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# BONN ECON DISCUSSION PAPERS

Discussion Paper 30/2006

## Multiple Periods Destroy the Axiomatic Base of Expected Utility Theory and its Standard Generalisations

by

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## Multiple Periods Destroy the Axiomatic Base of Expected Utility Theory and its Standard Generalisations

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### Abstract

It is here shown that the extension of expected utility theory to multiple periods destroys the axiomatic base by introducing timing contradictions in what the chooser knows at a single time point. It is shown that some of these timing contradictions remain even if, as Samuelson (1952) proposed, no segment of the outcome space (to which utility attaches) commences before all risk is passed.

JEL Classification: D81, D90

Key Words: expected utility theory, rank dependent theories, axioms, single period, multiple period, timing contradictions, degree of knowledge ahead, SKAT, the Stages of Knowledge Ahead Theory, atemporal, temporal, probability, stochastic dominance.

Standard theories for decisions under risk introduce timing contradictions if they attribute utility to any segment of the outcome flow that occurs *before* all risk and uncertainty is passed or have more than one outcome segment *after* all risk is passed. This limits these theories to a single period, and thus excludes their application to problems concerning loans, since loans require two periods, one of loan usage and a sequel of (contingent) loan repayment, and also excludes their application to other problems involving more than one period such as a consumption flow. The limitation is independent of whether probabilities are objective or subjective. This is because the limitation stems from what the chooser perceives as his degree of knowledge ahead at distinct time points – not from whether his perceptions are correct, nor from whether his perceptions are formed "objectively" or subjectively".

Under EUT, expected utility theory, and its standard rank dependent extensions such as anticipated utility, Quiggin (1982, 1993), cumulative prospect theory, Tversky and Kahneman (1992) and the Lopes aspiration theory, Lopes and Oden (1999), the axiomatic implications include the following.

- (i) The outcome flow must be evaluated "as if certain", even though uncertain at the point of choice, Friedman and Savage (1948).<sup>1</sup>
- (ii) The outcomes space must comprise all conceivable outcomes, Savage (1954, 1972).
- (iii) Each outcome stands in a one-to-one correspondence to a sure alternative, Savage (1954), Harsanyi (1977a).
- (iv) A risky alternative is a probability mix of sure outcomes, Harsanyi (1977a).
- (v) Any probability distribution can be applied to the outcomes space and constitutes an alternative with a ranking within the chooser's complete ranking of all conceivable alternatives, Savage (1954, 1972).
- (vi) Utility can attach only to those segments of the outcome flow that occur *after* all risk is resolved, Samuelson (1952).
- (vii) There can only be one post-risk segment of the outcome flow, the final (ie eschatological) segment.
- (viii) Compound gambles are treated as though the risk resolved at the successive time points were in fact resolved at the same time point.

Some readers might think that Samuelson made a mistake in claiming (vi), namely that the EUT outcome flow has to start *after* all risk is resolved, and some who grant (vi), might think that EUT can include

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<sup>1</sup> See Pope (2004) on the alternative Ramsey version of EUT which has the like property of precluding attaching a different utility to an outcome depending on its degree of risk, uncertainty.

multiple post-risk outcome segments. Indeed the author earlier thought this also. Let us therefore briefly demonstrate why the axioms truncate the outcome flow in ways that exclude both (vi) and (vii).

**1 The Exclusion of Periods Prior to the Resolution of all Risk in the Chosen Alternative**

This "as if certain" constraint (i) excludes consideration of emotional or material-financial utility arising out of the choosers' limited degree of knowledge ahead in the period before all risk is in the past. The constraint (i) excludes those sources of utility that are *based* on degree of knowledge ahead – those sources of satisfaction that in the terminology of Pope (2001) are termed *secondary* satisfactions. But it might be thought that utility can still attach to the earlier segment of the outcome flow that arises out of *primary* satisfactions, namely satisfactions that are *independent* of knowledge ahead, and therefore are evaluated "as if certain". There normally are such primary satisfactions in the earlier period, that accord with EUT's "as if certain" utility mapping restriction.

But to include such a segment is to violate EUT's restriction (ii), namely the restriction that *any* probability distribution be conceivable and applicable over the outcome space. To illustrate, suppose the outcomes in each segment of a two tuple-outcomes flow space are in bFF (billions of French francs) and include those of Table 1.

Table 1  
Segments of Two-Tuple Outcome Flows in bFF  
1st Segment Prior to the Resolution of Risk

| Outcome<br>Tuples | Segment <i>prior</i><br>to resolution of all risk | Segment <i>after</i><br>all risk is passed |
|-------------------|---|--|
| 3, 30             | 3   | 30   |
| 2, 20             | 2   | 20   |
| 3, 10             | 3   | 10   |

Then one sure alternative is the first two-tuple (3, 30) guaranteed. A second sure alternative is the second two-tuple (2, 20) guaranteed. Since under EUT, a risky alternative is a probability mix of the sure acts, restriction (iv), an example of a risky act is a probability of 0.5 of the two-tuples (3, 30) and (2, 20). We can select this particular probability distribution over the outcomes space under EUT restriction (v). But now consider the meaning of the risky alternative with a probability of 0.5 of these first two tuples / sure acts. It means that at the point of choice  $t=0$ , the chooser has only a fifty-fifty degree of knowledge ahead about what the outcome will be. But at the point of choice  $t=0$  the chooser already knows the segment before the risk is resolved. The chooser thus must already be experiencing either the 3 bFF or the 2 bFF. If experiencing the 2 bFF, the chooser knows this is followed by 20 bFF, ie the chooser has a sure act, contradicting the assumption that the chooser has only a 0.5 degree of knowledge of the final segment of the outcome flow. Likewise if the chooser is already experiencing the 3 bFF, the chooser knows that in the final segment of the outcome flow there will be 30 bFF.

The only way for the chooser to be ignorant of the final segment, is for the earlier segment before that risk is resolved to be *common*, Pope (1983). Restriction (ii) implies that the earlier segment would have to be common to *all* conceivable alternatives, not merely to probability mixes of the first and third tuples listed in Table 2, Pope (2005). The only way to have it common – without violating restriction (ii) and eliminating the 2nd 2-tuple in Table 2 – is to have the outcomes flow beginning after all risk is passed. Thus it can be seen that Samuelson (1952) is correct. The outcome flow to which utilities attach under EUT begins *after* all risk is passed.

In this regard, it is conspicuous for instance that in standard game theory, all the payoffs are timewise dimensionless end-points. The same restriction holds for the rank generalizations of EUT, as these also impose the dominance principle. The dominance principle is a preference for first order stochastically dominant distributions of outcomes. Specification of the dominance principle requires all outcomes segments to be after all risk is passed and also "as if certain". This is because to determine which decumulative distribution lies above the other, the outcome flows have to be ranked independently of the chooser's knowledge of whether they would occur. This pair of constraints limits the outcome flow that is evaluated to the final, eschatological time. It excludes most of agents' lives and experiences, and all emotional and financial effects from living through a period of risk and uncertainty, and all its aftermaths, even when the risk is resolved.

**2 The Exclusion of Multiple Periods After the Resolution of All Risk**

Beginning with Marschak (1950), the view arose that EUT was not as static as von Neumann and Morgenstern had declared it to be. The view arose that it was dynamic in the sense that *after* the resolution of all risk, there could be multiple segments of the outcomes flow to which utilities attach. This however is also incorrect. To illustrate, let the outcomes space include the two-tuples of Table 2.

Table 2  
 Segments of Two-Tuple Outcome Flows in bFF  
 Both Segments *after* the Resolution of *all* Risk

| Outcome Tuples | Date when 1st Segment begins ie upon the Resolution of all Risk | Date when 2nd Segment begins |
|----------------|---|------------------------------|
| 3, 30          | 2007  | 2010                         |
| 2, 20          | 2009  | 2010                         |
| 3, 10          | 2007  | 2008                         |

Under EUT axiomatic restriction (iii), an alternative is a 50/50 chance of the first two outcome tuples. This 50/50 chance however is inconceivable and cannot constitute an alternative since there is no date at which the chooser can choose this risky alternative. The chooser must choose it before 2007, for the risk to be passed by 2007, as required for the 1st two-tuple. But if he chooses it before 2007, then he must be receiving 3 FF in 2008 and knows in 2008 that he does not get the 2nd tuple, contradicting the specification of the 2nd tuple, namely that the chooser only learns whether he gets that outcome flow in 2009. Note that EUT axiomatic restriction (ii) precludes eliminating the 2nd two-tuple if the outcomes space permits temporally divisible outcomes. The outcomes flow has to be temporally indivisible (as in the axioms themselves), and thus pertaining to eschatology. For an alternative proof that the EUT outcome segment is limited to the eschatological period via its compound gamble axiom, see Pope (2005).

Much of the writing in defense of EUT is of this eschatological ilk, eg that of Harsanyi (1977b, 1978) in stating that under EUT, the process does not matter, only the end, and that of Hammond (1988a, 1988b) in stating that EUT concerns only consequences, consequences that, on inspection, ignore the process, the history of the chooser through life, and consider only his life's end point, Munier (1994). As before, the same restriction holds for any standard rank dependent generalisation of EUT, since the restriction of outcomes to an indivisible time unit is simply a feature of decision theories that impose the dominance principle.

Most economists have been innocent about the axiomatic restrictions under EUT (and its standard rank dependent generalisations) that each outcome must be evaluated "as if certain", and must be a timewise indivisible outcome segment beginning not at the point of choice  $t=0$ , but *later*, at  $t=k$ , *after* all risk.

### 3 Overview

Figure 1  
The Jump Through of the Prior Periods of Uncertainty to Certainty  
that occurs under EUT and its Standard Rank Dependent Generalisations

| $t=0$<br><i>Choice</i>   | $0 \leq t < k$<br><i>Pre-outcome period</i><br>of risk, of limited knowledge<br>ahead                                    | $t \geq k$<br>anticipated possible<br>utilities in the<br><i>post-outcome period</i>  |
|--|--|---|
| a risky act as at least two<br>possible final segments of<br>the outcome flow, $Y_1$ or ...<br>$Y_N$ | These segments of the<br>outcome flow are <b>jumped</b><br><b>over</b> and ignored even though<br>they occur after $t=0$ | $Y_i$<br>knowledge ahead<br>independent sources of<br>utility $U(Y_1), \dots, U(Y_N)$ |

The mapping of outcomes  $Y_i$  into utilities  $U(Y_i)$  depicted in Figure 1 is independent of risk – in other words, independent of the chooser's degree of knowledge ahead at time  $t=0$ . This independence can be seen from anticipated utility (the right hand column of Figure 1). The probability distribution – that denotes the chooser's degree of knowledge ahead – does *not* enter that utility mapping. It is a simple mapping from the final outcome segment alone, ie the "as if certain" mapping of Friedman and Savage (1948) and equation (1). The utility  $U(V)$  of a risky act  $V$  is defined by

$$U(V) = \sum_i p_i U(Y_i) \tag{1}$$

atemporal aggregation weight for  $U(Y_i)$ , **outside time**
anticipated utility of outcome  $Y_i$  in **real time**

Nothing that happens to people in *real time* concerning risk is considered in EUT's equation (1). The utilities  $U(Y_i)$  denote primary satisfactions exclusively, and not all primary satisfactions, only those of the final temporally indivisible portion of the choosers' future. The only way risk enters is *atemporally*, in how probabilities concerning the mutually exclusive outcomes aggregated to attain a single overall value of the alternative. Under EUT the atemporal aggregation rule is simple probability weights. Under its standard rank dependent generalisations, the atemporal aggregation rule is a more complex (de-) cumulative probability function. But still no real time risk effects are included.

### 4 Efforts to Extend EUT to Include Risk Effects

The issue of extending EUT to include real time risk effects, and to permit outcome segments prior to when all risk is passed, has been investigated from various angles.

#### 4.1 A Coincidence with Variance Effects

An early proposal was that of Friedman and Savage (1948) who argued that the choices made under EUT that ignores real time risk effects could coincide with those proposed by the older asset theory in which variance and other measures of dispersion, ie of risk. It could coincide, they proposed, since if choosers have concave "as if certain" utility functions, then other things equal, they will choose alternatives with less disperse outcomes. Some of the restrictions ensuring coincidence were identified in Borch (1969), Feldstein (1969) and other much more stringent ones – ie demonstrating that the conditions in Borch and

Feldstein were insufficient – discovered in Schneeweiß (1968a, 1969b, (1972a, 1973b). For other restrictions that enable EUT to coincide in capital markets, see eg Cochrane (2005). None of the restrictions ensuring coincidence, however, are plausible.

#### 4.2 Elaborated Outcomes

A second effort to in effect permit EUT to include outcome segments concerning events *before* all risk is passed is to specify the decision situation more fully or to redefine or elaborate the outcomes, eg Samuelson (1952), Markowitz (1959, 1991) and Caplin and Leahy (2001). These sorts of moves make it impossible to form preferences over all possible probability distributions over outcome flows since many of these distributions involve contradictions of outcome segments being simultaneously known and not known, simultaneously risky and certain (Pope 1983, 1984). These elaborations or redefinitions of the outcomes space destroy EUT's axiomatic base and preclude the derivation of its representation theorem in the usual sense, Pope (2000).

#### 4.3 Temporal, Recursive EUT

Kreps and Porteus (1978) discern that if one is to have an outcome segment to which utility attaches *prior* to the resolution of all risk, it will be necessary to expand the atemporal EUT axioms with ones that specify when each risk begins, and when that uncertainty is resolved. Their seminal contribution is to seek to do this, and axiomatically derive a dated version of the atemporal EUT property. Klibanoff and Oxdenoren (2006) extend Kreps and Porteus to an axiomatisation of subjective probabilities and utilities, analogous to Savage's extension of earlier axiomatisations of EUT with objective probabilities to subjective ones. (Such an extension raises issues of whether all events postulated can coexist that are discussed in eg Aumann and Karni but not discussed here.) Klibanoff and Oxdenoren (2006) make the analogies with individual axioms in atemporal EUT pleasingly transparent.

But the temporal or recursive EUT axioms, being analogous to those of EUT, exclude numerous risk effects that a reasonable decision maker would include, and thus in this way fail to retrieve EUT's axiomatic base. For examples of omitted effects from Kreps and Porteus' axioms, see Pope, Leitner and Leopold (2006).

#### 4.4 The Axiom Trap

The reason so many real time risk effects get omitted – and none are consistently included in temporal EUT – is that seeking an axiomatic derivation of EUT's mechanical atemporal probability weights rule itself imposes timing fallacies. One way of showing this is to note that in order to derive the EUT property – whereby utilities are aggregated up mechanically by their probability weights to obtain a single value for each act – any temporal or atemporal EUT axiomatisation needs at least one "mixing" axiom. Examples are Axiom 2.3 in Kreps and Porteus (1978) and Axiom 4 in Klibanoff and Oxdenoren (2006).

Now "mixing" is constructing compound gambles. The fraction in these "mixing" relations is treated as if it could be an atemporal fraction, when in fact it is a probability, and thus denotes the chooser's degree of knowledge ahead of which of the following alternatives (lotteries, acts) will ensue. Each such probability compound introduces a new time period, as proven in Pope (1985). By ignoring these extra periods in temporal or recursive EUT axiomatisations embed timing fallacies of probabilities being simultaneous degenerate and non-degenerate, and thereby destroy the axiomatic base.



Kreps and Porteus are aware that the "mixing" axioms in fact concern compound gambles. Their conclusion mentions that "reduction of compound uncertainty at a *single* time is implicit" in their axiomatisation, (1978, p199, emphasis added). But they are unaware of the contradiction this assumption of a *single* time entails of a probability being simultaneously degenerate and non-degenerate. This in turn relates to a timing fallacy in their introductory comments wherein timeless lotteries exist, and temporal lotteries are an extension. As shown in Pope (1985), timeless lotteries involve the contradiction of the chooser simultaneously knowing and not knowing the outcome at the point of choice.

## 5 The Way Ahead

The axiomatisations of EUT and its extensions were designed to achieve a mechanical way of doing the atemporal aggregation of the mutually exclusive outcomes using simple probability weights. This mechanical answer has come at the cost of scientists confusing (a) events in real time and their attendant risk effects with (b) atemporal aggregation weights. The confusion has arisen from too much focus on the algebra (done flawlessly), and not enough focus on the decision theoretic denotation of terms like probabilities. When we keep the denotation in mind, we see that doing the derivation – ie grinding out the mechanical EUT answer of atemporal aggregation via probability weights – requires imposing at least one false simultaneity postulate. It requires at least one set of relations in which *sequential* events are treated as *simultaneous*. (A corresponding situation of at least one false simultaneity postulate is required in other standard rank dependent generalisations whose axioms yield a mechanical atemporal aggregation rule.) The false simultaneity postulate is needed to generate a logical category error and conflate a) effects anticipated in real time and preferences concerning these, with b) the atemporal aggregation weights.

SKAT, the Stages of Knowledge Ahead Theory, of Pope (1983), in more detail in Pope (1995) and Pope, Leitner and Leopold (2006) allows an intuitive separation of (a) from (b), and avoidance of timing fallacies. SKAT paves the way for reasoned approaches to discerning a) the real time effects and b) how a model's atemporal aggregation rules should vary with the chooser and the context. There may then be value in axiomatising a) or b) for some contexts. But for understanding decision making under risk and uncertainty, as in much of physics, it may transpire that axioms have a limited role in enhancing our theoretical understanding and obtaining robust predictions.

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