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## How reasonable is the reasonable royalty rate?: Damage rules and probabilistic intellectual property rights

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HOW REASONABLE IS THE 'REASONABLE'  
ROYALTY RATE?  
DAMAGE RULES AND PROBABILISTIC  
INTELLECTUAL PROPERTY RIGHTS

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# HOW REASONABLE IS THE ‘REASONABLE’ ROYALTY RATE? DAMAGE RULES AND PROBABILISTIC INTELLECTUAL PROPERTY RIGHTS

## Abstract

This paper investigates how different damage rules in patent infringement cases shape competition when intellectual property rights are *probabilistic*. I develop a simple model of oligopolistic competition to compare two main liability doctrines that have been used in the US to assess infringement damages – the unjust enrichment rule and the lost profit rule. It also points out the logical inconsistency in the concept of the “reasonable royalty rates” when intellectual property rights are not ironclad.

JEL Code: O3, L1, L4, D8, K4.

Keywords: probabilistic intellectual property rights, damage rules, reasonable royalty rates.

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## I. Introduction

This paper investigates how different damage rules in patent infringement cases shape competition when intellectual property rights are *probabilistic*. Most of the literature on patent protection assumes ironclad patents and no uncertainty regarding patent claims.<sup>1</sup> The analysis of damage rules in the literature also seems to implicitly assume no uncertainty. This is puzzling in that there would be no infringement to speak of with ironclad patents under the damage rules adopted in the US and analyzed below. Therefore, I develop a simple model of oligopolistic competition that incorporates the probabilistic nature of patents. I also point out the logical inconsistency in the concept of the “reasonable royalty rates” when intellectual property rights are not ironclad.

Patent infringement damages are intended to protect intellectual property rights and compensate for the pecuniary loss that the patentholder has suffered from the infringement. In the US, there are two main liability doctrines that have been used to assess infringement damages. The “unjust enrichment” rule aims at deterring theft of intellectual property right by punishing the infringer who is required to disgorge all the profits from the infringement. This doctrine was mainly used in the assessment of damages up until the 1946 Amendment of Patent Act. Since then US courts have shifted towards the “lost profit” doctrine that is compensatory in nature. It intends to make the patentee “whole” by enforcing the defendant to make up for the difference between the patentee’s pecuniary condition that would have been without infringement and the one after the infringement.<sup>2</sup> Often the courts seem to conclude that all these approaches yield more or less the same estimate or similar effects, if implemented correctly. The aim of

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<sup>1</sup> Recently, however, more attention has been paid to the probabilistic nature of patent protection and its implications for competition. See Lemley and Shapiro (2005) for recent analyses of probabilistic patent protection.

<sup>2</sup> Currently in the US, economic damages in patent infringement litigation are based on Title 35, Section 284 of U.S. Code, which mandates that damages be adequate to compensate for the infringement, but no less than a reasonable royalty rate for the use of the subject patented invention.

this paper is to analyze how these different damage rules affect competition in different ways and to understand what factors derive the differences.

Considering the recent explosion in patent litigation and increasingly important role of intellectual property rights as a competitive strategy, it is important to understand the impact of different damage rules on market competition.<sup>3</sup> Even though there is a long standing interest and extensive discussions on patent damage rules in the law literature, formal and rigorous economics analyses on this issue are virtually non-existent with the exceptions of Shankerman and Scotchmer (2001) and Anton and Yao (forthcoming).

More specifically, I consider a duopolistic competition with a patent holder of product innovation and a potential infringer. Until recently, the existing literature on innovation typically assumed ironclad patents that are assumed to be valid with certainty and a well-defined scope of protection. In reality, however, most patents issued face a significant amount of uncertainty in terms of their commercial value, validity, and scope of protection. I thus develop a model that explicitly accounts for the uncertain nature of patents.<sup>4</sup> In fact, in my basic model which assumes product innovation and equal production cost, there will be no infringement under the damage rules analyzed below if ironclad patents are assumed. Both the patent holder and potential infringer are aware of the probabilistic nature of patents and compete in the shadow of litigation in that the amount of damages to be paid in the case of infringement depend on the strategies taken in the market place. The set-up of the model reflects the fact that a significant number of infringements can go undetected for more than a nominal period of time and the resolution of disputes entails significant delays in the court system.<sup>5</sup>

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<sup>3</sup> See Bessen and Meurer (2005).

<sup>4</sup> See Lemley and Shapiro (2005) for an excellent discussion of probabilistic patents. They discuss implications of patent litigation uncertainty and potential reforms of the current patent system in the US. However, they do not analyze and compare the effects of different damage rules on market competition.

<sup>5</sup> See Crampes and Langinier (2002) for a model in which the patentholder invests in monitoring to supervise the market and detect infringement.

The main model of Shankerman and Scotchmer (2001) considers a vertical relationship in which a patent on research tools is licensed to a potential infringer who can develop a commercial product. However, they analyze ironclad patents and show that the lost profit/reasonable royalty rate damage rule suffers from a multiplicity of equilibrium due to the circularity of logic inherent in the concept. In contrast, I consider a *probabilistic* patent and the non-existence of a “reasonable” royalty rate that is consistent with the logic.<sup>6</sup>

This paper is very closely related to Anton and Yao (forthcoming) who independently developed an equilibrium oligopoly model of patent infringement in which they analyze the impact of patent infringement damages on market competition. They consider a process innovation and provide an in-depth analysis of the lost profit measure of damages. In contrast, I analyze a product innovation and the focus of my paper is on the comparison of different damage rules.<sup>7</sup> The difference in the nature of innovation – product or process innovation – across these two papers turns out to be important. The process innovation implicitly assumes the availability of substitute technologies. In particular, it allows a “passive” form of infringement under the lost profit rule, in which the imitator infringes and produces at a lower cost, but at the output level that would have been produced without infringement. This type of infringement can lead to no lost profits for the patentholder and complicates their analysis. However, such infringement strategy is ruled out under product innovation. In sum, my paper and Anton and Yao (2005) focus on different types of innovation and different aspects of damage rules. These two papers

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<sup>6</sup> Shankerman and Scotchmer (2001) also consider a model in which the patentholder is able to develop the commercial product herself and infringement can lead to a race between the patentholder and a potential infringer. However, competition takes place in the R&D stage and there is no competition in the product market.

<sup>7</sup> Even though Anton and Yao (forthcoming) also analyze alternative damage rules, they are of secondary importance.

thus can be viewed as complementary in that taken together they provide a more complete picture of the impact of damage rules on competition.

In a different vein, Ayres and Klemperer (1999) argue in favor of denying immediate injunctive relief and substituting delayed probabilistic determination with monetary damages. They show that delay and uncertainty restrict patentees' market power and induce limited infringement without substantially undermining patentees' incentives to innovate. In addition, any shortfall in the patentees' profits due to limited infringement can be compensated by lengthening the length of the patent. Their argument is based on the logic of the envelope theorem and the "Ramsey intuition."<sup>8</sup> In this paper, I do not address the relative merits of delayed damage rules with uncertainty vis-à-vis injunctive relief. Instead, I take the uncertainty associated with the current damage system and substantial delay until the resolution of dispute as given, and compare the effects of different damage rules on interim competition.

The remainder of the paper is organized in the following way. In section II, I set up the basic model of competition with probabilistic patents under various rules of damages. I also extend and check the robustness of the basic model by considering the possibility of asymmetric cost structure between the patentholder and the infringer. Section III analyzes the reasonable royalty rate rule and points out the logical inconsistency of the doctrine with uncertain patents. Section IV allows ex ante licensing and analyzes how different damage rules affect the terms of ex ante licensing contracting. Concluding remarks follow in section V.

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<sup>8</sup> Ramsey pricing suggests that the optimal tax structure that minimizes the deadweight loss for generating a given amount of revenue tends to tax as many goods as possible to create small distortions in broad markets. Allowing a monopoly power through the patent system is similar to imposing a tax. If each year is viewed as separate product markets, this suggests that the patent length should be infinite with the scope of patent appropriately adjusted to generate the same discounted value of monopoly profit.

## II. The Model

I consider a situation called “two-supplier world,” in which a patentholder (firm 1) and a potential competitor/infringer (firm 2) are the two suppliers of the patented product. These two firms compete in the Cournot fashion with both firms simultaneously choosing quantities as a strategic variable. Let  $\pi_1(q_1, q_2)$  and  $\pi_2(q_1, q_2)$  be the profit level for the patentholder and the infringer, respectively, when they produce  $q_1$  and  $q_2$  before any ruling on damages. As standard in the literature, we assume that the strategic variables  $q_1$  and  $q_2$  are strategic substitutes, that is,  $\frac{\partial^2 \pi_i(q_1, q_2)}{\partial q_i \partial q_j} < 0$ .

Without any intellectual property right involved, each firm  $i$  maximizes  $\pi_i(q_1, q_2)$  with the following first order conditions:

$$\frac{\partial \pi_i(q_1, q_2)}{\partial q_i} = 0, \text{ for } i=1, 2 \quad (1),$$

which implicitly defines each firm’s reaction function  $q_i = R_i(q_j)$ , where  $i=1, 2$  and  $i \neq j$ . In the absence of IPR, the Nash equilibrium outputs  $(q_1^*, q_2^*)$  are at the intersection of these two reaction functions. We assume that the Nash equilibrium is well-defined, unique, and interior with  $q_1^* > 0$  and  $q_2^* > 0$ .

Now I introduce intellectual property rights in the model. Firm 1 has a patent for a product innovation. Firm 2 can produce the good either with license or by infringing the patent. We first analyze how market competition plays out assuming that firm 2 decides to infringe the patent. After the competition the court decides if firm 2 has infringed the patent. We assume that the IPR is uncertain in the sense that the infringement is found with probability  $\alpha$ , which is assumed to be common knowledge between the patent holder and potential infringer.<sup>9</sup> There are many reasons for this uncertainty. For instance, the patent may be declared invalid by the courts. According to U.S. patent law, the issuance of a patent does no more than confer a patent right that is “presumed” valid (35 U.S.C.A. Sec. 282). The final responsibility for validating or invalidating the patent resides with the courts. In addition, the “doctrine of equivalents”

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<sup>9</sup> See Lemley and Shapiro (2005).



entitles the patented invention to cover a certain range of equivalents. However, the exact boundary of the equivalents is impossible to draw. The matter of infringement can be reasonably assumed to be decided case by case. Finally, for process innovations, infringement may not be detected. In such a case, the probability of detection is also reflected in the parameter  $\alpha$ . Once the infringement is found, the court requires the infringer to compensate the patent holder for his transgression. I consider two damage rules, the unjust enrichment and the lost profit, and investigate how they affect market competition.

## II. 1. Competition with Uncertain Patents under the Unjust Enrichment Rule

In this subsection, I analyze how the patent-holder (firm 1) and the potential infringer (firm 2) compete in the product market under the “unjust enrichment” (UE) damage rule. According to the UE rule, the patentholder is entitled to recover profits earned by the infringer. Since the probability that the patent is deemed to be valid and the infringement is found is  $\alpha$ , the patentholder solves the following problem:

$$\text{Max}_{q_1} \Pi_1^{UE}(\alpha) = \pi_1(q_1, q_2) + \alpha \pi_2(q_1, q_2) \quad (2)$$

The first order condition for firm 1’s optimal output  $q_1$  is given by

$$\frac{\partial \Pi_1^{UE}(\alpha)}{\partial q_1} = \frac{\partial \pi_1(q_1, q_2)}{\partial q_1} + \alpha \frac{\partial \pi_2(q_1, q_2)}{\partial q_1} = 0, \quad (3)$$

which implicitly defines firm 1’s reaction function  $q_1 = R_1^{UE}(q_2; \alpha)$ .

The potential infringer solves the following problem.

$$\text{Max}_{q_2} \Pi_2^{UE} = (1 - \alpha) \pi_2(q_1, q_2) \quad (4)$$

Notice that the potential infringer’s profit under the UE rule is a scaled down version of the profit in the absence of any intellectual property rights. Thus, given  $q_1$ , the optimal choice for firm 2 is the same as in the normal Cournot competition. More precisely, the first order condition for firm 2’s optimal output  $q_2$  is given by

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_2} = 0, \quad (5)$$

which implicitly defines firm 2's reaction function  $q_2 = R_2^{UE}(q_1) = R_2(q_1)$ . The Nash equilibrium royalty rates  $[q_1^{UE*}(\alpha), q_2^{UE*}(\alpha)]$  are at the intersection of these two reaction functions. We assume that the Nash equilibrium is unique and satisfies the stability condition.

**Lemma 1.**  $\frac{q_1^{UE*}(\alpha)}{d\alpha} < 0$  and  $\frac{q_2^{UE*}(\alpha)}{d\alpha} > 0$ .

*Proof.* See the Appendix.

Lemma 1 reveals an interesting strategic effect under the UE rule. As the strength of the patent (parametrized by  $\alpha$ ) increases and it becomes more likely that the patent will be upheld in the court, the infringer competes more aggressively whereas the patentholder plays the role of an accommodator. The intuition is that the patent holder receives firm 2's profit when the patent is held valid. As a result, firm 1 behaves as if it had partial ownership ( $\alpha$  share) of firm 2. Firm 1 internalizes the effects of its output on firm 2's profit and behaves less aggressively compared to the standard Cournot competition. This effect is represented by an inward shift of firm 1's reaction function with  $R_1^{UE}(q_2; \alpha) < R_1(q_2)$ . In response, firm 2 behaves more aggressively with strategic complements.

## II. 2. Competition with Uncertain Patents under the Lost Profit Damage Rule

I now analyze how the patent-holder (firm 1) and the potential infringer (firm 2) compete under the alternative rule of "lost profit" (LP). Under this rule, the patentholder is entitled to recover lost profits due to infringement.<sup>10</sup> Let  $\pi^M$  be the monopoly profit

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<sup>10</sup> In actual patent infringement cases, lost profits are considered an appropriate measure of patent infringement damages if the following four conditions can be established: 1. demands for the subject of intellectual property exist, 2. acceptable noninfringing alternatives do not exist, 3. the plaintiff have the capacity to manufacture and market the infringing products, and 4. economic damages can be quantified with reasonable probability. These four conditions are called the *Panduit* test. The set-up in my model satisfies all these conditions.

that the patent holder would have received in the absence of any entry. Then, the patentholder solves the following problem under the LP rule:

$$\text{Max}_{q_1} \Pi_1^{LP}(\alpha) = \pi_1(q_1, q_2) + \alpha[\pi^M - \pi_1(q_1, q_2)] = (1 - \alpha)\pi_1(q_1, q_2) + \alpha\pi^M \quad (6)$$

The first order condition for firm 1's optimal output  $q_1$  is given by

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = 0, \quad (7)$$

which implicitly defines firm A's reaction function  $q_1 = R_1^{LP}(q_2) = R_1(q_2)$ .

The potential infringer solves the following problem.

$$\text{Max}_{q_2} \Pi_2^{LP} = \pi_2(q_1, q_2) - \alpha[\pi^M - \pi_1(q_1, q_2)] \quad (8)$$

The first order condition for firm 2's optimal output  $q_2$  is given by

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_2} + \alpha \frac{\partial \pi_1(q_1, q_2)}{\partial q_2} = 0, \quad (9)$$

which implicitly defines firm 2's reaction function  $q_2 = R_2^{LP}(q_1; \alpha)$ . Thus, the strategic incentives facing the imitator in the LP regime is isomorphic to those facing a firm that has partial ownership of  $\alpha$  in the rival firm. As a result, the market outcome in the LP regime is a mirror image of the one in the UE regime with the roles of the firms reversed. The Nash equilibrium royalty rates  $[q_1^{LP*}(\alpha), q_2^{LP*}(\alpha)]$  are at the intersection of these two reaction functions.

**Lemma 2.**  $\frac{q_1^{LP*}(\alpha)}{d\alpha} > 0$  and  $\frac{q_2^{LP*}(\alpha)}{d\alpha} < 0$ .

*Proof.* It can be proved by proceeding in a similar way as in the proof of lemma 1.

**Corollary.** Let  $(q_1^*, q_2^*)$  be the unique Nash equilibrium of the game in the absence of IPR. Then, we have  $q_1^{UE*}(\alpha) < q_1^* < q_1^{LP*}(\alpha)$  and  $q_2^{LP*}(\alpha) < q_2^* < q_2^{UE*}(\alpha)$  for all  $\alpha \in (0, 1]$ .

*Proof.* The absence of IPR corresponds to the case of  $\alpha = 0$ , that is,  $q_i^* = q_i^{LP^*}(0) = q_i^{UE^*}(0)$ ,  $i = 1, 2$ . The corollary follows immediately from Lemmas 1 and 2.

### II. 3. Comparison of the UE and LP Rules

**Lemma 3.**  $\pi_1(q_1^{UE^*}(\alpha), q_2^{UE^*}(\alpha)) < \pi_1(q_1^*, q_2^*) < \pi_1(q_1^{LP^*}(\alpha), q_2^{LP^*}(\alpha))$  and  $\pi_2(q_1^{LP^*}(\alpha), q_2^{LP^*}(\alpha)) < \pi_2(q_1^*, q_2^*) < \pi_2(q_1^{UE^*}(\alpha), q_2^{UE^*}(\alpha))$  for all  $\alpha \in (0, 1]$ .

*Proof.* We know that  $q_1^{UE^*}(\alpha) < q_1^* < q_1^{LP^*}(\alpha)$  and  $q_2^{LP^*}(\alpha) < q_2^* < q_2^{UE^*}(\alpha)$ . This implies that  $\pi_1(q_1^{UE^*}(\alpha), q_2^{UE^*}(\alpha)) < \pi_1(q_1^{UE^*}(\alpha), q_2^*) < \pi_1(q_1^*, q_2^*)$ , where the first inequality comes from the fact that  $q_2^{UE^*}(\alpha) > q_2^*$  and the second inequality follows from the definition of the Nash equilibrium. Similarly,  $\pi_1(q_1^*, q_2^*) < \pi_1(q_1^*, q_2^{LP^*}(\alpha)) < \pi_1(q_1^{LP^*}(\alpha), q_2^{LP^*}(\alpha))$ . Once again the first inequality is due to the fact that  $q_2^{LP^*}(\alpha) < q_2^*$  and the second inequality follows from the observation that firm 1's reaction function under the lost profit doctrine is the same as the reaction function in the absence of IPR, that is,  $q_1^{LP^*}(\alpha) = R_1^{LP}(q_2^{LP^*}(\alpha)) = R_1(q_2^{LP^*}(\alpha)) = \arg \max_{q_1} \pi_1(q_1, q_2^{LP^*}(\alpha))$ . Taken together,

I have proved the first part of Lemma 3.

The second part can be proved in an analogous manner. We have  $\pi_2(q_1^{LP^*}(\alpha), q_2^{LP^*}(\alpha)) < \pi_2(q_1^*, q_2^{LP^*}(\alpha)) < \pi_2(q_1^*, q_2^*)$  with the first inequality coming from  $q_1^{LP^*}(\alpha) > q_1^*$  and the second inequality resulting from the definition of the Nash equilibrium. By the same token,  $\pi_2(q_1^*, q_2^*) < \pi_2(q_1^{UE^*}(\alpha), q_2^*) < \pi_2(q_1^{UE^*}(\alpha), q_2^{UE^*}(\alpha))$ . The second inequality is from the fact that firm 2's reaction function under the unjust enrichment doctrine is the same as the reaction function in the absence of IPR, that is,  $q_2^{UE^*}(\alpha) = R_2^{UE}(q_1^{UE^*}(\alpha)) = R_2(q_1^{UE^*}(\alpha)) = \arg \max_{q_2} \pi_2(q_1^{UE^*}(\alpha), q_2)$ .

*Q.E.D.*

**Proposition 1.** With the symmetric production cost structure across firms, we have  $\Pi_1^{UE} < \Pi_1^{LP}$  and  $\Pi_2^{UE} > \Pi_2^{LP}$ , that is, the lost profit damage rule protects the patent holder

better than the unjust enrichment rule whereas the potential infringer prefers the unjust enrichment rule to the lost profit rule.

*Proof.* With equal production efficiency between the patent holder and the potential infringer, we have

$$\begin{aligned}
\Pi_1^{LP}(\alpha) - \Pi_1^{UE}(\alpha) &= \{ \pi_1(q_1^{LP*}(\alpha), q_2^{LP*}(\alpha)) + \alpha[\pi^M - \pi_1(q_1^{LP*}(\alpha), q_2^{LP*}(\alpha))] \} \\
&\quad - \{ \pi_1(q_1^{UE*}(\alpha), q_2^{UE*}(\alpha)) + \alpha\pi_2(q_1^{UE*}(\alpha), q_2^{UE*}(\alpha)) \} \\
&= \alpha\{ \pi^M - [\pi_1(q_1^{UE*}(\alpha), q_2^{UE*}(\alpha)) + \pi_2(q_1^{UE*}(\alpha), q_2^{UE*}(\alpha))] \} \\
&\quad + (1-\alpha)[\pi_1(q_1^{LP*}(\alpha), q_2^{LP*}(\alpha)) - \pi_1(q_1^{UE*}(\alpha), q_2^{UE*}(\alpha))] > 0
\end{aligned}$$

The inequality above follows since  $\pi^M \geq \pi_1(q_1, q_2) + \pi_2(q_1, q_2)$  for any combination of  $(q_1, q_2)$  and  $\pi_1(q_1^{LP*}(\alpha), q_2^{LP*}(\alpha)) > \pi_1(q_1^{UE*}(\alpha), q_2^{UE*}(\alpha))$  by lemma 3.

Similarly, we have

$$\begin{aligned}
\Pi_2^{UE}(\alpha) - \Pi_2^{LP}(\alpha) &= [(1-\alpha)\pi_2(q_1^{UE*}(\alpha), q_2^{UE*}(\alpha))] \\
&\quad - \{ \pi_2(q_1^{LP*}(\alpha), q_2^{LP*}(\alpha)) - \alpha[\pi^M - \pi_1(q_1^{LP*}(\alpha), q_2^{LP*}(\alpha))] \} \\
&> (1-\alpha)\pi_2(q_1^*, q_2^*) - \{ \pi_2(q_1^*, q_2^*) - \alpha[\pi^M - \pi_1(q_1^{LP*}(\alpha), q_2^{LP*}(\alpha))] \} \\
&= \alpha[\pi^M - [\pi_1(q_1^{LP*}(\alpha), q_2^{LP*}(\alpha)) + \pi_2(q_1^*, q_2^*)] \\
&> \alpha[\pi^M - [\pi_1(q_1^*, q_2^*) + \pi_2(q_1^*, q_2^*)]] > 0.
\end{aligned}$$

The first two inequalities follow from lemma 3 and the last inequality follows from the fact that  $\pi^M > \pi_1(q_1^*, q_2^*) + \pi_2(q_1^*, q_2^*)$ .

**Corollary.** The LP rule provides more R&D incentives than the UE rule.

I have not specified how the innovation takes place. I can consider two alternative scenarios. In the non-tournament case in which only one firm can invest in R&D, the innovation incentives depend on the patentee's expected profit. In the

tournament case in which the two firms engage in an R&D race to be the first firm to innovate, the R&D incentives typically depend on the *difference* between the patentee's and potential entrant's profits.<sup>11</sup> In both scenarios, the incentives to innovate are higher under the LP rule.

## II. 4. Unique Efficiencies

The result that the lost profit damage rule provides better protection for the patent holder in subsection II.3 hinges crucially on the assumption that the two firms are equally efficient and produce a homogenous product. If the potential imitator is much superior in its production cost or produces a sufficiently differentiated product thereby expanding the market demand, the patent holder may prefer the lost profit damage rule.

To see this possibility with sufficiently differentiated products, consider an extreme case in which the product produced by the imitator creates a new market and its introduction does not affect the profit of the patent holder. In this case, there will be no lost profit for the patent holder and the imitator is not liable for any damage. In contrast, under the unjust enrichment damage rule, the patent holder would be able to recover the profit of the imitator with probability  $\alpha$ .

The same is true if the imitator is much superior in production efficiency vis-à-vis the patent holder. More specifically, consider a situation in which both firms produce a homogeneous product with a linear market demand of  $p = A - Q$ , where  $Q = q_1 + q_2$ . Their respective constant marginal costs are given by  $c_1$  and  $c_2$  for firm 1 and firm 2. If they compete in the Cournot fashion, it can be easily verified that the following holds:

$$q_1^{LP*}(\alpha) = \frac{A - 2c_1 + c_2}{3 - \alpha}, \quad q_2^{LP*}(\alpha) = \frac{(1 - \alpha)A - 2c_2 + (1 + \alpha)c_1}{3 - \alpha}$$

$$q_1^{UE*}(\alpha) = \frac{(1 - \alpha)A - 2c_1 + (1 + \alpha)c_2}{3 - \alpha}, \quad q_2^{UE*}(\alpha) = \frac{A - 2c_2 + c_1}{3 - \alpha}$$

The patent holder's equilibrium profits under each regime are given by:

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<sup>11</sup> See Reinganum (1988) for a survey of R&D models.

$$\Pi_1^{LP} = \alpha \left( \frac{A - c_1}{2} \right)^2 + (1 - \alpha) \left( \frac{A - 2c_1 + c_2}{3 - \alpha} \right)^2$$

$$\Pi_1^{UE} = \left[ \frac{(1 - \alpha)A - 2c_1 + (1 + \alpha)c_2}{3 - \alpha} \right]^2 + \alpha \left( \frac{A - 2c_2 + c_1}{3 - \alpha} \right)^2$$

Let  $d = c_1 - c_2 \geq 0$  denote the efficiency advantage of the imitator and normalize  $c_2 = 0$ , which implies that  $c_1 = d$ . It can be easily verified that  $\Pi_1^{LP}(\alpha) - \Pi_1^{UE}(\alpha) < 0$  if  $d \geq A/2$ . In addition, we know that  $\Pi_1^{LP}(\alpha) - \Pi_1^{UE}(\alpha) > 0$  if  $d = 0$  by Proposition 1. A straightforward calculation shows that  $\frac{\partial[\Pi_1^{LP}(\alpha) - \Pi_1^{UE}(\alpha)]}{\partial d} < 0$ , which implies that there is a unique  $d^* \in (0, A/2)$  such that  $\Pi_1^{LP}(\alpha) - \Pi_1^{UE}(\alpha) \geq 0$  if and only if  $d \leq d^*$ .

This result is consistent with Shankerman and Scotchmer (2000) who show that the unjust-enrichment doctrine does a better job of protecting the patent-holder in a vertical relationship between the patent-holder and the potential imitator in which only the latter has the capability to develop a commercially marketable product.<sup>12</sup> The case considered in the Shankerman and Scotchmer is formally equivalent to the case of  $d \geq A$  ( $> d^*$ ) in my setup.

### III. Welfare Analysis

In the previous section, I investigated how the patent holder and the potential imitator behave in the output market under the damage rules of “lost profit” and “unjust enrichment” and compared their profits under the respective regimes. In this section, I analyze welfare implications of the two regimes.

I define social welfare as the sum of consumer surplus and producer surplus. Let  $Q^{LP} = q_1^{LP*}(\alpha) + q_2^{LP*}(\alpha)$  and  $Q^{UE} = q_1^{UE*}(\alpha) + q_2^{UE*}(\alpha)$  be the aggregate market output under the LP and UE regimes, respectively. Then

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<sup>12</sup> Shankerman and Scotchmer’s (2000) result holds if infringement would not be deterred under the unjust enrichment rule in their model. They need this additional condition since they consider a certain patent with  $\alpha=1$ , and thus the potential infringer is indifferent between infringement and non-infringement in their setup. With a probabilistic patent, the potential imitator always has incentives to infringe under the unjust enrichment rule.

$$SW^i = \left[ \int_0^{Q^i} P(x)dx - P(Q^i)Q^i \right] + [\Pi_1^i + \Pi_2^i] = \int_0^{Q^i} P(x)dx - c_1 q_1^{i*}(\alpha) - c_2 q_2^{i*}(\alpha),$$

where  $i = \text{LP or UE}$ .

### III. 1. Equal Efficiencies between the Patentholder and the Potential Imitator

Suppose that the two firms have the same production cost of  $c_1 = c_2 = c$ . Notice that with the symmetric cost structure,  $R_1^{LP}(\cdot; \alpha) = R_1(\cdot) = R_2(\cdot) = R_2^{UE}(\cdot; \alpha)$  and  $R_1^{UE}(\cdot; \alpha) = R_2^{LP}(\cdot; \alpha)$ , that is, the reaction function of the patentholder under LP is identical to that of the potential imitator under UE and the reaction function of the patentholder under UE is identical to that of the potential imitator under LP. As a result, we have  $q_1^{LP*}(\alpha) = q_2^{UE*}(\alpha)$  and  $q_2^{LP*}(\alpha) = q_1^{UE*}(\alpha)$ . In other words, only the roles of the firms are reversed across the two regimes. As a result, the total outputs in both regimes are identical, i.e.,  $Q^{LP} = q_1^{LP*}(\alpha) + q_2^{LP*}(\alpha) = q_1^{UE*}(\alpha) + q_2^{UE*}(\alpha) = Q^{UE}$ .

**Proposition 2.** *Ex post* innovation, the two damage rules yield the same social surplus.

So far, I have not focused on development incentives and have taken the innovation as given. Since the two damage rules provide the same social surplus with equal efficiencies between the patent holder and potential imitator, I can conclude that LP is superior to UE if the development incentives are taken into account.

### III.2. Unique Efficiencies

I now investigate welfare implications in the event of cost asymmetry between the patentholder and the potential imitator. With cost asymmetry, there are two complications. First, the total output need not be identical across the two regimes. In addition, even if the aggregate outputs are the same, the distribution of the market shares affect the total production costs.

It is difficult to have any analytical results on the comparison of the total outputs and welfare across the regimes in the general set-up. By making assumptions about the specific form of the market demand curve, one can make quantitative assessments of the



aggregate outputs across the regimes. I thus consider a linear demand curve to address the welfare question. Assume  $p = A - Q$ , where  $Q = q_1 + q_2$  as in subsection II.4. Then,

$$Q^{LP} = q_1^{LP*}(\alpha) + q_2^{LP*}(\alpha) = \frac{(2-\alpha)A - (1-\alpha)c_1 - c_2}{3-\alpha}$$

$$Q^{UE} = q_1^{UE*}(\alpha) + q_2^{UE*}(\alpha) = \frac{(2-\alpha)A - c_1 - (1-\alpha)c_2}{3-\alpha}$$

The following result immediately follows.

**Lemma 4.**  $Q^{LP} = q_1^{LP*}(\alpha) + q_2^{LP*}(\alpha) > Q^{UE} = q_1^{UE*}(\alpha) + q_2^{UE*}(\alpha)$  if and only if  $c_1 > c_2$ .

As a result, with a linear demand curve, the market price under the LP regime is lower than that under the UE regime if the patentholder has a higher production cost than the potential imitator. However, this does not imply that the welfare under the LP regime is higher than that under the UE regime when  $c_1 > c_2$ . The reason is that the patentholder produces a disproportionately larger share than the potential imitator under the LP regime, which is relatively inefficient when the patent holder has a higher production cost. Indeed, as the next Proposition demonstrates, with a linear demand curve the inefficiency in production outweighs allocative efficiency of the LP regime vis-à-vis the UE regime when  $c_1 > c_2$ .

**Proposition 3.** With a linear demand, social welfare is higher in the LP (UE) regime if  $c_1 < (>) c_2$ .

*Proof.* With a linear demand of  $p = A - Q$ , we have

$$SW^{LP} = \int_0^{Q^{LP}} P(x)dx - c_1 q_1^{LP*}(\alpha) - c_2 q_2^{LP*}(\alpha)$$

$$= \frac{[(4-\alpha)A + (1-\alpha)c_1 + c_2][(2-\alpha)A - (1-\alpha)c_1 - c_2]}{2(3-\alpha)^2}$$

$$- c_1 \left( \frac{A - 2c_1 + c_2}{3-\alpha} \right) - c_2 \left( \frac{(1-\alpha)A - 2c_2 + (1+\alpha)c_1}{3-\alpha} \right)$$

$$\begin{aligned}
SW^{UE} &= \int_0^{Q^{UE}} P(x)dx - c_1 q_1^{UE*}(\alpha) - c_2 q_2^{UE*}(\alpha) \\
&= \frac{[(4-\alpha)A + c_1 + (1-\alpha)c_2][(2-\alpha)A - c_1 - (1-\alpha)c_2]}{2(3-\alpha)^2} \\
&\quad - c_1 \left( \frac{(1-\alpha)A - 2c_1 + (1+\alpha)c_2}{3-\alpha} \right) - c_2 \left( \frac{A - 2c_2 + c_1}{3-\alpha} \right)
\end{aligned}$$

Therefore, we have

$$SW^{LP} - SW^{UE} = \frac{-\alpha[2(2-\alpha)A + (2+\alpha)(c_1 + c_2)]}{2(3-\alpha)^2} (c_1 - c_2), \text{ which proves the claim.}$$

The result in Proposition 3 also has implications for welfare analysis of the effects of partial ownership of competitors' assets in an industry.<sup>13</sup> It suggests that if firms are asymmetric in their efficiencies and sizes, it would be better for social welfare for a small and inefficient firm to have partial ownership of a large and efficient firm rather than the other way around, as usually is the case. The reason is that the small firm will restrict its output after acquiring partial ownership and the large firm will expand its output in response with industry output being shifted toward the firm with lower marginal costs.<sup>14</sup>

#### IV. How Reasonable is the “Reasonable” Royalty Rate?

When the lost profits or actual damages from the infringement cannot be proved or deemed to be too speculative, the court accepts a “reasonable” royalty rate as an alternative measure of damage.<sup>15</sup> *Georgia-Pacific Corp. v. United States Plywood Corp.* established 15 factors that can be considered in determining the reasonable royalty rate. Not surprisingly, in actual patent cases licensing experts on the plaintiff side tend to identify the factors that lead to high royalty rates while the infringer side points towards

<sup>13</sup> For an analysis of the competitive effects of partial equity interests in an oligopolistic industry, see Reynolds and Snapp (1986), Farrell and Shapiro (1990), Kwoka (1992), and Reitman (1994).

<sup>14</sup> Farrell and Shapiro (1990) make a similar observation. The most literature on partial ownership assumes symmetric firms and emphasizes the potential for collusion that such ownership entails. In contrast, Farrell and Shapiro show that welfare may well rise as ownership becomes more concentrated with a small firm buys part of a bigger firm, due to more efficient output distributions between heterogeneous firms.

<sup>15</sup> More precisely, the law specifies that any award cannot be lower than the reasonable royalty rate.

the factors that lead to low royalty rate. As a result, this doctrine has proved difficult to implement in a consistent and predictable manner (Conley, 1987). However, the essence of the rule is considered as a “hypothetical license” approach that defines the reasonable royalty rate as “[t]he amount that a licensor (such as the patentee) and a licensee (such as the infringer) would have agreed upon (at the time of the infringement began) if both had been reasonably and voluntarily trying to reach an agreement.”

In considering the counterfactual scenario to calculate what the infringer should and would have paid with a hypothetical negotiation between the infringer and patent holder, it should be recognized that the negotiation between them takes place in the shadow of litigation and the damage rule in case of infringement. More precisely, let  $\tilde{r}^e$  be the “reasonable” royalty rate that is expected to be paid by the infringer without licensing. This expected royalty rate sets the expected payoffs of each party in the absence of licensing, which serves as the threat point in the bargaining. The potential infringer will choose its output to solve:

$$\begin{aligned} \text{Max}_{q_2} \Pi_2^{RR}(\alpha) &= \pi_2(q_1, q_2) - \alpha \tilde{r}^e q_2 = [P_2(q_1, q_2) - c_2]q_2 - \alpha \tilde{r}^e q_2 \\ &= [P_2(q_1, q_2) - (c_2 + \alpha \tilde{r}^e)]q_2 \end{aligned}$$

Thus, firm 2 behaves as if its marginal cost were  $(c_2 + \alpha \tilde{r}^e)$ . Let  $\pi_i^*(c_1, c_2)$  be the standard Cournot equilibrium profit for firm  $i$  when firm 1’s and 2’s costs are given by  $c_1$  and  $c_2$ , respectively. Then, firm 2’s expected payoff from infringement under the reasonable royalty rate regime is given by  $\pi_2^*(c_1, c_2 + \alpha \tilde{r}^e)$ . This payoff serves as a reference point in the bargaining between the patentholder and the potential infringer. For simplicity and concreteness, let me assume that the negotiation takes place in the form of a take-it-or-leave-it offer by the patent holder.<sup>16</sup> Then, the royalty rate in a hypothetical negotiation will be set to maximize:

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<sup>16</sup> Allowing a more balanced bargaining power between the patent holder and the potential infringer as in the generalized Nash bargaining solution does not change any qualitative results.

$$\text{Max}_r \Pi_1^{RR}(\alpha) = \pi_1^*(c_1, c_2 + r) + r q_2(c_1, c_2 + r)$$

subject to

$$\pi_2^*(c_1, c_2 + r) \geq \pi_2^*(c_1, c_2 + \alpha \tilde{r}^e)$$

Notice that  $\frac{d\Pi_1^{RR}(\alpha)}{dr} = \frac{\partial \pi_1^*(c_1, c_2 + r)}{\partial c_2} + q_2(c_1, c_2 + r) + r \frac{\partial q_2(c_1, c_2 + r)}{\partial c_2}$ . Notice that

$$\frac{\partial \pi_1^*(c_1, c_2 + r)}{\partial c_2} = P, \frac{\partial q_2(c_1, c_2 + r)}{\partial c_2} q_1(c_1, c_2 + r) \text{ by the envelope theorem and the first}$$

order condition for firm 1's profit maximization. Thus, we have

$$\frac{d\Pi_1^{RR}(\alpha)}{dr} = -(P - c - r) \frac{\partial q_2(c_1, c_2 + r)}{\partial c_2} + q_2(c_1, c_2 + r) > 0. \text{ This implies that the constraint}$$

will be binding. Let the solution to the above problem be  $\tilde{r}$ . Then, it is clear that  $\tilde{r} =$

$\alpha \tilde{r}^e$ . The “reasonable” royalty rate requires that  $\tilde{r}^e = \tilde{r}$ , which implies that  $\tilde{r} = \alpha \tilde{r}$ .

This condition can be satisfied only when  $\alpha = 1$ , that is, patent protection is perfect and there is no uncertainty about the validity of the patent. In fact, when  $\alpha = 1$ , we have a continuum of “reasonable” royalty rates that are consistent with the logic. However, if the patent is probabilistic with  $\alpha \in (0, 1)$ , the concept of a “reasonable” royalty rate that presumes a hypothetical negotiation is flawed since there is no “reasonable” royalty rate that is consistent with the logic.

**Proposition 4.** When the patent is probabilistic, there is no “reasonable” royalty rate that is consistent with the logic. When the patent is ironclad ( $\alpha = 1$ ), the concept suffers from an opposite problem, that is, a multiplicity of reasonable royalty rate.

Shankerman and Scotchmer (2001) also recognize the circularity and self-referential nature of equilibrium in the logic of this doctrine.<sup>17</sup> In their model, they consider ironclad patents in which intellectual property rights are enforced with certainty. In such a framework, they show that licensee fees and prospective damages are equal and self-enforcing. Due to this bootstrapping nature of equilibrium, there is a continuum of

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<sup>17</sup> To emphasize the circularity of the logic, they avoid using the term “reasonable royalty” and instead refer to “lost royalty.”

reasonable royalty rates that are consistent with the logic. However, when there is uncertainty about the validity of patents, I point out that there is a more serious and opposite problem arises, that is, there is no reasonable royalty rate that is consistent with the logic.

The inconsistency of the logic in the case of *uncertain* patents is not difficult to understand. The hypothetical *ex ante* negotiation is supposed to take place under uncertainty about the validity of the patent (i.e.  $\alpha \in (0,1)$ ), whereas the damage liability consideration is relevant only in the *ex post* case that the patent is found to be valid ( $\alpha = 1$ ). As the value of a winning lottery ticket cannot be equal to the value of a lottery ticket before the drawing, the value of the patent that is certified to be valid in the court cannot be equal to the value of the patent with uncertain validity. However, the equivalence between these two is exactly what the “reasonable” royalty rate doctrine implicitly requires.

#### IV. Ex Ante Licensing Contract

In section II, I analyzed how the lost-profit and unjust-enrichment rules affect market competition between the patentholder and the infringer. In this section, I allow *ex ante* licensing and analyze how different damage rules affect the terms of *ex ante* licensing contracting with frictionless bargaining. In this case, the equilibrium profits under respective damage rules serve as the threat points in the bargaining game between the patentholder and the potential infringer as in the analysis of Shankerman and Scotchmer (2001).

As in the previous section, let me assume that the negotiation takes place in the form of a take-it-or-leave-it offer by the patent holder.<sup>18</sup> The royalty rate in a hypothetical negotiation will be set to maximize:

$$\underset{r,F}{\text{Max}} \Pi_1^L(\alpha) = \pi_1^*(c_1, c_2 + r) + r q_2(c_1, c_2 + r) + F$$

subject to

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<sup>18</sup> Once again, allowing a more balanced bargaining power between the patent holder and the potential infringer as in the generalized Nash bargaining solution does not change any qualitative results.

$$\pi_2^*(c_1, c_2 + r) - F \geq \Pi_2^K(\alpha), K = \text{UE, LP}$$

It is clear that the incentive constraint holds with equality with  $F =$

$\pi_2^*(c_1, c_2 + r) - \Pi_2^K(\alpha)$ . Thus, we can rewrite the problem as:

$$\text{Max}_r \Pi_1^L(\alpha) = \pi_1^*(c_1, c_2 + r) + \pi_2^*(c_1, c_2 + r) + r q_2(c_1, c_2 + r) - \Pi_2^K(\alpha)$$

**Lemma 5.** Let  $\bar{r}$  be the lowest  $r$  such that  $q_2(c_1, c_2 + r) = 0$ , that is, the minimum royalty rate that induces exit by firm 2. Then,

$\Pi_1^L(\alpha) = \pi_1^*(c_1, c_2 + r) + \pi_2^*(c_1, c_2 + r) + r q_2(c_1, c_2 + r) - \Pi_2^K(\alpha)$  is strictly increasing in  $r$  for  $r \in [0, \bar{r})$

*Proof.* The first order condition with respect to  $r$  is given by:

$$\frac{d\Pi_1^L(\alpha)}{dr} = \frac{\partial \pi_1^*(c_1, c_2 + r)}{\partial c_2} + \frac{\partial \pi_2^*(c_1, c_2 + r)}{\partial c_2} + q_2(c_1, c_2 + r) + r \frac{\partial q_2(c_1, c_2 + r)}{\partial c_2} = 0$$

By the envelope theorem,  $\frac{\partial \pi_1^*(c_1, c_2 + r)}{\partial c_2} = P' \frac{\partial q_2(c_1, c_2 + r)}{\partial c_2} q_1(c_1, c_2 + r)$  and

$\frac{\partial \pi_2^*(c_1, c_2 + r)}{\partial c_2} = P' \frac{\partial q_1(c_1, c_2 + r)}{\partial c_2} q_2(c_1, c_2 + r) - q_2(c_1, c_2 + r)$ . Therefore, we have

$$\frac{d\Pi_1^L(\alpha)}{dr} = P' \frac{\partial q_2(c_1, c_2 + r)}{\partial c_2} q_1(c_1, c_2 + r) + P' \frac{\partial q_1(c_1, c_2 + r)}{\partial c_2} q_2(c_1, c_2 + r) + r \frac{\partial q_2(c_1, c_2 + r)}{\partial c_2}.$$

By the first order condition for profit maximization, we know that  $P' q_1(c_1, c_2 + r) = c$  and

$P' q_2(c_1, c_2 + r) = c + r$ . Therefore, we can rewrite  $\frac{d\Pi_1^L(\alpha)}{dr}$  as

$$\frac{d\Pi_1^L(\alpha)}{dr} = -(P - c - r) \left[ \frac{\partial q_1(c_1, c_2 + r)}{\partial c_2} + \frac{\partial q_2(c_1, c_2 + r)}{\partial c_2} \right], \text{ which is positive since}$$

$P - c - r > 0$  for all  $r < \bar{r}$  and the expression in the square bracket is negative. This result implies that the patentholder offers a very high royalty rate to induce firm 2 to exit from the market in exchange for a reverse lump sum payment of  $\Pi_2^K(\alpha)$  to the potential

infringer. This contract will sustain a monopoly outcome in the market.<sup>19</sup> I thus assume that antitrust authorities do not allow reverse payments (negative  $F$ ).

With the constraint that  $F \geq 0$ , the patentholder will set  $F = 0$  and choose the highest  $r$  such that  $\pi_2^*(c_1, c_2 + r) \geq \Pi_2^K(\alpha)$ . Since  $\pi_2^*(c_1, c_2 + r)$  is decreasing in  $r$ , we can find a unique  $r$  that satisfies  $\pi_2^*(c_1, c_2 + r) = \Pi_2^K(\alpha)$ .

**Proposition 5.** Let  $(r^{UE}, F^{UE})$  and  $(r^{LP}, F^{LP})$  denote the equilibrium ex ante licensing contract under alternative regimes of UE and LP. If the reverse payment is not allowed by antitrust authorities,  $F^{UE} = F^{LP} = 0$  and  $r^K$  is uniquely defined by  $\pi_2^*(c_1, c_2 + r^K) = \Pi_2^K(\alpha)$ , where  $K = \text{UE, LP}$ , with  $r^{UE} < r^{LP}$ .

## V. Concluding Remarks

I have developed a simple model of product innovation in which I analyzed how different damage rules in patent infringement cases shape competition when intellectual property rights are *probabilistic*. In particular, I compared two infringement damage rules used in the US – the unjust enrichment rule and the lost profit rule. *Ex post* innovation, these two rules are equivalent in terms of outputs and social welfare if the patentholder and the potential infringer are equally efficient. However, with asymmetric inefficiency and a linear demand, the LP rule generates higher social welfare than the UE rule if and only if the imitator is more efficient than the patentholder. The analysis has implications for the effects of partial ownership in the industry since the competitive effects of damage rules are isomorphic to those of partial ownership. It also points out the logical inconsistency in the concept of the “reasonable royalty rates” when intellectual property rights are not ironclad.

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<sup>19</sup> See Farrell and Shapiro (2005).

## Appendix

### Proof of Lemma 1:

Totally differentiating (A1) and (A2) gives us the following expressions.

$$\frac{\partial^2 \pi_1}{\partial q_1^2} dq_1 + \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} dq_2 + \alpha \left[ \frac{\partial^2 \pi_2}{\partial q_1^2} dq_1 + \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} dq_2 \right] + \frac{\partial \pi_2}{\partial q_1} d\alpha = 0 \quad ()$$

$$\frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} dq_1 + \frac{\partial^2 \pi_2}{\partial q_2^2} dq_2 = 0$$

To derive comparative statics result with respect to  $\alpha$ , we can write the expressions above in the following matrix form.

$$\begin{bmatrix} \frac{\partial^2 \pi_1}{\partial q_1^2} + \alpha \frac{\partial^2 \pi_2}{\partial q_1^2} & \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} + \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} & \frac{\partial^2 \pi_2}{\partial q_2^2} \end{bmatrix} \begin{bmatrix} \frac{dq_1^{UE}}{\partial \alpha} \\ \frac{dq_2^{UE}}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} -\frac{\partial \pi_2}{\partial q_1} \\ 0 \end{bmatrix}$$

By applying Cramer's rule, we can derive

$$\frac{q_1^{UE*}(\alpha)}{d\alpha} = \frac{\begin{vmatrix} -\frac{\partial \pi_2}{\partial q_1} & \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} + \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} \\ 0 & \frac{\partial^2 \pi_2}{\partial q_2^2} \end{vmatrix}}{|A|} = \frac{-\frac{\partial \pi_2}{\partial q_1} \frac{\partial^2 \pi_2}{\partial q_2^2}}{|A|}$$

$$\frac{q_2^{UE*}(\alpha)}{d\alpha} = \frac{\begin{vmatrix} \frac{\partial^2 \pi_1}{\partial q_1^2} + \alpha \frac{\partial^2 \pi_2}{\partial q_1^2} & -\frac{\partial \pi_2}{\partial q_1} \\ \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} & 0 \end{vmatrix}}{|A|} = \frac{\frac{\partial \pi_2}{\partial q_1} \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2}}{|A|},$$



where  $|A| = \begin{vmatrix} \frac{\partial^2 \pi_1}{\partial q_1^2} + \alpha \frac{\partial^2 \pi_2}{\partial q_1^2} & \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} + \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} & \frac{\partial^2 \pi_2}{\partial q_2^2} \end{vmatrix} > 0$  by the stability condition. Since

$\frac{\partial^2 \pi_2}{\partial q_2^2} < 0$  by the second order condition and  $\frac{\partial^2 \pi_2}{\partial q_1 \partial q_2} < 0$  by strategic substitutes, we have

$$\frac{q_1^{UE^*}(\alpha)}{d\alpha} < 0 \text{ and } \frac{q_2^{UE^*}(\alpha)}{d\alpha} > 0.$$

Q.E.D.

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