 1, a private system with optimally chosen  $\rho^o$  and  $\tau = 1$  dominates a public system with optimal uniform replacement rates; the welfare gain amounts to 0.8 percent of consumption. More substantial welfare gains are obtained by also differentiating between  $\rho^u$  and  $\rho^n$ . The increase in welfare relative to the reference case amounts to 1.6 percent of consumption. Note that the rankings of the optimal nonwork replacement rates are  $\rho^u > \rho^s > \rho^n$ . There is a case for a relatively generous

unemployment compensation since it provides incentives to substitute active search for inactive nonparticipation. Of course, this type of differentiation presumes that monitoring of job search is feasible, at least to some degree.<sup>18</sup>

## 5 Concluding Remarks

The paper has offered a general equilibrium framework suitable for analyzing absenteeism along with employment, unemployment and nonparticipation. Welfare policy interdependencies arise naturally in the model and can be analyzed in a unified and coherent fashion. It comes as no surprise that nonemployment benefits have adverse employment effects, but it is less obvious that the propensity to be present at work also declines. Higher sickness benefits result in higher absenteeism, as should be expected. However, such policies will also generally affect labor market tightness, wages and employment.

The credibility of the numerical exercises would be much improved if we had better empirical knowledge about the key parameters. Better empirical knowledge about individual responsiveness to changes of sick pay, or other parameters of the sickness insurance system, is crucial for the design of policy. We also need a better understanding of firms' behavior, including knowledge of how firms respond to alternative financing schemes. Experience rating of sickness insurance has sometimes been suggested as a means to encourage firms to offer better workplaces with lower absenteeism. Skeptics have noted that policies that make absenteeism more costly to firms may induce them to discriminate against more sickness-prone individuals in their hiring decisions. To our knowledge, there is so far very little relevant knowledge that can be used for policy evaluation and policy guidance in this area.

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<sup>18</sup>The optimality of  $\rho^u > \rho^n$  can be shown to hold analytically in a simplified version of the model with exogenous tightness.

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## Appendix A. Value Functions

Time is discrete, future sickness status is uncertain and the time horizon is infinite. Tomorrow is another day and each morning involves a draw from the distribution  $F(\xi)$ . Let  $P(\xi)$  denote the expected present value of being present at work,  $S(\xi)$  the value of being on sick leave,  $U(\xi)$  the value of being unemployed, and  $N(\xi)$  the value of being a nonparticipant. These present values are computed after a particular realization of  $\xi$  and involves optimal behavior with respect to future shocks. The value functions are written as follows:

$$P(\xi) = [\ln w - a\xi + \int_0^\infty \phi^p \{\max [U(x), N(x)]\} dF(x) + \int_0^\infty (1 - \phi^p) \{\max [P(x), S(x)]\} dF(x)] \frac{1}{1+r} \quad (\text{A1})$$

$$S(\xi) = [\ln \rho^s w - \xi + \int_0^\infty \phi^s \{\max [U(x), N(x)]\} dF(x) + \int_0^\infty (1 - \phi^s) \{\max [P(x), S(x)]\} dF(x)] \frac{1}{1+r} \quad (\text{A2})$$



$$U(\xi) = [\ln \rho^u w - a\xi + \int_0^\infty \alpha(\theta) \{\max [P(x), S(x)]\} dF(x)] \quad (\text{A3})$$

$$+ \int_0^\infty (1 - \alpha(\theta)) \{\max [U(x), N(x)]\} dF(x) \frac{1}{1+r}$$

$$N(\xi) = [\ln \rho^n w - \xi + \int_0^\infty \{\max [U(x), N(x)]\} dF(x)] \frac{1}{1+r} \quad (\text{A4})$$

The present value of being employed and working involves a flow return given by  $\ln w - a\xi$  as well as changes in utility caused by sickness and labor market shocks. The probability of job loss is  $\phi^p$  and the probability of retaining the job is  $1 - \phi^p$ . If the worker loses the job he decides whether to choose unemployment or nonparticipation, i.e., he takes  $\max [U(x), N(x)]$ . If the job is retained the choice is between work and sick leave and the worker thus takes  $\max [P(x), S(x)]$ . End-of-period discounting is applied at the rate  $r > 0$ . Analogous interpretations hold for the other value functions. Note from (A3) and (A4) that job finding takes place only when unemployed; we have thus ignored transitions from nonparticipation to employment.

The decision rules are such that sufficiently serious sickness makes the worker more inclined to prefer inactivity to activity, i.e., sick-leave is preferred to work and nonparticipation is preferred to unemployment. Consider an individual at work who observes a new value of  $\xi$  and decides to remain at work as long as  $\xi$  does not exceed a critical value,  $Q$ . That is, work is chosen for  $\xi \leq Q$  and sickness absence for  $\xi > Q$ . Analogous rules apply to nonemployed individuals. Let  $R$  denote the critical value of sickness for a nonemployed person. Search unemployment is chosen for  $\xi \leq R$  and nonparticipation for  $\xi > R$ . A reservation sickness strategy is optimal for the employed worker when  $P(\xi) \geq S(\xi)$  for  $\xi \leq Q$ , and  $P(\xi) < S(\xi)$  for  $\xi > Q$ . Note that both  $P(\cdot)$  and  $S(\cdot)$  are decreasing in  $\xi$ , with  $P'(\xi) < S'(\xi)$ :

$$P'(\xi) = -a/(1+r), \quad S'(\xi) = -1/(1+r)$$

which guarantees the optimality of the reservation sickness rule since  $a > 1$ . For a nonemployed person, the optimality of the reservation sickness rule requires that  $U(\xi) \geq N(\xi)$  for  $\xi \leq R$ , and  $U(\xi) < N(\xi)$  for  $\xi > R$ .  $U(\cdot)$  and  $N(\cdot)$  are both decreasing in  $\xi$ , with slopes:

$$U'(\xi) = -a/(1+r), \quad N'(\xi) = -1/(1+r)$$

so the inequality  $U'(\xi) < N'(\xi)$  holds.

The reservation sickness conditions imply that we can define the following maximum value functions for employment and nonemployment:

$$M^e \equiv \int_0^Q P(x)dF(x) + \int_Q^\infty S(x)dF(x) \quad (\text{A5})$$

$$M^o \equiv \int_0^R U(x)dF(x) + \int_R^\infty N(x)dF(x) \quad (\text{A6})$$

where  $M^e$  pertains to employment (work and sick leave) and  $M^o$  to non-employment (unemployment and nonparticipation).  $M^e$  and  $M^o$  are ex ante expected present values of employment and nonemployment in the sense that they correspond to present values before the veil of ignorance concerning  $\xi$  is lifted, given that optimal decision rules are followed in the future. The reservation sickness conditions, given by eqs. (5) and (6) in the main text, are obtained by imposing the indifference condition  $P(Q) = S(Q)$  for the employed worker and the analogous condition  $U(R) = N(R)$  for a worker who is not employed.

## Appendix B: Uniqueness with Endogenous Sick Pay

The three equations that determine  $\rho^s$ ,  $w_c$  and  $\theta$  are:

$$\Gamma(\rho^s, w_c) \equiv \tau\rho^s - \frac{F(Q) - [(y/w_c) - 1]\varepsilon^s}{F(Q) + \varepsilon^s} = 0 \quad (\text{B1})$$

$$Z(w_c, \rho^s, \theta) \equiv F(Q)(y - w_c) - \tau[1 - F(Q)]\rho^s w_c - \kappa y \phi / q(\theta) = 0 \quad (\text{B2})$$

$$\hat{W}(\rho^s, \theta) \equiv \frac{\tilde{v}^e - \tilde{v}^o}{\phi + \tilde{\alpha}(\theta, R)} - \left( \frac{\eta}{1 - \eta} \right) \left[ \frac{\kappa}{F(Q)q(\theta) - \kappa\phi} \right] = 0 \quad (\text{B3})$$

where  $Q = Q(\rho^s)$  and  $\eta$  is taken to be constant. Our strategy is to substitute out  $w_c$  so as to get a system with two equations that determine  $\rho^s$  and  $\theta$ . First, use (B1) and (B2) to get a “zero profit contract curve” of the form:

$$\tilde{\Gamma}(\theta, \rho^s) \equiv [F(Q) + \varepsilon^s](1 - \tau\rho^s) - \frac{F(Q) + \tau[1 - F(Q)]\rho^s}{F(Q) - \kappa\phi/q(\theta)}\varepsilon^s = 0 \quad (\text{B4})$$

where  $\tilde{\Gamma}_\theta < 0$  and  $\tilde{\Gamma}_{\rho^s} < 0$ . As a second step, use (B3) in conjunction with (B4) to get a “modified wage equation” of the form:

$$\tilde{W}(\theta, \rho^s) \equiv \frac{\tilde{v}^e - \tilde{v}^o}{\phi + \tilde{\alpha}(\theta, R)} - \left( \frac{\eta}{1 - \eta} \right) \left[ \frac{\kappa(F(Q) + \varepsilon^s)}{\left( F(Q) + \frac{\tau \rho^s}{1 - \tau \rho^s} \right) \varepsilon^s q(\theta)} \right] = 0 \quad (\text{B5})$$

where  $\tilde{W}_\theta < 0$  and  $\tilde{W}_{\rho^s} \gtrless 0$ .

Lemma 5 in the main text can be proved by establishing that the following inequality holds:

$$\left( \frac{\partial \theta}{\partial \rho^s} \right)_{\tilde{\Gamma}} < \left( \frac{\partial \theta}{\partial \rho^s} \right)_{\tilde{W}} \quad (\text{B6})$$

where  $(\partial \theta / \partial \rho^s)_{\tilde{\Gamma}} = -(\tilde{\Gamma}_{\rho^s} / \tilde{\Gamma}_\theta) < 0$  and  $(\partial \theta / \partial \rho^s)_{\tilde{W}} = -(\tilde{W}_{\rho^s} / \tilde{W}_\theta) \gtrless 0$ . After some tedious calculations it can be shown that this inequality does indeed hold; the details are available on request. In graphical terms, the slope of  $\tilde{\Gamma}(\theta, \rho^s)$  is more negative in the  $\theta, \rho^s$ -space than the slope of  $\tilde{W}(\theta, \rho^s)$ ; as noted, the slope of  $\tilde{W}(\theta, \rho^s)$  can take either sign.

### Appendix C: Calibration of the model

We have calibrated the model assuming that the density  $f(\xi)$  is exponential, i.e.,  $f(\xi) = \lambda \exp(-\lambda \xi)$ ,  $\lambda > 0$ . The absence rate is then given as  $s^r = \exp(-\lambda Q)$ . Since  $Q = -\ln \rho^s / (a - 1)$  when  $\phi^p = \phi^s$ , we have  $\ln s^r = [\lambda / (a - 1)] \ln \rho^s$  in this case. The parameters  $\lambda$  and  $a$  enter into the model through the ratio  $\lambda / (a - 1)$ , i.e., the elasticity of  $s^r$  with respect to  $\rho^s$ . When  $\phi^p < \phi^s$ , the elasticity expression takes the form

$$\frac{d \ln s^r}{d \ln \rho^s} = \left( \frac{\lambda}{a - 1} \right) \left( \frac{\tilde{\alpha} + \phi^p}{\tilde{\alpha} + \tilde{\phi}} \right)$$

which is smaller than  $\lambda / (a - 1)$  since  $\phi^p < \tilde{\phi}$ . However, the difference between  $\phi^p$  and  $\tilde{\phi}$  is negligible for realistic values of the absence rate ( $s^r \leq 0.1$ , say) and  $\lambda / (a - 1)$  is therefore generally a good approximation of the elasticity. We have set  $a = 2$  and the choice of  $\lambda$  is then equivalent to choosing the elasticity of  $s^r$  with respect to  $\rho^s$ . Parameters are chosen so as to get  $s^r = 0.10$ , which corresponds to work hours lost due to absence as a fraction of contractual hours among Swedish employees in 2004, excluding absence due to holidays and similar “predetermined” causes of absence.<sup>19</sup> We set

<sup>19</sup>Sickness absence accounts for over 90 percent of this measure of absence. Source: Labor force surveys, Statistics Sweden.

$\lambda = 2$ , arguably on the high side of estimates of  $d \ln s^r / d \ln \rho^s$ , and require a rather low value for  $\rho^s$  to get  $s^r = 0.10$ . We have set  $\rho^s = \rho^u = \rho^n = 0.325$ , which yields an absence rate around 10 percent.<sup>20</sup> The other parameters were chosen so as to get an unemployment rate of 6.5 percent and an average annual job separation rate of roughly 25 percent. Consistency with empirical observations also requires that the expected duration of unemployment,  $1/\alpha(\theta)$ , should be substantially higher than the expected duration of vacancies,  $1/q(\theta)$ . Moreover, some (scanty) empirical evidence suggests  $Q > R$ , implying less sickness reporting among employees than among nonemployed workers.

Taking a day as the time unit, we set  $\phi^p = 0.25/365$ ,  $\phi^s = 0.35/365$ ,  $y = 1$ ,  $\kappa = 1.85$  and use a matching function of the form  $\alpha(\theta) = 0.015\theta^{0.5}$ . The rate of time preference is set to zero throughout. The implications of these choices are set out in the first column of Table C1. Sickness absence is 10 percent and unemployment is 6.5 percent of the labor force. The expected duration of unemployment is close to 14 weeks whereas the expected duration of vacancies is 6.6 weeks. The average (annual) separation rate is 26 percent. The inactivity rate,  $n/(n + u)$ , is 0.12, thus slightly higher than  $s^r$  (so we have  $Q > R$ ). We have also repeated the exercise under the assumption that there is no excess firing risk associated with absence. That is, we set  $\phi^p = \phi^s = 0.26/365$ , which corresponds to the average separation risk in the previous specification. All other parameters are the same. The results are very similar.

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<sup>20</sup>Replacement rates around 30 percent may seem implausibly low. However, these rates pertain to systems without time limits, whereas existing systems generally involve time limits on benefit receipt. Existing systems also involve some degree of monitoring of sickness status and job search, something that will tend to make higher replacement rates feasible.

Table C1. Calibration of the model.

	Exogenous $\rho^s$		Endogenous $\rho^s$
	$\phi^s > \phi^p$	$\phi^s = \phi^p$	$\phi^s = \phi^p$
$\rho^s$	–	–	.257
$w_c$	.933	.933	.918
$w$	.876	.874	.894
$\theta$	.473	.470	.482
$e$	.927	.927	.927
$p$	.834	.829	.866
$s$	.093	.098	.061
$u$	.064	.064	.063
$n$	.009	.009	.009
$u^r$	.065	.064	.064
$s^r$	.100	.106	.066
$1/\alpha(\theta)$ (in weeks)	13.8	13.9	13.7
$1/q(\theta)$ (in weeks)	6.6	6.5	6.6
$\tilde{\phi}$ (annualized)	.260	.260	.260

The third column of Table C1 shows the outcomes when sick pay is endogenously provided by firms (and  $\tau = 1$ ). All relevant parameters are those that apply in the second column of the table. It turns out that the optimal sick pay chosen by firms is somewhat lower than the benchmark case. Workers are compensated for the lower level of sick pay by higher wages. The lower level of sick pay implies lower sickness absence than in the previous cases. The other outcomes are close to those given in the first two columns.