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July 2007

Forschungsinstitut zur Zukunft der Arbeit Institute for the Study of Labor

# Optimal Taxation and Monopsonistic Labor Market: Does Monopsony Justify the Minimum Wage?

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# ABSTRACT

### Optimal Taxation and Monopsonistic Labor Market: Does Monopsony Justify the Minimum Wage?<sup>\*</sup>

We analyze optimal taxation in an economy with monopsonistic labor markets. The individuals, whose only decisions are whether to work, or not, have heterogeneous productivities and opportunity costs of work. Given its preferences for redistribution, the government, which does not observe the opportunity costs of work, chooses a tax scheme implementing the second best allocation. We compare the optima in the competitive and monopsonistic environments. We find that the government can always implement the second best allocation of the competitive economy in the monopsonistic environment. The optimal tax schedule comprises employment subsidies financed by taxes on profits. In this setup, there is no room for a minimum wage.

JEL Classification: H31, J30, J42

Keywords: minimum wage, optimal taxation, monopsony

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### 1 Introduction

A popular justification of the minimum wage is that it strengthens the hand of the low skilled workers who are exploited by monopsonist employers. As stressed by Dolado et al. (2000), proponents of the minimum wage take the competitive working of the labor market as the exception, rather than the rule, arguing that in many reasonable instances "monopsony" corresponds to the rule. Then, the minimum wage seems to be useful because it increases both employment and the income of low wage workers. This view had a strong influence on economic policy in the last decade. For instance, in 1994, the OECD Jobs Study was arguing that there was a need to "reassess the role of statutory minimum wages as an instrument to achieve redistributive goals, and switch to more direct instruments" (OECD, 1994). Four years later, after the publication of a set of papers and a book arguing that minimum wage increases could benefit low skilled employment according to the predictions of the monopsony model of the labor market (Card and Krueger, 1995), the perspective was quite different: the OECD Employment Outlook stressed that "a well-designed policy package of economic measures, with an appropriately set minimum wage in tandem with in-work benefits, is likely, on balance, to be beneficial in moving towards an employment-centered social policy" (OECD, 1998).

Although the recent minimum wage research finds a wide range of estimates on the overall effects on low-wage employment of an increase in the minimum wage (Neumark and Wascher, 2006), the monopsony model remains influential. For instance, in year 2005, it is still argued by the OECD that "the main impact of downward wage flexibility may be to worsen inactivity, unemployment and low-pay traps." (OECD, 2005, p 142). As a matter of fact, today, statutory or quasi-statutory minimum wages are in place in 21 OECD countries (Immervoll, 2007).

Since the minimum wage is so widespread, it could be thought that economic theory has clearly shown that tax schemes should be supplemented with minimum wages. It is striking to notice that it is very far from being the case. Actually, this issue has not received much attention in the literature. The papers which look at the minimum wage in labor markets with imperfect competition typically ask whether minimum wage increases can improve employment or welfare in the absence of other policy tools.<sup>1</sup> The efficiency of the minimum wage when there are taxes has mostly been considered in labor markets

 $<sup>^{1}</sup>$ See, among others Robinson (1933), Stigler (1946), Drazen (1986), Jones (1987), Manning, (1995, 2003), Rebitzer and Taylor (1995), Bhaskar and To (1999), Masters (1999), Cahuc et al. (2001), Flinn (2006).

with perfect competition.<sup>2</sup> In this context, it turns out that the minimum wage can be welfare improving when tax schemes are constrained (Allen, 1984, Guesnerie and Roberts, 1987). For instance, Allen (1984) shows that the minimum wage may be desirable when taxes are constrained to be linear, but that it becomes redundant with non linear tax schedules. Boadway and Cuff (2001) show that minimum wages can fulfill a useful role as part of an optimal tax transfer scheme if the government can force all the individuals eligible to public transfers to accept any job offer.

Therefore, this literature suggests that minimum wages are only useful in competitive markets when the government does not have access to the full range of tax instruments. If one does not put any restriction on the shape of taxes, a minimum wage  $\underline{W}$  can be mimicked by imposing very high taxes on wages below  $\underline{W}$ . Building on the previous literature, a natural strategy is to look for the second best allocation without any a priori restriction on the shape of the tax function. To implement the resulting optimal allocation, a minimum wage can help when agents of different productivities are paid the same wage at the bottom of the distribution of productivities of employees. It can also be useful when the tax function has a decreasing part for low wages.

The purpose of the present paper is to analyze the scope for minimum wages when labor markets are monopsonistic. The paper focuses on labor supply at the extensive margin where the agents' decision is zero-one, to work or not to work, as in the studies of Diamond (1980), Beaudry and Blackorby (1997), Saez (2002), Choné and Laroque (2005, 2006) and Laroque (2005).

We consider the standard optimum tax environment of these studies. There is a large population of workers who have heterogeneous productivities and opportunity costs of work which are independently distributed in the population. The government is all powerful but is limited by his lack of information on the characteristics of the private agents. Given its preferences for redistribution, it chooses the optimal tax scheme implementing the second best allocation.

We first recall the case of competitive labor markets. Then, we study the optimal tax scheme when the labor market is monopsonistic instead of perfectly competitive. Compared with the *laisser-faire*, apart from the allocative distortions (wage and employment are lower than under perfect competition), the monopsonist' profits create a specific redistributive issue which we analyze in two stages. First, we suppose that the government has the information and power to fully tax profits. We show that there is a labor tax sched-

<sup>&</sup>lt;sup>2</sup>The recent paper of Hungerbühler and Lehmann (2007), where the usefulness of the minimum wage is analyzed in a search and matching model are analyzed, is an exception.

ule that implements the second best allocation of the *competitive* model. Taxing profits and giving employment subsidies yields the optimal allocation and there is no room for minimum wage. Second, we assume that the monopsonist has private information which limits the power of the government in grabbing profits. The government can use corporate taxes on top of the labor tax schedule. Strikingly, the previous result still holds: the minimum wage is useless. The tax tools are enough to reach the best feasible allocation. Accordingly, in our framework, monopsonistic competition does not justify the introduction of a minimum wage. The intuition is that the minimum wage is not useful in the presence of other appropriate fiscal tools.

The paper is organized as follows. The model with heterogeneous skills and heterogeneous opportunity costs of work is presented in section 2. The textbook model of the monopsony, which corresponds to the case of a single skill, is presented in section 3. We recall that the optimum can be implemented either with a minimum wage or with taxes and that the equivalence between the two instruments holds only in the single skill model. Taxes cannot be dispensed with when there are several skills. In the rest of the paper, skill heterogeneity is accounted for. Section 4 describes the optimal tax schemes when the labor market is perfectly competitive. Then, section 5 derives properties of the optimal tax schedule in an economy with a monopsonistic labor market. Finally, section 6 provides some concluding comments.

### 2 The model

We consider an economy made of a continuum of agents of measure 1. A typical agent is described by a couple of exogenous characteristics, denoted by  $\theta = (\omega, \alpha)$ . The first component  $\omega$  denotes her productivity when working full time in market activities, producing an undifferentiated desirable commodity. The second component,  $\alpha$ , is a fixed cost of participating in the labor market, also measured in commodity units. In the economy there are profit maximizing firms that allow the transformation of the agents' labor into commodity, and a benevolent government with a redistributive social aim, who can raise taxes or distribute subsidies and set a minimum wage. The general structure of the economy and the distribution of agents' characteristics are common knowledge.

The labor market works as follows. The government cannot observe the individual characteristics: it only sees whether an agent works or not, and in the former case, the wage paid by her employer. An employer observes the productivities y's of his employees, but not their opportunity costs of work. When she works, the type- $(\omega, \alpha)$  agent produces

a quantity y, at most equal to  $\omega$  (the opportunity cost of work  $\alpha$  is *fixed*: it does not depend on the difference  $(\omega - y)$ ). When working and producing y, an employee gets a net wage W(y), possibly subject to a minimum wage constraint  $W(y) \geq \underline{W}$ . The tax schedule, denoted by T(W) (if negative, the absolute value of T is a subsidy to work), yields a labor cost C(W) = W + T(W).

Production may generate profits. We assume that after tax profits, if any, are dissipated by the owners of the firms with no contribution to social welfare. Under full information, we consider both the normal case where profits are taxed away in a lump sum fashion and the situation where they are not taxed at all. In section 5.5 we introduce an information based model of profit taxation.

The tax receipts are then used to give a subsistence income r to the unemployed agents.

We assume that  $\omega$  and  $\alpha$  are independently distributed. The cumulative distributions of  $\alpha$  and  $\omega$  are denoted  $F(\alpha)$  and  $G(\omega)$  respectively. F has support  $[\underline{\alpha}, \overline{\alpha}]$  while G has support  $[\underline{\omega}, \overline{\omega}]$ . We suppose

$$0 \leq \underline{\alpha} < \overline{\alpha} \leq \infty$$
 and  $0 \leq \underline{\omega} < \overline{\omega} \leq \infty$ .

F and G have continuous derivatives, denoted by f and g respectively, which are strictly positive everywhere on their support. A part of the analysis carries through with an unrestricted distribution for the couple  $(\alpha, \omega)$ , involving mass points and correlation between the two characteristics: we shall point out specifically when the independence assumption is needed.

The agents have a simple choice criterion, linear in income. They decide to produce an output y rather than stay on the dole whenever their financial incentive to work, W(y) - r, is larger than their work opportunity cost  $\alpha$ . Their choice follows from:<sup>3</sup>

$$u(W, r; \alpha, \omega) = \max_{0 \le y \le \omega} [r, W(y) - \alpha].$$
(1)

Let  $y(\omega)$  be the production of an agent of productivity  $\omega$  who works. The proportion of agents of productivity  $\omega$  that are employed is  $F[W(y(\omega)) - r]$ .

The preferences of the government are represented by a social welfare function

$$\underline{\int \int \Psi(u(W,r;\alpha,\omega)) \,\mathrm{d}F(\alpha) \,\mathrm{d}G(\omega)},$$

 $<sup>^3\</sup>mathrm{In}$  case of indifference between several maxima, we suppose that the worker chooses the largest production.

where  $\Psi$  is a non decreasing concave function.

To be feasible, the quadruple (W, T, r, y) must satisfy the budget constraint of the government. When the profits of the firms are not taxed, the budget constraint takes the form

$$\int_{\underline{\omega}}^{\overline{\omega}} T\left\{W(y(\omega))\right\} F[W(y(\omega)) - r)] \,\mathrm{d}G(\omega) = r \int_{\underline{\omega}}^{\overline{\omega}} \left\{\left[1 - F[W(y(\omega)) - r)\right]\right\} \,\mathrm{d}G(\omega).$$
(2)

The left hand side represents the collected taxes, while the right hand side measures the unemployment benefits. When the firms profits are taxed away, the government collects taxes T and profits y - T - W on each job, so that the budget constraint becomes:

$$\int_{\underline{\omega}}^{\bar{\omega}} \{y(\omega) - W(y(\omega))\} F[W(y(\omega)) - r)] \, \mathrm{d}G(\omega) = r \int_{\underline{\omega}}^{\bar{\omega}} \{[1 - F[W(y(\omega)) - r)]\} \, \mathrm{d}G(\omega).$$
(3)

The sequence of decisions is such that the government announces its policy, the tax function, the subsistence income and possibly the minimum wage at the beginning of the period, while anticipating the budget constraint. Then the firms choose the net wage function which relates productivity to net wage. Finally the workers decide on their labor supply.

Through the tax function T and the subsistence income r, the government has powerful policy tools at his disposal. Is a minimum wage useful in this circumstance? In a sense, if one does not put any restriction on the shape of the function T, a minimum wage  $\underline{W}$  indeed is superfluous: for instance, the constraint  $W \geq \underline{W}$  can be mimicked with  $T(W) = +\infty$  for  $W < \underline{W}$ . In what follows we look for the second best allocation without any a priori restriction on the shape of the tax function. Then, given plausible restrictions on the shape of the tax function, we analyze whether a minimum wage may be useful in implementing the second best optimum.

From an institutional viewpoint, it may make sense to separate the enforcement of the tax scheme at the bottom of the income distribution, when it presents features of a minimum wage, from the general accounting rules associated with a tax schedule. For instance, in the US, the minimum wage can be enforced through monetary, civil or criminal penalties. Employers who willfully or repeatedly violate the minimum wage requirements are subject to a civil money penalty of up to \$1,000 for each violation. Such considerations are absent from our theoretical model.

### 3 The case with a single skill

Our aim is to analyze the role of the minimum wage and taxes when labor markets are monopsonistic and when there are workers with heterogeneous productivity. It is worth briefly recalling the justification of the minimum wage in the textbook model of monopsony introduced by Robinson (1933). In the textbook model, all workers are homogeneous with respect to productivity: there is a single productivity level  $\omega$ . Moreover, taxes and transfers on labor are usually not considered, so that we can set r and T equal to zero for now.

The monopsonist is assumed to face a labor supply curve that relates the wage, W, to the level of employment, equal to F(W). The monopsonist maximizes his profit, knowing the shape of the supply curve, i.e. the quantity

$$\Pi = (\omega - W)F(W).$$

This leads to the first order condition<sup>4</sup>

$$\omega = W + \frac{F(W)}{F'(W)}.$$
(4)

The left-hand side of this equation is the marginal productivity of labor and the righthand side is the marginal cost of hiring an extra worker. The marginal cost is higher than the wage W because the employer computes the overall effect of the increase on his wage bill, knowing the labor supply schedule: the derivative of the wage bill WF(W) with respect to W is WF'(W) + F(W), implying a cost per worker of W + F(W)/F'(W) (there are F'(W) extra workers). The solution is represented graphically on Figure 1, where the marginal cost of labor (MCL) is represented as a function of the employment level. The equilibrium employment level is at the intersection of (MCL) with the horizontal line of intercept  $\omega$ . The wage chosen by the monopsonist, denoted by  $W_M$ , is read on the labor supply curve: it is lower than the marginal productivity  $\omega$ . Both wage and employment are lower than under perfect competition. The employer is making positive profits on workers who are all paid below their productivity.

As Robinson (1933, p 295) argued, "monopsonistic exploitation of this type can be removed by the imposition of a minimum wage". It suffices to set a minimum wage up to the competitive wage  $\omega$  to force the monopsonist to the competitive level of employment, equal to  $F(\omega)$ . Notice that the minimum wage eats away the profits of the monopsonist.

<sup>&</sup>lt;sup>4</sup>When the function F is log concave, an assumption which is often made, the logarithm of profit is concave so that there is a single maximum characterized by the first order condition.

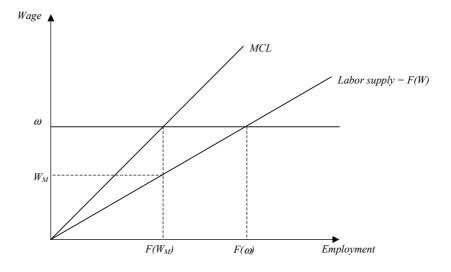


Figure 1: The textbook model of monopsony

A government who does not care about redistribution (whose preferences are represented by a linear function  $\Psi$ ) would systematically choose this minimum wage level in order to maximize production. But the government can also use taxes to implement this optimum. The profit of the firm, when wages are taxed (the typical net wage W bears a tax t), is

$$(\omega - t - W) F(W).$$

Profit maximization yields the same first order condition as (4) where  $\omega - t$  is substituted for  $\omega$ . Accordingly, the employer is induced to choose the competitive wage  $W = \omega$  and the competitive employment level if  $t = -F(\omega)/F'(\omega)$ , i.e. the tax is a subsidy. Assume that profits can be taxed away in a lump sum way: this allows the government to finance the subsidies. Therefore, a complete system of taxes is equivalent to the minimum wage in this model. This is reminiscent of the subsidy proposed by A. Robinson, (Robinson, 1933, p. 163), to lead a monopolist to produce the competitive output.

To summarize, the textbook model of monopsony shows that efficiency can be reached either with the minimum wage or with taxes on profits whose proceeds are used to finance employment subsidies. In this set up, the minimum wage seems easier to implement, rather than levying taxes on profits and redistributing the proceeds of these taxes under the form of employment subsidies. In what follows, we show that the minimum wage does not perform so well when there is a diversity of skills.

# 4 Second best tax schemes with perfectly competitive labor markets

Before studying the monopsony, it is useful to recall the properties of optimal taxation and of the second best allocations in a perfectly competitive economy with heterogeneous skills.

A second best optimum is a triple (W, r, y), such that

1. the workers choose whether to work or not, and the amount they produce  $y(\omega)$ , taking as given (W, r), according to

$$\max_{0 \le y \le \omega} [r, W(y) - \alpha], \tag{5}$$

2. anticipating the behavior of the agents, the government chooses a tax schedule (W, r) which maximizes the social welfare function:

$$\int_{\underline{\omega}}^{\overline{\omega}} \left\{ \int_{\underline{\alpha}}^{W[y(\omega)]-r} \Psi\left[W(y(\omega)) - \alpha\right] \mathrm{d}F(\alpha) + \left\{1 - F[W(y(\omega)) - r]\right\} \Psi(r) \right\} \mathrm{d}G(\omega),$$
(6)

subject to the budget constraint:

$$\int_{\underline{\omega}}^{\overline{\omega}} \{y(\omega) - W[y(\omega)] + r\} F[W(y(\omega)) - r] dG(\omega) = r.$$
(7)

The optima, as defined by 1. and 2., are studied in some details in Choné and Laroque (2006). Following the definition, second best optima are found as follows. First (5) implies that without loss of generality one can restrict the attention to non decreasing functions W and that workers work at their full productivity. Second, one solves the optimization problem of the government, maximizing (6) with respect to (W, r) subject to the constraint (7) on the set of non decreasing functions W. The tax wedge is then merely the difference between the productivity  $\omega$  and the net wage  $W(\omega)$ .

At the optimum the wage function is uniquely defined on an interval  $[\omega_{inf}, \overline{\omega}]$  of productivities, for which there are working agents. It can be given any arbitrary value, at most equal to  $r + \underline{\alpha}$  for smaller productivities. Minimum wages have no room here, in a competitive environment. If binding, they would prevent mutually beneficial transactions and destroy jobs. This argument relies on the competitiveness of the market and does not carry over when the labor market is monopsonistic.

## 5 Second best tax schemes with monopsonistic labor markets

We consider now the case where the government faces a large firm, which is the sole buyer on the labor market. We first describe some basic properties of second best allocations. Second, we prove the main result: when profits can be fully taxed the government can bypass the monopsonist and implement the second best allocation of the competitive economy. Then we show that the optimal tax schedules do not have the features of a minimum wage and we present two simple examples that illustrate the shapes of such schedules. Finally, we extend the model to a situation where a lack of information prevents the government to fully tax profits. It turns out that the previous results carry over to this case.

#### 5.1 Properties of second best allocations

Formally, a second best optimum is a quadruple (W, T, r, y), such that

1. the workers choose whether to work or not, and the amount they produce  $y(\omega)$ , taking as given (W, r), according to

$$\max_{0 \le y \le \omega} [r, W(y) - \alpha]; \tag{8}$$

2. the monopsonist chooses the net wage, taking as given (T, r), and anticipating the reactions of the workers to its choice, by maximizing

$$\max \int_{\underline{\omega}}^{\overline{\omega}} \left[ y(\omega) - W(y(\omega)) - T(W(y(\omega))) \right] F[W(y(\omega)) - r] \mathrm{d}G(\omega); \tag{9}$$

3. the government chooses (T, r), anticipating the behavior of the monopsonist and of the workers, maximizing the social welfare function:

$$\int_{\underline{\omega}}^{\overline{\omega}} \left\{ \int_{\underline{\alpha}}^{W(y(\omega))-r} \Psi(W(y(\omega)) - \alpha) \mathrm{d}F(\alpha) + \left\{ 1 - F[W(y(\omega)) - r] \right\} \Psi(r) \right\} \, \mathrm{d}G(\omega),$$
(10)

subject to the feasibility constraint

$$\int_{\underline{\omega}}^{\overline{\omega}} \left\{ y(\omega) - W(y(\omega)) + r \right\} F(W(y(\omega)) - r) \, \mathrm{d}G(\omega) = r, \tag{11}$$

when profits are taxed, or

$$\int_{\underline{\omega}}^{\overline{\omega}} \left\{ T(W(y(\omega))) + r \right\} F(W(y(\omega)) - r) \, \mathrm{d}G(\omega) = r, \tag{12}$$

when they are not. We assume that the government chooses a continuous and bounded below tax function T.

In order to simplify the resolution of this problem, it is useful to show that one can restrict the analysis to net wages that increase with productivity.

**Lemma 1** At a second best optimum, without loss of generality:

- 1. the monopsonist can choose a net wage function W that is non decreasing with respect to productivity;
- 2. the government can choose a tax schedule T such that the function  $x \to x + T(x)$  is everywhere non decreasing;
- 3. all the employees work at their full productivity  $\omega$ .

**Proof**: see appendix.

This lemma shows that any second best allocation can be reached with non decreasing net wage and cost schedules. At a second best optimum when the labor market is dominated by a monopsonist, individuals work at their full time productivity and the net wage is a non decreasing function of productivity, as in the competitive case.

#### 5.2 The main result

The description of the programs of the government (compare 2. in the definition of second best optima in section 4 with 3. in the definition of second best optima in section 5) makes it clear that, when profits can be fully taxed, the only difference between the competitive and monopsony problems comes from the (possible) restrictions imposed by the behavior of the monopsonist (9). As a consequence, the second best optima of the monopsonistic economy cannot Pareto dominate those of the competitive economy. They can at best coincide with them if the government manages to undo the wrongs caused by the monopsonist. We are going to show that this is indeed the case.

The Lagrangian of the program of the government in a competitive economy is

$$L = \int_{\underline{\omega}}^{\overline{\omega}} \left\{ \int_{\underline{\alpha}}^{W(\omega)-r} \Psi(W(\omega) - \alpha) dF(\alpha) + \Psi(r) \{1 - F[W(\omega) - r]\} + \lambda \left\{ [\omega - W(\omega) + r] F[W(\omega) - r] - r \right\} \right\} dG(\omega), \quad (13)$$

to be maximized over  $(W(.), \lambda, r)$ , for non decreasing W's. Let  $\tilde{\Omega} = \{\omega | \omega \in [\underline{\omega}, \overline{\omega}], W(\omega) - r > \underline{\alpha}\}$  be the endogenous set of productivities for which there are a positive number of employees at the optimum. The Lagrangian can be rewritten equivalently as

$$\frac{L}{\lambda} = -r + \frac{\Psi(r)}{\lambda} +$$

$$\int_{\tilde{\Omega}} \left\{ \omega - W(\omega) + r + \int_{\underline{\alpha}}^{W(\omega) - r} \frac{\Psi(W(\omega) - \alpha) - \Psi(r)}{\lambda F[W(\omega) - r]} \, \mathrm{d}F(\alpha) \right\} F[W(\omega) - r] \, \mathrm{d}G(\omega).$$
(14)

Now, the objectives of the government (14) and of the monopsonist (9) are aligned provided that to any value  $W, W > r + \underline{\alpha}$ , of the net wage corresponds a value  $\overline{C}(W) = W + \overline{T}(W)$  of the labor cost such that

$$\bar{C}(W) = W - r - \int_{\underline{\alpha}}^{W-r} \frac{\Psi(W - \alpha) - \Psi(r)}{\lambda F[W - r]} \mathrm{d}F(\alpha).$$
(15)

Note that  $\overline{C}$  can take any value when wages are smaller than  $r + \underline{\alpha}$  since nobody works for such wages. Therefore

**Theorem 1.** The second best optimal allocations in a monopsonistic economy where the profits can be fully taxed are identical to that of a competitive economy. An optimum in the monopsonistic economy can be implemented through a tax wedge  $\overline{T}(W)$  which satisfies

$$\bar{T}(W) = -r^c - \frac{1}{\lambda^c} \left\{ \int_{\underline{\alpha}}^{W-r^c} \frac{\Psi(W-\alpha)}{F[W-r^c]} \, \mathrm{d}F(\alpha) - \Psi(r^c) \right\} \text{ for } W \ge r^c + \underline{\alpha},$$

where  $r^c$  and  $\lambda^c$  are respectively the optimal subsistence income and marginal cost of public funds of the competitive economy.

Theorem 1 states that all allocations that can be reached in a competitive economy can also be reached, with different tax schedules, when the labor market is monopsonistic. It is a striking result which means that the government can systematically undo the wrongs caused by the monopsonist at no cost. Actually, this is already the case in the textbook example, discussed above, where the government can use employment subsidies financed by taxes on profits to reach the desired allocation. Theorem 1 shows that the solution is similar when there is heterogeneity in skills. Also it is easily checked that

#### **Corollary 2.** The tax function $\overline{T}$ is negative for $W \ge r^c + \underline{\alpha}$ .

Employment subsidies are efficient because they counteract the natural inclination of the monopsonist to reduce its demand for labor<sup>5</sup>. It should be stressed that the proof of Theorem 1 relies on the independence of the distributions of productivities and work opportunity costs. Indeed, in case of dependence, the argument does not go through: the number of workers, that is equal to  $F(W(\omega) - r)$  in equation (14), becomes a function  $F(W(\omega) - r|\omega)$  that depends on  $\omega$ . Then, the expression of the cost that aligns the objective of the government and the monopsonist, defined in equation (15), has no economic meaning since it depends on the unobserved  $\omega$ .

It would be of interest to know whether Theorem 1 extends to situations where productivity is correlated with work opportunity cost. We do not have a general answer to this question. However, there is an extreme polar case which is easily dealt with: this is the situation where the work opportunity cost is a continuous increasing function of productivity, say  $a(\omega)$ . It can be shown that the first best optimum consists in putting to work all individuals of productivity  $\omega$  greater than or equal to  $a(\omega)$ , while distributing welfare equally with a utility level equal to r for everyone where

$$r = \int_{\omega \ge a(\omega)} [\omega - a(\omega)] G(\omega).$$

The government can implement this optimum in a monopsonistic economy, when the monopsony profits are taxed away. Let C(W) = W - r (compare with (15)). Then the monopsonist pays the workers at the lowest acceptable net wage,  $W(\omega) = a(\omega) + r$ , and employs all individuals who bring a non negative profit, i.e. such that  $\omega - C(W(\omega)) = \omega - a(\omega) \ge 0$ , the same individuals as in the first best optimum.

### 5.3 On taxation and the minimum wage

Through the tax function T(W), the tax on profits and the subsistence income  $r^c$ , the government can reach the same second best allocations in the monopsonistic and the

<sup>&</sup>lt;sup>5</sup>Note that the function  $\bar{C}(W)$ , defined in (15), can have decreasing parts. The construction of Lemma 1 allows us to define an equivalent non decreasing cost function  $C(W) = \min_{x \ge W} \bar{C}(x)$ . It is easy to see that the associated tax schedule, T, is also negative, as the original function  $\bar{T}$ , for all  $W > r^c + \underline{\alpha}$ .

competitive economy. The question that we have to address now is whether a minimum wage may be helpful to implement a second best allocation when the labor market is monopsonistic.

A first interpretation of the existence of a minimum wage is related to the properties of the wage distribution i.e. the function  $W(\omega)$  for  $\omega > \omega_{inf}$ . The minimum wage can be helpful if agents with different productivities are paid the same wage at the bottom of the second best allocation. Such a situation with pooling may arise. But it is not specifically related to monopsony since this is a property of the second best allocation which is identical in the competitive and the monopsony models. According to this interpretation, monopsonistic competition does not justify the introduction of a minimum wage.

Another interpretation is related to the properties of the tax function and the associated cost function. It can be considered that there is no need for a minimum wage when the allocation can be implemented through a labor cost function which is equal to the *net* wage for all  $W \leq W_{inf}$ . Indeed, in that situation tax authorities may leave to *laisser-faire* all labor contracts for wages smaller than  $W_{inf}$  (i.e. labor cost C(W) is equal to net wage W in this region), and use the second best tax function above. Since labor is always subsidized when the labor market is monopsonistic, as shown in Corollary 2, with this interpretation, there is no need for a minimum wage. The government just announces that only net wages above  $W_{inf}$  are entitled to subsidies.

Our conclusion may look surprising to those who think that minimum wages are justified, even if it is possible to use labor subsidies, because monopsonistic employers take subsidies in their pocket instead of giving wage increases. At first sight, this argument looks particularly convincing in the simple case where labor supply is infinitely elastic, as in the remark at the end of Section 5.2. In that case, the monopsonist always sets the net wage at the reservation level  $a(\omega) + r$ , whatever the labor tax or subsidy. Employment subsidies serve to implement the optimal level of employment, and the minimum wage is not needed. Indeed the objectives of the monopsonist and of the government are aligned when the elasticity of labor supply is infinite as far as the determination of the net wage is concerned.

Let us now look more precisely at the shape of the second best tax schedules in the monopsonistic model.

#### 5.4 The shape of optimal tax schedules: two examples

The tax policy is not the same in the competitive and the monopsonistic case. The properties of the tax schedule in these two cases can be illustrated by looking at two polar assumptions about the objective of the government: first an output maximizing government, whose preferences are represented by the social welfare function  $\Psi(x) = x$ ; then a Rawlsian government which maximizes the welfare of the most disadvantaged agents.

#### Output maximizing government

When the labor market is competitive, the equilibrium without tax and subsistence income,  $r^c = T(W) = 0$ , yields a first best allocation. It is easy to check, from (13), that the marginal cost of public funds  $\lambda^c$  is equal to 1. An agent with characteristics  $(\omega, \alpha)$  gets a utility level max  $(0, \omega - \alpha)$ . When the labor market is monopsonistic, the equilibrium without taxes does not yield a first best allocation any more. The previous section has shown that the first best allocation can nevertheless be implemented. The tax system which supports it can be found by rewriting the Lagrangian of the government program as  $\int_{\omega}^{\overline{\omega}} [\omega - C(W(\omega))] F(W(\omega)) dG(\omega)$ , with  $C(W) = W - \int_{\alpha}^{W} \frac{W-\alpha}{F(W)} dF(\alpha)$ . The tax, equal to the difference between labor cost C(W) and net wage W, is negative, equal to  $-\int_{\alpha}^{W} (W - \alpha) dF(\alpha)/F(W)$ .

The solution is represented on Figure 2 in the case where F and G are uniform over  $[0, \bar{\alpha}]$  and  $[0, \bar{\omega}]$  respectively. Then, measured as a function of net wage, the subsidy is equal to W/2, so that the labor cost associated with a net wage W is also W/2 and the monopsony eventually chooses the net wage schedule  $W(\omega) = \omega$ .

#### Rawlsian government

A Rawlsian government maximizes the value of the subsistence income provided to the unemployed agents. Noting  $I(\omega) = W(\omega) - r$  the incentive to work, the program of the Rawlsian government can be written as

$$\max_{I(\cdot)} r = \int_{\underline{\omega}}^{\overline{\omega}} \left\{ \omega - I(\omega) \right\} F[I(\omega)] \, \mathrm{d}G(\omega).$$
(16)

The comparison of this program with the program of the monopsonist (9) immediately shows that the objectives of the Rawlsian government and the monopsonist are aligned if the labor cost schedule  $C(\cdot)$  is equal to the incentive to work W - r. As shown in figure

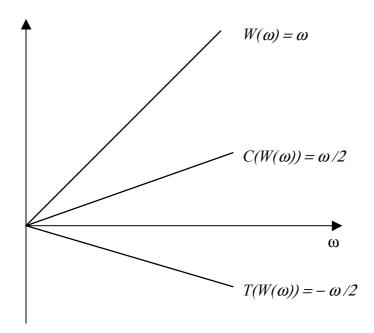


Figure 2: The net wage schedule  $W(\omega)$ , the tax schedule  $T(W(\omega))$ , the labor cost schedule  $C(W(\omega))$  when there is an output maximizing government in a monopsonistic economy with uniform distribution functions.

3, when F and G are uniform over  $[0, \bar{\alpha}]$  and  $[0, \bar{\omega}]$  respectively, the resulting allocation provides net wages  $W(\omega) = r + (\omega/2)$  and a subsistence income  $r = \bar{\omega}/8\bar{\alpha}$ .

#### 5.5 What happens when profits cannot be fully taxed away?

So far, it has been assumed that profits are entirely taxed. This is consistent with an all powerful government who knows the level of monopsony profits. To study the second best allocations when profits are not automatically taxed, we introduce information asymmetries which prevent the government from confiscating the profits of the monopsonist. A simple way to proceed is to consider a situation where the government regulates a continuum of separate local labor markets. In all these identical labor markets, there is a local monopsonist which faces the same labor supply as the one of the previous section: these markets work independently and there is no labor mobility across markets. The government announces the same tax schedule over the whole territory. The local monopsonist supports an entry cost h, which is drawn independently across regions with the c.d.f. H. At the outset of the game, the government announces, on top of (T, r), an operating tax t to be paid by the functioning firms. Since the government does not observe the entry costs of the firms, the operating tax can only be constant. Every firm computes the before

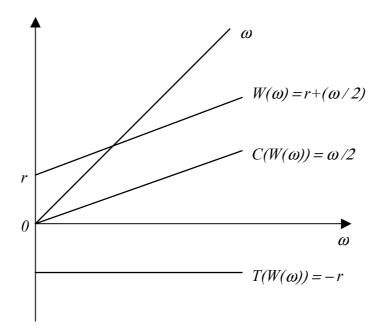


Figure 3: The net wage schedule  $W(\omega)$ , the tax schedule  $T(W(\omega))$ , the labor cost schedule  $C(W(\omega))$  when there is a Rawlsian government in a monopsonistic economy with uniform distribution functions.

tax operating profit  $\pi$ , which reads

$$\pi = \int_{\underline{\omega}}^{\overline{\omega}} \left\{ \omega - W(\omega) - T(W(\omega)) \right\} F(W(\omega) - r) \, \mathrm{d}G(\omega).$$

under (T, r), and decides to operate when  $\pi - t - h$  is non negative, i.e. with probability  $H(\pi - t)$ . All the workers in the regions where entry is not profitable stay unemployed.

The program of the government becomes

$$\max_{[T(\cdot),r,t]} H(\pi-t) \int_{\underline{\omega}}^{\bar{\omega}} \left\{ \int_{\underline{\alpha}}^{W(\omega)-r} \Psi(W(\omega) - \alpha) \, \mathrm{d}F(\alpha) + \{1 - F[W(\omega)) - r]\} \, \Psi(r) \right\} \, \mathrm{d}G(\omega) + [1 - H(\pi - t)] \Psi(r),$$

subject to the feasibility constraint

$$H(\pi - t) \int_{\underline{\omega}}^{\overline{\omega}} \{\omega - W(\omega) + r\} F(W(\omega) - r) dG(\omega) = r + (\pi - t)H(\pi - t),$$

and the behavior of the monopsonist

$$W(\cdot) = \arg \max_{S(\cdot)} \int_{\underline{\omega}}^{\overline{\omega}} \left\{ \omega - S(\omega) - T(S(\omega)) \right\} F(S(\omega) - r) \, \mathrm{d}G(\omega).$$

While the profit  $\pi$ , which depends on T, shows up in the objective and the feasibility constraint of the government, this does not create difficulties since it only appears through the after tax profit  $\pi - t$ . Ignoring the behavior of the monopsonist, one can write an auxiliary problem for the government:

$$\max_{[W(\cdot),r,\pi-t]} H(\pi-t) \int_{\underline{\omega}}^{\bar{\omega}} \left\{ \int_{\underline{\alpha}}^{W(\omega)-r} \Psi(W(\omega) - \alpha) \, \mathrm{d}F(\alpha) + \left\{ 1 - F[W(\omega)) - r \right] \right\} \Psi(r) \right\} \, \mathrm{d}G(\omega) + \left[ 1 - H(\pi-t) \right] \Psi(r),$$

subject to the feasibility constraint

$$H(\pi - t) \int_{\underline{\omega}}^{\overline{\omega}} \{\omega - W(\omega) + r\} F(W(\omega) - r) dG(\omega) = r + (\pi - t)H(\pi - t).$$

The solution of this auxiliary problem yields a value of the objective function of the government at least as high as that of the original problem.

As before, let  $\lambda$  be the Lagrangian multiplier associated with the feasibility constraint. Let us denote by  $W^*(\cdot), r^*, z^*, \lambda^*$  a solution of the auxiliary problem, where  $z^*$  stands for the after tax profit. As in the proof of Theorem 1, it can be shown that the objective of the government and of the monopsonist are aligned if one defines

$$T^*(W) = -r^* - \int_{\underline{\alpha}}^{W(\omega)-r^*} \frac{\Psi(W-\alpha) - \Psi(r^*)}{\lambda^* F[W-r^*]} \,\mathrm{d}F(\alpha).$$

To implement the solution of the auxiliary problem, let  $\pi^*$  be the before tax profit of the monopsonist under  $\{T^*(\cdot), r^*\}$ , and the entry tax be  $t^* = \pi^* - z^*$ .

Looking at the feasibility constraint it looks like the allocation of a competitive economy where a fraction  $H(z^*)$  of firms are operating and where a lump sum income  $z^*$ is dissipated. Therefore, it is likely that the marginal cost of public funds,  $\lambda^*$ , is larger than  $\lambda^c$ , the marginal cost of public funds of the competitive economy where all firms are operating with zero profits. Similarly the subsistence income  $r^*$  is likely to be smaller than  $r^c$ .

It is worth noting that the function  $T^*$  has the same analytical expression as in Theorem 1, but different values of the marginal costs of public funds and the subsistence income. Therefore, as discussed in section 5.3, there is no room for a minimum wage even though profits cannot be fully taxed. With the instruments adapted to its information structure, the government has all the necessary fiscal tools to reach the second best without using the minimum wage. It could be worth analyzing the degree of generality of this result.

### 6 Conclusion

In this paper we analyze income redistribution when labor markets are monopsonistic. The government has a large range of instruments at its disposal, which includes wages and profits taxes and the minimum wage. In our framework, the minimum wage appears to be of no use: it cannot be substituted to the tax schedule at the bottom of the wage distribution.

This result holds when abilities are distributed independently of work opportunity costs. It is also satisfied when work opportunity cost is a deterministic increasing function of ability. We do not know how to handle the intermediate cases.

The conclusion that monopsony does not justify the minimum wage does not mean that the minimum wage is useless as a part of efficient redistributive schemes in all circumstances. However, typically, there must be restrictions on the set of available tax instruments, so that the minimum wage somehow is a substitute to another missing tool. More research is needed in this area, to explain which instruments are likely to be missing and why the minimum wage may be useful.

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## A Appendix: Proof of Lemma 1

1) This is a consequence of the behavior of workers. When confronted with a net wage schedule  $\overline{W}$ , workers' choice of y follows from:

$$\sup_{0 \le y \le \omega} [r, \overline{W}(y) - \alpha].$$

Let

$$W(y) = \sup\{\overline{W}(z)|0 \le z \le y\}.$$

By construction W is non decreasing, the utility levels attained by the agents under  $\overline{W}$  and W are identical, while the labor supply under  $\overline{W}$  is a subset of the labor supply under W. Therefore, the monopsonist can choose a non decreasing net wage schedule without loss of generality.

2) This follows from a study of the behavior of the monopsonist. Consider any bounded below function  $\overline{C}$ , and let

$$C(w) = \inf_{x \ge w} \overline{C}(x).$$

Define  $C = \{w | C(w) < \overline{C}(w)\}$ , or equivalently  $\{w | \text{ there exists } z > w \text{ such that } \overline{C}(z) < \overline{C}(w)\}$ . This set is empty if  $\overline{C}$  is non decreasing, and C coincides with  $\overline{C}$  outside of C.

We show that a monopsonist facing the tax schedule leading to  $\overline{C}$  will never employ a worker at a net wage in  $\mathcal{C}$ , so that C and  $\overline{C}$  lead to the same allocation.

Let  $\overline{W}$  be a net wage schedule. The monopsony profits are

$$\int_0^{\omega} \left[ \overline{y}(\omega) - \overline{C}(\overline{W}(\overline{y}(\omega))) \right] F\left( \overline{W}(\overline{y}(\omega)) - r \right) \, \mathrm{d}G(\omega),$$

where  $\overline{y}(\omega)$  is the production of a worker of productivity  $\omega$  who faces the net wage schedule  $\overline{W}$ .

**Property**: Assume that  $\overline{C}$  is a continuous bounded below function on  $\mathbb{R}_+$ . There is an optimal non decreasing net wage schedule W such that

$$W(\mathbb{R}_+) \cap \mathcal{C} = \emptyset.$$

**Proof of the property**: Take a W associated with  $\overline{C}$ , which is non decreasing by 1), whose range may have a non empty intersection with C. We modify it into a new function W whose range does not intersect C, while weakly increasing the profits of the monopsonist.

By continuity of  $\overline{C}$ , the set C is made of the union of disjoint intervals, say  $(w_0, w_1)$ . One of the two following situations arises:

- a) either  $\overline{C}(z) > \overline{C}(w_1)$  for all z smaller than  $w_1$ ;
- b) or there exists  $w_0$  such that  $\overline{C}(w_0) = \overline{C}(w_1)$ , and  $\overline{C}(z) > \overline{C}(w_1)$  for all z such that  $w_0 > z > w_1$ .

We treat the two cases in turn, supposing that there is some z in the interval and some y such that z = W(y).

a) In the first case, let

$$y_1 = \inf\{y | W(y) \ge w_1\}.$$

We modify W for all  $y \leq y_1$  through define:

$$W(y) \le \underline{\alpha} + r \quad \text{for } y < \overline{C}(w_1) W(y) = w_1 \qquad \text{for } \overline{C}(w_1) < y < y_1$$

By construction the new W is non decreasing. It leads to profits at least as large as the original W: if there are points such that  $y < \overline{C}(w_1)$ , the pointwise profit initially equal to  $[y-\overline{C}(W(y))]F(W(y))$ , is negative, but becomes null because nobody works when  $W(y) \leq \underline{\alpha} + r$ ; when there are points such that  $\overline{C}(w_1) < y < y_1$ , the pointwise profit is initially positive, but is at least as large after the transformation since  $\overline{C}(w_1) \leq \overline{C}(\overline{W}(y))$  and  $F(w_1) \geq F(\overline{W}(y))$ . The range of the modified W has an empty intersection with  $[0, w_1)$  as desired.

b) In the second case, let

$$y_0 = \sup\{y|W(y) \le w_0\},\$$

and

$$y_1 = \inf\{y | W(y) \ge w_1\}.$$

We modify W on the interval  $(y_0, y_1)$  through

$$W(y) = \begin{cases} w_0 & \text{for } y_0 < y < \overline{C}(w_1) \\ w_1 & \text{for } \overline{C}(w_1) < y < y_1. \end{cases}$$

By construction, the new W is non decreasing. It leads to profit sat least as large as the original W: for the y's such that  $y_0 < y < \overline{C}(w_1)$ , if any, this follows from the fact that the pointwise profit  $[y - \overline{C}(W(y))]F(W(y) - r)$  is negative, associated with the inequalities  $\overline{C}(\overline{W}(y)) > \overline{C}(w_0)$  and  $F(w_0 - r) \leq F(W(y) - r)$ ; similarly for the y's such that  $\overline{C}(w_1) < y < y_1$ , the pointwise profit is positive and  $F(w_1 - r) \geq F(\overline{W}(y) - r)$ . The range of the modified W has an empty intersection with  $(w_0, w_1)$ .

This completes the proof of the Property.