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Solow Residuals without Capital Stocks

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Solow Residuals without Capital Stocks*

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Abstract

For more than fifty years, the Solow decomposition (Solow 1957) has served as the standard measurement of total factor productivity (TFP) growth in economics and management, yet little is known about its precision, especially when the capital stock is poorly measured. Using synthetic data generated from a prototypical stochastic growth model, we explore the quantitative extent of capital measurement error when the initial condition is unknown to the analyst and when capacity utilization and depreciation are endogenous. We propose two alternative measurements which eliminate capital stocks from the decomposition and significantly outperform the conventional Solow residual, reducing the root mean squared error in simulated data by as much as two-thirds. This improvement is inversely related to the sample size as well as proximity to the steady state. As an application, we compute and compare TFP growth estimates using data from the new and old German federal states.

Key Words: Total factor productivity, Solow residual, generalized differences, measurement error, Malmquist index.

JEL classification: D24, E01, E22, O33, O47.

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1 Introduction

For more than fifty years, the Solow decomposition has served as the standard measurement of total factor productivity (TFP) growth in economics and management.¹ Among its central attractions are its freedom - as a first approximation - from assumptions regarding the form of the production function, statistical model or econometric specification.² In his seminal paper, Robert Solow (Solow (1957)) demonstrated the limits of using changes in observable inputs to account for economic growth. In macroeconomics, the Solow residual and its estimated behavior has motivated a significant body of research, not only on the sources of long-run economic growth, but also on the sources of macroeconomic fluctuations.³ A recent citation search reveals that this paper has been referenced more than 1,100 times since its publication.⁴

Despite the considerable prominence attached to it, the goodness or robustness of the Solow residual measurement tool has yet to be evaluated. This is because the "true" evolution of total factor productivity is fundamentally unknown; the Solow approach, which defines TFP growth as the difference between observed output growth and a weighted average of observable inputs, cannot be validated with real data in any meaningful way. Yet there are several reasons to suspect the quality of TFP measurements. First, the capital stock is fundamentally unobservable; in practice, it is estimated as a perpetual-inventory function of past investment expenditures plus an estimate of a unknown initial condition. Uncertainty surrounding the initial condition as well as the depreciation, obsolescence and decommissioning of subsequent investment is bound to involve significant measurement errors. Second, as many scholars of productivity analysis have stressed, the Solow residual assumes full efficiency, and thus really represents a mix of changes in total factor productivity and efficiency of factor utilization.⁵ Particularly in macroeconomic analysis of capital stocks, intertemporal variation of the utilization of capital will bias an unadjusted calculation of the Solow residual as a measure of total factor productivity.

In this paper, we exploit advances in quantitative macroeconomic theory to assess the extent of the stock measurement problem directly using data generated by the prototypical stochastic growth model. In particular, we use the stochastic growth model as a laboratory to study the robustness of the Solow residual computed with capital stocks constructed, as is the case in reality, from relatively short series of observed investment expenditures and a fundamentally unobservable capital stock. We explore a more general case involving an endogenous rate of depreciation or obsolescence for all capital in place. Using synthetic data generated by a known stochastic growth model, we can show that measurement problems are generally most severe for "young economies" which are still far from some steady state. This drawback of the Solow residual is thus most acute in applications in which accuracy is most highly valued.

To deal with capital stock measurement error, we propose two alternative measurements of TFP growth. Both involve the elimination of capital stocks from the Solow calculation. Naturally, both alternatives introduce their own, different sources of measurement error. One

¹See, for example Jorgenson and Griliches (1967), Kuznets (1971), Denison (1972), Maddison (1992), Hulten (1992), O'Mahony and van Ark (2003).

²See Griliches (1996).

³See the references in Hulten, Dean, and Harper (2001).

⁴Source: Social Sciences Citation Index.

⁵For a representation of the Solow residual as difference between TFP growth and efficiency, see Mohnen and ten Raa (2002).

requires an estimate of the rental price of capital, while the other requires an initial condition for TFP growth (as opposed to an initial condition for the capital stock). Toward this end, we improve on the choice of starting value of TFP growth by exploiting the properties of the Malmquist index. We then evaluate the extent of these competing errors by pitting the alternatives against the Solow residual in a horse race. In almost all cases, our measures outperform the traditional Solow residual as an estimate of total factor productivity growth, and reduce the root mean squared error in some cases by as much as two-thirds.

The rest of paper is organized as follows. In Section 2, we review the Solow residual and the relationship between the Solow decomposition and the capital measurement problem. Section 3 proposes a prototypical stochastic dynamic general equilibrium model - the stochastic growth model - as a laboratory for evaluating the quality of the Solow residual as a measure of TFP growth. In Section 4, we propose our two alternative methods of TFP growth computation and present the results of a comparative quantitative evaluation of these measurements. Section 5 applies the new methods to the federal states of Germany after unification, which present an unusual application of TFP growth measurement to regional economies which are both close to and far from presumed steady-state values. Section 6 concludes.

2 The Solow Residual and the Capital Measurement Problem

2.1 The Solow Residual at 50: A Brief Review

Solow (1957)⁶ considered a standard neoclassical production function $Y_t = F(A_t, K_t, N_t)$ expressing output (Y_t) in period t as a linear homogeneous function of a single homogeneous composite physical capital stock in period (K_t) and employment during the period (N_t), while A_t represents the state of total factor productivity. He then approximated TFP growth as the difference of the observable growth rate of output and a weighted average of the growth of the two inputs, weighted by their respective local output elasticities α_t and $1 - \alpha_t$, i.e.

$$\frac{\dot{A}_t}{A_t} = \frac{\dot{Y}_t}{Y_t} - \alpha_t \frac{\dot{K}_t}{K_t} - (1 - \alpha_t) \frac{\dot{N}_t}{N_t},$$

where dots are used to denote time derivatives (e.g. $\dot{A} = dA/dt$). In practice, the Solow decomposition is generally implemented in discrete time as (see Barro (1999) and Barro and Sala-I-Martin (2003)):

$$\frac{\Delta A_t}{A_{t-1}} = \frac{\Delta Y_t}{Y_{t-1}} - \alpha \frac{\Delta K_t}{K_{t-1}} - (1 - \alpha) \frac{\Delta N_t}{N_{t-1}} \quad (1)$$

⁶Because it is so widely used in economics and management, we offer only a cursory survey of the growth accounting technique conventionally called the Solow decomposition, pioneered by Solow (1957), but in fact anticipated by Tinbergen (1942), and highlight the capital stock as a source of measurement error in this framework. For more detailed reviews of the Solow decomposition, see Diewert and Nakamura (2003, 2007) and ten Raa and Shestalova (2006).

where K_t denotes capital available at the beginning of period t , and Y_t and N_t stand for output and employment during period t . In competitive factor markets, output elasticities of capital and labor will equal income shares of these factors in aggregate output. In the case of Cobb-Douglas technology, these shares are constant over time; for other constant returns technologies which allow for factor substitution, equation (1) gives a reasonable first order approximation. A central reason for the Solow residual's enduring popularity as a measure of TFP growth is its robustness; it measures the contribution of observable factor inputs to output growth solely on the basis of theoretical assumptions, (perfect competition in factor markets, constant returns) and external information (factor income shares) and without recourse to statistical techniques.

Yet the Solow residual itself is hardly free of measurement error, and over the past half-century researchers have wrestled with the role of measurement error in the Solow residual.⁷ Jorgenson and Griliches (1967) and (1972) argued that the Solow residual is only a "measure of our ignorance" and necessarily contaminated by measurement error and model misspecification. In contrast, Denison (1972) and others extended the TFP measurement paradigm to a larger set of production factors, and confirmed that "the residual" is the most important factor explaining output growth. Christensen, Jorgenson, and Lau (1973) raised concerns about the choice of weights α and $1 - \alpha$; since then it has become common to employ the so-called Thörnqvist index specification of the Solow residual, presented here as a logarithmic approximation:

$$\ln \left(\frac{A_t}{A_{t-1}} \right) = \ln \left(\frac{Y_t}{Y_{t-1}} \right) - \bar{\alpha}_{t-1} \ln \left(\frac{K_t}{K_{t-1}} \right) - (1 - \bar{\alpha}_{t-1}) \ln \left(\frac{N_t}{N_{t-1}} \right) \quad (2)$$

where $\bar{\alpha}_{t-1} = \frac{\alpha_{t-1} + \alpha_t}{2}$ (see Thörnqvist (1936)). This formulation reduces measurement error and is exact if the production function is translog. Hall and Jones (1999) and Caselli (2005) have employed the Solow approximation across space as opposed to time to assess the state of technical progress relative to a technological leader, usually taken to be the United States.

Researchers in macroeconomics - especially real models of business cycles and growth - have been attracted to the Solow residual as a proxy for TFP growth as a driving force in the business cycle (Prescott (1986), King and Rebelo (1999) and Christiano, Eichenbaum, and Evans (2005)). Yet measurement error is likely to be important for a number of reasons in addition to the initial condition problem. While output and employment are directly observable and quantifiable, capital must be estimated in a way which involves leaps of faith and has been subject of substantial criticism. In this context it is worth recalling the famous capital controversy between Cambridge University, led by Joan Robinson, and the Massachusetts Institute of Technology and in particular, Paul Samuelson. Essentially our paper lends more credence to the position taken by Robinson, for reasons different from those she proposed (see Robinson (1953)).

⁷In his seminar contribution, Solow himself recognized the existence of measurement issues:

"[L]et me be explicit that I would not try to justify what follow by calling on fancy theorems on aggregation and index numbers. Either this kind of aggregate economics appeals or it doesn't.[...] If it does, one can draw some useful conclusions from the results." Solow (1957: 312).

2.2 The Capital Measurement Problem

Of all the variables employed in growth accounting, the capital stock poses a particular problem because it is not measured or observed directly, but rather is constructed by statistical agencies using time series of investment expenditures. For this purpose they employ the perpetual inventory method (PIM), which simply integrates the "Goldsmith equation" (Goldsmith (1955))

$$K_{t+1} = (1 - \delta_t) K_t + I_t \quad (3)$$

forward from some initial condition K_0 , given a sequence of depreciation rates $\{\delta_t\}$ applied to the entire capital stock for $t = 0, \dots, t$. Formally, (3) can be solved forward to period $t + 1$ to yield

$$K_{t+1} = \left[\prod_{i=0}^t (1 - \delta_{t-i}) \right] K_0 + \sum_{j=0}^t \left[\prod_{i=0}^j (1 - \delta_{t-i}) \right] I_{t-j} \quad (4)$$

The current capital stock is the weighted sum of an initial capital value, K_0 , and intervening investment expenditures, with weights corresponding to their undepreciated components. If the depreciation rate is constant and equal to δ , (4) collapses to

$$K_{t+1} = (1 - \delta)^{t+1} K_0 + \sum_{j=0}^t (1 - \delta)^j I_{t-j}. \quad (5)$$

which is identical to Hulten (1990).

From the perspective of measurement theory, four general problems arise from using capital stock data estimated by statistical agencies.⁸ First, the construction of capital stocks presumes an accurate measurement of the initial condition. The shorter the series under consideration, the more likely such measurement error regarding the capital stock will affect the construction of the Solow residual. Second, it is difficult to distinguish truly utilized capital at any point in time from that which is idle. Solow (1957) also anticipated this issue, arguing that the appropriate measurement should be of "*capital in use, not capital in place*". Third, for some sectors and some types of capital, it is difficult if not impossible to apply an appropriate depreciation rate; this is especially true of the retail sector. Fourth, many intangible inputs such as cumulated research and development expenditures and advertising goodwill are not included in measured capital.

The Goldsmith equation (3) implies that mismeasurement of the initial capital condition casts a long shadow on the current estimate of the capital stock as well as the construction of the Solow residual. This is especially true with respect to long-lived assets such as buildings and infrastructure. The problem can only be solved by pushing the initial condition sufficiently back into the past; yet with the exception of a few countries,⁹ it is impossible to find sufficiently long time series for investment. The perpetual inventory approach to constructing capital series was thus criticized by Ward (1976) and Mayes and Young (1994), who proposed

⁸See Diewert and Nakamura (2007) for more a detailed discussion.

⁹For example, Denmark and the US Statistical Office have respectively data on investment from 1832 and 1947; most industrialized economies only report data since the 1960s.

alternative approaches grounded in estimation methods.

In practice, different approaches are used to estimate a starting value for the capital stock. As suggested by the OECD (2001), the initial capital value can be compared with 5 different types of benchmarks: 1) *population census* take into account different types of dwellings from the Census; 2) *fire insurance records*; 3) *company accounts*; 4) *administrative property records*, which provides residential and commercial buildings at values at current market prices; and 5) *company share valuation*. Yet in the end, extensive data of this kind are unavailable, so such benchmarks are used to check the plausibility of estimates constructed from investment time series, which generally make use of investment time series I_t and some initial value I_0 . Table 1 summarizes some of the most commonly employed techniques. Employing long time series for the US, Gollop and Jorgenson (1980) set the initial capital at time $t = 0$ equal to investment in that period. This procedure can generate significant measurement error in applications with short time series. Jacob, Sharma, and Grabowski (1997) attempt to avoid this problem by estimating the initial capital stock with artificial investment series for the previous century assuming that the investment grows at the average same rate of output. The US Bureau of Economic Analysis (BEA)¹⁰ assumes that investment in the initial period I_0 , represents the steady state in which expenditures grow at rate g and are depreciated at rate δ , so a natural estimate of K_0 is given by $I_0 \left(\frac{1+g}{\delta+g} \right)$. Griliches (1980) proposed an initial condition $K_0 = \rho \frac{I_0}{Y_0}$ for measuring R&D capital stocks, where ρ is a parameter to be estimated. Over long enough time horizons and under conditions of stable depreciation, the initial condition problem should become negligible. Caselli (2005) assesses the quantitative importance of the capital measurement problem by the role played by the surviving portion of the initial estimated capital stock at time t as a fraction of the total, assuming a constant depreciation rate. He finds that measurement error induced by the initial guess is most severe for the poorest countries. To deal with this problem, he proposes two different approaches: for the richest countries the initial capital is approximated by a steady-state condition $K_0 = \frac{I_0}{(g+\delta)}$ where g is the investment growth rate; for the poorest countries, he applies a "lateral Solow decomposition", following Denison (1962) and Hall and Jones (1999) to the US production function corrected for the human capital, and estimates the capital stock as

$$K_0 = K_{US} \left(\frac{Y_0}{Y_{US}} \right)^{\frac{1}{\alpha}} \left(\frac{N_{US}}{N_0} \right)^{\frac{1-\alpha}{\alpha}} \quad (6)$$

where the index US refers to data to the first observation for the American economy in 1950. Caselli's innovative approach will lose precision if the benchmark economy is far from its steady state. In particular, the key assumption in (6) that total factor productivity levels are identical to those in the US in the base year appears problematic, and are inconsistent with the findings of Hall and Jones (1999). Most important, there is little reason to believe that K_{US} was free of measurement error in 1950.

¹⁰See, for example, Reinsdorf and Cover (2005) and Sliker (2007).

2.3 Measurement Error, Depreciation and Capital Utilization

The initial condition problem identified by Caselli (2005) applies *a fortiori* to a more general setting in which the initial value of capital is measured with error, if depreciation is stochastic, or is unobservable. Rewrite (4) as

$$K_{t+1} = \left[\prod_{i=0}^t \frac{(1 - \delta_{t-i})}{(1 - \delta)} \right] (1 - \delta)^{t+1} K_0 + \sum_{j=0}^t \left[\prod_{i=0}^j \frac{(1 - \delta_{t-i})}{(1 - \delta)} \right] (1 - \delta)^{j+1} I_{t-j} \quad (7)$$

Suppose that the sequence of time depreciation rates $\{\delta_t\}$ moves about some arbitrary constant δ , either deterministically or stochastically. Then K_{t+1} can be decomposed as:

$$\begin{aligned} K_{t+1} = & (1 - \delta)^{t+1} K_0 + \sum_{j=0}^t (1 - \delta)^{j+1} I_{t-j} \\ & + \left[\prod_{i=0}^t \frac{(1 - \delta_{t-i})}{(1 - \delta)} - 1 \right] (1 - \delta)^{j+1} K_0 \\ & + \sum_{j=0}^t \left[\prod_{i=0}^j \frac{(1 - \delta_{t-i})}{(1 - \delta)} - 1 \right] (1 - \delta)^{t+j} I_{t-j} \end{aligned} \quad (8)$$

Equation (8) expresses the true capital stock available for production tomorrow as the sum of three components: 1) an initial capital stock, net of assumed depreciation at a constant rate δ , plus the contribution of intervening investment $\{I_s\}_{s=0}^t$, also expressed net of depreciation at rate δ ; 2) mismeasurement of the initial condition's contribution due to fluctuation of depreciation about the assumed constant value; and 3) mismeasurement of the contribution of intervening investment from period 0 to t . Each of these three components represents a potential source of measurement error. The first component contains all errors involving the initial valuation of the capital stock. For the most part, the second and third components are unobservable. Ignored in most estimates of capital, they represent a potentially significant source of mismeasurement which would spill over into a Solow residual calculation.

The interaction between the depreciation of capital and capacity utilization is also important for both macroeconomic modeling and reality. From a growth accounting point of view, Hulten (1986) criticized the assumption that all factors are fully utilized because they will lead to erroneous TFP computation. Time-varying depreciation rates implies changing relative weights of old and new investment in the construction of the capital stock. In dynamic stochastic general equilibrium models, the depreciation rate is generally assumed constant. This seems in contrast with the empirical evidence, which suggests that the depreciation of capital goods is time dependent.¹¹ In addition, as argued by Corrado and Matthey (1997) and Burnside, Eichenbaum, and Rebelo (1995), capacity utilization seems to have pronounced cyclical variability. While Kydland and Prescott (1988) and Ambler and Paquet (1994) introduced respectively stochastic capital utilization and depreciation rate, other authors (as Wen (1998) and Harrison and Weder (2002)) extend the RBC models assuming capacity utilization to be a convex, increasing function of the depreciation rate. More recently, some macro models

¹¹See the OECD (2001) manual on capital stock.

have employed adjustment costs proportional to the growth rate of investment (Christiano, Eichenbaum, and Evans (2005)).

3 Capital Measurement and the Solow Residual: A Quantitative Assessment

3.1 The Stochastic Growth Model as a Laboratory

The central innovation of this paper is its assessment of alternative TFP growth measurement methods using synthetic data generated by a known, prototypical model of economic growth and fluctuations. The use of the neoclassical stochastic growth model in this research should not be seen as an endorsement, but rather as a tribute to its microeconomic foundations.¹² In this section we briefly describe this standard model and the data which it generates.

3.1.1 Technology

Productive opportunities in this one-good economy are driven by a single stationary stochastic process, total factor productivity $\{A_t\}$ embedded in a standard constant returns production function in effective capital and labor inputs. In particular, output of a composite output good is given by the Cobb-Douglas production technology proposed by Wen (1998)

$$Y_t = A_t ((U_t K_t)^\alpha N_t^{1-\alpha}) \quad (9)$$

where $U_t \in (0, 1)$ denotes the capital utilization rate. The rate of depreciation is an increasing, convex function of capacity utilization

$$\delta_t = \frac{B}{\chi} U_t^\chi \quad (10)$$

where $B > 0$, $\chi > 1$. As before, the capital stock evolves over time according to a modified version of (4) with K_0 given. Total factor productivity evolves according to

$$A_t = A_t^\rho \bar{A}_t^{1-\rho} e^{\epsilon_t} \quad (11)$$

where $\bar{A}_t = \bar{A}_0 \psi^t$ represents a deterministic trend. We assume that $\psi > 1$, and ϵ_t is a mean zero random variable in the information set at time t and $\bar{A} > 0$. It follows that total factor productivity can be written as $A_t = \bar{A}_t \gamma_t$, where $\ln \gamma_t$ follows a stationary AR(1) process with

$$\ln \gamma_t = \rho \ln \gamma_{t-1} + \epsilon_t, \quad (12)$$

¹²See King and Rebelo (1999) for a forceful statement of this view.

with $-1 < \rho < 1$, and given an initial condition $\ln \gamma_0$.

3.1.2 Households

Household own both factors of production and rent their services to firms in a competitive factor market. Facing sequences of factor prices for labor $\{\omega_t\}_{t=0}^{\infty}$ and capital services $\{\kappa_t\}_{t=0}^{\infty}$, the representative household chooses paths of consumption $\{C_t\}_{t=0}^{\infty}$, labor supply $\{N_t\}_{t=0}^{\infty}$, capital $\{K_{t+1}\}_{t=0}^{\infty}$, and capital utilization $\{U_t\}_{t=0}^{\infty}$ to maximize the present discounted value of lifetime utility (see King and Rebelo (1999), Cooley and Prescott (1995) and Prescott (1986) for surveys).

$$\max_{\{C_t\}, \{N_t\}, \{K_{t+1}\}, \{U_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t + \frac{\theta}{1-\eta} [(1-N_t)^{1-\eta} - 1] \right\} \quad (13)$$

subject to an initial condition for the capital stock held by household K_0 and the periodic budget restriction for $t = 0, 1, \dots$

$$C_t + K_{t+1} - (1 - \delta_t)K_t = \omega_t N_t + \kappa_t U_t K_t. \quad (14)$$

and to the dependence of capital depreciation on utilization given by (10). The period-by-period budget constraint restricts consumption and investment to be no greater than gross income from labor ($\omega_t N_t$) and capital ($\kappa_t U_t K_t$), where the latter includes the return of underpreciated capital. This constraint is always binding, since the assumed forms of utility and production functions rule out corner solutions. In the steady state, all variables grow at a constant rate $g = \psi^{\frac{1}{1-\alpha}} - 1$, with the exception of total factor productivity, which grows at rate $\psi - 1$, and employment, capital utilization and interest rates, which are trendless.

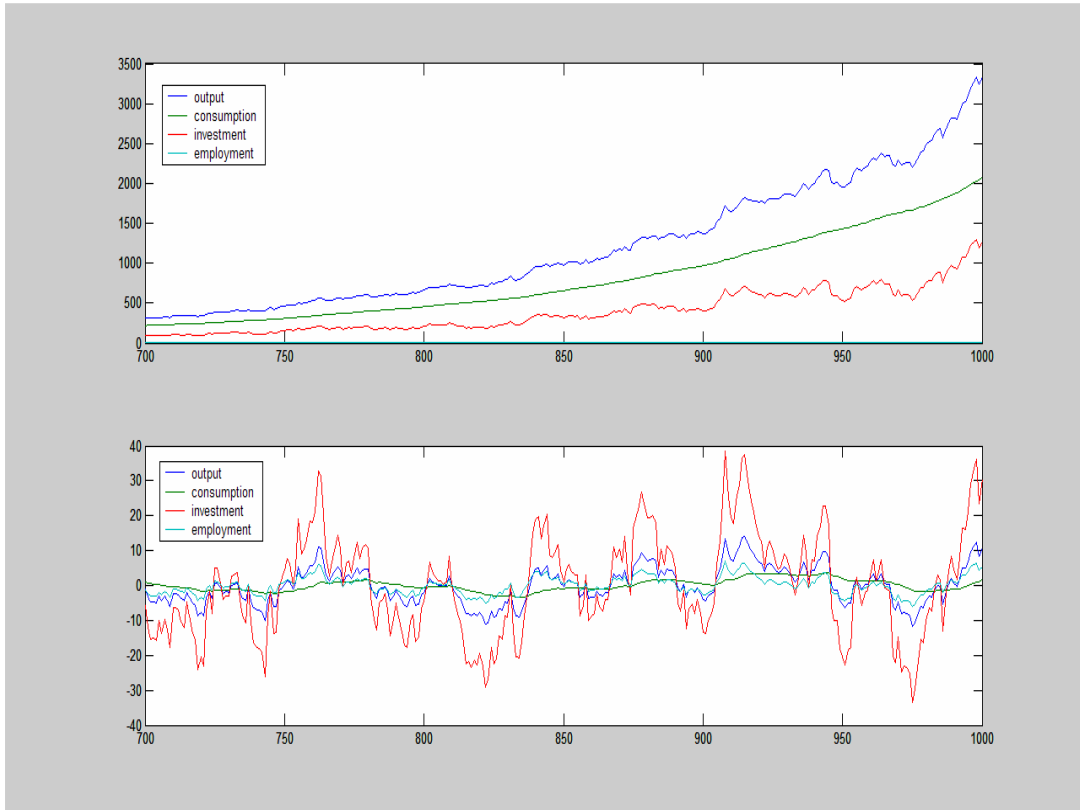
3.1.3 Firms

Firms in this perfectly competitive economy are owned by the representative household. The representative firm employs labor input N_t and capital services $U_t K_t$ to maximize profits subject to the constant returns production function given by (11). At optimal factor inputs, the representative firm sets the marginal product of labor equal to the real wage ω_t and the marginal product of capital services to the real user costs κ_t . Note that for the firm, capital service input is the product of the capital stock and its utilization rate; the firm is indifferent as to whether the capital services come from extensive or intensive use of the capital stock. In Appendix 1 we summarize the first order conditions and decentralized market equilibrium, the description of the log-linearized equations and the calibration values used. In Figure 1 we display a representative time series realization of the economy in trended and H-P detrended form.

3.1.4 First Order Conditions and Decentralized Equilibrium

In Appendix 1 we summarize the first order conditions for optimal behavior of households and firms and characterize the decentralized market equilibrium. In this regular economy, the model's equilibrium is unique and its dynamic behavior can be approximated by log-linearized versions of these equilibrium conditions. The model was then simulated for a calibrated version also described in Appendix 1. Models with both time-varying (endogenous) and constant depreciation were used to generate the synthetic data. Each realization is a time series of 1,000 observations with an initial condition for TFP drawn from a normal distribution with mean zero and standard deviation one. The true capital stock in period zero is set to zero; the model is allowed to run 200 periods before samples were drawn to be independent on an arbitrary set of initial conditions. Samples were drawn for both "mature" and for "transition" economies. A mature economy is derived from this stochastic growth model, while a transition economy has a capital stock which is equal to 50% of the value of the mature economy at the 200th observation. In Figure 1 we display a representative time series realization of the economy in original and H-P detrended form. 100 of these realizations are stored, with each realization containing 1,000 observations.

Figure 1: A typical time series realization (above) and in H-P detrended form (below), $T=1000$



3.2 Assessing Measurement Error of the Solow Residual for different initial conditions

The artificial data generated by a calibrated version of the model described in the preceding sections and summarized in Table 1 will now be used to investigate the precision of the Solow residual as a measurement of TFP growth. The basis of comparison is the root mean squared error (RMSE) for sample time series of either 50 or 200 observations taken from 100 independent realizations of the stochastic growth model described in subsection 3.3.

The Solow residual measure is calculated as a Thörnqvist index described in equation (2). (note that for the Cobb-Douglas production and competitive factor markets, factor shares and output elasticities are constant and the Thörnqvist and lagged factor share versions are equivalent). As in reality, the central assumption is that the true capital stock data are unobservable to the analyst, who computes them by applying the perpetual inventory method from to investment data series and some initial capital stock, which is in turn estimated using various methods described in Table 1.

The results of this first evaluation are presented in Table 2 as the average RMSE (in percent) for the estimate. Standard errors computed over the 100 realizations are presented in parentheses. The results show that the initial condition is an important source of error. The last line of the table shows that, if the initial capital stock is known perfectly, RMSE of the Solow residual collapses to the logarithmic approximation with a negligible error. Of the different methods, the BEA and Caselli approaches perform the best. As would be expected, as the sample size grows, the average RMSE declines. Yet even at a sample length of 50 years, the root mean squared error is quite high at about 2%.

We can use the artificial data generated by the stochastic growth model to evaluate the quantitative extent of measurement error of the initial condition of the capital stock. For conventionally assumed rates of depreciation, errors in estimating the initial stock of capital can have long-lasting effects on reported capital stocks. Figure 2 contains two graphs that illustrate this point. On the left side we display capital stock time series constructed using investment series generated from the stochastic growth model using eq.(3) with different initial values of K_0 . These were chosen between 0 and the maximum level of investment derived from the numerical simulations generated by the stochastic growth model. The initial condition is particularly relevant for the first periods: it takes more than 100 periods to reach a convergence within 10%. As a results the error induced in TFP measurement are severe and die out only after several decades.

To illustrate the impact of the initial capital value on productivity, we insert estimate of the capital stock into (2) to calculate a Thörnqvist Index version of the Solow residual. Measurement error in K_0 will bias total factor productivity (TFP) growth computations when 1) depreciation δ is low and 2) the time series under consideration is short ($t - j$ is low). On the right side we show the TFPG, expressed by $\ln\left(\frac{A_t}{A_{t-1}}\right)$, considering different values of K_0 and, similarly to the capital series represented in Figure 2, also TFPG has biased results dependent on the initial K_0 : it takes more than 30 periods to reach the convergence within 10%.

Figure 2: The consequences of different initial capital value and their impact on the Solow residual.

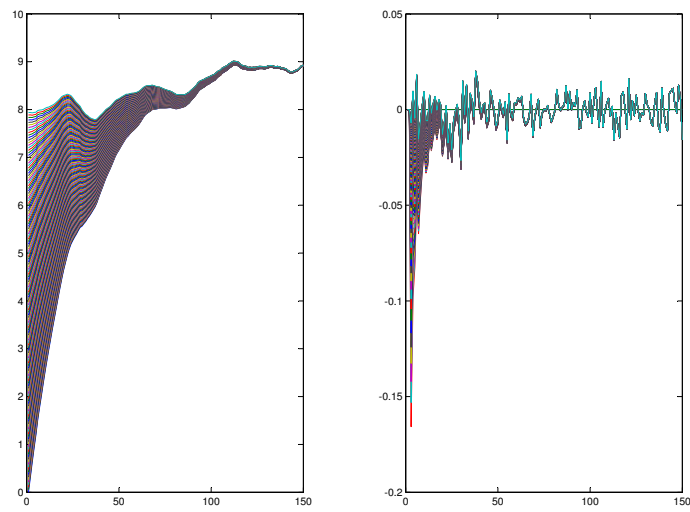


Table 1: Assumptions and methods used in constructing initial values of the capital stock

Method	K_0	Note
Gollop and Jorgenson (1980)	I_0	
Griliches (1980): steady state method	$\frac{I}{g+\delta}$	\bar{I} is estimated
Caselli (2005)	$K_{US} \left(\frac{Y_0}{Y_{US}} \right)^{\frac{1}{\alpha}} \left(\frac{N_{US}}{N_0} \right)^{\frac{1-\alpha}{\alpha}}$	US statistics
BEA	$I_0 \frac{g+1}{g+\delta}$	g growth of investment rate

Table 2: Root Mean Squared Errors and Standard Errors (in % per annum) for Solow Residuals with Different Capital Stock estimates, Artificial Quarterly Data.

Mature Economy(100 realizations, standard errors in parentheses)					
Computation Method for Initial Capital Stock	Endogenous Depreciation		Constant Depreciation		
	Avg. RMSE (%) T=50	Avg. RMSE (%) T=200	Avg. RMSE (%) T=50	Avg. RMSE (%) T=200	
<i>Traditional Solow Residual</i>					
-BEA	2.73 (0.10)	2.04 (0.08)	3.20 (0.83)	1.88 (0.50)	
-Caselli (2005)	1.88 (0.17)	1.90 (0.14)	1.65 (0.79)	1.66 (0.13)	
-Gollop and Jorgenson (1980)	3.86 (0.16)	2.58 (0.10)	4.65 (0.12)	2.54 (0.08)	
-Griliches (1980)	3.90 (0.09)	2.60 (0.07)	4.69 (0.59)	2.56 (0.40)	
Transition Economy (100 realizations, standard errors in parentheses)					
Method	Endogenous Depreciation		Constant Depreciation		
	Avg. RMSE (%) T=50	Avg. RMSE (%) T=200	Avg. RMSE (%) T=50	Avg. RMSE (%) T=200	
<i>Traditional Solow Residual</i>					
-BEA	3.33 (0.11)	2.11 (0.08)	3.16 (0.09)	1.87 (0.06)	
-Caselli (2005)	1.87 (0.18)	1.90 (0.14)	1.63 (0.16)	1.66 (0.13)	
-Gollop and Jorgenson (1980)	4.73 (0.20)	2.71 (0.10)	4.61 (0.15)	2.52 (0.07)	
-Griliches (1980)	4.79 (0.10)	2.73 (0.38)	4.65 (0.81)	2.54 (0.50)	

4 TFP Growth Measurement without Capital Stocks: Two Alternatives

In the previous section, we demonstrated the considerable measurement error that exists with the Solow residual. In the following two sections, we propose two stock-free alternatives to the Solow residual. The first, the DS method, is appropriate when far away to a steady-state. The second, the GD method, relies on proximity to a steady-state path.

4.1 Elimination of capital via direct substitution (DS)

The first strategy for estimating TFP relies on direct substitution to eliminate the capital stock from the equation generally used to construct the Solow residual. Differentiate the constant returns production function $Y_t = F(A_t, K_t, N_t)$ with respect to time to obtain

$$\dot{Y}_t = F_A + F_K \dot{K}_t + F_N \dot{N}_t. \quad (15)$$

Substitute the transition equation for capital $\dot{K}_t = I_t - \delta_t K_t$ and rearranging yields

$$\frac{\dot{Y}_t}{Y_t} = F_K \frac{I_t}{Y_t} - \alpha_t \delta_t + (1 - \alpha_t) \frac{\dot{N}_t}{N_t}.$$

where α_t is, as before, the point elasticity of output with respect to capital. The modified version of the Solow residual is given by

$$\frac{\dot{A}_t}{A_t} = \frac{\dot{Y}_t}{Y_t} - F_K \frac{I_t}{Y_t} + \alpha_t \delta_t - (1 - \alpha_t) \frac{\dot{N}_t}{N_t}. \quad (16)$$

In an economy with competitive conditions in factor markets, as assumed by Solow (1957), the marginal product of capital F_K is equated to κ_t , the user cost of capital in t . This equation is adapted to a discrete time context as

$$\frac{\Delta A_t}{A_{t-1}} = \frac{\Delta Y_t}{Y_{t-1}} - \kappa_{t-1} \frac{I_{t-1}}{Y_{t-1}} + \alpha_{t-1} \delta_{t-1} - (1 - \alpha_{t-1}) \frac{\Delta N_t}{N_{t-1}}. \quad (17)$$

The substitution eliminates the capital stock completely from the TFP calculation. The DS approach will be a better measurement of TFP growth to the extent that 1) capital depreciation is variable from period to period and is better measured from other sources than as an assumed constant proportion of an unobservable stock and 2) the last gross increment to the capital stock is more likely to be completely utilized than existing capital. Once TFP growth is estimated, it is possible to calculate the total contribution of capital to growth as a second residual

$$\alpha_{t-1} \frac{\widehat{\Delta K_{t-1}}}{K_{t-2}} = \frac{\Delta Y_t}{Y_{t-1}} - (1 - \alpha_{t-1}) \frac{\Delta N_t}{N_{t-1}}. \quad (18)$$

4.2 Generalized differences of deviations from the steady state (GD)

If an economy is close to its steady state – as would be the case in a mature economy or sector – it will be more appropriate to measure growth in total factor productivity in terms of deviations from a long-term deterministic trend path estimated using the entire available data set, e.g. trend regression estimates or Hodrick-Prescott filtered series. If \hat{X}_t denotes the deviation of X_t around a steady state value \bar{X}_t , then the production function (1) and the Goldsmith equation (3) can be approximated by

$$\hat{Y}_t = \hat{A}_t + s_K \hat{K}_t + (1 - s_K) \hat{N}_t \quad (19)$$

and

$$\hat{K}_t = \frac{(1 - \delta)}{(1 + g)} \hat{K}_{t-1} + \iota \hat{I}_{t-1}, \quad (20)$$

where $\iota = \frac{\overline{(I/K)}}{(1+g)}$, g is the deterministic steady state growth rate, and the capital elasticity $s_K \equiv \frac{F_K(A_t, K_t, N_t)K}{Y_t}$ is assumed constant, following the widely-accepted steady state restrictions on grand ratios emphasized by King, Plosser, and Rebelo (1988). Application of the transformation $\left(1 - \frac{(1-\delta)}{(1+g)}L\right)$ to both sides of (19) yields the following estimate of a generalized difference of the Solow residual:

$$\left(1 - \frac{(1-\delta)}{(1+g)}L\right) \hat{A}_t = \left(1 - \frac{(1-\delta)}{(1+g)}L\right) \hat{Y}_t - \iota s_K \hat{I}_{t-1} - \left(1 - \frac{(1-\delta)}{(1+g)}L\right) (1 - s_K) \hat{N}_t \quad (21)$$

In (21) the capital stock has been eliminated completely from the computation. To recover TFP growth estimates from (24), apply the logarithmic approximation in each period to obtain recursively for $t = 2, \dots, T$:

$$\ln \left(\frac{A_t}{A_{t-1}} \right) = \Delta\theta_t + \left(\frac{1-\delta}{1+g} \right) \ln \left(\frac{A_{t-1}}{A_{t-2}} \right) \quad (22)$$

where $\Delta\theta_t = \left(1 - \frac{(1-\delta)}{(1+g)}L\right) \hat{Y}_t - \iota s_K \hat{I}_{t-1} - \left(1 - \frac{(1-\delta)}{(1+g)}L\right) (1 - s_K) \hat{N}_t$.

In practice, the computation of productivity growth estimates using the GD procedure requires an independent estimate of the initial condition, $\ln \left(\frac{A_1}{A_0} \right)$. One approach is to set $\ln \left(\frac{A_1}{A_0} \right) = 0$, i.e. assuming no TFP growth in the first period. We propose exploiting the properties of the Malmquist index, which we illustrate in Appendix 2.

4.3 Need for numerical evaluation

The central difference the two methods is the point around which the approximation is taken. In the DS approach, that point is production function evaluated at factor inputs in the

previous period. In the GD approach, the point of approximation is a balanced growth path for which the capital elasticity, s_K , the growth rate g , and the grand ratio I/K are constant. These are very different points of departure and will have advantages and disadvantages which depend on the application at hand. If the economy is far from the steady state, the GD approach is likely to yield a poor approximation. On the other hand, it is likely to be more appropriate for business cycle applications involving OECD countries.

While both measurements eliminate capital from the TFP measurement, they substitute one form of measurement error for another. The DS method substitutes a small marginal contribution of new investment plus a depreciation which may or may not be time-varying. The capital rental price κ_t can be derived from completely independent sources or using economic theory but is likely to be measured with error. Similarly, the GD procedure measures the marginal contribution but substitutes another form of measurement (the growth of TFP in the first period). Given that the GD method necessarily assumes a constant rate of depreciation, it will tend to do badly when the depreciation rate is in fact endogenous and procyclical. It should perform badly for economies or sector which are far from their steady states. In the end, it is impossible to see which type of measurement error is lower without resorting to simulation methods. This is what we do in the next section.

4.4 Assessing the Accuracy of the Solow Residual: Results of a Horse Race

We now employ the same artificial data generated with the stochastic growth model in Section 3.1 to compare the most precise versions of the Solow residual calculation, which estimate initial capital stocks along the lines of the BEA (Reinsdorf and Cover (2005) and Sliker (2007)) and Caselli (2005), with the two alternative measurements we propose in Section 4.

It is important to state carefully the assumptions behind into the construction of the TFP growth measures using the synthetic data. For the DS method, we assume that the analyst can observe the user cost of capital (κ) generated directly by the model simulation in each period. In one variant of the calculation, the period-by-period use cost κ_{t-1} is assumed to be observable, as in equation (17). Because it is notoriously difficult in practice to measure the user cost of capital at high sampling frequencies, we employ also evaluate the measure when a constant value of κ is employed, set equal to its average value over the entire sample realization. The analyst is assumed *not* to observe the true rate of capacity utilization or the depreciation rate in each period, but does observed gross investment. A constant quarterly value of the depreciation rate was assumed, equal to 0.015.

The GD method estimates were computed under two alternative assumptions regarding growth of TFP in the period 0. In the first, the Malmquist index described in Section 4.2 was employed to construct the TFP growth estimate. In the second variant, equation (22) was employed. In all cases, values of the constants δ and ι set equal to 0.015 and 0.0112. respectively.

As in the previous section, the basis of comparison is the root mean squared error (RMSE) for sample time series of 50 or 200 observations taken from 100 independent realizations of the stochastic growth model already described in subsection 3.2. The RMSE results of this

horse race along with standard errors are presented for both the "mature" economy (Table 3) as well as the "transition economy" (Table 4). Recall that the mature economy comes from a realization after the 200th observation from the 1000 which has completed at least 50% of the convergence to the steady state. The transition economy is one which begins the sample at least 50% away from its steady state value for its value of capital.

Table 3: A Horse Race: Stock-less versus Traditional Solow-Thörnquist estimates of TFP Growth.

Mature Economy(100 realizations, standard errors in parentheses)					
Computation Method for Initial Capital Stock	Endogenous Depreciation		Constant Depreciation		
	Avg. RMSE (%) T=50	Avg. RMSE (%) T=200	Avg. RMSE (%) T=50	Avg. RMSE (%) T=200	
<i>Alternative Method</i>					
-Direct Substitution (DS)					
<i>Known κ_{t-1}</i>	1.50 (0.17)	1.50 (0.13)	1.18 (0.17)	1.16 (0.12)	
<i>Average κ</i>	1.50 (0.17)	1.50 (0.13)	1.18 (0.16)	1.16 (0.11)	
-Generalized Differences (GD)					
<i>Malmquist Initial Value</i>	2.13 (0.18)	2.15 (0.11)	1.71 (0.20)	1.67 (0.10)	
$\ln\left(\frac{\hat{A}_1}{\hat{A}_0}\right) = 0$	3.13 (0.30)	3.16 (0.20)	2.40 (0.35)	2.33 (0.19)	
<i>Traditional Solow Residual</i>					
-BEA	3.33 (0.11)	2.11 (0.08)	3.16 (0.09)	1.87 (0.06)	
-Caselli (2005)	1.87 (0.18)	1.90 (0.14)	1.63 (0.16)	1.66 (0.13)	

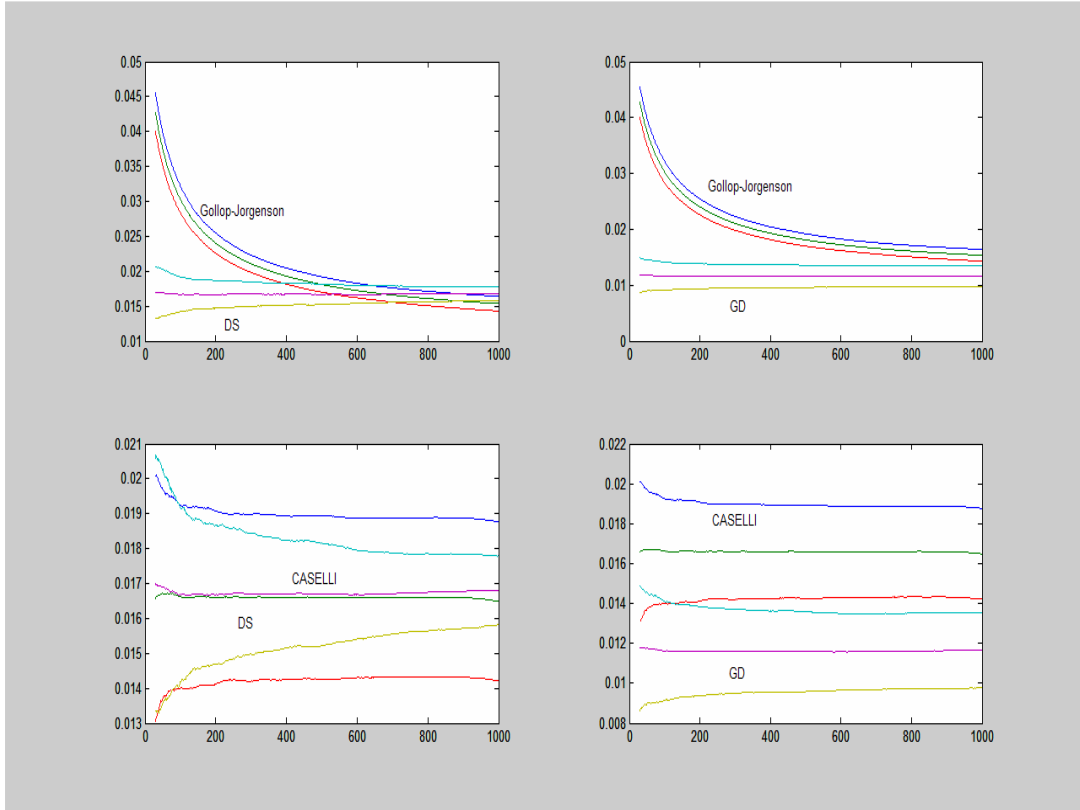
Table 4: A Horse Race: Stock-less versus Traditional Solow-Thörnquist estimates of TFP Growth.

Transition Economy(100 realizations, standard errors in parentheses)					
Computation Method for Initial Capital Stock	Endogenous Depreciation		Constant Depreciation		
	Avg. RMSE (%) T=50	Avg. RMSE (%) T=200	Avg. RMSE (%) T=50	Avg. RMSE (%) T=200	
<i>Alternative Method</i>					
-Direct Substitution (DS)					
<i>Known κ_{t-1}</i>	1.49 (0.18)	1.50 (0.13)	1.13 (0.15)	1.16 (0.12)	
<i>Average κ</i>	1.49 (0.18)	1.50 (0.13)	1.13 (0.15)	1.16 (0.12)	
-Generalized Differences (GD)					
<i>Malmquist Initial Value</i>	2.12 (0.21)	2.15 (0.12)	1.63 (0.17)	1.67 (0.10)	
$\ln \left(\frac{\hat{A}_1}{\hat{A}_0} \right) = 0$	3.12 (0.36)	3.17 (0.21)	2.28 (0.31)	2.34 (0.23)	
<i>Traditional Solow Residual</i>					
-BEA	3.33 (0.11)	2.11 (0.08)	3.16 (0.09)	1.87 (0.06)	
-Caselli (2005)	1.87 (0.18)	1.90 (0.14)	1.63 (0.16)	1.66 (0.13)	

The horse race results suggest a substantial performance improvement of the DS and GD approaches to TFP growth measurement, compared with methods using conventional capital stock estimates. This improvement, is significant for samples of both 50 and 200 observations, and for both mature and traditional economies. The DS outperforms both alternatives under all assumptions, and by as much as 63% (BEA versus DS, $T=50$). The GD measure is outperforms all alternatives with the exception of the endogenous depreciation case. The employment of the Malmquist index estimate of initial growth makes a substantial contribution to the RSME.

As would be expected, the RMSE improvement of the stock-less measures over the conventional Solow residual estimates is inversely related to the relative importance of the initial condition and thus to the length of the sample time series. This relationship in our synthetic data set is displayed in Figure 3. which presents four comparisons of average RMSE for the mature economy case, along with confidence bands of one-standard deviation, as a function of the sample size from the same 100 realizations of 1000 (quarterly) observations. The graphs demonstrate how the stock-less measurements perform significantly better in a root mean squared error sense but this advantage tends to die out, but only at a very slow rate (at a sample size of 400 or more observations - a century of data- are the two equally inaccurate).

Figure 3: Dependence of Measurement Error on Sample Size



5 Application: TFP Growth in the Old and New German Federal States

We now apply the two new TFP growth measures to study the source of GDP growth in the federal states of Germany after reunification. To this purpose, GDP and "national" income account data are available beginning with 1992 for 16 states: 11 "old" states (Bavaria, Baden-Württemberg, Bremen, Hamburg, Hesse, Lower Saxony, North Rhine-Westphalia, Rhineland-Palatinate, Saarland, Schleswig-Holstein), 6 "new" states (Berlin, Brandenburg, Mecklenburg-West Pomerania, Saxony-Anhalt, Saxony, and Thuringia).¹³ We employ the income and product accounts and capital stock estimates at the level of the federal states published by the Working Group for State Income and Product Accounts (*Volkswirtschaftliche Gesamtrechnung der Länder* in Stuttgart).¹⁴ This dataset allows us to revisit the findings of Burda and Hunt (2001), who assessed the widely divergent evolution of labor productivity and total factor productivity between East and West and within the two groups of states using the conventional Solow residual measure. Because the capital stock data for the new states are poor, especially for structures, the alternative DS and GD methods offer an opportunity to investigate TFP growth measurements with a "treatment" group (East Germany) as well as a "control" group (West Germany), where the "treatment" is an unusually bad measurement of initial capital stocks. Reunification - both market competition and the revaluation of the east German mark - rendered about 80% of East German production noncompetitive (Akerlof, Rose, Yellen, and Hessenius (1991)), implying a large loss of value of existing equipment and structures. At the same time, many structures long carried at minimal book value were suddenly activated and employed by businesses, implying higher value of the capital stock.

In Table 5, we compare TFP growth using the Solow method and our stock-free TFP measurements for both the new and the old German states over two sub-periods: 1992-1997 and 1998-2003. The Solow residual estimates utilize an estimate of capital stocks provided by the state statistical agencies and the working group involved in collecting and standardizing the state income and product accounts. A constant capital share (0.33) was assumed. For the DS method, the annual rental price of capital (κ) was set to be constant over the entire period at a value of 0.0752. For the GD approach, a simple two-sided moving average of 2 years was used to estimate the trend. For both approaches, a constant rate of capital depreciation δ equal to 0.0754 was employed.

¹³Berlin is counted as a "new state" consisting of the union of East and West Berlin, because the western half of Berlin, while under the protection and economic aegis of Western Germany until 1989, never enjoyed full status as a *Bundesland*.

¹⁴The data can be downloaded at the website <http://www.vgrdl.de/ArbeitskreisvGR/ergebnisse.asp>

Table 5: TFP Measurement in German Federal States: A Comparison

State	Solow Residual		DS		GD	
	1992-1997	1998-2003	1992-1997	1998-2003	1992-1997	1998-2003
<i>Eastern states</i>						
<i>Berlin</i>	2.6	0.7	1.7	-0.4	0.6	-0.1
<i>Brandenburg</i>	7.3	2.0	5.7	0.3	3.4	1.2
<i>Macklenburg-Western Pomerania</i>	6.6	1.2	4.7	-0.4	3.4	0.6
<i>Saxony</i>	7.8	2.1	6.2	0.5	3.3	1.1
<i>Saxony-Anhalt</i>	7.2	2.0	5.4	0.5	3.4	1.0
<i>Thuringia</i>	9.3	2.1	7.3	0.6	3.4	1.1
Western States						
<i>Baden Wuerttemberg</i>	2.2	2.1	1.0	1.5	0.5	0.6
<i>Bavaria</i>	2.5	3.0	1.1	2.2	0.7	1.2
<i>Bremen</i>	2.5	2.4	1.6	1.9	0.6	0.7
<i>Hamburg</i>	3.0	1.5	2.0	1.0	0.8	0.3
<i>Hessen</i>	2.6	1.9	1.5	1.3	0.6	0.5
<i>Lower Saxony</i>	1.8	1.1	0.6	0.3	0.3	0.2
<i>North Rhine-Westphalia</i>	2.1	1.3	1.1	0.7	0.3	0.2
<i>Rhineland-Palatinate</i>	1.5	1.2	0.3	0.4	0.3	0.3
<i>Saarland</i>	1.4	1.6	0.3	0.8	0.2	0.5
<i>Schleswig-Holstein</i>	2.4	1.5	1.3	0.7	0.6	0.4
All Germany	2.9	1.9	1.6	1.1	0.8	0.6

Table 6: Growth Accounting Using the Three Methods for the period 1992-1997: A Comparison

State	$\frac{\Delta Y}{Y}$	$(1 - \alpha) \frac{\Delta N}{N}$	$\frac{\Delta A^{Solow}}{A^{Solow}}$	$\frac{\Delta K^{Solow}}{K^{Solow}}$	$\frac{\Delta A^{DS}}{A^{DS}}$	$\alpha \frac{\Delta K^{DS}}{K^{DS}}$	$\frac{\Delta A^{GD}}{A^{GD}}$	$\alpha \frac{\Delta K^{GD}}{K^{GD}}$
<i>Eastern states</i>								
<i>Berlin</i>	0.9	-1.1	2.6	-0.5	1.7	0.4	0.6	1.5
<i>Brandenburg</i>	7.3	-1.8	7.3	1.8	5.7	3.4	3.4	5.7
<i>Macklenburg-Western Pomerania</i>	7.2	-1.8	6.6	2.4	4.7	4.3	3.4	5.6
<i>Saxony</i>	7.2	-2.2	7.8	1.6	6.2	3.2	3.3	6.0
<i>Saxony-Anhalt</i>	6.9	-2.6	7.2	2.2	5.4	4.0	3.4	6.0
<i>Thuringia</i>	8.3	-2.9	9.3	1.9	7.3	3.9	3.4	7.8
Western States								
<i>Baden Wuerttemberg</i>	0.9	-0.1	2.2	-1.2	1.0	0.0	0.5	0.5
<i>Bavaria</i>	1.4	-0.1	2.5	-1.0	1.1	0.4	0.7	0.8
<i>Bremen</i>	0.3	-0.8	2.5	-1.4	1.6	-0.4	0.6	0.6
<i>Hamburg</i>	1.2	-0.3	3.0	-1.5	2.0	-0.5	0.8	0.7
<i>Hessen</i>	1.1	-0.1	2.6	-1.4	1.5	-0.2	0.6	0.6
<i>Lower Saxony</i>	0.8	0.2	1.8	-1.2	0.6	-0.1	0.3	0.3
<i>North Rhine-Westphalia</i>	0.5	-0.2	2.1	-1.4	1.1	-0.4	0.3	0.4
<i>Rhineland-Palatinate</i>	0.4	0.0	1.5	-1.1	0.3	0.1	0.3	0.2
<i>Saarland</i>	0.2	-0.1	1.4	-1.1	0.3	0.0	0.2	0.2
<i>Schleswig-Holstein</i>	1.2	0.0	2.4	-1.2	1.3	0.0	0.6	0.6
All Germany	1.5	-0.5	2.9	-0.9	1.6	0.4	0.8	1.2

Table 7: Growth Accounting Using the Three Methods for the period 1992-1997: A Comparison

State	$\frac{\Delta Y}{Y}$	$(1 - \alpha) \frac{\Delta N}{N}$	$\frac{\Delta A^{Solow}}{A^{Solow}}$	$\frac{\Delta K^{Solow}}{K^{Solow}}$	$\frac{\Delta A^{DS}}{A^{DS}}$	$\alpha \frac{\Delta K^{DS}}{K^{DS}}$	$\frac{\Delta A^{GD}}{A^{GD}}$	$\alpha \frac{\Delta K^{GD}}{K^{GD}}$
<i>Eastern states</i>								
<i>Berlin</i>	-0.5	-0.4	0.7	-0.8	-0.4	0.3	-0.1	0.0
<i>Brandenburg</i>	1.7	-1.0	2.0	0.7	0.3	2.3	1.2	1.5
<i>Macklenburg-Western Pomerania</i>	0.7	-0.9	1.2	0.4	-0.4	2.0	0.6	1.0
<i>Saxony</i>	1.8	-0.6	2.1	0.2	0.5	1.8	1.1	1.3
<i>Saxony-Anhalt</i>	1.1	-1.4	2.0	0.5	0.5	2.0	1.0	1.5
<i>Thuringia</i>	1.9	-0.3	2.1	0.2	0.6	1.6	1.1	1.2
Western States				0.0		0.0		
<i>Baden Wuerttemberg</i>	2.0	1.0	2.1	-1.1	1.5	-0.5	0.6	0.3
<i>Bavaria</i>	3.0	0.9	3.0	-0.9	2.2	-0.1	1.2	0.9
<i>Bremen</i>	1.4	0.1	2.4	-1.1	1.9	-0.6	0.7	0.6
<i>Hamburg</i>	1.1	0.7	1.5	-1.0	1.0	-0.6	0.3	0.1
<i>Hessen</i>	1.4	0.7	1.9	-1.1	1.3	-0.5	0.5	0.3
<i>Lower Saxony</i>	1.0	0.9	1.1	-1.0	0.3	-0.2	0.2	-0.1
<i>North Rhine-Westphalia</i>	1.0	0.9	1.3	-1.2	0.7	-0.6	0.2	-0.1
<i>Rhineland-Palatinate</i>	1.2	1.0	1.2	-0.9	0.4	-0.1	0.3	0.0
<i>Saarland</i>	1.6	0.9	1.6	-0.9	0.8	-0.1	0.5	0.2
<i>Schleswig-Holstein</i>	1.0	0.3	1.5	-0.9	0.7	0.0	0.4	0.3
All Germany	1.5	0.6	1.9	-0.9	1.1	-0.1	0.6	0.4

First, we note that in all cases the the DS and GD estimates are less volatile than the respective Solow-residual type estimates. This observation holds both for the old and new German states. This implies that the cyclical fluctuations characteristic of TFP estimates - total factor productivity as measured by the Solow residual is the most important evidence invoked by the RBC school as the primary source of macroeconomic fluctuations - are exaggerated due to fluctuations in the rate of capacity utilization. Both DS and GD methods show TFP growth to be much more stable over the two halves of the sample (1992-7 and 1998-2003). Second, the dispersion of TFP growth across the states is much less volatile in both subsets. This appears more plausible for a variety of reasons. Finally, the estimates of TFP using the DS and GD methods can be used to back out an implied contribution of capital to real growth, or, given a capital share, to growth in the true (i.e. actually utilized) capital stock. These estimates are also presented in Table 6. They show indeed a larger degree of fluctuation than that implied by official estimates of capital stock growth. They support the notion, already due to Burnside, Eichenbaum, and Rebelo (1995) and others, that the fluctuation of capital in use is an important source of measurement error and should be considered carefully when computing the solow residual. Our proposed alternatives have the advantage of shutting down this source of mismeasurement, to the extent that the utilization of the most recent investment more closely tracks the "true" utilization rate.

6 Conclusion

Over the past half-century, the Solow residual has achieved widespread use in economics and management as a measurement of total factor productivity. Its acceptability is attributable to its simplicity and independence from statistical methods. Despite this acceptance, there has been no effort to evaluate systematically the quality of this measurement tool. This complacency is remarkable in light of potentially severe measurement problems associated with capital stock data. We have documented that the error, as measured by the root mean squared error - associated with the capital stock mismeasurement can be significant in a synthetic data set. These measurements can be characterized most generously as educated guesses of quantities for which reliable measurement or complete markets simply do not exist.

While the measurement error of the Solow residual decreases with sample size, it remains especially acute for short data sets or economies in transition. Thus, the Solow residual is least accurate in applications for which TFP measurements are most valuable. Such applications the transition to a market economy, the introduction of ICT capital in the production process, and the increasing employment of weightless assets such as advertising goodwill and research and development knowledge. (Corrado, Hulten, and Sichel (2006))

Both proposed alternatives to the Solow-Thörnqvist measures can be thought of as a "marginalization" of the error carried forward by the capital stock across time. Most recent investment is most likely to be properly valued at acquisition cost and to be fully utilized. Our results suggest that these methods could be applied to a number of investment context and types, thus widening the scope and appeal of applied TFP measurement.

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Appendix 1: The Stochastic Growth Model

First Order Conditions and Decentralized Market Equilibrium

Let λ_t denote the Lagrange multiplier corresponding to the periodic resource constraint (14)

The first-order conditions for the household are, for $t \geq 0$:

$$C_t : \lambda_t = \frac{1}{C_t}$$

$$K_{t+1} : \lambda_t = \beta E_t [\lambda_{t+1} (1 - \delta + U_{t+1} \kappa_{t+1})] \quad (23)$$

$$N_t : \theta(1 - \bar{N})^{-\eta} = \lambda_t \omega_t \quad (24)$$

$$U_t : BU_t^{\chi-1} = \kappa_{t+1} \quad (25)$$

First-order conditions for the firms

$$N_t : (1 - \alpha) A_t (U_t K_t)^\alpha N_t^{-\alpha} = \omega_t \quad (26)$$

$$K_t : \alpha A_t U_t^\alpha K_t^{\alpha-1} N_t^{1-\alpha} = \kappa_t \quad (27)$$

the production function

$$Y_t = A_t K_t^\alpha N_t^{-\alpha} \quad (28)$$

and the aggregate resource constraint (since $\omega_t N_t + \kappa_t U_{t-1} K_{t-1} = Y_t$).

$$K_{t+1} = (1 - \delta_t) K_t + Y_t - C_t \quad (29)$$

The equilibrium of this decentralized economy is defined as the sequences of wages $\{\omega_t\}$, rental prices for capital $\{\kappa_t\}$, output $\{Y_t\}$, consumption, $\{C_t\}$, employment $\{N_t\}$, capital stocks $\{K_{t+1}\}$, and the capacity utilization rate $\{U_t\}$ such that the above equations hold for $t \geq 0$ plus a suitable transversality condition to guarantee that the capital stock path is indeed consistent with utility maximization. The equilibrium of the problem will be, due to the first and second welfare theorems, unique and equivalent to the one chosen by a social planner with the objective of maximizing the utility of the representative household.

Detrended version of Equilibrium

Define detrended values of the variables of interest such that $\tilde{X}_t \equiv X_t / \bar{X}_t$. In equilibrium, capital and labor are paid their respective marginal products in each period. The following equations characterize the equilibrium of this economy:

$$\frac{\theta \tilde{C}_t}{(1 - N_t)^\eta} = (1 - \alpha) \gamma_t U_t^\alpha \tilde{K}_t^\alpha N_t^{-\alpha} \quad (30)$$

$$1 = E_t \left[\beta \frac{\tilde{C}_t}{\psi \tilde{C}_{t+1}} R_{t+1} \right] \quad (31)$$

$$\alpha \gamma_t \left(\frac{\tilde{K}_t}{N_t} \right)^{\alpha-1} = U_t^{\chi-\alpha+1} \quad (32)$$

$$\psi \tilde{K}_{t+1} = (1 - \delta_t) \tilde{K}_t + \tilde{Y}_t - \tilde{C}_t.$$

The first equation characterizes intratemporal optimality of time across alternative uses in production and leisure; the second is the familiar Euler equation which arbitrages expected intertemporal rates of substitution and transformation in expectation, where the latter is defined by $R_{t+1} = (1 - \alpha) \gamma_t \tilde{K}_t^\alpha N_t^{-\alpha} + 1 - \delta_t$ is the gross rate of return on holding a unit of capital K from period t to period $t + 1$. The last equation is the periodic resource constraint of the economy, given the production function and competitive factor remuneration. Given that this economy fulfills the conditions of the first welfare theorem, it would also characterize the optimal choice of a central planner maximizing (13) subject to the resource constraints (14) and the initial condition K_0 .

The Steady State

To solve for the non stochastic steady state, let $\gamma_t = 1$ and $\tilde{K}_{t+1} = \tilde{K}_t = \bar{K}$. We obtain the following equations:

$$\frac{\theta \bar{C}}{(1 - \bar{N})^\eta} = (1 - \alpha) \bar{U}^\alpha \bar{K}^\alpha \bar{N}^{-\alpha} \quad (33)$$

$$1 = \beta \bar{R} \quad (34)$$

$$\alpha \left(\frac{\bar{K}}{\bar{N}} \right)^{\alpha-1} = \bar{U}^{\chi-\alpha+1} \quad (35)$$

$$0 = (1 - \delta - \psi) \bar{K} + \bar{Y} - \bar{C}$$

Log Linearization

Using the convention that $\hat{x} = (x - \bar{x})/\bar{x}$ denote deviations from steady state values, the log-linearized first order condition for labor supply can be written as

$$\hat{c}_t - \left(\alpha + \frac{N}{1 - N} \eta \right) \hat{n}_t = \gamma_t + \alpha (\hat{u}_t + \hat{k}_t) \quad (36)$$

The resource constraint as

$$\frac{\bar{C}}{\bar{K}}\hat{c}_t + \psi\hat{k}_{t+1} = (1 - \delta)\hat{k}_t - \chi\hat{u}_t + \alpha\frac{\bar{Y}}{\bar{K}}\hat{k}_t + (1 - \alpha)\frac{\bar{Y}}{\bar{K}}\bar{N}\hat{n}_t + \frac{\bar{Y}}{\bar{K}}\hat{\gamma}_t \quad (37)$$

and the Euler equation

$$0 = E_t\beta \left[\hat{c}_t + \hat{c}_{t+1} + \beta\bar{r} \left[\hat{\gamma}_{t+1} - (1 - \alpha) \left(\hat{k}_{t+1} - \hat{n}_{t+1} \right) - \chi\hat{u}_t \right] \right] \quad (38)$$

Model Calibration and Generation of the Synthetic Dataset

We calibrate the model to a quarterly setting using values typically used for simulating the US time series in the literature and discussed in Prescott (1986), Rebelo and King (1999). The value chosen for the parameters are presented in Table 8.

Table 8: Stochastic growth model: parameters and calibration values

Parameter	Definition	Value	Source
β	utility discount factor (quarterly)	0.985	Data
\bar{R}	average real interest factor (quarterly)	1.015	Data
$\bar{\gamma}$	technology	1	Theory
$\bar{\delta}$	depreciation rate of physical capital	0.015	Data
α	capital elasticity in production	0.36	Data
η	elasticity of periodic utility to leisure	0.85	Theory
θ	utility weight for leisure/consumption	2.1	Theory
$\psi = (1 + g)^{1-\alpha}$	constant growth factor of technology	1.0075	Data
B	level parameter for capital depreciation rate	4.04	Data
χ	elasticity of depreciation to capacity utilization	11.08	Data
ρ	autocorrelation of TFP term A_t	0.95	Theory

Table 9 and Table 10 compare statistics from our synthetic data set with those from the US economy reported by Stock and Watson (1999) and from synthetic economies with divisible and indivisible labor reported by Hansen (1985). In the first two columns we compare our stochastic growth model respectively with endogenous and constant depreciation rate.

Table 9: Cross-correlations with output $corr(x_t, y_t)$ observations

Series	Stochastic Growth Model Endogenous Depreciation	Stochastic Growth Model Constant Depreciation	Div. Labor Hansen (1985)	Indiv. Labor Hansen (1985)	US DATA 1953Q1-1996Q4 Stock and Watson (1999)
Consumption	0.98	0.86	0.89	0.87	0.90
Investment	0.99	0.92	0.99	0.99	0.89
Employment	0.99	0.76	0.98	0.98	0.89
Productivity	0.97	0.99	0.98	0.87	0.77

Table 10: Stochastic Growth Model in comparison: standard deviations normalized by standard deviation of output

Series	Stochastic Growth Model End. Depreciation	Stochastic Growth Model Cons. Depreciation	Divisible Labor Model	Indivisible Labor Model	US DATA-Stock 1953Q1-1996Q4	US DATA-Dejong 1948Q1-2004Q4
Consumption	0.45	0.46	0.68	0.29	0.76	0.46
Investment	2.18	2.38	2.01	3.24	2.99	4.23
Employment	0.32	0.34	0.54	0.77	1.56	1.05

Appendix 2: The Malmquist Index

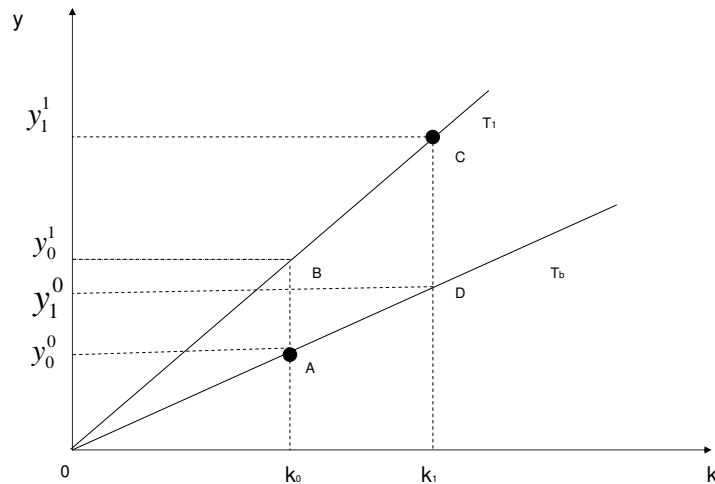
The Malmquist index originates in measurement theory and is frequently applied to productivity estimation problems (see Malmquist (1953), Färe (1989) and Färe, Grosskopf, Norris, and Zhang (1994)). Normalize observed output and capital by labour input $y_t = \frac{Y_t}{N_t}$ and $k_t = \frac{K_t}{N_t}$. If $f_t(k)$ defines the efficient level of production using k in time t , use the distance function $D_t(k_t, y_t) = y_t/f_t(k_t)$ to construct Malmquist index between periods 0 and 1 (Shephard (1970)):

$$M_0^1 = \underbrace{\sqrt{\frac{D_1(k_1, y_1)}{D_1(k_0, y_0)}}}_{\Delta(\text{efficiency})} \underbrace{\sqrt{\frac{D_0(k_1, y_1)}{D_0(k_0, y_0)}}}_{\Delta(\text{technology})} \quad (39)$$

Following Färe (1989), the Malmquist index can be decomposed as a product of change in efficiency at given technology, and technological change. Because the Solow decomposition assumes full efficiency, the Malmquist index is simply $\sqrt{\frac{D_0(k_1, y_1)}{D_0(k_0, y_0)}} = \sqrt{\frac{A_{t+1}}{A_t}}$. Figure 4 depicts two data points $y_0^0 \equiv f_0(k_0)$ (point A) and $y_1^1 \equiv f_1(k_1)$ (point C), and two counterfactuals $y_0^1 \equiv f_1(k_0)$ (point B) and $y_1^0 \equiv f_0(k_1)$ (point D). Assuming constant returns and full efficiency, the log of the Malmquist index equals the log of the geometric mean of the average products in the two periods, or .

$$\ln M_0^1 = \frac{1}{2} \ln \left(\frac{y_1^1 y_1^0}{y_0^0 y_0^1} \right) = \underbrace{\frac{1}{2} \ln \left(\frac{y_1^1}{y_0^0} \right)}_{\text{KNOWN}} + \underbrace{\frac{1}{2} \ln \left(\frac{y_1^0}{y_0^1} \right)}_{\text{UNKNOWN}} \quad (40)$$

Figure 4: Construction of the Malmquist index in the full efficiency case.



The Malmquist index puts a bound on possible evolution of TFP from period 0 to period 1, even when the capital stock is poorly measured or unobservable. Consider first the extreme case in which there no capital accumulation in period 0, i.e. $k_0 = k_1$ and $\ln M_0^1 = \frac{1}{2} \ln \left(\frac{y_1^1}{y_0^0} \right)$; in the other extreme, capital accumulation is identical to the growth of labor productivity, i.e. $\ln M_0^1 = \ln \left(\frac{y_1^1}{y_0^0} \right)$. We will employ the midpoint between these two values.

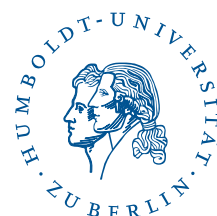
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