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## Comparison of panel cointegration tests

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# Comparison of Panel Cointegration Tests

Deniz Dilan Karaman Örsal\*

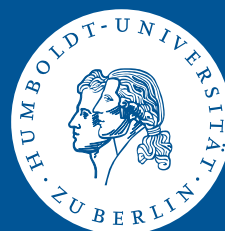


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# Comparison of Panel Cointegration Tests \*

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## Abstract

The main aim of this paper is to compare the size and size-adjusted power properties of four residual-based and one maximum-likelihood-based panel cointegration tests with the help of Monte Carlo simulations. In this study the panel- $\rho$ , the group- $\rho$ , the panel- $t$ , the group- $t$  statistics of Pedroni (1999) and the standardized LR-bar statistic of Larsson et al. (2001) are considered. The simulation results indicate that the panel- $t$  and standardized LR-bar statistic have the best size and power properties among the five panel cointegration test statistics evaluated. Finally, the Fisher Hypothesis is tested with two different data sets for OECD countries. The results point out the existence of the Fisher relation.

*Keywords:* Panel Cointegration tests, Monte Carlo Study, Fisher Hypothesis

*JEL classification:* C23, C33, C15

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# 1 Introduction

During the last two decades cointegration techniques have been widely used in the empirical studies. However, the difficulties in finding long time series and the low power of the ADF (augmented Dickey-Fuller) and DF (Dickey-Fuller) unit root tests for the univariate case let researchers to develop unit root and cointegration tests for the panel data. This approach brought the advantage of using the growing multiple cross-sectional dimension. Following the extension of the univariate unit root tests to the panel data by Levin et al. (2002), Quah (1994), Jörg & Meyer (1994) and Im et al. (2003), the application of the cointegration tests to the panel data has grasped a wide interest in the literature.

There are mainly two different approaches for the panel cointegration tests, residual-based and maximum-likelihood-based. Residual-based panel cointegration test statistics were introduced by McCoskey & Kao (1998), Kao (1999), Pedroni (1995, 1997, 1999) and maximum-likelihood-based panel cointegration test statistics were introduced by Groen & Kleibergen (2003), Larsson & Lyhagen (1999) and Larsson et al. (2001).

McCoskey & Kao (1998) derived a panel cointegration test for the null of cointegration which is an extension of the LM test and the locally best unbiased invariant (LBUI) test for an MA root. Kao (1999) considered the spurious regression for the panel data and introduced the DF and ADF type tests. He proposed four different DF type test statistics, and used the sequential limit theory of Phillips & Moon (1999) to derive the asymptotic distributions of these statistics.

Groen & Kleibergen (2003) presented how homogenous and heterogeneous cointegration vectors are estimated within a maximum-likelihood framework using the GMM procedure. They also proposed a likelihood ratio test for the common cointegration rank, which is based on these GMM estimators and the cross-sectional dependence. On the other hand, Larsson et al. (2001) suggested panel cointegration test statistic based on cross-sectional independence.

In this paper, the properties of the residual-based panel cointegration tests of Pedroni (1999) and the maximum-likelihood-based panel cointegration rank test of Larsson et al. (2001) will be compared with the help of a Monte Carlo simulation study.

In this simulation study we focus on the changes in size and size-adjusted power of the panel cointegration tests when time and cross-section dimensions and various parameters in the data generating process vary, e.g. the correlation parameter between the disturbances to stationary and non-stationary parts of the DGP for each cross-section. To our knowledge size-adjusted power properties of the panel cointegration test statistics are presented for the first time in the literature for the

test statistics considered here.

The results of the simulation study, which are based on a DGP with three variables demonstrate that the panel- $t$  statistic has the best size and power properties. The size of the panel- $t$  and standardized LR-bar test statistic approach the nominal size of 5% when time ( $T$ ) and cross-section ( $N$ ) dimensions increase. With the increase of  $N$  the size-adjusted power of the panel- $t$  statistic gets close to unity even when there is high correlation between the disturbances to the stationary and non-stationary parts of the DGP. Additionally, we test the Fisher relation among the OECD countries for different time spans using the tests considered in this paper.

The second part of the paper involves the panel cointegration tests of Pedroni (1999) and Larsson et al. (2001). In the third section we present the way how the DGP of Toda (1995) is modified for the panel data, and the fourth section gives a description of the simulation study. The fifth section is devoted to the interpretation of the simulation results. The validity of the Fisher relation is the subject of the sixth section. Conclusions are given in the seventh section. The simulation results are presented in Appendix A, B and C.

## 2 Panel Cointegration Tests

### 2.1 Pedroni (1999)

Following the introduction of the residual-based panel cointegration tests in 1995, Pedroni (1999) extended his panel cointegration testing procedure for the models, where there are more than one independent variable in the regression equation. In this study two within-dimension-based<sup>1</sup> (panel- $\rho$  and panel- $t$ ) and two between-dimension-based<sup>2</sup> (group- $\rho$  and group- $t$ ) null of no cointegration panel cointegration statistics of Pedroni (1999) will be compared with the maximum-likelihood-based panel cointegration statistic of Larsson et al. (2001). The panel- $\rho$  statistic is an extension of the non-parametric Phillips-Perron  $\rho$ -statistic, and the parametric panel- $t$  statistic is an extension of the ADF  $t$ -statistic. Between-dimension-based statistics are just the group mean approach extensions of the within-dimension-based ones. The group- $\rho$  statistic is chosen, because Gutierrez (2003) demonstrated that this test statistic has the best power among the test statistics of Pedroni (1999), Larsson et al. (2001) and Kao (1999). The group- $t$  statistic is selected, because the data generating process, which we will use for the simulation study, is appropriate for parametric ADF-type tests. In addition to this, the within-dimension versions

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<sup>1</sup>Within-dimension-based statistics are calculated by summing the numerator and the denominator over  $N$  cross-sections separately.

<sup>2</sup>Between-dimension-based statistics are calculated by dividing the numerator and the denominator before summing over  $N$  cross-sections.

of these statistics (i.e. panel- $\rho$  and panel- $t$ ) are considered in order to be able to compare them with their between-dimension versions.

The starting point of the residual-based panel cointegration test statistics of Pedroni (1999) is the computation of the residuals of the hypothesized cointegrating regression<sup>3</sup>,

$$y_{i,t} = \alpha_i + \beta_{1i}x_{1i,t} + \beta_{2i}x_{2i,t} + \dots + \beta_{Mi}x_{Mi,t} + e_{i,t}, \quad t = 1, \dots, T; \quad i = 1, \dots, N \quad (1)$$

where  $T$  is the number of observations over time,  $N$  denotes the number of individual members in the panel, and  $M$  is the number of independent variables. It is assumed here that the slope coefficients  $\beta_{1i}, \dots, \beta_{Mi}$ , and the member specific intercept  $\alpha_i$  can vary across each cross-section.

To compute the relevant panel cointegration test statistics, first the cointegration regression in (1) is estimated by OLS, for each cross-section.

In addition to this, the within-dimension based test statistics i.e. panel- $\rho$  and panel- $t$  statistics are computed by taking the first-difference of the original series and estimating the residuals of the following regression:

$$\Delta y_{i,t} = b_{1i}\Delta x_{1i,t} + b_{2i}\Delta x_{2i,t} + \dots + b_{Mi}\Delta x_{Mi,t} + \pi_{i,t} \quad (2)$$

Using the residuals from the differenced regression (2), with a Newey & West (1987) estimator, the long run variance of  $\hat{\pi}_{i,t}$  is calculated, which is represented as  $\hat{L}_{11i}^2$ .

To calculate the non-parametric statistics, panel- $\rho$  and group- $\rho$ , the regression  $\hat{e}_{i,t} = \hat{\gamma}_i \hat{e}_{i,t-1} + \hat{u}_{i,t}$  is estimated using the residuals  $\hat{e}_{i,t}$  from the cointegration regression (1). Then the long-run variance ( $\hat{\sigma}_i^2$ ) and the contemporaneous variance ( $\hat{s}_i^2$ ) of  $\hat{u}_{i,t}$  is computed. For the calculation of  $\hat{\sigma}_i^2$  Pedroni (1995) used  $4 \left(\frac{T}{100}\right)^{2/9}$  as the lag truncation function for the Newey-West kernel estimation as recommended in Newey & West (1994). The nearest integer is taken as the lag length for different  $T$  dimensions.

The parametric test statistics, panel- $t$  and group- $t$ , are estimated with the help of the the residuals  $\hat{e}_{i,t}$  from cointegration regression (1),  $\hat{e}_{i,t} = \hat{\gamma}_i \hat{e}_{i,t-1} + \sum_{k=1}^{K_i} \hat{\gamma}_{i,k} \Delta \hat{e}_{i,t-k} + \hat{u}_{i,t}^*$  and the variance of  $\hat{u}_{i,t}^*$  is computed, which is denoted as  $\hat{s}_i^{*2}$ . To determine the lag truncation order of the ADF  $t$ -statistics, the step-down procedure and the Schwarz lag order selection criterion are used. Using the following expressions the relevant test statistics can be constructed:

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<sup>3</sup>In this study the regression equation with an heterogeneous intercept will be considered. Note that it could also be estimated without a heterogeneous intercept, or with a time trend and/or common time dummies.

a. Panel- $\rho$  statistic

$$T\sqrt{N}Z_{\hat{\rho}_{N,T-1}} \equiv T\sqrt{N} \left( \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \hat{e}_{i,t-1}^2 \right)^{-1} \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \left( \hat{e}_{i,t-1} \Delta \hat{e}_{i,t} - \hat{\lambda}_i \right)$$

b. Panel- $t$  statistic

$$Z_{t_{N,T}}^* \equiv \left( \tilde{s}_{N,T}^{*2} \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \hat{e}_{i,t-1}^2 \right)^{-1/2} \sum_{i=1}^N \sum_{t=1}^T \hat{L}_{11i}^{-2} \hat{e}_{i,t-1} \Delta \hat{e}_{i,t}$$

c. Group- $\rho$  statistic

$$TN^{-1/2} \tilde{Z}_{\hat{\rho}_{N,T-1}} \equiv TN^{-1/2} \sum_{i=1}^N \left( \sum_{t=1}^T \hat{e}_{i,t-1}^2 \right)^{-1} \sum_{t=1}^T \left( \hat{e}_{i,t-1} \Delta \hat{e}_{i,t} - \hat{\lambda}_i \right)$$

d. Group- $t$  statistic

$$N^{-1/2} \tilde{Z}_{t_{N,T}}^* \equiv N^{-1/2} \sum_{i=1}^N \left( \sum_{t=1}^T \hat{s}_i^{*2} \hat{e}_{i,t-1}^2 \right)^{-1/2} \sum_{t=1}^T \hat{e}_{i,t-1} \Delta \hat{e}_{i,t}$$

where  $\hat{\lambda}_i = \frac{1}{2} (\hat{\sigma}_i^2 - \hat{s}_i^2)$  and  $\tilde{s}_{N,T}^{*2} \equiv \frac{1}{N} \sum_{i=1}^N \hat{s}_i^{*2}$ .

After the calculation of the panel cointegration test statistics the appropriate mean and variance adjustment terms are applied, so that the test statistics are asymptotically standard normally distributed.

$$\frac{\varkappa_{N,T} - \mu\sqrt{N}}{\sqrt{\nu}} \Rightarrow N(0, 1),$$

where  $\varkappa_{N,T}$  is the standardized form of the test statistic with respect to  $N$  and  $T$ ,  $\mu$  and  $\nu$  are the functions of moments of the underlying Brownian motion functionals. The appropriate mean and variance adjustment terms for different number of regressors and different panel cointegration test statistics are given in Table 2 in Pedroni (1999)<sup>4</sup>.

The null hypothesis of no cointegration for the panel cointegration test is the same for each statistic,

$$H_0 : \gamma_i = 1 \text{ for all } i = 1, \dots, N,$$

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<sup>4</sup>This table contains the mean and variance values for the cases when there is no heterogeneous intercept, or when there is a heterogeneous intercept or/and a time trend in the heterogeneous regression equation.  $m$  is the number of regressors without taking the heterogeneous deterministic terms into account.

whereas the alternative hypothesis for the between-dimension-based and within-dimension-based panel cointegration tests differs. The alternative hypothesis for the between-dimension-based statistics is

$$H_1 : \gamma_i < 1 \text{ for all } i = 1, \dots, N,$$

where a common value for  $\gamma_i = \gamma$  is not required. For within-dimension-based statistics the alternative hypothesis

$$H_1 : \gamma_i = \gamma < 1 \text{ for all } i = 1, \dots, N$$

assumes a common value for  $\gamma_i = \gamma$ .

Under the alternative hypothesis, all the panel cointegration test statistics considered in this paper diverge to negative infinity. Thus, the left tail of the standard normal distribution is used to reject the null hypothesis.

## 2.2 Larsson, Lyhagen and Løthgren (2001)

Larsson et al. (2001) presented a maximum-likelihood-based panel test for the cointegrating rank in heterogeneous panels. They proposed a standardized LR-bar test based on the mean of the individual rank trace statistic of Johansen (1995) .

The panel data set consists of  $N$  cross-sections observed over  $T$  time periods, where  $i$  is the index for the cross-section,  $t$  represents the index for the time dimension and  $p$  is the number of variables in each cross-section. The following heterogeneous  $VAR(k_i)$  model,

$$Y_{it} = \sum_{k=1}^{k_i} A_{ik} Y_{i,t-k} + \varepsilon_{it} \quad i = 1, \dots, N, \quad (3)$$

is considered for each cross-section under the assumptions that  $\varepsilon_{it}$  is Gaussian white noise with a nonsingular covariance matrix  $\varepsilon_{it} \sim N_p(0, \Omega_i)$ , and the initial conditions  $Y_{i,-k_i+1}, \dots, Y_{i,0}$  are fixed. However, this model allows neither an intercept nor a time trend in the VAR model. The error correction representation for (3) is

$$\Delta Y_{it} = \Pi_i Y_{i,t-1} + \sum_{k=1}^{k_i-1} \Gamma_{ik} \Delta Y_{i,t-k} + \varepsilon_{it}, \quad i = 1, \dots, N, \quad (4)$$

where  $\Pi_i$  is of order  $(p \times p)$ . In the reduced rank form it is possible to write the matrix  $\Pi_i$  as  $\Pi_i = \alpha_i \beta_i'$ , where  $\alpha_i$  and  $\beta_i$  are of order  $(p \times r_i)$  and have full column rank.



Larsson et al. (2001) considered the null hypothesis that all of the  $N$  cross-sections have at most  $r$  cointegrating relationships among the  $p$  variables. Thus, the null hypothesis for the panel cointegration test can be expressed as

$$H_0 : \text{rank}(\Pi_i) = r_i \leq r \quad \text{for all } i = 1, \dots, N,$$

where

$$H_1 : \text{rank}(\Pi_i) = p \quad \text{for all } i = 1, \dots, N.$$

They defined the standardized LR-bar statistic for the panel cointegration rank test as:

$$\gamma_{LR}\{H(r)|H(p)\} = \frac{\sqrt{N} \left( \frac{1}{N} \sum_{i=1}^N \left( -T \sum_{j=r_i+1}^p \ln(1 - \hat{\lambda}_{i,j}) \right) - E(Z_k) \right)}{\sqrt{\text{Var}(Z_k)}}, \quad (5)$$

where  $\hat{\lambda}_{i,j}$  is the  $j$ th eigenvalue of the  $i$ th cross-section to the eigenvalue problem given in Johansen (1995),  $E(Z_k)$  is the mean and  $\text{Var}(Z_k)$  is the variance of the asymptotic trace statistic  $Z_k$ .

$$Z_k \equiv \text{tr} \left\{ \int_0^1 (dW) W' \left( \int_0^1 W W' \right)^{-1} \int_0^1 W (dW)' \right\},$$

where  $W$  is a  $k = (p - r)$  dimensional Brownian motion. Larsson et al. (2001) have simulated the mean and variance of the asymptotic trace statistic for different  $k$  values using the simulation procedure described in Johansen (1995) <sup>5</sup>.

Under certain assumptions<sup>6</sup> the standardized LR-bar statistic is standard normally distributed as  $N$  and  $T \rightarrow \infty$  in such a way that  $\sqrt{N}T^{-1} \rightarrow 0$ .

The panel cointegration rank test of Larsson et al. (2001) is a one-sided test  $H_0 : \text{rank}(\Pi_i) = r_i \leq r$ , which is rejected for all  $i$ , if the standardized LR-bar statistic is bigger than the  $(1 - \alpha)$  standard normal quantile, where  $\alpha$  is the significance level of the test. The sequential procedure of Johansen (1988) is used as the testing procedure. First  $H_0 : r = 0$  is tested, and if  $r = 0$  is rejected  $H_0 : r = 1$  is tested. The procedure continues until the null hypothesis is not rejected or the  $H_0 : p - 1$  is rejected.

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<sup>5</sup>Simulated mean and variance values of the asymptotic trace statistic can be found in Table 1 of Larsson et al. (2001)

<sup>6</sup>Assumption 1-3' in Larsson et al. (2001)

### 3 Data Generating Process

The Monte Carlo study is based on the data generating process of Toda (1995), which has been used in several papers in the literature<sup>7</sup>. The canonical form of the Toda process allows us to see the dependence of the test performance on some key parameters. In the following lines you can find the modified data generating process of Toda for the panel data.

Let  $y_{i,t}$  be a  $p$ -dimensional vector, where  $i$  is the index for the cross-section,  $t$  is the index for the time dimension and  $p$  denotes the number of variables in the model. The data generating process has the form of a VAR(1) process. The general form of the modified Toda process for a system of three variables in the absence of a linear trend in the data is,

$$y_{i,t} = \begin{pmatrix} \psi_a & 0 & 0 \\ 0 & \psi_b & 0 \\ 0 & 0 & \psi_c \end{pmatrix} y_{i,t-1} + \varepsilon_{i,t} \quad t = 1, \dots, T; \quad i = 1, \dots, N, \quad (6)$$

where the initial values of  $y_{i,t}$ , which can be represented as  $y_{i,0}$  are zero. The error terms for each cross-section has the following structure:

$$\varepsilon_{i,t} = \begin{pmatrix} \varepsilon_{1i,t} \\ \varepsilon_{2i,t} \end{pmatrix} \equiv i.i.d.N \left( 0, \begin{pmatrix} I_r & \Theta \\ \Theta' & I_{p-r} \end{pmatrix} \right).$$

The true cointegrating rank of the process is denoted by  $r$  and  $\varepsilon_{1i,t}$ ,  $\varepsilon_{2i,t}$  are the disturbances to the stationary and non-stationary parts of the data generating process, respectively.  $\Theta$  represents the vector of instantaneous correlation between the stationary and non-stationary components of the relevant cross-section.

Taking into account (6), when  $\psi_a = \psi_b = \psi_c = 1$ , a cointegrating rank of  $r = 0$  is obtained. Thus, the data generating process becomes,

$$y_{i,t} = I_3 y_{i,t-1} + \varepsilon_{i,t}, \quad (7)$$

where  $\varepsilon_{i,t} \equiv i.i.d.N(0, I_3)$ , which means that the process consists of three non-stationary components and these components are instantaneously uncorrelated. The VEC representation of (7) is:

$$\Delta y_{it} = \Pi_{i,t} y_{i,t-1} + \varepsilon_{i,t}. \quad (8)$$

Here,  $\Pi_{i,t} = -(I_3 - A_{i1})$  and  $A_{i1} = I_3$  represents the coefficient matrix of the VAR(1) process from (7). As  $\Pi_{i,t}$  is a null matrix, (8) turns into:

$$\Delta y_{it} = \varepsilon_{i,t}.$$

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<sup>7</sup>Lütkepohl & Saikkonen (2000), Saikkonen & Lütkepohl (1999, 2000), Hubrich et al. (2001), Trenkler (2006), Trenkler et al. (2006) etc.

With  $|\psi_a| < 1$  and  $\psi_b = \psi_c = 1$  the true cointegrating rank of the DGP is 1, and it is composed of one stationary and two non-stationary components, which can be formulated as,

$$y_{i,t} = \begin{pmatrix} \psi_a & 0 \\ 0 & I_2 \end{pmatrix} y_{i,t-1} + \varepsilon_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \equiv i.i.d.N \left( 0, \begin{pmatrix} 1 & \Theta \\ \Theta' & I_2 \end{pmatrix} \right), \quad (9)$$

where  $\Theta = (\theta_a, \theta_b)$  and  $|\theta_a|, |\theta_b| < 1$ .

The cointegrating rank of the process is  $r = 2$ , when  $\psi_a$  and  $\psi_b$  are less than unity in absolute value and  $\psi_c = 1$ . This can be represented in matrix form as,

$$y_{i,t} = \begin{pmatrix} \psi_a & 0 & 0 \\ 0 & \psi_b & 0 \\ 0 & 0 & 1 \end{pmatrix} y_{i,t-1} + \varepsilon_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \equiv i.i.d.N \left( 0, \begin{pmatrix} I_2 & \Theta' \\ \Theta & 1 \end{pmatrix} \right), \quad (10)$$

where  $\Theta = (\theta_a, \theta_b)$  and  $\theta_a, \theta_b$  are less than unity in absolute value. This process consists of one non-stationary and two stationary components and these components are correlated, when at least  $\theta_a$  or  $\theta_b$  is different from zero.

When  $|\psi_a|, |\psi_b|$  and  $|\psi_c| < 1$  the DGP is an  $I(0)$  process, and the cointegrating rank is  $r = 3$ , which can be represented as,

$$y_{i,t} = \Psi y_{i,t-1} + \varepsilon_{i,t}, \quad (11)$$

where  $\Psi = \text{diag}(\psi_a, \psi_b, \psi_c)$  and  $\varepsilon_{i,t} \equiv i.i.d.N(0, I_3)$ .

## 4 Simulation Study

In order to see the performance of the tests to the changes in some key parameters, throughout the simulation study the time and cross-section dimensions,  $\psi_a, \psi_b, \psi_c$  parameters and the correlation parameters  $\theta_a$  and  $\theta_b$  will vary.

The correlation parameters  $\theta_a$  and  $\theta_b$  take the values  $\{0, 0.4, 0.7\}$  and  $\psi$  parameters take the values  $\{0.5, 0.8, 0.95, 1\}$ . The value 0.95 for  $\psi$  parameters will help us to see how the tests react when the cointegrating rank of the process is near zero. The performance of the tests under the assumption of no instantaneous correlation between the disturbances is checked by  $\theta_a = \theta_b = 0$ .

To compare the results with Larsson et al. (2001), for the cross section dimension  $N = \{1, 5, 10, 25, 50\}$  and for the time dimension  $T = \{10, 25, 50, 100, 200\}$  are taken into account. The total number of replications is 1000. While generating the random error terms, seeded values are used and the first 100 observations are deleted, so that

the starting values are not anymore zero. The tests were programmed in GAUSS 5.0.

The maximum lag order for the panel- $t$  and group- $t$  statistics is limited to 3, because this was the maximum lag order allowing an efficient estimation for small time dimensions, e.g.  $T = 10$ . To select the lag order for the non-parametric panel- $\rho$  and group- $\rho$  statistics a kernel estimator is used, as explained in Section 2. For the maximum-likelihood-based test statistic no VAR model lag order selection criterion is used, because the data is generated using a  $VAR(1)$  process. Only the null of no cointegration hypothesis is tested, because the residual-based tests cannot test for the order of panel cointegrating rank.

In the next sections the simulation results for the empirical size and the size-adjusted power of the panel cointegration test statistics will be discussed. The figures<sup>8</sup> for the simulation results are presented in the Appendix A and B.

## 5 Interpretation of the Simulation Results

The importance of this simulation study lies in the DGP. In this study, the DGP is based on AR processes and covers the small sample properties of the residual-based tests, when there is more than one independent variable in the DGP, which was not done by Pedroni (1995). The most interesting results for empirical size and power properties of the panel cointegration tests are presented in Appendix A<sup>9</sup>.

### 5.1 Empirical Size Properties

Figures 1 and 2 demonstrate that the empirical sizes of group- $\rho$  and panel- $\rho$  statistics are always zero for  $T = 10, 25$  and  $N \geq 1$ , which means that the true hypothesis of no cointegration can never be rejected. The severe size distortions for the other test statistics when  $T$  is small and  $N$  is large, can easily be recognized from Figures 1 and 2, (e.g. the empirical sizes of the test statistics except for the panel- and group  $\rho$  statistics are unity when  $T = 10$  and  $N \geq 25$ ). It is also obvious from the figures that when  $T$  is small, the test statistics become more oversized with increasing  $N$ . The reason for this may be the fact that we are using the asymptotic first and second moments in order to standardize the statistics. Thus, we should use the appropriate moments from the finite sample distribution of the test statistics for the small time

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<sup>8</sup>Please note that the scaling of the figures differs.

<sup>9</sup>In the figures, “sc” is the abbreviation for Schwarz lag selection Criterion, whereas “sd” denotes the step-down lag selection method.

series<sup>10</sup>. This points out the fact that these tests are not appropriate if the time dimension is much smaller than the cross-section dimension. However, in Figures 4 and 5 it is clear that when  $T$  and  $N$  dimensions increase, the empirical sizes of the standardized LR-bar and panel- $t$  test statistics approach to the nominal size level of 5%, especially for  $T = 200$  and  $N \geq 5$ . On the other hand, the empirical sizes of the group- $\rho$  and panel- $\rho$  statistics are around 5% when  $T = 100$ ,  $N \geq 5$  and  $T = 50$ ,  $N \geq 5$ , respectively. The size distortions of group- $t$ , panel- $t$  and standardized LR-bar test statistics decrease for fixed  $N$  when  $T$  increases.

## 5.2 Size-Adjusted Power Properties

The relevant graphs for the size-adjusted power results are demonstrated in Appendix B starting from Figure 6.

When  $\psi_a, \psi_b, \psi_c \in \{0.5, 1\}$  and there is no correlation, just the graphs for  $T = 10$  will be discussed, because the powers of all the test statistics approach unity for  $T \geq 25$  and  $N \geq 10$ . If  $T = 10$ , standardized LR-bar and group- $t$  statistics have the lowest power for the true cointegrating ranks of 1 and 2. In Figures 6 and 7 the panel- $\rho$  statistic has the highest power reaching 0.891 and 0.681 for  $r = 1$  and 2, respectively. If the true cointegrating rank is 3 as in Figure 8, the power of rejecting the null-hypothesis of no cointegration is the highest for the standardized LR-bar statistic (0.53), whereas the group- $t$  statistics have the lowest power (0.04).

As there is not much difference in the size-adjusted power results when  $\psi$  parameters increase to 0.8, we do not present this case in this paper<sup>11</sup>.

If the  $\psi$  parameters are near unity with 0.95 and  $T = 10$ , the powers of all the test statistics are at most 0.074 for  $r = 1, 2$  and 3, which can be observed on Figures 9, 13 and 17. Figures 10 and 11 indicate that the standardized LR-bar test statistics have the lowest power and the panel- $\rho$  and panel- $t$  test statistics have the highest power. With the true cointegrating assumption of  $r = 2$ , Figures 14 and 15 present that the maximum-likelihood-based test statistic has the lowest power again. The powers of all the test statistics converge to unity for high  $T$  and  $N$  dimensions, which proves what the theory concludes. One interesting outcome of the Monte Carlo study belongs to the case when  $T = 100$  and  $r = 3$ . For this case the standardized LR-bar test statistic has the highest power among all the test statistics. This eye-catching difference can be observed in Figure 19.

In order to understand how the test statistics behave under the assumption of correlated error terms, only the case with the highest correlation parameters is discussed because the power results do not change drastically if the correlation

<sup>10</sup>In addition to this, Hanck (2007, 2006) explains the increase of the size distortion with the increase in  $N$  as the cumulative effect of the small size distortions in the time series.

<sup>11</sup>The results can be supplied by the author on request.

between the error terms is not high or  $\psi$  parameters are low. For  $\psi_a, \psi_b, \psi_c \in \{0.95, 1\}$  the powers of the standardized LR-bar and the panel- $t$  statistics approach unity even if  $T = 10$ . On the other hand, the powers of the other test statistics are near zero for small  $T$  dimensions (Figures 21 and 24). For  $T \in \{100, 200\}$  only the group- $\rho$ , panel- $\rho$  and group- $t$  statistics are illustrated, because the other test statistics converge to unity faster. The power of rejecting the cointegrating rank of zero for group- $\rho$  and group- $t$  statistics cannot go to unity when the true rank is 1, even if  $T$  and  $N$  dimensions are high. The power of all the test statistics converge to unity if the true cointegrating rank is 2 (Figure 26).

The powers of group- $t$  and panel- $t$  test statistics are not much different for Schwarz and step-down lag selection methods, when  $T$  and  $N$  increase.

## 6 Empirical Example: Fisher Hypothesis

In this section, we try to find out whether the panel cointegration analysis gives different results than the results of the usual cointegration techniques to an empirical example. For this purpose, the Fisher Hypothesis is considered, which is a widely tested economic relation in the macroeconomic literature.

There are controversial conclusions in the empirical literature for Fisher effect. The non-stationarity of the nominal interest and inflation rates made the application of the cointegration techniques possible in order to test for the long-run relation between the nominal interest and inflation rates. The studies which find evidence for Fisher relation using the unit root and cointegration techniques are: Atkins (1989), Evans & Lewis (1995), Crowder & Hoffman (1996), Crowder (1997), whereas the studies of Rose (1988), MacDonald & Murphy (1989), Mishkin (1992) and Dutt & Gosh (1995) cannot find any evidence for the Fisher effect.

The panel data study by Crowder (2003) with 9 industrialized countries concluded that the Fisher effect exists.

The Fisher Hypothesis states that the real interest rate ( $r_{it}$ ) is the difference between the nominal interest rate ( $n_{it}$ ) and the expected inflation rate ( $\pi_{it}^e$ ),

$$r_{it} = n_{it} - \pi_{it}^e, \quad (12)$$

which means that no one will lend at a nominal rate lower than the expected inflation, and the nominal interest rate will be equal to the cost of borrowing plus the expected inflation.

$$n_{it} = r_{it} + \pi_{it}^e. \quad (13)$$

Another aspect of the Fisher relation is that the real interest rates are constant or show little trend in the long run. This can be explained with the phenomena that the nominal interest rate absorbs all the changes in the expected inflation rate

when the change in the growth rate of the money supply alters the inflation rate. If the real interest rate changes with a change in the expected inflation, then the Fisher Hypothesis will not hold. When stationarity of the real interest rates with a positive constant ( $r^*$ ) and a normally distributed error term ( $u_{it} \sim N(0, \sigma_{iu}^2)$ ) is assumed, the equation for  $r_{it}$  becomes,

$$r_{it} = r^* + u_{it}. \quad (14)$$

In addition to this, with the assumption that the agents do not make systematic errors and the actual inflation rate ( $\pi_{it}$ ) differs from the expected inflation rate ( $\pi_{it}^e$ ) with a stationary process ( $\xi_{it} \sim N(0, \sigma_{i\xi}^2)$ ), the equation for  $\pi_{it}$  is,

$$\pi_{it} = \pi_{it}^e + \xi_{it}. \quad (15)$$

When we insert (14) and (15) into (13), the Fisher equation for the cointegration analysis becomes,

$$n_{it} = a + b\pi_{it} + \varepsilon_{it},$$

where  $a = r^*$ ,  $\varepsilon_{it} = u_{it} - \xi_{it}$  and according to the theory  $b = 1$ . We search for the existence of a cointegrating relation between the nominal interest rate and the inflation rate in the panel data, in order to see if the Fisher relation holds.

To test the Fisher Hypothesis two different data sets consisting of quarterly nominal interest and inflation rates are considered. The first data set is the monthly data for 19 OECD countries<sup>12</sup> from 1986:06 to 1998:12. The second data set consists of monthly data for 11 OECD countries<sup>13</sup> from 1991:02 to 2002:12. The results of the panel cointegration tests for the Fisher Hypothesis are presented in Appendix C.

While testing the Fisher Hypothesis with the panel cointegration tests, we face a problem. Standardized LR-bar test cannot be applied to the VAR models with an intercept. Therefore we have to limit our attention to the case where there is no intercept in the VAR model for the maximum-likelihood-based panel cointegration test.

To standardize the test statistics of Pedroni, mean and variance values when there is one independent variable in the system ( $M = 1$ ) are required. These values can be found in Pedroni (1995). The lag selection criterion for the maximum-likelihood-based panel cointegration test is the Schwarz Criterion and the maximum lag order is set equal to 6. For the ADF  $t$ -statistic based panel cointegration tests of Pedroni, two lag selection methods will be considered: Step-down method and Schwarz Criterion. The maximum lag order for these methods is limited to 12, because the data sets consist of monthly data.

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<sup>12</sup>Germany, France, Italy, Netherlands, Spain, Finland, Austria, Ireland, Portugal, Belgium, US, Japan, UK, Denmark, Mexico, Norway, Iceland, Sweden, Canada.

<sup>13</sup>US, Korea, Japan, UK, Denmark, Mexico, Norway, Iceland, Hungary, Sweden, Canada.

When the first data set (1989 : 06 – 1998 : 12) is considered, country-by-country trace tests point out the existence of one cointegrating relation, except for Austria and UK. On the other hand, if the whole panel data is tested, standardized LR-bar test cannot reject the null hypothesis of all the countries having at most cointegrating rank of 2. This means that the underlying heterogeneous VAR model is stable and nominal interest and inflation rates are stationary processes. This result is also valid for the cases when the standardized LR-bar statistic is applied to the data set without Austria or to the data set without Austria and UK.

Country-by-country test results for the second data set (1991 : 01 – 2002 : 12) cannot reject the null hypothesis of  $r_i = 1$ , except for Japan. Standardized LR-bar test accepts the hypothesis of cointegrating rank of two, even when we test the relation without taking Japan into account.

The residual-based panel cointegration tests allow us to consider two different cases for the regression equation, i.e. the case where there is a heterogeneous intercept and the case where there is no heterogeneous intercept. The results of Pedroni's tests are demonstrated in the Appendix C (Table III-VI), which are different from the maximum-likelihood-based test results. For the residual-based panel cointegration tests, the rank order of the cointegrating matrix cannot be tested. By testing the null hypothesis of no cointegration it can just be determined whether there is a cointegration relation.

All of the residual-based panel cointegration tests reject the null of no cointegration for both data sets, when there is no heterogeneous intercept in the panel regression equation. This is also the same situation when we exclude Austria and UK from the first data set, and Japan from the second data set. However, some of the test statistics give different results for both data sets with the assumption of a heterogeneous intercept in the panel regression equation. The panel- $\rho$  test statistic cannot reject the null of no cointegration, if the first data set is considered as a whole, which means that the Fisher Hypothesis does not hold. This is also valid for the panel- $t$  statistics if the tests are undertaken for the second data set excluding Japan.

## 7 Conclusions

With the extensive simulation study in Section 5, which covers the empirical size and size-adjusted power of five panel cointegration tests, it can be concluded that the panel- $t$  test statistic has the best size and size-adjusted power properties. We found out that the size-adjusted power of the panel- $t$  statistic approaches unity for small  $T$  and  $N$  dimensions, even when there is strong correlation between the innovations to the non-stationary and stationary part of the data generating process, while the empirical size of it is around the nominal size of 5% when  $T = 200$  and



$N \geq 5$ . On the other hand, the other three residual-based panel cointegration test statistics; group- $\rho$ , panel- $\rho$  and group- $t$  have poor size-adjusted power results if the correlation and  $\psi$  parameters are high (e.g. when  $\theta_a = \theta_b = 0.7$  and  $\psi_a = \psi_b = 0.95$ , respectively).

The second best test statistics which has the best size and power properties is the standardized LR-bar statistic. It has better size-adjusted power if the correlation parameter is high and the  $\psi$  parameter is around unity. The empirical size of the standardized LR-bar statistic is around 5% like the panel- $t$  test statistic when both  $T$  and  $N$  increase, especially if the time dimension increases faster than the cross-section dimension just as the theory points out. The standardized LR-bar statistic has also high size-adjusted power, mainly when  $T$  is large. It should also be emphasized that the size and size-adjusted power results of the residual-based panel cointegration tests can depend on the choice of the dependent variable. In this paper the first variable of the DGP has been taken as the dependent variable for the residual-based panel cointegration tests.

In Section 6 while we were testing the Fisher hypothesis with the panel cointegration test statistics, we were able to present the results of the residual-based panel cointegration tests under the assumption of a heterogeneous intercept in the panel regression equation, whereas the maximum-likelihood-based statistic had to be considered without a heterogeneous intercept in the VAR model. Residual-based panel cointegration tests of Pedroni pointed out the existence of the Fisher relation for two different data sets consisting of OECD countries. On the other hand, the standardized LR-bar statistic emphasizes that nominal interest and inflation rate are stationary processes. For a future study the procedure in Larsson et al. (2001) can be extended to a new maximum-likelihood-based panel cointegration test statistic with a constant and a linear trend in the data.

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# Appendix A

Figure 1: Empirical Size Results for T=10

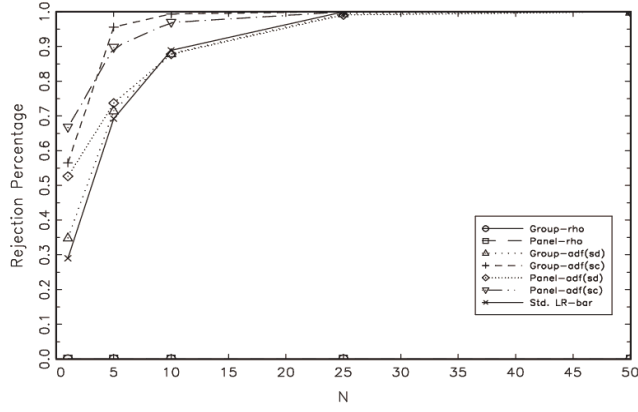


Figure 2: Empirical Size Results for T=25

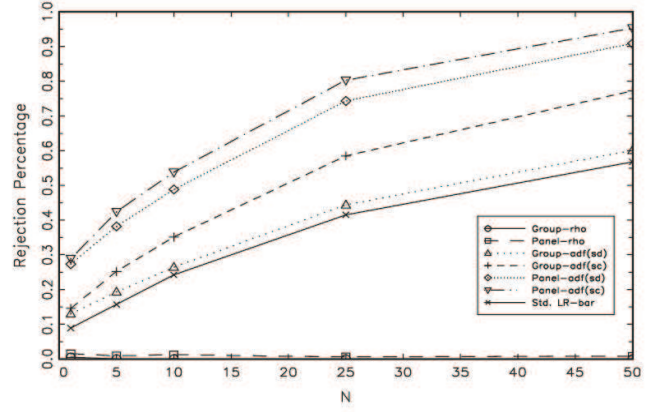


Figure 3: Empirical Size Results for T=50

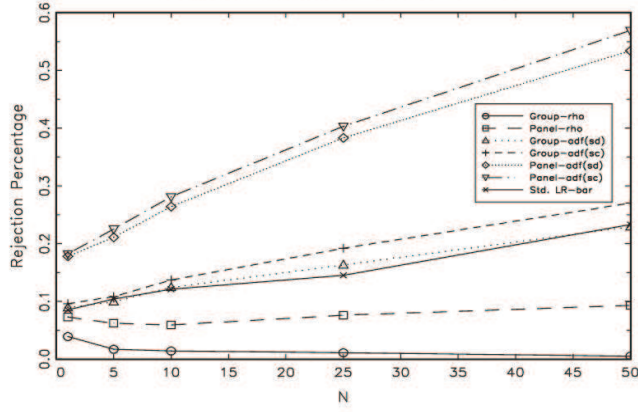


Figure 4: Empirical Size Results for T=100

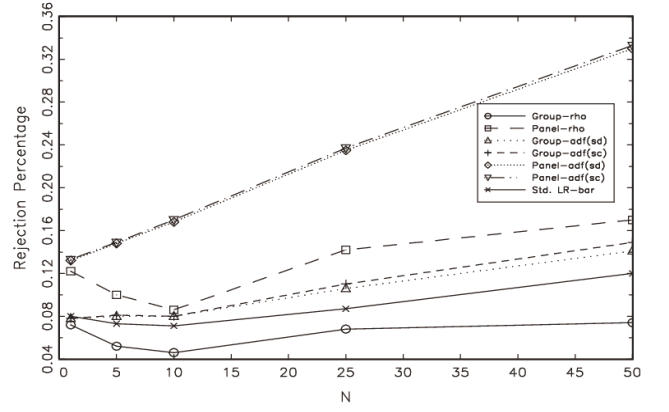
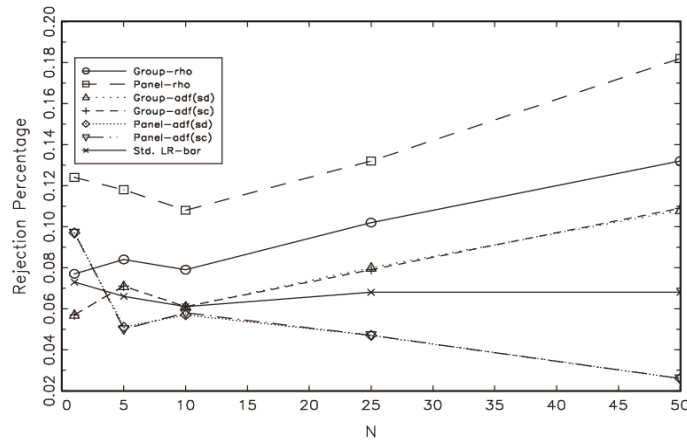
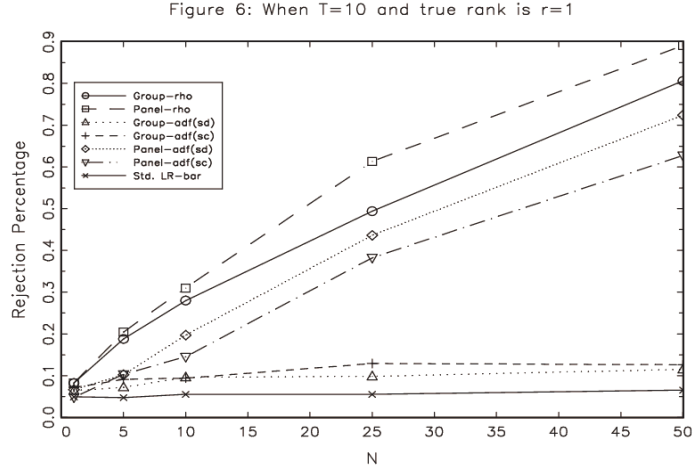


Figure 5: Empirical Size Results for T=200

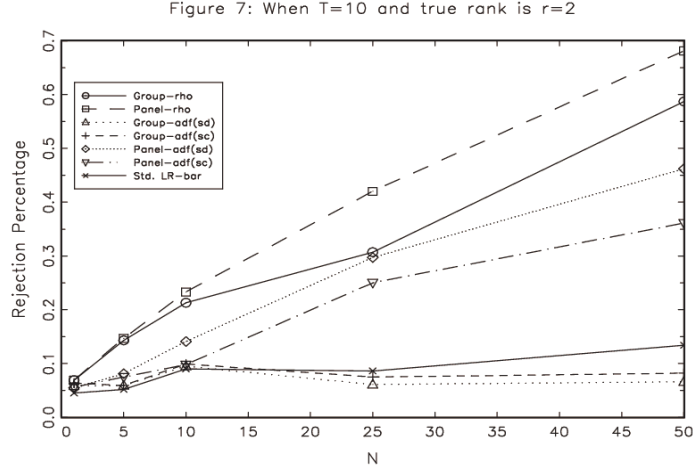


## Appendix B

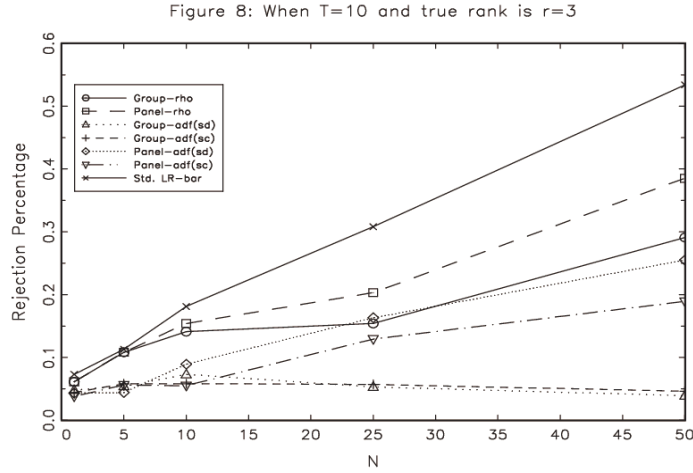
Size-Adjusted Power Results for  $\theta_a = \theta_b = 0$ ,  $\psi_a = 0.5$ ,  $\psi_b = \psi_c = 1$



Size-Adjusted Power Results for  $\theta_a = \theta_b = 0$ ,  $\psi_a = \psi_b = 0.5$ ,  $\psi_c = 1$



Size-Adjusted Power Results for  $\theta_a = \theta_b = 0$  and  $\psi_a = \psi_b = \psi_c = 0.5$



Size-Adjusted Power Results for  $\theta_a = \theta_b = 0$ ,  $\psi_a = 0.95$  and  $\psi_b = \psi_c = 1$

Figure 9: When  $T=10$  and true rank is  $r=1$

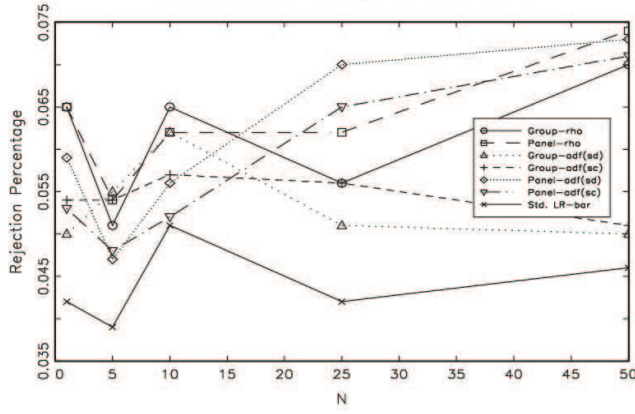


Figure 10: When  $T=25$  and true rank is  $r=1$

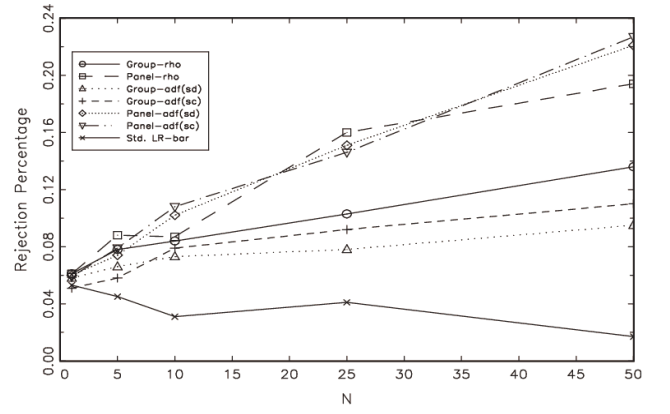


Figure 11: When  $T=100$  and true rank is  $r=1$

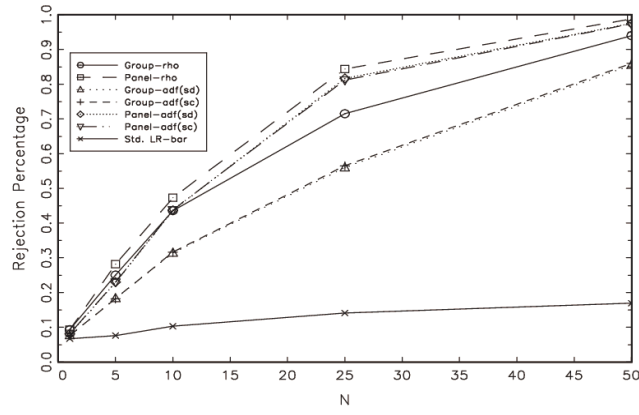
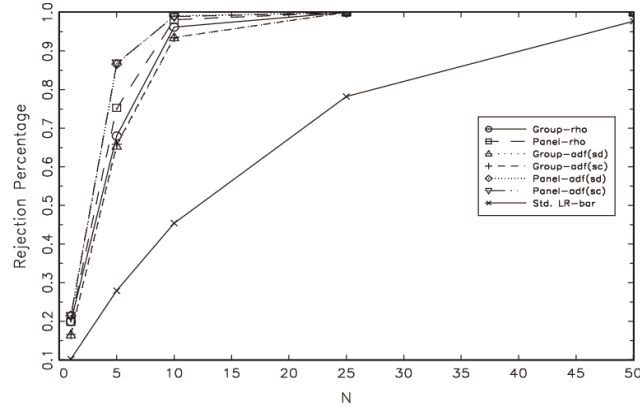


Figure 12: When  $T=200$  and true rank is  $r=1$



Size-Adjusted Power Results for  $\theta_a = \theta_b = 0$ ,  $\psi_a = \psi_b = 0.95$  and  $\psi_c = 1$

Figure 13: When  $T=10$  and true rank is  $r=2$

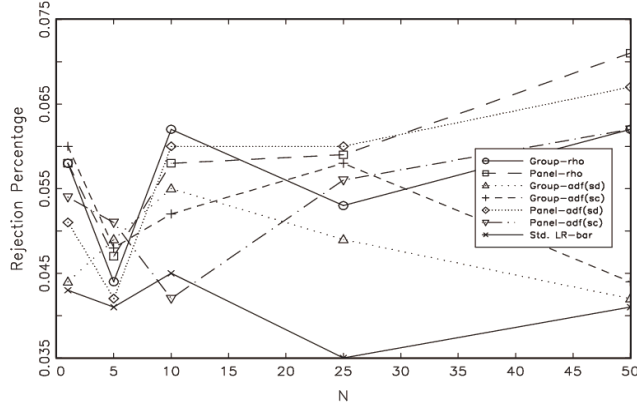


Figure 14: When  $T=25$  and true rank is  $r=2$

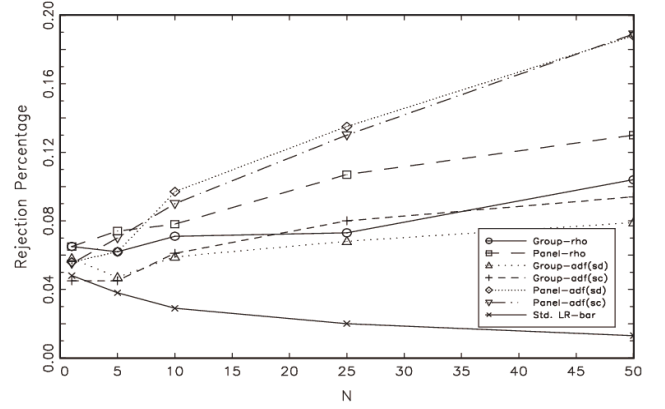


Figure 15: When  $T=100$  and true rank is  $r=2$

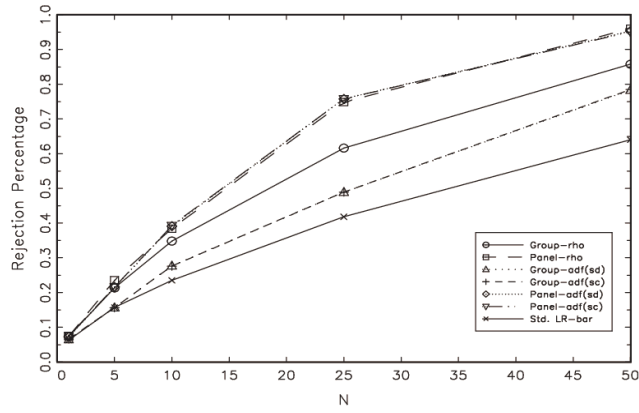
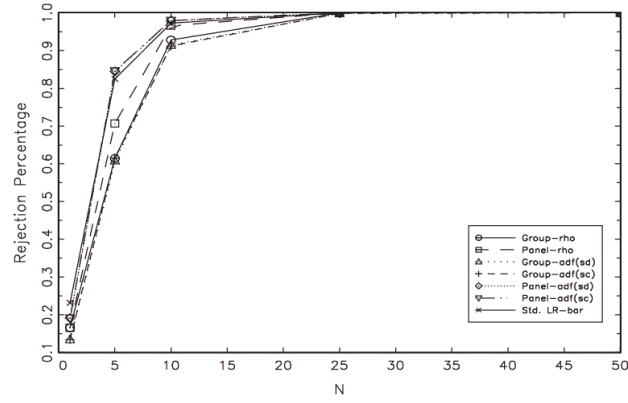


Figure 16: When  $T=200$  and true rank is  $r=2$





Size-Adjusted Power Results for  $\theta_a = \theta_b = 0$  and  $\psi_a = \psi_b = \psi_c = 0.95$

Figure 17: When  $T=10$  and true rank is  $r=3$

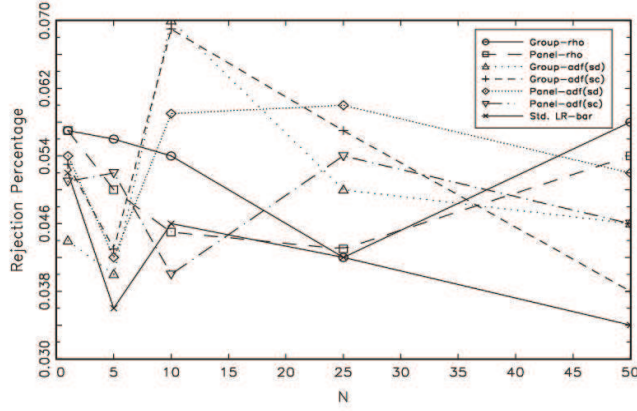


Figure 18: When  $T=25$  and true rank is  $r=3$

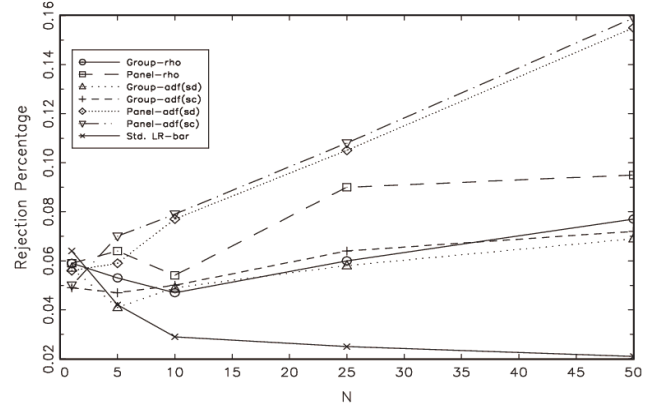


Figure 19: When  $T=100$  and true rank is  $r=3$

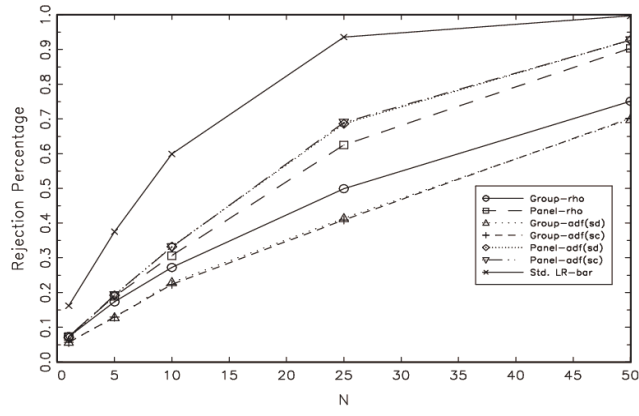
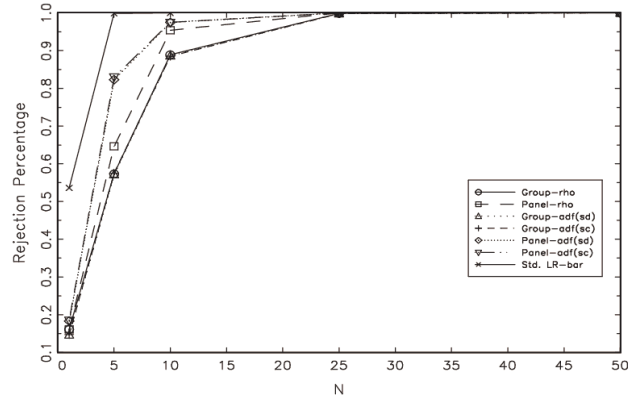


Figure 20: When  $T=200$  and true rank is  $r=3$



Size-Adjusted Power Results for  $\theta_a = \theta_b = 0.7$ ,  $\psi_a = 0.95$  and  $\psi_b = \psi_c = 1$

Figure 21: When  $T=10$  and true rank is  $r=1$

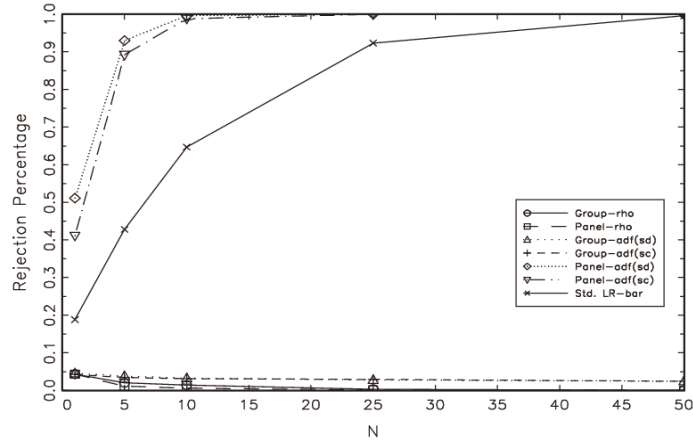


Figure 22: When  $T=100$  and the true rank is  $r=1$

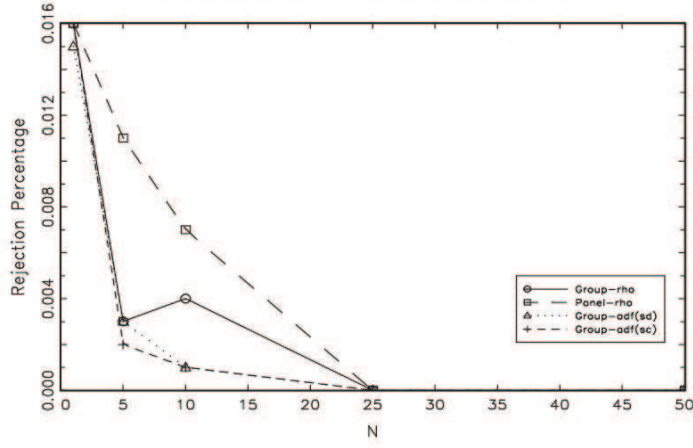
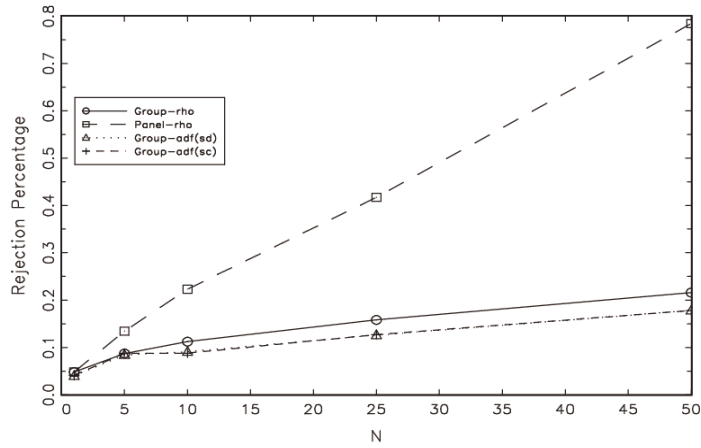


Figure 23: When  $T=200$  and the true rank is  $r=1$



Size-Adjusted Power Results for  $\theta_a = \theta_b = 0.7$ ,  $\psi_a = \psi_b = 0.95$  and  $\psi_c = 1$

Figure 24: When  $T=10$  and true rank is  $r=2$

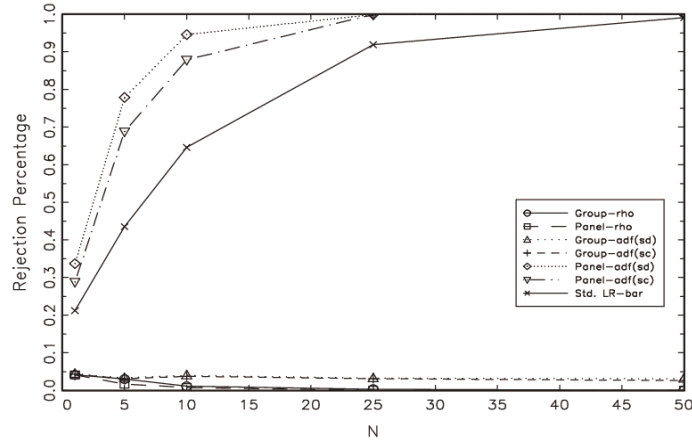


Figure 25: When  $T=100$  and the true rank is  $r=2$

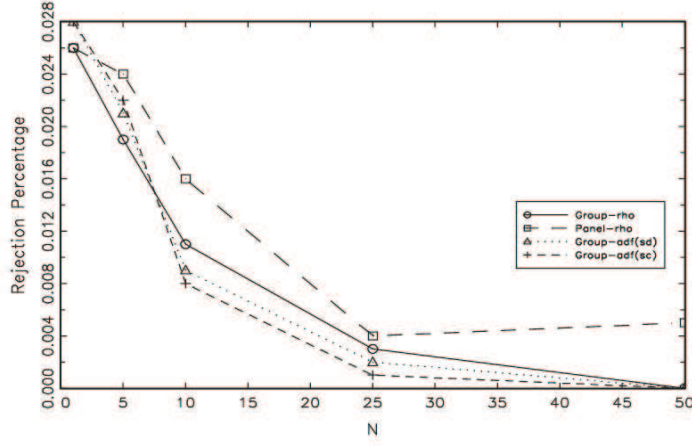
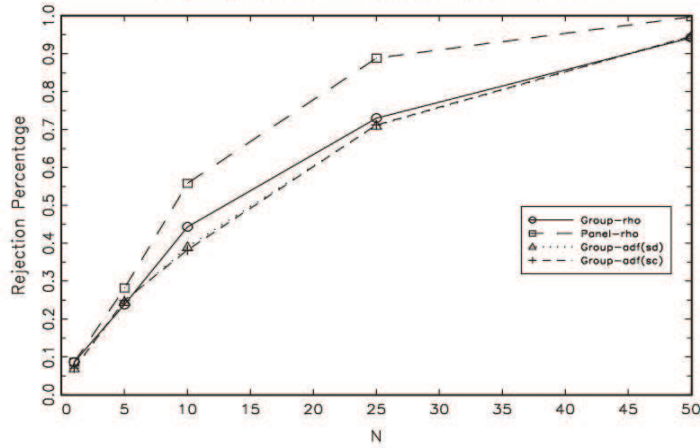


Figure 26: When  $T=200$  and the true rank is  $r=2$



# Appendix C

Table I: Empirical results of the trace test. Monthly data from 1989:06 through 1998:12 is used. All tests are performed at 5% level. For country by country tests the critical values are 12.53 and 3.84 for testing  $r = 0$  and  $r = 1$ , respectively. The panel rank test has a critical value of 1.645. There is neither in the VAR model nor in the cointegrating equation an intercept.

Country by Country Trace Test Statistics				
Country	lag	r=0	r=1	rank
Germany	2	48.01	1.57	1
France	2	63.31	1.44	1
Italy	1	15.75	2.04	1
Netherlands	4	17.37	1.98	1
Spain	5	30.16	2.97	1
Finland	2	26.82	1.61	1
Austria	5	36.16	4.81	2
Ireland	2	37.45	1.91	1
Portugal	4	18.36	3.48	1
Belgium	2	75.39	1.43	1
USA	2	36.48	2.14	1
Japan	2	94.15	2.01	1
UK	5	30.25	6.46	2
Denmark	4	13.98	1.61	1
Mexico	3	25.09	1.52	1
Norway	4	21.00	1.11	1
Iceland	4	17.94	1.99	1
Sweden	1	20.67	1.38	1
Canada	1	38.94	2.81	1
Panel Tests	r=0		r=1	
	$\overline{\gamma_{LR}}$	39.00	3.50	2
		38.42	3.75	2*
		37.17	2.23	2**

Notes:

\* indicates results for the panel cointegration test without Austria

\*\* indicates results for the panel cointegration test without Austria and UK.

Table II: Empirical results of the trace test. Monthly data from 1991:01 through 2002:12 is used. All tests are performed at 5% level. For country by country tests the critical values are 12.53 and 3.84 for testing  $r = 0$  and  $r = 1$ , respectively. The panel rank test has a critical value of 1.645. There is neither in the VAR model nor in the cointegrating equation an intercept.

Country by Country Trace Test Statistics				
Country	lag	r=0	r=1	rank
USA	2	52.05	1.53	1
Korea	2	91.14	1.67	1
Japan	2	130.53	27.34	2
UK	4	31.97	3.18	1
Denmark	4	14.94	2.66	1
Mexico	2	58.34	1.77	1
Norway	4	28.83	1.02	1
Iceland	5	20.14	3.66	1
Hungary	5	66.64	3.16	1
Sweden	1	28.77	3.12	1
Canada	4	20.03	3.07	1
Panel Tests		r=0	r=1	
	$\overline{\gamma_{LR}}$	44.26	8.04	2
		34.29	2.86	2*

Notes:

\* indicates results for the panel cointegration test without Japan.

Table III: Empirical results of Pedroni's panel cointegration tests without an intercept in the regression equation. Monthly data from 1989:06 through 1998:12 is used. All tests are performed at 5% level. The panel rank test has a critical value of -1.645.

Country	Slope	$\rho$ -stat	$t$ -stat(sc)	$t$ -stat(sd)	lag(sc)	lag(sd)	
Germany	596.56	-0.19	-5.05	-3.92	9	1	
France	1024.76	-0.24	-6.14	-6.15	5	5	
Italy	846.66	-0.23	-4.64	-3.80	9	1	
Netherlands	493.55	-0.16	-10.98	-4.17	9	1	
Spain	780.86	-0.34	-9.21	-5.48	9	2	
Finland	867.21	-0.23	-5.22	-3.67	9	0	
Austria	436.18	-0.14	-9.36	-5.72	9	3	
Ireland	77.77	-0.04	-2.46	-2.51	1	2	
Portugal	594.51	-0.22	-5.50	-3.94	9	1	
Belgium	776.12	-0.22	-6.75	-6.38	9	8	
USA	606.97	-0.19	-5.59	-4.00	10	1	
Japan	251.62	-0.08	-5.83	-3.38	11	2	
UK	533.78	-0.22	-10.09	-4.29	10	1	
Denmark	792.24	-0.19	-8.80	-5.27	9	2	
Mexico	491.30	-0.27	-5.19	-4.98	2	1	
Norway	826.55	-0.23	-5.64	-4.07	9	1	
Iceland	616.94	-0.16	-3.86	-3.32	3	0	
Sweden	414.31	-0.12	-4.16	-3.03	9	0	
Canada	695.63	-0.21	-4.95	-3.75	9	0	
Group Tests			Panel Tests				
Group- $\rho$	-12.92 <sup>*</sup>	-12.85 <sup>1*</sup>	-12.39 <sup>2*</sup>	Panel- $\rho$	-15.57 <sup>*</sup>	-16.00 <sup>1*</sup>	-15.27 <sup>2*</sup>
Group- $t$ (sc)	-24.16 <sup>*</sup>	-22.70 <sup>1*</sup>	-20.97 <sup>2*</sup>	Panel- $t$ (sc)	-31.92 <sup>*</sup>	-29.12 <sup>1*</sup>	-26.60 <sup>2*</sup>
Group- $t$ (sd)	-14.38 <sup>*</sup>	-13.62 <sup>1*</sup>	-13.22 <sup>2*</sup>	Panel- $t$ (sd)	-23.60 <sup>*</sup>	-21.57 <sup>1*</sup>	-20.34 <sup>2*</sup>

Notes:

<sup>1</sup> indicates results for the panel cointegration tests without Austria

<sup>2</sup> indicates results without Austria and UK

\* indicates rejection of the null of no cointegration.

Table IV: Empirical results of Pedroni's panel cointegration tests without an intercept in the regression equation. Monthly data from 1991:01 through 2002:12 is used. All tests are performed at 5% level. The panel rank test has a critical value of -1.645.

Country	Slope	$\rho$ -stat	$t$ -stat(sc)	$t$ -stat(sd)	lag(sc)	lag(sd)
US	581.63	-0.21	-6.16	-4.77	9	1
Korea	609.56	-0.18	-5.84	-4.43	9	2
Japan	131.37	-0.05	-6.02	-4.55	9	3
UK	503.50	-0.21	-11.35	-4.89	10	1
Denmark	562.19	-0.13	-8.62	-4.80	9	2
Mexico	482.24	-0.25	-4.44	-4.44	1	1
Norway	654.62	-0.19	-5.04	-4.00	9	1
Iceland	619.07	-0.15	-3.82	-3.56	3	1
Hungary	392.18	-0.24	-8.51	-4.88	12	4
Sweden	0.20	-0.02	-1.91	-1.91	0	0
Canada	0.00	-0.33	-7.35	-7.35	7	5
Group Tests			Panel Tests			
Group- $\rho$	-11.87*	-12.41 <sup>1*</sup>		Panel- $\rho$	-11.72*	-12.82 <sup>1*</sup>
Group- $t$ (sc)	-18.36*	-17.60 <sup>1*</sup>		Panel- $t$ (sc)	-15.43*	-14.81 <sup>1*</sup>
Group- $t$ (sd)	-11.70*	-11.14 <sup>1*</sup>		Panel- $t$ (sd)	-11.18*	-10.73 <sup>1*</sup>

Notes:

<sup>1</sup> indicates results for the panel cointegration tests without Japan.

\* indicates rejection of the null of no cointegration.

Table V: Empirical results of Pedroni's panel cointegration tests with an intercept in the regression equation. Monthly data from 1989:06 through 1998:12 is used. All tests are performed at 5% level. The panel rank test has a critical value of -1.645.

Country	Intercept	Slope	$\rho$ -stat	$t$ -stat(sc)	$t$ -stat(sd)	lag(sc)	lag(sd)
Germany	4.86	202.11	-0.08	-2.69	-2.44	3	2
France	5.29	362.34	-0.08	-3.40	-3.40	5	5
Italy	5.97	381.25	-0.15	-2.71	-2.71	1	1
Netherlands	5.76	49.36	-0.09	-0.94	-0.69	6	1
Spain	6.34	349.90	-0.15	-5.37	-3.24	9	0
Finland	5.72	419.24	-0.14	-3.56	-2.82	6	0
Austria	5.68	91.19	-0.03	-3.09	-2.17	12	3
Ireland	8.46	-21.18	-0.32	-5.37	-5.53	1	3
Portugal	6.97	298.00	-0.13	-3.62	-2.75	9	1
Belgium	5.43	205.31	-0.05	-2.39	-2.12	3	2
USA	4.30	163.63	-0.06	-2.76	-2.52	3	2
Japan	2.91	105.10	-0.02	-3.09	-1.85	11	2
UK	6.97	172.93	-0.09	-5.68	-2.86	10	1
Denmark	6.83	117.12	-0.03	-1.86	-1.55	5	0
Mexico	12.01	310.17	-0.28	-5.28	-5.15	2	1
Norway	6.83	140.90	-0.06	-1.84	-1.84	0	0
Iceland	5.81	419.83	-0.31	-5.20	-4.90	3	1
Sweden	7.45	151.48	-0.13	-2.97	-2.98	0	0
Canada	5.32	279.66	-0.13	-3.44	-3.02	3	0
Group Tests				Panel Tests			
Group- $\rho$	-3.23*	-3.55 <sup>1*</sup>	-3.62 <sup>2*</sup>	Panel- $\rho$	-1.50	-1.99 <sup>1*</sup>	-1.86 <sup>2*</sup>
Group- $t$ (sc)	-7.53*	-7.43 <sup>1*</sup>	-6.56 <sup>2*</sup>	Panel- $t$ (sc)	-6.10*	-6.02 <sup>1*</sup>	-4.98 <sup>2*</sup>
Group- $t$ (sd)	-4.50*	-4.59 <sup>1*</sup>	-4.47 <sup>2*</sup>	Panel- $t$ (sd)	-5.00*	-4.98 <sup>1*</sup>	-4.17 <sup>2*</sup>

Notes:

<sup>1</sup> indicates results for the panel cointegration tests without Austria

<sup>2</sup> indicates results without Austria and UK.

\* indicates rejection of the null of no cointegration.



Table VI: Empirical results of Pedroni's panel cointegration tests with an intercept in the regression equation. Monthly data from 1991:01 through 2002:12 is used. All tests are performed at 5% level. The panel rank test has a critical value of -1.645.

Country	Intercept	Slope	$\rho$ -stat	$t$ -stat(sc)	$t$ -stat(sd)	lag(sc)	lag(sd)
USA	4.29	71.99	-0.03	-1.11	-1.11	1	1
Korea	9.53	189.63	-0.08	-2.65	-2.43	3	2
Japan	1.64	94.53	-0.05	-5.59	-4.31	11	3
UK	6.28	59.24	-0.08	-6.10	-4.10	12	1
Denmark	5.75	38.84	-0.03	-1.75	-1.75	0	0
Mexico	9.00	332.18	-0.28	-4.77	-4.77	1	1
Norway	6.84	10.97	-0.06	-2.33	-2.33	0	0
Iceland	7.15	195.55	-0.11	-3.58	-3.01	8	1
Hungary	12.97	178.22	-0.13	-4.62	-3.08	12	4
Sweden	6.69	-1.26	-0.05	-2.14	-2.14	0	0
Canada	0.0005	0.0009	-0.33	-7.41	-7.41	5	5
Group Tests				Panel Tests			
Group- $\rho$	-3.84*	-4.11 <sup>1</sup> *		Panel- $\rho$	-2.22*	-2.82 <sup>1</sup> *	
Group- $t$ (sc)	-7.32*	-6.29 <sup>1</sup> *		Panel- $t$ (sc)	-2.98*	-1.17 <sup>1</sup>	
Group- $t$ (sd)	-5.24*	-4.61 <sup>1</sup> *		Panel- $t$ (sd)	-2.29*	-0.65 <sup>1</sup>	

Notes:

<sup>1</sup> indicates results for the panel cointegration tests without Japan.

\* indicates rejection of the null of no cointegration.

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