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# Market Segmentation and the Response of the Real Interest Rate to Monetary Policy Shocks

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## Abstract

Following a contractionary monetary policy shock, the aggregate output decreases over time for six to eight quarters, while the real interest rate increases immediately and remains high for three quarters. Full participation models can hardly replicate the joint response of the aggregate output and the real interest rate, while limited participation models can do so only in the impact period. This paper adopts a segmented markets framework where some households are permanently excluded from financial markets. The monetary authority controls the short-term nominal interest rate, and lets the money supply be determined by the bond market. The aggregate output and the nominal interest rate are modeled as exogenous autoregressive processes, while the real interest rate is determined endogenously. When markets are segmented enough, the model is able to account for both the persistent decreasing path of the aggregate output and the persistent increase in the real interest rate which follow an unanticipated increase in the nominal interest rate. The sign, the size and the persistence of the responses of the real interest rate and the money growth rate are close to those in the data.

*Keywords:* limited participation, market segmentation, monetary policy shocks, real interest rate.

*JEL Classification Number:* E52.

## 1 Introduction

In their comprehensive synthesis, Christiano, Eichenbaum and Evans (1999) document that the following evidence is robust across different identification schemes. Following a contractionary monetary policy shock, the aggregate output decreases over time for six to eight quarters, the federal funds rate increases immediately and remains high for three quarters,

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M1 and M2 decrease over time respectively for two and three quarters, the GDP deflator does not respond immediately and declines after five quarters<sup>1</sup>. The Fisher equation, which states that the real interest rate is approximately equal to the nominal interest rate minus the expected inflation rate, implies that the real interest rate increases immediately and remains high for three quarters, like the nominal interest rate.

Common full participation monetary models cannot replicate the joint response of the aggregate output and the real interest rate. In those models, the consumption Euler equation implies that the equilibrium real interest rate is an increasing function of the expected aggregate consumption growth rate. In most models, the aggregate consumption increases with the aggregate output, so the equilibrium real interest rate is an increasing function of the expected aggregate output growth rate. Full participation models, then, cannot predict that, following a contractionary monetary policy shock, the expected aggregate output growth rate is low and the real interest rate is high for several quarters. Benchmark sticky-price models, like the one in Clarida, Gali' and Gertler (1999), suffer from this important shortcoming. In those models, a contractionary shock is, by definition, a persistent increase in the real interest rate. As a result, they counterfactually predict that the aggregate output drops in the impact period and increases, instead of decreasing, over time.

Unlike full participation models, benchmark limited participation models, like the one in Fuerst (1992), are able to replicate the joint response of the aggregate output and the real interest rate in the impact period of a contractionary shock. The key assumption is that the representative household makes its decision on bond and money holdings before the realization of the shock. In their impact period, monetary policy shocks have a liquidity effect on interest rates, a contractionary shock increasing the real interest rate above its fundamentals. In the periods following the shock, however, the representative household makes its decisions taking into full account the effect of the shock, the liquidity effect vanishes, and the economy's response is the same as in full participation models. Although Christiano and Eichenbaum (1992), Alvarez and Atkeson (1997) and Alvarez, Atkeson and Kehoe (2002) have proposed economies where the liquidity effect lasts longer, how to model the economy's response in the periods *after* a monetary policy shock occurs remains an open and central research issue.

This paper adopts a heterogenous agents variant of the limited participation framework, the *segmented markets* model, previously studied by Alvarez and Atkeson (1996), Occhino (2000), Alvarez, Lucas and Weber (2001), Lahiri, Singh and Vegh (2003), and Occhino (2004). The central feature is that some households are permanently excluded from financial markets. Open market operations affect, then, the distribution of money and consumption expenditures across households. In turn, the distribution of consumption expenditures affects the real interest rate, since the equilibrium real interest rate is determined by the intertemporal marginal rate of substitution of the subset of households participating in the financial markets. When the monetary policy variable is serially correlated, a monetary policy shock persistently affects the monetary policy variable, the distribution of money and consumption expenditures across households, and the real interest rate.

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<sup>1</sup>The works of Gordon and Leeper (1994), Strongin (1995), Bernanke and Mihov (1998) and Leeper, Sims and Zha (1996) also support the claim, while Uhlig (2001) identifies a monetary policy shock with sign restrictions on the response of money, prices and interest rate, and finds that a contractionary monetary policy shock has no clear effect on real GDP.

In this paper, the monetary authority controls the short-term nominal interest rate, and lets the money supply be determined by the bond market. The aggregate output and the nominal interest rate are modeled as exogenous autoregressive processes, while the real interest rate is determined endogenously. When markets are segmented enough, the model is able to account for both the persistent decreasing path of the aggregate output and the persistent increase in the real interest rate which follow an unanticipated increase in the nominal interest rate. The sign, the size and the persistence of the responses of the real interest rate and the money growth rate are close to those in the data.

The paper is organized as follows. Section 2 describes the economy and defines the competitive equilibrium, Section 3 explains the numerical solution method, Section 4 describes and comments the results, and Section 5 concludes.

## 2 Model

The model is a cash-in-advance endowment economy, with a large number of households and a monetary authority. Time is discrete and is indexed by  $t \geq 0$ . There are a single non-durable consumption good, money, and one-period nominal bonds, which are claims to one unit of money payable at the end of the period. Households are of two types, traders and non-traders. Let  $\omega > 0$  and  $\omega^* \geq 0$  be respectively the number of traders and non-traders. We will refer to the case where  $\omega^* = 0$  and  $\omega^* > 0$  respectively as the full participation model and the segmented markets model.

Households of the same type are identical in all respects. The crucial difference between the two types of households is that non-traders spend all their money purchasing consumption goods, while traders can purchase bonds as well.

Households start each period with cash balances from the previous period. Then, two markets meet in sequence, a bond market and a goods market.

In the bond market, the monetary authority sells one-period nominal bonds to the traders, at the bond price  $q_t > 0$ . Open market operations are conducted in terms of the short-term nominal interest rate  $i_t > 0$  defined by

$$i_t \equiv -\log(q_t) \tag{1}$$

The monetary authority announces the bond price and stands ready to issue and sell any number of bonds to clear the market at that price. Monetary policy is, then, an exogenous stochastic process for the interest rate, while the bond supply and the money supply are determined endogenously. By assumption, the interest rate is strictly positive, and the bond price is strictly less than one.

After the bond market, all households participate in the goods market. Each trader and each non-trader respectively receive constant fractions  $\Lambda > 0$  and  $\Lambda^* > 0$  of the exogenous stochastic aggregate endowment  $Y_t > 0$ , with  $\omega\Lambda + \omega^*\Lambda^* = 1$ . The endowment cannot be consumed directly, and must be sold in exchange of money at the price  $P_t > 0$ . Households can only consume goods purchased with money held before the goods market session. The money supply

$$M_t \equiv P_t Y_t \tag{2}$$

is defined as the amount of dollars  $P_t Y_t$  spent in the goods market. Bonds are redeemed after the goods market closes.

The aggregate endowment  $Y_t$  and the nominal interest rate  $i_t$  are the only sources of uncertainty in the economy, and their joint dynamics is exogenously modeled as follows. Let  $\{\bar{Y}_t, \bar{i}_t\}_{t=0}^{\infty}$  be the non-stochastic steady state values of the aggregate endowment and the interest rate, and let us assume that  $\bar{Y}_{t+1}/\bar{Y}_t = \alpha$  and  $\bar{i}_t = i$  are constant over time. We assume that  $\hat{z}_t \equiv [\log(Y_t) - \log(\bar{Y}_t), i_t - i]$  follows the AR(N) process,

$$\hat{z}_t = \sum_{n=1}^N \hat{z}_{t-n} B_n + \eta_t C, \quad (3)$$

where  $B_n$  and  $C$  are  $2 \times 2$  matrices,  $C$  is upper triangular,  $\eta_t$  is a  $1 \times 2$  vector of independently and identically distributed standard Gaussian shocks.

Each trader chooses consumption  $C_t$ , bonds  $B_t$ , and next-period cash balances  $A_{t+1}$  to solve

$$\begin{aligned} & \max_{\{C_t > 0, B_t, A_{t+1} > 0\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t) \right] \\ & \text{subject to:} \\ & q_t B_t + P_t C_t \leq A_t \\ & A_{t+1} = A_t - q_t B_t - P_t C_t + P_t \Lambda Y_t + B_t, \end{aligned} \quad (4)$$

given the trader's initial cash balances  $A_0 > 0$  in period zero, where  $E_0$  is the expectation conditional on information available after  $\hat{z}_0$  has been revealed, the period utility function  $u(C) \equiv C^{1-1/\epsilon}/(1-1/\epsilon)$  is constant elasticity of substitution, and the preferences discount factor satisfies  $\beta \alpha^{1-1/\epsilon} \in (0, 1)$ .

Since the bond price  $q_t$  is strictly less than one for all  $t$ , holding idle cash balances is never optimal for traders, so the traders' cash-in-advance constraint always holds with equality. Then, the two constraints in the above maximization problem (4) can be substituted with the constraints

$$\begin{aligned} q_t B_t + P_t C_t &= A_t \\ A_{t+1} &= P_t \Lambda Y_t + B_t \end{aligned} \quad (5)$$

Non-traders spend all their initial cash balances purchasing consumption goods. Under this assumption, the behavior of a non-trader is simply described by constraints

$$\begin{aligned} P_t C_t^* &= A_t^* \\ A_{t+1}^* &= P_t \Lambda^* Y_t \end{aligned} \quad (6)$$

given the non-traders' initial cash balances  $A_0^* > 0$  in period zero.

The economy is described by the traders' initial assets  $A_0 > 0$ , the non-traders initial assets  $A_0^* > 0$ , the initial exogenous state  $\hat{z}_0$ , and the law of motion (3) for the exogenous state  $\hat{z}_t$ . An equilibrium is a set of contingent sequences  $\{C_t > 0, B_t, A_{t+1} > 0\}_{t=0}^{\infty}$  of consumption demand, bonds demand and cash balances for traders,  $\{C_t^* > 0, A_{t+1}^* > 0\}_{t=0}^{\infty}$  of consumption

demand and cash balances for non-traders, a contingent sequence  $\{D_t\}_{t=0}^{\infty}$  of bonds supplied by the monetary authority, and a contingent sequence  $\{P_t > 0\}_{t=0}^{\infty}$  of prices such that, given the prices, the traders' contingent sequence solves the traders' optimization problem (4), the non-traders' contingent sequence satisfies the non-traders constraints (6), and the following bonds and goods market equilibrium condition hold:

$$\begin{aligned}\omega B_t &= D_t \\ \omega C_t + \omega^* C_t^* &= Y_t\end{aligned}\tag{7}$$

The necessary first-order conditions for the traders' optimization problem are

$$\begin{aligned}\beta^t u'(C_t) - \nu_t^1 P_t &= 0 \\ -q_t \nu_t^1 + \nu_t^2 &= 0 \\ -\nu_t^2 + E_t[\nu_{t+1}^1] &= 0\end{aligned}\tag{8}$$

and the transversality condition is

$$\lim_{t \rightarrow \infty} E_0 [\nu_t^1 A_t] = 0\tag{9}$$

where  $\nu_t^1$  and  $\nu_t^2$  are the Lagrange multipliers associated with the two constraints (5). From the first-order conditions, it follows that

$$\begin{aligned}\beta^t u'(C_t) &= \nu_t^1 P_t \\ q_t \nu_t^1 &= E_t[\nu_{t+1}^1]\end{aligned}\tag{10}$$

The system describing the equilibrium is, then, made of the identities (1) and (2), the law of motion 3 for the exogenous state, the traders' first-order conditions 10, the traders' constraints 5, the non-traders' constraints 6, and the equilibrium conditions 7.

We finally define the inflation rate  $\pi_t$  and the real interest rate  $r_t$  by

$$\begin{aligned}\pi_{t+1} &\equiv \log(P_{t+1}) - \log(P_t) \\ r_t &\equiv i_t - E_t[\pi_{t+1}]\end{aligned}\tag{11}$$

and we define the money growth rate similarly to the inflation rate.

The economy is similar to the one in Alvarez and Atkeson (1996), Occhino (2000), Lucas, Alvarez and Weber (2001), and Occhino (2004). The main difference is that, in this paper, the aggregate endowment is stochastic. Another difference with the previous first three works is the behavior of the monetary authority. There, monetary policy is set in terms of a money growth rate, while, here, is set in terms of a short-term nominal interest rate. Modeling monetary policy in terms of a short-term interest rate is important for two reasons. First of all, this assumption best models the operating procedure of the monetary authorities of most OECD countries. Also, Bernanke and Blinder (1992) show that the federal funds rate is an excellent indicator of the stance of monetary policy, so innovations to the federal funds rate can be identified with monetary policy shocks.

### 3 Solution

For convenience, variables are normalized as follows. As in Lucas (1990), nominal variables are normalized by aggregate cash balances available at the beginning of the period. Let  $\bar{A}_t \equiv \omega A_t + \omega^* A_t^*$  be the initial aggregate cash balances. Then,  $y_t \equiv Y_t/\bar{Y}_t$ ,  $\nu_t \equiv u'(\omega)\nu_t^1 \bar{A}_t / \beta^t u'(\bar{Y}_t)\bar{Y}_t$ ,  $c_t \equiv \omega C_t/\bar{Y}_t$ ,  $b_t \equiv \omega B_t/\bar{A}_t$ ,  $a_t \equiv \omega A_t/\bar{A}_t$ ,  $c_t^* \equiv \omega^* C_t^*/\bar{Y}_t$ ,  $a_t^* \equiv \omega^* A_t^*/\bar{A}_t$ ,  $d_t \equiv D_t/\bar{A}_t$ ,  $\gamma_t \equiv \bar{A}_{t+1}/\bar{A}_t$ ,  $p_t \equiv P_t \bar{Y}_t/\bar{A}_t$ ,  $m_t \equiv M_t/\bar{A}_t$ . Also, let us define the traders' share of the aggregate endowment as  $\lambda \equiv \omega\Lambda = 1 - \omega^*\Lambda^*$ .  $\lambda$  is equal to 1 in the full participation model, and  $\lambda \in (0, 1)$  in the segmented markets model. It might be helpful to consider the case where the endowment received by a trader is the same as the one received by a non-trader, so  $\Lambda = \Lambda^*$ . In this case,  $\Lambda = 1/(\omega + \omega^*)$ , and  $\lambda = \omega/(\omega + \omega^*)$ , so  $\lambda$  is the proportion of traders in the economy.

The system describing the equilibrium can then be written as

$$i_t \equiv -\log(q_t) \quad (12a)$$

$$u'(c_t) = \nu_t p_t \quad (12b)$$

$$q_t \gamma_t \nu_t = \beta E_t[\nu_{t+1}] u'(\alpha) \alpha \quad (12c)$$

$$q_t b_t + p_t c_t = a_t \quad (12d)$$

$$\gamma_t a_{t+1} = p_t \lambda y_t + b_t \quad (12e)$$

$$p_t c_t^* = a_t^* \quad (12f)$$

$$\gamma_t a_{t+1}^* = p_t (1 - \lambda) y_t \quad (12g)$$

$$b_t = d_t \quad (12h)$$

$$c_t + c_t^* = y_t \quad (12i)$$

$$m_t \equiv p_t y_t \quad (12j)$$

$$a_t + a_t^* = 1 \quad (12k)$$

together with the law of motion (3) for the exogenous state. The transversality condition (9) can be written as

$$\lim_{t \rightarrow \infty} E_0 [\beta^t u'(\bar{Y}_t) \bar{Y}_t \nu_t a_t / u'(\omega) \omega] = 0. \quad (13)$$

It is convenient to derive an equivalent system as follows. From the households' budget constraints (12e) and (12g), it follows that

$$\begin{aligned} \gamma_t a_{t+1} + \gamma_t a_{t+1}^* &= p_t \lambda y_t + b_t + p_t (1 - \lambda) y_t \\ q_t \gamma_t [a_{t+1} + a_{t+1}^*] &= q_t p_t y_t + q_t b_t \end{aligned}$$

Then, using the households' cash-in-advance constraints (12d) and (12f), the equation (12a), and the goods market equilibrium condition (12i)

$$\begin{aligned} q_t \gamma_t [a_{t+1} + a_{t+1}^*] &= q_t p_t y_t + a_t - p_t c_t + a_t^* - p_t c_t^* \\ q_t \gamma_t &= q_t p_t y_t + 1 - p_t c_t - p_t c_t^* \\ q_t \gamma_t + (1 - q_t) p_t y_t &= 1 \end{aligned}$$

which we use in place of the traders' budget constraint 12e in the previous system 12.

In the non-stochastic steady state, all normalized variables are constant over time, and  $y_t = 1$ . The non-stochastic steady state can, then, be derived from the system:

$$\begin{aligned}
i &\equiv -\log(q) \\
q\gamma &= \beta u'(\alpha)\alpha \\
q\gamma + (1-q)py &= 1 \\
m &\equiv py \\
pc^* &= a^* \\
\gamma a^* &= p(1-\lambda)y \\
c + c^* &= y \\
u'(c) &= \nu p \\
a + a^* &= 1 \\
qb + pc &= a \\
b &= d
\end{aligned} \tag{14}$$

where the variables without the time subscript are the non-stochastic steady state values. Notice that, since  $\beta\alpha^{1-1/\epsilon} \in (0, 1)$ , the transversality condition is satisfied in the non-stochastic steady state.

Log-linearizing the system around the non-stochastic steady state yields

$$\hat{i}_t \equiv -\hat{q}_t \tag{15a}$$

$$-\frac{1}{\epsilon}\hat{c}_t = \hat{\nu}_t + \hat{p}_t \tag{15b}$$

$$\hat{q}_t + \hat{\gamma}_t + \hat{\nu}_t = E_t[\hat{\nu}_{t+1}] \tag{15c}$$

$$qb[\hat{q}_t + \hat{b}_t] + pc[\hat{p}_t + \hat{c}_t] = a\hat{a}_t \tag{15d}$$

$$q\gamma[\hat{q}_t + \hat{\gamma}_t] + (1-q)py\left[-\frac{q}{1-q}\hat{q}_t + \hat{p}_t + \hat{y}_t\right] = 0 \tag{15e}$$

$$\hat{p}_t + \hat{c}_t^* = \hat{a}_t^* \tag{15f}$$

$$\hat{\gamma}_t + \hat{a}_{t+1}^* = \hat{p}_t + \hat{y}_t \tag{15g}$$

$$\hat{b}_t = \hat{d}_t \tag{15h}$$

$$c\hat{c}_t + c^*\hat{c}_t^* = y\hat{y}_t \tag{15i}$$

$$\hat{m}_t \equiv \hat{p}_t + \hat{y}_t \tag{15j}$$

$$a\hat{a}_t + a^*\hat{a}_t^* = 0 \tag{15k}$$

where the variables without the time subscript are the non-stochastic steady state values, while the variables with the hat are the percentage deviations from the steady state values, except for  $\hat{i}_t \equiv i_t - i$ .

The system (15) together with the law of motion (3) for the exogenous state can be reduced to a four equations system in the two exogenous variables  $\hat{y}_t$  and  $\hat{i}_t$ , the endogenous state variable  $\hat{a}_t^*$  and the control variable  $\hat{\nu}_t$ . With standard methods, we, then, derive the linear system describing the equilibrium evolution of the three state variables  $\hat{y}_t$ ,  $\hat{i}_t$ , and  $\hat{a}_t^*$ ,



and linking all the other variables to the three state variables<sup>2</sup>.

The equilibrium is determinate even though monetary policy is an exogenous process for the interest rate. To see why, it is best to consider the full participation economy where all households are traders. In this case, the real allocation is determinate and the only issue regards whether nominal variables are determined. Recall that taxes and transfers are equal to zero. Given the traders' initial level of nominal assets, and the real variables stochastic processes, the traders' intertemporal budget constraint pins down the initial price level. Numerical methods show that the equilibrium is determinate in the segmented markets case as well, although the previous argument does not apply directly since changes in the money supply can affect the real allocation between traders and non-traders. The assumption that the fiscal variables do not adjust to balance the intertemporal budget constraint, so the fiscal policy is *active* in the terminology of Leeper (1991), plays an important role in the argument. In models where the equilibrium is indeterminate, in each period, the revenue from seigniorage is rebated to the households through lump-sum transfers. Equivalently, money is introduced into the economy through helicopter-drops. In these models, then, transfers are a function of open-market operations, the intertemporal budget constraint can hold for several possible path of money, prices and seigniorage, and the initial price level cannot be determined. Since we focus on the effects of monetary policy shocks for up to 2 years, we see the assumption that the fiscal variables are exogenous as far more plausible than assuming that they depend on current open market operations.

After solving the model, we can derive the following rates. From the definitions of the inflation rate and the real interest rate,

$$\begin{aligned}\pi_{t+1} &\equiv \log(p_{t+1}) + \log(\gamma_t) - \log(p_t) \\ r_t &\equiv i_t - E_t[\pi_{t+1}]\end{aligned}$$

Their non-stochastic steady state values are given by

$$\begin{aligned}\pi &\equiv \log(\gamma) \\ r &\equiv i - \pi\end{aligned}$$

and their linear approximations by

$$\begin{aligned}\hat{\pi}_{t+1} &\equiv \hat{p}_{t+1} + \hat{\gamma}_t - \hat{p}_t \\ \hat{r}_t &\equiv \hat{i}_t - E_t[\hat{\pi}_{t+1}]\end{aligned}\tag{16}$$

where  $\hat{\pi}_t \equiv \pi_t - \pi$  and  $\hat{r}_t \equiv r_t - r$ . The linear approximations of the money growth rate is derived similarly to the inflation rate.

Before turning to the analysis of the effects of a monetary policy shock, let us derive a more intuitive version of the traders' Euler equation. From the traders' first-order condition

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<sup>2</sup>The solution method is based on the eigenvalue decomposition of the matrix describing the evolution of the state and control variables. As a check, the model has been solved using MATLAB files written by Chris Sims and Paul Klein as well. We thank them for making the files available at the web address <http://www.ssc.uwo.ca/economics/faculty/klein/>. Their solution method is based on the Schur decomposition of the matrix describing the evolution of the state and control variables. The two methods yield identical solutions.

for consumption (15b) and the traders' Euler equation (15c), it follows that

$$\begin{aligned}\hat{q}_t + E_t[\hat{p}_{t+1}] + \hat{\gamma}_t - \hat{p}_t &= -\frac{1}{\epsilon} E_t[\hat{c}_{t+1} - \hat{c}_t] \\ \hat{q}_t + E_t[\hat{\pi}_{t+1}] &= -\frac{1}{\epsilon} E_t[\hat{c}_{t+1} - \hat{c}_t]\end{aligned}$$

where the last equation follows from the inflation rate definition in (16). Then, using the interest rate definition (15a) and the real interest rate definition in (16), the following version of the traders' Euler equation follows

$$\begin{aligned}\hat{i}_t - E_t[\hat{\pi}_{t+1}] &= \frac{1}{\epsilon} E_t[\hat{c}_{t+1} - \hat{c}_t] \\ \hat{r}_t &= \frac{1}{\epsilon} E_t[\hat{c}_{t+1} - \hat{c}_t]\end{aligned}\tag{17}$$

which implies that the equilibrium real interest rate is equal to the inverse of the intertemporal elasticity of substitution  $\epsilon$  times the traders' expected consumption growth rate.

## 4 Results

The economy's response to a monetary policy shock depends in a crucial way on the traders' share  $\lambda$  of the aggregate endowment.  $\lambda$  is a measure of the traders' economic weight. When  $\lambda$  is equal to one, the economy is the benchmark full participation representative agent economy with cash-in-advance constraints and stochastic aggregate endowment. The lower  $\lambda$ , the lower the traders' economic weight, the higher markets segmentation. Results are shown below for values of  $\lambda$  equal to 0.1 and 1.

The traders' marginal utility of consumption  $u'(c)$  is set equal to  $c^{-1/\epsilon}$ , so the intertemporal elasticity of substitution is constant and equal to  $\epsilon$ . Results are shown below for values of  $\epsilon$  equal to 1 and 5.

To obtain the other parameters in the model, we use quarterly data for the period 1955:I-1999:IV from the FRED II Database of the Federal Reserve Bank of St. Louis. Rates are expressed in annual percentage points. Logarithms are multiplied by 100, so the impulse response of a logarithm can be interpreted as the percentage response of the underlying variable.

Each period is one quarter. The aggregate endowment growth rate  $\alpha - 1$  in the non-stochastic steady state is set equal to 0.0082, to match the 3.25% average yearly growth rate of Real Gross Domestic Product. The inverse of the gross real interest rate  $\beta u'(\alpha)$  in the non-stochastic steady state is set equal to 0.9828, so the average annual real rate of return is 7% and matches approximately the average real rate of return of the S&P500 stock index over the sample period. The value of  $\beta$ , then, varies with the elasticity  $\epsilon$ .

To obtain the joint process for the detrended logarithm of the aggregate endowment and the nominal interest rate, we estimate a VAR with  $N = 2$  lags of the linearly-detrended logarithm of Real Gross Domestic Product and the Effective Federal Funds Rate. Varying the number of lags  $N$  does not affect the main result regarding the real interest rate response, so we only report the results for  $N = 2$ . Using the Hodrick-Prescott filtered logarithm of

Real GDP (with smoothing parameter 1600) does not affect the main result either. Shocks are identified using the Cholesky decomposition with the aggregate endowment ordered first and the nominal interest rate ordered second and last. A monetary policy shock is a one percent unanticipated increase in the nominal interest rate with no contemporaneous effect on the aggregate endowment. Equivalently, it is a one percent innovation in the nominal interest rate.

## 4.1 Full Participation

Figure 1 plots the model response to a contractionary monetary policy shock. The dashed line refers to a full participation representative agent economy ( $\lambda = 1$ ), while the solid line refers to a segmented markets economy where  $\lambda = 0.1$ . The intertemporal elasticity of substitution  $\epsilon$  is set equal to one (logarithmic utility function) for both economies. The six subplots show the impulse response functions of the nominal interest rate, the logarithm of the aggregate endowment, the logarithm of the share  $a_t^*$  of aggregate cash balances held by the non-traders at the beginning of each period, the money growth rate, the logarithm of the traders' consumption, and the real interest rate.

The responses of the nominal interest rate and the aggregate endowment are, of course, the same for both economies, independently of  $\lambda$ , since the shock is an innovation in the nominal interest rate, and the process for the nominal interest rate and the aggregate endowment is entirely exogenous. Since the nominal interest rate is strongly positively autocorrelated, its response remains high for ten quarters. The aggregate endowment decreases over time for several quarters, with a trough after twelve quarters. If the logarithm of Real GDP is Hodrick-Prescott filtered instead of being detrended linearly, the responses of the aggregate endowment and the nominal interest rate are qualitatively the same. The main difference is that the size of the aggregate endowment response is smaller, and the trough occurs after six quarters.

The model responses of the aggregate endowment and the nominal interest rate are qualitatively the same as the empirical responses documented in the VAR literature. The nominal interest rate is persistently high, and the aggregate endowment decreases over time with a trough after several quarters. Two differences, which are not relevant for the purpose of this study, are that the nominal interest rate response is more persistent, and the aggregate endowment response has a trough later. The differences are due to different choices of variables, number of lags and identification criterion.

The full participation economy clearly fails to predict the response of the real interest rate. The real interest rate response, which is positive in the data, is negative in the model. The reason is that, in full participation economies, the traders' consumption is the same as the aggregate consumption, which, in equilibrium, is equal to the aggregate endowment. Since the expected aggregate endowment growth rate is negative, the traders' consumption Euler equation implies that the response of the real interest rate is negative as well.

Most full participation monetary models, like the benchmark sticky-price model of Clarida, Gali' and Gertler (1999), suffer from this important shortcoming. They counterfactually predict that the responses of the real interest rate and the expected aggregate endowment growth rate have the same sign. The reason why, in those models, the response of the real interest rate is positive is that they counterfactually predict that the response of the

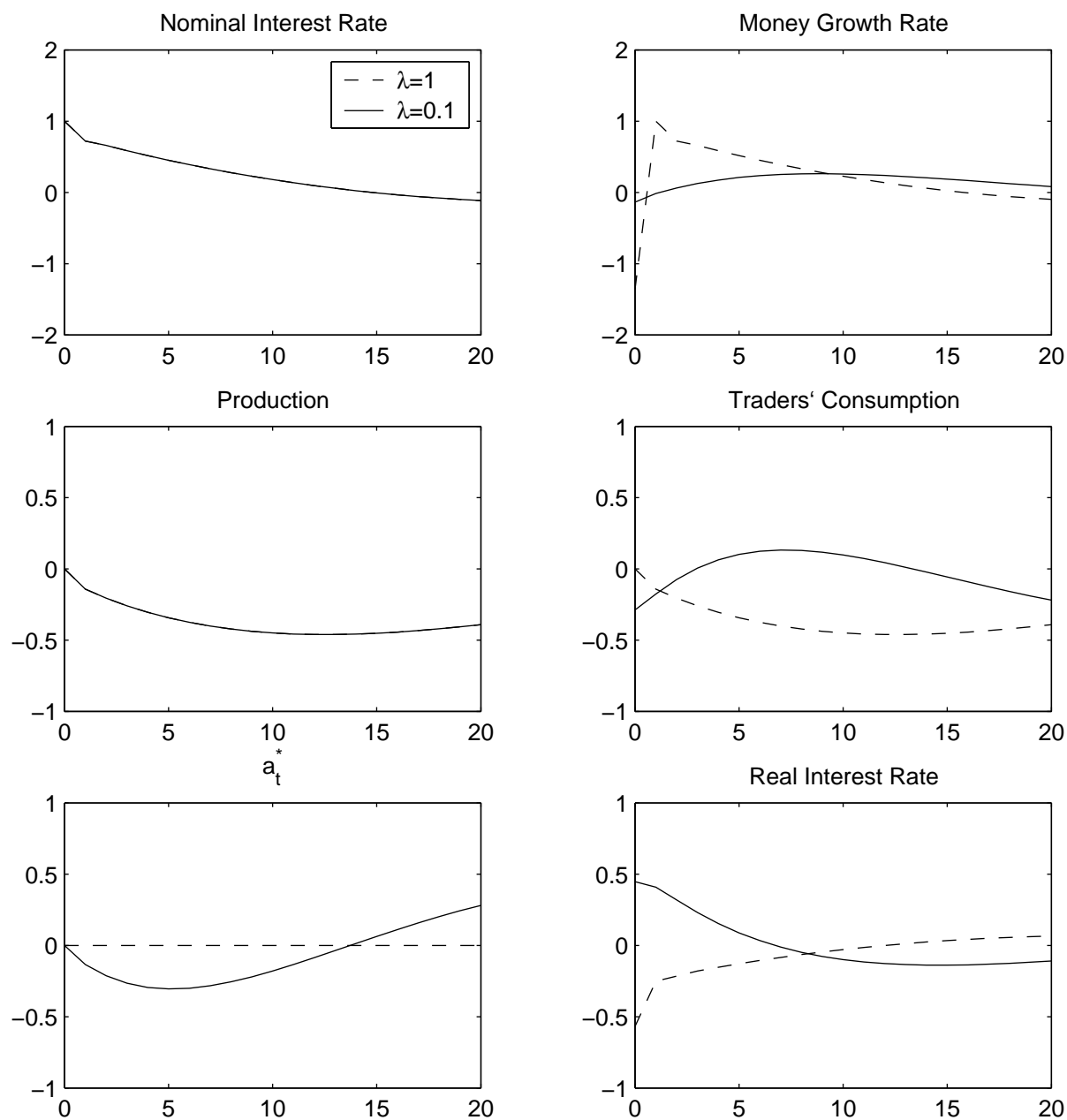


Figure 1: Model response to a contractionary monetary policy shock. The dashed and solid lines refer respectively to a full participation economy ( $\lambda = 1$ ) and a segmented markets economy ( $\lambda = 0.1$ ). The period utility function is logarithmic. The three rates are expressed in annual percentage points. The other variables are logarithms multiplied by 100.

expected aggregate endowment growth rate is positive. Once they predict the correct sign of the response of the expected aggregate endowment growth rate, they fail to predict the correct sign of the response of the real interest rate.

In the full participation model, moreover, the response of the money growth rate is negative in the impact period of a contractionary shock, and is positive in all the following periods. The evidence in Christiano, Eichenbaum and Evans (1999) indicates, however, that the response of the M1 and M2 growth rate are negative respectively for two and three quarters.

In the case of constant aggregate endowment, the response of the full participation economy to a contractionary monetary policy shock can be characterized analytically. We briefly present the solution because it is instructive and allows to verify analytically the determinacy of the equilibrium in the simplest case.

Since the aggregate endowment is constant,  $\hat{y}_t = 0$ , and, from the money definition equation (15j),  $\hat{m}_t = \hat{p}_t$ , money is proportional to prices. Since all households are traders,  $c^* = 0$ . Then, the goods market equilibrium condition (15i) implies that  $\hat{c}_t = 0$ , the traders' consumption is constant. From the traders' Euler equation (17),

$$\begin{aligned}\hat{i}_t - E_t[\hat{\pi}_{t+1}] &= 0 \\ \hat{r}_t &= 0\end{aligned}$$

deviations of the nominal interest rate and the expected inflation rate are the same, and the real interest rate is constant. Since the expected money growth rate is the same as the expected inflation rate, the expected money growth rate increases as the interest rate  $\hat{i}_t$  increases.

For simplicity, let us assume that the nominal interest rate follows the AR(1) process

$$\hat{i}_{t+1} = \rho \hat{i}_t + \sigma \eta_{t+1}$$

Since the system is linear, the expected response of the economy to a shock is the same as the deterministic evolution of the economy in the case that  $\eta_0 = 1$  and  $\eta_t = 0$  for all  $t > 0$ . We set the initial values of the state variables  $\hat{i}_{-1}$  and  $\hat{a}_0^*$  equal to zero, although, in the full participation case, the evolution of the system does not depend on the endogenous state variable  $\hat{a}_t^*$ . From the previous interest rate process, the deviation of the nominal interest rate from the steady state follows the exogenous evolution  $\hat{i}_t = \sigma \rho^t$ .

Since  $\hat{c}_t = 0$ , the traders' first-order condition for consumption (15b) implies that  $\hat{v}_t = -\hat{p}_t$ . The traders' Euler equation (15c) is then

$$E_t[\hat{p}_{t+1}] = \hat{p}_t - \hat{q}_t - \hat{\gamma}_t$$

The equation (15e) implies

$$\begin{aligned}q\gamma(\hat{q}_t + \hat{\gamma}_t) + (1-q)py \left( -\frac{q}{1-q}\hat{q}_t + \hat{p}_t \right) &= 0 \\ \hat{\gamma}_t + \hat{q}_t &= \frac{(1-q)py}{q\gamma} \left( \frac{q}{1-q}\hat{q}_t - \hat{p}_t \right)\end{aligned}$$

Plugging the expression for  $\hat{\gamma}_t + \hat{q}_t$  into the traders' Euler equation, we obtain the following first-order difference equation in  $E_t[\hat{p}_t]$ :

$$\begin{aligned} E_t[\hat{p}_{t+1}] &= \hat{p}_t - \frac{(1-q)py}{q\gamma} \left( \frac{q}{1-q} \hat{q}_t - \hat{p}_t \right) \\ E_t[\hat{p}_{t+1}] &= \left( 1 + \frac{(1-q)py}{q\gamma} \right) \hat{p}_t - \frac{py}{\gamma} \hat{q}_t \end{aligned}$$

The equation has an infinite number of solutions, depending on the initial value  $\hat{p}_0$ . However, since the coefficient of  $\hat{p}_t$  is strictly greater than one and the coefficient of  $E_t[\hat{p}_{t+1}]$  is equal to one, only one solution is stable.

Recall that  $\hat{i}_t = \sigma\rho^t$ , so, using the bond price identity (15a),  $\hat{q}_t = -\sigma\rho^t$ . The particular solution can be easily found guessing that  $E_t[\hat{p}_t] = K\rho^t$  for some constant  $K$ , and verifying the guess. Plugging the guess into the first-order difference equation,

$$\begin{aligned} K\rho^{t+1} &= \left( 1 + \frac{(1-q)py}{q\gamma} \right) K\rho^t + \frac{py}{\gamma} \sigma\rho^t \\ K &= -\frac{py}{\gamma} \sigma / \left( 1 - \rho + \frac{(1-q)py}{q\gamma} \right) < 0 \end{aligned}$$

Since  $K$  is negative,  $\hat{p}_t$  has the same dynamics as  $\hat{q}_t$ . Since  $\hat{i}_t \equiv -\hat{q}_t$ , prices (and money) decrease when the nominal interest rate increases. Once the price level has been determined, all the other variables can be derived. Notice that they all have the same functional form as  $\hat{q}_t$ , with different coefficients.

## 4.2 Market Segmentation

Figure 1 shows that, differently from the full participation model, the response of the segmented markets model is consistent with the evidence documented in the VAR literature. The real interest rate increases and remains positive. The money growth rate decreases and remains negative for three quarters. The size and the persistence of both responses are empirically plausible.

The reason why the real interest rate increases in the impact period is the following. As in the full participation model, the money supply decreases in the impact period of a contractionary monetary policy shock. In the segmented markets model, however, changes in the money supply have the following additional effect on the distribution of money and consumption across households, and on the real interest rate, an effect first described by Grossman and Weiss (1983) in a different limited participation model. Since the traders are a subset of all households, when the money supply decreases, the traders' money demand decreases more than proportionally. The goods price decreases proportionally because of the binding cash-in-advance constraints, so the traders' real money demand and consumption decreases. The traders' expected consumption growth rate increases, and the traders' consumption Euler equation implies that the real interest rate increases as well. The lower  $\lambda$ , the higher markets segmentation, the stronger the distributional effect of the money supply on the real interest rate.

The persistence of the responses of both the money growth rate and the real interest rate increases with markets segmentation. In the case of constant aggregate endowment, the following steps show analytically that the segmented markets model generates endogenous persistence.

In the case of market segmentation, the behavior of the economy depends on the endogenous state variable  $\hat{a}_t^*$ . In this case, we cannot find an analytical expression for the equilibrium price level, and we cannot fully characterize analytically the response of the economy to a monetary policy shock. However, the analysis of the full participation case suggests that, at least for moderate levels of market segmentation, money and prices still decrease after a contractionary monetary policy shock, that is the sign of  $\hat{m}_t$  and  $\hat{p}_t$  is negative while the sign of  $\hat{i}_t$  is positive in the impact period of the shock. This is what happens in all the numerical simulations presented in this paper. In this case, several important analytical results follow.

From the non-traders' constraints (15f), the goods market equilibrium condition (15i), it follows that

$$\hat{c}_t = \frac{c^*}{c}(\hat{p}_t - \hat{a}_t^*)$$

so the traders' consumption decreases with money and prices in the impact period of a contractionary monetary policy shock. A monetary policy shock, then, affects the distribution of cash balances and consumption between traders and non-traders. This effect, first described by Grossman and Weiss (1983), is crucially present in all limited participation economies.

In turn, changes in the traders' consumption affect the real interest rate. From the traders' Euler equation (17), it follows that

$$\begin{aligned}\hat{r}_t &= \frac{1}{\epsilon} E_t[\hat{c}_{t+1} - \hat{c}_t] \\ \hat{r}_t &= \frac{1}{\epsilon} \frac{c^*}{c} E_t[\hat{p}_{t+1} - \hat{a}_{t+1}^* - \hat{p}_t + \hat{a}_t^*] \\ \hat{r}_t &= \frac{1}{\epsilon} \frac{c^*}{c} E_t[\hat{p}_{t+1} - \hat{p}_t + \hat{\gamma}_t - \hat{p}_t + \hat{a}_t^*] \\ \hat{r}_t &= \frac{1}{\epsilon} \frac{c^*}{c} E_t[\hat{\pi}_{t+1} - \hat{p}_t + \hat{a}_t^*]\end{aligned}$$

where the second equation follows from the previous result, the third equation from the non-traders' budget constraint (15g), and the fourth equation from the inflation rate definition in (16). Then, using the real interest rate definition in (16),

$$\begin{aligned}\hat{r}_t &= \frac{c^*}{\epsilon c}(\hat{i}_t - \hat{r}_t - \hat{p}_t + \hat{a}_t^*) \\ \left(1 + \frac{c^*}{\epsilon c}\right) \hat{r}_t &= \frac{c^*}{\epsilon c}(\hat{i}_t - \hat{p}_t + \hat{a}_t^*) \\ \hat{r}_t &= \frac{c^*}{\epsilon c + c^*}(\hat{i}_t - \hat{p}_t + \hat{a}_t^*)\end{aligned}$$

so the real interest rate increases in the impact period of a contractionary monetary policy shock. The magnitude of this effect increases both with market segmentation through  $c^*$

and with the inverse of the intertemporal elasticity of substitution  $\epsilon$ , as previously pointed out by Alvarez and Atkeson (1996).

Finally, market segmentation introduces endogenous persistence into the dynamics of the model. For all periods  $t > 0$ ,

$$\begin{aligned}\hat{r}_t &= \frac{c^*}{\epsilon c + c^*}(\hat{i}_t - \hat{p}_t + \hat{a}_t^*) \\ \hat{r}_t &= \frac{c^*}{\epsilon c + c^*}(\hat{i}_t - \hat{\pi}_t)\end{aligned}$$

where the last equation follows from the non-traders' budget constraint (15g), and the inflation rate definition in (16). Then, substituting the expression for the real interest rate from the last equation into the real interest rate definition in (16),

$$\begin{aligned}E_t[\hat{\pi}_{t+1}] &= \hat{i}_t - \hat{r}_t \\ E_t[\hat{\pi}_{t+1}] &= \hat{i}_t - \frac{c^*}{\epsilon c + c^*}(\hat{i}_t - \hat{\pi}_t) \\ E_t[\hat{\pi}_{t+1}] &= \frac{\epsilon c}{\epsilon c + c^*}\hat{i}_t + \frac{c^*}{\epsilon c + c^*}\hat{\pi}_t\end{aligned}$$

so the expected inflation rate is a weighted average of the nominal interest rate and current inflation. The weight on current inflation increases with market segmentation and decreases with the intertemporal elasticity of substitution. Given the nominal interest rate process, the persistence of the inflation rate is passed on the other processes.

Hence, the inflation rate is endogenously persistent only if markets are segmented, and the endogenous persistence increases as markets segmentation increases. The same holds for the other variables. In the case of constant aggregate endowment, the money growth rate is the same as the inflation rate. Also, the Fisher equation determines the real interest rate as a function of the nominal interest rate, which is exogenous, and the expected inflation rate. Hence, the endogenous persistence of both the money growth rate and the real interest rate increases with markets segmentation as well.

### 4.3 Intertemporal Elasticity of Substitution

The two columns of figure 2 respectively plot the response of a full participation economy and a segmented markets economy to a contractionary monetary policy shock. For each economy, the figure shows the cases where the intertemporal elasticity of substitution  $\epsilon$  is set equal to 1 and 5.

In the full participation economy, changing the elasticity does not have any effect on the traders' consumption, since the traders' consumption is the same as the aggregate consumption, which is equal, in equilibrium, to the aggregate endowment. For the traders' consumption response to be the same in economies with different intertemporal elasticity of substitution, the higher the intertemporal elasticity of substitution, the smaller the real interest rate response. The full participation economy fails to predict the sign of the real interest rate response for any values of the intertemporal elasticity of substitution.

When markets are segmented, however, the monetary policy shock has the additional distributional effect on the real interest rate which has been described above. The resulting



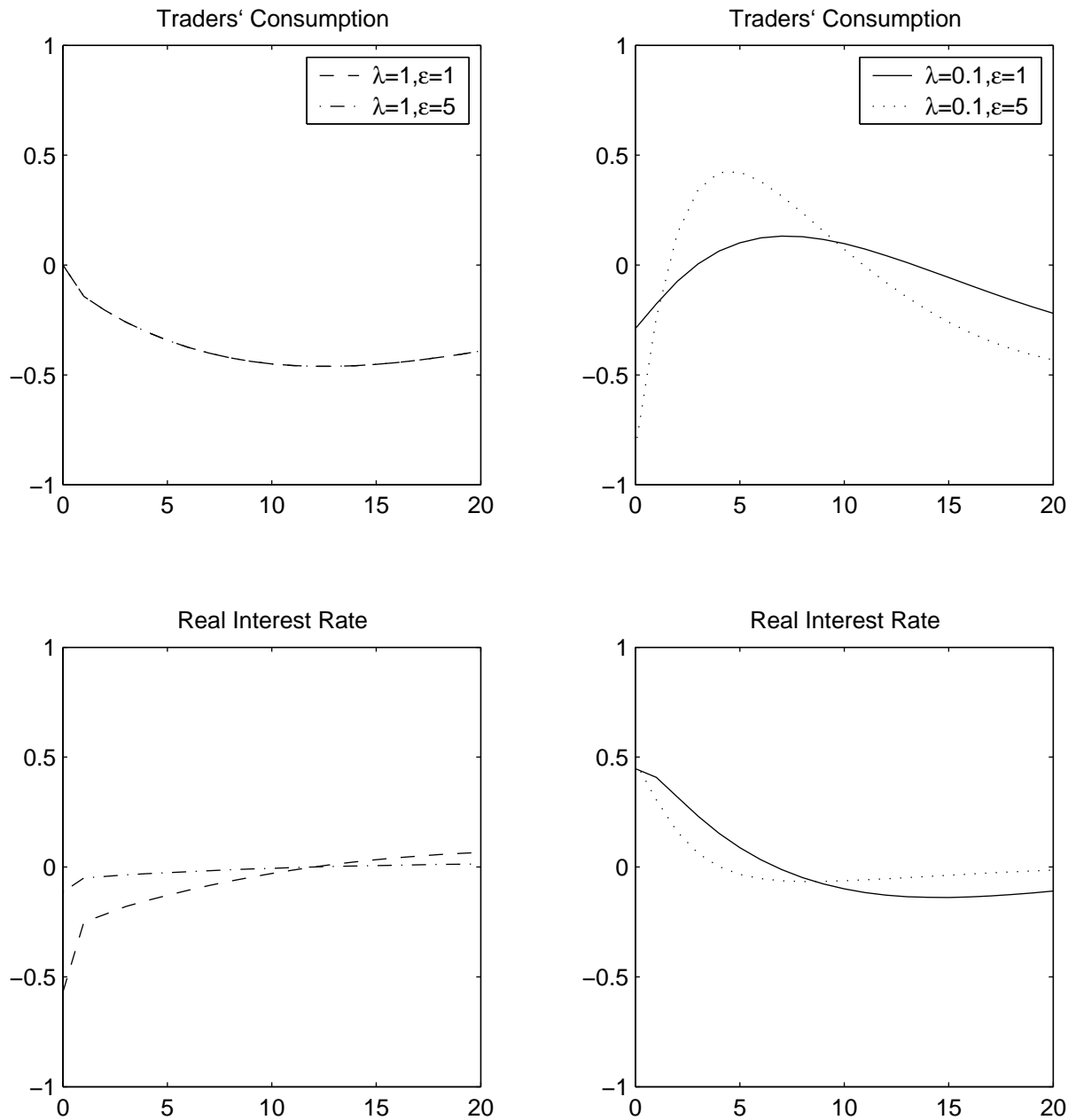


Figure 2: Model response to a contractionary monetary policy shock. The left column refers to a full participation economy with  $\lambda = 1$ , the right column refers to a segmented markets economy with  $\lambda = 0.1$ . The intertemporal elasticity of substitution  $\epsilon$  takes the values of 1 and 5. The real interest rate is expressed in annual percentage points. The logarithm of the traders' consumption is multiplied by 100.

real interest rate response to a contractionary monetary policy shock is positive, as it is empirically. Intuitively, the lower the traders' intertemporal elasticity of substitution, the smoother their consumption over time. When the traders' elasticity is lower, the response of the traders' consumption to a contractionary monetary policy shock is more persistent over time. From the traders' consumption Euler equation, the response of the real interest rate is approximately equal to the response of the expected traders' consumption growth rate divided by the traders' intertemporal elasticity of substitution. Hence, the real interest rate response is more persistent over time, like the traders' consumption response, which is consistent with the previous analytical result that the endogenous persistence of the model increases with market segmentation and decreases with the intertemporal elasticity of substitution.

## 5 Conclusion

Christiano, Eichenbaum and Evans (1999) argue that comparing the model response to a monetary policy shock with the empirical response is an important criterion for selecting a framework for monetary analysis. Following a contractionary monetary policy shock, the aggregate endowment decreases over time for several quarters, while the real interest remains high persistently, which can hardly be replicated by existing monetary models. This paper models exogenously the dynamics of the aggregate output and the nominal interest rate, and shows that the segmented markets model is able to account for the previous evidence. The sign, the size and the persistence of the responses of the real interest rate and the money growth rate are close to those in the data.

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