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**Working Paper**

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Working Paper, No. 2004-10

**Provided in Cooperation with:**

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*Suggested Citation:* Occhino, Filippo (2004) : Market Segmentation and the 'Hump-Shaped' Response of Output to Monetary Policy Shocks, Working Paper, No. 2004-10, Rutgers University, Department of Economics, New Brunswick, NJ

This Version is available at:

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# Market Segmentation and the ‘Hump-Shaped’ Response of Output to Monetary Policy Shocks

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August 2004

## Abstract

In the data, after a contractionary monetary policy shock aggregate output decreases over time, with a trough after four to eight quarters. This paper replicates the ‘hump-shaped’ response of output with a segmented markets model where part of the households are excluded from financial markets. A contractionary monetary policy shock is modeled as an unanticipated increase in the short-term nominal interest rate. Since households and firms need cash-in-advance to purchase consumption and hire labor, an increase in the nominal interest rate discourages the households’ consumption demand and labor supply, and the firms’ labor demand. In a benchmark full participation model, the effect is strongest in the impact period, and decays over time. When markets are segmented, however, the shock has an additional liquidity effect, increasing the real interest rate above fundamentals, and decreasing the growth rate of the participants’ labor supply. As a result, the response of the aggregate labor and output has a trough several quarters after the shock. The model is able to replicate the sign, the magnitude and the persistence of the responses of output, money, prices and interest rates. It can generate a positive response of the real interest rate together with a negative response of the output growth rate.

*Keywords:* limited participation, segmented markets, hump-shaped delayed response, monetary policy shocks, persistence.

*JEL Classification Number:* E52.

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# 1 Introduction

After a contractionary monetary policy shock, aggregate output decreases over time, with a trough after four to eight quarters. This evidence seems robust to different ways of identifying monetary policy shocks, as documented by Christiano, Eichenbaum and Evans (1999)<sup>1</sup>. The delayed response of output to monetary policy is, also, central in the analysis by most monetary economists at central banks and academics. Replicating this fundamental robust feature, however, has proven to be challenging for existing representative agent monetary models, both sticky-price and limited participation.

This paper replicates the ‘hump-shaped’ response of output using a simple segmented markets model where part of the households are excluded from financial markets. The model departs from the limited participation framework of Lucas (1990), Fuerst (1992) and Christiano and Eichenbaum (1992) reintroducing the distributional effects of monetary policy. It is most closely related with the segmented markets models of Alvarez and Atkeson (1996), Alvarez, Lucas and Weber (2001), and Occhino (2004), although it differs from them because production is endogenous.

In the model, the only source of uncertainty are monetary policy shocks. The monetary policy variable is the short-term nominal interest rate, so monetary policy shocks are unanticipated increases in the interest rate. A reason why we choose an interest rate monetary policy is that Bernanke and Blinder (1992) show that the federal funds rate, an overnight interest rate, is an excellent indicator of the stance of monetary policy, and innovations to the federal funds rate can be identified with monetary policy shocks. More generally, monetary policy is defined in terms of a short-term nominal interest rate in most OECD countries. We assume that the interest rate follows an exogenous stochastic process such that the response of the interest rate itself to a monetary policy shock matches the empirical response. There is no need to specify an interest rate feedback rule since the private sector, which acts in a non-strategic way, makes its decisions only based on the expected evolution of the monetary policy variable, independently of the specific monetary policy feedback rule. With regard to seigniorage revenue, we assume that it is not rebated to the households, which we see as the most plausible assumption since we focus on the effects of monetary policy shocks for up to 2 years. This is the assumption which leads to equilibrium determinacy.

Monetary policy affects production in two ways. First, households need cash-in-advance to purchase consumption goods, and wages are received with delay. An increase in the nominal interest rate, then, increases the price of consumption in terms of leisure, and discourages the labor supply. Also, firms need cash-in-advance to hire labor, an assumption that follows Fuerst (1992) and the limited participation literature, so an increase in the nominal interest rate increases the cost of wages, and discourages the labor demand. An

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<sup>1</sup>The works of Gordon and Leeper (1994), Strongin (1995), Bernanke and Mihov (1998) and Leeper, Sims and Zha (1996) also support the claim, while Uhlig (2001) identifies a monetary policy shock with sign restrictions on the response of money, prices and interest rate, and finds that a contractionary monetary policy shock has no clear effect on real GDP.

increase in the nominal interest rate, then, decreases aggregate labor and output.

In a benchmark full participation model, the effect is strongest in the impact period and decays over time. When markets are segmented, however, the monetary policy shock has an additional liquidity effect, originally described by Grossman and Weiss (1983), and further studied by Lucas (1990), Fuerst (1992), Alvarez and Atkeson (1996) and Occhino (2004) among others. The real interest rate increases above fundamentals, so the growth rates of consumption and leisure of participating households increase, and the growth rate of their labor supply decreases. The higher the degree of market segmentation, the stronger and more persistent the liquidity effect. As a result of the composition of the general contractionary effect and the segmented markets liquidity effect, the trough of the equilibrium labor and output response occurs after several quarters.

The paper is organized as follows. Section 2 describes the economy and defines the competitive equilibrium, Section 3 explains the solution method, Section 4 describes and comments the results, and Section 5 concludes.

## 2 Economy

The model is a cash-in-advance production economy, with a large number of households, a large number of firms owned by households, and a monetary authority. The number of firms is normalized to one. Each household is composed by a worker-shopper pair, as in standard cash-in-advance production models. Time is discrete and is indexed by  $t \geq 0$ . There are a single non-durable consumption good produced by labor, money, and one-period nominal bonds, which are claims to one unit of money payable at the end of the period. Monetary policy shocks are the only source of uncertainty in the economy.

Households are of two types, traders and non-traders. Let  $\lambda \in (0, 1]$  and  $\lambda^* \equiv 1 - \lambda$  be respectively the proportion of traders and non-traders. Households of the same type are identical in all respects. The difference between the two types of households is that traders choose bond demand, consumption demand and labor supply optimally, while non-traders cannot purchase bonds, they spend all their initial cash balances purchasing consumption goods, and they supply a constant amount of labor. We will refer to the case where  $\lambda^* = 0$  and  $\lambda^* > 0$  respectively as the full participation model and the segmented markets model.

Households start each period with cash balances from the previous period. Then, three markets meet in sequence, the bond market, the labor market and the goods market.

In the bond market, the firms and the monetary authority sell respectively  $D_t$  and  $D_t^*$  one-period nominal bonds to the traders at the bond price  $q_t > 0$ . Open market operations are conducted in terms of the short-term nominal interest rate  $i_t > 0$  defined by

$$i_t \equiv -\log(q_t) \tag{1}$$

The monetary authority announces the bond price and stands ready to issue and sell any number of bonds to clear the market at that price. Monetary policy is, then, an exogenous

stochastic process for the interest rate, while the bond supply and the money supply are both determined endogenously.

Let  $i > 0$  be the non-stochastic steady state value of the nominal interest rate. The nominal interest rate follows the process

$$i_{t+1} - i = \rho(i_t - i) + \sigma\epsilon_{t+1} \quad (2)$$

where  $\rho \in [0, 1)$ ,  $\sigma > 0$ , and  $\epsilon_{t+1}$  is a monetary policy shock independently and identically distributed as standard normal.

After the bond market, the labor market meets where firms hire labor with the money borrowed in the bond market. This assumption follows Fuerst (1992) and the limited participation literature. The role that banks play in limited participation models is played by traders here. We assume that firms need to borrow a fraction  $\delta \in [0, 1]$  of the cost of wages. The case  $\delta = 1$  corresponds to Fuerst's assumption, while setting  $\delta = 0$  implies that firms do not need to borrow money to finance production. Firms hire labor from workers at the wage rate  $W_t > 0$ .

After the labor market, firms produce consumption goods using a linear technology in labor. Units are normalized so that  $N_t \geq 0$  denotes both the aggregate labor demand and the aggregate production. Firms, then, enter the goods market, and sell consumption goods to shoppers at the goods price  $P_t > 0$ . By assumption, the money earned by workers are not available for shoppers' purchases, which is a standard assumption in cash-in-advance production models. The money supply

$$M_t \equiv P_t N_t \quad (3)$$

is the amount of dollars spent in the goods market.

Finally, after the goods market, households consume the goods purchased by shoppers, workers bring their wages back to the household, firms redeem their bonds and distribute profits to the households, and the monetary authority redeems its bonds. Since the production technology is linear, equilibrium profits are zero, and the equilibrium does not depend on the firms' ownership. For notational convenience, then, we assume that profits are destroyed rather than distributed to households.

Each trader chooses contingent sequences of consumption demand  $C_t$ , labor supply  $L_t$ , bond demand  $B_t$  and next-period cash balances  $A_{t+1}$ , to solve

$$\begin{aligned} & \max_{\{C_t > 0, L_t > 0, B_t, A_{t+1} > 0\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\alpha}}{1-\alpha} - \Phi \frac{L_t^{1+\varphi}}{1+\varphi} \right) \right] \\ & \text{subject to:} \\ & q_t B_t + P_t C_t = A_t \\ & A_{t+1} = W_t L_t + B_t \end{aligned} \quad (4)$$

given the trader's initial cash balances  $A_0 > 0$  in period zero, where  $E_0$  is the expectation conditional on information available after the monetary policy shock  $\epsilon_0$  has been revealed,

$\beta \in (0, 1)$ ,  $\alpha > 0$ ,  $\Phi > 0$ , and  $\varphi > 0$ . The case  $\alpha = 1$  is interpreted as the logarithmic utility case.

The necessary first-order conditions for the traders' optimization problem are

$$\begin{aligned}\beta^t C_t^{-\alpha} - \nu_t^1 P_t &= 0 \\ -\beta^t \Phi L_t^\varphi + \nu_t^2 W_t &= 0 \\ -\nu_t^1 q_t + \nu_t^2 &= 0 \\ -\nu_t^2 + E_t[\nu_{t+1}^1] &= 0\end{aligned}$$

and the transversality condition is

$$\lim_{t \rightarrow \infty} E_0[\nu_t^1 A_t] = 0$$

where  $\nu_t^1$  and  $\nu_t^2$  are the Lagrange multipliers associated with the two constraints in the traders' problem (4). It follows that

$$\begin{aligned}\beta^t C_t^{-\alpha} &= \nu_t^1 P_t \\ \beta^t \Phi L_t^\varphi &= \nu_t^1 q_t W_t \\ q_t \nu_t^1 &= E_t[\nu_{t+1}^1]\end{aligned}\tag{5}$$

and

$$\lim_{t \rightarrow \infty} E_0[\nu_t^1 A_t] = 0\tag{6}$$

Non-traders do not optimize. They spend all their initial cash balances purchasing consumption goods, and they supply a constant amount of labor  $L^* > 0$ . The behavior of a non-trader is described by the constraints

$$\begin{aligned}P_t C_t^* &= A_t^* \\ A_{t+1}^* &= W_t L^*\end{aligned}\tag{7}$$

given the non-trader's initial cash balances  $A_0^* > 0$  in period zero<sup>2</sup>.

Firms choose labor demand  $N_t$  and bonds supply  $D_t$  to solve the profits maximization problem

$$\begin{aligned}\max_{\{N_t \geq 0, D_t\}} & P_t N_t - W_t N_t - (1 - q_t) D_t \\ \text{subject to:} & \\ & q_t D_t = \delta W_t N_t\end{aligned}\tag{9}$$

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<sup>2</sup>Alternatively, one could restrict the relative risk aversion  $\alpha$  to be one, and assume that the non-traders solve the same problem as the traders except that they cannot purchase bonds. They would then solve

$$\begin{aligned}\max_{\{C_t^* > 0, L_t^* > 0, A_{t+1}^* > 0\}_{t=0}^\infty} & E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \log(C_t^*) - \Phi \frac{L_t^{*1+\varphi}}{1+\varphi} \right) \right] \\ \text{subject to:} & \\ & P_t C_t^* = A_t^* \\ & A_{t+1}^* = W_t L_t^*\end{aligned}\tag{8}$$

given the trader's initial cash balances  $A_0^* > 0$  in period zero. In this case, from the necessary first-order

The zero-profit condition for the firms' optimization problem is

$$\begin{aligned}
P_t N_t - W_t N_t - (1 - q_t) D_t &= 0 \\
P_t N_t - W_t N_t - \frac{1 - q_t}{q_t} \delta W_t N_t &= 0 \\
P_t &= \left(1 + \frac{1 - q_t}{q_t} \delta\right) W_t \approx W_t / q_t^\delta
\end{aligned} \tag{10}$$

where the approximation follows from

$$1 + \frac{1 - q_t}{q_t} \delta \approx \left(1 + \frac{1 - q_t}{q_t}\right)^\delta = \left(\frac{1}{q_t}\right)^\delta$$

The approximation is exact both for  $\delta = 0$  and  $\delta = 1$ .

The bond market, labor market and goods market equilibrium conditions are

$$\begin{aligned}
D_t + D_t^* &= \lambda B_t \\
\lambda L_t + \lambda^* L^* &= N_t \\
\lambda C_t + \lambda^* C_t^* &= N_t
\end{aligned} \tag{11}$$

Let the traders' initial assets  $A_0$ , the non-traders initial assets  $A_0^*$  and the values of all variables dated at periods earlier than zero be given. Histories are made of all possible realizations of the monetary policy shock  $\epsilon_t$ . An equilibrium is a set of contingent sequences of: consumption, labor supply, bond demand and cash balances for traders  $\{C_t > 0, L_t > 0, B_t, A_{t+1} > 0\}_{t=0}^\infty$ ; consumption, and cash balances for non-traders  $\{C_t^* > 0, A_{t+1}^* > 0\}_{t=0}^\infty$ ; labor demand and bond supply for firms  $\{N_t \geq 0, D_t\}_{t=0}^\infty$ ; bond supply for the monetary authority  $\{D_t^*\}_{t=0}^\infty$ ; and prices  $\{q_t > 0, i_t > 0, W_t > 0, P_t > 0\}_{t=0}^\infty$ ; satisfying the identity (1), the process (2), the traders' first-order conditions (5), the constraints in the traders' problem (4), the non-traders' constraints (7), the firms' zero profit condition (10), the constraint in the firms' problem (9), and the equilibrium conditions (11).

We finally define the inflation rate  $\pi_t$  and the real interest rate  $r_t$  by

$$\begin{aligned}
\pi_{t+1} &\equiv \log(P_{t+1}) - \log(P_t) \\
r_t &\equiv i_t - E_t[\pi_{t+1}]
\end{aligned} \tag{12}$$

and we define the money growth rate and the wage growth rate similarly to the inflation rate.

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conditions and the two constraints, it would follow that

$$\begin{aligned}
\Phi L_t^{*\varphi} / W_t &= E_t[\beta / C_{t+1} P_{t+1}] \\
\Phi L_t^{*\varphi} / W_t &= \beta / A_{t+1}^* \\
\Phi L_t^{*\varphi} / W_t &= \beta / W_t L_t^* \\
L_t^* &= (\beta / \Phi)^{1/(1+\varphi)}
\end{aligned}$$

so the non-traders would optimally choose to supply a constant amount of labor, and the analysis in this paper would apply. This is particularly noteworthy since all the numerical experiments in this paper are conducted for the case of  $\alpha = 1$ .

### 3 Solution

As in Lucas (1990), nominal variables are normalized by the aggregate cash balances  $\bar{A}_t \equiv \lambda A_t + \lambda^* A_t^*$  available at the beginning of the period. Also, we introduce aggregate variables for each of the two types of households, and we normalize variables as follows:  $\nu_t \equiv \nu_t^1 \bar{A}_t / \beta^t$ ,  $c_t \equiv \lambda C_t$ ,  $l_t \equiv \lambda L_t$ ,  $b_t \equiv \lambda B_t / \bar{A}_t$ ,  $a_t \equiv \lambda A_t / \bar{A}_t$ ,  $c_t^* \equiv \lambda^* C_t^*$ ,  $l^* \equiv \lambda^* L^*$ ,  $a_t^* \equiv \lambda^* A_t^* / \bar{A}_t$ ,  $n_t \equiv N_t$ ,  $d_t \equiv D_t / \bar{A}_t$ ,  $d_t^* \equiv D_t^* / \bar{A}_t$ ,  $w_t \equiv W_t / \bar{A}_t$ ,  $p_t \equiv P_t / \bar{A}_t$ ,  $m_t \equiv M_t / \bar{A}_t$ , and  $\gamma_t \equiv \bar{A}_{t+1} / \bar{A}_t$ .

The system describing the equilibrium can then be written as

$$i_{t+1} - i = \rho(i_t - i) + \sigma \epsilon_{t+1} \quad (13a)$$

$$i_t \equiv -\log(q_t) \quad (13b)$$

$$m_t \equiv p_t n_t \quad (13c)$$

$$\left(\frac{c_t}{\lambda}\right)^{-\alpha} = \nu_t p_t \quad (13d)$$

$$\Phi\left(\frac{l_t}{\lambda}\right)^\varphi = \nu_t q_t w_t \quad (13e)$$

$$q_t \gamma_t \nu_t = \beta E_t[\nu_{t+1}] \quad (13f)$$

$$q_t b_t + p_t c_t = a_t \quad (13g)$$

$$\gamma_t a_{t+1} = w_t l_t + b_t \quad (13h)$$

$$p_t c_t^* = a_t^* \quad (13i)$$

$$\gamma_t a_{t+1}^* = w_t l^* \quad (13j)$$

$$p_t = \left(1 + \frac{1 - q_t}{q_t} \delta\right) w_t \approx w_t / q_t^\delta \quad (13k)$$

$$q_t d_t = \delta w_t n_t \quad (13l)$$

$$d_t + d_t^* = b_t + b_t^* \quad (13m)$$

$$l_t + l^* = n_t \quad (13n)$$

$$c_t + c_t^* = n_t \quad (13o)$$

$$a_t + a_t^* = 1 \quad (13p)$$

The transversality condition (2) can be written as

$$\lim_{t \rightarrow \infty} E_0[\beta^t \nu_t a_t] = 0 \quad (14)$$

Notice that the system is the same as in an economy with one trader, one non-trader and one firm, with different preferences parameters.

It is now convenient to substitute the traders' budget constraint (13h) with the equation derived as follows. From the households' cash-in-advance constraints (13g) and (13i), and from the households' budget constraints (13h) and (13j), it follows that

$$\begin{aligned} q_t b_t + p_t c_t + p_t c_t^* &= a_t + a_t^* \\ q_t (\gamma_t a_{t+1} - w_t l_t + \gamma_t a_{t+1}^* - w_t l^*) + p_t c_t + p_t c_t^* &= a_t + a_t^* \end{aligned}$$



Then, using the identity (13p),

$$q_t \gamma_t - q_t w_t (l_t + l^*) + p_t (c_t + c_t^*) = 1$$

Finally, using the labor and goods markets equilibrium conditions (13n) and (13o), and the zero-profit condition (13k)

$$\begin{aligned} q_t \gamma_t - q_t w_t n_t + p_t n_t &= 1 \\ q_t \gamma_t - q_t w_t n_t + p_t n_t - q_t p_t n_t + [q_t + (1 - q_t) \delta] w_t n_t &= 1 \\ q_t \gamma_t + (1 - q_t) (p_t n_t + \delta w_t n_t) &= 1 \end{aligned}$$

which we use in place of the traders' budget constraint (13h) in the previous system (13).

In the non-stochastic steady state, all normalized variables are constant over time. Given the non-stochastic steady state value of the nominal interest rate  $i$ , the non-stochastic steady state can be computed numerically from the system

$$\begin{aligned} i &\equiv -\log(q) \\ m &\equiv pn \\ \left(\frac{c}{\lambda}\right)^{-\alpha} &= \nu p \\ \Phi \left(\frac{l}{\lambda}\right)^\varphi &= \nu qw \\ q\gamma &= \beta \\ qb + pc &= a \\ q\gamma + (1 - q)(pn + \delta wn) &= 1 \\ pc^* &= a^* \\ \gamma a^* &= wl^* \\ p &= \left(1 + \frac{1 - q}{q} \delta\right) w \approx w/q^\delta \\ qd &= \delta wn \\ d + d^* &= b \\ l + l^* &= n \\ c + c^* &= n \\ a + a^* &= 1 \end{aligned} \tag{15}$$

where the variables without the time subscript are the non-stochastic steady state values. Notice that, since  $\beta \in (0, 1)$ , the transversality condition for the traders' optimization problem is satisfied in the non-stochastic steady state.

Log-linearizing the system around the non-stochastic steady state yields

$$\hat{i}_{t+1} = \rho \hat{i}_t + \sigma \epsilon_{t+1} \quad (16a)$$

$$\hat{i}_t \equiv -\hat{q}_t \quad (16b)$$

$$\hat{m}_t \equiv \hat{p}_t + \hat{n}_t \quad (16c)$$

$$-\alpha \hat{c}_t = \hat{v}_t + \hat{p}_t \quad (16d)$$

$$\varphi \hat{l}_t = \hat{v}_t + \hat{q}_t + \hat{w}_t \quad (16e)$$

$$\hat{q}_t + \hat{\gamma}_t + \hat{v}_t = E_t[\hat{v}_{t+1}] \quad (16f)$$

$$qb(\hat{q}_t + \hat{b}_t) + pc(\hat{p}_t + \hat{c}_t) = a\hat{a}_t \quad (16g)$$

$$q\gamma(\hat{q}_t + \hat{\gamma}_t) + (1-q)pn \left( -\frac{q}{1-q} \hat{q}_t + \hat{p}_t + \hat{n}_t \right) + (1-q)\delta wn \left( -\frac{q}{1-q} \hat{q}_t + \hat{w}_t + \hat{n}_t \right) = 0 \quad (16h)$$

$$\hat{p}_t + \hat{c}_t^* = \hat{a}_t^* \quad (16i)$$

$$\hat{\gamma}_t + \hat{a}_{t+1}^* = \hat{w}_t \quad (16j)$$

$$\hat{p}_t = \hat{w}_t - \frac{\delta}{q + (1-q)\delta} \hat{q}_t \approx \hat{w}_t - \delta \hat{q}_t \quad (16k)$$

$$\hat{q}_t + \hat{d}_t = \hat{w}_t + \hat{n}_t \quad (16l)$$

$$d\hat{d}_t + d^* \hat{d}_t^* = b\hat{b}_t \quad (16m)$$

$$l\hat{l}_t = n\hat{n}_t \quad (16n)$$

$$c\hat{c}_t + c^* \hat{c}_t^* = n\hat{n}_t \quad (16o)$$

$$a\hat{a}_t + a^* \hat{a}_t^* = 0 \quad (16p)$$

where the variables without the time subscript are the non-stochastic steady state values, while the variables with the hat are the percentage deviations from the steady state values, except for  $\hat{i}_t \equiv i_t - i$ .

Using the system (16), we can express all variables in terms of the exogenous state variable  $\hat{i}_t$ , the endogenous state variable  $\hat{a}_t^*$ , and the control variable  $\hat{v}_t$ , so the system (16) can be reduced to a first-order system of difference equations in the two state variables and the control variable. With standard methods, we then derive the control variable  $\hat{v}_t$  as an equilibrium function of the two state variables and determine the equilibrium evolution of the system<sup>3</sup>.

The equilibrium is determinate even though monetary policy is an exogenous process for the interest rate. To see why, it is best to consider the full participation economy where all

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<sup>3</sup>The solution method is based on the eigenvalue decomposition of the matrix describing the evolution of the state and control variables. In addition, the model has been solved using MATLAB files written by Chris Sims and Paul Klein. We thank them for making the files available at the web address <http://www.ssc.uwo.ca/economics/faculty/klein/>. Their solution method is based on the Schur decomposition of the matrix describing the evolution of the state and control variables. The two methods yield identical solutions.

households are traders. In this case, the real allocation is determinate and the only issue regards whether nominal variables are determined. Recall that taxes and transfers are equal to zero. Given the traders' initial level of nominal assets, and the real variables stochastic processes, the traders' intertemporal budget constraint pins down the initial price level. Numerical methods show that the equilibrium is determinate in the segmented markets case as well, although the previous argument does not apply directly since changes in the money supply can affect the real allocation between traders and non-traders. The assumption that the fiscal variables do not adjust to balance the intertemporal budget constraint, so the fiscal policy is *active* in the terminology of Leeper (1991), plays an important role in the argument. In models where the equilibrium is indeterminate, in each period, the revenue from seigniorage is rebated to the households through lump-sum transfers. Equivalently, money is introduced into the economy through helicopter-drops. In these models, then, transfers are a function of open-market operations, the intertemporal budget constraint can hold for several possible path of money, prices and seigniorage, and the initial price level cannot be determined. Since we focus on the effects of monetary policy shocks for up to 2 years, we see the assumption that the fiscal variables are exogenous as far more plausible than assuming that they depend on current open market operations.

After solving the model, we can derive the following rates. From the definitions of the inflation rate and the real interest rate,

$$\begin{aligned}\pi_{t+1} &\equiv \log(p_{t+1}) + \log(\gamma_t) - \log(p_t) \\ r_t &\equiv i_t - E_t[\pi_{t+1}]\end{aligned}$$

Their non-stochastic steady state values are given by

$$\begin{aligned}\pi &\equiv \log(\gamma) \\ r &\equiv i - \pi\end{aligned}$$

and their linear approximations by

$$\begin{aligned}\hat{\pi}_{t+1} &\equiv \hat{p}_{t+1} + \hat{\gamma}_t - \hat{p}_t \\ \hat{r}_t &\equiv \hat{i}_t - E_t[\hat{\pi}_{t+1}]\end{aligned}\tag{17}$$

where  $\hat{\pi}_t \equiv \pi_t - \pi$  and  $\hat{r}_t \equiv r_t - r$ . The linear approximations of the money growth rate and the wage growth rate are derived similarly to the inflation rate.

Before turning to the analysis of the effects of a monetary policy shock, let us derive a more intuitive version of the traders' Euler equation. From the traders' first-order condition for consumption (16d) and the traders' Euler equation (16f), it follows that

$$\begin{aligned}\hat{q}_t + E_t[\hat{p}_{t+1}] + \hat{\gamma}_t - \hat{p}_t &= -\alpha E_t[\hat{c}_{t+1} - \hat{c}_t] \\ \hat{q}_t + E_t[\hat{\pi}_{t+1}] &= -\alpha E_t[\hat{c}_{t+1} - \hat{c}_t]\end{aligned}$$

where the last equation follows from the inflation rate definition in (17). Then, using the interest rate definition (16b) and the real interest rate definition in (17), the following version

of the traders' Euler equation follows

$$\begin{aligned}\hat{i}_t - E_t[\hat{\pi}_{t+1}] &= \alpha E_t[\hat{c}_{t+1} - \hat{c}_t] \\ \hat{r}_t &= \alpha E_t[\hat{c}_{t+1} - \hat{c}_t]\end{aligned}\tag{18}$$

which implies that the equilibrium real interest rate is equal to the relative risk aversion  $\alpha$  times the traders' expected consumption growth rate.

## 4 Results

We now turn to the expected response of the economy to a contractionary monetary policy shock which increases the nominal interest rate by  $\sigma$ . Since the system is linear, we equivalently study the deterministic evolution of the economy in the case that  $\epsilon_0 = 1$  and  $\epsilon_t = 0$  for all  $t > 0$ . We set the initial values of the state variables  $\hat{i}_{-1}$  and  $\hat{a}_0^*$  equal to zero. Then, from the interest rate process (16a), the deviation of the nominal interest rate  $\hat{i}_t$  from steady state follows the exogenous evolution  $\hat{i}_t = \sigma \rho^t$ . To gain intuition, we consider in sequence the four cases of full and limited participation with constant labor supply, and full and limited participation with endogenous labor supply.

As in Cooley and Hansen (1989) model, the length of a period plays an important role in the model because of the presence of cash-in-advance constraints. As explained below, the longer the length of the period, the larger the impact of a nominal interest rate increase on labor and production. We set one period equal to one quarter, but we show what happens as the length increases up to one year. The average interest rate  $i$  is set to  $6.14\%/4$ , to match the average federal funds rate<sup>4</sup>. The preferences discount rate  $-\log(\beta)$  is set to  $2.38\%/4$ , so the average inflation rate  $\pi = i + \log(\beta)$  takes the value  $3.76\%/4$ , and matches the average inflation rate of the GDP price deflator. The quarterly first-order autocorrelation  $\rho$  of the nominal interest rate is set to 0.84, which implies a half-life of one year and is in the range of typical estimates of the interest rate smoothing parameter (see Clarida, Gali' and Gertler (1999), page 1687–1688)<sup>5</sup>. The benchmark values for the other parameters are the following. The relative risk aversion  $\alpha$  and the inverse of the labor supply elasticity  $\varphi$  are both equal to 1. We show, however, what happens as the labor supply elasticity increases. The fraction of wages  $\delta$  that firms must hold in cash is 1. The constant labor supply  $L^*$  of one non-trader is normalized to 1. The parameter  $\Phi$  in the traders' preferences is set so that the steady state labor supply of one trader is equal to the constant labor supply of one non-trader. Then, the proportion of traders  $\lambda$  is also the traders' share of aggregate labor supply  $l/(l + l^*)$ . The analysis below focuses on how the market segmentation parameter

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<sup>4</sup>Data are for the period 1955:I–1999:IV, from the FRED II database of the Federal Reserve Bank of St. Louis.

<sup>5</sup>If one estimates the interest rate process (16a) directly from data on the federal funds rate, the first-order autocorrelation  $\rho$  is 0.92, and the conditional standard deviation  $\sigma$  is 1.29. Using the H-P filtered series,  $\rho$  is 0.71, and  $\sigma$  is 1.19.

$\lambda$  affects the economy's response to a monetary shock. In related work, Landon-Lane and Occhino (2004) have estimated an endowment segmented markets economy with data on the money growth rate and the inflation rate, and have obtained a maximum likelihood estimate of  $\lambda$  at 0.13. We then set  $\lambda = 0.1$  as benchmark value for the segmented markets economy.

## 4.1 Constant labor supply

Let us start considering the economy where the traders' labor supply  $l_t$  is held constant at the value  $l$ , so  $\hat{l}_t = 0$ . The equilibrium is still described by the system (16) without the traders' first-order condition for the labor supply (16e). Equivalently, the equilibrium is described by the system (16) with  $\varphi \rightarrow \infty$ , so the traders' labor supply elasticity tends to zero.

Using the labor market equilibrium condition (16n), it follows that  $\hat{n}_t = 0$ , the aggregate labor demand and the aggregate production are both constant in equilibrium. Then, from the money definition equation (16c),  $\hat{m}_t = \hat{p}_t$ , money is proportional to prices.

In the case of  $\delta = 0$ , the firms' zero-profit condition (16k) implies  $\hat{p}_t = \hat{w}_t$ , and the model effectively becomes the benchmark segmented markets model with constant endowment of Alvarez and Atkeson (1996), Alvarez, Lucas and Weber (2001) and Occhino (2004), except for the specification of the monetary policy.

### 4.1.1 Full participation

In the full participation case, all households are traders and  $c^* = 0$ . Then, the goods market equilibrium condition (16o) implies that  $\hat{c}_t = 0$ , the traders' consumption is constant. From the traders' Euler equation (18),

$$\begin{aligned}\hat{i}_t &= E_t[\hat{\pi}_{t+1}] \\ \hat{r}_t &= 0\end{aligned}$$

deviations of the nominal interest rate and the expected inflation rate are the same, and the real interest rate is constant.

In the full participation case, the evolution of the system does not depend on the endogenous state variable  $\hat{a}_t^*$ . In the case that  $\delta = 0$ , the response of the economy to a monetary policy shock can be derived analytically. Although the assumption of full participation is needed, the assumptions of constant labor supply and  $\delta = 0$  could probably be relaxed. We briefly present the solution because it is instructive and allows to verify analytically the determinacy of the equilibrium in the simplest case.

Since  $\hat{c}_t = 0$ , the traders' first-order condition for consumption (16d) implies that  $\hat{\nu}_t = -\hat{p}_t$ . The traders' Euler equation (16f) is then

$$E_t[\hat{p}_{t+1}] = \hat{p}_t - \hat{q}_t - \hat{\gamma}_t$$

Since  $\delta = 0$ , the equation (16h) implies

$$q\gamma(\hat{q}_t + \hat{\gamma}_t) + (1 - q)pn \left( -\frac{q}{1 - q}\hat{q}_t + \hat{p}_t \right) = 0$$

$$\hat{\gamma}_t + \hat{q}_t = \frac{(1 - q)pn}{q\gamma} \left( \frac{q}{1 - q}\hat{q}_t - \hat{p}_t \right)$$

Plugging the expression for  $\hat{\gamma}_t + \hat{q}_t$  into the traders' Euler equation, we obtain the following first-order difference equation in  $E_t[\hat{p}_t]$ :

$$E_t[\hat{p}_{t+1}] = \hat{p}_t - \frac{(1 - q)pn}{q\gamma} \left( \frac{q}{1 - q}\hat{q}_t - \hat{p}_t \right)$$

$$E_t[\hat{p}_{t+1}] = \left( 1 + \frac{(1 - q)pn}{q\gamma} \right) \hat{p}_t - \frac{pn}{\gamma}\hat{q}_t$$

The equation has an infinite number of solutions, depending on the initial value  $\hat{p}_0$ . However, since the coefficient of  $\hat{p}_t$  is strictly greater than one and the coefficient of  $E_t[\hat{p}_{t+1}]$  is equal to one, only one solution is stable.

Recall that  $\hat{i}_t = \sigma\rho^t$ , so, using the bond price identity (16b),  $\hat{q}_t = -\sigma\rho^t$ . The particular solution can be easily found guessing that  $E_t[\hat{p}_t] = K\rho^t$  for some constant  $K$ , and verifying the guess. Plugging the guess into the first-order difference equation,

$$K\rho^{t+1} = \left( 1 + \frac{(1 - q)pn}{q\gamma} \right) K\rho^t + \frac{pn}{\gamma}\sigma\rho^t$$

$$K = -\frac{pn}{\gamma}\sigma \left/ \left( 1 - \rho + \frac{(1 - q)pn}{q\gamma} \right) \right. < 0$$

Since  $K$  is negative,  $\hat{p}_t$  has the same dynamics as  $\hat{q}_t$ . Since  $\hat{i}_t \equiv -\hat{q}_t$ , prices (together with money and wages) decrease when the nominal interest rate increases. Once the price level has been determined, all the other variables can be derived. Notice that they all have the same functional form as  $\hat{q}_t$ , with different coefficients.

In summary, in the full participation economy with constant labor supply, money and prices decrease in the impact period of a contractionary shock. In the following periods, however, both the money growth rate and the inflation rate increase. The real interest rate does not respond. The pattern of these responses can be observed in Figure 1 which plots the response of the full participation economy with constant labor supply to a contractionary monetary policy shock which increases the nominal interest rate by one percent. The two cases of  $\delta = 0$  and  $\delta = 1$  differ only with regard to the response of the wage growth rate, but the difference is quantitatively and qualitatively insignificant.

#### 4.1.2 Segmented markets

In the case of market segmentation, the behavior of the economy depends both on the exogenous state variable  $\hat{i}_t$  and the endogenous state variable  $\hat{a}_t^*$ . In this case, we cannot

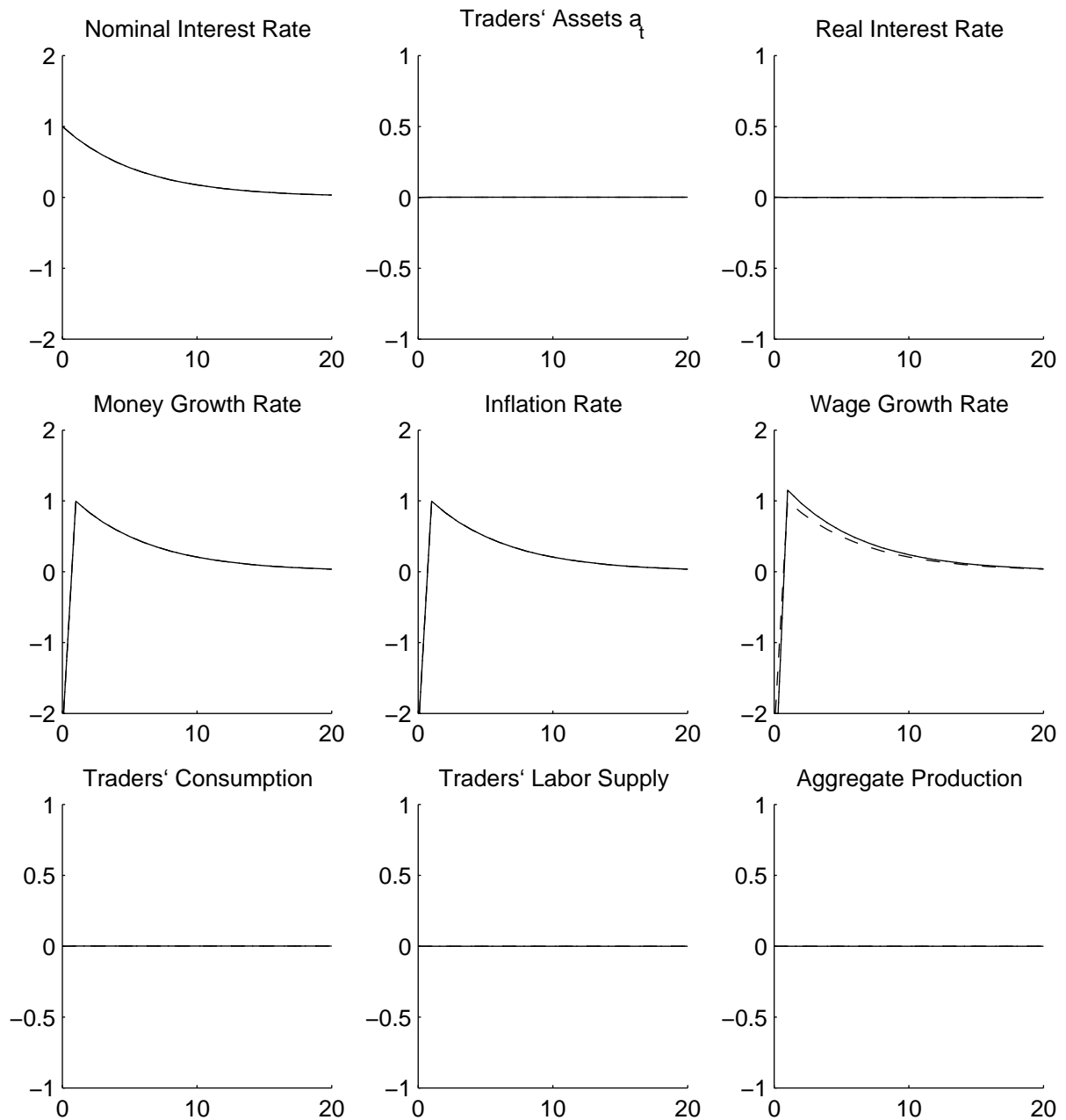


Figure 1: Response of the full participation economy ( $\lambda = 1$ ) with constant labor supply to a contractionary monetary policy shock.

The dashed and solid lines refer respectively to  $\delta = 0$  and  $\delta = 1$ .

One period is one quarter. The five rates are expressed in annual percentage points. The other variables are percent logarithms.

find an analytical expression for the equilibrium price level, and we cannot fully characterize analytically the response of the economy to a monetary policy shock. However, the analysis of the full participation case suggests that, at least for moderate levels of market segmentation, money and prices still decrease after a contractionary monetary policy shock, that is the sign of  $\hat{n}_t$  and  $\hat{p}_t$  is negative while the sign of  $\hat{i}_t$  is positive in the impact period of the shock. This is what happens in all the numerical simulations presented in this paper. In this case, several important analytical results follow.

From the non-traders' constraints (16i), the goods market equilibrium condition (16o), and  $\hat{n}_t = 0$ , it follows that

$$\hat{c}_t = \frac{c^*}{c}(\hat{p}_t - \hat{a}_t^*)$$

so the traders' consumption decreases with money and prices in the impact period of a contractionary monetary policy shock. A monetary policy shock, then, affects the distribution of cash balances and consumption between traders and non-traders. This effect, first described by Grossman and Weiss (1983), is crucially present in all limited participation economies.

In turn, changes in the traders' consumption affect the real interest rate. From the traders' Euler equation (18), it follows that

$$\begin{aligned}\hat{r}_t &= \alpha E_t[\hat{c}_{t+1} - \hat{c}_t] \\ \hat{r}_t &= \frac{\alpha c^*}{c} E_t[\hat{p}_{t+1} - \hat{a}_{t+1}^* - \hat{p}_t + \hat{a}_t^*] \\ \hat{r}_t &= \frac{\alpha c^*}{c} E_t[\hat{p}_{t+1} - \hat{w}_t + \hat{\gamma}_t - \hat{p}_t + \hat{a}_t^*] \\ \hat{r}_t &= \frac{\alpha c^*}{c} E_t[\hat{\pi}_{t+1} - \hat{p}_t + \hat{a}_t^*]\end{aligned}$$

where the second equation follows from the previous result, the third equation from the non-traders' budget constraint (16j), and the fourth equation from the zero profit condition (16k), and from the inflation rate definition in (17). Then, using the real interest rate definition in (17),

$$\begin{aligned}\hat{r}_t &= \frac{\alpha c^*}{c}(\hat{i}_t - \hat{r}_t - \hat{p}_t + \hat{a}_t^*) \\ \left(1 + \frac{\alpha c^*}{c}\right)\hat{r}_t &= \frac{\alpha c^*}{c}(\hat{i}_t - \hat{p}_t + \hat{a}_t^*) \\ \hat{r}_t &= \frac{\alpha c^*}{c + \alpha c^*}(\hat{i}_t - \hat{p}_t + \hat{a}_t^*)\end{aligned}$$

so the real interest rate increases in the impact period of a contractionary monetary policy shock. The magnitude of this effect increases both with market segmentation through  $c^*$  and with the relative risk aversion  $\alpha$ , as previously pointed out by Alvarez and Atkeson (1996).

Finally, market segmentation introduces endogenous persistence into the dynamics of the



model. For all periods  $t > 0$ ,

$$\hat{r}_t = \frac{\alpha c^*}{c + \alpha c^*} (\hat{i}_t - \hat{p}_t + \hat{a}_t^*)$$

$$\hat{r}_t = \frac{\alpha c^*}{c + \alpha c^*} (\hat{i}_t - \hat{\pi}_t)$$

where the last equation follows from the non-traders' budget constraint (16j), the zero profit condition (16k), and the inflation rate definition in (17). Then, substituting the expression for the real interest rate from the last equation into the real interest rate definition in (17),

$$E_t[\hat{\pi}_{t+1}] = \hat{i}_t - \hat{r}_t$$

$$E_t[\hat{\pi}_{t+1}] = \hat{i}_t - \frac{\alpha c^*}{c + \alpha c^*} (\hat{i}_t - \hat{\pi}_t)$$

$$E_t[\hat{\pi}_{t+1}] = \frac{c}{c + \alpha c^*} \hat{i}_t + \frac{\alpha c^*}{c + \alpha c^*} \hat{\pi}_t$$

so the expected inflation rate is a weighted average of the nominal interest rate and current inflation. The weight on current inflation increases both with market segmentation and with the relative risk aversion. Given the nominal interest rate process, the persistence of the inflation rate is passed on the other processes.

All the previous effects, which occur only when markets are segmented, are visible in Figure 2 which plots the response of the segmented markets economy with constant labor supply to a contractionary monetary policy shock. A contractionary monetary policy shock, identified as an exogenous increase in the nominal interest rate, decreases money and prices in the impact period. In the segmented markets economy, it has an additional liquidity effect, originally described by Grossman and Weiss (1983), and then by Lucas (1990), Fuerst (1992), Alvarez and Atkeson (1996), and Occhino (2004). It affects the distribution of cash balances and consumption between traders and non-traders, increasing the slope of the traders' consumption response and thereby increasing the real interest rate. Since the nominal interest rate is exogenous, the Fisher equation implies that the expected inflation rate is lower than in the full participation economy. If markets are segmented enough, the liquidity effect is so strong that the expected inflation rate decreases. The money growth rate behaves as the inflation rate. The mechanism, whereby low current money and prices lead to low future money and prices, adds endogenous persistence in the model. The behavior of the endogenous state variable  $\hat{a}_t^*$  reveals that the distributional effects of monetary policy play a crucial role in propagating the effect of the shock beyond the impact period.

## 4.2 Endogenous labor supply

Let us turn, now, to the general case where the labor supply is endogenous. In this case, a contractionary monetary policy shock has two effects on the traders' labor supply. First, it discourages the traders' labor supply by increasing the relative price of consumption relative to leisure. Second, when markets are segmented, it increases the real interest rate and

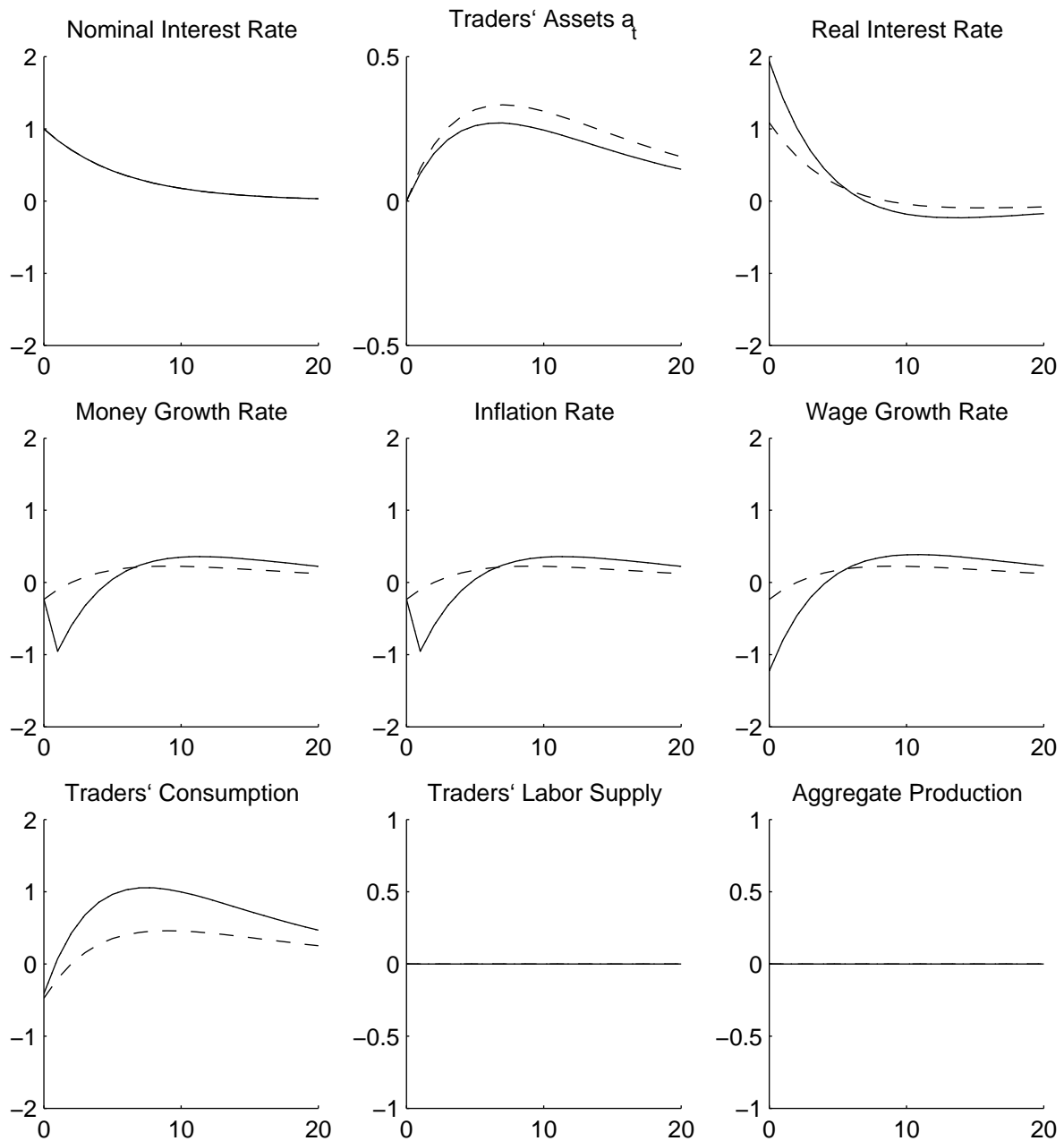


Figure 2: Response of the segmented markets economy ( $\lambda = 0.1$ ) with constant labor supply to a contractionary monetary policy shock.

The dashed and solid lines refer respectively to  $\delta = 0$  and  $\delta = 1$ .

One period is one quarter. The five rates are expressed in annual percentage points. The other variables are percent logarithms.

decreases the traders' labor supply growth rate. Let us consider separately each of these two effects.

#### 4.2.1 Full participation

The first effect is best seen in the case of full participation. An increase in the nominal interest rate discourages the labor supply through two channels. First, from the traders' first-order conditions (16d) and (16e), it follows that

$$\begin{aligned}\alpha\hat{c}_t + \varphi\hat{l}_t &= \hat{q}_t + \hat{w}_t - \hat{p}_t \\ \alpha\hat{c}_t + \varphi\hat{l}_t &= \hat{w}_t - \hat{p}_t - \hat{i}_t\end{aligned}$$

so the marginal rate of substitution between consumption and leisure increases with the real wage and decreases with the nominal interest rate. As in other cash-in-advance production economies, since wages cannot be spent to purchase consumption goods in the same period, the nominal interest rate is a wedge between the marginal rate of substitution and the real wage. It increases the relative price of consumption relative to leisure, acting as a tax on consumption. To gain intuition, one can think that traders can increase their current consumption purchases by borrowing money at the current nominal interest rate, increasing their current labor supply, and repaying their debt in the next period with the wages received. An increase in the nominal interest rate then discourages current consumption and labor supply.

Also, when  $\delta > 0$ , the nominal interest rate acts as a tax on labor demand by firms. Because of constant returns to scale, from the firms' zero-profit condition (16k)

$$\begin{aligned}\hat{w}_t - \hat{p}_t &= \frac{\delta}{q + (1-q)\delta}\hat{q}_t \approx \delta\hat{q}_t \\ \hat{w}_t - \hat{p}_t &= -\frac{\delta}{q + (1-q)\delta}\hat{i}_t \approx -\delta\hat{i}_t\end{aligned}$$

the real wage decreases by the full amount of the tax,  $\delta$  times the increase in the nominal interest rate, which further discourages the traders' labor supply. Notice that, in the data, real wages decrease after a contractionary monetary policy shock, as shown by Christiano, Eichenbaum and Evans (1996).

Using the two results just obtained,

$$\alpha\hat{c}_t + \varphi\hat{l}_t = -\frac{\delta}{q + (1-q)\delta}\hat{i}_t - \hat{i}_t \approx -(1 + \delta)\hat{i}_t \quad (19)$$

so the marginal rate of substitution between consumption and leisure decreases with the nominal interest rate. The larger  $\delta$ , the stronger this effect.

Let us now make use of the assumption of full participation. In this case, the market equilibrium conditions (16o) and (16n) imply that  $\hat{c}_t = \hat{n}_t = \hat{l}_t$ , so the previous result becomes

$$(\alpha + \varphi)\hat{l}_t = -\frac{\delta}{q + (1-q)\delta}\hat{i}_t - \hat{i}_t \approx -(1 + \delta)\hat{i}_t$$

so the traders' labor supply is a decreasing function of the nominal interest rate. When  $\delta = 0$ , there is only the first channel described above, when  $\delta > 0$ , the second channel is also present. Clearly, the effect increases with  $\delta$ , and decreases with the relative risk aversion  $\alpha$  and the inverse of the labor supply elasticity  $\varphi$ .

Figure 3 plots the response of the full participation economy to a contractionary monetary policy shock. The trough in the response of the traders' labor supply occurs in the impact period, like the peak of the nominal interest rate response. In the same period as the trough in the response of  $\hat{q}_t$ . Since the consumption growth rate increases in equilibrium, so does the real interest rate. Like in the case of constant labor supply, the money growth rate and the inflation rate decrease in the impact period but increase in the following periods. The first channel described above, then, can be seen as the composition of the effect of the interest rate on anticipated inflation, and the effect of anticipated inflation on consumption and labor. As Cooley and Hansen (1989) emphasize, anticipated inflation, operating through the inflation tax, discourages consumption, labor and production. The figure also shows that, intuitively, the magnitude of the effect of the nominal interest rate on production increases with the labor supply elasticity  $1/\varphi$ .

From the above discussion, it is clear that increases in the nominal interest rate further distort the economy and lower the welfare of households. However, the figure shows that both consumption and labor decrease after a contractionary monetary policy shock, so the magnitude of the welfare cost is not evident. We computed the welfare cost as the log-linear approximation to the percent consumption increase in the period of the shock required to keep the households' expected discounted utility constant. For the benchmark parameters  $\alpha = 1$  (log utility),  $\varphi = 1$  and  $\delta = 1$ , the welfare cost associated with a contractionary monetary policy shock increasing the nominal interest rate by one percent is negligible (0.04%).

#### 4.2.2 Segmented markets

Let us consider, now, the segmented markets economy with endogenous labor supply. We have seen earlier that, in the presence of market segmentation, a contractionary monetary policy shock has an additional liquidity effect, increasing the real interest rate above fundamentals. When the labor supply is endogenous, traders respond to the real interest rate increase by decreasing their labor supply growth rate. Using the equilibrium condition (19), the interest rate process (16a), and the traders' Euler equation (18),

$$\begin{aligned}\alpha E_t[\hat{c}_{t+1} - \hat{c}_t] + \varphi E_t[\hat{l}_{t+1} - \hat{l}_t] &\approx -(1 + \delta)E_t[\hat{i}_{t+1} - \hat{i}_t] \\ \alpha E_t[\hat{c}_{t+1} - \hat{c}_t] + \varphi E_t[\hat{l}_{t+1} - \hat{l}_t] &\approx (1 + \delta)(1 - \rho)\hat{i}_t \\ \varphi E_t[\hat{l}_{t+1} - \hat{l}_t] &\approx (1 + \delta)(1 - \rho)\hat{i}_t - \hat{r}_t\end{aligned}$$

Since the nominal interest rate is exogenous, the higher the real interest rate, the lower the expected traders' labor supply growth rate. When markets are segmented, then, after a contractionary monetary policy shock, the real interest rate increases above fundamentals,

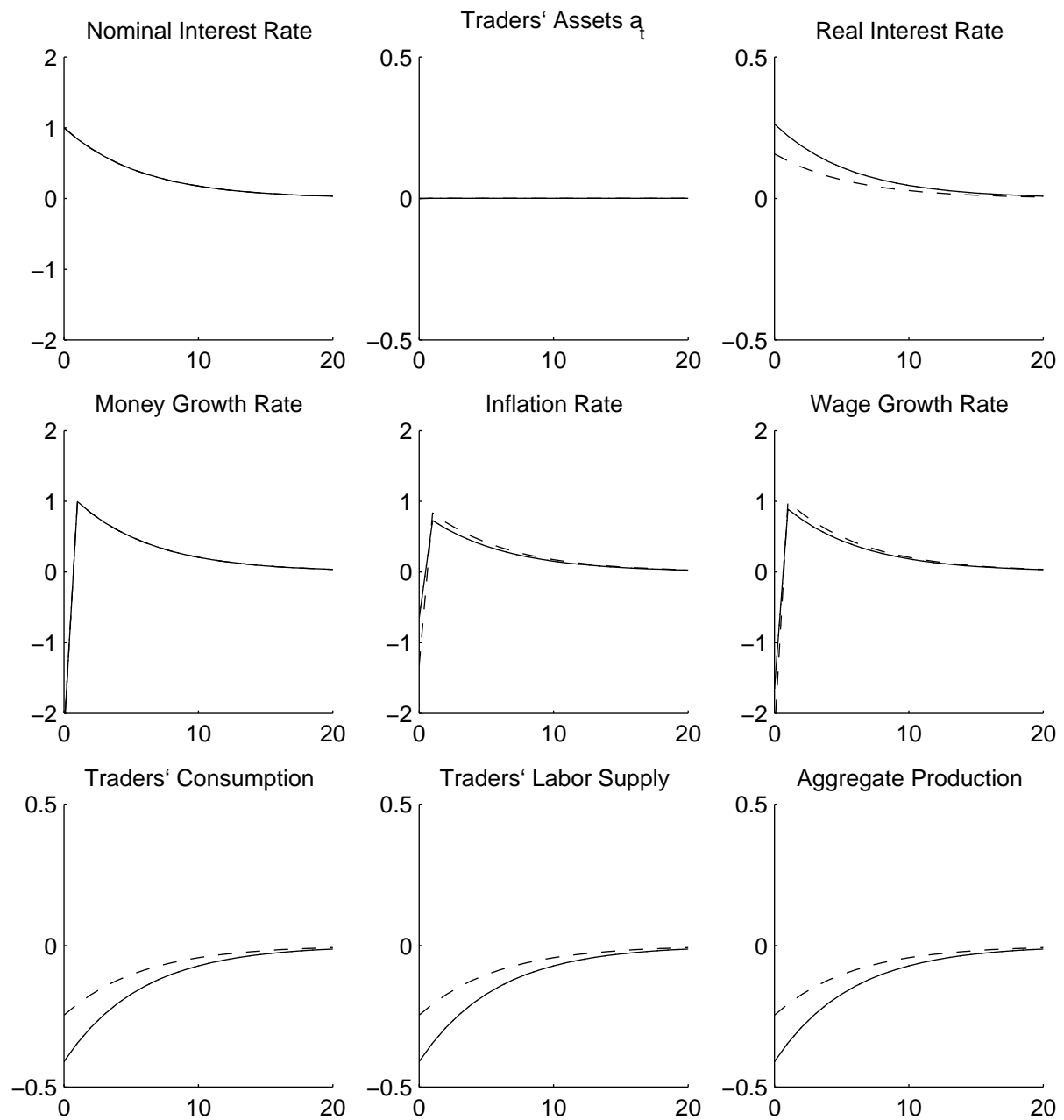


Figure 3: Response of the full participation economy ( $\lambda = 1$ ) with endogenous labor supply to a contractionary monetary policy shock.

The dashed and solid lines refer respectively to the cases where the labor supply elasticity  $1/\varphi$  is equal to 1 and 5.

One period is one quarter. The five rates are expressed in annual percentage points. The other variables are percent logarithms.

the slope of the traders' consumption and leisure responses increases, and the slope of their labor supply response decreases. If the liquidity effect is strong enough, the trough in the response of the traders' labor supply occurs several quarters after the shock.

Figure 4 plots the response of the segmented markets economy to a contractionary monetary policy shock. The figure shows the composition of the two effects, and the resulting 'hump-shaped' response of labor and output. After a contractionary monetary policy shock, interest rates increase, while output, money, prices and the real wage decrease.

Let us compare the model response with data. Consider, for instance, Figure 2 in Christiano, Eichenbaum and Evans (1999), which documents the empirical response to a contractionary monetary policy shock. The shock has an impact effect on the federal funds rate of about 75 basis points. The impact effect on the logarithm of M1 is less (in absolute values) than -0.1, and the impact effect on the logarithm of M2 is less than -0.3. There is almost no effect on prices. Equivalently, the impact effect on the annualized M1 and M2 growth rate is respectively less than -0.4% and -1.2%, while the impact effect on the inflation rate is about zero. The response of output is 'hump-shaped', it decreases over time with a trough after six quarters of -0.5%, and then increases back to its steady state.

The sign, the magnitude and the persistence of the model responses are realistic. Although the inflation rate does respond to the shock, the magnitude is not large. Also, the magnitude of the output response is about half of what is empirically. The real interest rate increases persistently, while the output growth rate decreases persistently. This joint response can hardly be obtained in full participation monetary models, like sticky-price models, where the Euler equation links directly the real interest rate with the expected output growth rate.

The figure also shows that the magnitude of the effect of the nominal interest rate on production increases with the labor supply elasticity  $1/\varphi$ . On the other hand, when the labor supply elasticity is higher, the responses are less persistent and the trough of the output response occurs earlier.

The responses of the traders' consumption and labor seem to indicate that a contractionary monetary policy shock increases their welfare. For the benchmark parameters  $\alpha = 1$  (log utility),  $\varphi = 1$  and  $\delta = 1$ , the welfare cost for traders associated with a contractionary monetary policy shock increasing the nominal interest rate by one percent is negative (-2.21% when  $\lambda = 0.5$ , and -17.04% when  $\lambda = 0.1$ ). We also computed the welfare cost for non-traders using the same utility function as for the traders, and we found it positive (2.40% both when  $\lambda = 0.5$ , and when  $\lambda = 0.1$ ). The social welfare cost, computed as the average welfare cost weighted by the proportion of traders and non-traders, is positive but small (0.10% when  $\lambda = 0.5$ , and 0.46% when  $\lambda = 0.1$ ). The main welfare effect of a contractionary monetary policy shock, then, is redistributive in favor of traders.

We now turn to what happens as we increase the length of a period. First, notice that both the steady state nominal interest rate  $i$  and the deviation of the nominal interest rate  $\hat{i}_t$  are expressed in units that depend on the length of the period. For a given increase in the yearly nominal interest rate, the longer the length of the period, the larger are both  $i$

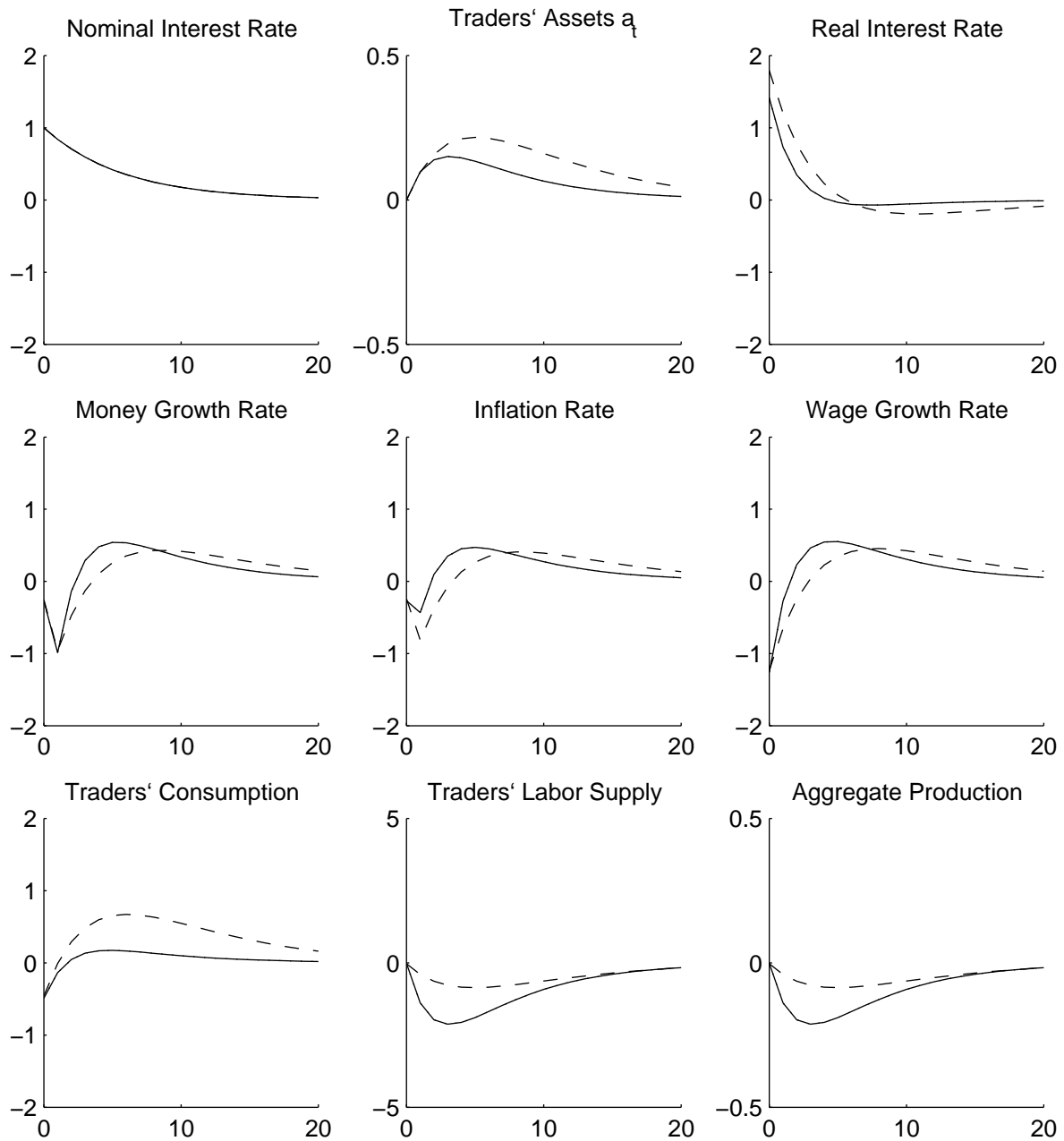


Figure 4: Response of the segmented markets economy ( $\lambda = 0.1$ ) with endogenous labor supply to a contractionary monetary policy shock.

The dashed and solid lines refer respectively to the cases where the labor supply elasticity  $1/\varphi$  is equal to 1 and 5.

One period is one quarter. The five rates are expressed in annual percentage points. The other variables are percent logarithms.

and  $\hat{i}_t$ . For instance, if the yearly nominal interest rate increases by one percentage point,  $\hat{i}_t$  increases by 25 basis points if a period is one quarter, and by 50 basis points if a period is one semester. Then, considering the equilibrium condition (19), it is clear that the impact of a given increase in the yearly nominal interest rate on the traders' consumption and labor supply increases linearly with the length of the period.

To see why, observe that a nominal interest rate increase is an increase in the cost of money. The cash-in-advance constraints imply that the longer the length of the period, the larger the money stock that households need to hold relative to their consumption flow, and the larger the money stock that firms need to hold relative to their cost of wages flow. Then, the effect of a given increase in the yearly nominal interest rate on interest costs increases linearly with the length of the period.

To show the potential of the model at matching the magnitude of the effect of a monetary policy shock on aggregate production, Figures 5 and 6 plot the response of the segmented markets economy to a contractionary monetary policy shock respectively in the case that a period is one semester and one year<sup>6</sup> For comparison with the other figures, however, the units of the x-axis are still quarters. All the comments previously made as to Figure 4 apply to these figures as well. This time, however, the magnitude of the output response is about right. The inflation rate, also, hardly responds to the shock.

Finally, let us briefly summarize what happens in the numerical experiments as the other parameters change. Given the nominal interest rate process, the preferences discount factor  $\beta$  determines the non-stochastic steady state values of the growth rate of nominal variables and the traders' share of aggregate cash balances, and affects this way the economy's response to a monetary policy shock, especially the response of nominal variables. Increasing  $\beta$ , however, does not affect significantly the economy's response, except that the output response becomes positive in the first periods for high values of  $\beta$  and low values of  $\delta$ . The threshold values depend on the degree of markets segmentation.

The effect of decreasing the relative risk aversion  $\alpha$  is similar to the effect of increasing the labor supply elasticity  $1/\varphi$ . The magnitude of the output response to monetary policy shocks increases. In addition, the responses become less persistent, and the trough of the output response occurs earlier. With regard to the exogenous process for the nominal interest rate, as the serial correlation of the nominal interest rate decreases, the persistence of all processes decrease, and the trough of the output response occurs earlier.

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<sup>6</sup>For a model with households' cash-in-advance constraints only, Cooley and Hansen (1989) point out that, in order to approximately match the ratio of money to consumption, one period should be set equal to one quarter (/one month) if money is defined as M1 (/the monetary base). Since the M2 money stock is more than four times the M1 money stock, one period should be set equal to one year if money is defined as M2. In our model, however, firms need cash-in-advance to hire labor, so the same line of argument would lead to setting the period about half as long.



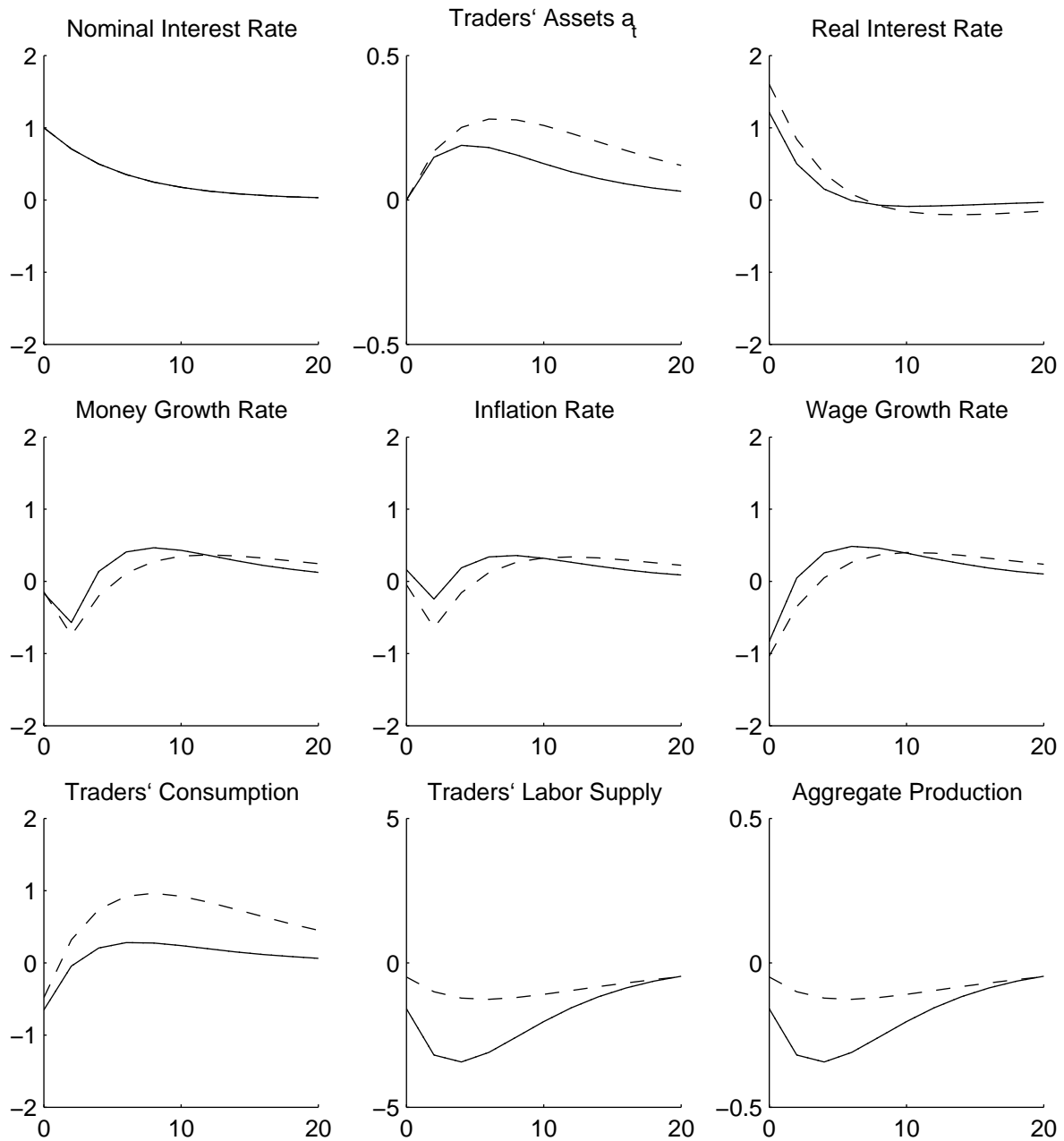


Figure 5: Response of the segmented markets economy ( $\lambda = 0.1$ ) with endogenous labor supply to a contractionary monetary policy shock.

The dashed and solid lines refer respectively to the cases where the labor supply elasticity  $1/\varphi$  is equal to 1 and 5.

One period is one semester. The units of the x-axis, however, are still quarters. The five rates are expressed in annual percentage points. The other variables are percent logarithms.

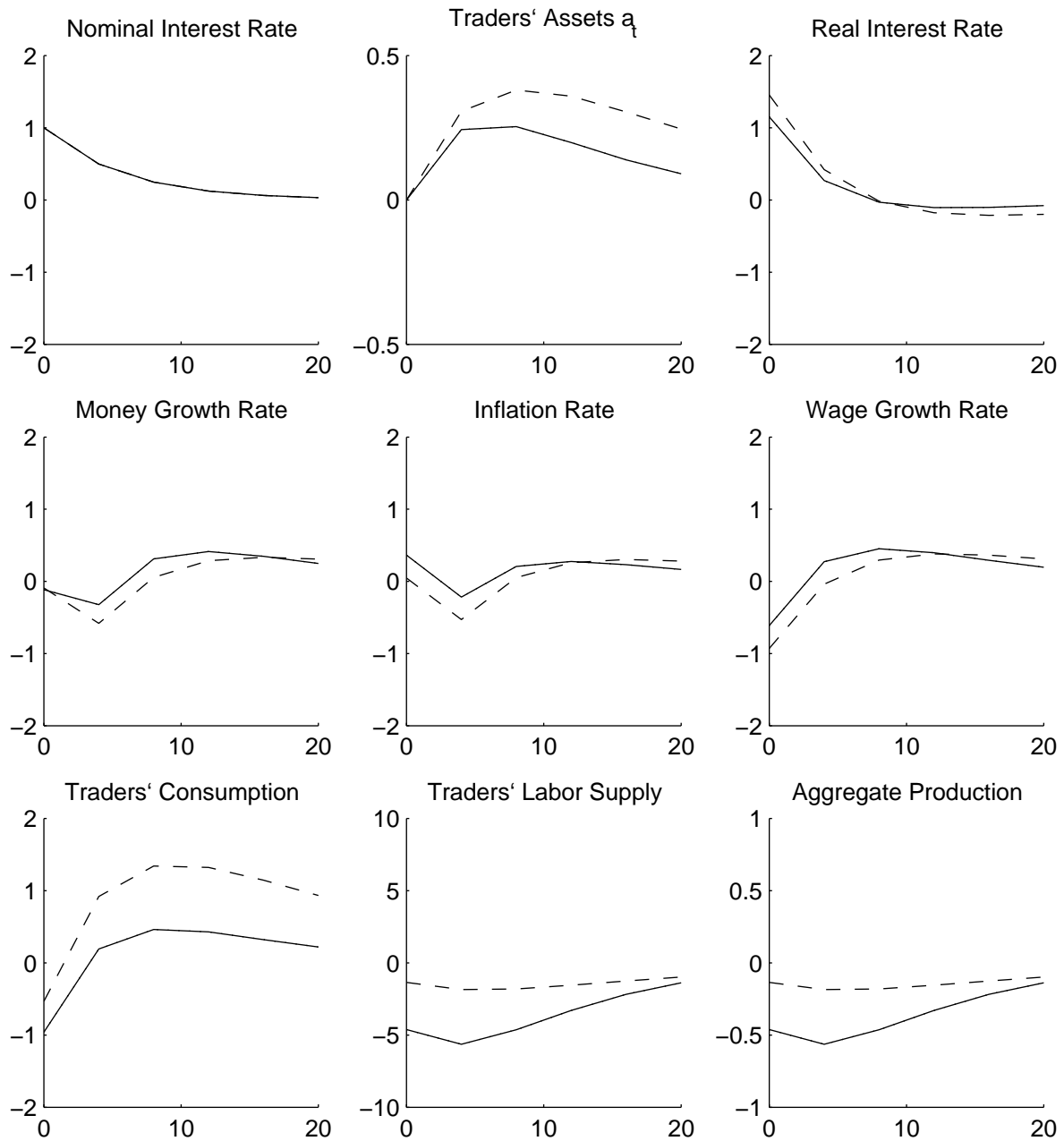


Figure 6: Response of the segmented markets economy ( $\lambda = 0.1$ ) with endogenous labor supply to a contractionary monetary policy shock.

The dashed and solid lines refer respectively to the cases where the labor supply elasticity  $1/\varphi$  is equal to 1 and 5.

One period is one year. The units of the x-axis, however, are still quarters. The five rates are expressed in annual percentage points. The other variables are percent logarithms.

## 5 Conclusion

Christiano, Eichenbaum and Evans (1999) argue that comparing the model response to monetary policy shocks with the empirical response is an important criterion for selecting a framework for monetary analysis. This paper shows that a segmented markets model with production is able to replicate the sign, the magnitude and the persistence of the responses of output, money, prices and interest rates. A contractionary shock increases the interest rates, and decreases output, money, and the real wage, while prices hardly respond. The output response is ‘hump-shaped’. The model can replicate the increase in the real interest rate together with the decrease in the output growth rate.

In future research, we plan to add technology shocks and a monetary policy rule, estimate the model and compare it with other monetary frameworks. The results from Landon-Lane and Occhino (2004) on a related segmented markets economy with exogenous stochastic endowment are very encouraging. We also would like to consider alternative assumptions on the interaction between monetary and fiscal policies. In the case that the revenue from seigniorage is rebated to the households through lump-sum transfers, an appropriate feedback monetary policy rule can insure equilibrium determinacy. Ultimately, we would like to evaluate the impact of adding the segmented markets friction in models like the one in Smets and Wouters (2003) where other standard frictions are present. In joint work with John Landon-Lane, we are conducting research along this direction, and we will report the results in later work.

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