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## **On the Informational Loss Inherent in Approximation Procedures: Welfare Implications and Impulse Responses**

by

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# On the Informational Loss Inherent in Approximation Procedures: Welfare Implications and Impulse Responses<sup>☆</sup>

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## Abstract

This paper shows the inappropriateness of approximation procedures for welfare rankings across suboptimal policy strategies. On the grounds of a simple general equilibrium model, we find that even commonly accepted techniques are not suitable to achieve accurate welfare orderings. This result points to a non-universality of these methods since we unveil welfare reversals when we compare them with the implications of the corresponding Ramsey problem. We infer that the spurious outcomes originate from restricting the approximations to only first and second-order moments. The order of approximation needed to obtain accuracy obviously depends on the underlying type of general equilibrium model and on its parameter values. But this creates uncertainty about the correct degree of approximation adopted by researchers in order to obtain clear welfare insights. Therefore, we strongly recommend that normative analyses should rely exclusively on the exact optimality conditions delivered by the Ramsey problem. Nonetheless, we are able to propose approximation methods in order to characterize macroeconomic fluctuations triggered by small disturbances.

*Jel classification:* C63; E52; E61

*Keywords:* Monetary Policy; Macroeconomic Policy Consistency; First-order approximation; Second-order approximation; Purely quadratic approach

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# 1 Introduction

When dealing with normative policy analysis, researchers clearly face the problem of choosing the approximation degree of general equilibrium models. Most importantly, several authors have established that a low degree of accuracy is connected with erroneous welfare rankings across alternative monetary strategies. Linearized settings do not seem at first sight to be suitable since they imply a so-called certainty equivalence by construction and eliminate potentially welfare-relevant information of higher-order terms. As an example, Chari et al. (1994) compute a first-order Taylor expansion in terms of logarithms of the optimality conditions characterizing the Ramsey policy<sup>1</sup>. This procedure is uncovered to be unreliable in the context of optimal tax policy problems. The most prominent example for this problem is delivered by Kim and Kim (2003). These authors find the spurious outcome of a higher welfare level in the autarky economy when compared to an international risk-sharing setting. These conclusions are reached by measuring absolute welfare levels with a standard second-order Taylor approximation of the households' utility function. Nonetheless, this kind of welfare measuring is erroneous since some second-order terms are absent while others are incorporated. Formally, the approximated welfare criterion displays linear terms, leading to a measure that is inaccurate to order two or higher<sup>2</sup>. The unwillingness to discard approximated model settings has led researchers to investigate for ways of accurate welfare measuring without using the underlying non-linear relationships explicitly. For example, Kim and Kim (2003, 2006) propose the approximation of the exact optimality conditions up to second-order through a perturbation method, while maintaining the simple second-order welfare measure. This should be able to account for higher moments which are essential for measuring risks, and therefore welfare. A further method advocated by

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<sup>1</sup> This policy strategy is given by the first-order conditions of a utility maximization problem subject to the non-linear model economy.

<sup>2</sup> Benigno and Woodford (2004, 2006) and Kim and Kim (2006) label these approaches as 'naive' because they deliver incorrect linear representations of the exact optimality conditions implied by the Ramsey optimization problem.

Kim and Kim (2006), Kahn et al. (2003), and King and Wolman (1999) consists in computing first-order approximations of the exact first-order conditions, but measuring welfare with the help of a purely quadratic welfare function along the lines of Benigno and Woodford (2004, 2006) and Woodford (1999, 2003). This welfare criterion is purely quadratic in the sense that all linear terms in the second-order approximation of the utility function are eliminated through second-order approximated model constraints<sup>3</sup>. However, these authors exclude this approach for welfare rankings among suboptimal policies like e.g. the ad hoc Taylor rule or the Ramsey policy under timeless perspective commitment. Instead, Benigno and Woodford (2004, 2006) and Woodford (1999, 2003) propose the minimization of the purely quadratic loss function using the first-order approximation of the model economy as a constraint. This allows the welfare evaluation to second-order using only a first-order approximation to the equilibrium evolution of inflation and output under a given and potentially suboptimal policy. The approach has become very popular for small shock disturbances because it is supposed to yield linear first-order conditions whose implications differ only slightly from applying second-order perturbation methods to the first-order conditions of the Ramsey problem (see Benigno and Woodford (2004, 2006) and Kim and Kim (2006)). At the same time, it provides identical approximate linear characterizations of the Ramsey optimality conditions. A recent contribution by Winkler (2009) demonstrates the potential welfare superiority of Taylor rules when compared with the Ramsey policy under timeless perspective commitment. The motivation for this work is the misperception in the literature about the optimality of the latter when considering time-invariant policy rules. The timeless perspective policy is, nonetheless, intrinsically not optimal with respect to the time zero conditional expectation target since optimality conditions

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<sup>3</sup> Debortoli and Nunes (2006) show that the timeless perspective commitment assumption is crucial for the derivation of a purely quadratic objective policy function in the presence of a distorted non-stochastic steady state. This is consistent with the fact that monopolistic distortions must be fully present in the non-linear model economy in order to achieve a Ramsey policy that is of timeless perspective.

concerning the shock period are neglected. This allows Taylor rules to deliver lower degrees of suboptimality for different persistent and non-persistent shocks. Blake (2001) and Jensen and McCallum (2002) show a similar result by using an ad hoc welfare loss function embedded in a canonical New Keynesian model, where the optimization target possesses the same structure as the purely quadratic criterion. The suboptimality of the timeless policy is established against other ad hoc policy rules than the Taylor rule. Naturally, one is forced to verify if the supposedly accurate approximation methods described above are able to reproduce the same welfare rankings as in Winkler (2009). By doing so, we conclude that this is not the case. This indicates that second-order procedures, including the prominent purely quadratic approach, may not be sufficient to replace a Ramsey-accurate welfare analysis in general equilibrium settings. As a complementing feature of our findings, we also compare the accuracy of the approximation procedures in terms of the time development of core variables after a shock impulse.

The remainder is as follows. In section 2, the underlying commercial banks-augmented New Keynesian framework is presented. It is related to the contributions of Henzel et al. (2009) and Hülsewig et al. (2009). The monetary authority is allowed to choose either an ad hoc Taylor rule or the optimal policy under timeless perspective commitment. Section 3 presents the welfare measures in order to obtain welfare rankings in both, the non-linear model and the corresponding approximations not exceeding the order of two. Section 4 calibrates the parameters which determine the model steady state. Section 5 outlines the computational simulation procedure and presents our results. Section 6 summarizes and concludes the discussion.

## 2 The Non-Linear Model

In this section, the underlying general equilibrium model is presented. Since no complications are needed to derive our insights, we lay down a very basic banks-augmented New Keynesian model for a cashless economy. All monetary non-neutralities stem from non-linear quadratic adjustment costs according to Rotemberg (1986) and Hairault and Portier (1993) in conjunction with monopolistic competition market structures. Since the model depicts a standard setup, our descriptions will be kept very brief.

### 2.1 Retailers

Retailers produce a homogeneous good  $Y_t$  sold under perfect market conditions with a corresponding aggregate price index  $P_t$ . They demand a monopolistically offered intermediate good  $Y_t(i)$  from a firm lying in the unit interval  $[0, 1]$ . The assembly across all types of intermediate goods towards the final good takes place according to a CES production technology:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\epsilon_{p,t}}{\epsilon_{p,t}-1}} di \right)^{\frac{\epsilon_{p,t}-1}{\epsilon_{p,t}}} \quad (1)$$

From equation (1) it is evident that the elasticity of substitution between input varieties  $\epsilon_{p,t}$  is allowed to vary over time. This takes place according to the following autoregressive process up to first-order with a shock impulse variable  $e_{p,t}$ :

$$\left( \frac{\epsilon_{p,t}}{\epsilon_p} \right) = \left( \frac{\epsilon_{p,t-1}}{\epsilon_p} \right)^{\rho_p} \exp \{e_{p,t}\} \quad (2)$$

The variable  $\epsilon_p$  denotes the steady state price elasticity of demand, while  $0 \leq \rho_p < 1$  gives the degree of persistence in the shock process.

The profit maximizing demand schedule for input variety  $i$  is:

$$\frac{Y_t(i)}{Y_t} = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_{p,t}} \quad (3)$$

Therefore, relative demand for the output of intermediate firm  $i$  is a decreasing function of its relative price. Inserting this equation into (1) yields the aggregate price index for the bundle  $Y_t$ :

$$P_t = \left( \int_0^1 P_t(i)^{1-\epsilon_{p,t}} di \right)^{\frac{1}{1-\epsilon_{p,t}}} \quad (4)$$

## 2.2 Intermediate Producers

An intermediate firm produces its own output level  $Y_t(i)$  with labor hours  $N_t(i)$ :

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad , \quad 0 < \alpha < 1 \quad (5)$$

The variable  $A_t$  gives the time varying technology level, while the parameter  $\alpha \in (0, 1)$  implies decreasing returns to labor. The technology level follows an autoregressive process up to first-order with a shock impulse variable  $e_{a,t}$ :

$$\left( \frac{A_t}{A} \right) = \left( \frac{A_{t-1}}{A} \right)^{\rho_a} \exp \{e_{a,t}\} \quad (6)$$

The variable  $A$  gives the steady state technology level and  $0 \leq \rho_a < 1$  is the degree of persistence in the technology shock process. The utilized employment can be seen as a CES bundle consisting of different labor types, each monopolistically offered by a household  $j$  lying in the unit interval  $[0, 1]$ :

$$N_t(i) = \left( \int_0^1 N_t(i, j)^{\frac{\epsilon_{w,t}}{\epsilon_{w,t}-1}} dj \right)^{\frac{\epsilon_{w,t}-1}{\epsilon_{w,t}}} \quad (7)$$



The demand of firm  $i$  for labor type  $j$  is given by  $N_t(i, j)$ . The variable  $\epsilon_{w,t}$  denotes the time-dependent elasticity of substitution between labor varieties. It is assumed to follow an autoregressive process up to first-order with a shock impulse variable  $e_{w,t}$ :

$$\left(\frac{\epsilon_{w,t}}{\epsilon_w}\right) = \left(\frac{\epsilon_{w,t-1}}{\epsilon_w}\right)^{\rho_w} \exp\{e_{w,t}\} \quad (8)$$

The variable  $\epsilon_w$  gives the steady state level of the elasticity, while  $0 \leq \rho_w < 1$  is the degree of persistence in the elasticity shock process. The profit maximizing demand schedule of firm  $i$  for labor variety  $j$  is:

$$\frac{N_t(i, j)}{N_t(i)} = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_{w,t}} \quad (9)$$

Reinserting this equation into (7) gives the aggregate nominal wage index:

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_{w,t}} dj\right)^{\frac{1}{1-\epsilon_{w,t}}} \quad (10)$$

The aggregate demand for a labor type supplied by a household  $j$  is given as a fraction of the total labor force  $N_t$ :

$$N_t(j) = \int_0^1 N_t(i, j) di = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_{w,t}} \underbrace{\int_0^1 N_t(i) di}_{=N_t} = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_{w,t}} N_t \quad (11)$$

Nominal labor cost  $W_t N_t(i)$  is assumed to be prefinanced at the beginning of a period  $t$  by the nominal loan volume  $L_t(i)$ <sup>4</sup>. Repayment takes place after one period at the gross loan rate  $R_t^L$ .

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<sup>4</sup> We therefore introduce credit lending as in Henzel et al. (2009).

The variable  $L_t(i)$  is a CES loan bundle of differentiated loan types demanded by firm  $i$ . Each loan type originates from one commercial bank  $l$  lying in the unit interval  $[0, 1]$ :

$$L_t(i) = \left( \int_0^1 L_t(i, l)^{\frac{\zeta_t}{\zeta_t-1}} dl \right)^{\frac{\zeta_t-1}{\zeta_t}} \quad (12)$$

Note that  $\zeta_t$  is the time varying elasticity of substitution between loan varieties. It is assumed that this variable evolves according to the following autoregressive process up to first-order:

$$\left( \frac{\zeta_t}{\zeta} \right) = \left( \frac{\zeta_{t-1}}{\zeta} \right)^{\rho_r} \exp \{e_{r,t}\} \quad (13)$$

The variable  $\zeta$  gives the steady state price elasticity of demand, while  $0 \leq \rho_r < 1$  represents, again, the degree of persistence in the shock process. The profit maximizing demand schedule of firm  $i$  for a loan variety  $l$  is:

$$\frac{L_t(i, l)}{L_t(i)} = \left( \frac{R_t^L(l)}{R_t^L} \right)^{-\zeta_t} \quad (14)$$

Reinserting this equation into the CES loan aggregator (12) gives the aggregate gross loan rate:

$$R_t^L = \left[ \int_0^1 R_t^L(l)^{1-\zeta_t} dl \right]^{\frac{1}{1-\zeta_t}} \quad (15)$$

The aggregate demand for a loan type  $l$  is given as a fraction of the total loan volume  $L_t$ :

$$L_t(l) = \int_0^1 L_t(i, l) di = \left( \frac{R_t^L(l)}{R_t^L} \right)^{-\zeta_t} \underbrace{\int_0^1 L_t(i) di}_{=L_t} = \left( \frac{R_t^L(l)}{R_t^L} \right)^{-\zeta_t} L_t \quad (16)$$

An intratemporal total cost minimization procedure of the intermediate firm yields the firm-specific real marginal cost with pre-financed nominal labor cost:

$$MC_t(i) = \left( \frac{1-\nu}{1-\alpha} \right) \frac{W_t}{P_t} R_t^L Y_t(i)^{\frac{\alpha}{1-\alpha}} A_t^{-\frac{1}{1-\alpha}} \quad (17)$$

The variable  $\nu$  enters the marginal cost as a government subsidy rate. An intermediate firm chooses its own price  $P_t(i)$  in order to maximize monopolistic real profits taking its own demand schedule (3) into account. Moreover, the firm faces real quadratic costs of price adjustment (Hairault and Portier (1993)) expressed as:

$$Q_t^p(i) = \frac{\psi_p}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \pi^p \right)^2 \quad (18)$$

The Parameter  $\psi_p$  gives the degree of quadratic cost reactions to deviations of intertemporal price relations  $\frac{P_t}{P_{t-1}}$  from the steady state gross inflation rate  $\pi^p$ . The profit maximization subject to these constraints yields after an aggregation around a symmetric equilibrium the following Phillips curve for the gross price inflation  $\pi_t^p$ :

$$\begin{aligned} \pi_t^p (\pi_t^p - \pi^p) = & \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^p - \pi^p) \pi_{t+1}^p \right] \\ & + \frac{Y_t \epsilon_{p,t}}{\psi_p} \left( MC_t - \frac{1}{\mu_{p,t}} \right) \end{aligned} \quad (19)$$

Note that setting  $\psi_p = 0$  gives the flexible price level under monopolistic distortions. The aggregate real marginal cost  $MC_t$  is structurally equivalent to the individual firm expression (17). The variable  $\mu_{p,t}$  is the time varying monopolistic markup of intermediate firms, which is given by:

$$\mu_{p,t} = \left( \frac{\epsilon_{p,t}}{\epsilon_{p,t} - 1} \right) \quad (20)$$

### 2.3 Commercial Banks

We introduce monopolistic loan supply by commercial banks as in Henzel et al. (2009) and Hülsewig et al. (2009). The differentiation and monopolistic competition in loan types may emerge from tight customer-banks relationships or specialization in certain geographical regions (Carletti et al. (2007)). We depart from the Calvo-type modeling of lending frictions (Calvo (1983)), and assume Rotemberg (1982) adjustment costs in the non-linear form postulated by Hairault and Portier (1993). The choice of a banks ( $l \in [0, 1]$ ) own gross loan rate  $R_t^L(l)$  is therefore costly in terms of:

$$Q_t^R(l) = \frac{\psi_r}{2} \left( \frac{R_t^L(l)}{R_{t-1}^L(l)} - 1 \right)^2 \quad (21)$$

The Parameter  $\psi_r$  gives the degree of gross loan rigidity. The balance sheet is assumed to be very simple and equalizes the nominal loan volume to the sum of nominal deposits  $d_t(l)$  and the net position in the money market  $B_t(l)$ :

$$L_t(l) = d_t(l) + B_t(l) \quad (22)$$

Deposits and money market credits are thought as perfect substitutes for refinancing, which implies that the gross deposit rate  $R_t^d$  always equals the gross money rate  $R_t^M$  (Freixas et. al (2000)):

$$R_t^d = R_t^M \quad (23)$$

Total real cost arising from the placing of loans is therefore given by:

$$TC_t^r(l) = \frac{d_t(l)}{P_t} R_t^d + \frac{B_t(l)}{P_t} R_t^M = \left( \frac{d_t(l) + B_t(l)}{P_t} \right) R_t^M = \left( \frac{L_t(l)}{P_t} \right) R_t^M \quad (24)$$

The real profits of a commercial bank can be written as:

$$E_t \sum_{k=0}^{\infty} \Delta_{t,t+k} \left( \frac{R_{t+k}^L(l) L_{t+k}(l)}{P_{t+k}} - TC_{t+k}^r(l) - Q_{t+k}^R(l) \right) \quad (25)$$

Since households own commercial banks, the term  $\Delta_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)$  represents the relevant stochastic discount factor for real profits. Profit maximization with respect to the control variable  $R_t^L(l)$  and subject to the loan demand schedule (16) yields after an aggregation around the symmetric equilibrium the following equation for the change of the gross loan rate  $\left( \frac{R_t^L}{R_{t-1}^L} \right)$ :

$$\begin{aligned} \left( \frac{R_t^L}{R_{t-1}^L} - 1 \right) \left( \frac{R_t^L}{R_{t-1}^L} \right) &= \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{R_{t+1}^L}{R_t^L} - 1 \right) \left( \frac{R_{t+1}^L}{R_t^L} \right) \right] \\ &+ \frac{N_t \zeta_t}{\psi_r} \frac{W_t}{P_t} \left( R_t^M - \frac{R_t^L}{\Xi_t} \right) \end{aligned} \quad (26)$$

Note that the monopolistic markup to the real marginal cost  $R_t^M$  is inversely time varying to the loan elasticity of substitution  $\zeta_t$ :

$$\Xi_t = \left( \frac{\zeta_t}{\zeta_t - 1} \right) \quad (27)$$

## 2.4 Households

There is a continuum of households in the unit interval  $[0, 1]$ , each supplying one specific variety of labor  $j$  in a monopolistic manner. Substitution of the labor demand schedule (11) into the widely applied CRRA utility function yields:

$$E_t \sum_{k=0}^{\infty} \beta^k \left[ \left( \frac{1}{1-\sigma} \right) C_{t+k}(j)^{1-\sigma} - \left( \frac{1}{1+\eta} \right) \left( \underbrace{\left( \frac{W_{t+k}(j)}{W_{t+k}} \right)^{-\epsilon_{w,t+k}}}_{=N_{t+k}(j)} N_{t+k} \right)^{1+\eta} \right] \quad (28)$$

The household can save in deposits  $d_t(j)$  at the gross deposit rate  $R_t^d$ . It owns firms and banks and therefore receives real dividends  $Div_t^r(j)$ . At the same time, the household has to pay real taxes  $T_t^r(j)$  to the government. Further, when choosing its monopolistic nominal wage, the household faces real quadratic adjustment costs denominated as  $Q_t^w(j)$ .

The periodical budget constraint is therefore given by:

$$C_t(j) + \frac{d_t(j)}{P_t} = \frac{W_t(j)}{P_t} \underbrace{\left( \frac{W_t(j)}{P_t} \right)^{-\epsilon_{w,t}} N_t + R_{t-1}^d}_{=N_t(j)} \frac{d_{t-1}(j)}{P_t} + Div_t^r(j) - T_t^r(j) - Q_t^w(j) \quad (29)$$

The real quadratic adjustment costs around the steady state wage inflation  $\pi^w$  are:

$$Q_t^w(j) = \frac{\psi_w}{2} \left( \frac{W_t(j)}{W_{t-1}(j)} - \pi^w \right)^2 \quad (30)$$

Maximization of (28) with respect to the control variable  $W_t(j)$  and subject to (29) and (30) yields after the symmetric equilibrium aggregation the following Phillips curve for the gross wage inflation  $\pi_t^w$ :

$$\pi_t^w (\pi_t^w - \pi^w) = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^w - \pi^w) \pi_{t+1}^w \right] + \frac{N_t (\epsilon_{w,t} - 1)}{\psi_w} \left( MRS_t \mu_{w,t} - \frac{W_t}{P_t} \right) \quad (31)$$

The variable  $MRS_t$  represents the marginal rate of substitution. Note that setting  $\psi_w = 0$  gives the flexible nominal wage level under monopolistic distortions. The variable  $\mu_{w,t}$  denotes the time varying monopolistic markup of the household:

$$\mu_{w,t} = \left( \frac{\epsilon_{w,t}}{\epsilon_{w,t} - 1} \right) \quad (32)$$

The marginal rate of substitution between consumption  $C_t$  and labor supply  $N_t$  is characterized by:

$$MRS_t = C_t^\sigma N_t^\eta \quad (33)$$

Combining the first-order conditions for the remaining control variables  $C_t(j)$  and  $B_t(j)$  leads after an aggregation to the standard Euler equation for the intertemporal consumption substitution:

$$C_t^{-\sigma} = \beta R_t^d E_t \left[ C_{t+1}^{-\sigma} \frac{1}{\pi_{t+1}^p} \right] \quad (34)$$

## 2.5 Government

The government enters the model implicitly with the only task of subsidising labor cost of intermediate firms and collecting net transfers from the household sector. The subsidy rate  $\nu$  could potentially be chosen in order to eliminate monopolistic distortions in the long run. The aggregate budget identity of the fiscal authority is given by:

$$P_t T_t^r = \nu W_t N_t \quad (35)$$

## 2.6 Aggregate Resource Constraint and Inflation Identity

The economy-wide resource utilization is derived by taking the aggregate version of the household budget constraint (29) into account. After eliminating all net transfers stemming from the private and the public sector, one arrives at:

$$Y_t = C_t + \frac{\psi_p}{2} (\pi_t^p - \pi^p)^2 + \frac{\psi_w}{2} (\pi_t^w - \pi^w)^2 + \frac{\psi_r}{2} \left( \frac{R_t^L}{R_{t-1}^L} - 1 \right)^2 \quad (36)$$

As usual, the existence of quadratic costs of adjustment restricts the full mapping of real income into aggregate consumption.

Note further that the gross price and nominal wage inflation rates are linked through the identity equation:

$$\frac{W_t}{P_t} = \frac{W_{t-1}}{P_{t-1}} \frac{\pi_t^w}{\pi_t^p} \quad (37)$$

## 2.7 Monetary Policy

The central bank is assumed to choose between the most simple Taylor rule (see Taylor (1993)) and an optimal policy strategy under timeless perspective commitment. Concerning the Taylor rule, one considers the non-linear deviation of the gross interest rate from its steady state counterpart  $R$  as the control instrument. The reaction parameters with respect to real output and inflation variations are given by  $\delta_\pi$ ,  $\delta_y$ , while  $\phi$  acts as a smoothing parameter:

$$\left(\frac{R_t}{R}\right) = \left(\left(\frac{\pi_t^p}{\pi^p}\right)^{\delta_\pi} \left(\frac{Y_t}{Y}\right)^{\delta_y}\right)^{1-\phi} \left(\frac{R_{t-1}}{R}\right)^\phi \quad (38)$$

A very simplifying assumption is the complete control of interbank money interest rates by the central bank instrument:

$$R_t = R_t^M \quad (39)$$

The Ramsey monetary policy is given by the first-order conditions of a constrained utility maximization problem<sup>5</sup>. An optimization of (28) subject to (5), (17), (19), (26), (31), (33), (36), and (37) with respect to the control variables  $C_t$ ,  $N_t$ ,  $\pi_t^p$ ,  $\pi_t^w$ ,  $\frac{R_t^L}{R_{t-1}^L}$ , and  $W_t/P_t$  delivers the following collection of first-order conditions<sup>6</sup>:

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<sup>5</sup> Applications can be found in Kahn et al. (2003), Yun (2005), Faia (2007, 2008), Faia and Monacelli (2004, 2007, 2008), and in Schmitt-Grohé and Uribe (2004a, 2005).

<sup>6</sup> Rational expectation operators are omitted for simplicity. Lagrange multipliers are unfactorized in order to directly recognize the outcome of the problem stated in the appendix section A.4.



$$\begin{aligned}
& C_t^{-\sigma} - \lambda_{1,t+1} - \lambda_{2,t+1} \beta \sigma \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{C_t} (\pi_{t+1}^p - \pi^p) \pi_{t+1}^p \\
& + \lambda_{2,t} \sigma \left( \frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{1}{C_t} (\pi_t^p - \pi^p) \pi_t^p \\
& - \lambda_{3,t+1} \beta \sigma \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{C_t} (\pi_{t+1}^w - \pi^w) \pi_{t+1}^w \\
& + \lambda_{3,t} \sigma \left( \frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{1}{C_t} (\pi_t^w - \pi^w) \pi_t^w \\
& - \lambda_{3,t+1} \sigma N^{1+\eta} C_t^{\sigma-1} \frac{\mu_{w,t} (\epsilon_{w,t} - 1)}{\psi_w} \\
& - \lambda_{5,t+1} \beta \sigma \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{C_t} \left( \frac{R_{t+1}^L}{R_t^L} - 1 \right) \left( \frac{R_{t+1}^L}{R_t^L} \right) \\
& + \lambda_{5,t} \sigma \left( \frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{1}{C_t} \left( \frac{R_t^L}{R_{t-1}^L} - 1 \right) \left( \frac{R_t^L}{R_{t-1}^L} \right) = 0
\end{aligned} \tag{40}$$

$$\begin{aligned}
& -N^\eta + \lambda_{1,t+1} (1-\alpha) A_t N_t^{-\alpha} + \lambda_{2,t+1} \left( \frac{\alpha(1-\nu)}{1-\alpha} \right) \frac{W_t \epsilon_{p,t}}{P_t \psi_p} R_t^L \\
& - \lambda_{2,t+1} \left( \left( \frac{1-\nu}{1-\alpha} \right) \frac{W_t}{P_t} \frac{1}{A_t} R_t^L N_t^\alpha - \frac{1}{\mu_{p,t}} \right) \frac{(1-\alpha) \epsilon_{p,t} A_t N_t^{-\alpha}}{\psi_p} \\
& - \lambda_{3,t+1} \eta N_t^\eta C_t^\sigma \frac{\mu_{w,t} (\epsilon_{w,t} - 1)}{\psi_w} \\
& - \lambda_{3,t+1} \left( \mu_{w,t} N_t^\eta C_t^\sigma - \frac{W_t}{P_t} \right) \left( \frac{\epsilon_{w,t} - 1}{\psi_w} \right) \\
& - \lambda_{5,t+1} (R_t^L - R_t^M \Xi_t) \frac{W_t}{P_t} \left( \frac{1-\zeta_t}{\psi_r} \right) = 0
\end{aligned} \tag{41}$$

$$\begin{aligned}
& - \lambda_{1,t+1} \psi_p (\pi_t^p - \pi^p) + \lambda_{2,t+1} (2\pi_t^p - \pi^p) \\
& - \lambda_{2,t} \left( \frac{C_t}{C_{t-1}} \right)^{-\sigma} (2\pi_t^p - \pi^p) + \lambda_{4,t+1} \left( \frac{W_{t-1}}{P_{t-1}} \right) \frac{\pi_t^w}{(\pi_t^p)^2} = 0
\end{aligned} \tag{42}$$

$$\begin{aligned}
 & -\lambda_{1,t+1} \psi_w (\pi_t^w - \pi^w) + \lambda_{3,t+1} (2\pi_t^w - \pi^w) \\
 & - \lambda_{3,t} \left( \frac{C_t}{C_{t-1}} \right)^{-\sigma} (2\pi_t^w - \pi^w) - \lambda_{4,t+1} \left( \frac{W_{t-1}}{P_{t-1}} \right) \frac{1}{\pi_t^p} = 0
 \end{aligned} \tag{43}$$

$$\begin{aligned}
 & -\lambda_{1,t+1} \psi_r \left( \frac{R_t^L}{R_{t-1}^L} - 1 \right) - \lambda_{2,t+1} \left( \frac{1-\nu}{1-\alpha} \right) \frac{W_t}{P_t} \frac{\epsilon_{p,t}}{\psi_p} N_t R_{t-1}^L \\
 & + \lambda_{5,t+1} \left( 2 \left( \frac{R_t^L}{R_{t-1}^L} \right) - 1 + \frac{W_t}{P_t} \frac{N_t \zeta_t}{\psi_r} \frac{R_{t-1}^L}{\Xi_t} \right) \\
 & - \lambda_{5,t} \left( \frac{C_t}{C_{t-1}} \right)^{-\sigma} \left( 2 \left( \frac{R_t^L}{R_{t-1}^L} \right) - 1 \right) = 0
 \end{aligned} \tag{44}$$

$$\begin{aligned}
 & -\lambda_{2,t+1} \left( \frac{1-\nu}{1-\alpha} \right) R_t^L \frac{\epsilon_{p,t}}{\psi_p} N_t + \lambda_{3,t+1} \left( \frac{\epsilon_{w,t} - 1}{\psi_w} \right) N_t \\
 & + \lambda_{4,t+1} - \lambda_{4,t+2} \beta \frac{\pi_{t+1}^w}{\pi_{t+1}^p} - \lambda_{5,t+1} \frac{N_t \zeta_t}{\psi_r} \left( R_t^M - \frac{R_t^L}{\Xi_t} \right) = 0
 \end{aligned} \tag{45}$$

The timeless perspective policy is attained by setting  $\nu = 0$  and, therefore, by letting all monopolistic distortions render an inefficiently low output level in the long run (Winkler (2009)). This implies steady state values of the predetermined costate variables  $\lambda_2$ ,  $\lambda_3$ , and  $\lambda_5$ , which are equalized to the steady state implied by the model equations. Wrapping up all the brief derivations, one can identify the non-linear structural model as consisting of equations (5), (17), (19), (20), (26), (27), (31), (32), (33), (34), (36), and (37). The autoregressive shock processes are given by (2), (6), (8), and (13). All relationships can be overviewed in the appendix section A.1. If the central bank does not follow the Taylor rule (38), this equation has to be replaced by the first-order conditions (40)-(45).

### 3 Welfare Measurement

The welfare measures in order to compare monetary policy strategies are chosen according to whether the model is approximated or not. Formally, the absolute (*abs.*) welfare measure in the non-approximated model reads according to Faia (2007), Schmitt-Grohé and Uribe (2004a, 2005), and Winkler (2009):

$$V_0^{abs.} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{1}{1-\sigma} \right) C_t^{1-\sigma} - \left( \frac{1}{1+\eta} \right) N_t^{1+\eta} \right] \quad (46)$$

In order to compare absolute welfare levels between two policy specifications, we construct a relative (*rel.*) measure of the form<sup>7</sup>:

$$V_0^{rel.} = \left( \frac{V_0^{abs.}(u) - V_0^{abs.}(w)}{V_0^{abs.}(w)} \right) 100 \quad (47)$$

Note that for  $\sigma > 1$  and  $\eta > 0$ , a policy  $u$  is superior to another policy  $w$  if  $V_0^{rel.} < 0\%$ . The first and second-order approximations of the non-linear model are suitable for a welfare analysis based on the quadratically approximated second-order aggregate utility function. The methodology is therefore in line with Benigno and Woodford (2004, 2006), Kim and Kim (2006), and Woodford (1999, 2003)<sup>8</sup>. The absolute welfare measure embedded in the approximated model versions is of the form:

$$J_0^{abs.} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \Omega (\hat{Y}_t - \gamma_1 \hat{Y}_t^{eff.} - \gamma_2)^2 + \Gamma_1 (\hat{\pi}_t^p)^2 + \Gamma_2 (\hat{\pi}_t^w)^2 + \Gamma_3 (\hat{\pi}_t^r)^2 \right] \quad (48)$$

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<sup>7</sup> See Schmidt-Grohé and Uribe (2004a, 2005), as well as Winkler (2009).

<sup>8</sup> Note that these authors state that accurate welfare measurement can also be achieved by using the direct second-order Taylor approximation of aggregate utility if second-order approximations of the exact optimality conditions are undertaken. However, we stick to the purely quadratic criterion since this does not destroy informational contents from second-order moments.

A hat above a variable denotes its logarithmic deviation from the steady state level. Note that we defined  $\hat{\pi}_t^r \approx \frac{R_t^L - R_{t-1}^L}{R_{t-1}^L}$  for the approximated relative change of the aggregate gross loan rate. A derivation of this welfare measure along the lines of Galí (2008, chapter 5), Woodford (2003, chapter 6), and Krause and Lemke (2005) can be inspected in the appendix section A.5. The relative weights on the respective components are determined by the structural model parameters<sup>9</sup>:

$$\Omega = \frac{Y^{1-\sigma}}{2} \frac{\beta(1+\eta) - (1-\sigma)(1-\alpha)}{(1-\alpha)} \quad (49)$$

$$\gamma_1 = \frac{\beta(\alpha + \eta + \sigma(1-\alpha))}{\beta(1+\eta) - (1-\sigma)(1-\alpha)} \quad (50)$$

$$\gamma_2 = \frac{(1-\alpha)(1-\beta(1-\varphi))}{\beta(1+\eta) - (1-\sigma)(1-\alpha)} \quad (51)$$

$$\Gamma_1 = \frac{Y^{-\sigma}\psi_p}{2} \quad (52)$$

$$\Gamma_2 = \frac{Y^{-\sigma}\psi_w}{2} \quad (53)$$

$$\Gamma_3 = \frac{Y^{-\sigma}\psi_r}{2} \quad (54)$$

The appearing parameter  $\varphi = 1 - \frac{1}{\mu_p \mu_w \Xi (1-\nu)}$  comprises all monopolistic distortions in the economy at the steady state. Relative welfare between two policies  $u$  and  $w$  can analogously be compared with the following measure:

$$J_0^{rel.} = \left( \frac{J_0^{abs.}(u)}{J_0^{abs.}(w)} \right) 100 \quad (55)$$

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<sup>9</sup> Note that  $Y$  denotes the parameter-determined level of real output at the model steady state summarized in the appendix section A.2.

Since  $J_0^{abs.} > 0$ , a policy  $u$  is superior to another policy  $w$  if  $J_0^{rel.} < 100\%$ .

## 4 Calibration

The model parameters are calibrated according to a quarter time unit. Using the standard values found in the literature, we set the households subjective discount factor  $\beta$  equal to 0.99, implying an annualized steady state real interest rate of 4 percent. We assume  $\sigma = \eta = 2$  for the inverse intertemporal elasticity of consumption and the inverse of the Frisch elasticity of labor supply. The labor elasticity of real output  $(1 - \alpha)$  is set to 0.8 and the steady state technology parameter is  $A = 1$ . The steady state values of the substitution elasticities are  $\epsilon_p = \epsilon_w = 6$  and  $\zeta = 3.5$ . This implies steady state markups on firms' and households' marginal cost of 20 percent respectively whereas banks charge 40 percent on  $R^M$ . The parameters that determine the degree of price and wage rigidity are laid down according to the probabilities of non-adjustment in the staggered pricing model of Calvo (1983)<sup>10</sup>. If they comprise an average price duration of four quarters, then  $\psi_p = 119.6271$  and  $\psi_w = 293.2574$ . Furthermore, the degree of loan rigidity is set according to Hülsewig, et al. (2009) to  $\psi_r = 1.6014$ <sup>11</sup>. We might induce the distorted steady state by setting  $\nu = 0$ , which renders predetermined Lagrange multipliers equalized to the long run values implied by the model steady state. Therefore, letting all monopolistic distortions be at work in the long run renders a Ramsey monetary policy, which of timeless perspective commitment. This policy is naturally suboptimal with respect to the welfare functions (46) and (48).

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<sup>10</sup> If  $\theta_p = \theta_w = 0.75$  are the Calvo parameters, it has to hold that  $\psi_p = \frac{\theta_p [1 + \alpha(\epsilon_p - 1)] Y \epsilon_p}{(1 - \theta_p)(1 - \beta \theta_p)(1 - \alpha) \mu_p}$  and  $\psi_w = \frac{\theta_w (1 + \epsilon_w \eta)(\epsilon_w - 1)(W/P)N}{(1 - \beta \theta_w)(1 - \theta_w)}$ , where variables without a time index denote steady state levels as listed in the appendix section A.2.

<sup>11</sup> These authors arrive at an empirical Calvo parameter of  $\tau = 0.41$  for the probability of gross loan rate adjustment. Using the relationship  $\psi_r = \frac{\tau \zeta (W/P) N R^M}{1 + \tau [\beta(\tau - 1) - 1]}$ , one arrives at the rigidity parameter value in the text.

However, in the case of a first-best long run equilibrium it is true that  $\varphi = 0$ , and therefore:

$$\nu = 1 - \left( \frac{1}{\mu_p \mu_w \Xi} \right) = \frac{1 + \epsilon_p (\epsilon_w - 1) + \zeta (\epsilon_p + \epsilon_w - 1) - \epsilon_w}{\epsilon_p \epsilon_w \zeta} \quad (56)$$

This subsidy rate would induce the first-best equilibrium allocation and deliver a Ramsey policy which is under time zero commitment. This policy is globally optimal in a sense that the predetermined Lagrange multipliers are equalized to zero before shocks occur (see Winkler (2009)). As for the target underlying the purely quadratic approach, the calibration values stated above imply  $\Omega = 2.8684$ ,  $\gamma_1 = 0.9979$ ,  $\gamma_2 = 0.1080$ ,  $\Gamma_1 = 88.6438$ ,  $\Gamma_2 = 217.3039$ , and  $\Gamma_3 = 1.1866$ . Since all weights in the welfare-theoretic loss criterion (48) are positive, an interior solution to the purely quadratic optimization procedure exists because the welfare loss function is rendered to be convex (Woodford (2006)). The autoregressive shock processes are assumed to be independent of one another and to die out very slowly. Therefore, we set  $\rho_p = \rho_w = \rho_r = 0.95$  for highly persistent shocks. The shock impulses  $e_{p,t=0}$ ,  $e_{w,t=0}$  and  $e_{r,t=0}$  start each autoregressive process only at  $t = 0$ , setting from then on the relevant equation system in motion. We assume on the one hand, that all shock impulses concerning the elasticities are negative, leading to an increase in the relevant markup in the shock period<sup>12</sup>. On the other hand, we let the impulse on the technology parameter to be positive, displaying a sudden increase of labor productivity. The elasticities and the technology parameter change by either one or ten percent in the impact period, where the variation is relative to their respective steady state value. Note that in the nonlinear model the impulses are deterministic, while for the approximated framework versions they are stochastic. Concerning the latter, they represent i.i.d. random variables. The reaction parameter values of the Taylor rule are standard and set to  $\delta_\pi = 1.5$  and  $\delta_y = 0.5$ . If needed, we set  $\phi = 0.85$  for interest rate smoothing behavior. The original Taylor Rule implies  $\phi = 0$ .

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<sup>12</sup> The monopolistic markup increases can be interpreted as elasticity-driven cost-push shocks.

## 5 Simulation Methodology and Results

We consider the non-linear environment as the point of reference for accurate welfare comparisons. That is, we first simulate the model equations summarized in the appendix section A.1 via one of the deterministic shock processes (69), (70), (71), or (72) jointly with the optimality conditions of the Ramsey policy (40), (41), (42), (43), (44), and (45)<sup>13</sup>. The latter is characterized by its timeless perspective specification achieved through  $\nu = 0$  since the Ramsey policy under time zero commitment is globally optimal and always welfare-dominant against any policy rule<sup>14</sup>. In contrast, if one assumes that the monetary authority follows a Taylor rule, the same simulation experiments are undertaken with equation (38) instead of using the optimality conditions of the Ramsey problem. Letting  $u$  : Taylor rule with or without smoothing and  $w$  : Ramsey monetary policy under timeless perspective commitment, the relative welfare measure (47) displays the numerical values stated in the first column of Table 1<sup>15</sup>. We obtain similar results as in Winkler (2009) for persistent cost-push shocks that lead to an impact decrease of the elasticities by one percent. In this case, the relative welfare measures reveal a lower degree of suboptimality of Taylor rules when compared to the timeless Ramsey policy. The opposite applies for the positive technology disturbance. However, the potential suboptimality of the timeless perspective policy contradicts its commonly believed welfare outperformance when considering the class of time-invariant policy rules.

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<sup>13</sup> Note that as the deterministic model has no close solution, one is forced to apply a non-linear approximation resolution routine. Nonetheless, no informational contents stemming from non-linearities are lost by this procedure. We employ the Newton-Raphson algorithm in the adaptation of Juillard (1996). This iterative computation and all algorithms for the stochastic environment explained below are implemented with the programs DYNARE version 4, in conjunction with Matlab version 7.

<sup>14</sup> A welfare comparison against optimal policies under time zero commitment would be pointless since this policy is always globally optimal with respect to the time zero welfare target (46) independently of the method used for welfare rankings.

<sup>15</sup> See section B for a list of tables.

Having set the benchmark for accurate welfare orderings, we now proceed to apply approximated techniques that are believed to be accurate as well. As a starting point, we compute first-order approximations of the same non-linear framework alternatives stated above, letting the shock impulses be stochastic. However, absolute welfare measuring takes place in terms of logarithms by means of the purely quadratic welfare loss function (48). Among others, Kim and Kim (2006) emphasize the suitability of this procedure. Our numerical simulation results under this environment can be verified in the second column of Table 1, where the relative welfare measure is now given by (55) with the same indexes for the policies as declared above. Evidently, the absolute welfare losses from Taylor rules are always higher when measured in terms of the losses connected with the timeless perspective policy. By comparing this result with the outcomes implied by the Ramsey approach, a reversal in welfare ordering is detected when considering elasticity-driven persistent cost-push shocks.

The next approach consists in computing second-order approximations of the above non-linear model under both policies and measuring welfare with a simple Taylor approximation of second-order of the aggregate utility<sup>16</sup>. According to Kim and Kim (2003, 2006), this should also deliver reliable welfare orderings. However, we maintain the purely quadratic criterion for measuring welfare instead of the linear quadratic utility approximation. This is done without loss of informational contents originating from second-order moments<sup>17</sup>. Benigno and Woodford (2006) find that the inclusion of second-order terms in the approximation of the Ramsey optimality conditions leads to a very small effect on the moments of the linearized conditions. According to these authors, this suggests that a log-linear approximation is fairly accurate in the case of small disturbances if welfare is measured with the purely quadratic criterion. Therefore, the numerical results in the third column of Table

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<sup>16</sup> Second-order model versions are computed via the perturbation method and computer code developed by Schmitt-Grohé and Uribe (2004b). Our numerical implications remain robust if the fixed-point perturbation algorithm by Collard and Juillard (2001) is applied instead.

<sup>17</sup> We obtain the same welfare orderings with the linear quadratic welfare measure.



1 resemble the previously explained outcomes of the second column. This does not only imply the inaccuracy of this method, but also of the one proposed by Kim and Kim (2003, 2006). This is because the latter only comprises a linear quadratic function to measure welfare, which contains less second-order information than the purely quadratic approach.

We move on by examining the most common application in the literature, namely the purely quadratic approach advocated by Benigno and Woodford (2004, 2006), Kim and Kim (2006), Woodford (1999, 2003), and many others. Until now, the purely quadratic welfare loss function (48) has only served as a welfare measure and not as a target. However, the typical procedure is to minimize this equation subject to the log-linearized equations that characterize the model economy in order to obtain the monetary policy under timeless perspective commitment<sup>18</sup>. An overview of the log-linear model is provided in the appendix section A.3. This procedure is absent when assuming a Taylor-type policy since we do not want to consider optimal simple rules, but only a calibrated ad hoc interest rate rule. However, welfare measurement for both policies still takes place by means of the purely quadratic criterion. We may anticipate in advance that the purely quadratic approach yields the same welfare implications as in the approximated approach presented above. This is because the procedure delivers first-order conditions to the minimization problem that represent a local linear approximation of the solution of the exact problem. The numerical results in Table 2 corroborate this fact since the relative welfare measures still imply an overall welfare dominance of the timeless policy across all persistent shock impulses. This leaves the welfare rankings from the last two columns of Table 1 unchanged. Therefore, the purely quadratic approach is not able to reproduce the welfare orderings of the Ramsey approach. We have to conclude that this method is inappropriate for normative policy analysis in our model context.

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<sup>18</sup> This procedure differs from the above in that we do not undertake approximations of the Ramsey optimality conditions, but only of the structural model equations.

The numerical results derived above shed a negative light on approximated techniques that are supposed to be accurate for welfare orderings. Most interestingly, the well-accepted purely quadratic approach is unsuccessful. Note all approaches share an approximation degree not exceeding the order of two. Among other authors, Collard and Juillard (2001) find in a stochastic environment that second-order expansions are more efficient than standard linear approximations because they are able to account for higher order moments of the probability distribution. The model used is a closed-form non-linear framework formulated by Burnside (1998). However, they also uncover on grounds of their second-order perturbation method that approximation procedures perform poorly for a high volatility of shock innovations and a high shock persistence, as well as for a high curvature of the utility function. Most importantly, the potential gains of moving to even higher orders of approximation are found to be large. This implies that in some cases higher-order approximations like e.g. kurtosis are urgently needed in order to achieve accurate approximations. The phenomenon of needing higher order terms is found to be very critical when dealing with highly autocorrelated shocks. We therefore infer that the approximated environments presented here are generally not suitable for comparative welfare analysis, as it may be the case that higher order moments are needed to exploit informational contents stemming from the model non-linearities. Since the number of moments needed to gain true welfare insights depends on the underlying model economy and its parameter values, the researcher is confronted with uncertainty regarding the correct degree of approximation. This uncertainty can be avoided by relying exclusively on the Ramsey approach.

Having discarded the approximated settings for accurate welfare rankings, we now undertake a comparison of their business cycle implications. This is simply visualized by the time development of core variables following a one percent shock decrease of the substitution elasticity between bank loan types  $\zeta_t$  relative to its steady state value  $\zeta$ . The underlying monetary strategy is given by the timeless perspective policy under commitment in order to compare the time path implications of the non-linear Ramsey-type deterministic economy to its first and second-order stochastic description. In addition, we compute the time paths resulting from the purely quadratic

procedure. Besides from obtaining the typical responses of the monetary authority to an inflationary cost-push shock, Figure 1 shows that the time paths of core variables are nearly identical<sup>19</sup>. Schmitt-Grohé and Uribe (2004b) state that if the shocks driving fluctuations are small and an interior solution exists, first and second-order approximations provide adequate answers to questions such as local existence, determinacy of equilibrium, and the size of the second moments of endogenous variables. Williams (2004) shows by using convergence methods in a stochastic growth model that trajectories for e.g. capital converge to the corresponding trajectories of a deterministic setting if the standard deviation of stochastic technology shocks converges asymptotically to zero. This provides analytic, theoretically justified approximations for stochastic models with small noise. For the sake of completeness, we also let the substitution elasticity between loan types  $\zeta_t$  decrease by ten percent relative to its steady state value  $\zeta$  in the shock period. As expected, the stochastic time paths tend to drift away from the deterministic variable developments<sup>20</sup>. We are therefore able to recommend the approximated methods examined in this paper when it comes to describe aggregate fluctuations triggered by small shock impulses that are stochastic.

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<sup>19</sup> See section C for a list of figures.

<sup>20</sup> Interestingly, the second-order time paths are outperformed by the first-order model approximation. This counterintuitive result does not hold for the one percent shock if one zooms into time paths of Figure 1.

## 6 Conclusions

We compared the accuracy of widely approved methods for normative policy exercises. This is done by means of Taylor rules and the optimal policy under timeless perspective commitment. As both policy rules are suboptimal with respect to the time zero welfare target, any of these strategies may deliver superior welfare results. As in Winkler (2009), the Ramsey approach uncovers a welfare outperformance of Taylor rules for persistent cost-push shocks. Conversely, apparently suitable approximation procedures are not able to replicate this result since our numerical simulations always display the welfare dominance of the optimal monetary policy under timeless perspective commitment. In particular, the widely applied and advocated purely quadratic approach fails to reproduce the welfare orderings resulting in the non-linear model. This would partly explain the misperception in the literature about the overall welfare dominance of the timeless perspective commitment policy when considering the class of time-invariant rules.

The welfare analysis methods in approximated environments only consider moments up to first and second-order. However, the approximation order needed for accurate welfare analyses depends on the specific non-linearities of the underlying model. Furthermore, the approximation and welfare measuring accuracy depend on the values taken by the model parameters. This exposes researchers to uncertainty concerning the correct order of approximation. We strongly recommend the non-linear Ramsey-type setting when working on welfare comparisons across suboptimal policy strategies. Approximated settings are found to be useful only for the characterization of macroeconomic fluctuations in the presence of small shock disturbances.

## A Appendix

### A.1 The Model in Terms of Non-Linear Equations

$$Y_t = A_t N_t^{1-\alpha} \quad , \quad 0 < \alpha < 1 \quad (57)$$

$$MC_t = \left( \frac{1-\nu}{1-\alpha} \right) \frac{W_t}{P_t} R_t^L Y_t^{\frac{\alpha}{1-\alpha}} A_t^{-\frac{1}{1-\alpha}} \quad (58)$$

$$MRS_t = C_t^\sigma N_t^\eta \quad (59)$$

$$C_t^{-\sigma} = \beta R_t^d E_t \left[ C_{t+1}^{-\sigma} \frac{1}{\pi_{t+1}^p} \right] \quad (60)$$

$$\begin{aligned} \left( \frac{R_t^L}{R_{t-1}^L} - 1 \right) \left( \frac{R_t^L}{R_{t-1}^L} \right) &= \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{R_{t+1}^L}{R_t^L} - 1 \right) \left( \frac{R_{t+1}^L}{R_t^L} \right) \right] \\ &+ \frac{N_t \zeta_t}{\psi_r} \frac{W_t}{P_t} \left( R_t^M - \frac{R_t^L}{\Xi_t} \right) \end{aligned} \quad (61)$$

$$\begin{aligned} \pi_t^p (\pi_t^p - \pi^p) &= \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^p - \pi^p) \pi_{t+1}^p \right] \\ &+ \frac{Y_t \epsilon_{p,t}}{\psi_p} \left( MC_t - \frac{1}{\mu_{p,t}} \right) \end{aligned} \quad (62)$$

$$\begin{aligned} \pi_t^w (\pi_t^w - \pi^w) &= \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^w - \pi^w) \pi_{t+1}^w \right] \\ &+ \frac{N_t (\epsilon_{w,t} - 1)}{\psi_w} \left( MRS_t \mu_{w,t} - \frac{W_t}{P_t} \right) \end{aligned} \quad (63)$$

$$Y_t = C_t + \frac{\psi_p}{2} (\pi_t^p - \pi^p)^2 + \frac{\psi_w}{2} (\pi_t^w - \pi^w)^2 + \frac{\psi_r}{2} \left( \frac{R_t^L}{R_{t-1}^L} - 1 \right)^2 \quad (64)$$

$$\frac{W_t}{P_t} = \frac{W_{t-1}}{P_{t-1}} \frac{\pi_t^w}{\pi_t^p} \quad (65)$$

$$\Xi_t = \left( \frac{\zeta_t}{\zeta_t - 1} \right) \quad (66)$$

$$\mu_{p,t} = \left( \frac{\epsilon_{p,t}}{\epsilon_{p,t} - 1} \right) \quad (67)$$

$$\mu_{w,t} = \left( \frac{\epsilon_{w,t}}{\epsilon_{w,t} - 1} \right) \quad (68)$$

$$\left( \frac{\zeta_t}{\zeta} \right) = \left( \frac{\zeta_{t-1}}{\zeta} \right)^{\rho_r} \exp \{e_{r,t}\} \quad , \quad 0 \leq \rho_r < 1 \quad (69)$$

$$\left( \frac{\epsilon_{p,t}}{\epsilon_p} \right) = \left( \frac{\epsilon_{p,t-1}}{\epsilon_p} \right)^{\rho_p} \exp \{e_{p,t}\} \quad , \quad 0 \leq \rho_p < 1 \quad (70)$$

$$\left( \frac{\epsilon_{w,t}}{\epsilon_w} \right) = \left( \frac{\epsilon_{w,t-1}}{\epsilon_w} \right)^{\rho_w} \exp \{e_{w,t}\} \quad , \quad 0 \leq \rho_w < 1 \quad (71)$$

$$\left( \frac{A_t}{A} \right) = \left( \frac{A_{t-1}}{A} \right)^{\rho_a} \exp \{e_{a,t}\} \quad , \quad 0 \leq \rho_a < 1 \quad (72)$$

## A.2 The Model Steady State

The stationary equilibrium of the dynamic model can be found by dropping all time indices and inserting the resulting relationships into one another. This is done until all endogenous variables solely depend on model parameters. We assume no trend inflation, which is equivalent to assuming inflation rates equal to zero or gross inflation rates equal to one:

$$\pi_p = \pi_w = 1 \quad (73)$$

Note that the model inefficiencies stemming from quadratic adjustment costs vanish in the long run. Therefore, the resource constraint (36) implies:

$$Y = C \quad (74)$$

From the Euler equation (34), it becomes clear that:

$$R = R^M = R^d = R^L = \frac{\pi_p}{\beta} = \frac{\pi_w}{\beta} = \frac{1}{\beta} \quad (75)$$

The steady state markups in terms of elasticities and the steady state real marginal cost are given by:

$$\mu_p = \frac{\epsilon_p}{\epsilon_p - 1} \quad (76) \quad \mu_w = \frac{\epsilon_w}{\epsilon_w - 1} \quad (77)$$

$$\Xi = \frac{\zeta}{\zeta - 1} \quad (78) \quad MC = \frac{1}{\mu_p} \quad (79)$$

Finally, by inserting the time-independent relationships into one another one arrives at the following long-run variable representations:

$$Y = \left( \frac{(1 - \alpha) \beta}{\mu_p \mu_w \Xi (1 - \nu)} \right)^{\frac{1 - \alpha}{\sigma(1 - \alpha) + \eta + \alpha}} \quad (80)$$

$$N = \left( \frac{(1 - \alpha) \beta}{\mu_p \mu_w \Xi (1 - \nu)} \right)^{\frac{1}{\sigma(1 - \alpha) + \eta + \alpha}} \quad (81)$$

$$\frac{W}{P} = \mu_w \left( \frac{(1 - \alpha) \beta}{\mu_p \mu_w \Xi (1 - \nu)} \right)^{\frac{\sigma(1 - \alpha) + \eta}{\sigma(1 - \alpha) + \eta + \alpha}} \quad (82)$$

$$V = \left( \frac{1}{1 - \beta} \right) \left( \frac{C^{1 - \sigma}}{1 - \sigma} - \frac{N^{1 + \eta}}{1 + \eta} \right) \quad (83)$$

The steady state variable in equation (83) stems from the objective function of the Ramsey planner and can be obtained by writing (46) in recursive form.

### A.3 The Linearized Model in Terms of Log-Deviations

The purely quadratic approach requires the log-linear version of the non-linear model, where  $\left(\frac{X_t - X}{X}\right) \approx \log(X_t) - \log(X) \equiv \hat{X}_t$ :

$$\hat{Y}_t = \hat{A}_t + (1 - \alpha) \hat{N}_t \quad , \quad 0 < \alpha < 1 \quad (84)$$

$$\hat{M}C_t = \hat{W}_t - \hat{P}_t + \hat{R}_t^L + \left(\frac{\alpha}{1 - \alpha}\right) \hat{Y}_t - \left(\frac{1}{1 - \alpha}\right) \hat{A}_t \quad (85)$$

$$M\hat{R}S_t = \sigma \hat{C}_t + \eta \hat{N}_t \quad (86)$$

$$\hat{C}_t = E_t[\hat{C}_{t+1}] - \frac{1}{\sigma} (\hat{R}_t^d - E_t[\hat{\pi}_{t+1}^p]) \quad (87)$$

$$\hat{\pi}_t^r = \beta E_t[\hat{\pi}_{t+1}^r] + \frac{N\zeta W}{\psi_r P} R^M (\hat{R}_t^M - \hat{R}_t^L + \hat{\varepsilon}_t) \quad (88)$$

$$\hat{\pi}_t^r = \hat{R}_t^L - \hat{R}_{t-1}^L \quad (89)$$

$$\hat{\pi}_t^p = \beta E_t[\hat{\pi}_{t+1}^p] + \frac{Y}{\psi_p} \frac{\epsilon_p}{\mu_p} (\hat{M}C_t + \hat{\mu}_{p,t}) \quad (90)$$

$$\hat{\pi}_t^w = \beta E_t[\hat{\pi}_{t+1}^w] + \frac{N(\epsilon_w - 1)}{\psi_w} \frac{W}{P} (\hat{\mu}_{w,t} + M\hat{R}S_t - (\hat{W}_t - \hat{P}_t)) \quad (91)$$

$$\hat{Y}_t = \hat{C}_t \quad (92)$$

$$\hat{W}_t - \hat{P}_t = \hat{W}_{t-1} - \hat{P}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t^p \quad (93)$$



$$\hat{\Xi}_t = -\frac{1}{\zeta - 1} \hat{\zeta}_t \quad (94)$$

$$\hat{\mu}_{p,t} = -\frac{1}{\epsilon_p - 1} \hat{\epsilon}_{p,t} \quad (95)$$

$$\hat{\mu}_{w,t} = -\frac{1}{\epsilon_w - 1} \hat{\epsilon}_{w,t} \quad (96)$$

$$\hat{\zeta}_t = \rho_r \hat{\zeta}_{t-1} + e_{r,t} \quad , \quad 0 \leq \rho_r < 1 \quad (97)$$

$$\hat{\epsilon}_{p,t} = \rho_p \hat{\epsilon}_{p,t-1} + e_{p,t} \quad , \quad 0 \leq \rho_p < 1 \quad (98)$$

$$\hat{\epsilon}_{w,t} = \rho_w \hat{\epsilon}_{w,t-1} + e_{w,t} \quad , \quad 0 \leq \rho_w < 1 \quad (99)$$

$$\hat{A}_t = \rho_a \hat{A}_{t-1} + e_{a,t} \quad , \quad 0 \leq \rho_a < 1 \quad (100)$$

## A.4 The Ramsey Problem

The first-order conditions describing the response of the Ramsey planner to orthogonal shocks are derived by differentiating the following Lagrange equation with respect to the control variables  $C_t, N_t, \pi_t^p, \pi_t^w, \frac{R_t^L}{R_{t-1}^L}$ , and  $\frac{W_t}{P_t}$ :

$$\begin{aligned}
 L\left(C_t, N_t, \pi_t^p, \pi_t^w, \frac{R_t^L}{R_{t-1}^L}, \frac{W_t}{P_t}\right) &= E_t \sum_{k=0}^{\infty} \beta^k \left[ \left(\frac{1}{1-\sigma}\right) C_{t+k}^{1-\sigma} - \left(\frac{1}{1+\eta}\right) N_{t+k}^{1+\eta} \right] \\
 &+ E_t \sum_{k=0}^{\infty} \beta^k \lambda_{1,t+k+1} \left[ A_{t+k} N_{t+k}^{1-\alpha} - C_{t+k} - \frac{\psi_p}{2} (\pi_{t+k}^p - \pi^p)^2 \right. \\
 &\quad \left. - \frac{\psi_w}{2} (\pi_{t+k}^w - \pi^w)^2 - \frac{\psi_r}{2} \left(\frac{R_{t+k}^L}{R_{t+k-1}^L} - 1\right)^2 \right] \\
 &+ E_t \sum_{k=0}^{\infty} \beta^k \lambda_{2,t+k+1} \left[ \pi_{t+k}^p (\pi_{t+k}^p - \pi^p) - \beta \left(\frac{C_{t+k+1}}{C_{t+k}}\right)^{-\sigma} (\pi_{t+k+1}^p - \pi^p) \pi_{t+k+1}^p \right. \\
 &\quad \left. - \frac{A_{t+k} N_{t+k}^{1-\alpha} \epsilon_{p,t+k}}{\psi_p} \left( \left(\frac{1-\nu}{1-\alpha}\right) \frac{W_{t+k}}{P_{t+k}} R_{t+k}^L N_{t+k}^\alpha \frac{1}{A_{t+k}} - \frac{1}{\mu_{p,t+k}} \right) \right] \\
 &+ E_t \sum_{k=0}^{\infty} \beta^k \lambda_{3,t+k+1} \left[ \pi_{t+k}^w (\pi_{t+k}^w - \pi^w) - \beta \left(\frac{C_{t+k+1}}{C_{t+k}}\right)^{-\sigma} (\pi_{t+k+1}^w - \pi^w) \pi_{t+k+1}^w \right. \\
 &\quad \left. - \frac{N_{t+k} (\epsilon_{w,t+k} - 1)}{\psi_w} \left( \mu_{w,t+k} N_{t+k}^\eta C_{t+k}^\sigma - \frac{W_{t+k}}{P_{t+k}} \right) \right] \\
 &+ E_t \sum_{k=0}^{\infty} \beta^k \lambda_{4,t+k+1} \left[ \frac{W_{t+k}}{P_{t+k}} - \frac{W_{t+k-1}}{P_{t+k-1}} \frac{\pi_{t+k}^w}{\pi_{t+k}^p} \right] \\
 &+ E_t \sum_{k=0}^{\infty} \beta^k \lambda_{5,t+k+1} \left[ \left(\frac{R_{t+k}^L}{R_{t+k-1}^L} - 1\right) \left(\frac{R_{t+k}^L}{R_{t+k-1}^L}\right) \right. \\
 &\quad \left. - \beta \left(\frac{C_{t+k+1}}{C_{t+k}}\right)^{-\sigma} \left(\frac{R_{t+k+1}^L}{R_{t+k}^L} - 1\right) \left(\frac{R_{t+k+1}^L}{R_{t+k}^L}\right) \right. \\
 &\quad \left. - \frac{N_{t+k} \zeta_{t+k}}{\psi_r} \frac{W_{t+k}}{P_{t+k}} \left( R_{t+k}^M - \frac{R_{t+k}^L}{\Xi_{t+k}} \right) \right]
 \end{aligned} \tag{101}$$

## A.5 Second-Order Welfare Measure

The starting point for the derivation of the approximated welfare measure is the periodical Dixit-Stiglitz aggregate utility function:

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \quad (102)$$

Inserting the aggregate resource constraint (36) and using the production function (5) in its aggregate version yields:

$$U_t = \frac{\left( Y_t - \frac{\psi_p}{2} (\pi_t^p - \pi^p)^2 - \frac{\psi_w}{2} (\pi_t^w - \pi^w)^2 - \frac{\psi_r}{2} (\pi_t^r - \pi^r)^2 \right)^{1-\sigma}}{1-\sigma} - \frac{Y_t^{\frac{1+\eta}{1-\alpha}} A_t^{-\frac{1+\eta}{1-\alpha}}}{1+\eta} \quad (103)$$

Note that for the change of the gross loan rate we simply wrote:  $\pi_t^r = \frac{R_t^L}{R_{t-1}^L}$ . The next step is to take a second-order approximation of equation (103). After applying the second-order Taylor expansion, we make use of the fact that:  $\left( \frac{X_t - X}{X} \right) \approx \log(X_t) - \log(X) + \frac{1}{2} (\log(X_t) - \log(X))^2 = \hat{X}_t + \frac{1}{2} (\hat{X}_t)^2$ . By doing this we arrive at:

$$\begin{aligned} U_t - U \approx & Y^{1-\sigma} \left[ \hat{Y}_t + \frac{1}{2} (1-\sigma) (\hat{Y}_t)^2 - \frac{\psi_p}{2Y} (\hat{\pi}_t^p)^2 - \frac{\psi_w}{2Y} (\hat{\pi}_t^w)^2 - \frac{\psi_r}{2Y} (\hat{\pi}_t^r)^2 \right] \\ & + \left( \frac{1}{1-\alpha} \right) Y^{\frac{1+\eta}{1-\alpha}} \left[ \hat{Y}_t - \hat{A}_t + \frac{1}{2} \left( \frac{1+\eta}{1-\alpha} \right) (\hat{Y}_t)^2 \right. \\ & \left. + \frac{1}{2} \left( \frac{1+\eta}{1-\alpha} \right) (\hat{A}_t)^2 - \left( \frac{1+\eta}{1-\alpha} \right) \hat{Y}_t \hat{A}_t \right] \end{aligned} \quad (104)$$

From the steady state real output relationship (80):

$$\left( \frac{1}{1-\alpha} \right) Y^{\frac{1+\eta}{1-\alpha}} = \beta \left( \frac{1}{\mu_p \mu_w \Xi (1-\nu)} \right) Y^{1-\sigma} = \beta (1-\varphi) Y^{1-\sigma} \quad (105)$$

Note that  $\varphi = 1 - \frac{1}{\mu_p \mu_w \Xi (1-\nu)}$  comprises all monopolistic distortions of the economy at the steady state. They are fully at work for  $\nu = 0$ . Inserting this equation into our approximated utility, multiplying all terms out, neglecting a host of policy-independent terms, and using  $\varphi (\hat{Y}_t)^2 = \varphi (\hat{A}_t)^2 \approx 0$  for small distortions (see Woodford (1999)) we arrive at:

$$\begin{aligned}
 J_t \approx Y^{1-\sigma} \frac{1}{2} (\alpha + \eta + \sigma (1 - \alpha)) & \left[ \frac{\beta (1 + \eta) - (1 - \sigma) (1 - \alpha)}{(1 - \alpha) [\alpha + \eta + \sigma (1 - \alpha)]} (\hat{Y}_t)^2 \right. \\
 & - 2 \left( \frac{1 - \alpha - \beta (1 - \varphi) (1 - \alpha) + \beta (1 + \eta) \hat{A}_t}{(1 - \alpha) [\alpha + \eta + \sigma (1 - \alpha)]} \right) \hat{Y}_t \\
 & + \frac{\psi_p}{Y [\alpha + \eta + \sigma (1 - \alpha)]} (\hat{\pi}_t^p)^2 \\
 & + \frac{\psi_w}{Y [\alpha + \eta + \sigma (1 - \alpha)]} (\hat{\pi}_t^w)^2 \\
 & \left. + \frac{\psi_r}{Y [\alpha + \eta + \sigma (1 - \alpha)]} (\hat{\pi}_t^r)^2 \right]
 \end{aligned} \tag{106}$$

The efficient long-run real output in non-linear terms can be achieved by assuming a subsidy according to (56). Approximating the resulting relationship up to first-order around the undistorted steady state yields<sup>21</sup>:

$$\hat{Y}_t^{eff.} = \left( \frac{1 + \eta}{\alpha + \eta + \sigma (1 - \alpha)} \right) \hat{A}_t \tag{107}$$

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<sup>21</sup> Note that in contrast to Woodford (2003, chap.6), the presence of elasticity-driven markups render together with  $\varphi > 0$  different fluctuations of natural output:  $\hat{Y}_t^n = \left( \frac{1 + \eta}{\alpha + \eta + \sigma (1 - \alpha)} \right) \hat{A}_t - \left( \frac{1 - \alpha}{\alpha + \eta + \sigma (1 - \alpha)} \right) (\hat{\mu}_{p,t} + \hat{\mu}_{w,t} + \hat{R}_t^L + \hat{\Xi}_t)$ .

Augmenting equation (106) by the term in brackets in (107) and inserting the latter in order to eliminate the productivity variable  $\hat{A}_t$  gives a final expression for the approximated welfare measure in period  $t$ :

$$\begin{aligned}
 J_t^{abs.} \approx & \underbrace{\frac{Y^{1-\sigma}}{2} \frac{\beta(1+\eta) - (1-\sigma)(1-\alpha)}{1-\alpha}}_{=\Omega} \left( \hat{Y}_t - \underbrace{\frac{\beta(\alpha + \eta + \sigma(1-\alpha))}{\beta(1+\eta) - (1-\sigma)(1-\alpha)}}_{=\gamma_1} \hat{Y}_t^{eff.} \right. \\
 & \left. - \underbrace{\frac{(1-\alpha)(1-\beta(1-\varphi))}{\beta(1+\eta) - (1-\sigma)(1-\alpha)}}_{=\gamma_2} \right)^2 \quad (108) \\
 & + \underbrace{\frac{Y^{-\sigma}\psi_p}{2}}_{=\Gamma_1} (\hat{\pi}_t^p)^2 + \underbrace{\frac{Y^{-\sigma}\psi_w}{2}}_{=\Gamma_2} (\hat{\pi}_t^w)^2 + \underbrace{\frac{Y^{-\sigma}\psi_r}{2}}_{=\Gamma_3} (\hat{\pi}_t^r)^2
 \end{aligned}$$

## B List of Tables

Impact Shock & Policy	Original	First-Order	Second-Order
$\zeta_{t=0}, \hat{\zeta}_{t=0} = -1\%$ ( $\rho_r = 0.95$ )			
<b>Timeless Perspective Policy</b>	– [–137.6868]	– [0.000073419]	– [0.000073419]
<b>Traditional Taylor Rule</b>	–0.0031% [–137.6825]	526.7710% [0.00038675]	526.7710% [0.00038675]
<b>Taylor Rule with Smoothing</b>	–0.0026% [–137.6831]	394.6254% [0.00028973]	394.6254% [0.00028973]
$\epsilon_{p,t=0}, \hat{\epsilon}_{p,t=0} = -1\%$ ( $\rho_p = 0.95$ )			
<b>Timeless Perspective Policy</b>	– [–137.6828]	– [0.000018087]	– [0.000018087]
<b>Traditional Taylor Rule</b>	–0.0014% [–137.6808]	534.5054% [0.000096676]	534.5054% [0.000096676]
<b>Taylor Rule with Smoothing</b>	–0.0012% [–137.6812]	401.4928% [0.000072618]	401.4928% [0.000072618]
$\epsilon_{w,t=0}, \hat{\epsilon}_{w,t=0} = -1\%$ ( $\rho_w = 0.95$ )			
<b>Timeless Perspective Policy</b>	– [–137.6807]	– [0.000015147]	– [0.000015147]
<b>Traditional Taylor Rule</b>	–0.000030964% [–137.6807]	414.6894% [0.000062813]	414.6894% [0.000062813]
<b>Taylor Rule with Smoothing</b>	–0.000018782% [–137.6807]	325.0017% [0.000049228]	325.0017% [0.000049228]
$A_{t=0}, \hat{A}_{t=0} = +1\%$ ( $\rho_a = 0.95$ )			
<b>Timeless Perspective Policy</b>	– [–137.4951]	– [0.00048531]	– [0.00048531]
<b>Traditional Taylor Rule</b>	0.0562% [–137.5724]	5213.2% [0.0253]	5213.2% [0.0253]
<b>Taylor Rule with Smoothing</b>	0.0514% [–137.5657]	3997.4% [0.0194]	3997.4% [0.0194]

Table 1: Absolute and relative welfare implied by the original and the approximated first-order conditions of the Ramsey problem due to a persistent one percent relative change of the respective shock variable at period  $t = 0$ . Values enclosed by [...] denote absolute welfare levels as defined in the text.

<b>Impact Shock (<math>\rho_i = 0.95</math>)</b>	<b>TPP</b>	<b>TTR</b>	<b>TRS</b>
$\zeta_{t=0}, \hat{\zeta}_{t=0} = -1 \%$	- [0.000071409]	541.5984% [0.00038675]	405.7332% [0.00028973]
$\epsilon_{p,t=0}, \hat{\epsilon}_{p,t=0} = -1 \%$	- [0.000017702]	546.1304% [0.000096676]	410.2248% [0.000072618]
$\epsilon_{w,t=0}, \hat{\epsilon}_{w,t=0} = -1 \%$	- [0.00024671]	1610.3% [0.000037035]	1262.0% [0.000038058]
$A_{t=0}, \hat{A}_{t=0} = +1 \%$	- [0.00047509]	5325.3% [0.0253]	4083.4% [0.0194]

Table 2: Absolute and relative welfare underlying the purely quadratic approach due to a persistent one percent relative change of the respective shock variable at period  $t = 0$ . TPP: Timeless Perspective Policy, TTR: Traditional Taylor Rule, TRS: Taylor Rule with Smoothing. Values enclosed by [...] denote absolute welfare levels as defined in the text.

## C List of Figures

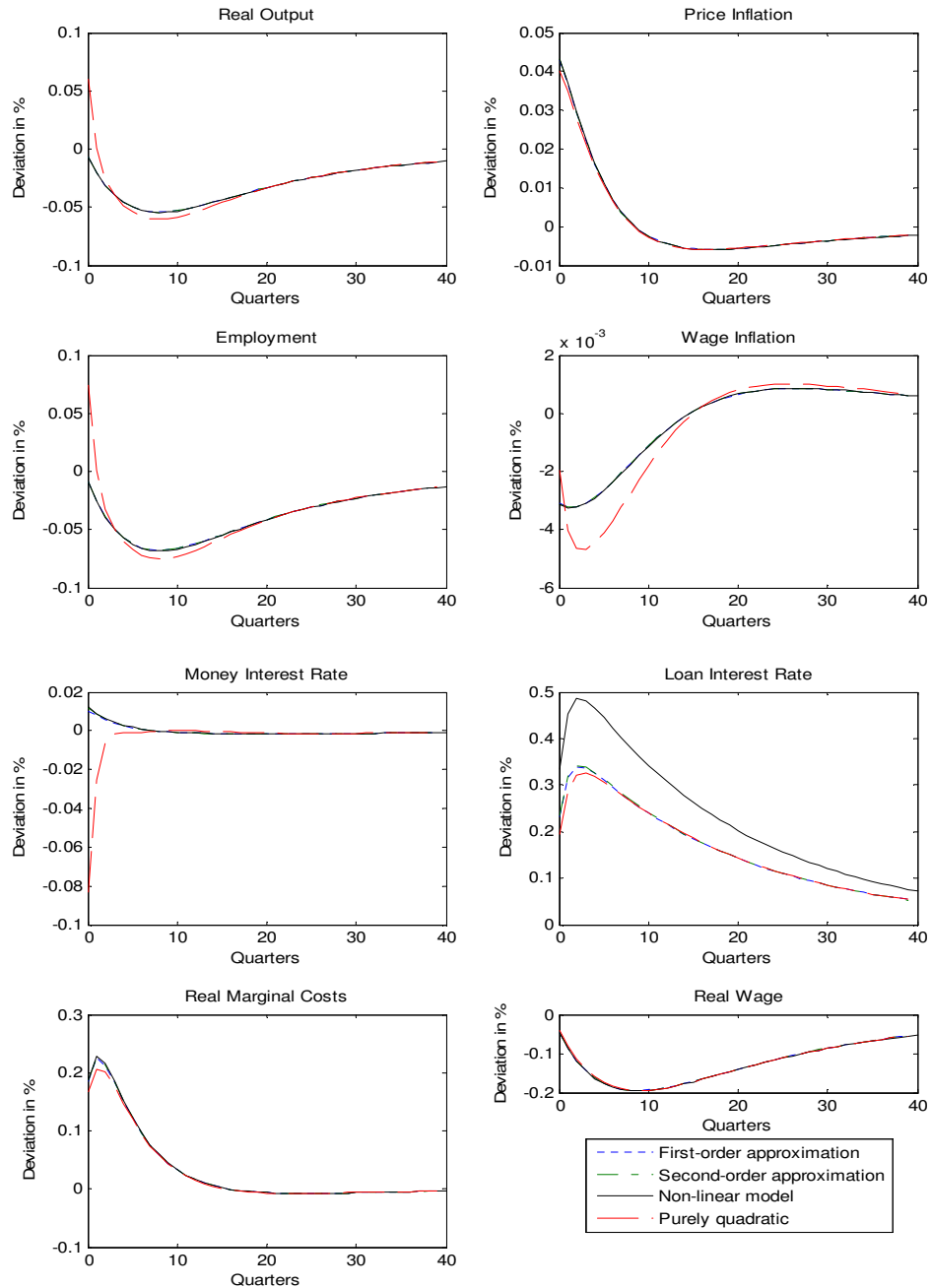


Figure 1: Responses of core variables over time in the original and the approximated model after a one percent decrease in  $(\zeta_t - \zeta)/\zeta$  and  $\hat{\zeta}_t$  at the shock period  $t=0$



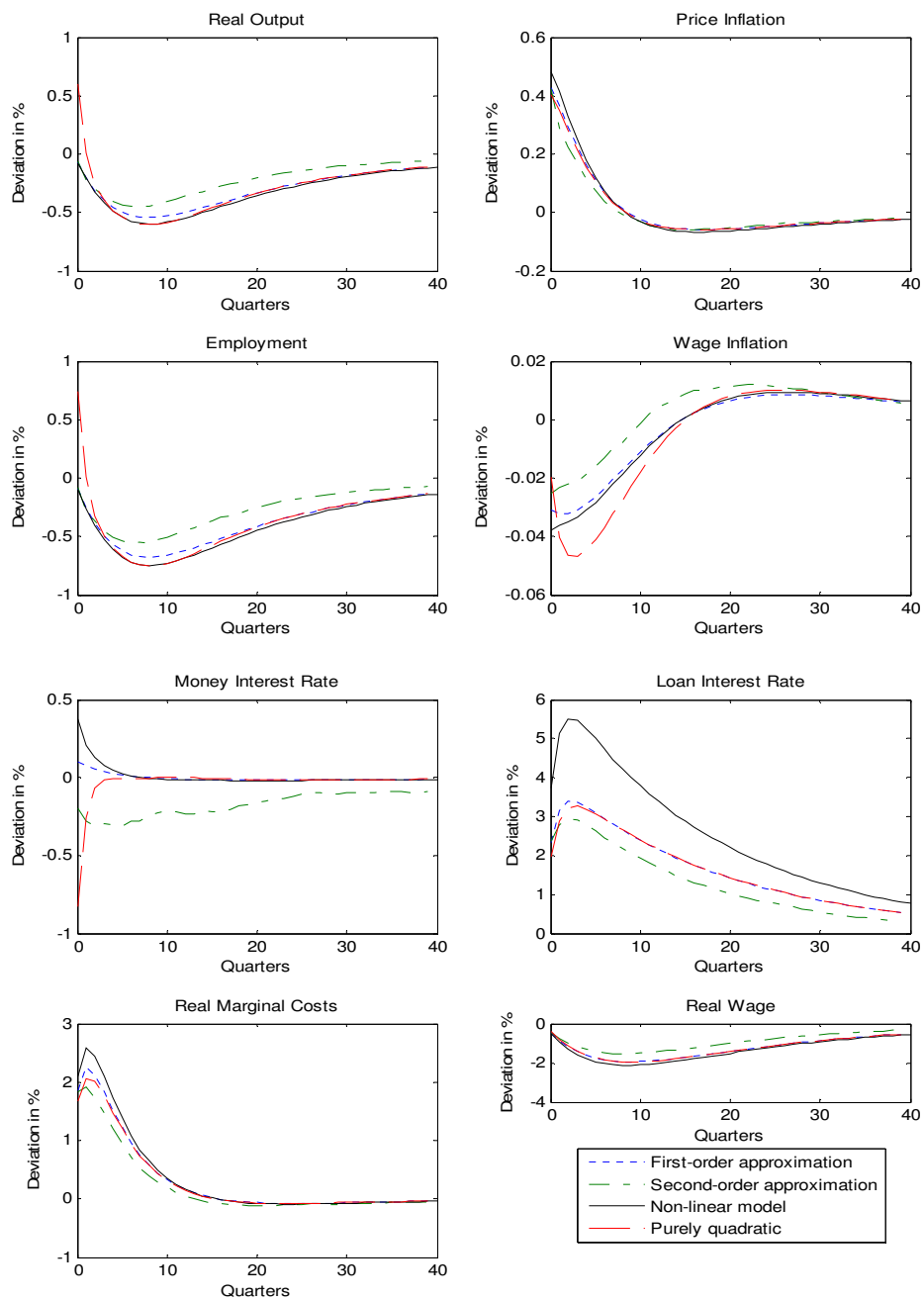


Figure 2: Responses of core variables over time in the original and the approximated model after a ten percent decrease in  $(\zeta_t - \zeta)/\zeta$  and  $\hat{\zeta}_t$  at the shock period  $t=0$

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