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Credit Constraints, Idiosyncratic Risks, and the Wealth Distribution in a Heterogeneous Agent Model

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Abstract

This paper examines the effects of credit market imperfections and idiosyncratic risks on occupational choice, capital accumulation, as well as on the income and wealth distribution in a two sector heterogeneous agent general equilibrium model. Workers and firm owners are subject to idiosyncratic shocks. Entrepreneurship is the riskier occupation. Compared to an economy with perfect capital markets, we find for the case of serially correlated shocks that more individuals choose the entrepreneurial profession in the presence of credit constraints, and that the fluctuation between occupations increases too. Workers and entrepreneurs with high individual productivity tend to remain in their present occupation, whereas low productivity individuals are more likely to switch between professions. Interestingly, these results reverse if we assume *iid* shocks, thus indicating that the nature of the underlying shocks plays an important role for the general equilibrium effects. In general, the likelihood of entrepreneurship increases with individual wealth.

Keywords: DSGE model, wealth distribution, occupational choice, credit constraints
JEL–classification: C68, D3 , D8, D9, G0, J24

1 Introduction

This paper examines the effects of credit market imperfections and idiosyncratic risks on occupational choice, capital accumulation, as well as on the income and wealth distribution. Our analysis contributes to recent literature on dynamic stochastic heterogeneous agent general equilibrium models concerned with risk and distributional dynamics, for instance, Quadrini (2000), Meh (2005), Boháček (2006) and Cagetti and De Nardi (2006).

We develop a model which combines the features of an Aiyagari (1994)–type economy with occupational choice under risk à la Kihlstrom and Laffont (1979) and Kanbur (1979*a*,*b*), and the two–sector approach of Romer (1990), but without endogenous growth. In each period of time, the risk–averse agents choose between between two alternative occupations. They either set up an enterprise in the intermediate goods industry which is characterized by monopolistic competition. Or, they supply their labor endowment to the production of a final good in a perfectly competitive market. Producers of the final good use capital and labor inputs, and differentiated varieties of the intermediate good. All households are subject to an income risk. Managerial ability and productivity as a worker follow independent random processes. Entrepreneurial activity is rewarded with a higher expected income.[1] Similar to Bewley (1977) and Lucas (1978), there is no aggregate risk.

The economic performance in the intermediate goods industry crucially depends on two factors: uncertainty and credit constraints. Business owners face an firm–specific productivity shock, and there are no markets available for pooling the idiosyncratic risks. Physical capital is the single input factor in the intermediate goods industry. Entrepreneurs maximize their profits if their business operates at the optimal firm size. For an individual wealth too small to maintain the optimal firm size, the firm–owner would want to borrow the remaining amount on the credit market, where he might be subject to financial constraints. If the entrepreneur is wealthy enough, he operates his business at the profit–maximizing level and supplies the rest of his wealth to the capital market. To this end, our approach draws a simple picture of the empirical result, stated by Heaton and Lucas (2000), that the entrepreneurial households' business wealth on average constitutes a relevant fraction of their total wealth. There is no further portfolio choice in our framework.

Capital accumulation plays a twofold role in the context outlined above: On the one hand, it endows individuals with the wealth necessary to set–up and operate a firm. On the other hand, buffer–stock saving provides a self–insurance on intertemporal markets against the non–diversifiable income risk. Accordingly, we find that wealthier households are more likely to be members of the entrepreneurial class than poorer ones and there is a marked concentration of wealth in the hands

[1] See also Clemens (2006*a*,*b*) and Clemens and Heinemann (2006) for entrepreneurial risk–taking in a general equilibrium context.

of entrepreneurs which is consistent with recent empirical findings (cf. Quadrini, 1999; Holtz-Eakin *et al.*, 1994*a*). Upward mobility of entrepreneurs in our model is primarily accumulation driven. The riskiness of entrepreneurial incomes looses its importance for occupational choice once the household's income share generated from profits declines relative to his capital income. Nevertheless, in accordance with Hamilton (2000), many entrepreneurs of our model enter and persist in business despite the fact that they have lower initial earnings than average wage incomes.

We are especially interested in the question of how tightening the credit constraints affects the macroeconomic general equilibrium regarding the sectoral allocation of capital and labor, factor prices, the income and wealth distribution, occupational choice as well as the between–group mobility of households. Regarding empirical evidence, there seems to be a strong support for the hypothesis that capital market imperfections are an impediment to entrepreneurship even after controlling for entrepreneurial ability; see Evans and Leighton (1989), Evans and Jovanovic (1989), Holtz-Eakin *et al.* (1994*b*), Blanchflower and Oswald (1998), Moskowitz and Vissing-Jørgensen (2002), as well as Desai *et al.* (2003). Gentry and Hubbard (2004) point out that external financing has important implications for individual investment and saving. This evidence is challenged by Hurst and Lusardi (2004), who find that the likelihood of entering entrepreneurship relative to initial wealth is flat over a large range of the wealth distribution and increasing only for higher wealth levels of workers. Our model is capable of reproducing theses findings for the case of uncorrelated shocks.

The general equilibrium nature of our approach generates surprising and almost counter–intuitive results regarding the impact of credit constraints on occupational choice under risk. If the idiosyncratic risks follow autoregressive processes, more households choose the entrepreneurial profession in the constrained compared to the unconstrained economy which is accompanied by a reduction in the average firm size, both results contradicting theoretical findings by Cagetti and De Nardi (2006). Wealth inequality does not necessarily decline if we relax borrowing constraints. Additionally, we observe an increase in between–group mobility, if credit constraints become more binding. Workers and entrepreneurs with high individual productivity tend to remain in their present occupation, whereas low productivity individuals are more likely to switch between professions.

These results reverse completely, if we consider *iid* shocks to individual productivity. In this case, credit constraints actually are an impediment to entrepreneurship. Only the wealthy workers tend to switch between occupations and between–group mobility drops down sharply for an increase in the tightness of credit constraints. Independent of the persistence of the underlying shocks, the likelihood of entrepreneurship increases in individual wealth. Regarding the functional distribution of income, we find that credit constraints have a redistributive effect by raising the profit income share at the cost of capital incomes. The results indicate that the

stochastic nature of the underlying idiosyncratic shocks also plays an important role for the explanation of the general equilibrium effects of financial constraints and credit market imperfections.

Recent contributions in this area of research suffer from several shortcomings which our approach aims to overcome. In Quadrini (2000), occupational choice and the level of entrepreneurship is (more or less) entirely governed by the underlying productivity shocks. Boháček (2006) discusses a one–sector economy which does not allow for factor movements between industries and therefore neglects factor substitution. In our model, producers of the intermediate and the final good are subject to competition, especially with respect to capital demand. Our approach does not have fixed entry costs (in terms of discrete investment projects) of entrepreneurship as in Ghatak *et al.* (2001) or Cagetti and De Nardi (2006). Instead, we have an endogenously determined optimal firm size and no discontinuities in individual credit demand. Occupational choice, entrepreneurial activity and performance crucially depend on monopoly profits, market shares and relative factor scarcity in the two sectors of production. Also different to Cagetti and De Nardi (2006), the entrepreneurs of our economy are essential for aggregate output. As will become obvious below, the interdependence of sectors is important for the general equilibrium results on occupational choice, between–group mobility and the income and wealth distribution, and contribute to the explanation of the sometimes counter–intuitive effects of borrowing constraints as outlined above.

The paper is organized as follows: Section 2 develops the two–sector model. We describe the equilibrium associated with a stationary earnings and wealth distribution in Section 3 and present benchmark results on static efficiency in the perfect risk–pooling economy. Since the formal structure of the model does not allow for analytical solutions, we perform numerical simulations of a calibrated model in order to examine the general equilibrium effects of an increase in the tightness of credit constraints. Section 4 gives information on the calibration procedure and discusses the simulation results. Section 5 concludes. Technical details are relegated to the Appendix.

2 The Model

We consider a neoclassical growth model with two sectors of production. The final goods industry consists of a large number of perfectly competitive firms who hire capital and labor services and use an intermediate good in order to produce a homogeneous output which can be consumed or invested respectively. The intermediate good is produced under the regime of monopolistic competition. Each firm in the intermediate goods industry is owned and managed by an entrepreneur. Both sectors of production are essential.

Market activity in the intermediate goods industry is constrained. In order to run the business at the profit–maximizing firm size, entrepreneurs either possess sufficient wealth of their own, or they need to compensate for their lack of equity by borrowing on the credit market, where they might be subject to credit constraints. In the latter case, remaining in the entrepreneurial profession might not prove worthwhile.

The economy is populated by a continuum $[0,1]$ infinitely–lived households, each endowed with one unit of labor. In each period of time, individuals decide whether to become producers of the intermediate good or to supply their labor services to the production of the final good. Labor efficiency as well as entrepreneurial productivity are idiosyncratic random variables. Regarding the associated income risk, we assume that wage incomes are less risky than profit incomes. There is no aggregate risk.

With respect to the timing of events, we assume that individual occupational choice takes place behind a veil of ignorance regarding the realization of the idiosyncratic shock. Once the draw of nature has occurred, entrepreneurs as well as workers in the final goods sector know their individual productivity. Those monopolists who now discover their own wealth being too low to operate at the optimal firms size, will express their capital demand on the credit market, probably become subject to credit–constraints, and then start production. After labor and profit income is realized, the households decide on how much to consume and to invest. There is no capital income risk.

Final goods sector The representative firm of the final goods sector produces a homogeneous good Y using capital K_F and varieties of an intermediate good $x(i), i \in [0,\lambda]$ as inputs. Production in this sector takes place under perfect competition and the price of Y is normalized to unity. The production function is of the generalized CES–form[2]

$$Y = A\left(K_F^{\gamma} L^{1-\gamma}\right)^{1-\alpha} \int_0^{\lambda} x(i)^{\alpha}\, \mathrm{d}i\,, \qquad 0<\alpha<1, \quad 0<\gamma<1, \quad A>0\,. \tag{1}$$

Each type of intermediate good employed in the production of the final good is identified with one monopolistic producer in the intermediate goods sector. Consequently, the number of different types is identical with the population share of entrepreneurs in the population. The number of entrepreneurs is determined endogenously through occupational choices of the agents, which will be described below. Additive–separability of (1) in intermediate goods ensures that the marginal product of input i is independent of the quantity employed of $i' \neq i$. Intermediate goods are close but not perfect substitutes in production.

[2]All macroeconomic variables are time–dependent. For notational convenience, we will drop the explicit time–notation unless necessary.

The profit of the representative firm in the final goods sector, π^F, is given in each period by

$$\pi_F = Y - wL - (r+\delta)K_F - \int_0^\lambda p(i)\,x(i)\,\mathrm{d}\,i\,, \tag{2}$$

where $p(i)$ denotes the price of intermediate good i. We further assume physical capital to depreciate over time at the constant rate δ, such that the interest factor is given by $R = 1 + r - \delta$. Optimization yields the profit maximizing factor demands consistent with marginal productivity theory

$$K_F = (1-\alpha)\gamma\,\frac{Y}{r+\delta}\,, \tag{3}$$

$$L = (1-\alpha)(1-\gamma)\frac{Y}{w} \tag{4}$$

$$x(i) = K_F^{\gamma} L^{1-\gamma}\left(\frac{\alpha A}{p(i)}\right)^{1/(1-\alpha)}\,. \tag{5}$$

The monopolistic producer of intermediate good $x(i)$ faces the isoelastic demand function (5), the direct price elasticity of demand given by $-1/(1-\alpha)$. Condition (4) describes aggregate labor demand in efficiency units. Equation (3) is the final good sector demand for capital services.

Production in the intermediate goods sector The intermediate goods sector consists of the population fraction λ of entrepreneurs who self–employ their labor endowment by operating a monopolistic firm. Each firm produces a single variety i of the differentiated intermediate good by using capital services according to the identical constant returns to scale technology of the form

$$x(i) = B\,\theta(i)_e\,k(i)\,, \qquad B > 0\,. \tag{6}$$

Firms differ with respect to the realization of an idiosyncratic productivity shock $\theta(i)_e$ which is assumed to be non–diversifiable and uncorrelated across firms. We will give a more detailed description of the distributional properties of this shock below. Entrepreneurs hire capital after the draw of nature has occurred. The firm problem essentially is a static one. Under perfect competition of the capital market, the producer treats the rental rate to capital as exogenously given and maximizes his profit

$$\pi(i) = \pi\left(k(i), \theta(i)_e\right) = p(i)\,x(i) - (r+\delta)\,k(i)\,. \tag{7}$$

Utilizing the demand function for intermediate good type–i, (5), and the production technology (6), the optimal firm decision can be expressed in terms of the optimal firm size as a function of capital input, which is given by:

$$k(i)^* = L\,(B\,\theta(i)_e)^{\frac{\alpha}{1-\alpha}}\left(\frac{\gamma w}{(1-\gamma)(r+\delta)}\right)^{\gamma}\,. \tag{8}$$

Because capital demand takes place after the draw of nature has occurred, there is no individual capital risk and no under–employment of input factors. The optimal firm size increases with random individual productivity $\theta(i)_e$, such that more productive business owners demand more capital on the capital market. Labor input in efficiency units determines the optimal firm size by means of the demand function for intermediate good type i. Aggregate employment is a weighted average and depends on the size of the labor force $1-\lambda$, i.e. the population fraction of agents choosing the occupation of a worker, and the idiosyncratic shock on labor productivity θ_w. The larger the labor force $1-\lambda$, the higher—*ceteris paribus*—will be aggregate employment L. This goes along with fewer monopolists in the intermediate goods industry, less competition, and a larger market share, as measured by the optimal firm size.

Capital market and credit constraints Let $k(i) = a(i) + b(i)$ be the firm size an entrepreneur is able to operate at from own wealth and borrowed resources. This operating capital $k(i)$ is not necessarily equal to the optimal firm size $k(i)^*$ determined in (8). An entrepreneur with individual wealth $a(i)$ lower than $k(i)^*$ would want to borrow the amount $k(i)^* - a(i)$. We assume that credit markets are imperfect with respect to lenders not being able to enforce loan–repayment due to limited commitment of borrowers (cf. Banerjee and Newman, 1993). In case of $k(i) < k(i)^*$ the firm faces a borrowing constraint. Without explicitly stating an incentive–compatibility constraint, we assume that the borrowing amount is limited such that the maximum possible loan is proportional to the borrowers individual wealth $a(i)$. Let $\phi a(i)$, $\phi \geqslant 0$, denote this upper limit to individual loans. The parameter ϕ can be interpreted as indirectly measuring the extent to which a lender can use the borrower's wealth and profit income as collateral, and therefore is able to enforce his claim, such that credit default does not occur in equilibrium. Credit constraints become less tight with rising ϕ and vanish for large ϕ. The limiting cases consequently reflect the two cases of either complete enforceability ($\phi \to \infty$) or no enforceability ($\phi = 0$), such that in the first case the borrower is considered solvent, whereas in the second one he is not.

Summing up, the operating firm size $k(i)$ of entrepreneur i with productivity $\theta(i)_e$ and wealth $a(i)$ can be written as:

$$k(i) = k\left(\theta(i)_e, a(i)\right) = \min\left[a(i), k(i)^*\right] + \min\left[\phi a(i), k(i)^* - \min\left[a(i), k(i)^*\right]\right]. \qquad (9)$$

The first term on the RHS of (9) reflects the size of a firm which does not have or does not need any access to the credit market and simply rests with its own wealth. The second term describes the amount an entrepreneur with wealth $a(i)$ will actually borrow. The subsequent numerical analysis will show that the high–productivity entrepreneurs are more likely subject to credit–constraints than the

low–productivity ones, because the optimal firm size and henceforth the capital demand are positively correlated with the productivity shock.

An entrepreneur whose individual wealth exceeds the wealth level needed to operate is business at the optimal firm size will lend the amount $a(i) - k^*(i)$ on the capital market at the equilibrium interest rate. There is no difference between borrowing and lending rates. The supply side of the capital market altogther consists of those entrepreneurs whose wealth exceeds their individual optimal firm size and of workers, who supply their savings. On the demand side we have the credit–constrained entrepreneurs and firms from the final goods industry. From this follows immediately that the size of the intermediate goods industry relative to the final goods sector essentially depends on occupational choice and individual wealth accumulation, both determined endogenously in equilibrium.

Idiosyncratic risks In each period of time, workers are endowed with one unit of raw labor and are subject to an idiosyncratic shock θ_w affecting labor supply in efficiency units, and exposing each of them to an uninsurable income risk. For simplicity, we assume that labor productivity θ_w evolves according to a first–order Markov process with $j = 1, \ldots, m$ states, and $\theta_{w,j} > 0$. The transition matrix associated with the Markov process is P_w.

Entrepreneurial productivity θ_e also evolves according to a first–order Markov process with $j = 1, \ldots, m$ different states $\theta_{e,1}, \ldots, \theta_{e,m}$; $\theta_{e,j} > 0$, and transition probability P_e. Since agents can either be workers or entrepreneurs, it is possible to identify the occupational status $s(j)$ of an agent with his productivity in the respective occupation, i.e. $s(j) = \{s(j)_w, s(j)_e\}$, where $s(j)_w = \left\{\theta_{w,j}\right\}_{j=1}^{m}$ and $s(j)_e = \left\{\theta_{e,j}\right\}_{j=1}^{m}$. We assume worker productivities to be more evenly distributed than managerial skills, such that profit incomes in general are more risky than wage incomes. As is well–known from the literature, entrepreneurs on average are compensated with a positive income differential (*aka* 'risk premium') for bearing the production risk.

By modeling two distinct random processes for workers and entrepreneurs, we take into account that the two professions demand different skills, for instance managerial ability. For this reason, we assume the processes θ_w and θ_e to be uncorrelated.

The conditional expectation of individual productivity as an entrepreneur is independent of the labor productivity, when being a worker. A high productivity as a worker in the present does not necessarily indicate an equivalently high future productivity as an entrepreneur, if the individual should decide to switch between occupations in the next period. The associated probabilities are summarized in a $m \times m$ transition matrices $P_{h,h'}$ describing the transition from productivity state $\theta_{h,j}$ to state $\theta_{h,j'}$ for $j, j' = 1, \ldots, m$, $h = e, w$ and $h \neq h'$.

We consider two different specifications regarding the Markov processes for entrepreneurial and worker productivity.[3] The shocks of the first setting are assumed to be *iid*, i.e. serially uncorrelated, such that an individual cannot infer from his present productivity how his future productivity in the same occupation will be. We then relax this assumption and allow for serial correlation of productivities, such that currently highly productive workers and entrepreneurs are more likely to be highly productive in the future.[4]

Intertemporal decision and occupational choice Each household has preferences over consumption and maximizes discounted lifetime utility

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \, U[c_t(i)] \qquad 0 < \beta < 1 \, .$$

E_0 is the expectation operator conditional on information at date 0 and β is the discount factor. Individuals are assumed to be identical with respect to their preferences regarding momentary consumption $c(i)$ which are described by constant relative risk aversion

$$U[c(i)] = \begin{cases} \dfrac{c(i)^{1-\rho}}{1-\rho} & \text{for } \rho > 0, \rho \neq 1 \\ \ln c(i) & \text{for } \rho = 1 \, , \end{cases}$$

where ρ denotes the Arrow/Pratt measure of relative risk aversion.

In each period, the single household is endowed with a unit of raw labor and—in addition to his intertemporal decision—makes a choice on his future occupation, which is either to become a self–employed producer of an intermediate good in the monopolistically competitive market or to supply his labor services in efficiency units inelastically to the production of the final good. Let $V^w(a(i), s(j)_w)$ denote the optimal value function of an agent currently being a worker with wealth $a(i)$, who is in productivity state $s(j)_w$. If he decides to remain a worker, his productivity evolves according to the transition matrix P_w of the underlying Markov process with states $\theta_{w,1}, \ldots, \theta_{w,m}$. If, instead, he decides to become an entrepreneur in the following period, his next period productivity θ'_e is determined by the transition matrix $P_{w,e}$.

[3] As the subsequential analysis will show, the dynamic properties of the underlying Markov processes turn out to be crucial for the observed between–group mobility.

[4] In our numerical simulations we follow Tauchen (1986) and build the discrete Markov processes for productivities such as to approximate $AR(1)$ processes of the general form

$$\ln \theta'_h = (\rho_h - 1)\frac{\sigma_h^2}{2} + \rho_h \ln \theta_h + \sigma_h \sqrt{1-\rho_h^2}\, \varepsilon \, , \qquad \varepsilon \sim \mathcal{N}(0,1) \, , \quad h = e, w \, .$$

The asymptotic distribution of productivities θ_h is lognormal with normalized mean, $\theta_h \sim \mathcal{N}\left(-\sigma_h^2/2, \sigma_h^2\right)$, such that the unconditional expectation is given by $\mathrm{E}[\theta_h] = 1$ for $h = e, w$.

The associated maximized value function for a typical individual currently being a worker is given by

$$
\begin{aligned}
&V^w\left(a(i), s(j)_w\right) = \\
&\qquad \max_{c(i)\geqslant 0, a(i)'\geqslant \underline{a}} \left\{ U[c(i)] + \beta \max_{\xi'\in\{0,1\}} \left\{ \mathrm{E}\left[V^w\left(a(i)', s(j)'_w\right) | s(j)_w\right], \mathrm{E}\left[V^e\left(a(i)', s(j)'_e\right)\right] \right\} \right\} \\
&\qquad\qquad \text{s.t.} \quad a(i)' = (1+r)\, a(i) + s(j)_w\, w - c(i)\ .
\end{aligned}
\tag{10}
$$

ξ is a boolean variable which takes on the values 0 or 1, depending on whether or not the agent decides to switch between occupations. r and w denote the equilibrium returns to capital and labor in efficiency units, which are constant over time for a stationary distribution of wealth and occupational statuses across agents. The optimal decision associated with the problem (10) is described by the two decision rules for individual asset holdings $a(i)'_w = a_w\left(a(i); s(j)_w\right)$ and the future professional state $\xi'_w = \xi_w\left(a(i); s(j)_w\right)$.

Let $V^e(a(i), s(j)_e)$ denote the maximized value function of an entrepreneur with wealth $a(i)$ in productivity state $s(j)_e$, who faces a decision problem similar to those of a worker. If he decides to remain an entrepreneur, his productivity evolves according to the transition matrix P_e of the underlying Markov process with states $s(j)_e = \theta_{e,1}, \ldots, \theta_{e,m}$. If, instead, he decides to switch between occupations by becoming a worker in the next period, his future productivity θ'_w is determined by the transition matrix $P_{e,w}$. With $k(s(j)_e)^*$ denoting the optimal firm size, the intertemporal problem of an entrepreneur currently in productivity state $s(j)_e$, can be written as

$$
\begin{aligned}
&V^e(a(i), s(j)_e) = \\
&\qquad \max_{c(i)\geqslant 0, a(i)'\geqslant \underline{a}} \left\{ U[c(i)] + \beta \max_{\xi'\in\{0,1\}} \left\{ \mathrm{E}\left[V^e\left(a(i)', s(j)'_e\right) | s(j)_e\right], \mathrm{E}\left[V^w\left(a(i)', s(j)'_w\right)\right] \right\} \right\} \\
&\qquad\qquad \text{s.t.} \quad a(i)' = (1+r)\, a(i) + \pi(s(j)_e, k(i)) - c(i) \\
&\qquad\qquad\qquad k(i) = \min\left[a(i), k(s(j)_e)^*\right] + \min\left[\phi\, a(i), k(s(j)_e)^* - \min\left[a(i), k(s(j)_e)^*\right]\right] \\
&\qquad\qquad \pi(s(j)_e, k(i)) = p(x(i))\, x(s(j)_e, k(i)) - (r+\delta)\, k(i)
\end{aligned}
\tag{11}
$$

Again, ξ is a boolean variable, indicating the agent's decision on leaving or remaining in his contemporaneous occupational status. The optimal decision associated with the problem (11) is described by the two decision rules for individual asset holdings $a(i)'_e = a_e(a(i); s(j)_e)$ and the future professional state $\xi'_e = \xi_e(a(i); s(j)_e)$.

In general, our model displays the same characteristics regarding individual savings and wealth accumulation under risk as, for instance, discussed in Aiyagari

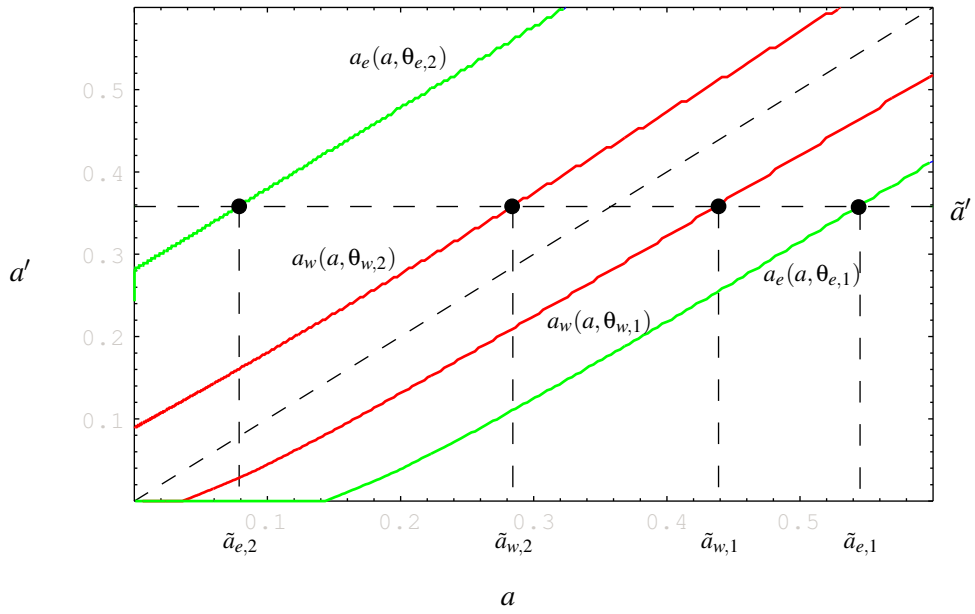


Figure 1: Policy functions in the unconstrained economy (*iid* shocks)

(1994) or Huggett (1996). Similar to Quadrini (2000) we additionally consider occupational choice. The shocks to worker and entrepreneurial productivity generate an income risk to which the households respond with precautionary saving. According to Leland (1968), decreasing absolute risk aversion is a necessary and sufficient condition to generate motives for precautionary savings in the presence of income risk, which is satisfied here for any coefficient of relative risk aversion of $\rho > 0$. In terms of Sandmo (1970) there is only an income but no capital risk in our model, such that the share of risky incomes in total household income declines with growing wealth. Accordingly, the importance of risky profits providing negative incentives for entrepreneurship fades for high wealth levels.

Figure 1 illustrates the individual occupational choice problem. It shows policy functions $a_w(a(i); s(j)_w)$ and $a_e(a(i); s(j)_e)$ for the simplified case of unconstrained entrepreneurs and serially uncorrelated shocks based on the numerically specified model of section 4. The corresponding decision rules $\xi_w(a(i), s(j)_w)$ and $\xi_e(a(i), s(j)_e)$ can be used to determine the next period critical wealth level $\tilde{a}'$ where a household currently in productivity state $s(j)$ decides to change his occupational status. In the special case of uncorrelated productivity shocks, this critical wealth level is identical over all productivity states and occupational statuses. For illustrative purposes we only show the policy functions for two (out of five) states, the subscripts $1, 2$ denoting the low and high productivity states respectively.

An agent, who currently is a worker with low productivity $\theta_{w,1}$ with wealth holdings below $\tilde{a}_{w,1}$, will never become an entrepreneur because his next period wealth will never exceed the critical level $\tilde{a}'$. In contrast to this, a worker currently in productivity state $\theta_{w,2}$ with wealth holdings below $\tilde{a}_{w,2}$ eventually becomes an entrepreneur because his wealth increases over time.

The line of argument is similar for households who currently are entrepreneurs. An individual, who is productivity state $\theta_{e,2}$ with wealth holdings above $\tilde{a}_{e,2}$, remains an entrepreneur in the next period, because his wealth holdings will never fall below the critical level $\tilde{a}'$. Contrary, an entrepreneur in the current productivity state $\theta_{e,1}$ with wealth holdings below $\tilde{a}_{e,1}$ eventually becomes a worker, because his wealth will decrease over time.

The major consequence of credit market imperfections for occupational choice is that they shift the critical level of next period's wealth upwards. This happens only if credit restrictions become actually binding. If the next period critical wealth level even exceeds the optimal (unconstrained) firm size of an entrepreneur with the highest productivity, no business owner will ever become credit constrained. Recall at this point that the firm size in the intermediate goods industry is endogenously determined by factor prices and the producers' market shares, the latter crucially depending on the number of firms in the market. If constraints become binding they have an aggravating effect on market dynamics in the case of uncorrelated shocks. Firms are forced to exit the market. The individual market share falling to the remaining producers grows. This leads to an increase in individual firm size and, in turn, exposes more entrepreneurs to credit constraints.

In the calibrated model underlying Figure 1, the optimal firm size of a producer with the highest productivity is by sixteen times larger than the critical wealth level making agents change their profession and by twelve times larger than equilibrium average wealth holdings. In this case less than 0.5% of the whole population own sufficient wealth not to demand any credit and business owners heavily rely on credit market access in order to operate their firms at the optimal firm size.[5]

Summarizing, our model allows us to predict the between–class mobility of agents in the case of uncorrelated shocks. Highly productive entrepreneurs remain in their occupational status, whereas low–productivity entrepreneurs quit and become workers. The reverse is true for workers. Here low–productivity households remain in their occupational status, whereas the highly productive workers decide to take their chances with entrepreneurship. As will become obvious below, the persistence of shocks leads to completely different and more complicated mobility patterns which are more difficult to predict, because not only individual wealth but also the time path of individual productivity is important for occupational choice.

[5]The equilibrium optimal firm size in the highest productivity state is $k^* = 6.072$, while the critical level of next period's wealth is given by $\tilde{a}' = 0.3581$. Average wealth holdings are reported in Table 2.

3 Stationary General Equilibrium

A general equilibrium is an allocation, where the equilibrium prices generate a distribution of wealth and occupational statuses across agents which is consistent with these prices given the exogenous process for the idiosyncratic shocks and the agents' optimal decision rules.

Let K_F, L and $x(i)^D$ denote the demands of capital, effective labor and intermediate goods in the final goods sector. We obtain aggregate labor supply by summing up individual labor supplies in efficiency units over the population fraction $1-\lambda$ of workers. Let, furthermore, $q_j, j = 1,\ldots,m$ denote the relative frequencies of states $\theta_{w,j}$ in the equilibrium distribution of labor productivities. The stationary recursive equilibrium is a set of value functions $V^w(a,s_w)$, $V^e(a,s_e)$, decision rules $a_w(a;s_w)$, $\xi_w(a;s_w)$ and $a_e(a;s_e)$, $\xi_e(a;s_e)$, prices w, r, p and a distribution $\lambda, 1-\lambda$ of households across occupations such that:

(i) the decision rules $a_w(a;s_w)$, $\xi_w(a;s_w)$ and $a_e(a;s_e)$, $\xi_e(a;s_e)$ solve the workers' and entrepreneurs' problems (10) and (11) at prices w, r, p,

(ii) the aggregate demands of consumption, labor, capital and intermediate goods are the aggregation of individual demands and markets clear at constant prices w, r, $p(i)$ where factor inputs are payed according to their marginal productivity:

$$
\begin{aligned}
Y &= C + \delta K \\
\int_0^1 k(i)\,\mathrm{d}i \equiv K &= K_F + \int_0^\lambda k(i)\,\mathrm{d}i \\
\int_\lambda^1 \sum_{j=1}^m q_j\,\theta_{w,j}\,\mathrm{d}i &= L \\
x(i)^S &= x(i)^D\,,
\end{aligned}
$$

(iii) the stationary distribution $\Gamma(\lambda, a, P_e, P_w, P_{e,w}, P_{w,e})$ of agents across individual wealth holdings, occupational statuses and associated productivities is the fixed point of the law of motion which is consistent with the individual decision rules and equilibrium prices. The distribution $\lambda, 1-\lambda$ of agents across occupations is time–invariant.

The decision rules for workers, $a_w(a,s_w)$, $\xi_w(a,s_w)$, and entrepreneurs, $a_e(a,s_e)$, $\xi_e(a,s_e)$, together with the stochastic processes for individual labor productivity and entrepreneurial productivity, determine the stationary distribution Γ at equilibrium prices w, r. The stationary distribution Γ governs the population share of entrepreneurs (i.e. the mass of firms in the intermediate goods sector), the efficiency units of labor supplied by workers, capital demand of the intermediate goods sector, and

the aggregate capital supply, the latter equaling the mean of individual wealth holdings. Once the population share of entrepreneurs λ is derived, this together with the stationary distribution of entrepreneurial productivities determines the supply of intermediate goods.

Equilibrium factor income shares In what follows, it will be convenient to have some information on the functional distribution of income. Households derive income from three sources: labor income, capital income and monopolistic profits. The two technology parameters α and γ entirely determine the division of aggregate income among the three income sources in the absence of credit constraints on entrepreneurial activity. In this case, each business in the intermediate goods sector operates at its optimal firm size (8). The factors of production are payed according to their marginal product. Eq. (1) immediately implies the labor income share $(1-\alpha)(1-\gamma)$ and the capital income share $(1-\alpha)\gamma$. The remaining income share α accrues to incomes generated in the intermediate goods sector, namely entrepreneurial profits and capital income. Since the direct price elasticity of demand is given by $-1/(1-\alpha)$, the profit income share equals $\alpha(1-\alpha)$. The residual α^2 then is the capital income share of capital employed in the production of intermediate goods, such that altogether the capital income share of the economy amounts to $(1-\alpha)\gamma+\alpha^2$.

Static efficiency in a risk–pooling economy The underlying model displays three types of inefficiencies. Monopolistic competition shifts rents towards the producers of the intermediate good. The presence of risk distorts the decisions of risk–averse household, and, finally, producers are credit constrained. We now briefly discuss the conditions for an efficient allocation implemented by a social planner, who allocates resources to sectoral production and individuals across occupations in order to maximize aggregate income. If aggregate income then is distributed uniformly among households, this is equivalent to an efficient allocation in a perfect risk–pooling economy. The analysis allows us to confront the subsequent calibration results from the decentralized economy with a reference allocation. For illustrative purposes, we confine our analysis two a simple 2–state setting. The productivity shocks are *iid*. Let $\theta_{w,1}$, $\theta_{w,2}$, with $\theta_{w,2} > \theta_{w,1}$, denote the two states of labor productivity, with associated probabilities $q_w, 1-q_w$. Let, accordingly, $\theta_{e,1}$,$\theta_{e,2}$, with $\theta_{e,2} > \theta_{e,1}$, denote the two states for entrepreneurial productivity, with associated probabilities $q_e, 1-q_e$, and output quantities of the intermediate good x_1 and x_2, the latter resulting as:

$$x_1 = \theta_{e,1}\, k_1 \,, \qquad\qquad x_2 = \theta_{e,2}\, k_2 \,,$$

where k_1 and k_2 denote the capital stocks allocated to entrepreneurs in state $\theta_{e,1}$ and $\theta_{e,2}$ respectively. With λ denoting the population share of entrepreneurs, aggregate

output in the intermediate goods industry can be written as:

$$\int_0^1 x(i)^\alpha \, \mathrm{d}i = \int_0^{\lambda q_e} x_1^\alpha \, \mathrm{d}i + \int_{\lambda q_e}^{\lambda} x_2^\alpha \, \mathrm{d}i = \lambda \left[q_e \left(\theta_{e,1} k_1 \right)^\alpha + (1 - q_e) \left(\theta_{e,2} k_2 \right)^\alpha \right] . \quad (12)$$

The population share of workers equals $1-\lambda$. Aggregate labor supply L in efficiency units is given by:

$$L = (1-\lambda) \left[q_w \theta_{w,1} + (1 - q_w) \theta_{w,2} \right] \equiv (1-\lambda) \bar{L}$$

Aggregate output can be derived as:

$$Y = F(K_F, (1-\lambda)\bar{L}) \, \lambda \left[q_e \left(\theta_{e,1} k_1 \right)^\alpha + (1 - q_e) \left(\theta_{e,2} k_2 \right)^\alpha \right]$$

where $F(K_F, L)$ is homogeneous of degree $1-\alpha$.

Static efficiency demands that λ, K_F, k_1 and k_2 are chosen such as to maximize Y subject to the the resource constraint $K_F + \lambda [q_e k_1 + (1-q_e) k_2] \leq K$, where K denotes the aggregate capital stock. With μ as the Lagrange multiplier, the necessary conditions for an efficient allocation are:

$$\begin{aligned}
\left(-F_L(K_F, L)\bar{L}\lambda + F(K_F, L)\right) \left[q_e \left(\theta_{e,1} k_1 \right)^\alpha + (1 - q_e) \left(\theta_{e,2} k_2 \right)^\alpha \right] &= \mu [q_e k_1 + (1-q_e) k_2] \\
F_{K_F}(K, L) \lambda \left[q_e \left(\theta_{e,1} k_1 \right)^\alpha + (1 - q_e) \left(\theta_{e,2} k_2 \right)^\alpha \right] &= \mu \\
F(K_F, L) \lambda q_e \left(\theta_{e,1} \right)^\alpha \alpha k_1^{\alpha - 1} &= \lambda \mu q_e \\
F(K_F, L) \lambda (1 - q_e) \left(\theta_{e,2} \right)^\alpha \alpha k_2^{\alpha - 1} &= \lambda \mu (1 - q_e)
\end{aligned}$$

From the latter two conditions follows the optimal allocation of capital over firms in the intermediate goods industry:

$$k_2^* = k_1^* \left(\theta_{e,2} / \theta_{e,1} \right)^{\frac{\alpha}{1-\alpha}} .$$

Using this in the derivative with respect to K_F and in the aggregate resource constraint, and combining both equations yields the optimal share of capital input in the final goods sector.

$$\frac{K_F^*}{K} = \frac{1}{1 + \alpha \frac{F(K_F, L)}{F_{K_F}(K_F, L) K_F}} .$$

For the first–order condition with respect to λ follows accordingly:

$$\frac{\lambda^*}{1 - \lambda^*} = \frac{1 - \alpha}{\frac{F_L(K_F, L)(1 - \lambda^*)\bar{L}}{F(K_F, L)}} .$$

The generalized CES–form of production technology in the final goods sector implies

$$\frac{K_F^*}{K} = \frac{(1-\alpha)\gamma}{\alpha + (1-\alpha)\gamma} \qquad \text{and} \qquad \frac{\lambda^*}{1 - \lambda^*} = \frac{1}{1 - \gamma} . \quad (13)$$

If we set the two technology parameter such as to yield realistic factor income shares, which is $\alpha = 0.3$ and $\gamma = 0.1$, about one fifth of the aggregate capital stock should be alloted to the final goods sector. Regarding the sectoral allocation of households we get a value of $\lambda^*/(1-\lambda^*) = 1.1$, stating that less workers are employed in the final goods production than we have business owners in the intermediate goods industry.

4 Model Calibration and Numerical Results

The economy is calibrated to match empirical evidence regarding the factor income shares and the households' preferences with respect to risk aversion and time preference. We focus on three benchmark models, the baseline model being the one without credit constraints ($\phi = \infty$). This is contrasted with (i) the case where the lower bound for the equity–ratio is one half of the operating capital ($\phi = 1$), and (ii) with perfect constraints ($\phi = 0$), which is the case of firms in the intermediate goods industry having no access to the capital market.

The first aspect we are interested in is whether our model is able to replicate empirical evidence on wealth distributions. We then examine how the presence of credit constraints affects the key macroeconomic variables, such as aggregate output, factor prices and factor income shares as well as individual incomes, household wealth and the degree of inequality, the latter measured by the Gini coefficient. We then analyze mobility between occupations and finally relate individual wealth levels to the probability of becoming an entrepreneur by providing the associated likelihood functions.

Numerical specification The baseline model is a model of perfect capital markets, such that credit constraints never become binding. Recalling the results from above on the functional distribution of income, the parameterization of the baseline model, as given in Table 1, is chosen in accordance with empirically observed income shares and plausible estimates for the coefficient of relative risk aversion and the rate of time preference (Mehra and Prescott, 1985; Obstfeld, 1994; King and Rebelo, 1999). The labor income share in this setting equals 0.63, the capital share is 0.16 and the profit income share amounts to 0.21. We use B as a scaling factor for entrepreneurial productivity.

Table 1: Parameters of the baseline model

α	γ	δ	β	ρ	ϕ	A	B
0.3	0.1	0.06	0.95	2.0	∞	1.0	10.0

The random processes of entrepreneurial and worker productivity are approximated by a five–state Markov chain. The numerical values of the five–state processes chosen for θ_w and θ_e respectively are:

$$\begin{aligned} \theta_w &= \begin{pmatrix} 0.537944 & 0.726149 & 0.980199 & 1.32313 & 1.78604 \end{pmatrix} \\ \theta_e &= \begin{pmatrix} 0.0036073 & 0.0342184 & 0.3246524 & 3.08022 & 29.2243 \end{pmatrix} \end{aligned} . \tag{14}$$

The transition matrices for individuals who decide to switch occupations are derived from the stationary distributions of the respective Markov processes. The probability for a worker of ending up in a specific state of entrepreneurial productivity $\theta_{e,1}, \ldots, \theta_{e,m}$ is given by the stationary (unconditional) probabilities for reaching this state. The numerical specification is chosen such as to generate a standard deviation of $\sigma_w = 0.2$ of labor productivity which is close to Aiyagari (1994) and a much higher standard deviation of $\sigma_e = 1.5$ for entrepreneurial risk. We assume the productivity shock affecting workers to be more evenly distributed than the productivity shock affecting entrepreneurs. For the case of correlated shocks, the underlying random process of worker productivity has a serial correlation of $\rho_w = 0.6$. Regarding entrepreneurs, we have $\rho_e = 0.8$, by this assuming a slightly larger degree of persistence.

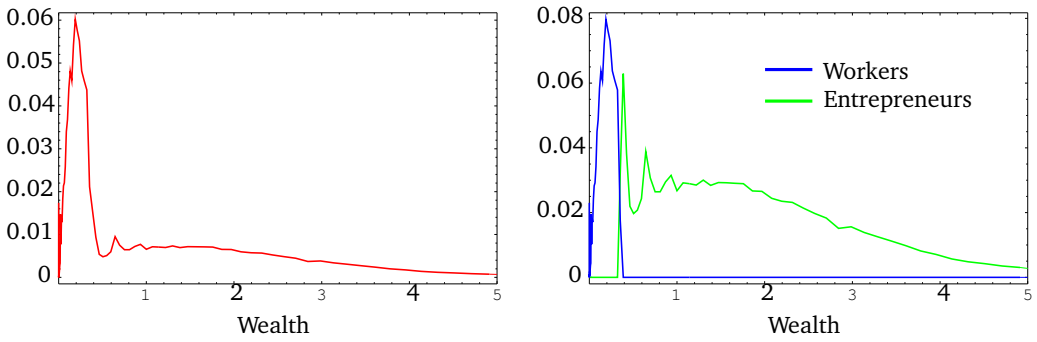
Tables 2(a) and 2(b) in Appendix A summarize the transition probabilities and the associated stationary distributions for transition between professions for the two cases of autocorrelated and uncorrelated shocks.

Calibration results For the case $\phi \to \infty$, Figures 2 and 3 show the stationary distribution of wealth (a) for the economy as a whole and (b) differentiated with respect to workers and entrepreneurs for the two types of the underlying shocks. In general, our model produces results similar to those found in the literature on heterogeneous agent models with entrepreneurial activity with respect to the distribution of wealth across all agents (cf. Quadrini, 1999, 2000). If we take a differentiated look at occupational statuses, we see that workers are more concentrated at lower wealth levels, and there exists a significant mass of wealthy entrepreneurs but also a comparably large share of poorer ones. This is in line with empirical findings by Gentry and Hubbard (2004); Hamilton (2000) as well as with related theoretical contributions (cf. Boháček, 2006; Cagetti and De Nardi, 2006).

Comparing Figure 2(b) to Figure 3(b) makes obvious how important the nature of the underlying shocks is for the between–group equilibrium wealth distribution. Although the aggregate wealth distributions roughly are of similar shape, there are striking differences, when it comes to the distribution of wealth in the two occupational classes.

If the shocks are serially uncorrelated, we observe a perfectly segregated economy where workers possess little wealth and all rich households are entrepreneurs. The picture is different if shocks are serially correlated. Here, the membership in wealth classes is not entirely related to the occupational status. We observe rich workers as well as poor entrepreneurs, although workers are more concentrated at lower wealth levels.

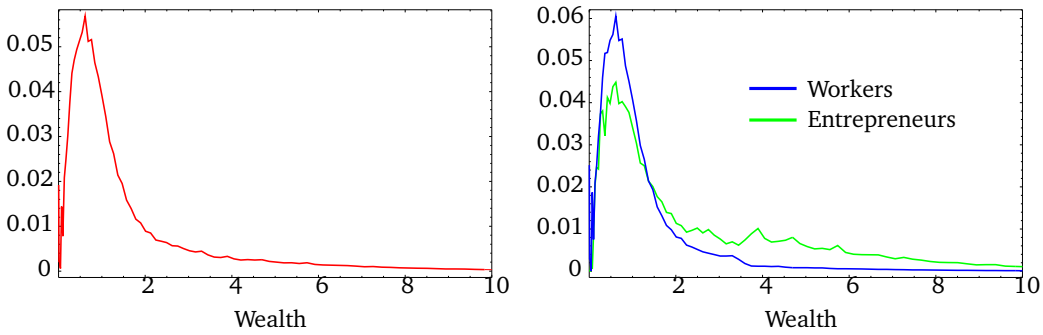
Uncorrelated shocks Table 2 summarizes the results for the macroeconomic key variables in the case of *iid* shocks. As the economic intuition would suggest, we



(a) PDF of individual wealth

(b) PDF of wealth: workers and entrepreneurs

Figure 2: Wealth distribution in the unconstrained economy — *iid* shocks



(a) PDF of individual wealth

(b) PDF of wealth: workers and entrepreneurs

Figure 3: Wealth distribution in the unconstrained economy — correlated shocks

observe a continuously declining population share of entrepreneurs, when access to the capital market becomes more and more limited. At this point it is important to notice that the presence of credit constraints not necessarily means that only those agents, who have sufficient own wealth to operate their business at the optimal firm size k^*, choose to become an entrepreneur. These are the only firms which actually maximize their profits, whereas the credit–constrained entrepreneurs are forced to operate at a suboptimally small business size. Consequently, the average firm size in the intermediate goods industry decreases. However, as more agents decide to become workers, the average profits of those remaining in the industry increase.

Table 2: Simulations results — *iid* shocks

		Tightness of credit constraints		
		$\phi \to \infty$	$\phi = 1$	$\phi = 0$
share of entrepreneurs	(%)	24.48	24.36	24.00
∅ firm size	total	1.127	1.060	0.979
∅ credit rationing	total	0	0.133	0.385
∅ profits	total	0.277	0.278	0.282
final	Y	0.323	0.314	0.299
goods	K_F (%)	43.73	45.87	48.92
sector	K_F	0.214	0.219	0.225
	L_F	0.758	0.759	0.763
factor	w	0.268	0.261	0.247
prices	r	0.045	0.041	0.033
	$w/(r+\delta)$	2.544	2.569	2.654
factor	labor	63.00	63.00	63.00
income	capital	16.00	15.41	14.31
shares	profits	21.00	21.58	22.69
∅ wealth	total	0.490	0.477	0.460
	workers	0.162	0.163	0.116
	entrepreneurs	1.502	1.453	1.550
∅ income	workers	0.268	0.278	0.258
	entrepreneurs	0.436	0.426	0.426
risk premium		0.028	0.063	0.140
wealth	total	0.636	0.625	0.677
inequality	workers	0.324	0.360	0.414
(Gini)	entrepreneurs	0.379	0.309	0.246

If there is only limited or no capital demand from the intermediate goods industry, more capital is employed in the final goods sector. With diminishing marginal returns, the equilibrium interest rate r and accordingly the factor price for capital $r+\delta$ decline in both sectors of the economy. Recalling that entrepreneurial households receive income from two sources, profits and capital incomes, the income share reflecting the user costs of capital declines for any given level of individual wealth, whereas the profit income share rises, which explains the above mentioned positive effect of credit constraints on average entrepreneurial profits. Altogether, we observe a shift in the functional income distribution from capital to profit incomes.

Wages as well as wage incomes decline in the presence of credit constraints. This result follows immediately from marginal productivity theory, because the population share of workers increases with less business owners remaining in the intermediate goods industry, also indicating that the average premium on entrepreneurial activity grows; in our calibrated model even by the factor five.

Aggregate output declines too if the tightness of credit constraints increases. The larger amounts of capital and labor employed (at decreasing marginal productivity) in the production of the final good do not compensate for the output loss associated with a decline in the supply of intermediate goods. This permanent loss in aggregate income is also reflected in lower average wealth holdings. Overall wealth inequality increases with more constrained access to the capital market. With respect to the within–group wealth distribution, we find that wealth becomes more unevenly distributed among workers, whereas wealth inequality among entrepreneurs declines.[6]

Table 3 summarizes our results on between–group mobility in a stationary equilibrium.[7] Irrespective of the degree to which credit constraints are binding, we find that switches between occupational statuses can only be observed for the highly productive workers, earning the highest wages, and the low–productivity entrepreneurs, earning the lowest profit incomes. Low and average productivity workers as well as the highly productive entrepreneurs never change their occupation. These results are in accordance with the economic intuition that earnings advantages translate into higher individual wealth, the latter being an important determinant of entrepreneurship, especially in the presence of credit constraints.

However, even in the unconstrained economy, only about 14% of those workers with highest current productivity actually decide to switch between occupations. This figure drops down to 2% in the case of perfect constraints. Related to the entire population, between–group mobility is steadily declining from around 2.5% in the unconstrained to 0.4% in the perfectly constrained economy, and altogether takes place at a very low scale. These numbers are far too low to match empirical findings. Evans (1987) reports entry and exit rates around 4.5%. Serially correlated shocks increase mobility considerably, even above the reported empirical values, as will become obvious below. From this we conclude that persistence of shocks is an important determinant of between–group mobility.

The critical wealth level, where an agent decides to change his occupation, is inversely related to the productivity shock. Generally, we can say that the presence of credit constraints dampens mobility between occupations, and that the threshold wealth levels at which households are willing to change their next period occupation rise for each given state of productivity.

Autocorrelated shocks Table 4 summarizes the results for the three settings $\phi \to \infty$, $\phi = 1$, and $\phi = 0$ with respect to the macroeconomic key variables for the case of serially correlated shocks. The most striking result is that the population share of entrepreneurs increases if credit constraints become more binding. Although the

[6]Notice, that the Gini coefficient does not allow for a simple decomposition of total inequality into inequality within and between subgroups.

[7]The within–group mobility can be inferred from the limiting distribution given in Table 2(a) in the Appendix.

Table 3: Mobility over occupational statuses — *iid* shocks

		Tightness of credit constraints								
		$\phi \to \infty$			$\phi = 1$			$\phi = 0$		
	state	total	change	$a_{crit}{}^{\star}$	total	change	$a_{crit}{}^{\star}$	total	change	$a_{crit}{}^{\star}$
workers	$\theta_{w,1}$	0.0092	0	0.486	0.0092	0	0.533	0.0093	0	0.532
	$\theta_{w,2}$	0.1619	0	0.438	0.1622	0	0.486	0.1629	0	0.486
	$\theta_{w,3}$	0.4130	0	0.372	0.4136	0	0.426	0.4156	0	0.426
	$\theta_{w,4}$	0.1619	0.0202	0.284	0.2622	0.0116	0.337	0.1629	0.0031	0.344
	$\theta_{w,5}$	0.0092	0.0045	0.164	0.0092	0.0028	0.221	0.0093	0.0010	0.232
	total	0.7552	0.0247		0.7564	0.0144		0.7600	0.0041	
entre-preneurs	$\theta_{e,1}$	0.0030	0.0006	0.593	0.0030	0.0005	0.643	0.0029	0.0001	0.625
	$\theta_{e,2}$	0.0525	0.0096	0.546	0.0522	0.0066	0.591	0.0515	0.0024	0.575
	$\theta_{e,3}$	0.1338	0.0145	0.415	0.1332	0.0073	0.455	0.1312	0.0016	0.451
	$\theta_{e,4}$	0.0525	0	0.078	0.0522	0	0.232	0.0515	0	0.269
	$\theta_{e,5}$	0.0030	0	–	0.0030	0	0.057	0.0029	0	0.086
	total	0.2448	0.0247		0.2436	0.0144		0.2400	0.0041	

* a_{crit} is the level of wealth at which an agent changes his occupational status. $\underline{a}$ denotes the minimum level of individual wealth, '–' means that a switches in the occupational status never occurs.

effect in total is quantitatively small, the population share increasing by less than one and a half percentage point, it is obvious that credit constraints do not turn out an impediment to entrepreneurship as one might have expected from the previous analysis.

This somewhat counter–intuitive result can be traced back to the general equilibrium nature of our approach. Credit constraints are only one out of several determinants of occupational choice. Entrepreneurs compete with firms from the final goods industry for capital and the expected income premium on entrepreneurial profits also affects the households' decisions. Besides these factors it is important to bear in mind that households continuously decide between two lotteries and possess (at least subjective) knowledge regarding the stochastic properties of the underlying shocks.

In the case of serially correlated shocks, a low–productivity worker is aware of the fact that being also lowly productive in the future is a more probable outcome than otherwise. Consequently he might be inclined to take his chances with entrepreneurship, knowing that his current productivity as a worker is not related to his future productivity as a business owner.[8]

One major effect, shown in Table 4, is that average profits generated from entrepreneurial activity are smaller if business owners are barred from the capital market

[8]Relaxing this assumption is left for future research.

Table 4: Simulation results — autocorrelated shocks

		Tightness of credit constraints		
		$\phi \to \infty$	$\phi = 1$	$\phi = 0$
share of entrepreneurs	(%)	23.75	25.01	25.19
∅ firm size	total	2.620	2.063	1.531
∅ credit rationing	total	0	0.865	1.952
∅ profits	total	0.569	0.551	0.527
final	Y	0.644	0.606	0.538
goods	K_F (%)	43.77	49.15	56.67
sector	K_F	0.484	0.499	0.505
	L_F	0.811	0.791	0.780
factor	w	0.500	0.482	0.435
prices	r	0.033	0.025	0.015
	$w/(r+\delta)$	5.372	5.673	5.825
factor	labor	63.00	63.00	63.00
income	capital	16.00	14.24	12.35
shares	profits	21.00	22.76	24.64
∅ wealth	total	1.107	1.015	0.890
	workers	0.895	0.741	0.568
	entrepreneurs	1.787	1.836	1.848
∅ income	workers	0.615	0.572	0.496
	entrepreneurs	0.736	0.707	0.665
risk premium		0.070	0.073	0.161
wealth	total	0.512	0.502	0.578
inequality	workers	0.457	0.441	0.555
(Gini)	entrepreneurs	0.551	0.476	0.437

(i.e. $\phi = 0$). As in the case of uncorrelated shocks the average firm size decreases, but here this result can partly be traced back to an increase in competition among monopolists, whose population share in the intermediate goods industry increases, such that each of them ends up with a smaller market share. In addition, we have the already mentioned effect that credit–constrained entrepreneurs are forced to operate at a suboptimally small business size which lets average profits decline accordingly. This, *ceteris paribus*, destroys the incentives of becoming an entrepreneur.

A counteracting impulse comes from the final goods sector which—in the constrained economy—employs more capital at diminishing marginal returns, such that the user costs of capital decline for any given level of individual wealth. This increases the attractivity of entrepreneurship. As before, we observe a redistributive effect in the functional income distribution, with a profit income share increasing at the cost of capital incomes.

Table 4 also reveals that aggregate output declines if the tightness of credit constraints increases. While capital input rises, labor input in efficiency units declines,

both by (5) having counteracting effects on the demand for the differentiated intermediate goods. There is a larger number of entrepreneurs in the economy, each of them having a market share in the intermediate goods industry smaller than in a world without constraints, because it is impossible to lure away capital inputs from the final goods production. In the end the aggregate supply of intermediate goods is smaller too. Substituting capital for labor in the final goods sector does not compensate for the output loss accompanying the reduction in labor input. Moreover, this result reflects the inefficiency arising from the fact the marginal product of capital in the intermediate goods industry is larger than in the final goods sector.

Wages as well as average incomes of workers decline too if credit constraints become more tight. Usually one would expect the equilibrium wage rate to rise as more agents opt for the entrepreneurial profession. However, the reduction in the aggregate input of intermediate goods negatively effects the equilibrium wage rate by (4) and ultimately leads to a decline in average labor incomes. Table 4 shows that the risk premium on entrepreneurial activity—as measured by the average income differential—grows if credit constraints become more severe. It more than doubles as ϕ approaches nil.

Wealth holdings on average decrease if credit constraints become more tight. This, however, stems from lower wealth holdings of workers, whereas the average wealth of entrepreneurs increases. Despite this decrease in overall average wealth holdings, not only the fraction of total capital allocated to the final goods sector increases, the level of capital rises too, thereby indirectly indicating the lack of capital in the intermediate goods industry. The remaining average excess demand for capital in the intermediate goods sector, as measured by average credit rationing in Table 4, exceeds the resulting average firm size.

Regarding the wealth distribution, we find that more tight credit constraints lead to a more unequal distribution of wealth among the whole population. Interestingly, wealth becomes more unequally distributed among workers, while wealth inequality among entrepreneurs declines. If we confront our results with the empirical findings, the baseline specification implies a too low degree of wealth inequality. This may be overcome by assuming a larger variance for the entrepreneurial productivities, i.e. a larger entrepreneurial risk and is left for future research. However, the general picture is in accordance with empirical results. The wealth distribution of Figure 2(a) contains mass at high wealth levels and Figure 2.(b) shows that entrepreneurs are located in the upper tail of the wealth distribution.

Next, we are interested in the mobility between occupations taking place under the stationary distribution. Table 5 presents the quantitative results. As can be seen, 10.4% of the population change their occupational status from period to period if credit constraints are practically absent, i.e. $\phi \to \infty$ which is a considerably larger share than in the case of uncorrelated shocks. We find a pronounced mobility across occupations in the case of perfect capital markets. This overall mobility

Table 5: Mobility over occupational states — autocorrelated shocks

		Tightness of credit constraints								
		$\phi \to \infty$			$\phi = 1$			$\phi = 0$		
	state	total	change	$a_{crit}{}^{\star}$	total	change	$a_{crit}{}^{\star}$	total	change	$a_{crit}{}^{\star}$
workers	$\theta_{w,1}$	0.0030	0.0030	$\underline{a}$	0.0049	0.0024	0.098	0.0074	0.0021	0.133
	$\theta_{w,2}$	0.1019	0.0944	0.043	0.1117	0.0791	0.178	0.1280	0.0573	0.224
	$\theta_{w,3}$	0.4171	0.0066	3.740	0.4034	0.0280	1.330	0.3944	0.0508	0.861
	$\theta_{w,4}$	0.2224	0	–	0.2125	0	–	0.2017	0.0	9.079
	$\theta_{w,5}$	0.0180	0	–	0.0174	0	–	0.0166	0	–
	total	0.7625	0.1040		0.7499	0.1095		0.7481	0.1102	
entre-preneurs	$\theta_{e,1}$	0.0024	0.0024	$\underline{a}$	0.0025	0.0025	$\underline{a}$	0.0025	0.0025	$\underline{a}$
	$\theta_{e,2}$	0.0240	0.0240	$\underline{a}$	0.0253	0.0253	$\underline{a}$	0.0255	0.0254	$\underline{a}$
	$\theta_{e,3}$	0.0776	0.0776	$\underline{a}$	0.0817	0.0817	$\underline{a}$	0.0823	0.0823	$\underline{a}$
	$\theta_{e,4}$	0.1160	0	–	0.1221	0	0.0622	0.1230	0	0.204
	$\theta_{e,5}$	0.0176	0	–	0.0185	0	–	0.0186	0	0.021
	total	0.2375	0.1040		0.2501	0.1095		0.2519	0.1102	

* a_{crit} is the level of wealth at which an agent changes his occupational status. $\underline{a}$ denotes the minimum level of individual wealth, '–' means that a change of the occupational status never occurs.

even increases in the presence of credit constraints. For the case of $\phi = 0$, 11.02% of the whole population change their occupational status in each period. While this change in overall mobility might seem small from a quantitative perspective, it is nevertheless remarkable, since it indicates that credit constraints not only increase the population share of entrepreneurs but also the fluctuation between occupations.

A more detailed look at the mobility patterns shown in Table 5 reveals that generally workers and entrepreneurs who exhibit a low productivity in their current profession decide to switch between professions. For $\phi \to \infty$, only few workers are in the lowest labor productivity state (0.3% of the population) but all of them decide to take their chances with entrepreneurship in the next period. It is possible to determine the critical wealth level a_{crit} for each occupational status at which households are willing to switch between professions. As becomes obvious, the least productive workers change their occupation regardless of their individual wealth which equals the lowest possible wealth level $\underline{a}$.

If we take a look at workers in the second lowest productivity state, who make up for 10.19% of the whole population, the majority of this group changes occupations (which amounts to a population share of 9.44%) and has individual wealth holdings above the critical level of $a = 0.043$. The remaining entries into entrepreneurship are workers from the third productivity state, where we observe a large degree of persistence in the occupational status. Since workers from this group decide to

become business owners only if their wealth level is comparably high—i.e. amounts to more than three times the average wealth level of the economy—the percentage number of entries from this class is very small (0.7% of the population), despite the fact that this class is the largest subgroup with 41.71% of the population. Even more productive workers never become entrepreneurs in the next period.

With regard to exits from the entrepreneurial class, the mobility patterns are more simple: All entrepreneurs finding themselves in the lowest three productivity states (in total 10.4% of the population) change their occupational status regardless of their individual wealth levels. Analogously, each entrepreneur who finds himself in the highest two productivity states (in total about 13% of the population) remains in the intermediate goods sector. Thus, mobility across occupational statuses in our model is confined to agents who are not successful in their current professions.

This general picture of mobility between occupational classes is unchanged by the presence of credit constraints. For $\phi = 0$, comparably unproductive agents also decide to change their occupational status. With regard to the exits from entrepreneurship we find again that entrepreneurs from the lowest three productivity states leave the intermediate goods sector while more productive entrepreneurs from the two highest classes remain in their profession.[9] However, there are differences too. Most strikingly, the presence of credit constraints generally raises the critical wealth levels at which workers are willing to become entrepreneurs. Now a larger fraction of workers from the third productivity state becomes part of the overall entries into entrepreneurship (5.08% as compared to about 0.7% of the population).

Table 5 also shows how the distribution of workers and entrepreneurs across productivity states is affected by credit constraints. According to the above described mobility effects, the distribution of workers across labor productivity states becomes more concentrated at lower productivity levels, for $\phi = 0$. Accordingly, the share of workers with high labor productivity decreases.[10] With respect to entrepreneurs, the differences in the distribution across productivity states are negligent.

The last aspect to examine is the relationship between credit constraints, individual wealth levels, and entrepreneurial activity. Figure 4 shows the likelihood functions for entrepreneurship for the three cases $\phi \to \infty$, $\phi = 1$, and $\phi = 0$.[11] The graphs display the likelihood (Prob(E)) for an individual with wealth $a(i)$ to be an

[9]The critical wealth levels for the highly productive entrepreneurs displayed in the table never becomes effective because for none of these entrepreneurs the wealth holdings will ever fall below these critical levels.

[10]The population share of workers in the upper two productivity states decreases from 31.5% to 29.2%.

[11]The likelihood functions displayed in Figure 4 result from a smoothing procedure. We pooled wealth levels from the grid in wealth classes and computed averages. The remaining erratic shape of the likelihood functions should disappear for specifications with a larger number of grid points in the productivity/wealth space.

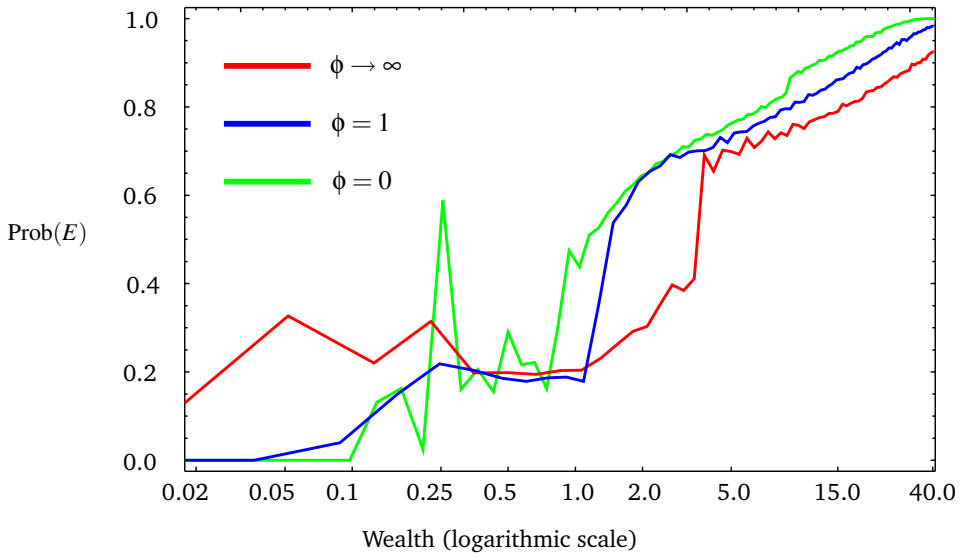


Figure 4: Likelihood function of entrepreneurship with and without credit constraints

entrepreneur for any given level of wealth. To certain degree, Figure 4 summarizes some our previously discussed results from Table 5 above.[12]

In an economy with credit constraints, agents only choose to become entrepreneurs once a higher level of wealth has been acquired. As a result, entrepreneurs are less frequent among the less wealthy and more frequent among the wealthier households. The pronounced spikes or kinks in the likelihood functions can be related to the critical wealth levels, a_{crit} from Table of 5, separating those agents who switch between occupations from those who remain in their profession for each productivity class. Because of the discrete number of productivity states and wealth levels underlying the numerical simulations of our model, the likelihood functions have a non–monotonic shape.

A more general result from Figure 4 is that the likelihood of entrepreneurship increases in individual wealth, regardless of whether or not the individuals are subject to credit constraints. From the viewpoint of our model it is not possible to infer from empirical results, stating a significant effect of individual wealth on entrepreneurship, that this can be interpreted as evidence for binding credit constraints.

[12]For the case of uncorrelated shocks, the likelihood function is a simple step–function assigning zero probability for entrepreneurship to all agents below the critical wealth level; see also Figure 2(b).

5 Concluding Remarks

This paper shows that the interdependence of markets in a dynamic stochastic general equilibrium model and the dynamic properties of the underlying idiosyncratic shocks play an important role in the explanation of the macroeconomic effects of imperfect capital markets in form of financial constraints on the allocation of input factors over sectors, entrepreneurial risk–taking, between–group mobility, wealth accumulation, and inequality.

The stationary wealth distribution generated in the model is consistent with empirical findings. Entrepreneurial households own a substantial share of household wealth and this share increases throughout the wealth distribution.

We derive ambiguous effects on equilibrium occupational choice and average profit incomes for an increase in the tightness of credit constraints, the results crucially depending on the dynamic properties of the idiosyncratic risks. The same is true for our findings regarding between–group mobility which differ strikingly between the two settings. This indicates that the degree of persistence of idiosyncratic shocks gives rise to interesting incentives for individual occupational choice and household accumulation behavior. Regarding exit and entry rates into entrepreneurship, we find that higher persistence of shocks generally increases between–group mobility. Uncorrelated shocks in tendency generate a too low level of mobility, whereas serially correlated shocks lead to mobility too large to match empirical evidence.

There are many important issues this paper does not address. The model lacks a fully microfounded formulation of credit constraints in terms of incentive–compatibility constraints and a more detailed modeling of financial intermediation. By simply stating the changes in the Gini coefficient, our results on inequality are still highly aggregated and should be decomposed in order to find out how good our calibration results on wealth concentration match the distributional data.

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A Computational Issues and Transition Probabilities

The state space of wealth is approximated by a grid of N wealth levels a_n for $n = 1, \cdots, N$ with $a_1 = \underline{a}$ and $a_N = \bar{k}$. The macroeconomic equilibrium is recursively computed. We start with a initial guess on factor prices $\tilde{w}$, $\tilde{r}$, and the equilibrium level of employment in efficiency units $\tilde{L}$. Let $\mu = \{\tilde{w}, \tilde{r}, \tilde{L}\}$ denote the vector of the initial guesses. We obtain factor proportions in the final goods sector from this first solution trial. The underlying production technology implies $\tilde{K}_F = \tilde{L} \frac{\tilde{w}}{\tilde{r}+\delta} \frac{\gamma}{1-\gamma}$. Moreover, $F(\tilde{K}_F, \tilde{L})$ equals $\tilde{L} \left(\frac{\tilde{w}}{\tilde{r}+\delta} \frac{\gamma}{1-\gamma} \right)^{\gamma}$.

Let $k(a_n, s(j)_e)$ denote the firm size of an entrepreneur with productivity $s(j)_e$ and wealth a_n is able to operate at for a given degree of borrowing constraints. His profit is given by

$$\pi[a_n, s(j)_e|\mu] = \alpha\,(B\theta(i)_e\,k(a_n, s(j)_e))^{\alpha}\,\tilde{L}\left(\frac{\tilde{w}}{\tilde{r}+\delta}\frac{\gamma}{1-\gamma}\right)^{\gamma} - (\tilde{r}+\delta)\,k(a_n, s(j)_e)\;.$$

Let $a_w(a_n, s(j)_w|\mu)$ and $\xi_w(a_n, s(j)_w|\mu)$ as well as $a_e(a_n, s(j)_e|\mu)$ and $\xi_e(a_n, s(j)_e|\mu)$ denote the policy functions associated with the optimization problems (10) and (11) for the given initial guess on prices and employment. We characterize agents by their wealth holdings a_n, their occupational status ζ, where $\zeta = 1$ denotes a worker and $\zeta = 2$ an entrepreneur, and their current productivity state $s(j)_h$, $h = e, w$.

Knowing the policy functions and transition matrices for the underlying productivity shocks, we are able to compute the probability for an agent to have wealth a_n, occupational status ζ and productivity state $s(j)$. Let $\psi_{n,\zeta,s}(\mu)$ denote the respective probability for $n = 1,\ldots,N$, $\zeta = 1,2$ and $s(j)_h = \theta_{h,j}$, $j = 1,\ldots,m$, $h = e, w$.

The probabilities $\psi_{n,\zeta,s}(\tilde{w}, \tilde{r}, \tilde{L})$ can be used to compute aggregate quantities. The aggregate capital stock (i.e. mean wealth holdings) can be determined as:

$$K(\mu) = \sum_{n=1}^{N}\sum_{\zeta=1}^{2}\sum_{j=1}^{m}\psi_{n,\zeta,s}(\mu)\,a_n$$

The population share of entrepreneurs results as

$$\lambda(\mu) = \sum_{n=1}^{N}\sum_{j=1}^{m}\psi_{n,2,s}(\mu)$$

while labor supply in efficiency units is given by

$$L(\mu) = \sum_{n=1}^{N}\sum_{j=1}^{m}\psi_{n,1,s}(\mu)\,\theta_{w,j}$$

Capital demand of the intermediate goods sector can be computed as:

$$K_I^D(\mu) = \sum_{n=1}^{N}\sum_{j=1}^{m}\psi_{n,2,s}(\mu)\,k(a_n, s(j)_e)$$

The supply of capital to the final goods sector is given by $K_F^S(\mu) = K(\mu) - K_I^D(\mu)$. Employment L and capital input K_F in the final goods sector generate an aggregate output of

$$Y(K_F, L|\mu) = \left(K_F^{\gamma}, L^{1-\gamma}\right)^{1-\alpha}\sum_{n=1}^{N}\sum_{j=1}^{m}\psi_{n,2,s}(\mu)\,(Bs(j)_e)\,k(a_n, s(j)_e))^{\alpha}$$

The initial solution guess only represents an equilibrium if the following conditions must hold:

Labor supply in efficiency units must equal the initial guess $\tilde{L}$

$$L(\mu) = \tilde{L} \qquad \text{(i)}$$

Labor demand and capital demand in the final goods sector equal their respective supplies:

$$L(\mu) = (1-\alpha)(1-\gamma)\,\frac{Y(K_F^S(\mu), L(\mu)\,|\,\mu)}{\tilde{w}} \tag{ii}$$

$$K_F^S(\mu) = (1-\alpha)\gamma\,\frac{Y(K_F^S(\mu), L(\mu)\,|\,\mu)}{\tilde{r}+\delta} \tag{iii}$$

The algorithm for finding the equilibrium values consists of three nested loops over $\tilde{L}$, $\tilde{w}$ and $\tilde{r}$. The first loop iteratively computes the value $\tilde{L}$ which meets condition (i) for given factor prices $\tilde{w}$ and $\tilde{r}$. Then, factor prices $\tilde{w}$ and $\tilde{r}$ are adjusted according to the resulting excess demands for labor and capital according to conditions (ii) and (iii). The whole procedure is repeated until the equilibrium conditions (i) to (iii) are satisfied, except or a tolerably small approximation error.

To implement the algorithm, we used the programming language C^{++}. The underlying source code is available from the authors upon request.

Table A.1: Transition probabilities of the Markov processes

$$P_w = P_e = \begin{pmatrix} 0.01222 & 0.21440 & 0.54676 & 0.21440 & 0.01222 \\ 0.01222 & 0.21440 & 0.54676 & 0.21440 & 0.01222 \\ 0.01222 & 0.21440 & 0.54676 & 0.21440 & 0.01222 \\ 0.01222 & 0.21440 & 0.54676 & 0.21440 & 0.01222 \\ 0.01222 & 0.21440 & 0.54676 & 0.21440 & 0.01222 \end{pmatrix}$$

(a) Transition probabilities — *iid* shocks

$$P_w = \begin{pmatrix} 0.28689 & 0.61844 & 0.09396 & 0.00072 & 0.00000 \\ 0.04575 & 0.52861 & 0.40605 & 0.01954 & 0.00001 \\ 0.00246 & 0.17179 & 0.65150 & 0.17179 & 0.00246 \\ 0.00001 & 0.01954 & 0.40605 & 0.52861 & 0.04575 \\ 0.00000 & 0.00072 & 0.09396 & 0.61844 & 0.28689 \end{pmatrix}$$

$$P_{w,e} = \begin{pmatrix} 0.02309 & 0.23025 & 0.49331 & 0.23025 & 0.02309 \\ 0.02309 & 0.23025 & 0.49331 & 0.23025 & 0.02309 \\ 0.02309 & 0.23025 & 0.49331 & 0.23025 & 0.02309 \\ 0.02309 & 0.23025 & 0.49331 & 0.23025 & 0.02309 \\ 0.02309 & 0.23025 & 0.49331 & 0.23025 & 0.02309 \end{pmatrix}$$

$$P_e = \begin{pmatrix} 0.59871 & 0.39831 & 0.00298 & 0.00000 & 0.00000 \\ 0.04006 & 0.73331 & 0.22605 & 0.00058 & 0.00000 \\ 0.00008 & 0.10556 & 0.78870 & 0.10556 & 0.00008 \\ 0.00000 & 0.00058 & 0.22605 & 0.73331 & 0.04006 \\ 0.00000 & 0.00000 & 0.00298 & 0.39831 & 0.59871 \end{pmatrix}$$

$$P_{e,w} = \begin{pmatrix} 0.01604 & 0.22153 & 0.52487 & 0.22153 & 0.01604 \\ 0.01604 & 0.22153 & 0.52487 & 0.22153 & 0.01604 \\ 0.01604 & 0.22153 & 0.52487 & 0.22153 & 0.01604 \\ 0.01604 & 0.22153 & 0.52487 & 0.22153 & 0.01604 \\ 0.01604 & 0.22153 & 0.52487 & 0.22153 & 0.01604 \end{pmatrix}$$

(b) Transition probabilities — autocorrelated shocks

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