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# Testing Multiplicative Error Models Using Conditional Moment Tests

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# Testing Multiplicative Error Models Using Conditional Moment Tests\*

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## Abstract

We suggest a robust form of conditional moment test as a constructive test for functional misspecification in multiplicative error models. The proposed test has power solely against violations of the conditional mean restriction but is not affected by any other type of model misspecification. Monte-Carlo investigations show that an appropriate choice of weighting function induces high power against various alternatives. We illustrate how to adapt the framework to test also out-of-sample moment restrictions, such as orthogonalities of prediction errors.

*Keywords:* Robust Conditional Moment Tests, Finite Sample Properties, Multiplicative Error Models, Prediction Errors

*JEL Classification:* C12, C22, C52

## 1 Introduction

The multiplicative error model (MEM) as discussed by Engle (2002) has become a workhorse for the modelling of serially dependent positive-valued random variables in financial time series. Though several specification tests for MEMs have been proposed in the recent literature, only a few approaches address the problem of explicitly testing the validity of the imposed conditional mean restriction. The latter condition is a prerequisite for consistent parameter estimation using quasi maximum likelihood (see Engle, 2000, and Drost and Werker, 2004). Meitz and Teräsvirta (2006) introduced various Lagrange multiplier (LM) tests against different forms of functional misspecification. These tests are constructive and have optimal power against specific (local) alternatives. More general approaches are

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omnibus tests (see de Jong, 1996, or Hong and Lee, 2003), which are generally consistent but typically have quite poor power properties in finite samples.

In this paper, we evaluate the performance of classical CM tests as a flexible alternative allowing to bridge the gap between generally inconsistent tests with optimal power against local alternatives (such as LM tests) and asymptotically consistent tests with poor finite-sample properties. In particular, we suggest a robust form of Newey’s (1985) conditional moment (CM) test which is asymptotically only sensitive to violations of the underlying conditional mean restriction but is not sensitive to any other type of model misspecification. Evaluating the finite-sample properties based on a Monte-Carlo study we show that the test has good power properties against various forms of misspecification if the imposed conditioning information is appropriately chosen. In this sense, CM tests serve as a flexible diagnostic tool replenishing the existing literature. We illustrate that the proposed framework is straightforwardly adapted to test also out-of-sample conditional moment restrictions such as the orthogonality of (out-of-sample) forecasting errors and possible predictors. In this context, CM tests can provide information on how to improve the forecasting power of MEM specifications.

The remainder of the paper is organized as follows. Section 2 introduces the MEM and illustrates how to construct robust tests for in-sample and out-of-sample CM restrictions. In Section 3, we present the results of a Monte-Carlo study analyzing the finite-sample properties of CM tests in the given context. Finally, Section 4 concludes.

## 2 Conditional Moment Tests for Multiplicative Error Models

### 2.1 Model Framework and Assumptions

Let  $\{y_t\}$ ,  $t = 1, \dots, T$ , denote a non-negative (scalar) random variable representing, e.g., price volatilities, trading intensities, volumes or trading costs. In general form, the MEM is given by

$$y_t = \mu_t \varepsilon_t, \quad \mathbb{E}[\varepsilon_t | \mathcal{F}_{t-1}] = 1,$$

where  $\mathcal{F}_t$  denotes the information set up to  $t$ ,  $\mu_t := \mu_t(\theta) = \mathbb{E}[y_t | \mathcal{F}_{t-1}]$  is a non-negative conditionally deterministic process given  $\mathcal{F}_{t-1}$ , and  $\theta$  is a  $p \times 1$  parameter vector. A linear MEM( $m, n$ ) specification is given by  $\mu_t = \omega + \sum_{j=1}^m \alpha_j y_{t-j} + \sum_{j=1}^n \beta_j \mu_{t-j}$ , where  $\omega > 0$ ,  $\alpha_j \geq 0$ ,  $\beta_j \geq 0$ .<sup>1</sup> It resembles the conditional variance equation of a GARCH model (Bollerslev, 1986) as long as  $y_t$  denotes the squared (de-meaned) log return. Alternatively, if  $y_t$

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<sup>1</sup>For a survey on extended specifications, see, e.g., Bauwens and Hautsch (2008).

corresponds to a (financial) duration, such as, e.g., the time between consecutive trades, the model is referred to an ACD specification as introduced by Engle and Russell (1998). Multivariate MEMs have been discussed by Manganelli (2005), Engle and Gallo (2006) and Hautsch (2008).

Define  $\rho_t := \rho_t(\theta)$ ,  $t = 1 \dots, T$ , as the  $s \times 1$  vector of conditional moment functions with the property  $E[\rho_t|w_t] = 0$ , where  $w_t$  is a  $s \times q$  matrix of instruments. Correspondingly, we obtain the  $q \times 1$  vector of unconditional moment functions as  $\tau_t := \tau_t(\theta) := w_t' \rho_t$ . Moreover, we define the  $q \times 1$  vector of sample moments  $\psi_T := T^{-1} \sum_{t=1}^T \tau_t$ . In the MEM framework, natural choices for  $\rho_t$  are  $(y_t - \mu_t)$  or  $(y_t/\mu_t - 1)$  allowing to test the null hypotheses

$$H_0 : E[y_t - \mu_t|w_t] = 0 \quad \text{or} \quad H_0^* : E[y_t/\mu_t - 1|w_t] = 0.$$

We assume that  $\theta$  is estimated by pseudo maximum likelihood (PML) using the exponential log likelihood function  $\mathcal{L}(\theta) = -\sum_{t=1}^T (\ln \mu_t + y_t/\mu_t)$ .<sup>2</sup> Correspondingly, we denote the  $p \times 1$  vector  $s_t := s_t(\theta)$  as the score associated with the  $t$ -th log likelihood contribution. Accordingly, we define the  $T \times p$  matrix  $s := s(\theta) := (s_1', \dots, s_T')$  and  $\mathcal{H}(\theta) := \frac{\partial \mathcal{L}(\theta)}{\partial \theta \partial \theta'}$  denotes the Hessian of the pseudo log likelihood. Furthermore, we make the following assumptions:

(A1)  $\tau_t(\theta_0)$  follows a stationary and ergodic process with  $\theta_0$  defining the true parameter.

(A2)  $\tau_t(\theta)$  is continuous differentiable in  $\theta$  with  $E[\tau_t(\theta)] < \infty$ .

(A3)  $\psi_T \xrightarrow{p} E[\tau_t]$  and  $T^{-1} \sum_{t=1}^T \partial \tau_t(\theta)/\partial \theta' \xrightarrow{p} E[\partial \tau_t(\theta_0)/\partial \theta']$ .

(A4)  $T^{1/2} \begin{bmatrix} T^{-1} \sum_{t=1}^T \tau_t \\ T^{-1} \sum_{t=1}^T s_t \end{bmatrix} \xrightarrow{d} \mathcal{N}(0, \Sigma)$  with  $\Sigma$  denoting a positive semi-definite covariance matrix of dimension  $p + q$ .

(A5) For some neighborhood  $\mathcal{N}$  of  $\theta_0$ :  $E[\sup_{\theta \in \mathcal{N}} \|\mathcal{H}(\theta)\|] < \infty$ .

## 2.2 A Robust Form of the Conditional Moment Test

In this section, we suggest a form of Newey's (1985) conditional moment test which is robust to any misspecification other than violations of the conditional mean restriction, as, e.g., distributional misspecification or conditional heteroscedasticity in the scores. The asymptotic distribution of  $T^{1/2} \hat{\psi}_T$  is derived by expanding  $\hat{\psi}_T$  around  $\theta_0$  using the mean

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<sup>2</sup>Obviously, we could also allow for alternative consistent (extremum) estimators, as, e.g., the semiparametrically efficient estimator proposed by Drost and Werker (2004).

value theorem,

$$T^{1/2}\hat{\psi}_T = T^{1/2} \left[ T^{-1} \sum_{t=1}^T \tau_t(\theta_0) + \left( \text{plim}_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T \partial \tau_t(\theta^*) / \partial \theta \right) (\hat{\theta} - \theta_0) \right], \quad (1)$$

where  $\theta^* := \theta_0 + \lambda(\hat{\theta} - \theta_0)$ ,  $0 \leq \lambda \leq 1$ . With  $\hat{\theta}$  being a PML estimator, we have

$$T^{1/2}(\hat{\theta} - \theta_0) = - [T^{-1} \mathcal{H}(\theta^*)]^{-1} T^{-1/2} \sum_{t=1}^T s_t(\theta_0).$$

Substituting back into (1) yields

$$T^{1/2}\hat{\psi}_T = T^{-1/2} \sum_{t=1}^T \tau_t(\theta_0) - \left( \text{plim}_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T \partial \tau_t(\theta^*) / \partial \theta \right) T^{1/2} \mathcal{H}(\theta^*)^{-1} \sum_{t=1}^T s_t(\theta_0). \quad (2)$$

This expression can be re-written as

$$T^{1/2}\hat{\psi}_T = B \begin{bmatrix} T^{-1/2} \sum_{t=1}^T \tau_t(\theta_0) \\ T^{-1/2} \sum_{t=1}^T s_t(\theta_0) \end{bmatrix}, \quad (3)$$

where the  $q \times (p + q)$  matrix  $B$  is given by

$$B = \begin{bmatrix} I_q & \vdots & \left( \text{plim}_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T \partial \tau_t(\theta^*) / \partial \theta \right) (T^{-1} \mathcal{H}(\theta^*))^{-1} \end{bmatrix}, \quad (4)$$

and  $I_q$  denotes a  $(q \times q)$  identity matrix. Then, we yield  $T^{1/2}\hat{\psi}_T \xrightarrow{d} \mathcal{N}(0, B \Sigma B')$  and thus

$$T[\hat{\psi}_T'(B \Sigma B')^{-1} \hat{\psi}_T] \overset{a}{\sim} \chi_q^2. \quad (5)$$

Under the given assumptions, we have  $\Sigma = \sum_{j=-T}^T \Gamma_j = \Gamma_0 + \sum_{j=1}^T (\Gamma_j + \Gamma'_j)$ , where  $\Gamma_j := \mathbb{E}[\phi_t(\theta_0) \phi_{t-j}(\theta_0)']$  and  $\phi(y_t, \theta_0) := \phi_t := (\tau_t(\theta_0), s_t(\theta_0))$  is the  $(q + p) \times 1$  vector of moment restrictions and scores in  $t$ . Then,  $\Sigma$  can be consistently estimated by a kernel-based estimator

$$\hat{\Sigma} = \sum_{j=-T+1}^{T-1} k(j/q(T)) \hat{\Gamma}_j, \quad (6)$$

where  $k(\cdot)$  is a kernel function and  $q(T)$  is a bandwidth depending on  $T$ . Natural choices are Bartlett kernels, quadratic spectral kernels or Parzen kernels as, e.g., suggested by Newey and West (1987) and Andrews (1991).

Estimating the matrix  $B$  requires consistently estimating  $\mathcal{H}(\theta)$  by the empirical Hessian which is ensured by the dominance condition (A5). Moreover,  $\text{plim}_{T \rightarrow \infty} T^{-1} \sum_t \partial \tau_t(\theta^*) / \partial \theta$  can be consistently estimated by

$$T^{-1} \sum_{t=1}^T \partial \tau_t(\hat{\theta}) / \partial \theta = T^{-1} \sum_{t=1}^T \left( w_t \partial \tau_t(\hat{\theta}) / \partial \theta + \tau_t(\hat{\theta}) \partial w_t / \partial \theta \right),$$

where

$$\partial\tau_t(\hat{\theta})/\partial\theta = \begin{cases} -y_t\hat{s}_t/(y_t - \hat{\mu}_t) & \text{in case of } H_0, \\ -\hat{s}_t\hat{\mu}_t^2/(y_t - \hat{\mu}_t) & \text{in case of } H_0^*. \end{cases}$$

Note that in case of i.i.d. observations,  $\Sigma$  is consistently estimated by  $T^{-1}\hat{\phi}_t'\hat{\phi}_t$ , whereas  $\text{plim}_{T \rightarrow \infty} \sum_t \partial\tau(\cdot)/\partial\theta$  can be consistently estimated by the outer product between score and moment vector (see Tauchen, 1985, or Newey, 1985). Then, we get the well-known expression (see, e.g., Pagan and Vella, 1989)

$$T[\hat{\psi}'_T(B\Sigma B')^{-1}\hat{\psi}_T] = \iota'R(R'R - R's(s's)^{-1}s'R)^{-1}R'\iota, \quad (7)$$

where  $\iota$  is a  $(T \times 1)$  vector of ones and  $R$  is the  $T \times q$  matrix with  $\tau'_t$  as  $t$ -th element.

### 2.3 Testing Out-of-Sample Moment Restrictions

The framework outlined above is straightforwardly extended to test the orthogonality of (out-of-sample) moment restrictions,

$$E[\rho_t|w_{T_1}] = 0, \quad t = T_1 + 1, \dots, T, \quad (8)$$

where  $\theta$  is consistently estimated using the sample  $t = 1, \dots, T_1$ , and predictions are computed from  $T_1 + 1$  to  $T$  capturing a period of  $T_2 := T - T_1$  observations. Then,  $s := (s'_1, \dots, s'_{T_1})$  is a  $T_1 \times p$  matrix, and  $R := (\tau'_{T_1+1}, \dots, \tau'_T)$  is a  $T_2 \times q$  matrix. Mimicking the proceeding above and assuming that

$$\begin{bmatrix} T_2^{-1/2} \sum_{t=1}^{T_2} \tau_t(\theta_0) \\ T_1^{-1/2} \sum_{t=1}^{T_1} s_t(\theta_0) \sqrt{\frac{T_1}{T_2}} \end{bmatrix} \xrightarrow{d} N(0, \tilde{\Sigma}), \quad \tilde{\Sigma} = \begin{bmatrix} \tilde{\Sigma}_{\tau\tau} & \tilde{\Sigma}_{\tau s} \\ \tilde{\Sigma}'_{\tau s} & \tilde{\Sigma}_{ss} \end{bmatrix},$$

yields

$$T_2[\hat{\psi}'_{T_2}(\tilde{B}\tilde{\Sigma}\tilde{B}')^{-1}\hat{\psi}_{T_2}] \stackrel{a}{\sim} \chi_q^2,$$

where  $\hat{\psi}_{T_2} := T_2^{-1} \sum_{t=1}^{T_2} \tau_t(\hat{\theta})$  and

$$\tilde{B} = \begin{bmatrix} I_q & : & \left( \text{plim}_{T_2 \rightarrow \infty} T_2^{-1} \sum_{t=1}^{T_2} \partial\tau_t(\theta^*)/\partial\theta \right) (T_2^{-1}\mathcal{H}(\theta^*))^{-1} \end{bmatrix}.$$

The elements of  $\tilde{\Sigma}$  can be consistently estimated by

$$\hat{\hat{\Sigma}}_{\tau\tau} = \sum_{j=-T_2+1}^{T_2+1} k(j/q(T_2)) \hat{\Gamma}_{\tau\tau,j}, \quad \hat{\hat{\Sigma}}_{ss} = \sum_{j=-T_1+1}^{T_1+1} k(j/q(T_1)) \hat{\Gamma}_{ss,j},$$

where  $\hat{\Gamma}_{\tau\tau,j} = T_2^{-1} \sum_{t=1}^{T_2} \hat{\tau}_t \hat{\tau}'_{t-j}$ ,  $\hat{\Gamma}_{ss,j} = T_1^{-1} \sum_{t=1}^{T_1} \hat{s}_t \hat{s}'_{t-j} T_1/T_2$ , whereas for  $T_1, T_2 \rightarrow \infty$ , we have  $\tilde{\Sigma}_{\tau s} \xrightarrow{p} 0_{q \times p}$ .

### 3 A Monte Carlo Study on Small Sample Properties

In order to gain deeper insights into the size and power properties of the proposed test, we conduct a Monte Carlo study. We draw samples of size 3000 which is still relatively small for high-frequency financial data and allows us to study the finite-sample properties. Each Monte Carlo experiment is repeated 500 times. We use 5 data generating processes (DGPs) based on the following MEM specifications ensuring  $E[\mu_t] = 1$ :

$$\mu_t = 0.1 + 0.1y_{t-1} + 0.8\mu_{t-1} \quad (9)$$

$$\mu_t = \exp(0.137 + 0.3\varepsilon_{t-1} + 0.8 \ln \mu_{t-1}) \quad (10)$$

$$\mu_t = (0.05\mu_{t-1} + 0.5)\varepsilon_{t-1} + 0.8\mu_{t-1} \quad (11)$$

$$\mu_t = \exp(-0.18 + 0.5\varepsilon_{t-1} - 0.48|\varepsilon_{t-1} - 1| + 0.8 \ln \mu_{t-1}) \quad (12)$$

$$\mu_t = \begin{cases} 0.05 + 0.20y_{t-1} + 0.85\mu_{t-1} & \text{if } y_{t-1} \leq 0.25, \\ 0.10 + 0.05y_{t-1} + 0.90\mu_{t-1} & \text{if } y_{t-1} \in (0.25, 1.5], \\ 0.20 + 0.03y_{t-1} + 0.80\mu_{t-1} & \text{if } y_{t-1} > 1.5, \end{cases} \quad (13)$$

where  $y_t = \mu_t \varepsilon_t$ ,  $\varepsilon_t \sim \text{Exp}(1)$ . Eq. (9) represents a linear MEM (Engle and Russell, 1998), (10) is a logarithmic MEM (Bauwens and Giot, 2000), whereas (11) includes innovations both multiplicatively *and* additively (Hautsch, 2004). Furthermore, eq. (12) allows for a kinked news impact function (Dufour and Engle, 2000) whereas (13) corresponds to a threshold specification as proposed and estimated by Zhang, Russell, and Tsay (2001).

**Table 1:** Choice of weighting functions  $w_t$  in the CM tests.

|           |  |  |  |  |  |  |
|-----------|--|--|--|--|--|--|
|           | $z_{t,1} = (\mathbf{1}_{\{\varepsilon_{t-1} < 1\}}, \mathbf{1}_{\{\varepsilon_{t-1} < 1\}}\varepsilon_{t-1}, \mathbf{1}_{\{\varepsilon_{t-1} \geq 1\}}\varepsilon_{t-1})'$<br>$z_{t,2} = (z'_{t,1}, \mathbf{1}_{\{\varepsilon_{t-2} < 1\}}, \mathbf{1}_{\{\varepsilon_{t-2} < 1\}}\varepsilon_{t-2}, \mathbf{1}_{\{\varepsilon_{t-2} \geq 1\}}\varepsilon_{t-2})'$<br>$z_{t,3} = (\mathbf{1}_{\{y_{t-1} < 1\}}, \mathbf{1}_{\{y_{t-1} < 1\}}y_{t-1}, \mathbf{1}_{\{y_{t-1} \geq 1\}}y_{t-1})'$<br>$z_{t,4} = (z'_{t,3}, \mathbf{1}_{\{y_{t-2} < 1\}}, \mathbf{1}_{\{y_{t-2} < 1\}}y_{t-2}, \mathbf{1}_{\{y_{t-2} \geq 1\}}y_{t-2})'$ |  |  |  |  |  |
|           | CM tests   |  |  |  |  |  |
| $CM_1$    | $w_{t,1} = (y_{t-1}, y_{t-1}^2, y_{t-1}^3, \varepsilon_{t-1}, \varepsilon_{t-1}^2, \varepsilon_{t-1}^3)'$  |  |  |  |  |  |
| $CM_2$    | $w_{t,2} = (w'_{t,1}, y_{t-2}, y_{t-2}^2, y_{t-2}^3, \varepsilon_{t-2}, \varepsilon_{t-2}^2, \varepsilon_{t-2}^3)'$  |  |  |  |  |  |
| $CM_3$    | $w_{t,3} = (y_{t-1}, z'_{t,1})'$   |  |  |  |  |  |
| $CM_4$    | $w_{t,4} = (y_{t-1}, y_{t-2}, z'_{t,2})'$  |  |  |  |  |  |
| $CM_5$    | $w_{t,5} = (\varepsilon_{t-1}, z'_{t,3})'$   |  |  |  |  |  |
| $CM_6$    | $w_{t,6} = (\varepsilon_{t-1}, \varepsilon_{t-2}, z'_{t,4})'$  |  |  |  |  |  |
| $CM_7$    | $w_{t,7} = (z'_{t,1}, z'_{t,3})'$  |  |  |  |  |  |
| $CM_8$    | $w_{t,8} = (z'_{t,2}, z'_{t,4})'$  |  |  |  |  |  |
| $CM_9$    | $w_{t,9} = (y_{t-1}, y_{t-2}, \dots, y_{t-10})'$   |  |  |  |  |  |
| $CM_{10}$ | $w_{t,10} = (\varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-10})'$  |  |  |  |  |  |
| $CM_{11}$ | bins for $\varepsilon_{i-1}$ and $\varepsilon_{i-2}$ : $[0, 0.1), [0.1, 0.2), [0.2, 0.5), [0.5, 0.8), [0.8, 1), [1.2, 1.5), [1.5, 2), [2, 3), [3, \infty)$   |  |  |  |  |  |
| $CM_{12}$ | bins for $y_{i-1}$ and $y_{i-2}$ : $[0, 0.1), [0.1, 0.2), [0.2, 0.5), [0.5, 0.8), [0.8, 1), [1.2, 1.5), [1.5, 2), [2, 3), [3, \infty)$   |  |  |  |  |  |



For each data generating process (DGP), we estimate a (linear) MEM(1,1) specification  $\mu_t = \omega + \alpha y_{t-1} + \beta \mu_{t-1}$ . We use the conditional moment function  $\rho_t = y_t/\mu_t - 1$  and 12 weighting functions  $w_{t,i}, i = 1, \dots, 12$ , based on functions of past durations, innovations, and indicator variables indicating possible nonlinear news impact effects (see Table 1). The CM tests are computed using a Bartlett kernel with optimal bandwidth to estimate  $\Sigma$  (see Newey and West, 1987). As a benchmark we compute a consistent integrated conditional moment (ICM) test as proposed by de Jong (1996). Here, we choose a setting which allows us to consistently test against any possible alternative involving 10 lags.<sup>3</sup>

**Table 2:** Rejection frequencies of the individual CM tests (see Table 1). Size of simulated samples: 3000. Number of replications: 500. Estimated model: MEM(1,1).

|           | DGP (9) |       | DGP (10) |       | DGP (11) |       | DGP (12) |       | DGP (13) |       |
|-----------|---------|-------|----------|-------|----------|-------|----------|-------|----------|-------|
|           | 5%      | 10%   | 5%       | 10%   | 5%       | 10%   | 5%       | 10%   | 5%       | 10%   |
| $CM_1$    | 0.066   | 0.126 | 1.000    | 1.000 | 0.498    | 0.605 | 1.000    | 1.000 | 0.140    | 0.212 |
| $CM_2$    | 0.076   | 0.142 | 0.994    | 1.000 | 0.526    | 0.670 | 1.000    | 1.000 | 0.132    | 0.210 |
| $CM_3$    | 0.074   | 0.146 | 1.000    | 1.000 | 0.454    | 0.591 | 1.000    | 1.000 | 0.156    | 0.250 |
| $CM_4$    | 0.070   | 0.148 | 1.000    | 1.000 | 0.443    | 0.584 | 1.000    | 1.000 | 0.162    | 0.246 |
| $CM_5$    | 0.068   | 0.138 | 1.000    | 1.000 | 0.464    | 0.581 | 1.000    | 1.000 | 0.168    | 0.254 |
| $CM_6$    | 0.064   | 0.136 | 1.000    | 1.000 | 0.436    | 0.584 | 1.000    | 1.000 | 0.162    | 0.266 |
| $CM_7$    | 0.074   | 0.116 | 1.000    | 1.000 | 0.485    | 0.591 | 1.000    | 1.000 | 0.182    | 0.282 |
| $CM_8$    | 0.076   | 0.130 | 1.000    | 1.000 | 0.447    | 0.567 | 1.000    | 1.000 | 0.186    | 0.284 |
| $CM_9$    | 0.072   | 0.120 | 0.998    | 1.000 | 0.488    | 0.601 | 1.000    | 1.000 | 0.168    | 0.274 |
| $CM_{10}$ | 0.068   | 0.122 | 0.996    | 1.000 | 0.440    | 0.564 | 1.000    | 1.000 | 0.188    | 0.286 |
| $CM_{11}$ | 0.064   | 0.104 | 1.000    | 1.000 | 0.519    | 0.615 | 1.000    | 1.000 | 0.210    | 0.314 |
| $CM_{12}$ | 0.066   | 0.126 | 1.000    | 1.000 | 0.450    | 0.574 | 1.000    | 1.000 | 0.222    | 0.338 |
| $ICM$     | 0.010   | 0.022 | 0.930    | 0.952 | 0.175    | 0.251 | 0.014    | 0.034 | 0.840    | 0.872 |

Table 2 gives the rejection rates of the individual tests. The first column shows the size since the estimated model and the DGP coincide. We find that the CM tests tend to be slightly oversized for the given sample size whereas the ICM test is strongly undersized. The power of the CM tests is generally quite high and increases with the strength of the deviation from linearity in  $\mu_t$ . Consequently, the tests have very high power against the DGPs (10) or (12). Lower rejection rates are shown for tests against additive stochastic components (DGP (11)) and regime switching behavior (DGP (13)). Both forms of misspecification are hard to detect since the deviation from a linear MEM is not too severe. In this sense, the test outcomes are quite promising. Overall, we find the highest power for conditional moment tests based on weighting functions which are particularly sensitive against nonlinearities in the news response function. These specifications seem to have power against a wide range of

<sup>3</sup>We compute the test based on different choices of underlying test functionals and tuning parameters as discussed by de Jong (1996) and observe that the test outcomes are very robust in this respect. Hence, the reported figures can be considered to be representative for various designs of the test.

possible misspecifications. In contrast, the power properties of the ICM test are very poor. This is a general finding for omnibus tests and is also confirmed by Meitz and Teräsvirta (2006) regarding the spectral density test proposed by Hong and Lee (2003).

## 4 Conclusions

We have proposed a robust form of Newey’s (1985) conditional moment test for functional misspecification in multiplicative error models. The proposed test is robust to any potential misspecification other than those violating the conditional mean restriction. It is shown that the proposed framework is easily adapted to test also out-of-sample moment restrictions. A Monte-Carlo study shows that the test has significantly better power properties as a corresponding consistent conditional moment test. The results indicate that an appropriate choice of the weighting functions induces consistency against a wide range of misspecification while preserving reasonable power properties in finite samples. Consequently, in real applications, CM tests seem to be clearly preferable compared to omnibus tests. As a result, we see them as valuable complements to LM type tests as proposed by Meitz and Teräsvirta (2006). Both kind of tests serve as constructive tests in the sense of Godfrey (1996) allowing to detect possible sources of model misspecification.

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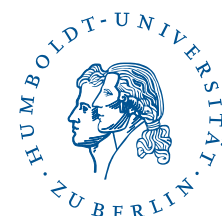
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