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Conditionally parametric fits for CAPM betas

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Abstract: The CAPM model assumes stock returns to be a linear function of the market return. However, there is considerable evidence that the beta stability assumption commonly used when estimating the model is invalid. Nonparametric regression methods are used to examine the stability of beta coefficients in German stock returns. Since local polynomial regression is used for estimation, known methods for testing the stability and for bandwidth choice can be used. For some returns the test indicates time-varying betas. For these returns conditionally parametric fits are calculated.

Keywords: CAPM, time-varying betas, conditionally parametric fits, nonparametric regression

1 Introduction

The CAPM beta (β) is a parameter which plays a central role in finance as a measure of an asset's risk. The CAPM basically postulates a stable linear relationship

between the expected risk and return of holding a financial asset. For stock markets the linear relationship between the expected return of a single stock, r , and its associated risk, measured by the standard deviation of returns, σ , can be written as

$$r - r_f = (\sigma/\sigma_m) \cdot (r_m - r_f), \quad (1)$$

where r_m and σ_m denote the expected overall market return and its standard deviation. r_f is the return on a risk free asset.

The beta is often estimated by regressing an assets return on the return of the market portfolio using time series data. I.e. the following model is estimated:

$$Y_t = \alpha + \beta X_t + \varepsilon_t. \quad (2)$$

However a series of studies show that the parameter is not stable over time. For example the articles of Fabozzi and Francis (1978), Bos and Newbold (1984), Bollerslev et al. (1988), Cheng (1997) and Neumann (2003) belong to these studies.

Various models and estimation techniques have been applied to analyze time-varying betas. For example, Kim (1993) focuses on structural breaks causing level shifts in betas. Cheng (1997) employs Goldfeld-Quandt switching regression method. Wells (1994) applies the Flexible Least Squares method. Also state space models with different types of coefficient variation have been employed, see e.g. Fabozzi and Francis (1978), Bos and Newbold (1984) and Wells (1994).

In this article nonparametric regression techniques are applied. A conditionally parametric model is fitted with the help of local polynomial regression. This kind of analysis assumes a gradual type beta-variation.

The application of nonparametric estimation methods permits the use of well established techniques for estimation and inference. These techniques are discussed in the next section. In Section 3 the nonparametric methods are applied to monthly stock returns contained in the German DAX100 index. Based on local linear regression beta-stability is tested and time -varying betas are estimated.

2 Conditionally parametric models

Conditionally parametric fits were introduced by Cleveland et al. (1991). This kind of model is part of the broader class of varying-coefficient models discussed in Hastie, Tibshirani (1993). Suppose there is one random variable Y and predictors X_1, \dots, X_p and R_1, \dots, R_p . A varying-coefficient model is of the form

$$Y = \beta_0 + X_1\beta_1(R_1) + X_2\beta_2(R_2) + \dots + X_p\beta_p(R_p) + \varepsilon, \quad (3)$$

with $E[\varepsilon] = 0$, $Var[\varepsilon] = \sigma^2$ and coefficients β_i which depend on R_i . Often the R_i s will be the same variable, a factor such as time or age, that we suspect could modify the effects of X_1, \dots, X_p . In the present application the data consists of two time series over n time points $t \in t_1, \dots, t_n$ and there is only one independent variable x . Then a suitable model is

$$Y_t = \alpha(t) + X_t\beta(t). \quad (4)$$

This is a type of model which is called “conditionally parametric“ by Cleveland et al. (1991). These authors treat $\alpha(t)$ and $\beta(t)$ as nonparametric functions. Given t the model is parametric, and hence the name “conditionally parametric“.

The model is estimated with nonparametric regression techniques. To be precise local linear regression is used. There are two predictors, namely t and X . Therefore a bivariate local linear regression approach is needed. Ignoring the conditionally

parametric factor X in the neighborhood weight leads to a conditionally linear fit. Fan and Gijbels (1996) and Loader (1999) are two very suitable monographs about local polynomial regression. Loader provides in addition a very powerful software for the calculation of local regression estimates. This software is called `locfit`. It is used here in connection with the statistical computer language R to compute the estimates.

With

$$\mu(x, t) = \alpha(t) + \beta(t)x, \quad (5)$$

the fit at (x, t) is computed by minimizing

$$\sum_{i=1}^n (y_i - \mu((x_i - x), (t_i - t)))^2 \cdot K\left(\frac{t_i - t}{h}\right), \quad (6)$$

with a kernel function $K(\cdot)$ and a bandwidth h . However, the specification of a bandwidth is needed. Leave-one-out cross validation is used for bandwidth choice here. The cross validation criterion is

$$CV(\hat{\mu}) = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{\mu}_{-1}(x_i, t_i))^2, \quad (7)$$

where $\hat{\mu}_{-1}(x_i, t_i)$ denotes the leave-one-out estimator of $\mu(x_i, t_i)$. That is, each (x_i, t_i) is deleted from the data set in turn, and the local regression estimate is computed from the remaining $n - 1$ data points.

Cross validation is a standard method for choosing the bandwidth. It is also a popular method for model selection in the statistical learning theory. It avoids the problem of overfitting the data (see e.g. Hastie et al. (2001), Chapter 7: Model assessment and selection).

After choosing a bandwidth, the model can be estimated. As a next step a test checking the significance of the conditionally parametric model is needed. The nonparametric fit will be compared with the simple linear model. For a broad class of smoothers, including kernels, nearest neighbors, as well as splines, the approximate methods described in Hastie and Tibshirani (1990) can be used. These rely on the unifying concept of the “approximate degrees of freedom“ of a linear smoother. For a linear smooth fit $\hat{y} = Sy$, Hastie and Tibshirani (1990) argue that the appropriate degrees of freedom for the fit is $\nu = \text{tr}(2S - SS^T)$ and the distribution of the test statistic

$$\frac{RSS(\text{linear}) - RSS(\text{conditionally parametric})(\nu - 2)}{RSS(\text{conditionally parametric})/(n - \nu)} \quad (8)$$

is roughly $F_{\nu-2, n-\nu}$ distributed. RSS means residual sum of squares.

The nonparametric estimation method, the cross validation technique and the approximate F-test are the instruments which are applied to stock return data in the following.

3 Application to German stock returns

The above discussed estimation methods are now applied to stock returns of the German DAX100 index. The returns are calculated by $y_i = \log P_i - \log P_{i-1}$, $i = 2, \dots, n$ and $x_i = \log DAX100_i - \log DAX100_{i-1}$, with P_i denoting the price of a stock at time i and “DAX100 $_i$ “ the DAX100 index at time i . The empirical analysis relies on monthly return data from January 1988 to October 2002 and is restricted to those 48 stocks of the DAX100 that have been listed at the German stock exchange for the entire sample period.

Table 1 shows the bandwidths for the conditionally parametric models chosen by the leave-one-out cross validation criteria (denoted $h(CV)$). The entry “limo“ in the $h(CV)$ column means that the criterion chooses the usual linear model without varying betas. The following columns contain the results for the approximate F-test. The results in this table altogether show that a flexible approach is need for modelling the returns. For some returns the usual linear model seems to be sufficient. For other returns the betas seem to be time-varying. However even for these returns the chosen bandwidths are very different.

Table 2 and 3 show the estimation results for the returns for which the approximate F-test is significant at least at the 0.1 level. Table 2 contains the estimated regression coefficients and the R^2 value for the linear model. Contained in Table 3 is the R^2 for the estimated conditionally parametric fit based on the cross-validation bandwidth. For these selected returns the R^2 s of the conditionally parametric fits are all higher than the R^2 s of the common linear models. Also listed in the tables are the leave-one-out cross validation values for both models (marked by CV_1). In addition leave-five-out cross-validation values (CV_5) are given. In this case subsequent data blocks of size 5 are left out in the estimation process. With this estimator the values of the five left out data points are then predicted. The resulting averaged squared error is denoted by CV_5 . With the cross validation technique one tries to capture the problem of overfitting (see Hastie et al. ,2001). Actually there is one return series for which both cross validation values are larger for the conditionally parametric model than for the linear model. There seems to be an overfitting problem for the Continatal data. For a collection of the remaining return series the resulting $\beta(t)$ estimates are plotted in Figures 1 and 2. In result the manner of beta-variation is very different: the graphs range from almost monotone to curvy.

Return	$h(CV)$	F-value	p-value	significance
Agiv	limo	-	-	
Allianz	limo	-	-	
Altana	26	2.632	0.001	***
AVA	114	2.763	0.022	**
BASF	94	1.466	0.207	
Hypovereinsbank	161	2.707	0.043	**
Bayer	159	1.551	0.200	
Beiersdorf	limo	-	-	
Bilfinger & Berger	32	1.937	0.021	**
BMW	69	1.236	0.286	
Boss	31	1.546	0.0888	*
Commerzbank	69	2.589	0.015	**
Continental	19	1.453	0.086	*
Degussa	55	2.314	0.024	**
Deutsche Bank	limo	-	-	
Douglas	152	2.028	0.103	
Draegerwerke	42	1.689	0.077	*
Dyckerhoff	110	3.164	0.012	**
Eon	113	3.324	0.008	***
Gehe	154	2.104	0.094	*
Gildemeister	121	1.955	0.097	*
Heidelbg. Zement	123	1.226	0.301	
Henkel	66	1.736	0.0999	*

Table 1: (a) Cross-validation bandwidths and approximate F-test for monthly German DAX100 returns

Return	$h(CV)$	F-value	p-value	significance
Hochtief	171	1.504	0.216	
Hornbach	111	1.615	0.161	
IKB Dt. Industriebk.	37	1.377	0.174	
IVG Immobilien	18	1.711	0.022	**
IWKA	60	2.479	0.014	**
Kali & Salz	161	1.844	0.137	
Karstadt Quelle	limo	-	-	
Kloeckner-Werke	limo	-	-	
Linde	31	1.580	0.079	*
Lufthansa	limo	-	-	
MAN	limo	-	-	
Mannheimer	limo	-	-	
MG Technologies	184	1.260	0.289	
Mnchner Rck	limo	-	-	
Phoenix	20	1.554	0.057	*
Puma	80	2.212	0.045	**
Rheinmetall	193	1.038	0.369	
RWE	136	1.635	0.169	
Schering	160	1.233	0.299	
Siemens	30	2.186	0.007	***
Suedzucker	115	1.859	0.109	
Thyssen Krupp	58	0.873	0.546	
TUI	limo	-	-	
Volkswagen	158	1.7897	0.146	
Wella	limo	-	-	

Table 1: (b) Cross-validation bandwidths and approximate F-test for monthly German DAX100 returns

Is in addition considered, that for some returns the linear model seems suitable, the fittings show how important a flexible approach is.

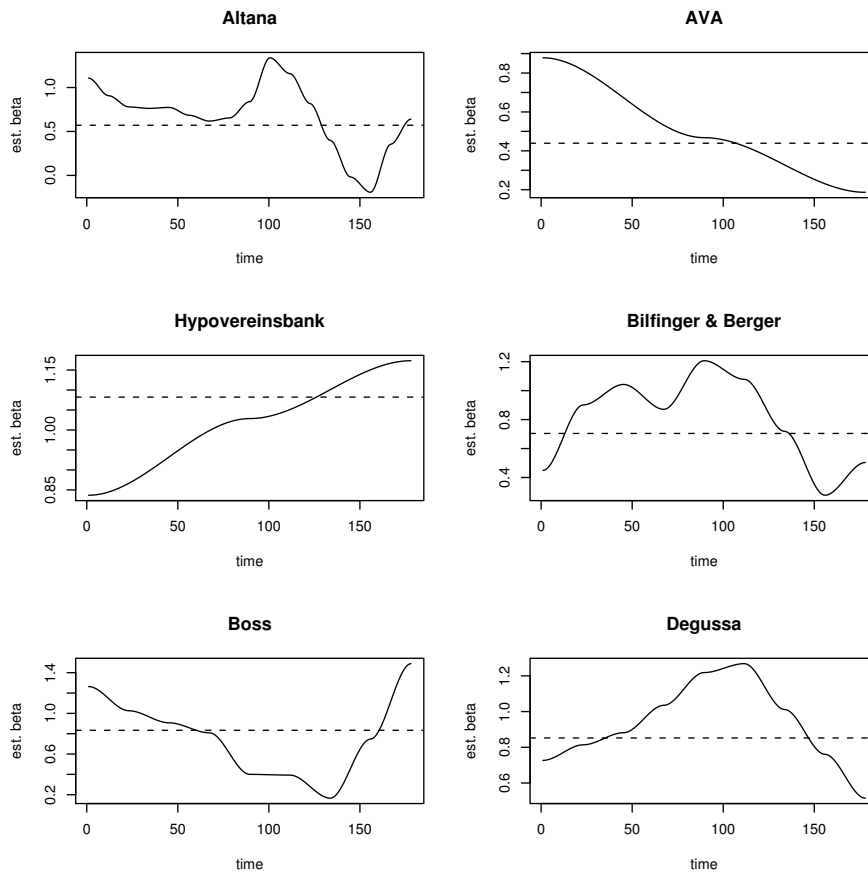


Figure 1: Estimated time-varying betas for selected German DAX100 returns (dotted line= constant OLS beta)

Return	Linear model				
	$\hat{\alpha}$	$\hat{\beta}$	R^2	$CV_1 \cdot 10^{-2}$	$CV_5 \cdot 10^{-2}$
Altana	0.0105	0.5706	0.1806	0.5481	0.5378
AVA	0.0048	0.4391	0.0912	0.7119	0.6306
Hypovereinsbank	-0.0074	1.0822	0.4762	0.4846	0.4688
Bilfinger & Berger	0.0007	0.7042	0.2124	0.6791	0.6993
Boss	0.009	0.8336	0.1967	1.0563	1.0924
Commerzbank	-0.0093	1.1299	0.5546	0.3888	0.3604
Continental	-0.0037	0.8271	0.3304	0.5126	0.4993
Degussa	-0.0026	0.8523	0.3662	0.4655	0.4539
Draegerwerke	-0.0017	0.6233	0.1801	0.6525	0.6525
Dyckerhoff	-0.0037	0.8502	0.2447	0.8294	0.7511
Eon	0.0034	0.5750	0.3116	0.2704	0.2646
Gehe	0.0104	0.3711	0.0858	0.5394	0.5316
Gildemeister	-0.0091	1.1999	0.2861	1.3207	1.3453
Henkel	0.0018	0.6430	0.3194	0.3256	0.3149
IVG Immobilien	0.0039	0.5300	0.1720	0.5003	0.5019
IWKA	-0.0035	0.8245	0.3479	0.4712	0.4764
Linde	-0.0015	0.7107	0.4264	0.2503	0.2464
Phoenix	-0.0042	0.7999	0.2923	0.5722	0.5704
Puma	0.0061	0.7134	0.1297	1.2639	1.2095
Siemens	-0.0012	1.3781	0.6737	0.3400	0.3271

Table 2: Linear regression results for selected German DAX100 returns

Return	Conditionally parametric model		
	R^2	$CV_1 \cdot 10^{-2}$	$CV_5 \cdot 10^{-2}$
Altana	0.3567	0.5173	0.4810
AVA	0.1552	0.6828	0.6285
Hypovereinsbank	0.5012	0.4728	0.4589
Bilfinger & Berger	0.3374	0.6355	0.6547
Boss	0.3054	1.0249	1.0652
Commerzbank	0.5977	0.3727	0.3516
Continental	0.4661	0.5128	0.5256
Degussa	0.4266	0.4557	0.4291
Draegerwerke	0.2661	0.6487	0.6121
Dyckerhoff	0.2956	0.8123	0.7083
Eon	0.3696	0.2571	0.2582
Gehe	0.1219	0.5294	0.5235
Gildemeister	0.3203	1.2763	1.2764
Henkel	0.3673	0.3156	0.3256
IVG Immobilien	0.3735	0.4869	0.5131
IWKA	0.4185	0.4422	0.4515
Linde	0.5056	0.2377	0.2413
Phoenix	0.4358	0.5523	0.5536
Puma	0.1916	1.2417	1.1969
Siemens	0.7348	0.3146	0.2983

Table 3: Conditionally parametric fit results for selected German DAX100 returns

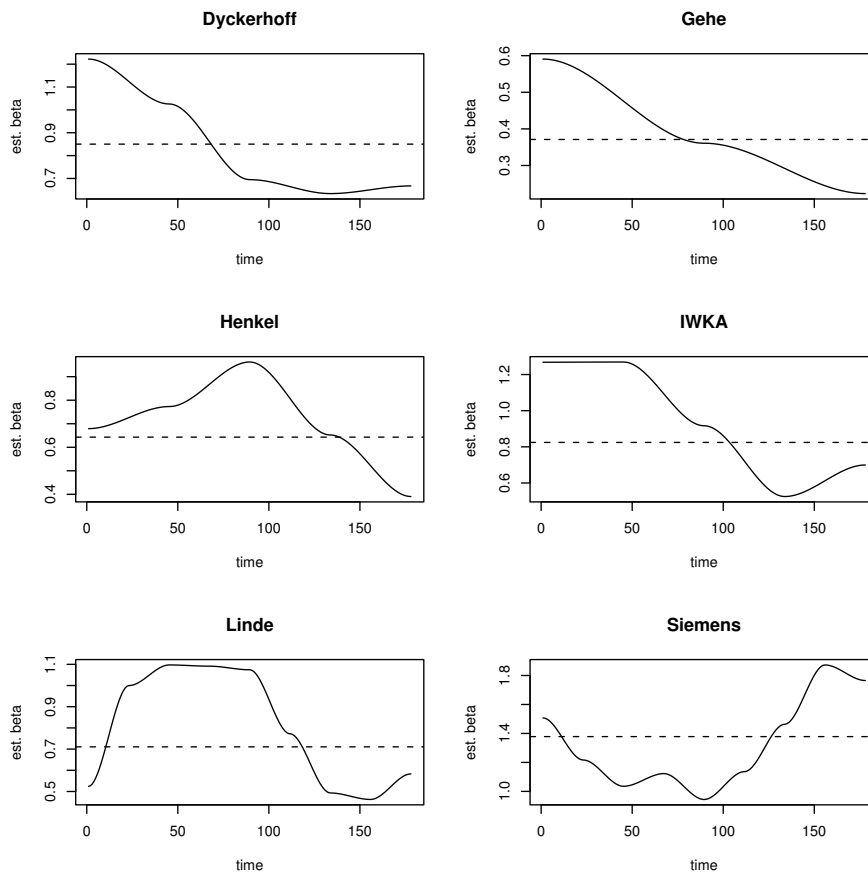


Figure 2: Estimated time-varying betas for selected German DAX100 returns (dotted line= constant OLS beta)

4 Conclusion

Many studies have shown that the CAPM beta for stock returns is time-varying. In this article nonparametric regression methods are used to estimate time-varying betas for German stock returns. Nonparametric regression estimators require the choice of a suitable smoothing parameter. Cross-validation is used for bandwidth choice here. Cross-validation is also a suitable method to capture the problem of overfitting. With the concept of approximate degree of freedom for linear smoothers tests can be calculated. Applied to German stock returns these test show that for some return series time varying-betas should be considered. The estimated beta functions reveal different courses. There arise very different behaviors of β for the examined returns. The courses range from constant over time, almost monoton to very variable.

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