

Gapeev, Pavel V.

Working Paper

Multiple disorder problems for Wiener and compound Poisson processes with exponential jumps

SFB 649 Discussion Paper, No. 2006,074

Provided in Cooperation with:

Collaborative Research Center 649: Economic Risk, Humboldt University Berlin

Suggested Citation: Gapeev, Pavel V. (2006) : Multiple disorder problems for Wiener and compound Poisson processes with exponential jumps, SFB 649 Discussion Paper, No. 2006,074, Humboldt University of Berlin, Collaborative Research Center 649 - Economic Risk, Berlin

This Version is available at:

<https://hdl.handle.net/10419/25157>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Multiple Disorder Problems for Wiener and Compound Poisson Processes With Exponential Jumps

Pavel V. Gapeev*



* Weierstrass Institute for Applied Analysis and Stochastics,
Berlin, Germany
and
Russian Academy of Sciences, Institute of Control Sciences,
Moscow, Russia

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

<http://sfb649.wiwi.hu-berlin.de>
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin



Multiple disorder problems for Wiener and compound Poisson processes with exponential jumps*

Pavel V. Gapeev

The multiple disorder problem consists of finding a sequence of stopping times which are as close as possible to the (unknown) times of 'disorder' when the distribution of an observed process changes its probability characteristics. We present a formulation and solution of the multiple disorder problem for a Wiener and a compound Poisson process with exponential jumps. The method of proof is based on reducing the initial optimal switching problems to the corresponding coupled optimal stopping problems and solving the equivalent coupled free-boundary problems by means of the smooth- and continuous-fit conditions.

1. Introduction

Assume that at time $t = 0$ we begin to observe a continuously updated process $X = (X_t)_{t \geq 0}$ which probability characteristics change at some unknown times $(\eta_n)_{n \in \mathbb{N}}$ when an unobservable (two-stated) continuous time Markov chain $\theta = (\theta_t)_{t \geq 0}$, called the *disorder process*, changes its state from one to another. Throughout the paper it is assumed that the process θ starts at 0 with probability $1 - \pi$, starts at 1 with probability π , and changes its state with intensity $\lambda > 0$. The multiple disorder problem (or the problem of quickest multiple disorder detection) is to decide by observing the process X at which time instants one should give alarms in order to indicate the occurrence of disorders $(\eta_n)_{n \in \mathbb{N}}$. In contrast to the problem of *single* disorder, in the *multiple* disorder problem one looks for an infinite sequence of alarm times which should be as close as possible to the times $(\eta_n)_{n \in \mathbb{N}}$ in the sense that the sum of probabilities of false alarms and the total average time between the occurrence of disorders and the alarms (when the latter are given correctly) should be minimal. The idea of consideration of multiple disorder problems in such formulation is due to A.N. Shiryaev. Note that the problem of quickest detection admits different formulations and appears in a number of applied sciences (see, e.g., [20] or [5]).

The problem of detecting a change in drift of a Wiener process was formulated and solved by Shiryaev [26]–[28] (see also [29] and [30; Chapter IV]). Some particular cases of the problem

* This research was supported by Deutsche Forschungsgemeinschaft through the SFB 649 Economic Risk.
Mathematics Subject Classification 2000. Primary 60G40, 62M20, 34K10. Secondary 62C10, 60J60, 60J75.

Key words and phrases: Multiple disorder problem, Wiener process, compound Poisson process, optimal switching, coupled optimal stopping problem, (integro-differential) coupled free-boundary problem, smooth and continuous fit, Itô-Tanaka-Meyer formula.

of detecting a change in the intensity of a Poisson process were considered in Gal'chuk and Rozovskii [13] and in Davis [6]. Peskir and Shiryaev [23] presented a complete solution of the disorder problem for a Poisson process in the Bayesian formulation. A complete solution to the problem for a compound Poisson process with exponential jumps in the Bayesian and variational formulations was derived in [14]. Recently, Dayanik and Sezer [7] obtained a solution to the disorder problem for a general compound Poisson process. A finite horizon version of the Wiener disorder problem was studied in [15]. In the present paper we formulate and solve the multiple disorder problem for observed Wiener and compound Poisson processes having exponentially distributed jumps. This problem can be reduced to an equivalent optimal switching problem.

Optimal switching problems are extensions of optimal stopping problems and optimal stopping games where one is looking for an infinite sequence optimal stopping times. A general approach for studying such problems was developed in Bensoussan and Friedman [2]-[3] and Friedman [11] (see also Friedman [12; Chapter XVI]). This investigation was continued in Brekke and Øksendal [4], Duckworth and Zervos [9], Hamadène and Jeanblanc [17] for the continuous-time case, and in Yushkevich [31] and Yushkevich and Gordienko [32] for the discrete-time case. A direct method for solving optimal switching problems for diffusion processes is described in Dayanik and Egami [8].

The paper is organized as follows. In Section 2, we give a formulation of the multiple disorder problem for a Wiener and a compound Poisson process with exponential jumps, and reduce it to the corresponding optimal switching problem. Then, using the strong Markov property of the a posteriori probability process, we construct an equivalent coupled optimal stopping problem and formulate the corresponding coupled free-boundary problem. In Section 3, we derive solutions to the coupled free-boundary problems for the both cases of Wiener and compound Poisson processes with exponential jumps, separately. In Section 4, we formulate and prove the main assertion of the paper showing that the specified solutions of the coupled free-boundary problems turn out to be solutions of the initial coupled optimal stopping problems. The main results of the paper are formulated in Theorem 4.1. The optimal switching procedure is displayed more explicitly in Remark 4.3.

2. Formulation of the problem

In order to simplify the further exposition, in this section we formulate the multiple disorder problem for the observed sum of a Wiener and a compound Poisson process having exponentially distributed jumps (see [30; Chapter IV, Sections 3-4] and [23] for the single disorder case).

2.1. For a precise formulation of the problem, it is convenient to assume that all our considerations take place on a probability space $(\Omega, \mathcal{F}, P_\pi)$ for $\pi \in [0, 1]$. Let $\theta = (\theta_t)_{t \geq 0}$ be a continuous time Markov chain with two states 0 and 1, initial distribution $[1 - \pi, \pi]$, transition-probability matrix $[e^{-\lambda t}, 1 - e^{-\lambda t}; 1 - e^{-\lambda t}, e^{-\lambda t}]$ for $t \geq 0$, and intensity-matrix $[-\lambda, \lambda; \lambda, -\lambda]$ with $\lambda > 0$. The process θ defined above is called a '*telegraphic signal*' (see [21; Chapter IX, Section 4]). It is assumed that the process θ is unobservable, so that, the switching times $\eta_n = \inf\{t \geq \eta_{n-1} \mid \theta_t \neq \theta_{\eta_{n-1}}\}$, when the process θ switches from 0 to 1 and from 1 to 0, are unknown (i.e., they cannot be observed directly).

It is further assumed that we observe a process $X = (X_t)_{t \geq 0}$ defined by:

$$X_t = \int_0^t \theta_{s-} dX_s^1 + \int_0^t (1 - \theta_{s-}) dX_s^0 \quad (2.1)$$

where $X_t^i = i\mu t + \sigma W_t + \sum_{j=1}^{N_t^i} Y_j^i$ for all $t \geq 0$. Here $W = (W_t)_{t \geq 0}$ is a standard Wiener process, $N^i = (N_t^i)_{t \geq 0}$ are Poisson processes with intensities $1/\lambda_i$, and $(Y_j^i)_{j \in \mathbb{N}}$ are sequences of independent random variables exponentially distributed with parameters $\lambda_i > 0$ for $i = 0, 1$, respectively. It is supposed that W , N^i , $(Y_j^i)_{j \in \mathbb{N}}$ and θ are independent for $i = 0, 1$.

Based upon the continuous observation of X , our task is to find a (nondecreasing) sequence of stopping times with respect to the natural filtration $\mathcal{F}_t^X = \sigma\{X_s \mid 0 \leq s \leq t\}$ generated by X for $t \geq 0$ being 'as close as possible' to the unknown switching times of the process θ . More precisely, the problem consists of computing the risk function:

$$R_*(\pi) = \min\{V_*(\pi), W_*(\pi)\} \quad (2.2)$$

for $\pi \in [0, 1]$, where

$$V_*(\pi) = \inf_{(\tau_n)} \sum_{n=1}^{\infty} \left(bP_{\pi}[\theta_{\tau_{2n-1}} = 0] + aP_{\pi}[\theta_{\tau_{2n}} = 1] + \sum_{i=0}^1 E_{\pi} \left[\int_{\tau_{2n-2+i}}^{\tau_{2n-1+i}} I(\theta_t = 1-i) dt \right] \right) \quad (2.3)$$

$$W_*(\pi) = \inf_{(\sigma_n)} \sum_{n=1}^{\infty} \left(aP_{\pi}[\theta_{\sigma_{2n-1}} = 1] + bP_{\pi}[\theta_{\sigma_{2n}} = 0] + \sum_{i=0}^1 E_{\pi} \left[\int_{\sigma_{2n-2+i}}^{\sigma_{2n-1+i}} I(\theta_t = i) dt \right] \right) \quad (2.4)$$

and finding the corresponding (nondecreasing) sequences of optimal stopping times $(\tau_n^*)_{n \in \mathbb{N}}$ and $(\sigma_n^*)_{n \in \mathbb{N}}$ at which the infimums in (2.3) and (2.4) are attained. In order to avoid difficulties with notations, we set $\tau_0 = \sigma_0 = 0$. Note that in (2.3) it is assumed that the process θ initially switches from 0 to 1 first, while in (2.4) it is assumed that θ initially switches from 1 to 0 first. Here $P_{\pi}[\theta_{\tau_n} = i]$ is the probability of a 'false alarm' and $E_{\pi} \left[\int_{\tau_{n-1}}^{\tau_n} I(\theta_t = 1-i) dt \right]$ is the 'average delay' in detecting the 'disorder' correctly after giving the alarm τ_{n-1} when the process θ switches from the state i to the state $1-i$ for $i = 0, 1$ and $n \in \mathbb{N}$, and $a > 0$ and $b > 0$ are given constants (costs of false alarms). It follows that if $V_*(\pi) < W_*(\pi)$ then $(\tau_n^*)_{n \in \mathbb{N}}$ is the optimal strategy in (2.2), while if $V_*(\pi) > W_*(\pi)$ then $(\sigma_n^*)_{n \in \mathbb{N}}$ is optimal in (2.2), and either solution is good if $V_*(\pi) = W_*(\pi)$.

2.2. Straightforward calculations based on the fact that $(\tau_n)_{n \in \mathbb{N}}$ is a nondecreasing sequence of stopping times with respect to the filtration $(\mathcal{F}_t^X)_{t \geq 0}$ show that in (2.3)-(2.4) we have:

$$\begin{aligned} E_{\pi} \left[\int_{\tau_{n-1}}^{\tau_n} I(\theta_t = i) dt \right] &= E_{\pi} \left[\int_0^{\infty} I(\tau_{n-1} \leq t) I(\theta_t = i) I(t \leq \tau_n) dt \right] \\ &= E_{\pi} \left[\int_0^{\infty} E_{\pi} [I(\tau_{n-1} \leq t) I(\theta_t = i) I(t \leq \tau_n) \mid \mathcal{F}_t^X] dt \right] = E_{\pi} \left[\int_{\tau_{n-1}}^{\tau_n} P_{\pi}[\theta_t = i \mid \mathcal{F}_t^X] dt \right] \end{aligned} \quad (2.5)$$

for $i = 0, 1$. Then, by means of similar arguments to those presented in [30; pages 195-197],

one can reduce the functions (2.3)-(2.4) to the form:

$$V_*(\pi) = \inf_{(\tau_n)} E_\pi \left[\sum_{n=1}^{\infty} \left(b(1 - \pi_{\tau_{2n-1}}) + a\pi_{\tau_{2n}} + \int_{\tau_{2n-2}}^{\tau_{2n-1}} \pi_t dt + \int_{\tau_{2n-1}}^{\tau_{2n}} (1 - \pi_t) dt \right) \right] \quad (2.6)$$

$$W_*(\pi) = \inf_{(\sigma_n)} E_\pi \left[\sum_{n=1}^{\infty} \left(a\pi_{\sigma_{2n-1}} + b(1 - \pi_{\sigma_{2n}}) + \int_{\sigma_{2n-2}}^{\sigma_{2n-1}} (1 - \pi_t) dt + \int_{\sigma_{2n-1}}^{\sigma_{2n}} \pi_t dt \right) \right] \quad (2.7)$$

where $\pi_t = P_\pi[\theta_t = 1 \mid \mathcal{F}_t^X]$ for $t \geq 0$ is the *a posteriori probability process* with $P_\pi[\pi_0 = \pi] = 1$, and we set $\tau_0 = \sigma_0 = 0$. Moreover, it is easily seen that the infimums in (2.6) and (2.7) are taken over all sequences of stopping times $(\tau_n)_{n \in \mathbb{N}}$ and $(\sigma_n)_{n \in \mathbb{N}}$ such that $E_\pi[\tau_n \vee \sigma_n] < \infty$ for all $n \in \mathbb{N}$.

2.3. It can be shown (see [21; Chapters IX, XVIII and XIX]) that the *a posteriori probability process* $(\pi_t)_{t \geq 0}$ solves the stochastic differential equation:

$$d\pi_t = \lambda(1 - 2\pi_t) dt + \frac{\mu}{\sigma} \pi_t(1 - \pi_t) d\bar{W}_t + \int_0^\infty \frac{\pi_{t-}(1 - \pi_{t-})(e^{-\lambda_1 x} - e^{-\lambda_0 x})}{\pi_{t-}e^{-\lambda_1 x} + (1 - \pi_{t-})e^{-\lambda_0 x}} \left(\mu^X(dt, dx) - (\pi_{t-}e^{-\lambda_1 x} + (1 - \pi_{t-})e^{-\lambda_0 x}) dt dx \right) \quad (\pi_0 = \pi) \quad (2.8)$$

where the innovation process $\bar{W} = (\bar{W}_t)_{t \geq 0}$ defined by:

$$\bar{W}_t = \frac{1}{\sigma} \left(X_t^c - \mu \int_0^t \pi_s ds \right) \quad (2.9)$$

is a standard Wiener process (see also [21; Chapter IX]). Here $X^c = (X_t^c)_{t \geq 0}$ denotes the continuous part and $\mu^X(dt, dx)$ is the measure of jumps of the process X (see [19; Chapters I and II]). It can be verified that $(\pi_t)_{t \geq 0}$ is a time-homogeneous (strong) Markov process under P_π with respect to its natural filtration. As the latter clearly coincides with $(\mathcal{F}_t^X)_{t \geq 0}$, it is also clear that the infimums in (2.6) and (2.7) can equivalently be taken over all stopping times of $(\pi_t)_{t \geq 0}$. This shows that the process $(\pi_t)_{t \geq 0}$ plays the role of a *sufficient statistic* in the problems (2.6) and (2.7).

2.4. Using the strong Markov property of the process $(\pi_t)_{t \geq 0}$, we can reduce the system (2.6)-(2.7) to the following *coupled optimal stopping problem*:

$$V_*(\pi) = \inf_{\tau} E_\pi \left[b(1 - \pi_\tau) + \int_0^\tau \pi_t dt + W_*(\pi_\tau) \right] \quad (2.10)$$

$$W_*(\pi) = \inf_{\sigma} E_\pi \left[a\pi_\sigma + \int_0^\sigma (1 - \pi_t) dt + V_*(\pi_\sigma) \right] \quad (2.11)$$

where the infimums in (2.10) and (2.11) are taken over all stopping times τ and σ such that $E_\pi[\tau \vee \sigma] < \infty$, respectively. By using the arguments in [30; pages 197-198] and [23] it can be verified that the function $V_*(\pi)$ from (2.10) is concave and decreasing, while the function $W_*(\pi)$ from (2.11) is concave and increasing on $[0, 1]$. Then it follows that the optimal stopping times in (2.10) and in (2.11) have the form:

$$\tau_* = \inf\{t \geq 0 \mid \pi_t \geq B_*\} \quad (2.12)$$

$$\sigma_* = \inf\{t \geq 0 \mid \pi_t \leq A_*\} \quad (2.13)$$

where B_* is the smallest number π from $[0, 1]$ such that $V_*(\pi) = b(1 - \pi)$, and A_* is the largest number π from $[0, 1]$ such that $W_*(\pi) = a\pi$. Hence, we may conclude that the sequence of stopping times $(\tau_n^*)_{n \in \mathbb{N}}$ given by:

$$\tau_{2n-1}^* = \inf\{t \geq \tau_{2n-2}^* \mid \pi_t \geq B_*\} \quad (2.14)$$

$$\tau_{2n}^* = \inf\{t \geq \tau_{2n-1}^* \mid \pi_t \leq A_*\} \quad (2.15)$$

is optimal in (2.6) and thus in (2.3), while the sequence of stopping times $(\sigma_n^*)_{n \in \mathbb{N}}$ given by:

$$\sigma_{2n-1}^* = \inf\{t \geq \sigma_{2n-2}^* \mid \pi_t \leq A_*\} \quad (2.16)$$

$$\sigma_{2n}^* = \inf\{t \geq \sigma_{2n-1}^* \mid \pi_t \geq B_*\} \quad (2.17)$$

is optimal in (2.7) and thus in (2.4). In order to avoid difficulties in notations, here we set $\tau_0^* = \sigma_0^* = 0$.

It is also seen that there exist a unique point $0 < \pi_* < 1$ such that $V_*(\pi_*) = W_*(\pi_*)$. Therefore, for a given number π from the interval $[0, 1]$ it follows that if $\pi_* < \pi \leq 1$ then the sequence (2.14)-(2.15) is optimal in the problem (2.2), while if $0 \leq \pi < \pi_*$ then the sequence (2.16)-(2.17) is optimal in (2.2), and either solution is good if $\pi = \pi_*$.

2.5. Standard arguments imply that the infinitesimal operator \mathbb{L} of the process $(\pi_t)_{t \geq 0}$ acts on a function $F \in C^2([0, 1])$ according to the rule:

$$\begin{aligned} (\mathbb{L}F)(\pi) = & \left(\lambda(1 - 2\pi) - \frac{\lambda_0 - \lambda_1}{\lambda_0 \lambda_1} \pi(1 - \pi) \right) F'(\pi) + \frac{\mu^2}{2\sigma^2} \pi^2(1 - \pi)^2 F''(\pi) \\ & + \int_0^\infty \left[F\left(\frac{\pi e^{-\lambda_1 x}}{\pi e^{-\lambda_1 x} + (1 - \pi)e^{-\lambda_0 x}} \right) - F(\pi) \right] (\pi e^{-\lambda_1 x} + (1 - \pi)e^{-\lambda_0 x}) dx \end{aligned} \quad (2.18)$$

for all $\pi \in [0, 1]$. In order to find the unknown value functions $V_*(\pi)$ and $W_*(\pi)$ from (2.10) and (2.11) as well as the unknown boundaries A_* and B_* from (2.12) and (2.13), using the general theory of optimal stopping problems for continuous time Markov processes (see, e.g., [16] and [30; Chapter III, Section 8]), we can formulate the following *coupled free-boundary problem*:

$$(\mathbb{L}W)(\pi) = -(1 - \pi) \quad \text{for } A < \pi < 1, \quad (\mathbb{L}V)(\pi) = -\pi \quad \text{for } 0 < \pi < B \quad (2.19)$$

$$W(A+) = aA + V(A+), \quad V(B-) = b(1 - B) + W(B-) \quad (2.20)$$

$$W(\pi) = a\pi + V(\pi) \quad \text{for } 0 \leq \pi < A, \quad V(\pi) = b(1 - \pi) + W(\pi) \quad \text{for } B < \pi \leq 1 \quad (2.21)$$

$$W(\pi) < a\pi + V(\pi) \quad \text{for } A < \pi < 1, \quad V(\pi) < b(1 - \pi) + W(\pi) \quad \text{for } 0 < \pi < B \quad (2.22)$$

with $0 < A_* < B_* < 1$, where the conditions (2.20), which are satisfied by virtue of the concavity arguments above, play the role of *instantaneous-stopping conditions*. Note that by the superharmonic characterization of the value function (see [10] or [30]) it follows that $V_*(\pi)$ from (2.10) and $W_*(\pi)$ from (2.11) are the largest functions satisfying (2.19)-(2.22). Moreover, we assume that the *smooth-fit conditions*:

$$(\text{if } \mu \neq 0 \text{ or } \lambda_0 > \lambda_1) \quad W'(A+) = a + V'(A+), \quad V'(B-) = -b + W'(B-) \quad (\text{if } \mu \neq 0 \text{ or } \lambda_0 < \lambda_1) \quad (2.23)$$

are satisfied. The latter can be explained by the fact that in these cases the process $(\pi_t)_{t \geq 0}$ can pass through the corresponding boundaries A_* and B_* continuously. Such property was earlier observed in [22]-[23] by solving some other optimal stopping problems for jump processes (see also [1] for necessary and sufficient conditions for the occurrence of smooth fit and references to the related literature, and [24] for an extensive overview).

In order to find the optimal boundaries A_* and B_* , let us introduce the *reference (difference) function* $U(\pi) = V(\pi) - W(\pi)$ for all $\pi \in [0, 1]$. Then from (2.19)-(2.22) and (2.23) it follows that $U(\pi)$ solves the system:

$$(\mathbb{L}U)(\pi) = 1 - 2\pi \quad \text{for } A < \pi < B \quad (2.24)$$

$$U(A+) = -aA, \quad U(B-) = b(1 - B) \quad (2.25)$$

$$U(\pi) = -a\pi \quad \text{for } 0 \leq \pi < A, \quad U(\pi) = b(1 - \pi) \quad \text{for } B < \pi \leq 1 \quad (2.26)$$

$$U(\pi) > -a\pi \quad \text{for } A < \pi \leq 1, \quad U(\pi) < b(1 - \pi) \quad \text{for } 0 \leq \pi < B \quad (2.27)$$

and the following conditions hold:

$$(\text{if } \mu \neq 0 \text{ or } \lambda_0 > \lambda_1) \quad U'(A+) = -a, \quad U'(B-) = -b \quad (\text{if } \mu \neq 0 \text{ or } \lambda_0 < \lambda_1). \quad (2.28)$$

3. Solutions of the coupled free-boundary problem

In this section we solve the systems (2.24)-(2.27)+(2.28) and (2.19)-(2.22)+(2.23) for the both cases $\mu \neq 0$ with $\lambda_0 = \lambda_1$ and $\mu = 0$ with $\lambda_0 \neq \lambda_1$, separately.

3.1. By means of straightforward calculations it can be checked that in case $\mu \neq 0$ and $\lambda_0 = \lambda_1$ the solution of the system (2.24)-(2.26)+(2.28) takes the form:

$$U(\pi; A, B) = \frac{b - a}{F_0(A) - F_0(B)} \int_A^\pi F_0(x) dx + \frac{\pi - A}{\lambda} - aA \quad (3.1)$$

for all $A_* < \pi < B_*$ and the boundaries A_* and B_* such that $0 < A_* < B_* < 1$ are uniquely determined by the following coupled system of equations:

$$bF_0(A) - aF_0(B) = \frac{1}{\lambda} (F_0(B) - F_0(A)) \quad (3.2)$$

$$(b - a) \int_A^B F_0(x) dx = \left(aA + b(1 - B) - \frac{1}{\lambda}(B - A) \right) (F_0(A) - F_0(B)) \quad (3.3)$$

with the function $F_0(x)$ defined by:

$$F_0(x) = \exp\left(\frac{2\lambda\sigma^2}{\mu^2 x(1 - x)}\right) \quad (3.4)$$

for all $0 < x < 1$ (see Figure 1 below).

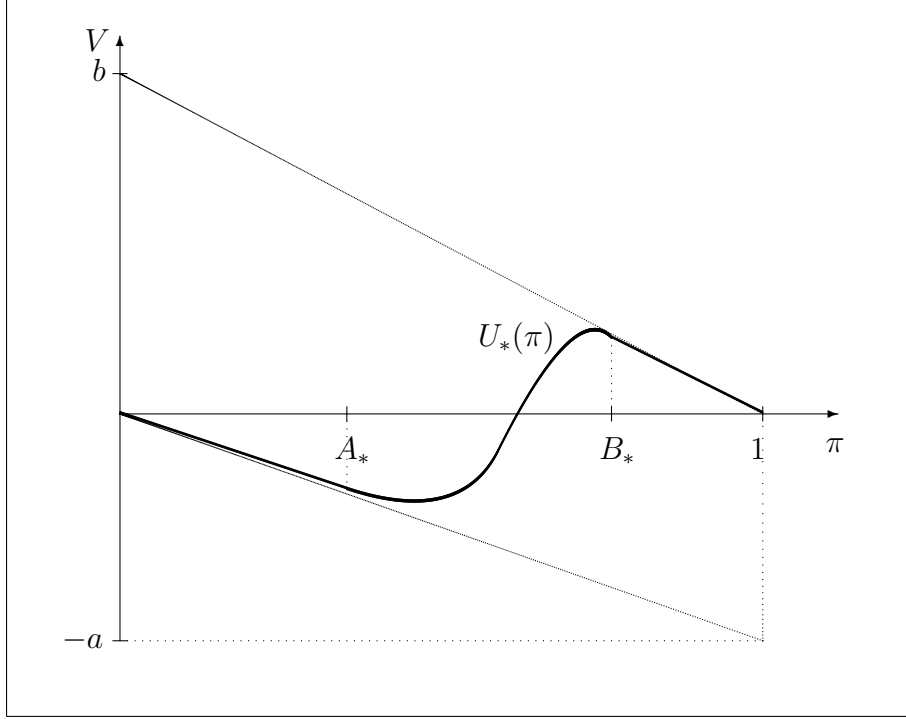


Figure 1. A computer drawing of the reference (difference) function $\pi \mapsto U_*(\pi)$ on $[0, 1]$.

Therefore, solving equations (2.19) and using conditions (2.20) for A and B fixed (as well as taking into account the fact that the value functions should be bounded), we obtain the expressions:

$$V(\pi; B) = b(1 - B) + \frac{2\sigma^2}{\mu^2} \int_{\pi}^B \int_0^x \frac{F_0(x)}{F_0(y)} \frac{dy}{y(1-y)^2} dx \quad (3.5)$$

$$W(\pi; A) = aA + \frac{2\sigma^2}{\mu^2} \int_A^{\pi} \int_x^1 \frac{F_0(x)}{F_0(y)} \frac{dy}{y^2(1-y)} dx \quad (3.6)$$

where the function $F_0(x)$ is defined in (3.4).

3.2. Let us now assume that $\mu = 0$ and $\lambda_0 \neq \lambda_1$. In this case, by making straightforward calculations it is shown that when $\lambda_0 > \lambda_1$ the solution of the system (2.24)-(2.26)+(2.28) takes the form:

$$U(\pi; A, B) = b(1 - B) - \int_{\pi}^B \frac{\gamma \lambda_1 H_1(x, B)(1-x)x^{\gamma}}{[\lambda_1 + (\lambda_0 - \lambda_1)x](1-x)^{\gamma}} dx \quad (3.7)$$

with

$$H_1(x, B) = \frac{1}{D(x)} \left(C_1(x, B) - \int_x^B \frac{C_1(y, B)G_1(y, B)}{D(y)G_1(x, B)} dy \right) \quad (3.8)$$

$$C_1(x, B) = \frac{bB(1-B)^\gamma}{\gamma(\gamma-1)B^\gamma} - \frac{\lambda_0(1-2x)(1-x)^\gamma}{\gamma(1-x)x^\gamma} \quad (3.9)$$

$$D(x) = \frac{x[\lambda'\gamma(\gamma-1)(1-2x) - x(1-x)]}{(1-x)(x+\gamma-1)} \quad (3.10)$$

$$G_1(x, B) = \exp \left(- \int_x^B \frac{dz}{D(z)} \right) \quad (3.11)$$

and $\gamma = \lambda_0/(\lambda_0 - \lambda_1) > 1$, $\lambda' = \lambda_0(\lambda_0 - \lambda_1) > 0$ as well as the boundaries A_* and B_* such that $0 < A_* < B_* < 1$ are uniquely determined by the following coupled system of equations:

$$\frac{\gamma\lambda_1 H_1(A, B)(1-A)A^\gamma}{[\lambda_1 + (\lambda_0 - \lambda_1)A](1-A)^\gamma} = -a \quad (3.12)$$

$$\int_A^B \frac{\gamma\lambda_1 H_1(x, B)(1-x)x^\gamma}{[\lambda_1 + (\lambda_0 - \lambda_1)x](1-x)^\gamma} dx = aA + b(1-B) \quad (3.13)$$

and when $\lambda_0 < \lambda_1$ the function $U(\pi; A, B)$ is given by:

$$U(\pi; A, B) = -aA + \int_A^\pi \frac{\gamma\lambda_1 H_2(x, A)(1-x)x^\gamma}{[\lambda_1 + (\lambda_0 - \lambda_1)x](1-x)^\gamma} dx \quad (3.14)$$

with

$$H_2(x, A) = \frac{1}{D(x)} \left(C_2(x, A) + \int_A^x \frac{C_2(y, A)G_2(y, A)}{D(y)G_2(x, A)} dy \right) \quad (3.15)$$

$$C_2(x, A) = -\frac{aA(1-A)^\gamma}{\gamma(\gamma-1)A^\gamma} - \frac{\lambda_0(1-2x)(1-x)^\gamma}{\gamma(1-x)x^\gamma} \quad (3.16)$$

$$G_2(x, A) = \exp \left(\int_A^x \frac{dz}{D(z)} \right) \quad (3.17)$$

and $\gamma = \lambda_0/(\lambda_0 - \lambda_1) < 0$, $\lambda' = \lambda_0(\lambda_0 - \lambda_1) < 0$ as well as the boundaries A_* and B_* such that $0 < A_* < B_* < 1$ are uniquely determined by the following coupled system of equations:

$$\frac{\gamma\lambda_1 H_2(B, A)(1-B)B^\gamma}{[\lambda_1 + (\lambda_0 - \lambda_1)B](1-B)^\gamma} = -b \quad (3.18)$$

$$\int_A^B \frac{\gamma\lambda_1 H_2(x, A)(1-x)x^\gamma}{[\lambda_1 + (\lambda_0 - \lambda_1)x](1-x)^\gamma} dx = aA + b(1-B). \quad (3.19)$$

Therefore, solving equations (2.19) and using conditions (2.20) for A and B fixed (as well as taking into account the fact that the value functions should be bounded), we obtain the expressions:

$$V(\pi; B) = b(1-B) - \int_\pi^B \frac{\gamma\lambda_1 F_1(x, B)(1-x)x^\gamma}{[\lambda_1 + (\lambda_0 - \lambda_1)x](1-x)^\gamma} dx \quad (3.20)$$

$$W(\pi; A) = aA + \int_A^\pi \frac{\gamma\lambda_1 F_2(x, A)(1-x)x^\gamma}{[\lambda_1 + (\lambda_0 - \lambda_1)x](1-x)^\gamma} dx \quad (3.21)$$

where when $\lambda_0 > \lambda_1$ we have:

$$F_1(x, B) = \frac{1}{D(x)} \left(C_3(x, B) - \int_x^B \frac{C_3(y, B)G_1(y, B)}{D(y)G_1(x, B)} dy \right) \quad (3.22)$$

$$F_2(x, A) = \frac{1}{D(x)} \left(C_4(x, A) + \int_A^x \frac{C_4(y, A)G_2(y, A)}{D(y)G_2(x, A)} dy \right) \quad (3.23)$$

$$C_3(x, B) = \frac{bB(1-B)^\gamma}{\gamma(\gamma-1)B^\gamma} - \frac{\lambda_0 x(1-x)^\gamma}{\gamma(1-x)x^\gamma} \quad (3.24)$$

$$C_4(x, A) = -\frac{aA(1-A)^\gamma}{\gamma(\gamma-1)A^\gamma} - \frac{\lambda_0 x(1-x)^\gamma}{\gamma(1-x)x^\gamma} \quad (3.25)$$

while when $\lambda_0 < \lambda_1$ we have:

$$F_1(x, B) = \frac{1}{D(x)} \left(C_5(x, B) - \int_x^B \frac{C_5(y, B)G_1(y, B)}{D(y)G_1(x, B)} dy \right) \quad (3.26)$$

$$F_2(x, A) = \frac{1}{D(x)} \left(C_6(x, A) + \int_A^x \frac{C_6(y, A)G_2(y, A)}{D(y)G_2(x, A)} dy \right) \quad (3.27)$$

$$C_5(x, B) = \frac{bB(1-B)^\gamma}{\gamma(\gamma-1)B^\gamma} - \frac{\lambda_0(1-x)(1-x)^\gamma}{\gamma(1-x)x^\gamma} \quad (3.28)$$

$$C_6(x, A) = -\frac{aA(1-A)^\gamma}{\gamma(\gamma-1)A^\gamma} - \frac{\lambda_0(1-x)(1-x)^\gamma}{\gamma(1-x)x^\gamma}. \quad (3.29)$$

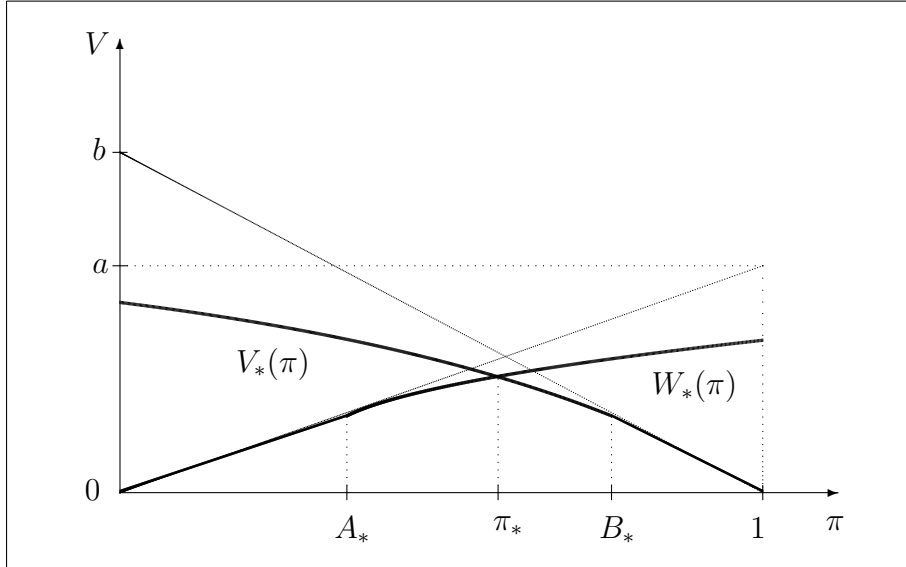


Figure 2. A computer drawing of the value functions $\pi \mapsto V_*(\pi)$ and $\pi \mapsto W_*(\pi)$ for $\pi \in [0, 1]$.

4. Main result and proof

Taking into account the facts proved above, we are now ready to formulate and prove the main assertion of the paper.

Theorem 4.1. *Let the process $X = (X_t)_{t \geq 0}$ be given by (2.1) with $\mu \neq 0$ or $\lambda_0 \neq \lambda_1$. Then the value functions (2.3) and (2.4) take the expressions:*

$$V_*(\pi) = \begin{cases} V(\pi; B_*), & \text{if } 0 \leq \pi < B_* \\ b(1 - \pi), & \text{if } B_* \leq \pi \leq 1 \end{cases} \quad (4.1)$$

and

$$W_*(\pi) = \begin{cases} W(\pi; A_*), & \text{if } A_* < \pi \leq 1 \\ a\pi, & \text{if } 0 \leq \pi \leq A_* \end{cases} \quad (4.2)$$

and the optimal stopping times $(\tau_n^*)_{n \in \mathbb{N}}$ and $(\sigma_n^*)_{n \in \mathbb{N}}$ have the structure (2.14)-(2.15) and (2.16)-(2.17), where the functions $V(\pi; B)$ and $W(\pi; A)$ and the boundaries A_* and B_* are specified as follows [see Figure 2 above]:

(i) if $\mu \neq 0$ and $\lambda_0 = \lambda_1$, then $V(\pi; B)$ and $W(\pi; A)$ are given by (3.5) and (3.6), as well as the optimal boundaries A_* and B_* satisfy the inequalities $0 < A_* < B_* < 1$ and are uniquely determined by the coupled system of equations (3.2)-(3.3);

(ii) if $\mu = 0$ and $\lambda_0 > \lambda_1$, then $V(\pi; B)$ and $W(\pi; A)$ are given by (3.20) and (3.21), as well as the optimal boundaries A_* and B_* satisfy the inequalities $0 < A_* < B_* < 1$ and are uniquely determined by the coupled system of equations (3.12)-(3.13);

(iii) if $\mu = 0$ and $\lambda_0 < \lambda_1$, then $V(\pi; B)$ and $W(\pi; A)$ are given by (3.20) and (3.21), as well as the optimal boundaries A_* and B_* satisfy the inequalities $0 < A_* < B_* < 1$ and are uniquely determined by the coupled system of equations (3.18)-(3.19).

Proof. In order to verify the related assertions, it remains to show that the functions (4.1) and (4.2) coincide with the value functions (2.10) and (2.11), respectively, and the stopping times τ_* and σ_* from (2.12) and (2.13) with the boundaries A_* and B_* specified above are optimal. For this, let us denote by $V(\pi)$ and $W(\pi)$ the right-hand sides of the expressions (4.1) and (4.2), respectively. In these cases, by means of straightforward calculations and the assumptions above it follows that the functions $V(\pi)$ and $W(\pi)$ solve the system (2.19)-(2.22), and conditions (2.23) are satisfied under the corresponding relationships on the parameters of the model. Note that from the formulas of the previous section it is seen that the both functions $V(\pi)$ and $W(\pi)$ are concave on $[0, 1]$. The latter can be shown directly by analyzing the expressions (3.5)-(3.6) and (3.20)-(3.21). Then, applying Itô-Tanaka-Meyer formula (see, e.g., [18; Chapter V, Theorem 5.52] or [25; Chapter IV, Theorem 51]) to $V(\pi_t)$ and $W(\pi_t)$, we obtain:

$$V(\pi_t) = V(\pi) + \int_0^t (\mathbb{L}V)(\pi_s) I(\pi_s \neq B_*) ds + M_t \quad (4.3)$$

$$W(\pi_t) = W(\pi) + \int_0^t (\mathbb{L}W)(\pi_s) I(\pi_s \neq A_*) ds + N_t \quad (4.4)$$

where the processes $(M_t)_{t \geq 0}$ and $(N_t)_{t \geq 0}$ defined by:

$$\begin{aligned} M_t &= \int_0^t V'(\pi_s) \frac{\mu}{\sigma} \pi_s (1 - \pi_s) d\bar{W}_s \\ &+ \int_0^t \int_0^\infty \left[V \left(\frac{\pi_{s-} e^{-\lambda_1 x}}{\pi_{s-} e^{-\lambda_1 x} + (1 - \pi_{s-}) e^{-\lambda_0 x}} \right) - V(\pi_{s-}) \right] \left(\mu^X(ds, dx) - \bar{\nu}(ds, dx) \right) \end{aligned} \quad (4.5)$$

$$N_t = \int_0^t W'(\pi_s) \frac{\mu}{\sigma} \pi_s (1 - \pi_s) d\overline{W}_s \quad (4.6)$$

$$+ \int_0^t \int_0^\infty \left[W \left(\frac{\pi_{s-} e^{-\lambda_1 x}}{\pi_{s-} e^{-\lambda_1 x} + (1 - \pi_{s-}) e^{-\lambda_0 x}} \right) - W(\pi_{s-}) \right] \left(\mu^X(ds, dx) - \overline{\nu}(ds, dx) \right)$$

are local martingales under the measure P_π with respect to $(\mathcal{F}_t^X)_{t \geq 0}$ and we set $\overline{\nu}(dt, dx) = (\pi_{t-} e^{-\lambda_1 x} + (1 - \pi_{t-}) e^{-\lambda_0 x}) dt dx$.

By the construction of $V(\pi)$ and $W(\pi)$ from the previous sections and by using the straightforward calculations it can be checked that $(\mathbb{L}V)(\pi) \geq -\pi$ for all $B < \pi < 1$ and $(\mathbb{L}W)(\pi) \geq -(1 - \pi)$ for all $0 < \pi < A$. Moreover, by means of standard arguments it can be shown that the function $V(\pi; B_*)$ is decreasing, while the function $W(\pi; A_*)$ is increasing on the intervals $(0, B_*)$ and $(A_*, 1)$, respectively, since for their derivatives we have $-b < V'(\pi; B_*) < 0$ and $0 < W'(\pi; A_*) < a$. Then the properties (2.22) also hold, that together with (2.20)-(2.21) yields $V(\pi) \leq b(1 - \pi) + W(\pi)$ and $W(\pi) \leq a\pi + V(\pi)$ for all $\pi \in [0, 1]$. Observe that by using (2.8) it is shown that the time spent by the process $(\pi_t)_{t \geq 0}$ at the points A_* and B_* is of Lebesgue measure zero. Hence, from the expressions (4.3)-(4.4) and the structure of stopping times in (2.12)-(2.13), by using the fact that $A_* \leq (a\lambda + 1)/(2a\lambda + 1)$, $B_* \geq b\lambda/(2b\lambda + 1)$ and $0 < A_* < B_* < 1$ it follows that the inequalities:

$$b(1 - \pi_\tau) + \int_0^\tau \pi_s ds + W(\pi_\tau) \geq V(\pi_\tau) + \int_0^\tau \pi_s ds \geq V(\pi) + M_\tau \quad (4.7)$$

$$a\pi_\sigma + \int_0^\sigma (1 - \pi_s) ds + V(\pi_\sigma) \geq W(\pi_\sigma) + \int_0^\sigma (1 - \pi_s) ds \geq W(\pi) + N_\sigma \quad (4.8)$$

hold for any stopping times τ and σ of the process $(\pi_t)_{t \geq 0}$.

Let $(\tau_n)_{n \in \mathbb{N}}$ and $(\sigma_n)_{n \in \mathbb{N}}$ be arbitrary localizing sequences of stopping times for the processes $(M_t)_{t \geq 0}$ and $(N_t)_{t \geq 0}$, respectively. Then, using (4.7)-(4.8) and taking the expectations with respect to P_π , by means of the optional sampling theorem (see, e.g., [19; Chapter I, Theorem 1.39]), we get:

$$E_\pi \left[b(1 - \pi_{\tau \wedge \tau_n}) + \int_0^{\tau \wedge \tau_n} \pi_s ds + W(\pi_{\tau \wedge \tau_n}) \right] \quad (4.9)$$

$$\geq E_\pi \left[V(\pi_{\tau \wedge \tau_n}) + \int_0^{\tau \wedge \tau_n} \pi_s ds \right] \geq V(\pi) + E_\pi [M_{\tau \wedge \tau_n}] = V(\pi)$$

$$E_\pi \left[a\pi_{\sigma \wedge \sigma_n} + \int_0^{\sigma \wedge \sigma_n} (1 - \pi_s) ds + V(\pi_{\sigma \wedge \sigma_n}) \right] \quad (4.10)$$

$$\geq E_\pi \left[W(\pi_{\sigma \wedge \sigma_n}) + \int_0^{\sigma \wedge \sigma_n} (1 - \pi_s) ds \right] \geq W(\pi) + E_\pi [N_{\sigma \wedge \sigma_n}] = W(\pi)$$

for all $\pi \in [0, 1]$. Hence, letting n go to infinity and using Fatou's lemma, for any stopping times τ and σ such that $E_\pi[\tau \vee \sigma] < \infty$ we obtain that the inequalities:

$$E_\pi \left[b(1 - \pi_\tau) + \int_0^\tau \pi_s ds + W(\pi_\tau) \right] \geq V(\pi) \quad (4.11)$$

$$E_\pi \left[a\pi_\sigma + \int_0^\sigma (1 - \pi_s) ds + V(\pi_\sigma) \right] \geq W(\pi) \quad (4.12)$$

are satisfied for all $\pi \in [0, 1]$.

By virtue of the fact that the functions $V(\pi)$ and $W(\pi)$ satisfy the system (2.19)-(2.22) with the boundaries A_* and B_* , by the structure of the stopping times τ_* in (2.12) and σ_* in (2.13) as well as by the expressions (4.3) and (4.4) it follows that the equalities:

$$V(\pi_{\tau_* \wedge \tau_n}) + \int_0^{\tau_* \wedge \tau_n} \pi_s ds = V(\pi) + M_{\tau_* \wedge \tau_n} \quad (4.13)$$

$$W(\pi_{\sigma_* \wedge \sigma_n}) + \int_0^{\sigma_* \wedge \sigma_n} (1 - \pi_s) ds = W(\pi) + N_{\sigma_* \wedge \sigma_n} \quad (4.14)$$

hold for all $\pi \in [0, 1]$. Note that, by means of standard arguments and using the structure of the process (2.8) and of the stopping times (2.12) and (2.13), we have $E_\pi[\tau_* \vee \sigma_*] < \infty$ for all $\pi \in [0, 1]$. Hence, letting n go to infinity in (4.13)-(4.14) and using conditions (2.21)-(2.22), by means of the Lebesgue bounded convergence theorem, we obtain the equalities:

$$E_\pi \left[b(1 - \pi_{\tau_*}) + \int_0^{\tau_*} \pi_s ds + W(\pi_{\tau_*}) \right] = V(\pi) \quad (4.15)$$

$$E_\pi \left[a\pi_{\sigma_*} + \int_0^{\sigma_*} (1 - \pi_s) ds + V(\pi_{\sigma_*}) \right] = W(\pi) \quad (4.16)$$

for all $\pi \in [0, 1]$, that together with (4.11)-(4.12) directly imply the desired assertion. \square

Remark 4.2. By means of straightforward calculations from the previous section it can be verified that in case $\mu = 0$ with $\lambda_0 > \lambda_1$ we have $V'(B_*-; B_*) > -b + W'(B_*-; A_*)$, while in case $\mu = 0$ with $\lambda_0 < \lambda_1$ we have $W'(A_*+; A_*) < a + V'(A_*+; B_*)$. According to the arguments in [22]-[23] such effects can be explained by the fact that in those cases the process $(\pi_t)_{t \geq 0}$ can pass through the corresponding boundaries B_* or A_* only by jumping. According to the results in [1] we may conclude that this property appears because of finite intensity of jumps and exponential distribution of jump sizes of the compound Poisson process J .

Remark 4.3. The results formulated above show that the following sequential procedure is optimal. Being based on the observations $X = (X_t)_{t \geq 0}$ we construct the sufficient statistic process $(\pi_t)_{t \geq 0}$ and stop the observations as soon as the latter process comes into the region $[0, A_*]$ or $[B_*, 1]$ and then conclude that the continuous Markov chain $\theta = (\theta_t)_{t \geq 0}$ has switched from 1 to 0 or from 0 to 1, respectively. Starting from one of those regions $[0, A_*]$ or $[B_*, 1]$, we stop the observations as soon as the process $(\pi_t)_{t \geq 0}$ comes to the opposite region and then conclude that θ has switched from 0 to 1 or from 1 to 0, respectively. Then we continue the procedure from the beginning.

Acknowledgments. The results of the paper were presented at INFORMS Applied Probability Conference in Ottawa, July 2005. The author is grateful to Savas Dayanik for invitation.

References

- [1] ALILI, L. and KYPRIANOU, A. E. (2005). Some remarks on first passage of Lévy processes, the American put and pasting principles. *Ann. Appl. Probab.* **15**(3) (2062–2080).

- [2] BENSOUSSAN, A and FRIEDMAN, A. (1974). Non-linear variational inequalities and differential games with stopping times. *J. Funct. Anal.* **16** (305–352).
- [3] BENSOUSSAN, A and FRIEDMAN, A. (1977). Nonzero-sum stochastic differential games with stopping times and free-boundary problems. *Trans. Amer. Math. Soc.* **231** (275–327).
- [4] BREKKE, J. A. and ØKSENDAL, B. (1994). Optimal switching in an economic activity under uncertainty. *SIAM J. Control Optim.* (1021–1036).
- [5] CARLSTEIN, E., MÜLLER, H.-G. and SIEGMUND, D. (eds) (1994). *Change-point problems*. IMS Lecture Notes Monogr. Ser. **23**.
- [6] DAVIS, M. H. A. (1976). A note on the Poisson Disorder Problem. *Banach Center Publ.* **1** (65–72).
- [7] DAYANIK, S. and SEZER, S. O. (2005). Compound Poisson disorder problem. *Preprint*, Princeton University (45 pp). To appear in *Math. Oper. Res.*
- [8] DAYANIK, S. and EGAMI, M. (2005). A direct solution method for optimal switching problems of one-dimensional diffusions. *Preprint*.
- [9] DUCKWORTH, K. and ZERVOS, M. (2001). A model for investment decisions with switching costs. *Ann. Appl. Probab.* **11**(1) (239–260).
- [10] DYNKIN, E. B. (1963). The optimum choice of the instant for stopping a Markov process. *Soviet Math. Dokl.* **4** (627–629).
- [11] FRIEDMAN, A. (1973). Stochastic games and variational inequalities. *Arch. Rational Mech. Anal.* **51** (321–346).
- [12] FRIEDMAN, A. (1976). *Stochastic Differential Equations and Applications II*. Academic Press, New York.
- [13] GAL’CHUK, L. I. and ROZOVSKII, B. L. (1971). The ‘disorder’ problem for a Poisson process. *Theory Probab. Appl.* **16** (712–716).
- [14] GAPEEV, P. V. (2005). The disorder problem for compound Poisson processes with exponential jumps. *Ann. Appl. Probab.* **15A** (487–499).
- [15] GAPEEV, P. V. and PESKIR, G. (2003). The Wiener disorder problem with finite horizon. *Research Report* No. 435, Dept. Theoret. Statist. Aarhus (22 pp). To appear in *Stoch. Proc. Appl.*
- [16] GRIGELIONIS, B. I. and SHIRYAEV, A. N. (1966). On Stefan’s problem and optimal stopping rules for Markov processes. *Theory Probab. Appl.* **11** (541–558).
- [17] HAMADÈNE, S. and JEANBLANC, M. (2005). On the starting and stopping problem: application in reversible investments. To appear in *Math. Oper. Res.*

- [18] JACOD, J. (1979). *Calcul Stochastique et Problèmes de Martingales*. Lecture Notes in Mathematics, Berlin.
- [19] JACOD, J. and SHIRYAEV, A. N. (1987). *Limit Theorems for Stochastic Processes*. Springer, Berlin.
- [20] KOLMOGOROV, A. N., PROKHOROV, YU. V. and SHIRYAEV, A. N. (1990). Methods of detecting spontaneously occurring effects. *Proceedings of the Steklov Mathematical Institute*. **1** (1–21).
- [21] LIPTSER, R. S. and SHIRYAEV, A. N. (1977). *Statistics of Random Processes I, II*. Springer, Berlin.
- [22] PESKIR, G. and SHIRYAEV, A. N. (2000). Sequential testing problems for Poisson processes. *Ann. Statist.* **28** (837–859).
- [23] PESKIR, G. and SHIRYAEV, A. N. (2002). Solving the Poisson disorder problem. *Advances in Finance and Stochastics*. Essays in Honour of Dieter Sondermann. Sandmann, K. and Schönbucher, P. eds. Springer (295–312).
- [24] PESKIR, G. and SHIRYAEV, A. N. (2006). *Optimal Stopping and Free-Boundary Problems*. Birkhäuser, Basel.
- [25] PROTTER, PH. (1990). *Stochastic Integration and Differential Equations*. Springer, New York.
- [26] SHIRYAEV, A. N. (1961). The problem of the most rapid detection of a disturbance in a stationary process. *Soviet Math. Dokl.* **2** (795–799).
- [27] SHIRYAEV, A. N. (1963). On optimum methods in quickest detection problems. *Theory Probab. Appl.* **8** (22–46).
- [28] SHIRYAEV, A. N. (1965). Some exact formulas in a 'disorder' problem. *Theory Probab. Appl.* **10** (348–354).
- [29] SHIRYAEV, A. N. (1967). Two problems of sequential analysis. *Cybernetics*. **3** (63–69).
- [30] SHIRYAEV, A. N. (1978). *Optimal Stopping Rules*. Springer, Berlin.
- [31] YUSHKEVICH, A. I. (2001). Optimal switching problem for countable Markov chains: average reward criterion. *Math. Meth. Oper. Res.* **53** (1–24).
- [32] YUSHKEVICH, A. I. and GORDIENKO, E. (2002). Average optimal switching of a Markov chain with a Borel state space. *Math. Meth. Oper. Res.* **55** (143–159).

Pavel V. Gapeev
 Weierstraß Institute
 for Applied Analysis and Stochastics (WIAS)
 Mohrenstr. 39, D-10117 Berlin, Germany
 e-mail: gapeev@wias-berlin.de

(Russian Academy of Sciences
 Institute of Control Sciences
 Profsoyuznaya Str. 65
 117997 Moscow, Russia)

SFB 649 Discussion Paper Series 2006

For a complete list of Discussion Papers published by the SFB 649, please visit <http://sfb649.wiwi.hu-berlin.de>.

- 001 "Calibration Risk for Exotic Options" by Kai Detlefsen and Wolfgang K. Härdle, January 2006.
- 002 "Calibration Design of Implied Volatility Surfaces" by Kai Detlefsen and Wolfgang K. Härdle, January 2006.
- 003 "On the Appropriateness of Inappropriate VaR Models" by Wolfgang Härdle, Zdeněk Hlávka and Gerhard Stahl, January 2006.
- 004 "Regional Labor Markets, Network Externalities and Migration: The Case of German Reunification" by Harald Uhlig, January/February 2006.
- 005 "British Interest Rate Convergence between the US and Europe: A Recursive Cointegration Analysis" by Enzo Weber, January 2006.
- 006 "A Combined Approach for Segment-Specific Analysis of Market Basket Data" by Yasemin Boztuğ and Thomas Reutterer, January 2006.
- 007 "Robust utility maximization in a stochastic factor model" by Daniel Hernández-Hernández and Alexander Schied, January 2006.
- 008 "Economic Growth of Agglomerations and Geographic Concentration of Industries - Evidence for Germany" by Kurt Geppert, Martin Gornig and Axel Werwatz, January 2006.
- 009 "Institutions, Bargaining Power and Labor Shares" by Benjamin Bental and Dominique Demougin, January 2006.
- 010 "Common Functional Principal Components" by Michal Benko, Wolfgang Härdle and Alois Kneip, January 2006.
- 011 "VAR Modeling for Dynamic Semiparametric Factors of Volatility Strings" by Ralf Brüggemann, Wolfgang Härdle, Julius Mungo and Carsten Trenkler, February 2006.
- 012 "Bootstrapping Systems Cointegration Tests with a Prior Adjustment for Deterministic Terms" by Carsten Trenkler, February 2006.
- 013 "Penalties and Optimality in Financial Contracts: Taking Stock" by Michel A. Robe, Eva-Maria Steiger and Pierre-Armand Michel, February 2006.
- 014 "Core Labour Standards and FDI: Friends or Foes? The Case of Child Labour" by Sebastian Braun, February 2006.
- 015 "Graphical Data Representation in Bankruptcy Analysis" by Wolfgang Härdle, Rouslan Moro and Dorothea Schäfer, February 2006.
- 016 "Fiscal Policy Effects in the European Union" by Andreas Thams, February 2006.
- 017 "Estimation with the Nested Logit Model: Specifications and Software Particularities" by Nadja Silberhorn, Yasemin Boztuğ and Lutz Hildebrandt, March 2006.
- 018 "The Bologna Process: How student mobility affects multi-cultural skills and educational quality" by Lydia Mechtenberg and Roland Strausz, March 2006.
- 019 "Cheap Talk in the Classroom" by Lydia Mechtenberg, March 2006.
- 020 "Time Dependent Relative Risk Aversion" by Enzo Giacomini, Michael Handel and Wolfgang Härdle, March 2006.
- 021 "Finite Sample Properties of Impulse Response Intervals in SVECMs with Long-Run Identifying Restrictions" by Ralf Brüggemann, March 2006.
- 022 "Barrier Option Hedging under Constraints: A Viscosity Approach" by Imen Bentahar and Bruno Bouchard, March 2006.

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".



- 023 "How Far Are We From The Slippery Slope? The Laffer Curve Revisited" by Mathias Trabandt and Harald Uhlig, April 2006.
- 024 "e-Learning Statistics – A Selective Review" by Wolfgang Härdle, Sigbert Klinke and Uwe Ziegenhagen, April 2006.
- 025 "Macroeconomic Regime Switches and Speculative Attacks" by Bartosz Maćkowiak, April 2006.
- 026 "External Shocks, U.S. Monetary Policy and Macroeconomic Fluctuations in Emerging Markets" by Bartosz Maćkowiak, April 2006.
- 027 "Institutional Competition, Political Process and Holdup" by Bruno Deffains and Dominique Demougin, April 2006.
- 028 "Technological Choice under Organizational Diseconomies of Scale" by Dominique Demougin and Anja Schöttner, April 2006.
- 029 "Tail Conditional Expectation for vector-valued Risks" by Imen Bentahar, April 2006.
- 030 "Approximate Solutions to Dynamic Models – Linear Methods" by Harald Uhlig, April 2006.
- 031 "Exploratory Graphics of a Financial Dataset" by Antony Unwin, Martin Theus and Wolfgang Härdle, April 2006.
- 032 "When did the 2001 recession *really* start?" by Jörg Polzehl, Vladimir Spokoiny and Cătălin Stărică, April 2006.
- 033 "Varying coefficient GARCH versus local constant volatility modeling. Comparison of the predictive power" by Jörg Polzehl and Vladimir Spokoiny, April 2006.
- 034 "Spectral calibration of exponential Lévy Models [1]" by Denis Belomestny and Markus Reiß, April 2006.
- 035 "Spectral calibration of exponential Lévy Models [2]" by Denis Belomestny and Markus Reiß, April 2006.
- 036 "Spatial aggregation of local likelihood estimates with applications to classification" by Denis Belomestny and Vladimir Spokoiny, April 2006.
- 037 "A jump-diffusion Libor model and its robust calibration" by Denis Belomestny and John Schoenmakers, April 2006.
- 038 "Adaptive Simulation Algorithms for Pricing American and Bermudan Options by Local Analysis of Financial Market" by Denis Belomestny and Grigori N. Milstein, April 2006.
- 039 "Macroeconomic Integration in Asia Pacific: Common Stochastic Trends and Business Cycle Coherence" by Enzo Weber, May 2006.
- 040 "In Search of Non-Gaussian Components of a High-Dimensional Distribution" by Gilles Blanchard, Motoaki Kawanabe, Masashi Sugiyama, Vladimir Spokoiny and Klaus-Robert Müller, May 2006.
- 041 "Forward and reverse representations for Markov chains" by Grigori N. Milstein, John G. M. Schoenmakers and Vladimir Spokoiny, May 2006.
- 042 "Discussion of 'The Source of Historical Economic Fluctuations: An Analysis using Long-Run Restrictions' by Neville Francis and Valerie A. Ramey" by Harald Uhlig, May 2006.
- 043 "An Iteration Procedure for Solving Integral Equations Related to Optimal Stopping Problems" by Denis Belomestny and Pavel V. Gapeev, May 2006.
- 044 "East Germany's Wage Gap: A non-parametric decomposition based on establishment characteristics" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.
- 045 "Firm Specific Wage Spread in Germany - Decomposition of regional differences in inter firm wage dispersion" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

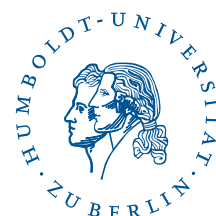
This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".



- 046 "Produktdiversifizierung: Haben die ostdeutschen Unternehmen den Anschluss an den Westen geschafft? – Eine vergleichende Analyse mit Mikrodaten der amtlichen Statistik" by Bernd Görzig, Martin Gornig and Axel Werwatz, May 2006.
- 047 "The Division of Ownership in New Ventures" by Dominique Demougin and Oliver Fabel, June 2006.
- 048 "The Anglo-German Industrial Productivity Paradox, 1895-1938: A Restatement and a Possible Resolution" by Albrecht Ritschl, May 2006.
- 049 "The Influence of Information Costs on the Integration of Financial Markets: Northern Europe, 1350-1560" by Oliver Volckart, May 2006.
- 050 "Robust Econometrics" by Pavel Čížek and Wolfgang Härdle, June 2006.
- 051 "Regression methods in pricing American and Bermudan options using consumption processes" by Denis Belomestny, Grigori N. Milstein and Vladimir Spokoiny, July 2006.
- 052 "Forecasting the Term Structure of Variance Swaps" by Kai Detlefsen and Wolfgang Härdle, July 2006.
- 053 "Governance: Who Controls Matters" by Bruno Deffains and Dominique Demougin, July 2006.
- 054 "On the Coexistence of Banks and Markets" by Hans Gersbach and Harald Uhlig, August 2006.
- 055 "Reassessing Intergenerational Mobility in Germany and the United States: The Impact of Differences in Lifecycle Earnings Patterns" by Thorsten Vogel, September 2006.
- 056 "The Euro and the Transatlantic Capital Market Leadership: A Recursive Cointegration Analysis" by Enzo Weber, September 2006.
- 057 "Discounted Optimal Stopping for Maxima in Diffusion Models with Finite Horizon" by Pavel V. Gapeev, September 2006.
- 058 "Perpetual Barrier Options in Jump-Diffusion Models" by Pavel V. Gapeev, September 2006.
- 059 "Discounted Optimal Stopping for Maxima of some Jump-Diffusion Processes" by Pavel V. Gapeev, September 2006.
- 060 "On Maximal Inequalities for some Jump Processes" by Pavel V. Gapeev, September 2006.
- 061 "A Control Approach to Robust Utility Maximization with Logarithmic Utility and Time-Consistent Penalties" by Daniel Hernández-Hernández and Alexander Schied, September 2006.
- 062 "On the Difficulty to Design Arabic E-learning System in Statistics" by Taleb Ahmad, Wolfgang Härdle and Julius Mungo, September 2006.
- 063 "Robust Optimization of Consumption with Random Endowment" by Wiebke Wittmüß, September 2006.
- 064 "Common and Uncommon Sources of Growth in Asia Pacific" by Enzo Weber, September 2006.
- 065 "Forecasting Euro-Area Variables with German Pre-EMU Data" by Ralf Brüggemann, Helmut Lütkepohl and Massimiliano Marcellino, September 2006.
- 066 "Pension Systems and the Allocation of Macroeconomic Risk" by Lans Bovenberg and Harald Uhlig, September 2006.
- 067 "Testing for the Cointegrating Rank of a VAR Process with Level Shift and Trend Break" by Carsten Trenkler, Pentti Saikkonen and Helmut Lütkepohl, September 2006.
- 068 "Integral Options in Models with Jumps" by Pavel V. Gapeev, September 2006.
- 069 "Constrained General Regression in Pseudo-Sobolev Spaces with Application to Option Pricing" by Zdeněk Hlávka and Michal Pešta, September 2006.

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".



- 070 "The Welfare Enhancing Effects of a Selfish Government in the Presence of Uninsurable, Idiosyncratic Risk" by R. Anton Braun and Harald Uhlig, September 2006.
- 071 "Color Harmonization in Car Manufacturing Process" by Anton Andriyashin, Michal Benko, Wolfgang Härdle, Roman Timofeev and Uwe Ziegenhagen, October 2006.
- 072 "Optimal Interest Rate Stabilization in a Basic Sticky-Price Model" by Matthias Paustian and Christian Stoltenberg, October 2006.
- 073 "Real Balance Effects, Timing and Equilibrium Determination" by Christian Stoltenberg, October 2006.
- 074 "Multiple Disorder Problems for Wiener and Compound Poisson Processes With Exponential Jumps" by Pavel V. Gapeev, October 2006.

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

