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Nested stochastic possibility frontiers with heterogeneous capital inputs

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**Georg Erber
Reinhard Madlener**



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for Economic Research

**Nested Stochastic Possibility Frontiers with
Heterogeneous Capital Inputs**

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**Nested Stochastic Possibility Frontiers with
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Berlin, August 2007

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Abstract

This paper studies the productivity impact of heterogeneous capital inputs of selected EU-15 member countries and of the U.S. at the macroeconomic level. The stochastic possibility frontiers approach of Battese and Coelli (1992) applied here is used to identify neutralities or non-neutralities between different heterogeneous capital and labor inputs. Owing to the introduction and estimation of two-stage nested translog possibility production frontiers, the otherwise huge parameter space for the seven input factors included in the model is reduced significantly. This gives more robust estimates of the remaining parameters. Due to the detailed data, specific types of biased technological change in heterogeneous capital inputs can be tested. Furthermore, time-varying inefficiency trajectories for each country are obtainable. Annual data from 1980 to 2004, calculated and published by the Groningen Growth and Development Centre, are used in the empirical analysis. The results obtained shed new light on how fast technological progress in a global economy can shift comparative advantages between countries. In particular the different factor specific impacts of ICT and non-ICT capital stocks give a more detailed picture of the structural dynamics between factor inputs than do most other empirical studies using more aggregate factor input data.

Keywords: nested production possibility frontiers, (in-)efficiency benchmarking, technology adoption, convergence

JEL classification: C23, C51, D24, E23, O33, O47, O57

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1 Introduction

Since the productivity upturn of the mid-1990s in the U.S. (see e.g. Jorgenson et al., 2000; Jorgenson, 2001, 2003), the resurgence of productivity growth has been attributed in particular to the increased production and usage of ICT capital goods. However, most econometric studies on capital inputs have dealt with only two types: on the one hand, information and communications technology (ICT) and, on the other, non-ICT capital. More detailed breakdowns of capital inputs into other heterogeneous types were missing in the majority of them. Whereas one reason was a lack of data, another was the problem of a rapidly increasing parameter space when translog functions are used as flexible functional forms.

The study presented here uses data calculated by the Groningen Growth and Development Centre (GGDC). It distinguishes six different types of capital inputs: (1) information technology (IT), (2) communications technology (CT) and (3) software capital inputs as components of ICT capital, and a breakdown of non-ICT capital into three other capital inputs, namely (4) non-residential structures, (5) transport equipment, and (6) non-ICT equipment capital inputs.

Most empirical studies on the impacts of ICT on productivity and growth are based on growth accounting methods which use Törnqvist indices. The underlying theoretical assumptions are not empirically tested for their validity with respect to the dataset employed. One key assumption of the theoretical models in such growth accounting calculations is that all observed factor inputs are used efficiently. In other words, there is no room for wasted inputs or underutilized factors. This, however, is a very strong assumption, which requires empirical testing.

Contrary to these types of efficient factor market allocation models, the possibility frontier approach used in this study allows some leeway for sticky input factor markets and, consequently, the emergence of inefficiencies in factor usage in production (see e.g. Farrell, 1957; Kumbhakar and Knox Lovell, 2003). Since productivity is defined as a ratio of an output indicator and an input indicator, excessive inputs relative to constant outputs indicate inefficiencies or lower productivity, respectively.

The remainder of this paper is organized as follows: section 2 discusses the theoretical framework, section 3 the models used, section 4 introduces the data, section 5 presents the results, and section 6 concludes.

2 Theoretical framework

The evolution of the theoretical framework that combines the concept of efficient factor allocation, which is based on the production function approach, with the production possibility frontier (PPF) approach, which tries to take into account the unequal real-world capabilities of companies, industries or whole societies in making the best use of available production opportunities, took place in two major steps.

In a first step, the deterministic possibility frontier approach dealt with the concept that there are both leaders and laggards in efficient production by introducing the concept of the “distance from an efficiency frontier”, i.e. the traditional production function, representing the best-practice producer as a measure for inefficiency.

In a second step, the stochastic production possibility frontier (SPF) approach gave way to the idea that even the best practice producers have room for improvement and that efficiency depends not only on deterministic but also on stochastic influences, which makes persistent best practice at the frontier more or less a most unlikely event, even for the leaders in a particular business field.

2.1 Production possibility frontiers

Due to indivisibilities of capital goods, or volatility in output demand production in particular, capacities related to fixed capital stocks cannot be adjusted instantaneously according to actual market conditions. This leads to at least temporary inefficiency until the adjustment process has worked out.

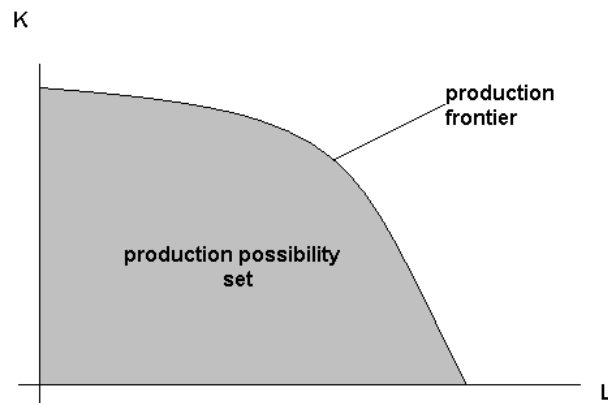
Another source of inefficiencies emerges from the unequal ability to organize the production at each plant with the same degree of efficiency. Furthermore, innovations need learning-by-doing (Arrow, 1962) and learning-by-using (Rosenberg, 1982) effects to become proficient in a certain technology. Entry and exit of new firms also plays an important role in the emergence of changes in the efficiency ranking between firms (see e.g. Aghion and Howitt, 2005).

Impediments at the social and institutional level of a country or region, influencing what is also referred to as the ‘social capability’ (see e.g. Abramovitz, 1986; Abramovitz and David, 1996), make it impossible to obtain efficiency levels and/or efficiency growth rates equal to those of environments elsewhere, which are better suited to encouraging innovation. Encouragement of innovation is traditionally measured by the rate of technical progress and by more

efficient allocation mechanisms (in particular more flexible markets and other institutions), leading to more rapid speeds of adjustment. This has raised the issue of the competitiveness of economic entities at the regional or national level. As a single indicator, productivity has become a common standard in the economics literature for measuring overall economic performance.

The production possibility frontier approach, in contrast to the more traditional production function approach, makes it possible to disentangle the overall productivity growth (see e.g. Acemoğlu et al., 2003) into two components: the rate of technological progress of the frontier, and the movements of single entities from inefficient usage towards the efficiency frontier (see figure 1).

Figure 1
Production possibility set and frontier



If, given the factor input set, the produced output level stays below the potential maximum level, then the respective inefficient use of resources indicates indirectly that the whole production system or, at the micro level the single producer, faces an inability to match the best available practice. Farrell (1957) was the first to distinguish between technical and allocative efficiency. Technical efficiency reflects the ability of a firm to obtain maximal output from a given set of inputs. Allocative efficiency is used for the ability of a firm to use the inputs in optimal proportions, given their respective prices. The combination of both gives a measure of the total economic efficiency.

At the outset of the literature on production possibility frontiers (see, e.g., Aigner and Chu, 1968; Afriat, 1972), it was assumed that the leader of a sample always reached the boundary

of the frontier. Therefore, the term “deterministic production possibility frontier” was used. The best producer, therefore, could not improve his or her performance any further. This view, however, is at least somewhat misrepresentative, as most managers would agree that even being a leader always leaves ample room for further improvement. Similarly, world champions in a sport would never allow themselves to believe that they could not improve their performance, or that others would never top them.

Another criticism relates to the sensitivity of such a frontier to the possible influence of measurement errors and other noise at the frontier (see Timmer, 1971). Estimating a deterministic possibility frontier would therefore not give robust results under such circumstances. Furthermore, excluding the best-practice firm from a random sample would lead to highly biased efficiency estimates. Therefore, it made sense to weaken the deterministic frontier approach by changing the deterministic frontier into a stochastic one (see Aigner et al., 1977).

2.2 Stochastic production possibility frontiers

A stochastic possibility frontier (SPF) introduces a theoretical benchmark which usually cannot be matched by any actual producer. It is a quasi-ideal production frontier which, due to all kinds of impediments in the particular situations of each producer, cannot be matched completely (at least permanently). This gives sufficient incentive for even the best-practice producer to search for further improvements. Assuming for the moment a log-linear production function where i firms produce their output given the technological parameter β , the stochastic possibility frontier is determined by two types of random errors. These are the always-positive new inefficiency random variable u_i and the usual random error term v_i , which has the standard properties of identical, independent, normally distributed errors with mean μ_v , and constant variance σ_v^2 .

The production frontier is therefore determined by the deterministic part plus a stochastic part consisting of a mixture of two probability distributions: a non-negative one, u_i , (e.g., a positive truncated normal distribution) representing stochastic inefficiencies, plus the usual normal distribution of the error term v_i , representing stochastic measurement errors in the data. As a result, the estimation of a stochastic possibility frontier has to address the parameters of the respective production function plus those of the two probability distributions simultaneously.

Non-neutrality of the different heterogeneous capital and labor inputs needs functional forms of production functions which are flexible enough to determine the necessary non-neutrality by introducing parameters to measure it.

The translog production function has become one of the most frequently applied functional structures to offer sufficient flexibility. In contrast, a Cobb-Douglas production function includes no appropriate parameters for modeling non-neutrality of technical change.

By estimating the parameters of a translog-possibility production frontier, all necessary parameters for testing neutrality or non-neutrality of factor usage are available. These two measures of inefficiency are also generated to measure the relative distance of an entity from the possibility frontier. Using this kind of integrated model, it is possible to test for the specific types of biased technological change present in the general macroeconomic possibility frontier under the assumption that inefficiencies are present.

Furthermore, the relative performance of different countries, the current entities, can be benchmarked in terms of their (in-)efficiency at a macroeconomic level.

It is important to distinguish heterogeneous capital inputs (such as IT, communications equipment, software, together with other capital inputs, such as non-ICT equipment and non-residential structures) because they contribute differently to the efficiency improvements of an economy and are, to a different degree, adjustable in the short- and medium-run.

In particular, in our analysis we apply Cobb-Douglas and translog model formulations for both constant and variable returns to scale in a nested two-stage model structure in order to measure the effects of these heterogeneous capital inputs. This is in contrast to other studies, which distinguish only two types of capital, i.e. ICT and non-ICT. The empirical estimates obtained from this type of analysis contribute to a better understanding of how fast technological progress in a global economy shifts the comparative advantages between countries due to both the different timing of ICT adoption and to the dynamics of ICT technology diffusion.

In efficient frontier estimation, different approaches have been used. Apart from the stochastic production possibility frontiers approach, SPF (e.g. Kumbhakar and Knox Lovell, 2003), data envelopment analysis, DEA (e.g. Cooper et al., 2004), has been applied in numerous studies. A more recent development has been to use the so-called generalized maximum entropy approach, GME (see e.g. Golan et al., 1996), which avoids more restrictive distributional assumptions on the stochastic inefficiency term (see section 2.4 below). In our present study, we

have chosen to apply the SPF approach, which tends to give similar results to those of the GME. For a comparison of the different approaches in efficient frontier estimation see e.g. Campell et al. (2005).

Commonly used production functions or possibility frontiers restrict the number of input factors to a small set, e.g. to two or three. The Solow model (1957), for instance, just distinguishes two primary input factors, labor, L , and capital, K , plus a time trend t to represent autonomous Harrod-neutral technical change. This model fitted empirical data for most countries quite well when a Cobb-Douglas production function was used as a specification, i.e.

$$Y_t = f(L_t, K_t, t) = A \cdot e^{\gamma \cdot t} \cdot L_t^\alpha \cdot K_t^{1-\alpha}, \quad (1)$$

where Y denotes output, A is a scaling parameter, γ the rate of technical progress, α the partial output to labor elasticity, and t denotes time (a proxy for autonomous technical change).

Usually, constant returns to scale (CRS) are assumed in macroeconomic production function specifications, which implies that the partial output elasticity to capital is equal to $1 - \alpha$. This assumption has been used with some success in a number of empirical studies (Heal, 1986; Mankiw et al., 1992; Hansen and Knowles, 1998; McCombie and Mark, 2007).

Taking logarithms of equation (1), we obtain the following linear form in the transformed variables and parameters:

$$\ln Y_t = \ln A + \alpha \cdot \ln L_t + (1 - \alpha) \cdot \ln K_t + \gamma \cdot t. \quad (2)$$

Adding the usual two random variables for a stochastic possibility frontier, with

$v_t \sim iid \ N(0; \sigma_v^2)$ denoting the error term, plus the inefficiency random variable

$u_t \sim iid \ N^+(\frac{1}{\theta_u}, \sigma_u^2)$, a term which exhibits a left-truncated normal distribution, and assum-

ing that v_t and u_t are distributed independently of each other and of the regressors (e.g. Kumbhakar and Knox Lovell, 2003, p.74), we obtain the stochastic Cobb-Douglas production frontier

$$\ln Y_t = \ln A + \alpha \cdot \ln L_t + (1 - \alpha) \cdot \ln K_t + \gamma \cdot t + v_t - u_t. \quad (3)$$

One shortcoming of extending the Cobb-Douglas function by including more than two factors is that the implicit substitution elasticity between all factors is always restricted to unity. In

order to avoid this highly restrictive assumption, the constant elasticity of substitution (CES) function, which was suggested as a useful alternative specification by Arrow et al. (1961), has an elasticity of substitution that is constant but not necessarily equal to one. This implies that the elasticity (or complementarity) between input factors becomes measurable.

However, extending this model to a multi-factor approach with $n > 2$, where n denotes the number of input factors, again causes the problem of all factors having a common constant elasticity of substitution. To avoid this situation, the flexible transcendental logarithmic ('translog') functional form introduced by Christensen et al. (1973), which uses a logarithmic Taylor-expansion up to the second order term in the input and output factors of an otherwise unknown function, gives sufficient flexibility to obtain a production function where the substitution elasticities may be different between all input factors:

$$\ln Y_t = \ln A + \alpha \cdot \ln L_t + \beta \cdot \ln K_t + \gamma \cdot t + \beta_{LL} \cdot \ln L_t \cdot \ln L_t + \beta_{LK} \cdot \ln L_t \cdot \ln K_t + \beta_{Lt} \cdot t \cdot \ln L_t + \beta_{Kt} \cdot t \cdot \ln K_t + \beta_{KK} \cdot \ln K_t \cdot \ln K_t + \beta_{tt} \cdot t^2 \quad (4)$$

This degree of generality, however, comes at a price. The parameter space of such translog production functions increases over-linearly and, thus, very often makes this flexible functional form 'too flexible' in empirical applications if the number of input factors increases beyond $n > 3$. In other words, the risk is high that a maximum-likelihood (ML) or least-squares (LS) estimation would fit the data with $n > 3$ too well. The flatness of the estimation function in some dimensions of the parameter space – similarly to the multicollinearity problem – yields parameter estimates that may be way off the true parameters. This is so because of the trade-off between some parameters. These are linked to each other in such a way that any combination of them changes the value of the ML or LS estimation very slightly, and hence the estimation function becomes indifferent inside a huge solution space.

To get rid of this problem, a more parsimonious modeling approach might be more helpful, even if some rather restrictive assumptions have to be imposed. Specifically, since in our investigation we want to investigate heterogeneous capital stocks with six different types of capital, we have to reduce the parameter space of an unrestricted translog production function with six different capital inputs plus labor and time from 44 parameters (i.e. $n = 7 + 1$ plus $n \cdot (n + 1) / 2$) sufficiently in order to avoid these difficulties.¹

¹ By including time as a variable for measuring technological change, the number of input factors increases from six to seven. Adding a constant term already gives eight parameters. Since the parameters of the quadratic-terms

2.3 Nested stochastic production possibility frontiers (NSPF)

The notion of nesting production functions was already emerging in the 1960s (Sato, 1967) in a quest for more flexible forms of multiple factor production functions. The general idea behind it was that there exists an aggregator function, g , which appropriately aggregates some individual factor inputs to an aggregate factor input (Berndt and Christensen, 1973). The overall capital stock is formed by m sub-aggregates, i.e.

$$K_t = g(K_1, \dots, K_m). \quad (5)$$

By substituting the aggregator function into the original multi-factor production function, we obtain a mapping from the higher dimensional space $(n + 1)$ into a lower dimensional space $(n - m + 2)$. If the aggregator function is sufficiently accurate, i.e. such that the input factors are weakly separable (e.g. Leontief, 1947; Blackorby et al., 1978), a perfect aggregator function would have to fulfill the weak separability condition, which can be tested empirically (Berndt and Christensen, 1974). Substituting the aggregator function into the original production possibility frontier, we obtain

$$Y_t = f(L_t, K_t, t) = f(L_t, g(K_1, \dots, K_m), t). \quad (6)$$

The next step we propose here is to substitute, instead of using an exact (i.e. deterministic) aggregator function, a stochastic aggregator function including a simple error term. Moreover, instead of adding the usual random error term, we include an inefficiency random variable again, as we did in the non-nested SPF above, and thus obtain

$$K_t = g(K_1, \dots, K_m) + w_t - z_t, \quad (7)$$

with $w_t \sim iid \ N(0; \sigma_w^2)$ denoting the error term, $z_t \sim iid \ N^+(\frac{1}{\theta_z}, \sigma_z^2)$ the inefficiency random variable exhibiting a left-truncated normal distribution, and the independence assumption between both of these terms and the regressors included in the aggregator function.

But what are the advantages of extending the approach towards nesting an aggregator SPF instead of a standard production function into our model framework? The key comparative advantage relates to the fact that we decompose the overall inefficiency term u_t into two

of a translog function for all seven explanatory variables can be written in a 7 x 7 triangular matrix form, it is easy to see that they can be calculated by the formula given above.

separate inefficiency components, $z_z + \bar{z}_t$, where the first measures the inefficiencies between the input factors included in the aggregator function, and \bar{z}_t the inefficiencies between the input factors of the first level production function and those of the input factor included in the aggregator function. This decomposition, however, is based on an appropriately chosen aggregator function which fulfils the weak separability conditions mentioned above. Such nesting of possibility frontiers in order to obtain decomposition of inefficiencies by different input factor groupings seems to be an original contribution to the existing literature.

Furthermore, if we do not impose assumptions concerning the independence of the respective error terms and inefficiency terms we can check *ex post* whether the estimates obtained exhibit large covariances between the respective random variables. Because the whole model structure is strictly recursive in the model variables, we do not have to worry about simultaneous equation biases resulting from interdependencies.

Summing up, the nested production possibility frontier approach, outlined above, offers several interesting new features compared to other approaches that try to derive a decomposition of inefficiency terms by single factors using a dual function of a cost or restricted profit function (see Kumbhakar and Knox Lovell, 2003, p.170 ff.). However, this particular approach will not even be theoretically equivalent to such simultaneous factor demand equation approaches. This is due to the lack of self-duality² of this general flexible functional form, and of NSPFs in particular.

2.4 Decomposition of inefficiencies in NSPFs

Taking a two-level NSPF model as described in section 2.2, we can write for the first stage a translog specification of the form

$$\ln Y_t = \ln A + \alpha \cdot \ln L_t + (1 - \alpha) \cdot \ln K_t + \gamma \cdot t + v_t - u_t \quad (8)$$

and for the second level

$$\ln K_t = g(\ln K_1, \dots, \ln K_m) + w_t - z_t. \quad (9)$$

² Self-duality is a term used for functional relations where the functional form, e.g. Cobb-Douglas, prevails in the quantity as well as in the price space. Cobb-Douglas or CES functional forms are self-dual when applied as production or minimum cost functions, e.g. $Y = A \cdot e^{\gamma \cdot t} \cdot L^\alpha \cdot K^{1-\alpha} \Leftrightarrow C = \tilde{A} \cdot e^{\tilde{\gamma} \cdot t} \cdot w^{\tilde{\alpha}} \cdot r^{1-\tilde{\alpha}} \cdot Y$. The variables w and r denote the respective factor prices for labour and capital.

By substituting the second level NSPF into the first-level NSPF, we obtain

$$\ln Y_t = \ln A + \alpha \cdot \ln L_t + (1 - \alpha) \cdot (\beta_1 \cdot \ln K_{1,t} + \beta_2 \cdot \ln K_{2,t} + w_t + z_t) + \gamma \cdot t + v_t - u_t \quad (10)$$

and by reparameterization

$$\ln Y_t = \ln A + \alpha \cdot \ln L_t + \tilde{\beta}_1 \cdot \ln K_{1,t} + \tilde{\beta}_2 \cdot \ln K_{2,t} + \gamma \cdot t + (1 - \alpha) \cdot w_t + (1 - \alpha) \cdot z_t + v_t - u_t \quad (11)$$

For reasons of expositional simplicity, we have omitted the quadratic terms for the factor inputs and the technical progress term t , but these can, of course, be added later with no difficulty.

Since the random variables w_t and v_t are normally distributed, the sum of both will add up to another normally distributed random variable x_t , $x_t = (1 - \alpha) \cdot w_t + v_t$, with an expectation

$$E(x_t) = (1 - \alpha) \cdot E(w_t) + E(v_t) = (1 - \alpha) \cdot \mu_w + \mu_v \quad (12)$$

and a variance

$$\begin{aligned} VAR(x_t) &= (1 - \alpha)^2 \cdot VAR(w_t) + VAR(v_t) + 2 \cdot (1 - \alpha) \cdot COV(w_t, v_t) \\ &= (1 - \alpha)^2 \cdot \sigma_w^2 + \sigma_v^2 + 2 \cdot (1 - \alpha) \cdot \sigma_{w,v}. \end{aligned} \quad (13)$$

Similarly, the half-normally distributed random variables, z_t and u_t , sum up to a random variable y_t , $y_t = (1 - \alpha) \cdot z_t + u_t$, which is again a half-normally distributed random variable with the expectation

$$E(y_t) = (1 - \alpha) \cdot E(z_t) + E(u_t) = \frac{1 - \alpha}{\theta_z} + \frac{1}{\theta_u}, \quad (14)$$

where θ_z and θ_u are the respective integration constants limited to the domain $u_t, z_t \in [0, \infty)$ of the univariate truncated normal distribution variance (see e.g. del Castillo, 1994). The variance of this random variable turns out to be

$$\begin{aligned} VAR(y_t) &= (1 - \alpha)^2 \cdot VAR(z_t) + VAR(u_t) + 2 \cdot (1 - \alpha) \cdot COV(z_t, u_t), \\ &= \frac{(1 - \alpha)^2 \cdot (\pi - 2)}{2 \cdot \theta_z} + \frac{(\pi - 2)}{2 \cdot \theta_u} + 2 \cdot (1 - \alpha) \cdot \sigma_{z,u}. \end{aligned} \quad (15)$$

Since the expectation of the inefficiency random variable y_i is equal to the sum of the expectation of the first-level SPF plus the expectation of the second-level SPF, we have obtained a decomposition of the overall inefficiency term into the inefficiency attributable to the first-level SPF factor allocation, and the inefficiency attributable to the second-level SPF factor allocation. Note that this result can easily be extended to more sophisticated NSPFs (i.e. those with more than two stages), or multiple aggregator functions at a particular level for different subsets. Thus, the introduction of an NSPF has a reasonable economic interpretation, since it provides a decomposition of the overall inefficiency into partial inefficiency components attributable to the different levels of the NSPF.

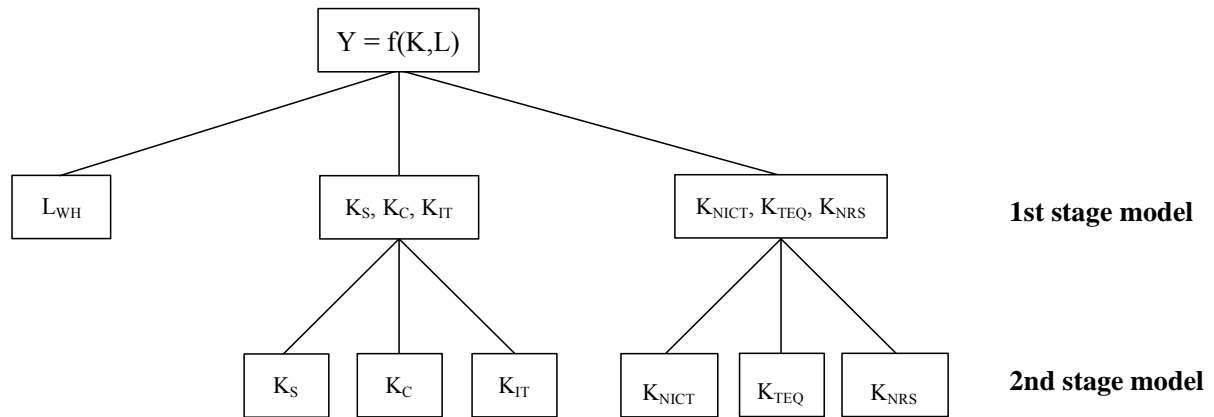
3 Models used

In our analysis, we applied several different models that are based on the translog functional approach. Since the macroeconomic dataset employed covers seven different input factors, and separates different types of capital inputs besides the labor input variable, we used a Cobb-Douglas specification for all seven input factors as a first-stage NSPF. In a second step, we separated the seven factors into three subsets. The first consists of the labor input variable, the total ICT capital stock, and the non-ICT capital stock as aggregates of the two subsets used in the second-stage stochastic frontier. For the second stage, we used two subsets of three separate capital stock variables in each of them. The first includes IT capital, communications technology capital, and software capital. The second contains non-ICT capital, transport equipment, and non-residential structures in each single economy (see figure 2).

For the econometric estimation with a 16-country balanced panel dataset plus the EU-15 aggregate, we organized the data separately for each subset of the NSPF (see section 4 and appendix A for a more detailed data description). For the estimation of the stochastic possibility frontiers, we used the software package Frontier (Coelli, 1996).³ Apart from the seven input factors, a time trend was included to account for Harrod-neutral technical change in each stochastic frontier equation.

³ For the empirical estimations we used both Frontier 4.1 and EViews 4. EViews was applied to check the results obtained by Frontier for determining the initial values from an OLS estimation of an ordinary production function. For reasons of convenience, in 'general-to-specific' modeling (by successively eliminating insignificant parameters) for the translog model specification, EViews was more helpful in this kind of specification search.

Figure 2

Nested structure of the SPF model

Notation: Y ... Output, K ... Capital, L ... Labor, WH ... Work hours, S ... Software, C ... Communication, IT ... Information technologies, NICT ... Non-ICT (non-information & communications technologies), TEQ ... Transport equipment, NRS ... Non-residential structures.

In order to obtain time-varying inefficiencies or efficiencies for each single country, we estimated the model by using the frontier error component model introduced by Battese and Coelli (1992). From previous studies with macroeconomic multi-country panel data on an industry level (cf. Erber, 2005), we know that inefficiencies can vary considerably over time. Estimating a simple static inefficiency model – and in doing so, just determining an average degree of inefficiency over the whole sample period – might be grossly misleading with respect to the inefficiency dynamics inherent in the data. Additionally, for estimating the Cobb-Douglas possibility frontier, the frontier error component model does not only have to estimate the parameter of the respective half-normal distribution (e.g. σ_u^2) for the first stage frontier but, additionally, has to estimate an adjustment parameter, η_u , that is related to the adjustment process of inefficiency for the respective country panel. Note that the different inefficiency trajectories obtained for each single country are formed by the general adjustment parameter, which is inherent to the production possibility frontier, and by the respective factor inputs for each single country, which jointly determine the inefficiency trajectories (see appendix B for a selection of these inefficiency time series obtained). Analogously, the two stochastic aggregator possibility frontiers were estimated separately. For the estimation, we did not impose constant returns to scale (CRS), but we estimated all models by imposing this restriction as well.

Additionally, in a further step, we extended the Cobb-Douglas frontier model to a full translog stochastic possibility frontier model for each single stage. Since in a multi-factor production function the substitution elasticities should not be restricted to unity, as is the case when using a Cobb-Douglas function, we expected to get more consistent results from the perspective of economic theory. However, the estimates for the inefficiencies turned out to be problematic because the models fitted the data without an inefficiency term so well that there was very little leeway left for inefficiency modeling. While this problem requires further investigation, an explanation and a solution are beyond the scope of the present paper.

4 Data

In our empirical estimations, we use a balanced 16-country panel dataset for the years 1980-2004, extracted from a database provided by the Groningen Growth and Development Centre (Timmer et al., 2005). Countries included in the panel data are the fifteen EU member states before the Eastern enlargement, i.e. Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the U.K. As another potential benchmark country, the U.S. was included as well.

Instead of using growth rates, we calculated from these cumulative indices for all output and input variables. Absolute level data are unavailable in the GGDC dataset. Even if they would have been supplied the problem of making the different time series for each country comparable as level data would have made it necessary to use purchasing power parities (PPPs) in order to shift the data to joint absolute levels of common PPPs for each input and output variable. The methodological problems to be solved for making this kind of analysis feasible are discussed in greater detail by Caves et al. (1982) (see also Conrad, 1985; Erber, 1993; Bernard and Jones, 1996; and Soerensen, 2001). As a reference system for a multilateral comparison, multilateral invariant PPPs are necessary instead of bilateral PPPs. Due to these still unsolved problems, the present analysis is less ambitious, but also faces less measurement problems. Therefore, our analysis studies the production possibilities of the different countries over time, but cannot calibrate these changes to a joint absolute level.

The data comprise the ICT capital stock, non-ICT capital stock, total factor productivity (TFP), labor input, and the change in the quality of labor input by indices with 1970 as the base year (see Appendix A for a more detailed data description). Growth in economic output

is measured in terms of the gross domestic product, or GDP (in 2000 prices). However, the output is, again, an index series calculated in the same fashion as the input series.

5 Empirical results

Due to the large size of the parameter estimates obtained and the inefficiency trajectories for the whole multi-country panel, we only present the most relevant results (more detailed results can be obtained from the authors upon request). Because the most interesting aspects of the dataset from our perspective are attributable to the heterogeneity of the different types of capital stocks, we focus on the results from the second-stage NSPFs. Table 1 depicts the estimation results for the first- and second-stage NSPF model (Cobb-Douglas error component specification).

Looking at the outcomes of the first- and second-stage estimates of the SPF using a Cobb-Douglas functional form, we notice that all parameters of the factor inputs have positive output elasticities (as expected by theory) that lie between zero and unity (see table 1). Adding up the three parameter estimates $\beta_1, \beta_2, \beta_3$, we obtain the point estimate for the scale elasticity r for the first stage.⁴ A value of 1.16 indicates significant increasing returns to scale with a likelihood-ratio (LR) test statistic of 0.998. However, the parameter of the Harrod-neutral technical change term turns out to be statistically insignificant and close to zero. This might be attributable to the static specification of our model which cannot properly disentangle short-run from long-run effects (see e.g. Meijers, 2007, Erber, 2005). As is well known from the literature, in general it is difficult to disentangle scale economies from the rate of Harrod-neutral technical progress. The model of the first stage could be modified by imposing constant returns to scale. Additionally, the parameter $\gamma = \sigma_u^2 / (\sigma_v^2 + \sigma_u^2)$, which is calculated instead of an explicit estimate for σ_u^2 , shows that there is a problem with the separation in the two stochastic variables u and v . A sufficient lack of variation in the data might cause problems when disentangling the different statistical effects as desired.

⁴ For a Cobb-Douglas production function, the scale elasticity r is determined by equation

$$\lambda^r \cdot Y = A \cdot e^{g \cdot t} \cdot (\lambda \cdot WH)^{\beta_1} \cdot (\lambda \cdot K_{ICT})^{\beta_2} \cdot (\lambda \cdot K_{NICT})^{\beta_3} = A \cdot e^{g \cdot t} \cdot \lambda^{\beta_1 + \beta_2 + \beta_3} \cdot WH^{\beta_1} \cdot K_{ICT}^{\beta_2} \cdot K_{NICT}^{\beta_3}.$$

Table 1

Parameter estimates of the first- and second-stage Cobb-Douglas NSPF error component model, ICT and non-ICT capital services, 1980 - 2004

1st Stage			2nd Stage, ICTS			2nd Stage, NICTTN		
Constant:	β_0	1.70 (4.1)	Constant:	β_0	2.03 (22.1)	Constant:	β_0	2.10 (24.0)
WH:	β_{1j}	0.67 (14.1)	IT Capital:	β_{1j}	0.32 (15.7)	NICT Capital:	β_{1j}	0.34 (18.7)
ICTS Capital:	β_{2j}	0.13 (8.2)	C Capital:	β_{2j}	0.49 (30.1)	TEQ Capital:	β_{2j}	0.47 (31.8)
NICTTN Capital:	β_{3j}	0.36 (8.7)	S Capital:	β_{3j}	0.16 (10.2)	NRS Capital:	β_{3j}	0.17 (11.1)
Time:	β_{4j}	0.003 (1.8)	Time:	β_{4j}	-0.025 (-7.1)	Time:	β_{4j}	-0.029 (-10.8)
	σ^2	1.95 (3.8)		σ^2	0.029 (3.8)		σ^2	0.033 (3.7)
	γ	0.999 (5.5)		γ	0.596 (5.5)		γ	0.648 (6.6)
	η	-0.0020 (9.8)		η	0.0497 (9.8)		η	0.0498 (11.3)
Log likelihood		576.3			308.7			308.3
No. of iterations		56			23			26

Notes: *t*-values in parentheses. ICTS denotes ICT capital services (information and communications technology plus software capital), NICTTN the category non-ICT capital services (i.e. non-IT equipment plus transport equipment and non-residential structures).

A look at the results for the second-stage possibility frontiers shows that the results are much more promising. The returns to scale obtained from the summation of the estimated betas are 0.97 for the ICT capital stock SPF, and 0.98 for the non-ICT capital stock SPF. Again, it would be justified to estimate the model by imposing constant returns to scale.

Next, we present the parameter estimates for the translog stochastic production frontier of the ICT capital stock. Table 2 shows the estimation results for the second-stage translog frontier of the ICT capital stock. As can be seen from the parameter estimates obtained, the returns to scale estimate is now 0.99, taking the first-order terms as a benchmark. However, because of the positive second-order terms on the quadratic approximation, this will tend to increase with increasing overall factor inputs.

The cross-product terms of the translog function show that some complementarity exists between the IT and the communication capital stocks. In contrast, both of them have a substitution elasticity with software that indicates a certain degree of substitutability between the IT capital and communication capital stocks on the one hand, and software capital on the other hand. This is a plausible outcome because, as is well known from anecdotic evidence at the micro level, it is common practice to substitute software and hardware solutions in IT and communication solutions. We will calculate the average substitution elasticities for the respective factor inputs in order to get a better understanding of the model structure and its economic implications. Note that, due to the unfavorable ratio between the number of parameters to be estimated (44) and the number of observations (425), we refrained from testing for weak separability.

Table 2

Parameter estimates of the second stage translog function for the ICT capital services stock, 1980 - 2004

	1 st order terms	2 nd order terms			
		IT Capital: β_{i1}	C Capital: β_{i2}	S Capital: β_{i3}	Time: β_{i4}
Constant: β_0	1.01 44.2				
IT Capital: $\beta_{1,j}$	0.20 (18.0)	0.0455 (21.9)	0.0148 (26.5)	-0.0011 (-3.1)	-0.0348 (-10.7)
C Capital: $\beta_{2,j}$	0.44 (34.4)		0.0723 (39.9)	-0.0134 (-24.5)	-0.0548 (-29.4)
S Capital: $\beta_{3,j}$	0.35 50.4			0.0724 (65.1)	-0.1011 (-60.3)
Time: $\beta_{4,j}$	-0.0071 (-5.6)				-0.0002 (-5.3)
σ^2	0.000167				
Log likelihood	1,252.8				

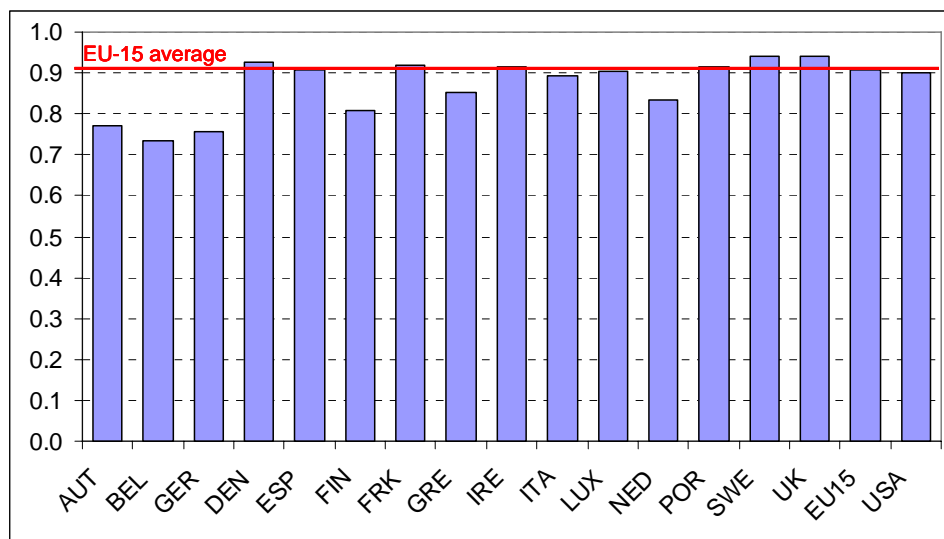
Notes: *t*-values in parentheses. IT ... information and communications technology, C ... communication equipment, S ... software equipment

Figures 3 and 4 show the technical efficiency estimates for the (selected) EU-15 countries plus the U.S. The efficiency frontiers are represented as averages over the time period 1980-2004. As can be seen from figure 3, the EU-15 as a whole and the U.S. exhibit an almost

equal average efficiency score. Within the EU-15, Denmark, France, Ireland, Portugal, Spain, Sweden, and the U.K. feature above-average efficiencies.

Figure 3

Technical efficiency estimates for the EU-15 Member States and the U.S., 1980 - 2004 (average values)



Source: GGDC data, own calculations

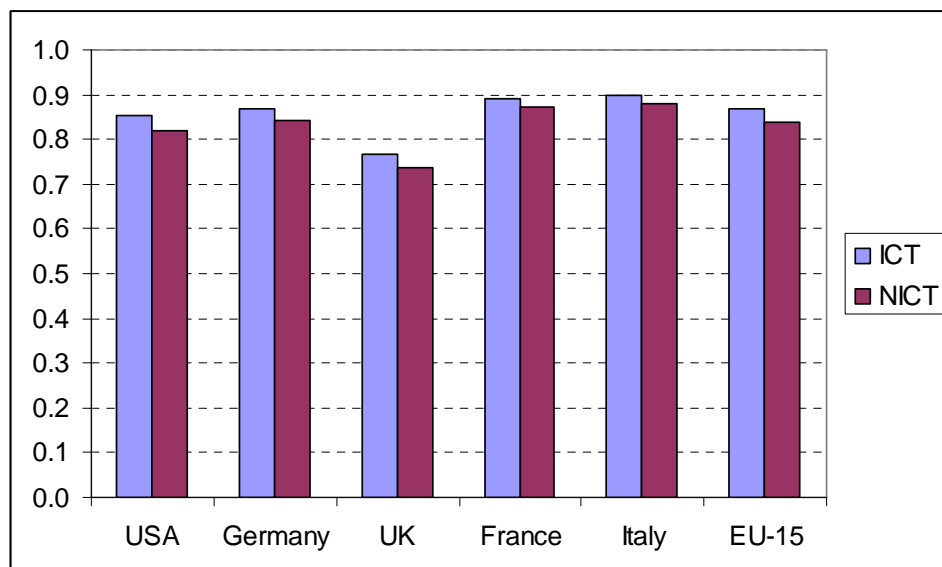
Note: Estimates based on model parameters of second-stage SPFs

Accordingly, the SPF approach can therefore be used for benchmarking a country's efficiency in the use of factor inputs, and thus for obtaining a ranking by country.

In figure 4 we compare the average efficiency difference between selected countries, in particular between the 'Big Four' in Europe (France, Germany, Italy, and the U.K.) with the U.S. in the two second-stage SPFs. An interesting result is that the efficiency levels of the ICT capital stocks are always higher than those of the non-ICT capital stocks. This seems to be a reasonable outcome because the factor allocation between non-ICT capital, transport equipment and non-residential structures is obviously much harder to accomplish than between the three ICT capital stock inputs.

Figure 4

Technical efficiency estimates for selected EU-15 Member States and the U.S., ICT vs. non-ICT capital services, 1980 - 2004 (average values)



Source: GGDC data, own calculations

Note: Estimates based on model parameters of second-stage SPFs

The results already look promising, in that the nested stochastic possibility frontier approach does indeed provide economically meaningful insights into the factor allocation process of an economy at the aggregate level.

6 Conclusions

The aim of this paper was to extend the stochastic possibility frontiers approach in the direction of breaking down a large set of factor inputs into several nested subsets. This would seem to be necessary in many situations where the number of input factors exceeds three, and where the application of a flexible functional form, such as the translog production function, is deemed desirable. Due to the large number of parameters the ability to estimate all these parameters in an economically meaningful way was not fruitful in most empirical applications in the past. In the case of an insufficient number of observations, alternative strategies have to be searched for. The nesting of production functions aimed at reducing the number of parameters to be estimated at each stage has been common practice in a number of applications. A reasonable theoretical foundation can be provided by the separability literature, as outlined in section 2 of this paper. We extend the stochastic possibility frontier approach in a similar

fashion by introducing the nested stochastic possibility frontiers (NSPFs) as an alternative method. Apart from the ability to reduce the parameter space, the NSPF approach offers new insights into the factor allocation problem, since the overall inefficiency in a production process can be decomposed into partial inefficiencies related to the allocation of particular factor input bundles.

One interesting finding of our analysis is that inefficiencies differ between the factor allocation of ICT capital inputs and of non-ICT capital inputs. Therefore, a shift of capital investment from non-ICT capital towards ICT capital will most likely tend to be efficiency-increasing overall. Thus, reducing inefficiencies in the overall capital stock allocation might contribute to faster productivity growth. This is in line with microeconomic studies, which found that ICT capital is significantly efficiency-enhancing at the firm level (Brynjolfsson and Hitt, 2003). Up to now, stochastic possibility frontiers have been used much more often in a microeconomic environment, and much less so for the analysis of subsectoral, industry, or even macroeconomic data. The results obtained from our analysis show that this new area of application can, in principle, produce meaningful results and could contribute to the empirical literature on (in-)efficiency benchmarking of national economies.

One first insight from the econometric analysis is that stochastic frontier estimations, and the inefficiency measures derived in particular, depend heavily on the quality of the parameter estimates of the underlying production function. If there is an over-fitting of the production function, as it often occurs with flexible functional forms, then the inefficiency estimates become highly sensitive to those parameter estimates determining the production frontier. In an extreme case the quadratic terms of the translog function are sufficient for explaining almost the entire variance contained in the data. This happened even in the case of the large GGDC data sample used in our investigation (425 observations). Why this happens will need further analysis, and might, in fact, be attributable to the consistent aggregation of all factor input variables by Tornqvist indices. As is well known from the literature, the Tornqvist index is an alternative representation of the translog production function (cf. Diewert, 1976, 1978), which might restrict the usability of the translog function in econometric analysis employing data constructed with the help of Tornqvist indices.

However, as other studies using the same dataset show (e.g. Venturini, 2006), it remains a difficult undertaking to obtain a suitable model for multi-country panel datasets because of possible weaknesses in the database as well as limitations regarding the size of the data sam-

ple. Venturini's findings suggest that trend-stationarity and short- and long-term substitution elasticities do not differ much, a result that underlines the suitability of our NSPF modeling approach, which essentially ignores dynamics. Therefore, our results seem to be robust over different model specifications and applications. In future research, we will aim at establishing a fully fledged translog nested possibility frontier (TNPF) model. The availability of a new dataset from the EU project KLEMS (www.euklems.net), where both the sample size and the number of countries included have increased, might help to get more efficient and consistent estimates from TNPF model approaches.

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Appendix A – Data description

The database maintained by the Groningen Growth and Development Centre (GGDC) comprises the following variables (for the period 1980-2004):

A) Basic Data for Growth Accounting

- Growth in labor input (total hours (in millions), annual hours per worker, total no. of workers) (in thousands);⁵
- Growth in output, measured by the gross domestic product (GDP), in 2000 prices (in millions €)⁶;
- Share of labor, IT capital, and non-IT capital in GDP;
- Growth in ICT capital service (computers, communication equipment and software);
- Growth in non-ICT capital services (non-IT equipment, non-residential structures, transport equipment);
- Growth of total factor productivity (TFP) (in %)⁷.

B) Gross Fixed Capital Formation, in constant 2000 prices and current prices (in millions €)

- IT equipment (*KIT*), communication equipment (*KC*), non-ICT equipment (*KNICT*), transport equipment (*KTEQ*), non-residential structures (*KNRS*), software (*KS*), total.

C) Gross Fixed Capital Stock, midyear, in constant 2000 prices (in millions €)

- IT equipment, communication equipment, non-ICT equipment, transport equipment, non-residential structures, software, total.

D) Factor input compensation shares.

Of the data listed, we have made use of the six capital stock variables listed under heading B, work hours, and GDP as index time series normalized to 100 for the first year of the observation period.

⁵ Labor input measures are based on total persons engaged, including own-account and family workers alongside employees.

⁶ In contrast to GDP at *current* prices, GDP at *constant* prices excludes imputed rents and rents paid.

⁷ All growth rates are exponential growth rates.

Appendix B – Estimation results

Table B.1

Technical efficiency estimates¹ in ICT capital stocks in the U.S. and selected EU member countries, 1980 - 2004

	USA	Germany	UK	France	Italy	EU15
1980	0.759	0.783	0.629	0.818	0.831	0.784
1981	0.769	0.792	0.643	0.826	0.839	0.793
1982	0.779	0.801	0.657	0.833	0.846	0.802
1983	0.788	0.810	0.671	0.841	0.853	0.811
1984	0.798	0.818	0.684	0.848	0.859	0.819
1985	0.806	0.826	0.696	0.855	0.866	0.827
1986	0.815	0.834	0.709	0.861	0.872	0.835
1987	0.823	0.841	0.721	0.867	0.877	0.842
1988	0.831	0.848	0.732	0.873	0.883	0.849
1989	0.838	0.855	0.743	0.879	0.888	0.856
1990	0.845	0.861	0.754	0.885	0.893	0.862
1991	0.852	0.868	0.764	0.890	0.898	0.868
1992	0.859	0.874	0.774	0.895	0.903	0.874
1993	0.865	0.879	0.784	0.900	0.907	0.880
1994	0.871	0.885	0.793	0.904	0.912	0.886
1995	0.877	0.890	0.802	0.909	0.916	0.891
1996	0.883	0.895	0.811	0.913	0.920	0.896
1997	0.888	0.900	0.819	0.917	0.923	0.901
1998	0.893	0.904	0.827	0.921	0.927	0.905
1999	0.898	0.909	0.835	0.925	0.930	0.909
2000	0.903	0.913	0.842	0.928	0.934	0.914
2001	0.907	0.917	0.849	0.931	0.937	0.918
2002	0.912	0.921	0.856	0.935	0.940	0.921
2003	0.916	0.925	0.862	0.938	0.943	0.925
2004	0.920	0.928	0.869	0.941	0.945	0.929
1980 - 2004	0.852	0.867	0.765	0.889	0.898	0.868

¹ Estimates based on model parameters of second-stage SPFs.

Table B.2

Technical efficiency estimates¹ in non-ICT capital stocks in the U.S. and selected EU member countries, 1980 - 2004

	USA	Germany	UK	France	Italy	EU15
1980	0.710	0.746	0.589	0.790	0.803	0.738
1981	0.721	0.756	0.605	0.799	0.811	0.749
1982	0.733	0.767	0.619	0.808	0.820	0.759
1983	0.744	0.777	0.634	0.816	0.828	0.769
1984	0.755	0.786	0.648	0.824	0.835	0.779
1985	0.765	0.795	0.662	0.832	0.843	0.789
1986	0.775	0.804	0.675	0.839	0.850	0.798
1987	0.785	0.813	0.688	0.847	0.856	0.807
1988	0.794	0.821	0.701	0.853	0.863	0.815
1989	0.803	0.829	0.713	0.860	0.869	0.823
1990	0.812	0.837	0.725	0.866	0.875	0.831
1991	0.820	0.844	0.736	0.872	0.881	0.839
1992	0.828	0.851	0.747	0.878	0.886	0.846
1993	0.836	0.857	0.758	0.884	0.891	0.853
1994	0.843	0.864	0.768	0.889	0.896	0.859
1995	0.850	0.870	0.778	0.894	0.901	0.866
1996	0.857	0.876	0.788	0.899	0.906	0.872
1997	0.863	0.882	0.797	0.904	0.910	0.878
1998	0.869	0.887	0.806	0.908	0.914	0.883
1999	0.875	0.892	0.814	0.912	0.918	0.888
2000	0.881	0.897	0.822	0.916	0.922	0.894
2001	0.886	0.902	0.830	0.920	0.926	0.899
2002	0.892	0.906	0.838	0.924	0.929	0.903
2003	0.897	0.911	0.845	0.928	0.932	0.908
2004	0.901	0.915	0.852	0.931	0.936	0.912
1980 - 2004	0.820	0.843	0.738	0.872	0.880	0.838

¹ Estimates based on model parameters of second-stage SPFs.