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## Modes of spousal interaction and the labor market environment

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## Modes of Spousal Interaction and the Labor Market Environment

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# Modes of Spousal Interaction and the Labor Market Environment

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## **ABSTRACT**

### **Modes of Spousal Interaction and the Labor Market Environment\***

We formulate a model of household behavior in which cooperation is costly and in which these costs vary across households. Some households rationally decide to behave noncooperatively, which in our context is an efficient outcome. An intriguing feature of the model is that, while the welfare of the spouses is continuous in the state variables, labor supply decisions are not. Small changes in state variables may result in large changes in labor supplies when the household switches its mode of behavior. We estimate the model using a nationally representative sample of Italian households and find that the costly cooperation model significantly outperforms a noncooperative model. This suggests the possibility of attaining large gains in aggregate labor supply by adopting policies which promote cooperative household behavior.

JEL Classification: C79, D19, J22

Keywords: household time allocation, Nash bargaining, Nash equilibrium, maximum likelihood

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# 1 Introduction

There is a long history of the theoretical and empirical investigation of married women's labor supply decisions. Perhaps the starting point for modern econometric analysis of this question is Heckman (1974), in which a neoclassical model of wives' labor supply was estimated using disaggregated data. The starting point of his analysis, and most of those that immediately followed, was the specification of a household utility function which included as arguments the leisure levels of wives and household consumption. With the addition of a wage function, Heckman was able to consistently estimate household preference parameters and the wage function in a manner that eliminated the types of endogenous sampling problems known to create estimator bias when the participation decision is ignored.<sup>1</sup>

Many researchers have estimated household labor supply functions in the intervening years using models based on household utility function specifications, though in many cases the husband's labor supply decision has been treated as predetermined or exogenous. Over the last fifteen years there has been a movement to view the family as a collection of agents with their own preferences who are united through the sharing of public goods, emotional ties, and production technologies. Household members are seen as behaving strategically with respect to one another given their rather complicated and interconnected resource constraints. Analysis of these situations has focused on describing and analyzing cooperative equilibrium outcomes. Though models using the cooperative approach (e.g., Manser and Brown (1980), McElroy and Horney (1981), Chiappori (1988)) differ in many respects, they share the common characteristic of generating outcomes that are Pareto-efficient (the primary distinction between them being the method for selecting a point on the Pareto frontier). The noncooperative approach, which uses Nash equilibrium as an equilibrium concept (e.g. Leuthold (1968), Bourguignon (1984), Del Boca and Flinn (1995)), leads to outcomes that are generally Pareto-dominated. The analytic attractiveness of noncooperative equilibrium models lies in the fact that equilibria are often unique, an especially distinct advantage when conducting empirical investigations.

A large number of empirical studies have tested whether observed household behavior is more consistent with a single household utility function or with a model that posits strategic interactions between household members. These studies have led to a decisive rejection of the "unitary" model. Unfortunately, there have been few empirical studies to date that have attempted to actually estimate a collective model of household labor supply (some notable exceptions include Kapteyn and Kooreman (1992), Fortin and Lacroix (1997), and Blundell et al. (2005)). Two of the more important reasons for the paucity of empirical studies are the stringent data requirements for estimation of such a model and lack of agreement regarding the "refinement" to utilize when selecting a unique equilibrium when a multiplicity exist (as is the case in virtually all cooperative models).

Some researchers have advocated using the hypothesis of Pareto efficiency in nonunitary models as an identification device (see, e.g., Bourguignon and Chiappori (1992) and Flinn (2000)). Our view in this paper is slightly more eclectic. We view labor supply outcomes as either being associated with a particular utility outcome on the Pareto frontier (the one chosen under symmetric Nash bargaining) or to be associated with the noncooperative equi-

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<sup>1</sup>While Heckman's model was based on an explicit model of utility maximization, it did assume that the labor supply decision of the husband was predetermined.

librium point. In reality there are a continuum of points that dominate the noncooperative equilibrium point and that do not lie on the Pareto frontier, however developing a model that allows such outcomes to enter the choice set of the household seems beyond our means.<sup>2</sup> Our paper expands the equilibrium choice set to two focal points, but it still represents a very restrictive view of the world.

Even under an assumption of efficiency there is wide latitude in modeling the mechanism by which a specific efficient outcome is implemented, as is evidenced by the lively debate between advocates of the use of Nash bargaining or other axiomatic systems (e.g., McElroy and Horney (1981, 1990), McElroy (1990)) and those advocating a more data driven approach (e.g., Chiappori (1988)). The use of an axiomatic system such as Nash bargaining requires that one first specify a “disagreement outcome” with respect to which each party’s surplus can be explicitly defined.<sup>3</sup> It has long been appreciated that the bargaining outcome can depend critically on the specification of this threat point. Most often in the household economics literature the threat point has been assumed to represent the value to each agent of living independently from the other. Lundberg and Pollak (1993) provide an illuminating discussion of the consequences of alternative specifications of the threat point on the analysis of household decision-making. In particular, instead of assuming the value of the divorce state as the disagreement point for each partner, they consider this point to be given by the value of the marriage to each given some default mode of behavior, which they call “separate spheres.” In this state, each party takes decisions and generally acts in a manner in accordance with “customary” gender roles. Lundberg and Pollak state that households will choose to behave in this customary way when the “transactions costs” they face are too high.

In this paper we develop and estimate a model of household labor supply which allows for both cooperative and noncooperative intrahousehold behavior.<sup>4</sup> We endow husbands and wives with individual utility functions defined over consumption and leisure, and for simplicity (and due to data limitations) we constrain utility interactions to be defined solely with respect to joint consumption of all market goods purchased by the household. We assume that the utility functions of spouses are Cobb-Douglas and that the Cobb-Douglas parameters are randomly distributed in the population according to a distribution function  $G(\alpha_1, \alpha_2)$ , where  $\alpha_i$  is the preference parameter attached to (private) leisure by spouse  $i$ . We adopt a neoclassical labor supply approach in that spouses are assumed to receive wage offers at the beginning of the period and are free to choose any level of labor supply in the feasible set  $[0, T_i]$ , where  $T_i$  is the time endowment of spouse  $i$ . Wage offers are drawn from the joint

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<sup>2</sup>We should note that the cooperative equilibria in our model do in fact generally lie inside the Pareto frontier, at least the Pareto frontier defined in the costless implementation case. Given the cost of non-compliance, characterized by  $\xi$ , our cooperative equilibria do belong to the Pareto frontier associated with  $\xi$ .

<sup>3</sup>The sharing rule approach advocated by Chiappori and others avoids the requirement of explicitly specifying a threat point by positing something of a reduced form approach to the selection of an outcome on the Pareto frontier. Though estimation of the sharing rule does not require specification of a disagreement outcome, it does require explicit assumptions regarding the arguments of the utility functions of the two spouses.

<sup>4</sup>Lugo-Gil (2003) contains an analysis of a model based on a similar idea. In her case, spouses decide on consumption allocations in a cooperative manner after the outside option is optimally chosen. All “intact” households chose a threat point either of divorce or noncooperative behavior. The choice of threat point has an impact on intrahousehold allocations.

distribution  $F(w_1, w_2)$ ; due to assortative mating, these wage draws may be correlated.<sup>5</sup> We begin by investigating the case in which spouses behave noncooperatively. We characterize the equilibrium type, i.e., whether both spouses work, one works, or neither works, in terms of critical wage values conditional on preference types and household nonlabor income.

Perhaps the most innovative feature of the analysis is that we allow for a mixture of cooperative and noncooperative equilibrium outcomes in the model. Under the preference structure we assume, we first show that whenever the noncooperative equilibrium results in at least one spouse supplying time to the labor market, there exists a cooperative equilibrium which Pareto dominates it and in which both spouses supply more time to the market than in the noncooperative equilibrium. Since virtually all households in our sample of young and middle-aged households have at least one person in the market, all would seem ‘eligible’ to improve the payoff of both spouses by moving to a cooperative equilibrium, yet they do not. To rationalize this outcome within our modeling framework, there must exist some cost, in some form, to implementing a cooperative equilibrium. We provide a discussion of the form such costs or constraints could take, before settling on one. We will assume the presence of a tax,  $1 - \xi$ , on the consumption gains from cooperative behavior, a tax which is household specific and that follows a distribution  $Q(\xi)$  in the population of households. We show that given household preference and choice set characteristics, there exists a unique value  $\xi^*$  such that all households with a value  $\xi < \xi^*$  behave in noncooperative manner while those with  $\xi \geq \xi^*$  behave cooperatively. Since cooperative behavior is costly, it is not always inefficient to behave noncooperatively. In our model of “costly cooperation,” all outcomes are efficient, including those which result in only one spouse working.

In our model, each household has a positive probability of being cooperative. The probability that a household is cooperative is an increasing function of the gains from cooperation, which is measured here as the product of the surpluses attained by the spouses under symmetric Nash bargaining. We assume that each household member faces a randomly-determined cost of cooperation  $\theta$  which has a population distribution function given by  $M(\theta; \zeta)$ . The parameter characterizing the distribution may be a function of household members’ characteristics as well as environmental variables. The cooperation cost reduces the surplus each individual gets from cooperation and, when  $\theta$  is large enough, can make noncooperation a preferable mode of behavior.

The empirical analysis is conducted using a relatively rich cross-sectional data set from the Bank of Italy for the year 1998. In the future, we plan to use our modeling framework to perform comparative analyses involving the Italian and U.S. labor markets. We view our set up as being a general one in which household behavior can be compared across labor market and cultural environments. In particular, we are able to account for differences in intra-household labor supply responses in terms of differences in (1) joint wage offer distributions, (2) intrahousehold preference distributions, and (3) costs of cooperative behavior. Taken together, all three sources of household differences are capable of generating very different patterns of cooperative behavior across labor markets, which is the main innovation in our modeling framework.

The plan of the paper is as follows. In Section 2 we briefly discuss some aspects of the Italian labor market and family structure. Section 3 contains a development of the

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<sup>5</sup>For the moment we do not allow for “assortative mating” in terms of the preference parameter distributions, i.e., we assume that the preference draws for a husband and wife are independent..

theoretical framework employed, including a discussion of noncooperative equilibrium, the cooperative equilibrium, and the cooperative equilibrium with coordination costs. In Section 4 we provide details of the maximum likelihood estimator employed to estimate the wage distribution, preference distribution, and distribution of coordination costs - which are the primitives of the model. In Section 5 we present the data and our (preliminary) empirical results. Section 6 presents some illustrative simulation results, and conclusions are offered in Section 7.

## 2 Some Features of the Italian Labor Market

The Italian labor market is considered to be one of the most highly regulated among Western European countries. Strict rules apply regarding the hiring and firing of workers and permissible types of employment arrangements. The hiring system and the high entry wage as well as very strict firing rules severely restrict employment opportunities for labor market entrants. These labor market regulations are considered to be largely responsible for the high unemployment rates of women and youth.

A particularly important aspect of the rigidity of the labor market especially as regards married women is the limited menu of available employment arrangements. Progression towards a more flexible working hours system has begun later in Italy than in other countries and has been much slower. This is partially due to the institutional constraints imposed upon employers; for example, under current regulations social contributions paid by employers are strictly proportional to the number of employees, not their hours worked, which makes the employment of two part-time workers more costly than one full-time employee. Moreover, the service sector, where part-time work is traditionally more widespread, has not developed as quickly in Italy as in other countries (Colombino and Del Boca (1990)). Some have argued that the strong influence of unions, largely representing the interests of a male membership, has skewed the labor market in the direction of offering mainly full-time jobs meant to be held for the majority of one's adult life.

Another factor discouraging the participation of mothers is the structure of the Italian child care sector. Public child care services are typically inexpensive relative to private sector alternatives, though their capacity in terms of number of children and hours per child is extremely limited, particularly for children under the age of four. While the public system is highly subsidized and provides a quality of service that is comparable with private alternatives, the hours in which it is available (assuming that a mother can find a slot) are incompatible with full-time work. Since part-time work options are limited, the combination of these factors make it quite difficult for mothers to find profitable employment opportunities (Del Boca (2003)).

Given these rigidities, the neoclassical model estimated below in which wages are treated as parametric and hours choices are "unconstrained" would appear to be a poor approximation to the actual situation on the ground. As is true even in the U.S., there is an extreme degree of clustering of hours outcomes around 40 hours per week (particularly for husbands), with the female hours distribution clustered around 40, 30, and 0 hours. Beginning from a smooth set of utility functions for spouses and a reasonably disperse joint wage offer distribution, it is difficult to produce this kind of clustering without resorting to institutional constraints, which are difficult to quantify and introduce into a modeling



framework, or an equilibrium analysis that specifically models the decisions of firms regarding the types of wage-hours bundles to optimally offer.<sup>6</sup> Both these routes offer their own challenges and have their own drawbacks.

The model developed here has some hope of producing the types of observed clustering of hours outcomes we observe in practice, at least within households in which both spouses work. The reason is that switches in behavioral mode within the household are associated with discontinuous hours “jumps” for both spouses. These types of discontinuities can possibly better fit the hours distributions we observe in practice. In the empirical work conducted below we hope to get some idea of the model’s ability to generate reasonable approximations to observed hours distributions.

### 3 Models of Household Labor Supply Decisions

Throughout the paper we assume that individuals have Cobb-Douglas preferences over their own leisure and the consumption of a market good. Because we don’t have access to consumption information, we will assume that all consumption within the household is public. While this assumption is less than ideal, there are few attractive alternatives to it because of the limitations of our data (it also considerably simplifies the analysis, it must be said). Let spouse  $i$  have a utility function given by

$$u_i(l_i, c) = \alpha_i \ln(l_i) + (1 - \alpha_i) \ln(c), \quad i = 1, 2,$$

where  $\alpha_i$  is the preference parameter of spouse  $i$ ,  $l_i$  is their leisure consumption, and  $c$  is the household consumption of a market good. The total amount of time available to spouse  $i$  is denoted  $T_i$ , and the wage rate of individual  $i$  is given by  $w_i$ . Total household consumption is then

$$c = Y + w_1(T_1 - l_1) + w_2(T_2 - l_2),$$

where  $Y = y_1 + y_2$ , with  $y_i$  denoting the nonlabor income of individual  $i$ . We have assumed that the price of  $c$  is equal to 1 without loss of generality.

#### 3.1 The Noncooperative Equilibrium

The noncooperative equilibrium solution is determined as follows. The objective functions of the household members are characterized by the preference parameter draws  $(\alpha_1 \alpha_2)$ . Household nonlabor income  $Y$  is predetermined. Given preferences and  $Y$ , we describe the noncooperative equilibrium as a function of the wage draws  $(w_1 w_2)$ . For any set of preferences and  $Y$ , the equilibrium solution may result in both spouses working, neither working, or one of the two working. We characterize the conditions on the wage draws that result in each type of equilibrium outcome, and then given the type, we present the actual labor supply choices made by any spouse who is working.

The reaction functions of the spouses are used to determine all noncooperative outcomes. Define the reaction function of spouse  $i$  given the labor supply choice of spouse  $i'$  as

$$\hat{h}_i(h_{i'}) = \arg \max_{h_i} \alpha_i \ln(T_i - h_i) + (1 - \alpha_i) \ln(Y + w_i h_i + w_{i'} h_{i'}),$$

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<sup>6</sup>For an example of an analysis along these lines, see Aaberge, Colombino, and Strom (1998)

which yields

$$\hat{h}_i(h_{i'}) = (1 - \alpha_i)T_i - \frac{\alpha_i}{w_i}[Y + w_{i'}h_{i'}], \quad i = 1, 2; \quad i' \neq i.$$

We begin by characterizing the critical wage values  $(w_1^{**}, w_2^{**})$  that have the property

$$w_1 < w_1^{**} \text{ and } w_2 < w_2^{**} \Leftrightarrow h_1^N = 0 \text{ and } h_2^N = 0,$$

where  $h_i^N$  is the noncooperative equilibrium leisure choice of spouse  $i$ . These critical values are determined as follows. If spouse 2 chooses not to supply time to the market then  $h_2 = 0$ . In this case, the reaction function of agent 1 yields a labor supply choice of

$$\hat{h}_1(0) = (1 - \alpha_1)T_1 - \alpha_1 \frac{Y}{w_1}. \quad (1)$$

In this case individual 1 will choose to supply no time to the market whenever

$$\begin{aligned} (1 - \alpha_1)T_1 - \alpha_1 \frac{Y}{w_1} &\leq 0 \\ \Rightarrow w_1 &\leq \frac{\alpha_1}{1 - \alpha_1} \frac{Y}{T_1} \\ w_1 &\leq w_1^{**}. \end{aligned}$$

We can perform the same analysis for individual 2 conditional on a leisure demand of individual 1 equal to  $T_1$ . This case is completely symmetric, and we find the individual 2 doesn't participate given that agent 1 doesn't participate when

$$w_2 \leq w_2^{**} \equiv \frac{\alpha_2}{1 - \alpha_2} \frac{Y}{T_2}.$$

From this result we know that if  $w_1 > w_1^{**}$  or  $w_2 > w_2^{**}$  then at least one of the two spouses will work. To determine the conditions under which spouse 1 only, spouse 2 only, or both spouses work, we return to the reaction functions of the two agents. Say that only agent 1 is working at the wage  $w_1$ . Then we know that the labor supply of spouse 1 is given by [1], and we define the critical value  $w_2^*(w_1)$  as that value of the wage offer to individual 2 at which they would just be willing to supply time to the market. This critical value is defined as

$$\begin{aligned} 0 &= (1 - \alpha_2)T_2 - \frac{\alpha_2}{w_2^*(w_1)}[Y + w_1\hat{h}_1(0)] \\ \Rightarrow w_2^*(w_1) &= \frac{\alpha_2(1 - \alpha_1)}{(1 - \alpha_2)T_2}[Y + w_1T_1] \end{aligned}$$

Then we have agent 1 works and agent 2 does not if and only if  $w_1 > w_1^{**}$  and  $w_2 \leq w_2^*(w_1)$ . If the wage offers satisfy these conditions, then the labor supply of spouse 1 is given by  $\hat{h}_1(0)$ .

The situation is symmetric with respect to the case of spouse 1 not working while spouse 2 supplies time to the market. In this case we define the critical value

$$w_1^*(w_2) = \frac{\alpha_1(1 - \alpha_2)}{(1 - \alpha_1)T_1}[Y + w_2T_2],$$

and in equilibrium spouse 2 will supply time to the market and spouse 1 will not if and only if  $w_1 \leq w_1^*(w_2)$  and  $w_2 > w_2^*$ .

If both spouses work, the unique noncooperative equilibrium  $(h_1^N, h_2^N)$  is given by

$$\begin{aligned} h_1^N &= \hat{h}_1(h_2^N) \\ h_2^N &= \hat{h}_2(h_1^N), \end{aligned}$$

which under our functional form assumptions implies

$$\begin{aligned} h_1^N &= T_1 - \frac{\alpha_1(1-\alpha_2)}{1-\alpha_1\alpha_2} \frac{FI}{w_1} \\ h_2^N &= T_2 - \frac{\alpha_2(1-\alpha_1)}{1-\alpha_1\alpha_2} \frac{FI}{w_2}, \end{aligned}$$

where  $FI = Y + w_1T_1 + w_2T_2$  denotes full (household) income. For the equilibrium to be of this type, it must be the case that  $w_1 > w_1^*(w_2)$  and  $w_2 > w_2^*(w_1)$ .

**Table 1**  
**Characterization of the Noncooperative Equilibrium**  
**Equilibrium Hours Choices**

<i>Equilibrium Type</i>	$w_1$	$w_2$	$h_1^N$	$h_2^N$
I	$w_1 \leq w_1^{**}$	$w_2 \leq w_2^{**}$	0	0
II	$w_1 > w_1^{**}$	$w_2 \leq w_2^*(w_1)$	$T_1 - \frac{\alpha_1}{w_1}[Y + w_1T_1]$	0
III	$w_1 \leq w_1^*(w_2)$	$w_2 > w_2^{**}$	0	$T_2 - \frac{\alpha_2}{w_2}[Y + w_2T_2]$
IV	$w_1 > w_1^*(w_2)$	$w_2 > w_2^*(w_1)$	$T_1 - \frac{\alpha_1(1-\alpha_2)}{1-\alpha_1\alpha_2} \frac{FI}{w_1}$	$T_2 - \frac{\alpha_2(1-\alpha_1)}{1-\alpha_1\alpha_2} \frac{FI}{w_2}$

The noncooperative equilibrium in terms of market hours choices is summarized in Table 1. For econometric purposes the fact that there is a unique equilibrium for every possible value of  $\Gamma$  is an extremely useful property. We forego any detailed discussion of the comparative statics properties of the equilibrium since all are standard.

### 3.2 Costless Cooperative Equilibria

It is clear that the equilibrium described in the previous section is inefficient due to the fact that neither spouse considers the welfare of the other when making their own labor supply decisions. In this section we consider the characteristics of cooperative equilibria given the environment we have described.

Prior to confronting the vexing issue of selecting from the set of cooperative equilibria, we first describe the set of such equilibria. Let the noncooperative labor supply equilibrium of household with characteristics  $\Gamma = (y_1 \ y_2 \ w_1 \ w_2 \ \alpha_1 \ \alpha_2 \ T_1 \ T_2)$  be given by  $(h_1^N \ h_2^N)(\Gamma)$ , and define the noncooperative utility values of the two spouses by  $V_i^N = \alpha_i \ln(T_i - h_i^N(\Gamma)) +$

$(1 - \alpha_i) \ln(Y + w_1 h_1^N(\Gamma) + w_2 h_2^N(\Gamma))$ ,  $i = 1, 2$ . In considering the cooperative problem, we will make extensive use of the Nash product,

$$S(h_1, h_2; \delta) = [\alpha_1 \ln(T_1 - h_1) + (1 - \alpha_1) \ln(Y + w_1 h_1 + w_2 h_2) - V_1^N]^\delta \quad (2) \\ \times [\alpha_2 \ln(T_2 - h_2) + (1 - \alpha_2) \ln(Y + w_1 h_1 + w_2 h_2) - V_2^N]^{1-\delta}.$$

Now given that a cooperative solution exists, if we simply impose the efficiency criterion (i.e., that the equilibrium utility yields for the spouses lie on the Pareto frontier) it is well known that the solution can be characterized as follows. Let  $(h_1^C(\delta), h_2^C(\delta))$  denote the arguments that maximize [2] for a given bargaining power parameter  $\delta \in [0, 1]$ . Then an efficient equilibrium labor supply choice is an element of the set  $(h_1^C(\delta), h_2^C(\delta))$ ,  $\delta \in [0, 1]$ . There is considerable debate as to the manner in which a particular outcome from this set is selected, both in theory and practice. We will see that in this particular model, there are situations in which the choice among cooperative equilibria is not an issue, since there exists only one.

Before proceeding to some formal results on this question, some intuition may be helpful. Given our specification of the utility functions of the spouses, it is clear that there exists a potential for substantial gains in the utility levels of each through cooperation. This is due to the fact that in making noncooperative decisions the impact of one spouse's labor supply choice on the welfare of the other spouse (through the publicly consumed good  $c$ ) is ignored. For the case in which the noncooperative equilibrium has both spouses participating in the market, a continuum of cooperative equilibria will always exist in which each supplies more time to the market, where each equilibrium in the set corresponds to a unique solution value to [2] associated with a distinct value of  $\delta \in [0, 1]$ .

When one or both spouses do not supply time to the market in the noncooperative equilibrium, a cooperative solution may not result in either spending more time in the market. When this is the case, we will say that the cooperative equilibrium is not Pareto-improving and is, in fact, identical to the noncooperative equilibrium. This occurs when one, or both spouses, even after taking into account the impact of their labor supply decision on the welfare of the other, would choose not to supply time to the market. As an extreme example, consider the case in which spouse 1 has a wage offer of 0. There will be a well-defined noncooperative equilibrium in this case in which  $h_1 = 0$ , but there will be no possibility of a cooperative solution existing since the impact of 1's supplying time to the market on agent 2's welfare is nil. Below we derive necessary and sufficient conditions on the set  $\Gamma$  for a cooperative solution to exist.

We now turn to the formalization of some of these results. We begin by stating and proving a result that will also be important when we consider the econometric implementation of the cooperative model.

**Proposition 1** *In any Pareto-improving cooperative equilibrium the labor supplies of both spouses are strictly positive.*

**Proof.** Assume that in the noncooperative equilibrium  $h_1^N > 0$  and  $h_2^N = 0$ . Assume that  $h_2^C = 0$ . In this case, for agent 2 to receive at least as high a level of utility under cooperation it must be the case that  $h_1^C \geq h_1^N$ . If  $h_1^C = h_1^N$  then the cooperative equilibrium is identical to the noncooperative equilibrium and there is no welfare improvement for

either spouse. If  $h_1^C > h_1^N$  then agent 1 has lower welfare under cooperation than under noncooperation. The symmetric argument applies for the case of  $h_1^N = 0$  and  $h_2^N > 0$ .

When  $h_1^N = 0$  and  $h_2^N = 0$ , a cooperative equilibrium must have both spouses working by a similar argument. If neither works under the cooperative equilibrium, there is no welfare gain for either spouse since the cooperative and noncooperative equilibria are identical. If only one works, for them to experience a welfare gain implies the other must experience a welfare loss. Then both must work in any Pareto-improving cooperative equilibrium. ■

When both spouses work in the noncooperative equilibrium, there exists a set of efficient equilibria in which they both sell more time to the market than they would under noncooperation. In the situation where at least one doesn't work, the above proposition implies that the cooperative equilibrium is unique and is identical to the noncooperative equilibrium.

In characterizing the conditions for a cooperative equilibrium to dominate the noncooperative one, conditions are once again imposed upon the state space  $\Gamma$ . The critical wage values that describe the subset of  $\Gamma$  associated with cooperation (which corresponds to both spouses working) will be less than those derived above which characterized the simultaneous participation decision in the noncooperative case.

When we allow for the possibility of cooperative behavior we immediately run into several disturbing empirical implications. We begin by establishing a useful characterization result.

**Proposition 2** *Let the noncooperative leisure choice of agent  $i$  be denoted  $l_i^N$ . Then a cooperative equilibrium exists if and only if*

$$\alpha_1 w_2 (T_2 - h_2^N) + \alpha_2 w_1 (T_1 - h_1^N) - \alpha_1 \alpha_2 (Y + w_1 T_1 + w_2 T_2) > 0.$$

*Proof:* Without loss of generality, assume that agent 1 is free to choose the hours  $h_1$  and  $h_2$  subject only to the condition that agent 2 be made no worse off than she would be in the noncooperative equilibrium. Define the choice of hours as the deviation from the noncooperative level, or  $h_i = h_i^N + \varepsilon_i$ , with  $\varepsilon_i \geq 0$ ,  $i = 1, 2$ . Then the constraint on the choice of hours facing agent 1 is given by

$$\alpha_2 \ln(T_2 - h_2^N - \varepsilon_2) + (1 - \alpha_2) \ln(Y + w_1(h_1^N + \varepsilon_1) + w_2(h_2^N + \varepsilon_2)) - V_2^N = 0.$$

Implicitly differentiating this function yields

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{(1 - \alpha_2)w_1(T_2 - h_2 - \varepsilon_2)}{\alpha_2(Y + w_1(h_1^N + \varepsilon_1) + w_2(h_2^N + \varepsilon_2)) - (1 - \alpha_2)w_2(T_2 - h_2^N - \varepsilon_2)}.$$

Evaluated at  $\varepsilon_1 = 0$  (which implies that  $\varepsilon_2 = 0$  by the definition of the constraint), this expression becomes

$$\frac{\frac{w_1}{w_2}}{\frac{\alpha_2}{(1 - \alpha_2)} \frac{c^N}{w_2(T_2 - h_2^N)} - 1},$$

where  $c^N = Y + w_1 h_1^N + w_2 h_2^N$ .

After substituting the constraint into the (pseudo) problem faced by 1, we have

$$\max_{\varepsilon_1} \alpha_1 \ln(T_1 - h_1^N - \varepsilon_1) + (1 - \alpha_1) \ln(Y + w_1(h_1^N + \varepsilon_1) + w_2(h_2^N + \varepsilon_2(\varepsilon_1))). \quad (3)$$

For an interior solution to exist, i.e., for  $\varepsilon_1^* > 0$ , it must be the case that the derivative of [3] evaluated at  $\varepsilon_1 = 0$  is positive. This condition is

$$\left\{ -\frac{\alpha_1}{(T_1 - h_1^N - \varepsilon_1)} + \frac{(1 - \alpha_1)}{(Y + w_1(h_1^N + \varepsilon_1) + w_2(h_2^N + \varepsilon_2))} \left\{ w_1 + w_2 \frac{d\varepsilon_2}{d\varepsilon_1} \right\} \right\} \Big|_{\varepsilon_1=0} > 0$$

$$\Rightarrow \alpha_1 w_2 (T_2 - h_2^N) + \alpha_2 w_1 (T_1 - h_1^N) - \alpha_1 \alpha_2 (Y + w_1 T_1 + w_2 T_2) > 0.$$

■

This result is very useful in characterizing the space of cooperative solutions. For example, consider the situation in which both individuals work in the noncooperative equilibrium. Unsurprisingly, there will always exist a cooperative equilibrium in this case, and as we have seen, in the cooperative equilibrium both will supply more time to the market.

**Corollary 3** *When  $h_1^N > 0$  and  $h_2^N > 0$  a cooperative equilibrium exists.*

*Proof:* Define the set of wages that produce a noncooperative equilibrium in which both spouses work given their preference types  $\alpha_1$  and  $\alpha_2$  and nonlabor income  $Y$  by  $\Omega^B(\alpha_1, \alpha_2, Y) \subset \{w_1 > w_1^{**}, w_2 > w_2^{**}\}$ . Then for any  $(w_1, w_2) \in \Omega^B(\alpha_1, \alpha_2, Y)$ , we substitute the interior solutions for  $h_1^N$  and  $h_2^N$  into  $CF$  to get

$$\alpha_1 \alpha_2 (Y + w_1 T_1 + w_2 T_2) \left\{ \frac{(1 - \alpha_1)}{(1 - \alpha_1 \alpha_2)} + \frac{(1 - \alpha_2)}{(1 - \alpha_1 \alpha_2)} - 1 \right\}$$

$$= \frac{\alpha_1 \alpha_2 (Y + w_1 T_1 + w_2 T_2)}{(1 - \alpha_1 \alpha_2)} (1 - \alpha_1)(1 - \alpha_2)$$

$$> 0, \forall (\alpha_1, \alpha_2) \in (0, 1)^2.$$

■

Perhaps the next result is slightly more surprising.

**Corollary 4** *When  $h_i^N > 0$  and  $h_{i'}^N = 0$ ,  $i \neq i'$ , a cooperative equilibrium exists.*

*Proof:* Without loss of generality, say that  $h_1^N > 0$  and  $h_2^N = 0$ , and define the set of wages that result in this outcome by  $\Omega^1(\alpha_1, \alpha_2, Y)$ . Then for any  $(w_1, w_2) \in \Omega^1(\alpha_1, \alpha_2, Y)$ , substitution of the equilibrium leisure demands of agents 1 and 2 yields the following value of  $CF$

$$\alpha_1 w_2 T_2 + \alpha_2 w_1 \left[ \frac{\alpha_1}{w_1} (Y + w_1 T_1) \right] - \alpha_1 \alpha_2 (Y + w_1 T_1 + w_2 T_2)$$

$$= \alpha_1 (1 - \alpha_2) w_2 T_2$$

$$> 0, \forall (\alpha_1, \alpha_2) \in (0, 1)^2.$$

■

Finally, we consider the case in which neither spouse works in the noncooperative equilibrium. In such a situation, a cooperative equilibrium may or may not exist. The relevant condition is easily derived.

**Corollary 5** When  $h_1^N = 0$  and  $h_2^N = 0$ , a cooperative equilibrium exists iff

$$\alpha_1(1 - \alpha_2)w_1T_1 + \alpha_2(1 - \alpha_1)w_2T_2 - \alpha_1\alpha_2Y > 0. \quad (4)$$

*Proof:* Let the set of wages that result in neither spouse working be defined by  $\Omega^0(\alpha_1, \alpha_2, Y)$ . Then for any  $(w_1, w_2) \in \Omega^0(\alpha_1, \alpha_2, Y)$ ,  $CF$  is given by

$$\begin{aligned} & \alpha_1w_2T_2 + \alpha_2w_1T_1 - \alpha_1\alpha_2(Y + w_1T_1 + w_2T_2) \\ = & \alpha_1(1 - \alpha_2)w_1T_1 + \alpha_2(1 - \alpha_1)w_2T_2 - \alpha_1\alpha_2Y. \end{aligned}$$

■

This series of corollaries leads us to the main result of this section.

**Proposition 6** In the cooperative equilibrium the labor supply choices  $(h_1^C, h_2^C)$  are either  $(0, 0)$  or  $(h_1^C > 0, h_2^C > 0)$ .

*Proof:* We have shown that all noncooperative equilibria in which at least one spouse works are dominated by a cooperative equilibrium in which both supply time to the market, and in which both work at more than the noncooperative level. A subset of noncooperative equilibria in which neither spouse works are dominated by cooperative equilibria in which both do. In the complement of this set in  $\Omega^0(\alpha_1, \alpha_2, Y)$  are all those wage pairs in which no cooperative equilibrium dominates the noncooperative one, and in this set of wage offers the equilibrium is the noncooperative one of  $(0, 0)$ . ■

This last result indicates why our model as it currently stands cannot be used to fit the patterns of household labor supply observed in a country like Italy. The majority of households in our sample have only one member supplying time to the market, almost always the husband. This pattern cannot be reproduced by our model, but once we consider the fact that the act of cooperation may only be possible if additional requirements are satisfied our model has the possibility of fitting the observed data more adequately.

### 3.3 Costly Cooperative Equilibria

What can account for the failure to cooperate as we have defined it? There are a number of possible reasons why households may fail to cooperate - as we have defined the term in our application. One natural way to account for “imperfect” cooperation is to posit the existence of fixed costs (see, e.g., Cogan (1981)). When applied to the household setting, it may be reasonable to assume that (quasi-) fixed costs are absorbed when both spouses supply time to the market. The reason for such a supposition is that when at least one spouse stays at home most mundane tasks associated with household maintenance can be performed by him or her. When both are in the market place the household must purchase at least some of these services in the market. If the quantity of these contracted services is more or less independent of the hours choices of the spouses, they can be considered quasi-fixed costs.

Another natural and realistic way to generate the types of choices observed in the data is to assume that in accepting employment individuals agree to work a minimum number of hours per week at their job. While the propositions above suggest that positive hours

will always be chosen to ensure cooperation, for many values of the state variables  $\Gamma$  these choices can involve the supply of a small number of hours by one or both of the spouses. A constraint that all employed individuals must work at least  $\underline{h}$  hours will force those who would choose some  $0 < h < \underline{h}$  so as to cooperate to select either 0 hours, and to resort to the noncooperative equilibrium, or to choose an  $h$  at least equal to  $\underline{h}$  so as to ensure cooperation.

In either of the above two cases for certain values of the state variables it will sometimes be too costly for the household to choose the “constrained” cooperative solution. Both of these two scenarios seems plausible to us, though from a modeling perspective both share the same difficult problem of nonuniqueness. In both cases it is easy to show that for certain values of  $\Gamma$  there can be two noncooperative equilibria and within the model there exists no nonarbitrary way to choose between them. That there exist multiple efficient equilibria is well-appreciated - in our case we have utilized a Nash bargaining framework to arbitrarily select one of the continuum of possible outcomes. The fact that there are multiple noncooperative equilibria complicates the problem further, since in that case the disagreement outcomes used to define the “unique” Nash bargaining outcomes assume multiple values.

Our solution to this conundrum is to follow the lead of Lundberg and Pollak (1993) and to allow for the existence of what they term “transactions costs” associated with cooperative behavior. Their claim is that cooperative solutions are costly in terms of communication, implementation, and monitoring. When such costs are “netted out” of the total surplus from cooperation for each spouse the value of the cooperative state may be less than the noncooperative “default” mode of behavior. While Lundberg and Pollak only posit the existence of such a cost in the development of a Nash bargaining model with a noncooperative equilibrium as a threat point (instead of the more common usage of the value of the divorce state as the threat point), in our case we will want to allow households to actually choose this behavioral mode.

How to include such a cost is admittedly problematic. For simplicity we will assume that the cost of cooperation is reflected in an decreased “enjoyment” of consumption. The interpretation may be that the time involved in coordination decreases the amount of time spent actively enjoying public consumption activities and thus operates effectively as a tax on consumption. Let 1 minus the “tax” rate be given by  $\xi$ , so that in a cooperative equilibrium the value of labor supplies  $h_1$  and  $h_2$  to agent  $i$  is given by

$$\begin{aligned} & \alpha_i \ln(T_i - h_i) + (1 - \alpha_i) \ln(\xi[Y + w_1 h_1 + w_2 h_2]) \\ = & \alpha_i \ln(T_i - h_i) + (1 - \alpha_i) \ln(Y + w_1 h_1 + w_2 h_2) + (1 - \alpha_i) \ln \xi \\ = & U_i(h_1, h_2) + (1 - \alpha_i) \ln \xi, \end{aligned}$$

where  $U_i(h_1, h_2)$  is the “normal,” or untaxed, utility individual  $i$  would receive from the hours choices of  $h_1$  and  $h_2$ , while the term  $(1 - \alpha_i) \ln \xi$ , which is less than or equal to 0 since  $\xi \in [0, 1]$ , measures the cost of cooperation to individual  $i$ . We treat the tax rate as a random variable in the population of married households, with the size of  $\xi$  reflecting the degree of trust, openness, etc. exhibited in the marriage. We denote the distribution of  $\xi$  by  $Q(\xi; \zeta)$ , with  $\zeta$  a finite-dimensional parameter vector, and assume it possesses a density  $q$  everywhere on  $[0, 1]$ . In the empirical analysis conducted below we assume that  $Q$  is a



power function distribution, with

$$Q(\xi; \zeta) = \xi^\zeta, \quad \xi \in [0, 1], \zeta > 0.$$

Given a draw from  $Q$  and the preference parameters  $\alpha_1$  and  $\alpha_2$ , we determine whether a cooperative equilibrium exists as follows. We can define a “modified” Nash bargaining objective function by

$$\tilde{S}(h_1, h_2; \xi, \delta) = \tilde{S}_1(h_1, h_2, \xi)^\delta \tilde{S}_2(h_1, h_2, \xi)^{1-\delta}$$

where

$$\tilde{S}_i(h_1, h_2, \xi) = \alpha_i \ln(T_i - h_i) + (1 - \alpha_i) \ln(Y + w_1 h_1 + w_2 h_2) - V_i^N + \chi[h_i \neq h_i^N](1 - \alpha_i) \ln \xi, \quad i = 1, 2.$$

The presence of the terms  $\chi[h_i \neq h_i^N](1 - \alpha_i) \ln \xi$  indicates that the cooperation costs are only born by agent  $i$  if his or her hours are not set at the noncooperative hours levels.<sup>7</sup>

Using the functions  $\tilde{S}_i$ ,  $i = 1, 2$ , we can now redefine the household’s cooperative choice problem as

$$\begin{aligned} (\tilde{h}_1, \tilde{h}_2)(\xi; \delta) &= \arg \max_{h_1 \geq h_1^N, h_2 \geq h_2^N} \tilde{S}(h_1, h_2; \xi, \delta) \\ &\text{s.t. } \tilde{S}_i(h_1, h_2, \xi) \geq 0, \quad i = 1, 2. \end{aligned} \tag{5}$$

We use the expression “cooperative choice” problem advisedly, since one outcome is in fact to choose the noncooperative solution. The crucial point is that all pairs  $(\tilde{h}_1, \tilde{h}_2)$  are *efficient*, even those corresponding to the noncooperative equilibrium, when we allow for “costly” cooperation.

Our first task is to characterize the decision to behave noncooperatively. Think of the modified Nash bargaining problem as one of dividing the surplus between the agents using the modified “threat points” of  $V_i^N - (1 - \alpha_i) \ln \xi$  for  $i = 1, 2$ . We wish to determine whether there exists a nonnegative surplus for each spouse, and hence for the pair since the Nash objective function is the product of a positive transformation of these surpluses, using these modified disagreement outcomes.

**Proposition 7** *There exists a unique  $\xi^*(\Gamma) \in (0, 1)$ , independent of  $\delta$ , such that the spouses cooperate if and only if  $\xi > \xi^*(\Gamma)$ .*

**Proof.** For any  $\Gamma$  we know that the cooperative Nash equilibrium dominates the non-cooperative equilibrium when  $\xi = 1$  for any  $\delta$ . Similarly, we know that the noncooperative equilibrium dominates the cooperative one for  $\xi = 0$  for any  $\delta$ .

Let the maximized value of the function  $\tilde{S}(h_1, h_2; \xi, \delta)$  be given by

$$\hat{S}(\xi; \delta) = \max_{h_1 \geq h_1^N, h_2 \geq h_2^N} \tilde{S}(h_1, h_2; \xi, \delta).$$

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<sup>7</sup>We know that in any cooperative equilibrium both agents will have to supply more time to the market than under noncooperation without costs of cooperation, and this is reinforced when cooperation costs are positive. Thus if a cooperative equilibrium is chosen both agents will choose hours greater than the noncooperative level and each will pay the cost  $(1 - \alpha_i) \ln \xi > 0$ . Thus the index functions do not have to explicitly include the condition that both spouses’ labor supply choices not be equal to their noncooperative choices.

Then the minimum value of  $\hat{S}(\xi; \delta)$  is equal to 0 when the noncooperative solution is chosen and is otherwise greater than 0. When  $\hat{S}(\xi; \delta) = 0$  this is true for all values of  $\delta$  since  $\hat{S}(\xi; \delta) = 0$  implies the noncooperative solution, which is independent of  $\delta$ . When the function  $\hat{S}(\xi; \delta) > 0$ , then it is greater than 0 for all values of  $\delta \in (0, 1)$  since each spouse must receive a positive surplus under the Nash bargaining solution. Starting from the value  $\xi = 1$ , which elicits a cooperation choice for all values of  $\delta \in (0, 1)$  and which produces the maximum value of  $\hat{S}(\cdot; \delta)$  for any fixed value of  $\delta$ ,  $\hat{S}(\xi; \delta)$  is strictly decreasing as we decrease  $\xi$  for all  $\delta$  as long as  $\hat{S}(\xi; \delta) > 0$ . Then there exists a connected set,  $(\xi^*, 1]$ , where  $\xi \in (\xi^*, 1) \Leftrightarrow \hat{S}(\xi; \cdot) > 0$ . ■

Let us note a few features of the “costly” cooperative solution. As mentioned above, all choices are efficient, even when the noncooperative solution is implemented. Thus the equilibrium outcome is consistent with Chiappori’s claim that only efficient outcomes should be considered as candidates for household equilibria.

In the empirical implementation of any cooperative equilibrium, costly or not, we will face the problem of multiplicity. The tact we take is to use a Nash bargaining solution with a prespecified value of the bargaining power parameter  $\delta$ . This contrasts with the “sharing rule” selection mechanism favored by Chiappori and his coauthors, though each are in the end somewhat ad hoc procedures. When there is no cost to implementing a cooperative solution, the usual case in the literature, then the actual solution will depend on the bargaining power parameter  $\delta$ .<sup>8</sup> In the costly cooperation case, matters are somewhat worse in that the solution will depend both on the bargaining power parameter and the level of the cooperation cost,  $\xi$ .

As demonstrated in the above proposition, the probability of cooperation does not depend on  $\delta$ , however. Given the state variables, the probability of costly cooperation is simply

$$\begin{aligned} P(C|\Gamma, \delta) &= P(C|\Gamma) \\ &= \tilde{Q}(\xi^*(\Gamma); \zeta), \end{aligned}$$

where  $\tilde{Q}$  denotes the survivor function of  $\xi$ . Thus the cooperation decision is robust with respect to  $\delta$ .

Finally, we point to the role of  $\xi$  in the determination of household labor supply. In models designed to investigate the formation and dissolutions of households (recent examples of which include Brien et al (2006) and Brown and Flinn (2005)), it is standard to include a common random variable in current period utility function of both spouses that describes the marriage match value at that point in time. Variations in match quality over time, in conjunction with changes in marriage specific capital values and/or the information sets of the spouses, result in changes in marital status and/or changes in the investment levels in marriage specific capital (as in Brown and Flinn (2005)). In our static model, we abstract from issues regarding divorce and the general tenability of the marriage. Our marriage specific random variable  $\xi$  does not directly contribute to the utility level of each individual, but does so in a more indirect way through the selection of behavioral mode and the payoff in the cooperative equilibrium (when  $\xi > \xi^*(\Gamma)$ ). Just as the value of marriage

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<sup>8</sup>In the sharing rule formulation instead, the equilibrium selected will depend on the econometric specification of the sharing rule and properties of the data. Since the econometric specification of the rule is not deduced from the primitives of the model, a degree of arbitrariness is always present.

to each spouse is monotonically increasing in a model in which the match quality variable enters linearly in the utility function, in this model as well the spouse-specific values of marriage are nondecreasing in  $\xi$ . Thus are random variable  $\xi$  shares much in common with less-subtle models of decision-making within marriage, though the interpretation of the random variable and its effect on behavior are quite different.

### 3.4 An Extended Example

To illustrate the main features of the model we present some graphical representations of how it works within the context of a particular example. We assume that the preference parameters of the spouses are  $\alpha_1 = .3$  and  $\alpha_2 = .4$ . The time endowment of each spouse is normalized to unity (i.e.,  $T_1 = T_2 = 1$ ). The weekly level of nonlabor income of the household is  $Y = 2$ . We begin by illustrating how the set of “efficient” solutions changes with increases in the cost of cooperation. We will then look at the characteristics of the set of wage offers that lead to costly cooperative behavior. Finally, we look at the labor supply behavior of the household as a function of the wage offer pair  $(w_1, w_2)$ , as well as the distribution of welfare within the household.

In Figure 1 we plot the “costly” Pareto frontiers for our model given a value of  $w_1 = 4$  and  $w_2 = 3$  at four values of the tax rate. When there is no tax, so that  $\xi = 1$ , then we know that there exists a Pareto efficient solution that dominates the noncooperative equilibrium payoffs for the spouses. The Pareto frontier for this case is the curve furthest from the noncooperative payoff point. When we increase the tax rate to 0.02, the Pareto frontier shifts inward but still lies substantially above the noncooperative payoff point. We see that even at a tax rate of 6 percent, the noncooperative solution is dominated by the costly cooperative one, though the “size” of the Pareto frontier is relatively small. As the tax rate increases, or as  $\xi$  decreases, the size of the Pareto frontier shrinks until it consists of a single point. The tax rate at which the Pareto frontier is a single point is given by  $\xi^*$ . For all values of  $\xi \leq \xi^*$ , the “efficient” outcome is the noncooperative equilibrium. Since the noncooperative equilibrium is unique, this poses no problem for implementation. However, when  $\xi > \xi^*$ , we face the issue of choosing a point to call “the” equilibrium. As is relatively standard, we use Nash bargaining with the modified outside options, those that include  $(1 - \alpha_i) \ln \xi$ , to determine labor supplies in this case.

Figure 2 contains a plot of the cooperation set of wage pairs, after we added  $\xi = 0.92$  to the other fixed parameter values. The wage rates we consider are elements of the set  $[2, 8]^2$ , and a value of ‘1’ on the  $z$  axis indicates that the wage pair produces a cooperative outcome. As a rule, cooperation occurs when wages are less disperse (if the preference weights were equal, i.e.,  $\alpha_1 = \alpha_2$ , the set would be symmetric around the 45 degree line. This suggests that positive assortative mating on both preferences and market productivity characteristics may be more likely to lead to cooperative outcomes.<sup>9</sup>

We have plotted the equilibrium labor supply functions and payoff functions for the two spouses as a function of the wage pairs in Figure 3. The top two figures exhibit the labor supply surfaces for the two spouses, while the figures underneath them map the spouse-

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<sup>9</sup>Our highly stylized model has made no provision for household production and the gains to specialization that such a generalization typically implies. Thus it is not too surprising that positive assortative mating would be beneficial in the framework considered here.

specific payoffs. The most important point to note is that the labor supply functions are discontinuous at points where the household shifts from one behavioral mode to another, while the payoff functions are everywhere continuous. Note that the payoff functions are monotone in the wage pairs, as we expect to be true under either the noncooperative or costly cooperative equilibrium. We do see “creases” in the surfaces, and these occur at wage pairs where the couple is switching between behavioral modes. At these points, the payoff functions are continuous, yet not differentiable in either  $w_1$  or  $w_2$ .

The labor supply surfaces present a more interesting story. Shifts between behavioral modes produce not only noncontinuous labor supply functions, but nonmonotonic ones, even under our Cobb-Douglas assumptions. Consider Figure 3.b, which plots the labor supply surface for spouse 2 as a function of the wage pairs. First, consider the labor supply response of spouse 2, in equilibrium, as her wage increases given the wage of her husband. If her husband has a high wage, say  $w_1 = 7$ , then we see that at  $w_2 = 2$  she does not work and the equilibrium is noncooperative. As her wage offer increases to around 2.25, she enters the market and “jumps” to a labor supply level of about 0.4. With further increases in her wage offer, up to  $w_2 = 8$ , her labor supply smoothly increases while the labor supply of her husband decreases. Thus given that her husband’s wage is 7, her labor supply is increasing, but not continuously so, in her own wage.

The situation is different when we fix  $w_1 = 3$ , for example. In this case, when her wage is 2, the pair cooperate. As her wage increases from 2, the couple initially engage in cooperative behavior. Up until her wage reaches around 4, her labor supply is smoothly increasing (and his smoothly decreasing). Past a critical wage rate, however, the wage disparities are too great to maintain the costly cooperative equilibrium, and the couple switches to the noncooperative equilibrium. At this point, the labor supply of both dramatically falls. Increases in her wage rate, keeping the husband’s fixed at 3, result in smooth increases in her equilibrium labor supply up to the maximum wage we consider. Thus within any regime, cooperative or noncooperative, her labor supply is smoothly increasing in her own wage. However, when switching from the cooperative to noncooperative equilibrium, there is a precipitous drop in the labor supply of both. In this case, her labor supply function is not only discontinuous, it is also nonmonotonic in  $w_2$ .

## 4 Econometric Specification

Whether estimating a noncooperative or (costly) cooperative version of the equilibrium household labor supply model, the underlying parameterization of the household’s problem is invariant. We let household preferences be determined by a draw from a joint distribution over  $\alpha$ , the c.d.f. of which we denote by  $G(\cdot; \nu)$ , where  $\nu$  is a finite dimensional parameter vector. Since  $\alpha$  are parameters of Cobb-Douglas utility functions, we restrict the support of  $G$  to be  $[0, 1]^2$ . For simplicity, in estimating the model we have assumed that preference draws are independent for husbands and wives, and that the distribution of utility for each spouse is a power distribution with

$$\begin{aligned} G_i(\alpha) &= G(\alpha; \nu_i) \\ &= \alpha^{\nu_i}, \alpha \in [0, 1], \nu_i > 0. \end{aligned}$$

Thus the probability that  $\alpha_1 \leq A_1$  and  $\alpha_2 \leq A_2$  is given by  $A_1^{\nu_1} A_2^{\nu_2}$ .<sup>10</sup>

We have assumed that the wage offer distribution is bivariate log normal, with

$$\begin{pmatrix} \ln w_1 \\ \ln w_2 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \Sigma\right).$$

In the actual estimation, we allow for individual-specific heterogeneity in the parameters  $\mu_i$ . In particular, we assume that  $\mu_i = \beta_i X_i$ , where  $X_i$  is a  $K \times 1$  vector of observable characteristics of spouse  $i$  and  $\beta_i$  is a conformable parameter vector. We allow  $\sigma_{12} \neq 0$ , which means that  $\ln w$  offers are not independently distributed (and of course, neither are the  $w = \exp(\ln(w))$ ). In this way we allow for the possibility of assortative mating, although in an admittedly crude manner.

The model is very parsimoniously specified, with the joint distribution of wages and labor supplies determined by the parameters  $\Omega = \{\nu_1, \nu_2, \beta_1, \beta_2, \Sigma\}$  in the case of noncooperation and  $\Omega' = \{\Omega, \zeta\}$  in the costly cooperation case.<sup>11</sup> We first consider the estimation of these parameters under the assumption that the household equilibrium is noncooperative.

#### 4.1 Estimation of the Noncooperative Model

We form maximum likelihood estimators for  $\Omega$ . The likelihood function consists of four components, one for each “type” of equilibrium represented in Table 1. We proceed through them sequentially.

The likelihood of neither spouse working (Type I) is computed as follows. Conditional on the preference parameter vector  $\alpha$ , the probability that neither works is the joint probability that the wage offer to spouse 1 is less than  $w_1^{**}(\alpha_1, T_1, Y)$  and the wage offer to spouse 2 is less than  $w_2^{**}(\alpha_2, T_2, Y)$ , or

$$F_{W_1, W_2}(w_1^{**}(\alpha_1, T_1, Y), w_2^{**}(\alpha_2, T_2, Y))$$

The marginal probability of this event, which is its likelihood, is obtained by multiplying the above expression by the density of  $\alpha$  and integrating over the interval  $[0, 1]^2$ , or

$$P_I(\nu, \mu, \Sigma) = \int_0^1 \int_0^1 F_{W_1, W_2}(w_1^{**}(\alpha_1, T_1, Y), w_2^{**}(\alpha_2, T_2, Y); \mu, \Sigma) dG(\alpha; \nu).$$

The second and third types of equilibrium in which only one spouse works are symmetric, so we will only discuss the case in which spouse 1 works and spouse 2 does not (Type II). In this case the wage offer to the first spouse must satisfy  $w_1 > w_1^{**}$ , while the wage offer to spouse 2 must be less than  $w_2^*(w_1)$ . Now when the second agent does not work, it is straightforward to compute the first agent’s preference parameter value since

$$\begin{aligned} h_1 &= T_1 - \frac{\alpha_1}{w_1} [Y + w_1 T_1] \\ \Rightarrow \alpha_1 &= \frac{w_1(T_1 - h_1)}{Y + w_1 T_1}, \end{aligned} \tag{6}$$

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<sup>10</sup>We are restricting the preference parameters to be independent mainly for computational simplicity at this point, though our intention is to allow some dependence in these parameters in our future research. For the moment, all assortative mating is captured through dependence in wage offers.

<sup>11</sup>The noncooperative case is actually nested (in a limiting sense) within the noncooperative case, as  $\zeta \rightarrow 0$ .

and all of the terms on the right hand side of [6] are observable. Thus the likelihood of agent 1's choice of hours and their wage rate is given by

$$\left[ \frac{w_1}{Y + w_1 T_1} \right] \left[ \nu_1 \left( \frac{w_1(T_1 - h_1)}{Y + w_1 T_1} \right)^{\nu_1 - 1} \right] f_{W_1}(w_1; \mu_1, \sigma_{11}),$$

where the first term in brackets is the absolute value of the Jacobian, which together with the second term in brackets comprises the density of hours of spouse 1 conditional on the wage rate  $w_1$ . The last term is the marginal density of individual 1's wage, so that the entire product gives the joint density of hours and the wage of spouse 1.

Turning to the information in individual 2's decision not to participate, we note that

$$w_2^*(w_1, \alpha_1, \alpha_2) = \frac{\alpha_2(1 - \alpha_1)}{(1 - \alpha_2)T_2} [Y + w_1 T_1]$$

gives the critical spouse 2 wage offer such that all  $w_2$  below this value lead to a corner solution for individual 2. Conditional on  $\alpha_2$  (in addition to  $w_1$  and  $\alpha_1$ ), the probability that agent 2 does not have an acceptable offer is given by

$$p(w_2 < w_2^*(w_1) | \alpha_1, \alpha_2, w_1) = F_{W_2|W_1}(w_2^*(w_1, \alpha_1, \alpha_2) | w_1).$$

Unconditional on  $\alpha_2$  (but conditional on  $w_1$  and  $\alpha_1$ ) then the probability that individual 2 does not work is

$$p(w_2 < w_2^*(w_1) | \alpha_1, w_1) = \int F_{W_2|W_1}(w_2^*(w_1, \alpha_1, \alpha_2) | w_1) dG_2(\alpha_2).$$

Finally, multiplying by the likelihood of  $\alpha_1$  and  $w_1$  we have the likelihood of the Type II outcome

$$\begin{aligned} P_{II}(h_1, w_1; \nu, \mu, \Sigma) &= \int F_{W_2|W_1}(w_2^*(w_1, \alpha_1, \alpha_2) | w_1; \mu, \Sigma) \nu_2 \alpha_2^{\nu_2 - 1} d\alpha_2 \\ &\times \left[ \frac{w_1}{Y + w_1 T_1} \right] \left[ \nu_1 \left( \frac{w_1(T_1 - h_1)}{Y + w_1 T_1} \right)^{\nu_1 - 1} \right] f_{W_1}(w_1; \mu_1, \sigma_{11}). \end{aligned}$$

The likelihood of a Type III equilibrium, in which spouse 1 does not work and spouse 2, is computed in an exactly symmetric way, so that

$$\begin{aligned} P_{III}(h_2, w_2; \nu, \mu, \Sigma) &= \int F_{W_1|W_2}(w_1^*(w_2, \alpha_1, \alpha_2) | w_2; \mu, \Sigma) \nu_1 \alpha_1^{\nu_1 - 1} d\alpha_1 \\ &\times \left[ \frac{w_2}{Y + w_2 T_2} \right] \left[ \nu_2 \left( \frac{w_2(T_2 - h_2)}{Y + w_2 T_2} \right)^{\nu_2 - 1} \right] f_{W_2}(w_2; \mu_2, \sigma_{22}). \end{aligned}$$

The likelihood of a Type IV equilibrium with observed values of  $h_1, h_2, w_1$ , and  $w_2$  is computed as follows. Define  $A_i = (Y + w_1 T_1 + w_2 T_2) / w_i$ ,  $i = 1, 2$ , and let

$$\begin{aligned} \tau &= \frac{l_1 l_2}{(A_1 - l_1)(A_2 - l_2)}, \\ \tau_1 &= \frac{\tau}{l_1} + \frac{\tau}{A_1 - l_1}, \\ \tau_2 &= \frac{\tau}{l_2} + \frac{\tau}{A_2 - l_2}. \end{aligned}$$

Then we can write the implied values of the preference parameters as

$$\begin{aligned}\alpha_1 &= \tau + (1 - \tau) \frac{l_1}{A_1}, \\ \alpha_2 &= \tau + (1 - \tau) \frac{l_2}{A_2}.\end{aligned}$$

The Jacobian is given by

$$J_N = - \left\| \begin{array}{cc} \tau_1(1 - \frac{T_1 - h_1}{A_1}) + \frac{1 - \tau}{A_1} & \tau_2(1 - \frac{T_1 - h_1}{A_1}) \\ \tau_1(1 - \frac{T_2 - h_2}{A_2}) & \tau_2(1 - \frac{T_2 - h_2}{A_2}) + \frac{1 - \tau}{A_2} \end{array} \right\|.$$

Thus, conditional on the wage draws  $(w_1, w_2)$ , the likelihood the hours choices  $(h_1, h_2)$  is

$$p(h_1, h_2 | w_1, w_2) = J_N \nu_1 (\tau + (1 - \tau) \frac{l_1}{A_1})^{\nu_1 - 1} \nu_2 (\tau + (1 - \tau) \frac{l_2}{A_2})^{\nu_2 - 1}.$$

After multiplying by the likelihood of the two wage draws, we get the total likelihood of the Type IV equilibrium draw as

$$\begin{aligned}P_{IV}(h_1, h_2, w_1, w_2; \nu, \mu, \Sigma) &= J_N \nu_1 (\tau + (1 - \tau) \frac{T_1 - h_1}{A_1})^{\nu_1 - 1} \nu_2 (\tau + (1 - \tau) \frac{T_2 - h_2}{A_2})^{\nu_2 - 1} \\ &\quad \times f_{W_1, W_2}(w_1, w_2; \mu, \Sigma).\end{aligned}$$

## 4.2 Estimation of the Costly Cooperation Model

The cooperative model is estimated only under the condition that there exist positive coordination costs for the reasons stated above. In terms of determining whether or not a data observation for a sample household corresponds to a noncooperative or a cooperative equilibrium, this is relatively straightforward in many cases given our modeling assumptions. In the sample utilized in the empirical analysis we have excluded all households in which neither spouse is employed; there were less than five households that were excluded on the basis of this criterion. Thus the relevant parts of the likelihood function pertain to households in which both spouses work or in which only one works.

We begin by considering the case in which spouse 1 works and spouse 2 does not, which was referred to as a Type II case above. In this situation we observe  $w_1$  and  $h_1$  as well as  $Y$ . From the theoretical analysis presented in the previous section, we know that in this case the household equilibrium must be of the noncooperative type. Thus, the likelihood contribution for this case shares much with  $P_{II}$  above, except that the likelihood of noncooperation must be explicitly included. Now the likelihood of noncooperation is given by

$$Q(\xi^*(\Gamma); \zeta),$$

where the “essential” elements of  $\Gamma$  include  $(w_1, w_2, \alpha_1, \alpha_2)$ ; we will omit the elements  $Y$  and  $T_1$  and  $T_2$  which are assumed to be observable for all sample members. As before, the data implies a unique value of  $\alpha_1$  in this case. Then the likelihood contribution for this case is

given by

$$R_{II}(h_1, w_1; \nu, \mu, \Sigma, \zeta) = \int \int^{w_2^*(w_1, \alpha_1, \alpha_2)} Q(\xi^*(w_1, w_2, \alpha_1, \alpha_2); \zeta) f_{W_2|W_1}(w_2|w_1; \mu, \Sigma) \nu_2 \alpha_2^{\nu_2-1} d\alpha_2 \\ \times \left[ \frac{w_1}{Y + w_1 T_1} \right] \left[ \nu_1 \left( \frac{w_1(T_1 - h_1)}{Y + w_1 T_1} \right)^{\nu_1-1} \right] f_{W_1}(w_1; \mu_1, \sigma_{11}).$$

In contrast to the expression for  $P_{II}$ , it is necessary to integrate over the unobserved wage  $w_2$  even after conditioning on a value of  $\alpha_2$ , since the critical value  $\xi^*$  required to compute the probability of noncooperation depends on both of these state variables in addition to  $w_1$  and  $\alpha_1$ .

The likelihood contribution for the case in which individual 2 works and individual 1 does not is exactly symmetric. We then have a likelihood contribution of

$$R_{III}(h_2, w_2; \nu, \mu, \Sigma, \zeta) = \int \int^{w_1^*(w_2, \alpha_1, \alpha_2)} Q(\xi^*(w_1, w_2, \alpha_1, \alpha_2); \zeta) f_{W_1|W_2}(w_1|w_2; \mu, \Sigma) \nu_1 \alpha_1^{\nu_1-1} d\alpha_1 \\ \times \left[ \frac{w_2}{Y + w_2 T_2} \right] \left[ \nu_2 \left( \frac{w_2(T_2 - h_2)}{Y + w_2 T_2} \right)^{\nu_2-1} \right] f_{W_2}(w_2; \mu_2, \sigma_{22}).$$

Lastly we consider the likelihood contribution of households for whom both spouses work. In this situation the equilibrium can be either noncooperative or (costly) cooperative. Our strategy will be to compute  $P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, N)$  and  $P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, C)$ , and then to compute the marginal likelihood contribution by summing over the behavioral modes, or

$$P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0) = P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, N) \\ + P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, C).$$

The computation of the term  $P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, N)$  is the most straightforward. Given that all of the dependent variables  $h_1, h_2, w_1, w_2$  are observed, under noncooperation the calculation of the likelihood of the event is identical to what appears in  $P_{IV}(h_1, h_2, w_1, w_2; \nu, \mu, \Sigma)$ , up to multiplication by the likelihood that the outcome is noncooperative. Given the observed  $w_1$  and  $w_2$  and the implied values of  $\alpha_1(w_1, w_2, h_1, h_2)$  and  $\alpha_2(w_1, w_2, h_1, h_2)$ , we know the critical value  $\xi^*(w_1, w_2, \alpha_1, \alpha_2)$ , so that the probability of noncooperation is simply  $Q(\xi^*(w_1, w_2, \alpha_1, \alpha_2); \zeta)$ . Thus we have

$$P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, N) = P_{IV}(h_1, h_2, w_1, w_2; \nu, \mu, \Sigma) \\ \times Q(\xi^*(w_1, w_2, \alpha_1(w_1, w_2, h_1, h_2), \alpha_2(w_1, w_2, h_1, h_2)); \zeta) \quad (7)$$

Calculation of the joint likelihood of  $h_1$  and  $h_2$  under cooperation is a bit more involved, simply because the likelihood of any given pair of  $(h_1, h_2)$ , given  $(w_1, w_2)$ , depends on the cost of cooperation,  $\xi$ . There is a 1 – 1 mapping between  $(h_1, h_2)$  and  $(\alpha_1, \alpha_2)$  in the cooperative equilibrium conditional on  $Z$  and  $\xi$ , where  $Z \equiv (w_1, w_2, Y)$ . Let us denote this dependence by  $(\alpha_1^C(\xi, h_1, h_2, Z), \alpha_2^C(\xi, h_1, h_2, Z))$ . We can compute the value of the noncooperative equilibrium for each individual,  $V_i^N(\alpha_1^C(\xi, h_1, h_2, Z), \alpha_2^C(\xi, h_1, h_2, Z), Z)$  given



$h_1, h_2$ , and  $Z$  for any given value of  $\xi$ ,  $i = 1, 2$ .<sup>12</sup> Then define  $\xi^C$  as that value of  $\xi$  such that

$$\xi > \xi^C \Leftrightarrow \hat{S}(\xi; \cdot, \alpha_1^C(\xi, h_1, h_2, Z), \alpha_2^C(\xi, h_1, h_2, Z), Z) > 0.$$

Due to the discontinuity in the labor supply functions at all points at which behavior switches between cooperation and noncooperation, it is not the case that  $\xi^*$  is the same as  $\xi^C$ . Under our distributional assumptions regarding the preference parameters, we can relatively easily write down the joint density for hours given wages and cooperation, where cooperation is assured by considering only values  $\xi > \xi^C$ . This joint conditional density of hours, at a given level of  $\xi > \xi^C$ , is given by

$$m(h_1, h_2 | w_1, w_2, \xi, \xi > \xi^C) = J_C(h_1, h_2, \xi, Z) \times \nu_1(\alpha_1^C(\xi, h_1, h_2, Z))^{\nu_1-1} \nu_2(\alpha_2^C(\xi, h_1, h_2, Z))^{\nu_2-1}$$

where the Jacobian is

$$J_C(h_1, h_2, \xi, Z) = \left\| \begin{array}{cc} \frac{\partial \alpha_1^C}{\partial h_1} & \frac{\partial \alpha_1^C}{\partial h_2} \\ \frac{\partial \alpha_2^C}{\partial h_1} & \frac{\partial \alpha_2^C}{\partial h_2} \end{array} \right\|.$$

Now we can complete the construction of the term  $P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, C)$ . It is given by

$$\int_{\xi^C}^1 m(h_1, h_2 | w_1, w_2, \xi, \xi > \xi^C) f_{W_1, W_2}(w_1, w_2; \mu, \Sigma) dQ(\xi; \zeta). \quad (8)$$

The sum of [7] and [8] yields  $R_{IV}(h_1, h_2, w_1, w_2; \nu, \mu, \Sigma, \zeta)$ , and completes the specification of the likelihood function associated with the model of costly cooperation.

The computation of the log likelihood associated with this model is significantly more involved and (computer) time intensive than the one associated with the noncooperative equilibrium model. The most intensive calculation is that of  $P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, C)$ , since we must compute the functions  $\alpha_i^C(\xi, h_1, h_2, Z)$ ,  $i = 1, 2$ , and the Jacobian  $J_C$  at a number of values of  $\xi$  in the interval  $(\xi^C, 1)$ . To speed up computation, we evaluate the conditional density  $m(h_1, h_2 | w_1, w_2, \xi, \xi > \xi^C)$  only at the midpoint of the interval  $(\xi^C, 1)$ . Define

$$\bar{\xi}(\xi^C) = \frac{1 - \xi^C}{2}.$$

Then we form

$$\hat{P}(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, C) = m(h_1, h_2 | w_1, w_2, \bar{\xi}(\xi^C), \xi > \xi^C) f_{W_1, W_2}(w_1, w_2; \mu, \Sigma) \tilde{Q}(\xi^C; \zeta).$$

The log likelihood is maximized using  $\hat{P}(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, C)$  in place of  $P(h_1, h_2, w_1, w_2, h_1 > 0, h_2 > 0, C)$  when forming  $R_{IV}(h_1, h_2, w_1, w_2; \nu, \mu, \Sigma, \zeta)$ . All other components of the log likelihood function are as before.

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<sup>12</sup>The value of the noncooperative equilibrium is not itself a function of  $\xi$ , which is only paid given cooperation, but the inferred values of the preference parameters given  $Z$  for a particular hours choice  $(h_1, h_2)$  depend on  $\xi$  given behavior is cooperative. This explains the presence of the term  $\xi$  in  $V^N$ .

### 4.3 Relationship between the Two Econometric Specifications

While the concept of a coordination cost is arguably somewhat vague, framing the choice between cooperation and noncooperation in this manner has distinct econometric advantages. As we remarked earlier, such a setup avoids problems of nonuniqueness of the noncooperative equilibrium associated with nonconvexities introduced into the model when allowing for the presence of fixed costs of work or minimum hours constraints. In additional advantage is that the noncooperative model is a special case of the cooperative-choice model. This is obvious through the following formal result.

**Proposition 8**  $\ln L^N(\nu, \mu, \Sigma) = \lim_{\zeta \rightarrow 0} L^C(\nu, \mu, \Sigma, \zeta)$  for all  $(\nu, \mu, \Sigma)$ .

No formal proof is required. For any values of  $\alpha$ ,  $w$ , and  $Y$ , there exists a critical tax rate  $\xi^* \in (0, 1)$ . Since the probability of noncooperation is given by  $Q(\xi^*; \zeta)$  and we have

$$\lim_{\zeta \rightarrow 0} Q(x; \zeta) = 1 \text{ for all } x \in (0, 1),$$

then the probability of cooperation is equal to zero for all values of the state variables  $(w, h, Y)$  and parameters  $(\nu, \mu, \Sigma)$  so that the cooperative log likelihood is identical to the noncooperative one term by term.

The above result suggests that we must be on guard for problem which can occur in the process of model estimation. If the noncooperative model fits the data more adequately than the costly cooperative model, the likelihood function associated with the later model is monotone in the parameter  $\zeta$ . This will result in increasingly values of  $\hat{\zeta}$  during the iterative process and may cause instability of the optimization algorithm.

## 5 The Data

The empirical analysis utilizes data from the Bank of Italy's Survey of Household Income and Wealth (SHIW 1998). The survey is conducted every two years, and gathers information on the incomes and wealth of family members, labor market hours and earning information, and sociodemographic characteristics of the households (e.g., age of all of the members of the family, schooling attainment). For purposes of our analysis we have selected a sample of households comprised of a married couple with or without children. The husbands are between 25 and 50 years of age (inclusive), while the wives are between 25 and 45 years of age. We have also excluded a few extremely high wage, hours, and nonlabor income observations from our sample. These last restrictions only resulted in the exclusion of approximately 20 households and left us with a sample size of 1575.

Means and standard deviations of the variables of interest are presented in Table 2. The first column contains information for the entire sample. In this table, all monetary amounts are expressed in thousands of 1998 lire (ITL). The weekly nonlabor income of sample households is only about 6100 ITL, but this figure is a bit misleading given that less than 15 percent of sample households report any nonlabor income. Of those that do, the mean weekly amount is 41000 ITL.

The average ages of the spouses differs by about 5 years, which reflects the fact that women tend to marry older men and given our sample selection criterion, which included

men up to age 50 and women only up to age 45. The schooling distributions for husbands and wives are similar, though, as is true in the U.S., women tend to have a slightly higher level of attainment than their husbands. The variable “Mid-level School” is coded one when the individual has attained any type of high school certificate, while “High-level School” assumes the value one when the individual has a laurea (college) degree or other type of post-high school qualification. University completion figures are very low in Italy, particularly in comparison with the U.S., and this is one reason that the higher education curriculum has recently been reformed to allow for three year, in addition to the standard five year, degree program. We see that only 11 percent of husbands and wives have completed a five year university program.

In column 2 we present descriptive statistics for the subsample of households in which the husband works and the wife does not. Husbands in these households work about 40 hours per week, though there is a nontrivial standard deviation. We note that spouses in these households tend to have the lowest schooling attainment levels exhibited by any of the three subsamples.

Column 3 contains the statistics for the small subsample of households in which only the wife works. The wife’s average hours of work are relatively low in this case. The average wage of women in these households is less than the average wage of men in households in which only they work. Women in these households tend to have the highest level of educational attainment among the three groups, though their husbands tend to be well-educated as well. Since the men are generally of pre-retirement age, perhaps their being not employed in the reference month is a temporary phenomenon (not considered in our competitive markets model).

The last column contains the means and standard deviations for the households in which both spouses work. This is a smaller group than the households in which only the husband works, which distinguishes the modern day Italian labor market from the U.S. and Northern European ones. The means and standard deviations of hours worked for the husbands and wives are similar to what was observed for the households in which only one worked. Average nonlabor income is greater in these households, but is still at insignificant levels for the majority of households in this group. These households have much higher levels of educational attainment than do households in which only the husband works.

## 5.1 Estimates of the Noncooperative Model

Computation of the maximum likelihood estimates for the noncooperative model takes less than one second of CPU time. Unfortunately, the current implementation of the costly cooperative equilibrium estimator consumes many orders of magnitude more time. As a result, in this version of the paper we estimate the model specifications only on 400 observations, which are a randomly selected subsample from the larger sample. Since we are only attempting to estimate a small number of parameters, there is no reason to believe that the estimates on the subsample will differ in any major way from what we will obtain when we use the entire sample.

Table 3 contains estimates of two specifications of the noncooperative equilibrium model. As noted previously, under our specification of the noncooperative model each household has a positive probability of being in any one of the three regimes observed in the data (i.e.,

$\{h_1 > 0, h_2 = 0\}, \{h_1 = 0, h_2 > 0\}, \{h_1 > 0, h_2 > 0\}$ .

The first column contains the most basic specification. In this case, we assume that all households draw wage pairs from the same lognormal distribution, and that the Cobb-Douglas parameters  $\alpha_1$  and  $\alpha_2$  are independently distributed, each according to a power distribution with parameter  $\nu_i$ . The parameters  $\beta_{0,i}$  are the population means of the logs of the wages of spouse  $i$ . We see that there is a slight (but statistically significant) difference in these parameters, with the husband having the advantage. The estimates of the covariance matrix of the log wage draws show that the variance of the wives' draws are significantly larger than the husbands'. This is commonly found when one side has many more censored observations than the other. There is a high degree of positive association in the wage draws of the spouses, with the estimate of  $\rho$  being 0.470. The last two parameters estimated are those that describe the preference parameter distribution. Since husbands and wives have similar population wage offer distributions, and since wives work much less than their husbands, it is not surprising that the wives' average valuation of leisure is estimated to be much greater than their husbands'. The average for the husbands is 0.454, while for wives it is 0.540.

In the second specification we condition the expected value of the log of the wage offer on schooling attainment of the relevant spouse. The parameters  $\beta_{1,i}$  are associated with a medium schooling level dummy and  $\beta_{2,i}$  with a high schooling level dummy. The estimates of the  $\beta_{1,i}$  and  $\beta_{2,i}$  are as expected, with there being a particularly large payoff to having completed university education (which is a relatively rare event in the sample). After conditioning on schooling attainment, the variances of the log wage draws decrease, as expected. Moreover, the correlation between the log wage draws is greatly reduced after conditioning on the schooling levels of the two spouses, though it is still statistically significant. The estimates of the preference distribution parameters are unchanged, reflecting the near orthogonality of the estimators for preferences and the choice set in this specification of the model.

We also estimated a specification, the results from which are not reported, in which we allowed the distributions of the preference parameters to depend on the total number of children in the family and the number of children less than seven years of age. Though our theoretical model makes no provision for household production or investment in children, we reasoned that such activities may appear as a high valuation for leisure, particularly among mothers. Thus in that specification we defined

$$\nu_i = \exp(\gamma_{i,0} + \gamma_{i,1}\text{kle6} + \gamma_{i,2}\text{nkids}), \quad i = 1, 2,$$

where  $\text{kle6}$  is the number of children 6 years of age or less in the household and  $\text{nkids}$  is the total number of children of the parents. Neither  $\gamma_{i,1}$  nor  $\gamma_{i,2}$  were significant for either parent. While it may well be the case that household production and child investment are important phenomena to consider when estimating household labor supply, this method of introducing family composition variables does not satisfactorily capture the role they play in household decision-making.

## 5.2 Estimates of the Costly Cooperation Model

Table 4 contains estimates of the two specifications of the costly cooperation model. The parameters estimated are the same as in Table 3 with the exception of  $\zeta$ , which indexes the

distribution of tax rates. Column 1 contains estimates from the specification that doesn't include schooling covariates. In comparison with estimates of the noncooperative equilibrium model, there are small but noticeable changes in the estimated wage offer distributions. The estimates of the means of the distributions are lower when cooperation is allowed, though not by large amounts. In terms of second moment properties of the wage offer distributions, the estimated variance (marginal) in the offers to husbands is greater under costly cooperation, but the variance in the wife's offer distribution is less. The estimated correlation in the offers is much greater under costly cooperation than under the noncooperative equilibrium assumption.

The biggest differences in the estimates obtained under the two models are in the parameters characterizing the preference distributions. The "costly cooperation" estimates indicate a much larger weight attached to leisure, on average, by both spouses. This result makes intuitive sense. When both spouses work, the only case in which the household can be cooperative, hours and wages of both are observed. Conditional on wage rates, a fixed level of hours implies a higher weight attached to leisure on the part of both spouses than is the case if the spouses are not cooperating. We see that the magnitude of this impact is large. The average weight attached to leisure by wives in the noncooperative equilibrium is about 0.540, while in the costly cooperative equilibrium it is 0.676. The increase is also substantial for the husband's average welfare weight, which increases from 0.454 to 0.523.

The second column of Table 4 contains the estimates of the specification that includes schooling attainment. As was true in the noncooperative equilibrium case, this specification fits the data much better than the one in which wage draws are i.i.d. Unlike the noncooperative equilibrium case, explicitly conditioning on schooling does not reduce the level of correlation in wage draws.

The estimate of the parameter indexing the cooperation tax distribution is about 42 under both specifications of the model. Since this tax rate is assumed to follow a power distribution, the mean amount of the tax on consumption under cooperation is about 2.3 percent, with the median tax rate being 1.2 percent. Since a substantial proportion of the sample is in the noncooperative regime, this indicates that typical gains from cooperation are not exceedingly large.

The likelihood values associated with the various specifications lead us to conclude that the cooperative model is a much better description of the household equilibrium than the strictly noncooperative model. It is not possible to carry out a standard likelihood ratio test since the null is associated with a value of  $\zeta$  on the boundary of the parameter space.

## 6 The Impact of Changes in Parameters on Household Labor Supply and Welfare

This section presents some illustrative calculations of the impacts of changes in the labor market and household environment on labor supply outcomes and the distribution of welfare within households. We present a few calculations to illustrate our areas of interest. In performing the simulations, we only use point estimates from the "homogeneous" specification of the costly cooperative model, which appear in the first column of Table 4.

In performing the simulations we utilized the following technique. In terms of the

homogeneous specification, all individuals are ex ante homogeneous except with respect to nonlabor income  $Y$ . To replicate the properties of the sample, under the model, we have generated 2500 simulation draws for each sample member (i.e.,  $Y_i$ ). The same “raw” sample draws were used in generating the baseline simulation and the experiments. Since we have 400 sample members, we have one million draws in total.

The two experiments we report on involve changing the preference distribution parameter of wives to match those of husbands (admittedly not an option directly available to a policy maker) and then changing the wage offer distribution of wives to match those of their husbands (more in the policy realm). We report average values of the mean labor supply and welfare levels of the spouses, the proportion of households cooperating, and the mean absolute difference in the welfare levels of the spouses.

With respect to the baseline, shifting the preference parameter distribution to match those of their husbands (though keeping the draws independent) results in the largest change in the outcomes we measure.<sup>13</sup> While the average labor supply of husbands decreases by a small amount, the labor supply of wives almost doubles. We cannot legitimately compare the payoffs of the wives before and after the change since we have changed their valuation function. However, the welfare of husbands has significantly improved by being married to someone with a higher valuation of the public good and who therefore supplies no less time to the market in any state of the world. We see a very large change in the proportion of cooperative outcomes, from 59 percent to 84 percent.

Partially because the mean differences in wage offers between spouses were not estimated to be very large, the impact of increasing the mean log wage offer of wives to their husband’s level has a very small effect on household outcomes. The increase in the mean log wage offer to wives is about 8 percent. The changes in all of the outcome measures are substantially less than this in percentage terms.

## 7 Conclusion

In this paper we have developed a model of “efficient” noncooperation, in which spouses choose their labor supplies and the mode in which they interact. By introducing a cost to cooperation, some households in which monitoring or communication costs are large will rationally decide to behave in a noncooperative manner, which we presume to require no monitoring (since it is a unique Nash equilibrium) and less communication. Under our specification of preferences and household technology, all households in which one spouse works must be in the noncooperative mode, while households in which both spouses work may be behaving cooperatively or noncooperatively. These theoretical results inform the construction of the costly cooperative equilibrium likelihood function.

A sample of households from a nationally representative Italian data set is used to estimate the model. We first estimate an equilibrium model in which households behave noncooperatively, and then add the possibility of cooperation. Indications from the log likelihood value suggest that the costly cooperative model is a much better description of the data.

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<sup>13</sup>To what extent this is true even in the noncooperative equilibrium we cannot say since we have not as yet performed the simulation exercises using that model.

The most intriguing aspect of the model from the policy point of view is the possibility that small changes in parameters that may be influenced by policy actions, such as those generating wage offer distributions, may lead to small welfare impacts on households but large changes in labor supply. This can occur due to the fact that household welfare is continuous in all of the relevant state variables, such as nonlabor income and wage offers, but labor supply is not. When households switch from a noncooperative to a cooperative mode of behavior, labor supplies of both spouses increase by a discrete amount. Thus policies which promote cooperative behavior can potentially have large impacts on aggregate labor supply.

**Table 2**  
**Descriptive Statistics**  
**Means and (Standard Deviations)**

<i>Variable</i>	<i>All</i>	$h_1 > 0, h_2 = 0$	$h_1 = 0, h_2 > 0$	$h_1 > 0, h_2 > 0$
Husband's Hours of Work		40.739 (8.080)		40.255 (7.800)
Wife's Hours of Work			34.707 (9.106)	33.235 (9.013)
Husband's Hourly Wage		14.234 (5.522)		14.866 (5.427)
Wife's Hourly Wage			13.669 (5.756)	13.560 (5.766)
Household Nonlabor Income	6.105 (20.911)	4.712 (19.068)	4.153 (19.465)	8.218 (23.027)
Husband's Age	40.080 (5.857)	40.239 (6.0510)	40.087 (5.538)	39.900 (5.801)
Wife's Age	36.607 (5.372)	36.382 (5.459)	37.185 (4.993)	36.703 (5.367)
Mid-level School (H)	0.439	0.393	0.451	0.487
Mid-level School (W)	0.462	0.361	0.570	0.546
High-level School (H)	0.105	0.040	0.169	0.148
Wife High-level School (W)	0.112	0.040	0.202	0.169
<i>N</i>	1575	735	183	657



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**Table 3**  
**Noncooperative Equilibrium Estimates (MLE)**  
**(Asymptotic Standard Error)**

<i>Parameter</i>	I	II
$\beta_{0,1}$	2.142 (0.017)	2.002 (0.021)
$\beta_{1,1}$		0.234 (0.032)
$\beta_{2,1}$		0.439 (0.050)
$\beta_{0,2}$	1.968 (0.028)	1.724 (0.046)
$\beta_{1,2}$		0.303 (0.055)
$\beta_{2,2}$		0.688 (0.078)
$\sigma_{11}$	0.109 (0.008)	0.084 (0.006)
$\sigma_{22}$	0.187 (0.018)	0.143 (0.014)
$\rho_{12}$	0.470 (0.057)	0.281 (0.077)
$\nu_1$	0.832 (.041)	0.832 (0.042)
$\nu_2$	1.174 (0.042)	1.164 (0.059)
$\ln L$	-1813.288	-1743.364

**Table 4**  
**Costly Cooperative Equilibrium Estimates (MLE)**  
**(Asymptotic Standard Error)**

<i>Parameter</i>	I	II
$\beta_{0,1}$	2.099 (0.019)	2.004 (0.023)
$\beta_{1,1}$		0.155 (0.030)
$\beta_{2,1}$		0.319 (0.049)
$\beta_{0,2}$	1.944 (0.020)	1.789 (0.024)
$\beta_{1,2}$		0.232 (0.031)
$\beta_{2,2}$		0.516 (0.053)
$\sigma_{11}$	0.139 (0.009)	0.118 (0.008)
$\sigma_{22}$	0.163 (0.011)	0.131 (0.009)
$\rho_{12}$	0.669 (0.028)	0.626 (0.031)
$\nu_1$	1.098 (.055)	1.099 (0.055)
$\nu_2$	2.089 (0.106)	2.089 (0.106)
$\zeta$	42.527 (5.809)	42.526 (5.809)
$\ln L$	-1367.200	-1309.524

**Table 5**  
**Simulation Results**

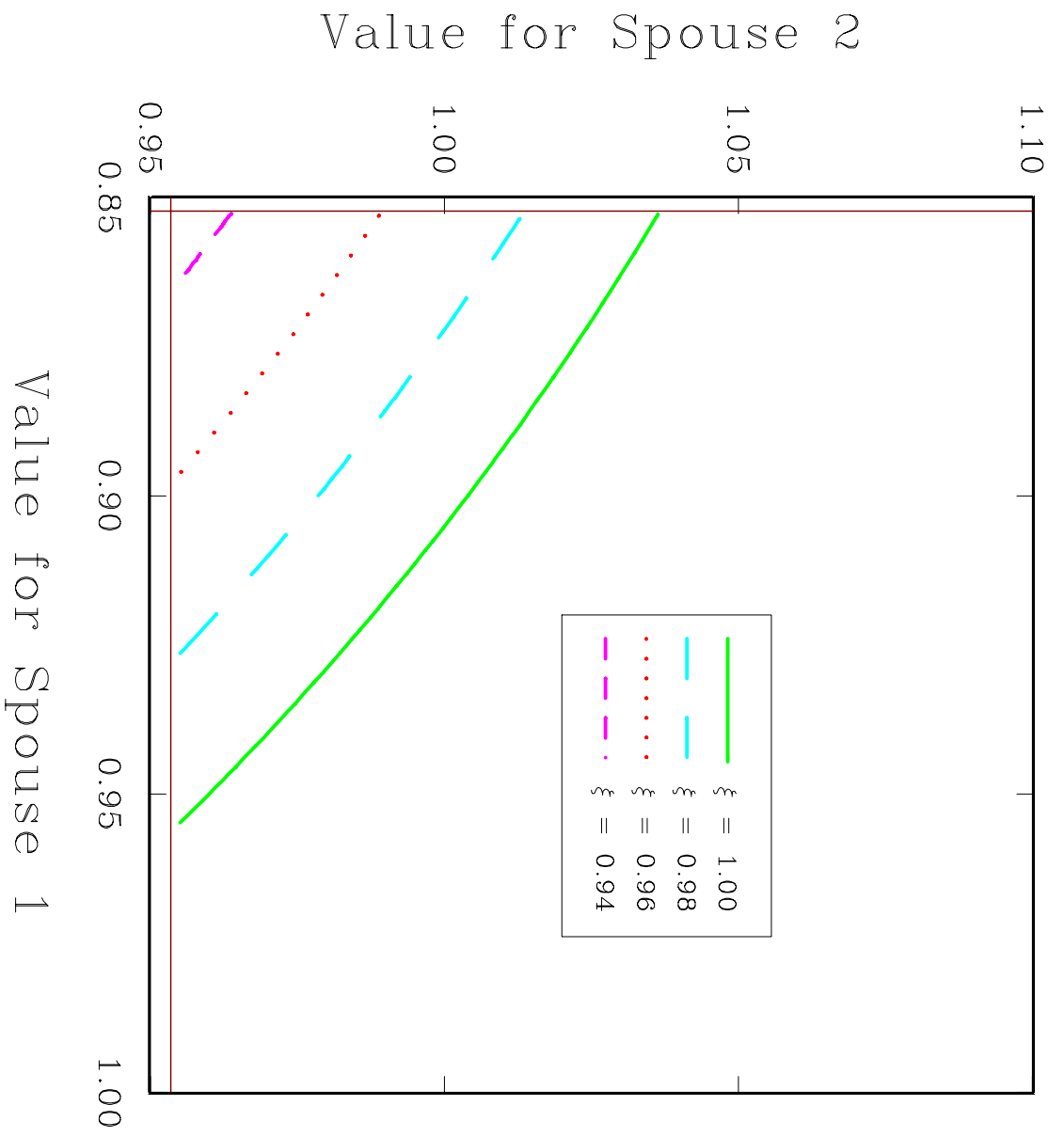
<i>Characteristic</i>	<i>Baseline</i>	$\nu_1 = \nu_2 = 1.098$	$\beta_{0,1} = \beta_{0,2} = 2.099$
$h_1$	0.468	0.443	0.470
$h_2$	0.238	0.420	0.257
$V_1$	0.638	0.735	0.665
$V_2$	0.438	0.737	0.455
$C = 1$	0.589	0.838	0.607
$ V_1 - V_2 $	0.375	0.417	0.383

## References

- [1] Aaberge, R., U. Colombino, and S. Strom (1999). "An Empirical Analysis of Joint Household Decisions, with Taxes and Quantity Constraints." *Journal of Applied Econometrics* 14, pp. 403-422.
- [2] Becker, G. (1981). *A Treatise on the Family*. Cambridge: Harvard University Press.
- [3] Blundell R. W. P. A Chiappori, T. Magnac, and C. Meghir (2005) "Collective Labour Supply: Heterogeneity and Nonparticipation." IZA Discussion Paper 1785.
- [4] Bourguignon, F. (1984) "Individual or Strategic Rationality: The Case of Family Labor Supply." (in French) *Revue Economique* 35, pp. 147-162.
- [5] Bourguignon, F. and P.A. Chiappori (1992) "Collective Models of Household Behavior: An Introduction." *European Economic Review* 36, pp. 355-364.
- [6] Brown, M. and C. Flinn. (2005) "Investment in Child Quality over Marital States." Unpublished Manuscript, New York University.
- [7] Brien, M., L. Lillard, and S. Stern (2006). "Cohabitation, Marriage, and Divorce in a Model of Match Quality." Forthcoming, *International Economic Review*.
- [8] Chiappori P.A. (1988). "Rational Household labor Supply" *Econometrica* 56, 63-89.
- [9] Cigno, A. (1991). *Economics of the Family*. Oxford: Oxford University Press.
- [10] Cogan, J. (1981). "Fixed Costs and Labor Supply." *Econometrica* 49, pp. 945-963.
- [11] Colombino, U. and D. Del Boca (1990). "The Effect of Taxation on Labour Supply in Italy." *Journal of Human Resources* 25, pp. 390-414.
- [12] Del Boca D. 2002 "The Effect of Child care and Part time on Participation and Fertility" *Journal of Population Economics* 15, 3.
- [13] Del Boca, D., 1997. "Intrahousehold Distribution of Resources and Labor Market Participation Decisions" in *Economics of the Family and Family Policies* (Persson, I., Jonung C. eds ) Routledge Press.
- [14] Del Boca, D. and C. Flinn (1995). "Rationalizing Child Support Orders." *American Economic Review* 85, pp. 1241-1262.
- [15] Flinn, C. (2000) "Modes of Interaction Between Divorced Parents." *International Economic Review* 41, pp. 545-578.
- [16] Fortin B. and G. Lacroix (1997) "A Test of Neoclassical Model and Collective Models of Household Labour Supply." *Economic Journal* 107, pp. 933-955.
- [17] Kapteyn, A. and P. Kooreman (1990). "On the empirical Implementation of some theoretic Models of Household Labour Supply" *Journal of Human Resources* 25.

- [18] Heckman, J. (1974). "Shadow Prices, Market Wages, and Labor Supply." *Econometrica* 42, pp. 679-694.
- [19] Leuthold, J. H. (1968) "An Empirical Study of Female Income Transfers and the Work Decision of the Poor." *Journal of Human Resources* 3, pp. 312-323.
- [20] Lugo-Gil, J. (2003). "Intrahousehold Bargaining and Welfare in Intact Families." Unpublished Manuscript, New York University.
- [21] Lundberg, S. and R. Pollak (1993). "Separate Spheres Bargaining and the Marriage Market." *Journal of Political Economy* 101, pp. 988-1010.
- [22] Manser, M. and M. Brown (1980). "Marriage and Household Decision Making: A Bargaining Analysis." *International Economic Review* 21, pp. 31-44.
- [23] McElroy M. and M. Horney (1981). "Nash-Bargained Household Decisions: Towards a Generalization of the Theory of Demand." *International Economic Review* 22, pp. 333-349.
- [24] McElroy, M. and M. Horney (1990). "Nash-Bargained Household Decisions: Reply." *International Economic Review* 31, pp. 237-242.

Figure 1  
Household Labor Supply Example



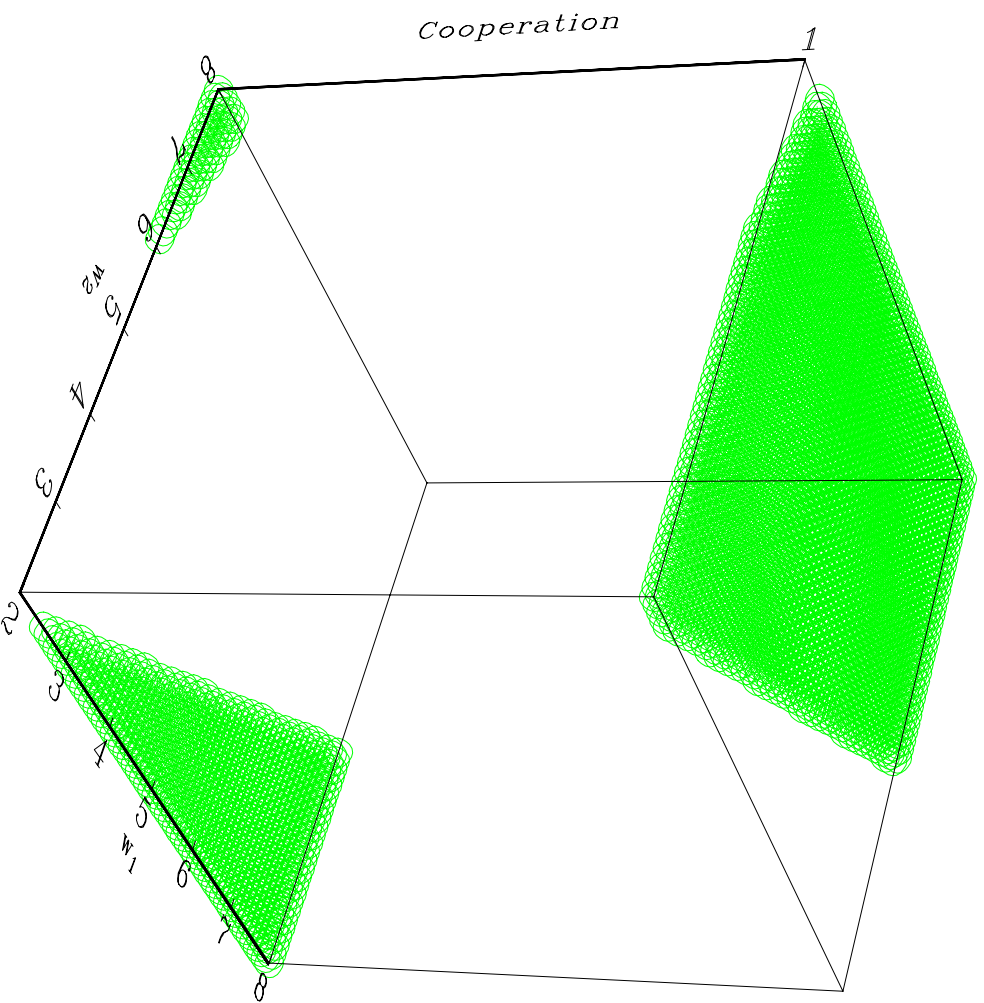


Figure 2  
Cooperation Set (Wages)



Figure 3.a  
 $h_1$

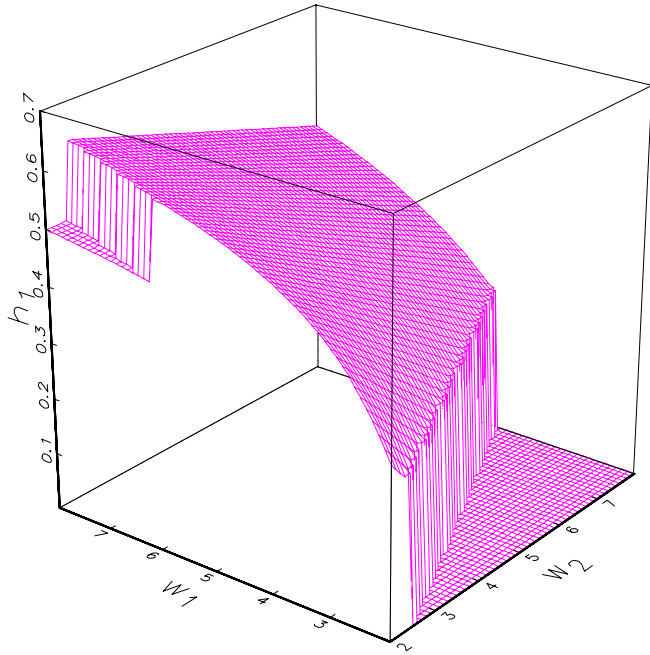


Figure 3.b  
 $h_2$

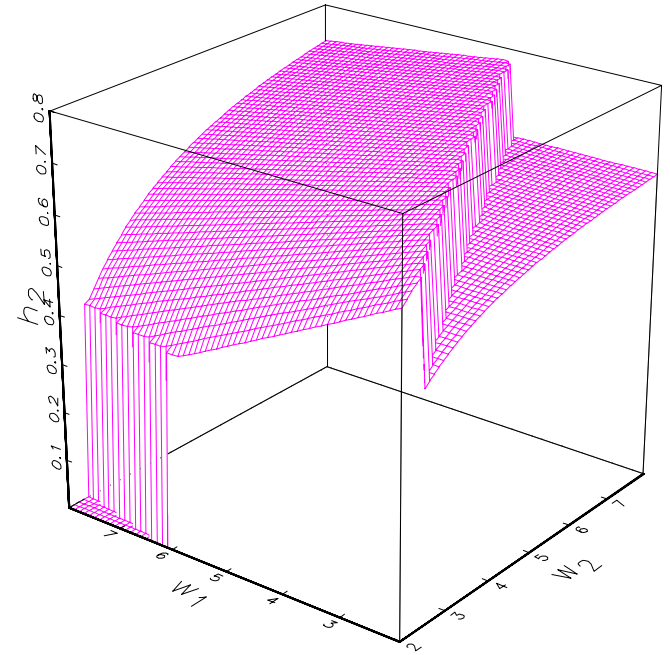


Figure 3.c  
 $V_1$

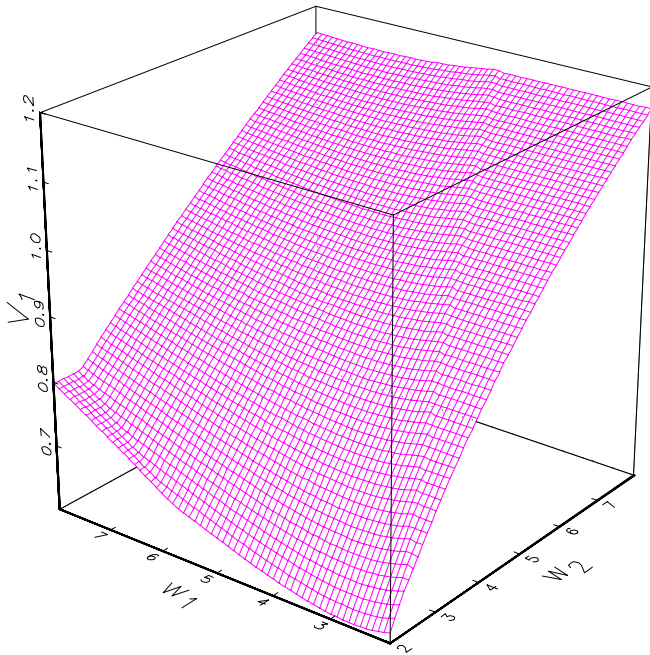


Figure 3.d  
 $V_2$

