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Assessing Central Bank Credibility During the EMS Crises: Comparing Option and Spot Market-Based Forecasts

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Abstract

Financial markets embed expectations of central bank policy into asset prices. This paper compares two approaches that extract a probability density of market beliefs. The first is a simulated moments estimator for option volatilities described in Mizrach (2002); the second is a new approach developed by Haas, Mittnik and Paolella (2004a) for fat-tailed conditionally heteroskedastic time series. We find, in an application to the ERM crises of 1992-93, that both the options and the underlying exchange rates provide useful information for policy makers.

Keywords: options; implied probability densities; GARCH; fat-tails; European Monetary System;

JEL Classification: G12, G14, F31.

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1. Introduction

A basic insight of financial economics is that asset prices should reflect views about the future. For this reason, many economists rely on market prices to make predictions. Even when these views are incorrect, policy makers may want to avoid changes that the market is not expecting.

In recent years, some novel techniques have been introduced to extract market expectations. This paper explores two of them: extracting implied probability densities from option prices and volatility modeling of the underlying. Both methods have the advantage of producing predictive densities rather than just point forecasts. These tools can, in principal, allow central bankers to examine the full range of risks facing their economies.

There are numerous approaches that generalize the Black-Scholes model to obtain probabilistic information from options. Merton (1976) and Bates (1991) allow sudden changes in the level of asset prices. Wiggins (1987), Hull and White (1987), Stein and Stein (1991), and Heston (1993) allow volatility to change over time. A related literature, with papers by Dumas, Whaley and Fleming (1998) and Das and Sundaram (1999), has looked at deterministic variations in volatility with the level of the stock price or with time.

We utilize a method first used in Mizrach (2002) that looks directly at the probability distribution. We parameterize the underlying asset as a mixture of log normals, as in Ritchey (1990), and Melick and Thomas (1997), and fit the model to options prices. In an application to the Enron bankruptcy, Mizrach found that investors were far too optimistic about Enron until days before the stock's collapse.

Our second approach tries to extract information directly from the underlying currencies. We utilize a general mixture of two normal densities to extract information from the spot foreign exchange market. In this model, both the mixing weights as well as the parameters of the component densities, i.e., component means and variances, are time–varying and may depend on past exchange rates as well as further explanatory variables, such as interest rates. The dynamic mixture model we specify is a combination of the logistic autoregressive mixture with exogenous variables, or LMARX, model investigated in Wong and Li (2001) and the mixed normal GARCH process recently proposed by Haas, Mittnik and Paolella (2004a). The predictive densities generated from the resulting LMARX–GARCH model exhibit an enormous flexibility, and they may be multimodal, for example, in times where a realignment becomes more probable.

In this paper, we utilize the two approaches to explore market sentiment prior to the exchange

rate crises of September 1992 and July-August 1993. In the first episode, the British Pound (BP) and Italian Lira withdrew from the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS). The Pound had traded in a narrow range against the German Deutschemark (DM) for almost two years and the Lira for more than five. The crisis threw the entire plan for European economic and financial integration into turmoil. The French Franc (FF) remained in the mechanism, but speculative pressures against it remained strong. In the second crisis we examine, the Franc, in August 1993, had to abandon its very close link with the DM (the "Franc fort") and widen it's fluctuation band.

Campa and Chang (1996) have looked at ERM credibility using arbitrage bounds on option prices. They find that option prices reflected the declining credibility of the Lira and Pound in 1992 and the Franc in 1993. Malz (1996) finds an increasing risk of BP devaluation starting in late August 1992. Christoffersen and Mazzotta (2004) find useful predictive information in 10 European countries' over-the-counter currency options.

We first examine the options markets' implied probability of depreciation in the FF and BP prior to the ERM crises. The model estimates reveal that the market anticipated both events. The devaluation risk with the Franc rises significantly 11 days in advance of the crisis. With the Pound, the risk is subdued until only five days before it devalued on "Black Tuesday" September 14.

Vlaar and Palm (1993) were the first to use the normal mixture density to model EMS exchange rates against the DM, noting that, in contrast to freely floating currencies, these often show pronounced skewness, due to jumps which occur in case of realignments, but also, for example, as a result of expected policy changes or speculative attacks. Although Vlaar and Palm (1993) noted that making the jump probability a function of explanatory variables such as inflation and interest rates may be a promising task, they did not undertake such analysis.

Neely (1994) surveys research on forecasting realignments in the EMS and reports evidence for realignments to be predictable to some extent from information such as interest rates and the position of the exchange rate within the band. Building both on the results surveyed in Neely (1994) and the work of Vlaar and Palm (1993) and Palm and Vlaar (1997), Bekaert and Gray (1998) and Neely (1999) employ more general dynamic mixture models of exchange rates in target zones. Thus, the model employed below has some similarities with those developed in these studies, as will be discussed below. The dynamic mixture model provides, as in the options-based approach, estimates of the probability of a depreciation. For the FF, the model indicates a considerable increase of this probability one week in advance of the crisis, and a further increase immediately before the de facto devaluation of the FF, when the bands of the target zone were widened to $\pm 15\%$.

For the BP, we can, in contrast to the options-based approach, not develop a promising dynamic mixture model, because the BP joined the EMS only in October 1990 and withdrew in September 1992. During this period there were no realignments or large jumps within the band, so that there is no information that could be used to fit a target zone mixture model. Consequently, the mixed normal GARCH model detects a rise in the devaluation probability only *after* the Pound was withdrawn from the ERM.

Both models provide a complete predictive density for the exchange rate, and the last part of the paper examines the fit of the entire density. We utilize the approach of Berkowitz (2001) to produce formal comparisons. In the options market, the predictive density become indistinguishable from the post crisis density on July 21 for the FF, 11 days before the crisis. For the BP, there are some early warning signals in mid-August and the beginning of September. In the FF spot market, the predictive density is consistent with the post-crisis data from the outset. For the BP, the result is similar to the options. There are some brief early signals, but the densities statistically differ until September 10th.

The paper continues with some discussion of the ERM. Section 3 describes the theory of implied density extraction from options. It also proposes a mixture of log normals specification which nests the Black-Scholes. We also develop a GARCH mixture model for the spot exchange rate. Section 4 contains some stylized features of the currency options, and some detailed issues in estimation for both models. From the two sets of parameter estimates, we compute implied devaluation probabilities. Section 5 compares the entire predictive density statistically. Section 6 concludes with directions for future research.

2. The ERM

The ERM began in 1979 with seven member countries.¹ The mechanism included a grid of fixed exchange rates with European Currency Unit (ECU) central parities and fluctuation bands. Prior to the crises, the FF had a target zone of $\pm 2.25\%$ and the BP $\pm 6\%$. Maintaining the parities

¹ Belgium, Denmark, France, Germany, Italy, Ireland, and the Netherlands.

requires policy coordination with the Bundesbank, and when necessary, intervention.

By the Spring of 1992, the momenta towards a single European currency seemed irreversible. Spain had joined the ERM in June of 1989. Great Britain finally overcame its resistance in October 1990. Portugal joined in April 1992 bringing the total membership to ten. In addition, Finland and Sweden had been following indicative DM targets. All the major European currencies, save the Swiss Franc, were incorporated in a system of apparently stable exchange rate bands. Almost five years had passed without devaluations.² The financial sector seemed poised for integration, the next logical step in the blueprint of the Maastricht treaty signed on December 10, 1991.

A swift sequence of events left the idea of currency union almost irretrievably damaged. The Danes rejected the Maastricht treaty in June of 1992. The Finnish Markaa and the Swedish Krona faced devaluation pressures in August which the Bank of Finland and the Swedish Riksbank actively resisted. The Markaa was allowed to float on September 8th, and it quickly devalued 15% against the DM. The Riksbank raised their marginal lending rate to 500% on September 16th.

Then some of the core ERM currencies came under speculative attack. The Bank of England briefly raised their base lending rates, but the British chose to withdraw from the ERM on September 16th rather than expending additional reserves.³ The Lira devalued by 7% on September 13th and withdrew from the mechanism on September 17.

A number of additional devaluations followed. The Krona was allowed to float on November 19th. The Spanish Peseta (in September and November 1992), the Portuguese Escudo (in November 1992), and then the Irish Punt (in February 1993) subsequently adopted new parities. The ERM remained in turmoil into the summer. France faced continued pressure and went through a de facto devaluation when the ERM bands were widened to $\pm 15\%$ on August 2, 1993.

In retrospect, the origins of these crises were evident. The Finnish and Swedish economies were weakened by recession and the collapse of the Soviet Union. Britain had probably overvalued the Pound when it entered the ERM. The Lira had appreciated 30% in real terms against the DM since 1987. Germany had raised interest rates to fight off inflationary pressures from unification, weakening the entire European economy in the process.

The folklore of this period suggests that some market participants anticipated the crisis, and may even have precipitated it. The hedge fund trader George Soros is rumored to have made some US\$1 billion speculating against the Pound and the Lira in 1992.

 $^{^{2}}$ There was a small devaluation of the Italian Lira when it moved to narrow bands in January 1990.

 $^{^3}$ The Bundesbank is reported to have spent DM92bn defending the Pound and Lira during this crisis.

The question we ask here is how well diffused was this information. Did either the options market or spot market anticipate these events and can our models extract these expectations?

3. Models for Currency Options and the Spot Rate

3.1 Implied Probability Densities from Options

The basic option pricing framework builds upon the Black-Scholes assumption that the underlying asset is log normal. Let $f(S_T)$ denote the terminal risk neutral probability that S = x at time T, and let $F(S_T)$ denote the cumulative probability. A European call option at time t, expiring at T, with strike price K, is priced

$$C(K,\tau) = e^{-i_d\tau} \int_K^\infty (S_T - K) f(S_T) dS_T,$$
(1)

where $\tau = T - t$, and i_d and i_f are the annualized domestic and foreign risk-free interest rates. In the case where $f(\cdot)$ is the log-normal density and volatility σ is constant with respect to K, this yields the Black-Scholes formula,

$$BS(S_t, K, \tau, i_f, i_d, \sigma) = S_t e^{-i_f \tau} \Phi(d_1) - K e^{-i_d \tau} \Phi(d_2),$$
(2)
$$d_1 = \frac{\ln(S_t/K) + (i_d - i_f + \sigma^2/2)\tau}{\sigma\sqrt{\tau}},$$
(2)
$$d_2 = d_1 - \sigma\sqrt{\tau},$$

where $\Phi(\cdot)$ is the cumulative normal distribution. In this benchmark case, implied volatility is a sufficient statistic for the entire implied probability density which is centered at the risk-free interest differential $i_d - i_f$.

Mizrach (2002) surveys an extensive literature and finds that option prices in a variety of markets appear to be inconsistent with the Black-Scholes assumptions. In particular, volatility seems to vary across strike prices often with a parabolic shape called the volatility "smile." The smile is often present on only one part of the distribution giving rise to a "smirk."

3.1.1 How volatility varies with the strike

Under basic no-arbitrage restrictions, we can consider more general densities than the log-normal for the underlying. Breeden and Litzenberger (1978) show that the first derivative is a function of the cumulative distribution,

$$\partial C/\partial K \mid_{K=S_T} = -\exp^{-i_d\tau} (1 - F(S_T)). \tag{3}$$

The second derivative then extracts the density,

$$\partial^2 C / \partial K^2 \mid_{K=S_T} = \exp^{-i_d \tau} f(S_T).$$
(4)

The principal problem in estimating f is that we do not observe a continuous function of option prices and strikes. Early attempts in the literature like Shimko (1994) simply interpolated between option prices. As Mizrach (2002) notes, this often leads to arbitrage violations.

Later attempts turned to either specifying a density family for f or a more general stochastic process for the spot price. Dupire (1994) shows that both approaches are equivalent; for driftless diffusions, there is a unique stochastic process corresponding to a given implied probability density. This paper follows Ritchey (1990) and Melick and Thomas (1997) by specifying f as a mixture of log normal distributions. The advantage of this specification is that the option prices are just probability weighted averages of the Black-Scholes prices for each mixture.

3.1.2 A Mixture of Log Normals Specification

We assume that the stock price process is a draw from a mixture of three normal distributions, $\Phi(\mu_j, \sigma_j), j = 1, 2, 3$ with $\mu_3 \ge \mu_2 \ge \mu_1$. Three additional parameters p_1, p_2 and p_3 define the probabilities of drawing from each log normal. To nest the Black-Scholes, we restrict the mean to equal the interest differential, $\mu_2 = i_d - i_f$. Risk neutral pricing then implies restrictions on either the other means or the probabilities. We chose to let μ_1, p_1 and p_3 vary, which implies

$$\mu_3 = \mu_1 p_1 / p_3, \tag{5}$$

and

$$p_2 = 1 - p_1 - p_3. \tag{6}$$

For estimation purposes, this leaves six free parameters $\theta = (\theta_1, \theta_2, \dots, \theta_6)$. We take exponentials of all the parameters because they are constrained to be positive. The left-hand mixture is given by

$$\Phi(\mu_1, \sigma_1) = \Phi(i_d - i_f - \exp(\theta_1), 100 \times \exp(\theta_2)).$$
(7)

The only free parameter of the middle normal density is the standard deviation,

$$\Phi(\mu_2, \sigma_2) = \Phi(i_d - i_f, 100 \times \exp(\theta_3)).$$
(8)

We use the logistic function for the probabilities to bound them on [0, 1],

$$p_1 = \exp(\theta_4) / (1 + \exp(\theta_4)), \tag{9}$$

$$p_3 = \exp(\theta_5) / (1 + \exp(\theta_5)). \tag{10}$$

The probability specification implies the following mean restrictions on the third normal,

$$\Phi(\mu_3, \sigma_3) = \Phi\left((i_d - i_f - \exp(\theta_1)) \times \frac{\exp(\theta_4)/(1 + \exp(\theta_4))}{\exp(\theta_5)/(1 + \exp(\theta_5))}, 100 \times \exp(\theta_6) \right).$$
(11)

Mizrach (2002) shows that this data generating mechanism can match a wide range of shapes for the volatility smile.

3.2 GARCH Mixture Model for the Spot Exchange Rate

The mixed normal GARCH process is the building block of our models for the spot rate.⁴ It was recently proposed by Haas, Mittnik and Paolella (2004a) and generalizes the classic normal GARCH model of Bollerslev (1986) to the mixture setting. The percentage change of the log-exchange rate, $r_t = 100 \times (s_t - s_{t-1}) = 100 \times \log(S_t/S_{t-1})$, where S_t is the exchange rate at time t and $s_t = \log(S_t)$, is said to follow a k-component mixed normal (MN) GARCH(p, q) process if the conditional distribution of r_t is a k-component MN, that is,

$$r_t | \Psi_{t-1} = MN(\lambda_{1,t}, \dots, \lambda_{k,t}, \mu_{1,t}, \dots, \mu_{k,t}, \sigma_{1,t}^2, \dots, \sigma_{k,t}^2),$$
(12)

where Ψ_t is the information at time t; $\lambda_j \in (0, 1)$, j = 1, ..., k, and $\sum_j \lambda_j = 1$. The $k \times 1$ vector of component variances, denoted by $\sigma_t^{(2)} = [\sigma_{1,t}^2, \ldots, \sigma_{k,t}^2]'$, evolves according to

$$\sigma_t^{(2)} = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^{(2)}, \tag{13}$$

where α_0 is a positive $k \times 1$ vector; α_i , i = 1, ..., q, are nonnegative $k \times 1$ vectors; and β_i , i = 1, ..., p, are nonnegative $k \times k$ matrices, and

$$\epsilon_t = r_t - E(r_t | \Psi_{t-1}) = r_t - \sum_{j=1}^k \lambda_{j,t} \mu_{j,t}.$$
 (14)

Haas, Mittnik and Paolella (2004a) considered the case where the mixing weights, $\lambda_{j,t}$, and the component means, $\mu_{j,t}$, $j = 1, \ldots, k$, are constant over time, but the generalization considered in equations (12)–(14), with these quantities being time–varying, is straightforward conceptually. The mixing weights and the component means may depend both on lagged values of r_t and on further explanatory variables, as in the LMARX model of Wong and Li (2001). Thus, the dynamic mixture model employed in the present paper is a combination of the MN–GARCH and the LMARX models,

⁴ For an application of a related model class, the Markov-switching GARCH model, to predicting exchange rate densities, see Haas, Mittnik and Paolella (2004b).

which will be termed LMARX–GARCH.

As with the classic GARCH model, the MN–GARCH(1,1) specification will usually be sufficient, and in most applications it will be reasonable to impose certain restrictions on the α_i 's and β_i 's in (13). However, the general formulation will be useful in discussing different versions of the MN–GARCH process corresponding to different restrictions imposed on the parameters.

3.2.1 Conditional density

We assume that the conditional density of the exchange rate return process, r_t , is a two-component normal mixture density, that is,

$$f(r_t|\Psi_{t-1}) = \frac{\lambda_t}{\sigma_{1,t}\sqrt{2\pi}} \exp\left\{-\frac{(r_t - \mu_{1,t})^2}{2\sigma_{1,t}^2}\right\} + \frac{1 - \lambda_t}{\sigma_{2,t}\sqrt{2\pi}} \exp\left\{-\frac{(r_t - \mu_{2,t})^2}{2\sigma_{2,t}^2}\right\},\tag{15}$$

where Ψ_t is the information available up to time t, which consists of the exchange rates up to time t as well as further explanatory variables, such as interest rates.

With probability λ_t , there is a jump in the exchange rate, due to a realignment or a relatively large movement within the target zone. As in Bekaert and Gray (1998) and Neely (1999), the mixing weight, or probability of a jump, λ_t , depends on the slope of the yield curve, $yc_t = i_t^3 - i_t^1$, where i_t^3 and i_t^1 are the three- and one-month interest rates, respectively. The functional relationship is specified in a logistic fashion. More specifically, we assume that

$$\lambda_t = \frac{1}{1 + \exp\{\gamma_0 + \gamma_1 y c_{t-1}^{\star}\}},\tag{16}$$

where $yc_t^* = sign(yc_t) \log(1 + |yc_t|)$. We have also considered a probit specification in (16), where $\lambda_t = \Phi(\gamma_0 + \gamma_1 y c_{t-1}^*)$, and $\Phi(z) = (2\pi)^{-1/2} \int_{-\infty}^z e^{-\xi^2/2} d\xi$ is the standard normal cumulative distribution function. This specification is used in Mizrach (1995), Bekaert and Gray (1998), and Neely (1999), but it yields to virtually the same relation between λ_t and yc_{t-1} for the data at hand.⁵ Beine and Laurent (2003) and Beine, Laurent, and Lecourt (2003) use the logistic specification in modeling returns of the US\$ against other major currencies, where the mixing weight depends on central bank interventions. In addition to using the probit specification, Bekaert and Gray (1998) and Neely (1999) work in terms of the untransformed variable yc_t , that is, they set $\lambda_t = \Phi(\gamma_0 + \gamma_1 y c_{t-1})$.⁶ The motivation for our use of the contracting transformation yc_t^* is illus-

 $[\]frac{5}{5}$ A generalization of the probit–approach to more than two mixture components is considered in Lanne and Saikkonen (2003).

 $^{^{6}}$ Actually, Neely (1999) uses short–term interest rate differentials as a second explanatory variable. The latter and the slope of the yield curve are highly correlated, however, with a correlation coefficient of -0.7591 in our training sample.

trated in Figure 5, which plots the next period's return r_t against $y_{c_{t-1}}$ and $y_{c_{t-1}}^{\star}$, respectively, for the 179 monthly observations that form our estimation period. Obviously, using $y_{c_{t-1}}$ directly, estimated relationships between $y_{c_{t-1}}$ and the next period's density of r_t will suffer from the single large "outlier" min{ y_{c_t} } = -32.

[Insert Figure 5]

The mean of the jump-component, μ_{1t} , is also assumed to depend on yc_{t-1} , where we let

$$\mu_{1,t} = \phi_0 + \phi_1 y c_{t-1}^\star. \tag{17}$$

The second mixture component in (15) represents the density of the exchange rate when the target zone is credible, so that, as in Neely (1999), it is plausible to let μ_{2t} depend on the position of the exchange rate within the target zone. More specifically,

$$\mu_{2,t} = \psi_0 + \psi_1 (S_{t-1} - P_{t-1}), \tag{18}$$

where P_t is the central parity at date t.

Finally, we discuss the conditional heteroskedasticity in the component variances $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$. To do so, we reproduce the defining equation of the MN–GARCH process specified by Haas, Mittnik, and Paolella (2004a) for the two–component GARCH(1,1) case, where (13) becomes

$$\begin{bmatrix} \sigma_{1,t}^2 \\ \sigma_{2,t}^2 \end{bmatrix} = \begin{bmatrix} \alpha_{01} \\ \alpha_{02} \end{bmatrix} + \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \end{bmatrix} \epsilon_{t-1}^2 + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} \sigma_{1,t-1}^2 \\ \sigma_{2,t-1}^2 \end{bmatrix}$$
(19)

where $\epsilon_t = r_t - E(r_t|\Psi_{t-1}) = r_t - \lambda_t \mu_{1t} - (1 - \lambda_t)\mu_{2t}$. Vlaar and Palm (1993) assume that, for all t, the difference between $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$ is equal to a constant jump size, δ^2 ; that is, they restrict, in (19), $\alpha_{01} = \alpha_{02} + \delta^2$, $\alpha_{11} = \alpha_{12}$, $\beta_{12} = \beta_{22}$, and $\beta_{21} = \beta_{11} = 0$, so that $\sigma_{1,t}^2 = \sigma_{2,t}^2 + \delta^2$ for all t. Vlaar and Palm (1993) argue that "this procedure is preferred to that of independent variances, since it seems reasonable to assume that the same GARCH effect is present in all variances." This specification is also adopted in Neely (1999) and Beine and Laurent (2003). We will, however, not use this, but employ a restricted version of (19), termed "partial MN–GARCH" in Haas, Mittnik, and Paolella (2004a), which sets $\alpha_{11} = 0$, and $\beta_{11} = \beta_{12} = \beta_{21} = 0$, so that $\sigma_{1,t}^2 = \sigma_1^2 = \alpha_{01}$ for all t. That is, only the variance in the "credibility regime" is driven by a GARCH process, while the variance in the jump component is constant. This specification seems more reasonable, given that, in a system of target zones, jumps are not expected to come clustered, so that a dynamic behavior of the jump component's variance would be difficult to interpret.

4. Data and Estimation Results

4.1 Options Market

4.1.1 Data

The majority of the intra-ERM derivatives trading is in the over-the-counter markets, and the data is not generally available to non-traders. The best publicly available data are for US dollar (US\$) exchange rates which are traded in Philadelphia. We focus on the US Dollar/British Pound (US\$/BP) and Dollar/French Franc (US\$/FF) contracts. We have data for the years 1992 and 1993, which encompass both major ERM realignments.

The US\$ appears to be an adequate proxy for the DM. During September 1992, the DM depreciated by -1.47% against the US\$, while the BP depreciated -11.51%. From July 1 to August 5, 1993, the DM was similarly stable, depreciating -0.83%, while the Franc devalued by -3.59% against the US\$.

Both American⁷ and European options are traded. The BP options are for 31,250 Pounds, and the FF options are for 250,000 Francs. We use daily closing option prices that are quoted in cents. Spot exchange rates are expressed as US\$ per unit foreign and are recorded contemporaneously with the closing trade. Foreign currency appreciation (depreciation) will increase the moneyness of a call (put) option. Interest rates are the Eurodeposit rates closest in maturity to the term of the option.

To obtain a rough idea about the implied volatility pattern in the currency options, we look at sample averages. We sort the data into bins based on the strike/spot ratio, S/K, and compute implied volatilities using the Black-Scholes formula. In Figures 1 and 2, we plot the data for all of 1992 and 1993, for the BP and FF, respectively.

[INSERT Figure 1 Here] [INSERT Figure 2 Here]

Both appear to display the characteristic pattern, with the minima of the implied volatility at the money, and with higher implied volatilities in the two tails.

⁷ Currency options may be thought of as options on a dividend paying stock where the dividend is equal to the foreign risk free rate. Early exercise is relevant for call options where the foreign risk free rate is high because this indicates that the currency is likely to devalue. The risk of devaluation will then be priced into American options of all maturities.

For estimation purposes, we excluded options that were more than 10% in or out of the money and with volumes less than 5 contracts. This seemed to eliminate most data points with unreasonably high implied volatilities. For the Pound, we looked at options from 5 to 75 days to maturity. Because the data were thinner with the Franc, we utilized all maturities greater than 5 days.

We will now try to infer whether changes in the smile signalled an impending crisis in the ERM.

4.1.2 Implied Density Estimation

There are two key issues in fitting the model. The first is to extend the analysis to American options which can be exercised before expiration. The second is choosing the loss function for estimation.

We approximate American puts and calls using the Bjerksund and Stensland (1993) approach. Hoffman (2000) shows that the Bjerksund-Stensland algorithm compares favorably in accuracy and computational efficiency to the Barone-Adesi and Whaley (1987) quadratic approximation. Our estimates were also quite similar using implied binomial trees.

Because $f(S_t)$ is the risk neutral density and is not directly observable, we must find a way to treat the options prices as sample "moments". Let

$$\{d_{j,t}\}_{j=1}^n = [\{c(\tau_1, K_1), \dots, c(\tau_m, K_m), p(\tau_{m+1}, K_{m+1}), \dots, p(\tau_n, K_n)\}]$$

denote a sample of size n of the calls c and puts p traded at time t, with strike price K_j and expiring in τ_j years. Denote the pricing estimates from the model as $\{d_{j,t}(\theta)\}$.

In matching model to data, Christoffersen and Jacobs (2001) emphasize that the choice of loss function is important. Bakshi, Cao and Chen (1997), for example, match the model to data using option prices. This can lead to substantial errors among the low priced options though. Since these options are associated with tail probability events, this is not the best metric for our exercise. We obtained the best fit overall using the implied Bjerksund-Stensland implied volatility,

$$\sigma_{j,t} = BJST^{-1}(d_{j,t}, S_t, i_t).$$
(20)

Let the estimated volatility be denoted by

$$\sigma_{j,t}(\theta) = BJST^{-1}(d_{j,t}(\theta), S_t, i_t).$$
(21)

We then minimize the sum of squared deviations from the implied volatility in the data,

$$\min_{\theta} \sum_{j=1}^{n} (\sigma_{j,t}(\theta) - \sigma_{j,t})^2.$$
(22)

As Christoffersen and Jacobs note, this is just a weighted least squares problem that, with the

monotonicity of the option price in θ , satisfies the usual regularity conditions.

We next fit (22) to daily option prices for the FF and BP in intervals around the two crises. We first extract the model's ability to predict a depreciation of at least 3% in a four-week horizon. We chose the jump size to be large enough to for the BP to escape from the midpoint of the upper half of the band. We defer discussion of the entire predictive density until Section 5 after we develop forecasts using both options and spot market models.

4.1.3 French Franc Options Estimates

We estimate the six parameter model day-by-day from July 18 to August 5, 1993 for the FF. We report coefficient estimates, *t*-ratios, and R^2 in Table 1. The model describes the option prices well with an average goodness of fit of 97%.

[INSERT Table 1]

The devaluation risk, depicted in Figure 3, starts at less than 1% on July 18, quickly rises to nearly 23% on July 20 and peaks at nearly 25% on July 26. The risk stays above 20% for 6 of the 7 days prior to the FF's de facto devaluation.

[INSERT Figure 3 Here]

This exercise, we feel, is largely successful. The model fits the data well and provides a sharp increase in devaluation risk 11 days before the FF bands widen. In principal, this could provide sufficient time for the central bank to react to market expectations.

4.1.4 British Pound Options Estimates

We next estimate the model for August 19 to September 29, 1992 for the BP. We report coefficient estimates, *t*-ratios, and R^2 in Table 2. The model again captures the data well with an average R^2 of 96%.

[INSERT Table 2]

The devaluation risk, which is graphed in Figure 4, starts at almost 15% on August 19, 1992. It rises steadily into the crisis, except for two steep declines on September 4 and September 11, 1992. The risk exceeds 20% for 17 out of 18 trading days prior to the BP devaluation on September 17, 1992.

[INSERT Figure 4 Here]

The options again provide a potential early warning signal to policy makers. The devaluation risk reaches 20%, the critical level for the Franc, 27 days before the British Pound leaves the ERM.

We now turn to the spot market volatility to search for possible signals of the crises.

4.2 The Spot Market

4.2.1 French Franc

As we do not model the dynamics of the interest rates, and are interested in one-month-ahead density forecasts, we estimate the LMARX-GARCH model with monthly data. For the FF, we use monthly percentage returns, $r_t = 100 \times \log(S_t/S_{t-1})$, from May 1979 to July 16, 1993, a total of 179 monthly observations. Maximum likelihood estimates⁸ are reported in Table 3.

[Insert Table 3]

As expected, $\gamma_1 > 0$ and $\phi_1 < 0$, so that both the probability of a jump, λ_t , as well as the expected jump size, $\mu_{1,t}$, increase when the yield curve inverts. Also, $\psi_1 < 0$, that is, there is mean reversion when the target zone is credible.

From the fitted model, we compute the four-week ahead densities for the period from July 16 to August 5, 1993. The implied densities of the percentage log-change of the FF against the DM four weeks from the trading date are summarized in Table 4.

[Insert Table 4]

To illustrate the time-variation of the density forecasts depending on interest rates and the past observations of the exchange rate, Figure 6 shows the predictive densities calculated for July 21 and July 30, respectively. While the density forecast of July 21 is somewhat skewed to the right, the density of July 30 exhibits a bimodal shape, three days before the bands were widened on August 2, corresponding to a de facto devaluation of the FF. The normal mixture densities extracted from the time series of currency prices demonstrate a considerable increase in downside risk at least a week before the de facto devaluation of the Franc, with a further sharp increase immediately before the widening of the target zone, that is, on July 30. The probabilities are shown in Figure 7.

⁸ See Haas, Mittnik and Paolella (2004a) for a discussion of maximum likelihood estimation. Alexander and Lazar (2004) provide further results on the specification and estimation of GARCH(1,1) mixture models.

[Insert Figure 6] [Insert Figure 7]

4.2.2 British Pound

Given the short period of time the BP belonged to the ERM, we do not necessarily expect to fit a meaningful model as we did for the FF. In contrast to the FF model, we assume for the BP that both the mixing weight, λ , as well as the component means, μ_1 and μ_2 , are constant. By estimating the *diagonal* version of the general specification (19), i.e., a model with $\beta_{12} = \beta_{21} = 0$, it turns out that the partial MN–GARCH model, as introduced in the discussion following equation (19), is the appropriate specification here.

We make use of pre–ERM data, that is, we use monthly returns from January 1979 to August 19, 1992 (177 observations), to fit the MN–GARCH model of equation (19). The parameter estimates for this model are reported in Table 5.

[Insert Table 5]

The first component's (constant) variance, σ_1^2 , is quite large, and its mixing weight, λ_1 , is relatively small. However, identifying this regime as the jump component is problematic, given that its mean, μ_1 , although estimated to be positive (devaluation of the BP), has a very large standard error and is not significantly different from zero.

The implied densities of the percentage log-change of the BP against the DM four weeks from the trading date, for the period August 19 to September 29, 1992 are summarized in Table 6, while the probabilities of a devaluation of at least 3% are shown in Figure 8.

As expected, lacking characteristic information in the sample used for estimation, the mixed normal GARCH model for the BP does not anticipate the withdrawal of the pound from the EMS, and so, the probability of a large devaluation rises only ex-post. The latter effect is due to the GARCH component in the mixture model, i.e., the relatively large changes of the exchange rate following the withdrawal induce a large $\sigma_{2,t}^2$, relative to its value preceding the crisis.

5. Comparison of Predictive Densities

In this section, we evaluate the forecast densities produced by our two models. The approach we

take is the one originally proposed by Berkowitz (2001). Let $f(s_t)$ be the probability density of the spot exchange rate, and let $F(s_t)$ be the cumulative distribution.

$$F(s_t) = \int_{-\infty}^{s_t} f(u) du.$$

Berkowitz notes that estimates $\hat{F}(s_t)$ are uniform, independent, and identically distributed under fairly weak assumptions.

Testing for an independent uniform density in small samples can be problematic, so Berkowitz suggests transforming the data into normal random variates,

$$z_t = \Phi^{-1}(\widehat{F}(s_t))$$

where $\Phi(\cdot)$ is the normal distribution.⁹ The likelihood ratio,

$$LR = \sum_{t=1}^{20} (z_t^2 / \hat{\sigma}^2 - 1), \qquad (23)$$

where $\hat{\sigma}$ is the forecast standard deviation, is then distributed $\chi^2(1)$ for the null hypothesis that the transformed forecast statistics, z_t , have mean zero.

5.1 Option forecasts

We test the forecast densities for the FF from July 20 to August 29, 1993. Likelihood-ratio statistics are in the last column of Table 1. At the 10% level, we can accept the null that our forecast could have generated the subsequent four weeks of trading data from July 21 through the rest of the crisis. After that point, our model is statistically indistinguishable from the post-crisis density except for two days in August.

We do the same exercise for the BP for the period August 20 to September 29, 1992. There are stronger rejections prior to this crisis. Nonetheless, on August 20, 21 and September 1 and 2, we have a forecast consistent with the four-week returns data at the 10% level.

5.2 Spot market forecasts

Ignoring non-trading days, as we do in model specification and estimation. the 20-trading dayahead forecast density is given by a mixture of two normals, namely,

$$f(r_{t+20}|\Psi_t) = \sum_{j=1}^2 \lambda_{j,t+20} \frac{1}{\sqrt{2\pi}\sigma_{j,t+20}} \exp\left\{-\frac{(r_{t+20} - \mu_{j,t+20})^2}{2\sigma_{j,t+20}^2}\right\},\tag{24}$$

where $r_{t+20} = 100 \times (\log S_{t+20} - \log S_t)$. Under constancy assumptions, we can scale the 20-day

 $^{^{9}}$ We use the numerical transformation for the inverse normal proposed by Wichura (1988).

ahead densities to obtain daily log-changes $r_{t+\tau}^d := 100 \times (\log S_{t+\tau} - \log S_{t+\tau-1}), \tau = 1, \dots, 20,$ implying a two-component normal mixture distribution, given by

$$f(r_{t+\tau}^d|\Psi_t) = \sum_{j=1}^2 \lambda_{j,t+20} \frac{1}{\sqrt{2\pi}\sigma_{j,t+20}/\sqrt{20}} \exp\left\{-\frac{(r_{t+\tau}^d - \mu_{j,t+20}/20)^2}{2\sigma_{j,t+20}^2/20}\right\}.$$
 (25)

Expression (25) can be used to compute the cumulative distribution function $F(r_{t+\tau}^d|\Psi_t)$, and transformation $z_t = \Phi^{-1}(F(r_{t+\tau}^d|\Psi_t)), \tau = 1, ..., 20$. Then, given that our density predictions are correctly specified, the likelihood ratio (23) again has an approximate $\chi^2(1)$ distribution. Using (23), we test for a correct specification of the mean of our forecast distribution. In the present situation, we would of course like to test for additional properties of the forecast density, such as skewness, reflecting in our mixture models some sense of the realignment risk, or kurtosis. However, with only 20 data points at hand, any test involving higher-order forecast moments is highly questionable.

The test results are reported in Tables 6 and 7 for the FF and BP, respectively. For the latter, the parameters of the 20–days–ahead forecast densities are not reported, given that they are all constant with the exception of $\sigma_{2,t}^2$. In terms of the LR test (23), the dynamic mixture model performs well for the FF, but exhibits a poor performance in predicting the crisis of the BP.

6. Conclusion

Viewed through some recent tools, asset prices can provide insights to about the entire probability distribution of future events. This paper has utilized the mixture of log normals in two separate contexts: with options and with the underlying currencies.

The ERM case was certainly an epochal event for the markets. Central bankers became aware, perhaps for the first time, that the markets might be an irresistible force.

Policy makers may find these tools and inference worthwhile in a variety of contexts. Their subjective weights between type I and type II errors should not only be tested ex-post but incorporated directly in the estimation. Both Skouras (2001) and Christoffersen and Jacobs (2001) have made progress along these lines. Loss aversion on the part of investors and traders may give them similar preferences.

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Date	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	R^2	LR
16-Jul-1993	-8.000 (0.00)	-5.346 (0.04)	$\underset{(0.00)}{3.766}$	-4.908 (12.53)	$\begin{array}{c} 3.232 \\ \scriptscriptstyle (6.50) \end{array}$	-7.740 (0.00)	0.910	
19-Jul-1993	-0.174 (0.00)	$\underset{(0.80)}{1.928}$	-5.876 (1.82)	$\mathop{5.848}\limits_{(0.00)}$	-2.269 (1.27)	-1.945 (0.00)	0.990	$\underset{(0.12)}{2.3752}$
20-Jul-1993	-1.019 (0.69)	-1.720 (108.26)	-0.191 (1.79)	-3.441 (0.00)	-4.006 (0.00)	-4.233 (0.00)	0.990	$\underset{(0.10)}{2.7213}$
21-Jul-1993	-1.125 (0.00)	-1.042 (0.01)	-3.579 $_{(0.01)}$	-0.474 (0.00)	-2.567 (0.13)	-2.056 (0.05)	0.999	$1.1477 \\ (0.28)$
22-Jul-1993	-1.505 $_{(0.02)}$	$\underset{(0.29)}{-3.058}$	-4.188 (0.00)	-6.801 (28.36)	$\begin{array}{c}9.809\\(0.00)\end{array}$	-2.147 (0.00)	0.997	$\underset{(0.19)}{1.7291}$
23-Jul-1993	-2.504 (0.03)	-2.460 (0.22)	-0.325 $_{(0.00)}$	-2.405 (0.01)	-1.034 (0.01)	-2.040 (0.07)	0.993	$\underset{(0.64)}{0.2145}$
26-Jul-1993	-0.393 (0.01)	-1.835 (0.09)	-1.047 (0.01)	-0.014 (0.01)	-2.218 (0.16)	-2.064 (0.05)	0.990	$\underset{(0.28)}{1.1618}$
27-Jul-1993	-3.945 (0.06)	-2.111 (36.96)	$\underset{(0.00)}{17.480}$	-4.701 (0.03)	-6.418 (0.00)	-4.795 (0.00)	0.994	$\underset{(0.14)}{2.3117}$
28-Jul-1993	$\underset{(0.00)}{0.394}$	-2.183 $_{(0.09)}$	-1.007 (0.00)	-1.524 (0.01)	-2.798 (0.06)	-1.955 $_{(0.09)}$	0.999	$\underset{(0.14)}{2.1560}$
29-Jul-1993	-1.231 (0.00)	-2.162 (0.02)	-1.040 (0.00)	-0.328 (0.00)	-2.110 (0.03)	-1.826 (0.01)	0.945	$\underset{(0.42)}{0.6379}$
30-Jul-1993	-1.355 (0.01)	-1.637 (0.05)	-1.490 (0.01)	-0.464 (0.00)	-2.386 (0.14)	-1.873 (0.05)	0.977	$1.1151 \\ (0.29)$
2-Aug-1993	-1.029 (0.01)	-0.035 $_{(0.01)}$	-4.371 (0.73)	-0.324 (0.00)	-2.139 (0.73)	-2.222 (1.20)	0.991	$\underset{(0.11)}{3.2378}$
3-Aug-1993	$\underset{(0.23)}{3.192}$	-0.562 (0.06)	-3.473 $_{(0.21)}$	-0.332 (0.07)	-3.207 (1.52)	-1.930 (1.14)	0.986	$\underset{(0.14)}{2.2335}$
4-Aug-1993	-1.007 (0.01)	-1.687 (0.02)	-1.547 (0.00)	-0.455 (0.00)	-2.472 (0.07)	-2.060 (0.04)	0.912	$\underset{(0.20)}{1.6081}$
5-Aug-1993	$\underset{(0.01)}{0.083}$	-3.169 (2.10)	-0.776 $_{(0.20)}$	-1.132 (0.01)	-1.880 (0.06)	-1.938 (0.12)	0.974	$\underset{(0.24)}{1.3703}$

 Table 1: French Franc Options Model

The $\theta's$ are estimates of the model (22). *t*-ratios are in parentheses. The *LR* statistic, with *p*-values underneath, is given by (23) and is distributed $\chi^2(1)$.

Date	θ_1	θ_2	θ_3	$ heta_4$	θ_5	θ_6	R^2	LR
19-Aug-1992	$\begin{array}{c}-30.857\\\scriptscriptstyle{(0.00)}\end{array}$	-2.468 (5.96)	$\begin{smallmatrix} 6.351 \\ \scriptscriptstyle (0.02) \end{smallmatrix}$	-5.617 $_{(5.51)}$	$\underset{(1.84)}{1.879}$	-1.982 (0.00)	0.995	
20-Aug-1992	$\begin{array}{c}-2.179\\\scriptscriptstyle(0.01)\end{array}$	$\underset{(0.15)}{-1.908}$	$\begin{smallmatrix} -0.816 \\ \scriptscriptstyle (0.01) \end{smallmatrix}$	-1.855 $_{(0.01)}$	$\underset{(0.07)}{-1.908}$	$\underset{(0.63)}{-2.440}$	0.935	$\begin{smallmatrix}1.5008\\(0.22)\end{smallmatrix}$
21-Aug-1992	-1.620 (0.01)	-1.623 (0.07)	$\underset{\scriptscriptstyle(0.02)}{-2.679}$	$\underset{\scriptscriptstyle(0.01)}{-1.099}$	-1.859 $_{(0.14)}$	$\underset{\scriptscriptstyle(0.90)}{-2.435}$	0.973	$\underset{(0.12)}{2.4618}$
24-Aug-1992	-2.374 (0.26)	$\begin{array}{c}-2.609\\\scriptscriptstyle (4.98)\end{array}$	$\left \begin{array}{c} -0.452 \\ \scriptscriptstyle (0.27) \end{array} \right $	$\underset{\scriptscriptstyle(0.08)}{-1.805}$	$\underset{(0.37)}{-1.409}$	$\underset{\scriptscriptstyle(1.01)}{-1.897}$	0.967	$\underset{(0.06)}{3.5960}$
25-Aug-1992	-17.296 (0.00)	-2.183 (15.64)	$\underset{(0.01)}{4.381}$	-4.843 (2.67)	$\underset{(0.63)}{0.987}$	$\underset{\scriptscriptstyle(0.00)}{-2.163}$	0.974	$7.8066 \\ \scriptscriptstyle (0.01)$
26-Aug-1992	-2.306 $_{(0.12)}$	-2.614 (1.10)	-0.467 $_{(0.05)}$	-2.018 $_{(0.05)}$	-1.595 $_{(0.30)}$	-1.865 $_{(0.60)}$	0.988	$\underset{(0.01)}{6.5960}$
27-Aug-1992	-2.601 (0.05)	-2.465 $_{(0.57)}$	-0.479 $_{(0.02)}$	-1.722 $_{(0.02)}$	-1.706 $_{(0.26)}$	-1.934 $_{(0.29)}$	0.990	$\begin{smallmatrix} 6.8458\\ \scriptscriptstyle (0.04) \end{smallmatrix}$
31-Aug-1992	-2.314 (0.00)	$\underset{(0.00)}{14.497}$	-7.251 $_{(11.92)}$	-4.856 (0.27)	$\begin{array}{c}-3.710\\\scriptscriptstyle(0.04)\end{array}$	-2.204 (0.00)	0.908	$\underset{(0.03)}{4.6372}$
1-Sep-1992	-2.165 $_{(0.05)}$	-2.541 (0.45)	-0.595 $_{(0.01)}$	-1.656 $_{(0.01)}$	-1.567 $_{(0.06)}$	-1.992 $_{(0.09)}$	0.948	$\underset{(0.10)}{2.6884}$
2-Sep-1992	-2.541 (0.13)	-2.194 (0.95)	$\underset{(0.01)}{0.930}$	$\begin{smallmatrix} -5.933 \\ \scriptscriptstyle (2.28) \end{smallmatrix}$	$\underset{(0.64)}{1.135}$	$\underset{(0.52)}{-2.000}$	0.974	$\underset{(0.10)}{2.6996}$
3-Sep-1992	-4.724 (0.02)	-2.178 (0.59)	$\begin{smallmatrix} -0.205 \\ \scriptscriptstyle (0.16) \end{smallmatrix}$	$\begin{array}{c} -3.812 \\ \scriptscriptstyle (0.18) \end{array}$	$\begin{smallmatrix}-0.685\\\scriptscriptstyle(0.07)\end{smallmatrix}$	-2.077 $_{(0.55)}$	0.945	$\underset{(0.04)}{4.1894}$
4-Sep-1992	$\underset{(0.66)}{2.824}$	-0.324 (0.27)	-3.620 $_{(1.15)}$	$\underset{(0.09)}{0.390}$	-2.823 (1.88)	-1.757 $_{(1.28)}$	0.990	$\underset{(0.07)}{3.3074}$
8-Sep-1992	-3.384 (0.13)	-2.216 (1.40)	$\underset{(0.01)}{0.734}$	-4.205 (1.85)	$\begin{array}{c} -0.028 \\ \scriptstyle (0.02) \end{array}$	-2.040 (0.51)	0.992	$\begin{smallmatrix} 5.4434\\ \scriptscriptstyle (0.02) \end{smallmatrix}$
9-Sep-1992	-0.651 (0.01)	$\underset{(0.00)}{43.422}$	-8.564 (16.27)	-2.607 (6.56)	-10.131 (0.00)	-1.936 (0.00)	0.960	$\underset{(0.03)}{4.9878}$
10-Sep-1992	-2.968 $_{(0.02)}$	-0.836 (0.13)	-2.859 (0.17)	$\underset{(0.11)}{0.017}$	-2.226 (0.94)	$\underset{(0.50)}{-2.019}$	0.990	$\underset{(0.06)}{3.4620}$
11-Sep-1992	-2.259 (0.06)	-3.154 (1.22)	-0.597 $_{(0.52)}$	-1.889 (0.79)	-0.879 $_{(1.43)}$	-1.891 (3.37)	0.965	$\underset{(0.05)}{3.9484}$
14-Sep-1992	-1.592 (0.06)	$\begin{array}{c}-2.197\\\scriptscriptstyle(0.84)\end{array}$	$\begin{smallmatrix} -0.517 \\ \scriptscriptstyle (0.01) \end{smallmatrix}$	$\begin{array}{c}-3.083\\\scriptscriptstyle{(0.16)}\end{array}$	$\begin{smallmatrix} -0.737 \\ \scriptscriptstyle (0.09) \end{smallmatrix}$	$\begin{array}{c}-2.011\\\scriptscriptstyle (0.55)\end{array}$	0.978	$\underset{(0.08)}{2.9796}$
15-Sep-1992	$\begin{array}{c}-3.353\\\scriptscriptstyle(0.01)\end{array}$	-1.991 (0.17)	-1.054 (0.01)	-1.719 $_{(0.01)}$	$\underset{(0.09)}{-1.801}$	-1.898 $_{(0.21)}$	0.980	$\underset{(0.18)}{1.7626}$
16-Sep-1992	-128.074 (0.00)	-2.008 (40.88)	$\begin{smallmatrix}16.878\\\scriptscriptstyle(0.00)\end{smallmatrix}$	-5.017 (53.64)	5.456 $_{(0.00)}$	$\underset{\scriptscriptstyle(0.00)}{-3.079}$	0.980	$\begin{smallmatrix} 0.8910\\ \scriptscriptstyle (0.35) \end{smallmatrix}$
17-Sep-1992	-0.882 (0.07)	$\begin{smallmatrix}-0.943\\\scriptscriptstyle(0.48)\end{smallmatrix}$	$\begin{array}{c} -3.634 \\ \scriptscriptstyle (0.48) \end{array}$	$\begin{array}{c} -3.108 \\ \scriptstyle (0.02) \end{array}$	$\begin{array}{c}-2.051\\\scriptscriptstyle(0.09)\end{array}$	$\underset{(2.12)}{-1.933}$	0.985	$\underset{(0.38)}{0.7756}$
18-Sep-1992	-5.051 (0.05)	-2.165 $_{(0.49)}$	$\left \begin{smallmatrix} -0.453 \\ \scriptscriptstyle (0.01) \end{smallmatrix} \right $	$\begin{array}{c} -3.130 \\ \scriptstyle (0.17) \end{array}$	-0.889 (0.19)	-1.735 $_{(0.49)}$	0.928	$\begin{smallmatrix} 0.8970 \\ \scriptscriptstyle (0.34) \end{smallmatrix}$
21-Sep-1992	-4.066 (0.01)	$\begin{smallmatrix} -2.235\\ \scriptscriptstyle (0.43) \end{smallmatrix}$	-0.436 $_{(0.03)}$	$\begin{smallmatrix} -1.379 \\ \scriptscriptstyle (0.08) \end{smallmatrix}$	$\begin{array}{c} -2.102 \\ \scriptstyle (0.13) \end{array}$	-1.601 (1.48)	0.908	$\begin{smallmatrix} 0.8004 \\ \scriptscriptstyle (0.37) \end{smallmatrix}$
22-Sep-1992	$\begin{array}{c} -6.819 \\ \scriptscriptstyle (0.00) \end{array}$	-2.284 (1.50)	-0.241 (0.07)	-2.746 (0.15)	$\begin{array}{c} -0.900 \\ \scriptscriptstyle (0.16) \end{array}$	-1.886 $_{(0.68)}$	0.975	$\underset{(0.50)}{0.4570}$
23-Sep-1992	-1.142 (0.00)	-1.678 $_{(0.05)}$	-4.145 $_{(0.01)}$	-6.490 (4.69)	$\underset{(0.01)}{3.387}$	-1.918 (2.35)	0.987	$\underset{(0.39)}{0.7244}$
24-Sep-1992	-2.741 (0.05)	-2.980 (1.07)	-0.844 (0.18)	-2.448 (0.10)	-0.977 (0.14)	-1.787 $_{(0.75)}$	0.955	$\underset{(0.51)}{0.4260}$
25-Sep-1992	$\begin{array}{c}-3.089\\\scriptscriptstyle(0.03)\end{array}$	-2.012 (0.38)	$\underset{(0.00)}{0.772}$	-4.469 (0.24)	$\underset{(0.01)}{0.121}$	-1.956 $_{(0.17)}$	0.984	$\begin{smallmatrix} 0.7130\\ \scriptscriptstyle (0.40) \end{smallmatrix}$
28-Sep-1992	-2.325 $_{(0.04)}$	-2.460 (0.53)	$\underset{(0.04)}{-0.781}$	-1.612 $_{(0.12)}$	-1.536 $_{(0.30)}$	-1.721 (0.38)	0.970	$\underset{(0.23)}{1.4356}$
29-Sep-1992	-1.886 (0.09)	-0.884 (0.39)	-2.202 (0.29)	-0.640 (0.15)	-2.703 (2.00)	-1.846 $_{(1.16)}$	0.975	$\underset{(0.12)}{2.3652}$

 Table 2: British Pound Options Model

The $\theta's$ are estimates of the model (22). *t*-ratios are in parentheses. The *LR* statistic, with *p*-values underneath, is given by (23) and is distributed $\chi^2(1)$.

σ_1^2	α_{02}	α_{12}	β_{22}	γ_0
$\underset{(1.135)}{2.892}$	$\underset{(0.013)}{0.026}$	$\underset{(0.031)}{0.028}$	$\underset{(0.115)}{0.744}$	$\underset{(0.389)}{2.170}$
γ_1	ϕ_0	ϕ_1	ψ_0	ψ_1
0.937	1.772	-1.128	0.074 (0.034)	-3.545 (0.983)

 Table 3: French Franc Spot Exchange Rate Model

Date	λ_t	μ_{1t}	μ_{2t}	σ_{1t}^2	σ_{2t}^2	$E_{t-1}(r_t)$	LR
16-Jul-1993	0.127	2.068	-0.149	2.892	0.196	0.134	
19-Jul-1993	0.128	2.076	-0.129	2.892	0.169	0.154	$\underset{(0.11)}{2.6009}$
20-Jul-1993	0.123	2.014	-0.137	2.892	0.175	0.127	$\underset{(0.20)}{1.6610}$
21-Jul-1993	0.103	1.772	-0.150	2.892	0.181	0.047	$\underset{(0.38)}{0.7741}$
22-Jul-1993	0.123	2.014	-0.153	2.892	0.171	0.113	$\underset{(0.57)}{0.3298}$
23-Jul-1993	0.234	2.953	-0.153	2.892	0.165	0.573	$\underset{(0.96)}{0.0026}$
26-Jul-1993	0.242	3.011	-0.142	2.892	0.153	0.622	$\underset{(0.82)}{0.0531}$
27-Jul-1993	0.242	3.011	-0.140	2.892	0.153	0.623	$\underset{(0.86)}{0.0297}$
28-Jul-1993	0.205	2.750	-0.124	2.892	0.140	0.464	$\underset{(0.66)}{0.1949}$
29-Jul-1993	0.196	2.686	-0.146	2.892	0.142	0.410	$\underset{(0.70)}{0.1532}$
30-Jul-1993	0.307	3.403	-0.195	2.892	0.158	0.910	$\underset{(0.87)}{0.0263}$
02-Aug-1993	0.196	2.686	-0.419	2.892	0.343	0.190	$\underset{(0.97)}{0.0017}$
03-Aug-1993	0.220	2.862	-0.443	2.892	0.385	0.286	$\underset{(0.71)}{0.1410}$
04-Aug-1993	0.250	3.059	-0.274	2.892	0.212	0.558	$\underset{(0.89)}{0.0196}$
05-Aug-1993	0.239	2.988	-0.322	2.892	0.227	0.468	0.0410 (0.84)

Table 4: French Franc Spot Exchange Rate Densities

The first five columns of the table report the parameters of the predictive four-week-ahead normal mixture density for the respective trading days. The sixth column reports the overall mean of the mixture, that is, $E_{t-1}(r_t) := E(r_t|\Psi_{t-1}) = \lambda_t \mu_{1,t} + (1-\lambda_t)\mu_{2,t}$. The last column shows the *LR* statistic (23), with *p*-values underneath, which is distributed $\chi^2(1)$.

σ_1^2	α_{02}	α_{12}	β_{22}
$\underset{(5.758)}{13.367}$	$\underset{(0.357)}{0.419}$	$\underset{(0.064)}{0.149}$	$\underset{(0.144)}{0.637}$
λ_1	μ_1	λ_2	μ_2
0.238	0.253	0.762	0.028

 Table 5: British Pound Spot Exchange Rate Model

Date	λ_t	μ_{1t}	μ_{2t}	σ_{1t}^2	σ_{2t}^2	$E_{t-1}(r_t)$	LR
19-Aug-1992	0.238	0.253	0.028	13.367	2.099	0.082	
20-Aug-1992	0.238	0.253	0.028	13.367	2.147	0.082	1.9320 (0.16)
21-Aug-1992	0.238	0.253	0.028	13.367	2.203	0.082	$\underset{(0.12)}{2.4582}$
24-Aug-1992	0.238	0.253	0.028	13.367	2.132	0.082	3.3641
25-Aug-1992	0.238	0.253	0.028	13.367	2.187	0.082	3.9034 $_{(0.05)}$
26-Aug-1992	0.238	0.253	0.028	13.367	2.225	0.082	$\underset{(0.13)}{2.3320}$
27-Aug-1992	0.238	0.253	0.028	13.367	2.127	0.082	$\underset{(0.09)}{2.9135}$
28-Aug-1992	0.238	0.253	0.028	13.367	2.228	0.082	3.6498 $_{(0.06)}$
31-Aug-1992	0.238	0.253	0.028	13.367	2.191	0.082	3.0300 (0.08)
1-Sep-1992	0.238	0.253	0.028	13.367	2.209	0.082	2.4438 $_{(0.12)}$
2-Sep-1992	0.238	0.253	0.028	13.367	2.167	0.082	$\underset{(0.08)}{3.1109}$
3-Sep-1992	0.238	0.253	0.028	13.367	2.010	0.082	$\underset{(0.04)}{4.2429}$
4-Sep-1992	0.238	0.253	0.028	13.367	1.691	0.082	$\underset{(0.01)}{6.3591}$
7-Sep-1992	0.238	0.253	0.028	13.367	1.712	0.082	$7.1571 \\ \scriptscriptstyle (0.01)$
8-Sep-1992	0.238	0.253	0.028	13.367	1.882	0.082	3.8208 $_{(0.05)}$
9-Sep-1992	0.238	0.253	0.028	13.367	1.952	0.082	3.0172
10-Sep-1992	0.238	0.253	0.028	13.367	1.858	0.082	$\underset{(0.13)}{2.3022}$
11-Sep-1992	0.238	0.253	0.028	13.367	1.825	0.082	$\underset{(0.20)}{1.6530}$
14-Sep-1992	0.238	0.253	0.028	13.367	1.839	0.082	$\underset{(0.15)}{2.0535}$
15-Sep-1992	0.238	0.253	0.028	13.367	1.807	0.082	1.2606 (0.26)
16-Sep-1992	0.238	0.253	0.028	13.367	1.943	0.082	$\underset{(0.23)}{1.4618}$
17-Sep-1992	0.238	0.253	0.028	13.367	7.433	0.082	$\underset{(0.26)}{1.2888}$
18-Sep-1992	0.238	0.253	0.028	13.367	9.309	0.082	$\underset{(0.25)}{1.3111}$
21-Sep-1992	0.238	0.253	0.028	13.367	14.220	0.082	$\underset{(0.33)}{0.9449}$
22-Sep-1992	0.238	0.253	0.028	13.367	16.640	0.082	0.2094
23-Sep-1992	0.238	0.253	0.028	13.367	12.457	0.082	0.6254 (0.43)
24-Sep-1992	0.238	0.253	0.028	13.367	13.875	0.082	0.3842 (0.54)
25-Sep-1992	0.238	0.253	0.028	13.367	18.375	0.082	0.0705 (0.79)
28-Sep-1992	0.238	0.253	0.028	13.367	16.852	0.082	$\underset{(0.57)}{0.3165}$
29-Sep-1992	0.238	0.253	0.028	13.367	15.345	0.082	$\begin{array}{c} 0.6004 \\ \scriptscriptstyle (0.44) \end{array}$

 Table 6: British Pound Spot Exchange Rate Densities

The first five columns of the table report the parameters of the predictive four-week-ahead normal mixture density for the respective trading days. The sixth column reports the overall mean of the mixture, that is, $E_{t-1}(r_t) := E(r_t|\Psi_{t-1}) = \lambda_t \mu_{1t} + (1-\lambda_t)\mu_{2t}$. The last column shows the *LR* statistic (23), with *p*-values underneath, which is distributed $\chi^2(1)$.

Figure 1: Averages of Implied Volatility USFF Options 1992 and 1993



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Implied Volatility (% Per Annum)

Figure 3: Options Probability of 3% Depreciation in French Franc Over Next 4 Weeks



Figure 4: Options Probability of 3% Depreciation in British Pound Over Next 4 Weeks





Figure 5: Scatter Plots of FF Returns Against Slope of Yield Curve



Figure 6: GARCH Model Density Predictions for the FF

Figure 7: GARCH Model Probability of 3% Depreciation in French Franc Over Next 4 Weeks



Figure 8: GARCH Model Probability of 3% Depreciation in British Pound Over Next 4 Weeks

