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# BONN ECON DISCUSSION PAPERS

Discussion Paper 2/2003

## Aggressiveness and Redistribution

by

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# Aggressiveness and Redistribution

Dieter Bös and Martin Kolmar

## 1 Introduction

In this paper we deal with voluntary and compulsory redistribution in an economy where the enforcement of property rights is costly. The definition and enforcement of property rights is one of the undisputed responsibilities of the government. However, the fact that the enforcement of property rights requires the investment of scarce resources has only rarely been systematically analyzed. In a world where the provision of “property rights” incurs opportunity costs, some of the basic economic principles concerning taxation and redistribution can be turned upside down.

We consider an economy where property rights for a particular good can only be imperfectly enforced by the government because of prohibitive public enforcement costs. The lack of public enforcement will motivate the individuals to invest resources to appropriate goods produced by other individuals, and at the same time to invest resources to defend themselves against appropriation. Hence, the private enforcement of property rights will act as a substitute for public enforcement. Private enforcement of property rights is a contest. Accordingly, economic agents will not only engage in productive activities, but will have to spend part of their time coping with conflict. These investments in conflict may imply defense or aggression.

The new developments in the internet are a good example of what we have in mind. The internet economy has the potential to reduce production and especially distribution costs on a large scale. However, the public and private enforcement of property rights in the internet is still an unresolved problem. The crisis of the music industry, for example, is at least partly a reaction to the development of the mp3 standard that allows the distribution of music much more efficiently than the CD, but which, at the same time,

makes the enforcement of property rights much more difficult. Accordingly, the industry reacts by developing new security standards, in other words, by the investment of resources in the private enforcement of property rights. In general, security concerns of the general public seem to be the most important impediment to the commercial development of the internet.

It is unclear whether the usual conjectures about the effects of policy measures carry over to an economy with partially self-enforced property rights. Skaperdas and Syropoulos (1997) analyze an anarchy contest and show that standard comparative-static results derived under conditions of perfect property rights can be reversed in an economy with conflict. This finding has far-reaching implications for our perception about, for example, the economic role of taxes and redistribution.

The basic logic behind Skaperdas and Syropoulos (1997) can be derived from the general idea of the second best: a government intervention like redistributive taxation, that is neutral with respect to efficiency, or even destroys it in a non-distorted economy, can be efficiency enhancing in an economy that is already distorted. In an economy with conflict this outcome can be based on two effects. First, government interventions can distort the economy at the margin and, thereby, bring it closer to the first-best marginal conditions. Second, government interventions can reduce a distortion resulting from appropriation by changing the equilibrium fraction of time that is devoted to conflict, that is, government interventions can be used to indirectly control the amount of resources invested in conflict. In this paper we concentrate on lump-sum redistribution and thereby focus on the second of the above-mentioned effects.

So far, we have concentrated on redistribution enforced by the government. However, in an economy with conflict *voluntary redistribution* of useful goods and resources can be a rational strategy of selfish individuals, in contrast to an economy with perfect property rights where individual interests prevent voluntary redistribution. Hence, we have to distinguish between two types of redistribution with different normative legitimization; the *ex-post* voluntary redistribution that needs no delegation of power to a government, and redistribution that is at most Pareto improving from an *ex ante* perspective (under a real or hypothetical veil of ignorance). This second type of

redistribution requires the delegation of power to a government.

The accentuation of private enforcement of property rights implies that our article must be seen in the tradition of the many recent papers that start from a situation of anarchy. In an anarchic economy, there is no coercive power that could enforce formal rules.<sup>1</sup> Most of the recent papers on anarchic economies use contest models, in particular, rent-seeking models. Particularly well-known papers are Hirshleifer (1995) who focuses on the dynamic stability of anarchy, and Skaperdas (1992) who analyzes technological prerequisites for the existence of cooperation in anarchy. Since our paper is interested in voluntary and compulsory redistribution under the threat of appropriative conflict, three recent papers should be mentioned that deal with that particular problem. In Grossman's (1995) theory of redistribution, capitalists pay workers a markup on their wage in order to reduce theft. In a similar spirit, in Acemoglu and Robinson (2000) the rich elite accepts a redistributive taxation in order to avoid revolution. In contrast to Grossman and Acemoglu-Robinson, the paper by Bös and Kolmar (2002) hypothesizes that redistribution is not linked to the avoidance of criminal behavior, but is considered to be a compensation that has to be paid for a Pareto-improving reallocation of productive resources. The present paper is a sequel to Bös and Kolmar (2002). It is much simpler, avoiding the complicated multi-stage setting and the repeated-game analysis.

The repeated-game analysis, in several recent papers on anarchic societies, was chosen to explain the emergence of constitutions, such as in Garfinkel (1990) and in Muthoo (2000). In Bös and Kolmar (2002) the founding of a constitution is a central point of the analysis and different types of constitutions are shown to emerge from one-shot games and repeated games.<sup>2</sup> In contrast, this paper does not concentrate on a theory that explains how a constitution comes about, and we do not deal with repeated games. Rather, the gist of an analysis of redistribution in an anarchic society is shown by

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<sup>1</sup>Bush and Mayer (1974) define an anarchic economy as a society of conflict where agents can rely neither on the voluntary respect for individual possession nor on the fulfillment of bilateral or multilateral arrangements.

<sup>2</sup>This differentiates Bös and Kolmar (2002) from the papers by Garfinkel (1990) and Muthoo (2000) who only concentrate on repeated games.

means of three case studies for various specified production functions and contest-success functions.

The paper proceeds as follows. In Section 2 we present various examples of ‘partial anarchy,’ a state of the world where property rights are guaranteed for some, but not for all goods. Contests in this setting of partial anarchy are then explicitly modelled in Section 3. The following Section 4 presents two types of examples: In the first there is no conflict at all; the interests of the agents point in the same direction. In the second there is a trade-off between joint and antagonistic interests. Both examples of section 4 exhibit a utilitarian optimum at the boundary, where one agent owns all of the basic resource. This is not the case in the example of section 5, which has an interior optimum. Section 6 presents a brief conclusion.

## 2 Examples of Partial Anarchy

Consider a situation where individuals cannot rely on the public enforcement of property rights, but where the government can, in principle, control the allocation of a basic resource. This setup corresponds in a stylized way to a situation where it is possible to guarantee property rights for some goods or resources but where it is impossible to guarantee property rights for some other goods (‘partial anarchy’). Examples for such a situation are (1) the assignment of property rights during the colonialization of the American West during the 19th century, (2) international trade and foreign investments that are neither backed by formal supra-national rules nor by reputation effects, and (3) particular types of internet transactions.

(1) During the colonialization of the American West there existed a so called ‘homestead principle:’ the first occupant of land, that had not yet been owned by a white settler, became its legal owner. However, this principle was unenforceable for a period of time because of lack of governmental control due to obvious logistical impediments. In this period of transition, therefore, property rights of land in the United States were legally enforceable in some states, but not in others – in these Western states any kind of redistribution of land had to be voluntary.

(2) International trade, by the same token, relies heavily on reputation effects between trading partners and governments because of an absence of a supra-national agency enforcing property rights. The non-ability of several governments in developing countries to commit to not expropriating foreign investors has been an important impediment to the development of these countries. In contrast, the ruling regime will forcefully and efficiently guarantee the property of their own clientele – a typical example of partial anarchy.

(3) The example of the internet economy shows that technological change need not have effects that are unambiguously positive. The development of new technologies for communication (for all goods) and for distribution (for information goods) substantially increases market transparency and reduces transportation costs. At the same time it has become increasingly difficult to enforce property rights. The general public does not really trust the security of internet data, and businesses also remain skeptical. Market places like *ebay* or *Amazon*, therefore, have to invest part of their resources in order to establish a reputation of reliability. These investments are private responses to the specific problems created by the ‘virtualization’ of market places. The institutions that emerge from this process, to some extent, compete with the classical institutions of public enforcement. The two-sided face of the internet becomes especially cumbersome for all goods that can be digitized. The digitization leads to a drastic reduction in transportation costs, but at the same time makes illegal appropriation very easy. The state as the traditional enforcement agency relies heavily on the help of the industry that develops new security standards like electronic watermarks. However, because all these standards are also comprised of digitized information, they can be overcome by the next generation of computer hackers. Note that the absence of public enforcement in some internet market does not imply that the government is impotent in other markets as well. Hence, it can use its power on these markets to indirectly control the conflict in the problematic market.

### 3 The Model

Let us now present a simple formal model. In a situation of ‘partial anarchy,’ there are two egoistic and risk neutral economic agents,  $i = 1, 2$ . Let us first



describe their productive activities. Each agent is endowed with one unit of time and spends  $\ell_i \in [0, 1]$  units for the production of a single output good (labor inputs). Moreover, agent 1 is initially endowed with  $r^o \in [0, \bar{r}]$  units of a basic resource (capital or land) that also determines the production of output. The remaining  $\bar{r} - r^o$  units of the resource are initially endowed to agent 2. The variable  $r$  is the instrument of redistribution. This redistribution can be based on a voluntary agreement of the agents. Sometimes, however, an intervention by a government may achieve higher utility for each agent than is attainable by voluntary agreement.

Total production  $x$  is additively linear in the labor inputs  $\ell_i$  and depends on the quantities  $r, \bar{r} - r$  of the basic resource that the agents possess after redistribution,

$$x(\ell_1, \ell_2, r) = g(r)\ell_1 + \ell_2, \quad g(r) > 0. \quad (1)$$

Accordingly, agent 1 produces  $g(r)\ell_1$  units of output, whereas agent 2 produces  $\ell_2$  units. What are the efficiency implications of this production function? Consider, first, the efficiency in the use of the basic resource,

$$\frac{\partial x}{\partial r} = g'(r)\ell_1. \quad (2)$$

If  $g' > 0$ , total production increases if agent 1 possesses more of the basic resource. The opposite holds if  $g' < 0$ . Consider next the labor productivity of the agents:

$$\frac{\partial x}{\partial \ell_1} = g(r), \quad (3)$$

$$\frac{\partial x}{\partial \ell_2} = 1. \quad (4)$$

If  $g(r) > 1$ , one unit of labor of agent 1 yields more output than one unit of labor of agent 2. The opposite holds if  $g(r) < 1$ . In this case agent 1 is less productive with respect to his use of labor. Note that there are four combinations of efficiency that result from the various realizations of resource and labor productivity. Agent 1 may be more efficient in both respects ( $g' > 0, g > 1$ ). However, it is also possible that he only excels in the use of the resource ( $g' > 0, g < 1$ ) or only in the use of labor ( $g' < 0, g > 1$ ). Finally, he could lose completely ( $g' < 0, g < 1$ ). A variety of these cases can also occur

for a single specification of the production function. By way of example, if  $g(r) = r^2$ , we have  $g' > 0$  for all possible realizations of  $r$ , but  $g < 1$  for  $r < 1$  and  $g > 1$  for  $r > 1$ .

If every agent gets what he has produced, agent 1 consumes  $g(r)\ell_1$  units, and agent 2 gets  $\ell_2$  units of the output good. However, in a situation with incomplete property rights it is not necessarily the case that each agent gets exactly what he has produced. Both agents may engage in conflict activities and, therefore, may eventually consume a share of output which differs from their own production. We analyze a “common-pool contest” in contrast to an “initial-endowment contest” (Grossman 2000). In this common-pool contest the total quantity of output is lumped together and the share of output an agent receives does not depend on the quantity he produced. As a consequence, we need not explicitly distinguish between defensive and appropriative investments, and assume that each agent invests  $a_i \in [0, 1]$  units of time in conflict, splitting his time endowment in productive and conflict activities,  $\ell_i + a_i = 1$ .<sup>3</sup> Agent  $i$ ’s investment in conflict,  $a_i$ , will be taken as measure of his “aggressiveness.” The fraction  $p$  of the total output that accrues to agent 1 is given by a modified Tullock *contest-success function*,<sup>4</sup> which depends on the agents’ conflict activities and on the distribution of the basic resource,

$$p(a_1, a_2, r) = \frac{f(r)a_1}{f(r)a_1 + a_2} = \frac{1}{1 + \frac{1}{f(r)} \cdot \frac{a_2}{a_1}}. \quad (5)$$

Note that in our model appropriation and defense refer only to the consumption good (output), not to the basic resource and, trivially, not to labor. The term  $1/f(r)$  measures the relative efficacy of agent 1’s compared to agent 2’s conflict activities. A formally similar term can be found in various papers on contests, for instance in Grossman (2001). The remaining fraction of output,  $1 - p$ , accrues to agent 2. The contest-success function can be interpreted

<sup>3</sup>This seems plausible in a model where all investments are manpower, which typically can be alternatively used for defense or appropriation. Compare Grossman and Kim (1995), p. 1278, footnote 4.

<sup>4</sup>For investments  $a_1$  and  $a_2$  the original Tullock contest-success function is defined as  $p = a_1/(a_1 + a_2)$ , see Tullock (1980, 1984). Note that this function is not well-defined if  $a_1 = a_2 = 0$ .

in the same way as a standard production function that abstracts from the exact technological and organizational process of production. Accordingly, the conflict technology  $p(a_1, a_2, r)$  exhibits marginal productivities, which characterize the agents' efficacy in conflict. Once again, we consider first the efficacy in the use of the basic resource,

$$\frac{\partial p}{\partial r} = \frac{f' a_1 a_2}{(f a_1 + a_2)^2} . \quad (6)$$

Therefore, if  $f' > 0$ , agent 1's share of output increases if he possesses more of the basic resource.<sup>5</sup> Increasing possession of the basic resource increases the appropriative abilities of agent 1; if he owns more of the basic resource, he becomes the better predator or defender. If  $f' < 0$ , the contrary holds. Next, we consider how the agents' investments in conflict influence their shares in output:

$$\frac{\partial p}{\partial a_1} = \frac{f a_2}{(f a_1 + a_2)^2} , \quad (7)$$

$$\frac{\partial(1-p)}{\partial a_2} = \frac{f a_1}{(f a_1 + a_2)^2} . \quad (8)$$

Therefore, whenever  $a_2 > a_1$ , agent 1's conflict investments are marginally more effective than those of agent 2.

The agents' utilities  $u_i$  are measured in terms of their consumption of output. Using  $\ell_i + a_i = 1$  we can eliminate  $\ell_i$ . The maximization problems are then given by

$$\begin{aligned} \max_{a_1} u_1(a_1, a_2, r) &= p(a_1, a_2, r) x(\ell_1, \ell_2, r) \\ &= \frac{f(r) a_1}{f(r) a_1 + a_2} (g(r)(1 - a_1) + (1 - a_2)) , \end{aligned} \quad (9)$$

$$\begin{aligned} \max_{a_2} u_2(a_1, a_2, r) &= (1 - p(a_1, a_2, r)) x(\ell_1, \ell_2, r) \\ &= \left( 1 - \frac{f(r) a_1}{f(r) a_1 + a_2} \right) (g(r)(1 - a_1) + (1 - a_2)) . \end{aligned} \quad (10)$$

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<sup>5</sup>Unless one of the agents owns all of the basic resource, that is, if either  $a_1 = 0$  or  $a_2 = 0$ .

The first-order conditions for the solution of the agents' problems are the following:

$$\frac{\partial p}{\partial a_1} x - p \frac{\partial x}{\partial a_1} \begin{cases} = 0 & \wedge & a_1 \in [0, 1] \\ < 0 & \wedge & a_1 = 0 \\ > 0 & \wedge & a_1 = 1 \end{cases}, \quad (11)$$

$$- \frac{\partial p}{\partial a_2} x - (1 - p) \frac{\partial x}{\partial a_2} \begin{cases} = 0 & \wedge & a_2 \in [0, 1] \\ < 0 & \wedge & a_2 = 0 \\ > 0 & \wedge & a_2 = 1 \end{cases}. \quad (12)$$

The first terms in (11) and (12) measure the marginal revenue of an increase in  $a_i$ , which is given by the change in the fraction of total production accruing to agent  $i$ . The second terms measure the marginal increase in production accruing to agent  $i$ .

Together, conditions (11) and (12) define reaction functions of the agents,  $a_1(a_2, r)$ ,  $a_2(a_1, r)$ . A Nash equilibrium of the game is a tuple  $a_1^N(r), a_2^N(r)$  such that  $a_1^N = \arg \max_{a_1} u_1(a_1, a_2^N, r)$  and  $a_2^N = \arg \max_{a_2} u_2(a_1^N, a_2, r)$ .<sup>6</sup> Given the functional specifications of the model, these values can be calculated as<sup>7</sup>

$$a_1^N(r) = \frac{1 + g}{2(g + f^{1/2}g^{1/2})}, \quad (13)$$

$$a_2^N(r) = \frac{(1 + g)f^{1/2}}{2(f^{1/2} + g^{1/2})}. \quad (14)$$

Note that here and in what follows we suppress the functional dependencies of  $f(r)$  and  $g(r)$ . This greatly simplifies the presentation. Given these values of  $a_1^N, a_2^N$ , it is straightforward to calculate the equilibrium fraction  $p^N(r)$  and the indirect utility functions  $v_1^N(r), v_2^N(r)$ :

$$p^N(r) = \frac{1}{1 + g^{1/2}/f^{1/2}}, \quad (15)$$

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<sup>6</sup>For the proofs of existence and uniqueness of this Nash equilibrium see Skaperdas (1992) and Bös and Kolmar (2002), Appendices 1 and 2, which will be sent to the reader on request.

<sup>7</sup>It can be easily verified that the second-order conditions are fulfilled:

$$\frac{d^2 u_1}{da_1^2} = - \frac{4f^{1/2}g^{3/2}}{1 + g} < 0, \quad \frac{d^2 u_2}{da_2^2} = - \frac{4g^{1/2}}{(1 + g)f^{1/2}} < 0.$$

$$v_1^N(r) = \frac{(1+g)f^{1/2}}{2(f^{1/2} + g^{1/2})}, \quad (16)$$

$$v_2^N(r) = \frac{(1+g)g^{1/2}}{2(f^{1/2} + g^{1/2})}. \quad (17)$$

## 4 Effects of Redistributive Policies

In Bös and Kolmar (2002) we derived a theory of redistribution as a theory of the emergence of institutions (the constitution). In this paper we argue from a complementary perspective where we assume that institutions supporting redistributive policies already exist. The main focus of this analysis then becomes to better understand the conditions under which the economy has to rely on compulsory redistribution and the conditions under which voluntary redistribution resolves frictions efficiently.

Hence, in this section we analyze the effect of a redistribution of  $r$  from one individual to the other for different functional specifications of our model. By means of comparative-static analysis we will consider the changes in the individual conflict activities and in the individual utilities which result from such an increase in  $r$ . We obtain

$$\frac{\partial a_1^N}{\partial r} = \frac{-g(1+g)f' + f(g-1)g' - 2f^{1/2}g^{1/2}g'}{4(f^{1/2} + g^{1/2})^2 f^{1/2}g^{3/2}}, \quad (18)$$

$$\frac{\partial a_2^N}{\partial r} = \frac{g(1+g)f' + f(g + 2f^{1/2}g^{1/2} - 1)g'}{4(f^{1/2} + g^{1/2})^2 f^{1/2}g^{1/2}}, \quad (19)$$

$$\frac{\partial v_1^N}{\partial r} = \frac{g(1+g)f' + f(g + 2f^{1/2}g^{1/2} - 1)g'}{4(f^{1/2} + g^{1/2})^2 f^{1/2}g^{1/2}}, \quad (20)$$

$$\frac{\partial v_2^N}{\partial r} = \frac{-g(1+g)f' + f^{1/2}(2g^{3/2} + f^{1/2}(1+3g))g'}{4(f^{1/2} + g^{1/2})^2 f^{1/2}g^{1/2}}. \quad (21)$$

The results of the comparative-static analysis, equations (18) to (21), are too complicated to derive general results. Instead we will consider particular specifications of the functions  $f(r)$  and  $g(r)$  and in most cases present the results in graphical form. For these calculations we assume that the parameter  $r$  is located in the  $[0, 1]$ -interval.

## 4.1 No Conflict

As a first example let us assume that each agent gets exactly what he has produced as long as both agents choose the same conflict investments,  $a_1 = a_2 = a$ : agent 1 gets  $g(r)\ell$  and agent 2 gets  $\ell$ . This specification can be interpreted as a local symmetry condition implying that they are equally effective in appropriation at the margin. Agent 1 gets a share of

$$p(a, a, r) = \frac{g(r)}{1 + g(r)},$$

whereas agent 2 gets  $1 - p$ . If the agents choose different conflict investments, a sufficient extension that preserves the desired property is

$$p(a_1, a_2, r) = \frac{g(r)a_1}{g(r)a_1 + a_2} = \frac{1}{1 + \frac{1}{g(r)} \cdot \frac{a_2}{a_1}}. \quad (22)$$

Therefore, in this example we have  $f(r) = g(r)$ . Substituting this into (13) and (14) we derive the following Nash-equilibrium strategies:<sup>8</sup>

$$a_1^N = \frac{1}{4} \left( 1 + \frac{1}{g(r)} \right), \quad a_2^N = \frac{1}{4} (1 + g(r)). \quad (23)$$

Substituting these values of the Nash conflict activities into the production function and into the conflict-success function, we obtain:

$$x = \frac{1}{2} (1 + g(r)), \quad (24)$$

$$p = 1/2. \quad (25)$$

Accordingly, the agents attain the following equal utilities

$$v_1^N = v_2^N = \frac{1}{4} (1 + g(r)). \quad (26)$$

Hence, there is no conflict of interest between the agents. The initial distribution of the resource may be inefficient, but the agents will fully redistribute the basic resource until  $r = 1$  if  $g' > 0$ , and choose  $r = 0$  if  $g' < 0$ : the better user of the basic resource will be given all of the resource. The unequal distribution of the consumption good that would result and that would eliminate

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<sup>8</sup>The equilibrium strategies are only characterized by the first-order conditions (23) if  $1/3 \leq g(r) \leq 3$ . Otherwise, we would obtain corner solutions.

the incentive for voluntary redistribution in a world without conflict<sup>9</sup> is corrected by the conflict activities that guarantee that both agents participate in the aggregate surplus. Note that the agents will voluntarily redistribute the basic resource. There is no need for any government intervention.

## 4.2 Joint versus Antagonistic Interests

We continue with the analysis of a particular combination of exponential specifications of  $f$  and  $g$ . For this purpose we assume that increasing the amount of the basic resource possessed increases agent 1's ability in production, but reduces his ability in conflict. This is attained by choosing the following production and conflict technologies:

$$x(\ell_1, \ell_2, r) = r^{1/2}\ell_1 + \ell_2; \quad p(a_1, a_2, r) = \frac{r^{-4}a_1}{r^{-4}a_1 + a_2}. \quad (27)$$

For these specifications we obtain the following Nash investments

$$a_1^N = \frac{1 + r^{1/2}}{2(r^{1/2} + r^{-7/4})}, \quad a_2^N = \frac{(1 + r^{1/2})r^{-2}}{2(r^{-2} + r^{1/4})}. \quad (28)$$

Substituting  $a_1^N, a_2^N$  into  $u_1$  and  $u_2$  yields the maximum utilities the agents can achieve for any given  $r$ . They are as follows:

$$v_1^N = \frac{(1 + r^{1/2})r^{-2}}{2(r^{-2} + r^{1/4})}, \quad v_2^N = \frac{(1 + r^{1/2})r^{1/4}}{2(r^{-2} + r^{1/4})}. \quad (29)$$

Figure 1 presents the agents' Nash investments in conflict for a parameter  $r \in [0, 1]$ . The specification of the conflict technology in (27) implies that agent 1's investment in conflict is weighted by  $r^{-4} \geq 1$ , a unit of agent 1 is only weighted by 1. This is a comparative advantage of agent 1 which implies that he can appropriate a fairly high amount of the output through relatively little time investment in conflict. Therefore, his investments  $a_1^N$  are lower than those of agent 2. However, this relative advantage is reduced if agent 1 gets more of the basic resource, because  $g' < 0$ . Accordingly, the more of the resource agent 1 owns, the more time he must dedicate (up to a

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<sup>9</sup>In a world with perfect property rights, voluntary redistribution would of course be replaced by trade.

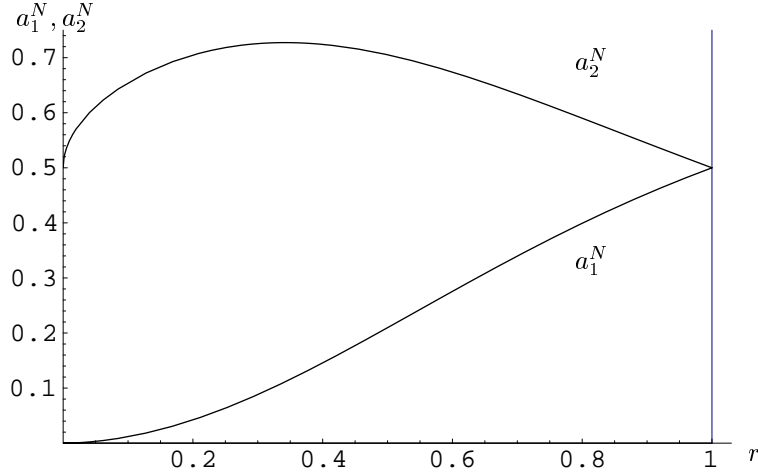


Figure 1: Conflict investments for varying allocations of the basic resource

maximum of 50 per cent if he owns all of the resource) to conflict. In contrast, agent 2 must always exert high conflict investments to cope with his weakness in the conflict technology.

The interpretation, which we have derived from the properties of the conflict technology, can be corroborated by an analogous interpretation on the basis of the production technology. The specification in (27) implies that a labor unit of agent 1 is weighted by only  $r^{1/2} \leq 1$ , whereas a labor unit of agent 2 is weighted by 1. Accordingly, agent 2 does not need much time to produce the output  $x$  and, therefore, spends more time in conflict, and vice versa for agent 1.

Figure 2 presents the Nash-equilibrium utilities which result from the agents' choice of investments in conflict and in production. The utility of agent 1 is first increasing and then decreasing in  $r$ , whereas the utility of agent 2, and the sum of both agents' utilities, are monotonically increasing in  $r$ . Now consider two cases. In the first case, the initial endowment  $r^0$  falls below the maximum utility of agent 1, for example,  $r^0 = 0.2$ . Every allocation in the interval  $[0.2, 0.5]$  constitutes a Pareto improvement and may, therefore, be the outcome of a voluntary redistribution of the resource. In the second



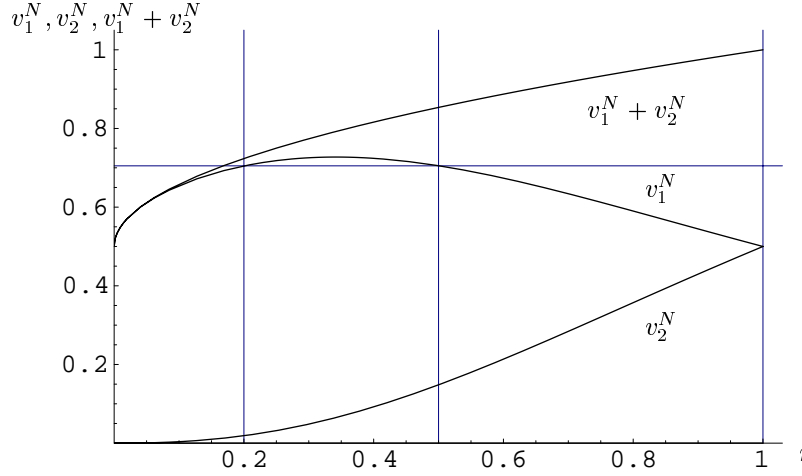


Figure 2: Nash utilities for varying allocations of the basic resource

case,  $r^0$  exceeds the maximum utility of agent 1. Hence, there is a direct conflict of interest and there is no voluntary redistribution of the resource. The agents remain at the initial distribution  $r^o$ .

### 4.3 Voluntary Redistribution and Voluntary Establishment of Compulsion

In our first example there was no conflict of interest between the agents: both were willing to accept a redistribution of the initial resource in order to increase effective consumption. This need no longer be the case in the second example. However, there exist potential Pareto improvements. In order to realize these improvements the agents would unanimously agree to establish a utilitarian government under a veil of ignorance.<sup>10</sup>

The possibility to realize these improvements largely depends on the abil-

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<sup>10</sup>Most contractarian models assume that the rules which are established in a social contract are enforceable. Compare, for instance, Brennan and Buchanan (1985). In this paper, we make the same assumption. In Börs and Kolmar (2002), we show how the social contract changes if the assumption of a priori enforceability of rules is given up.

ity of a government to enforce redistribution. As we have argued at the beginning of section 2 above, the limitations of a government to enforce property rights on one market do not imply that its hands are equally bound on other markets and with respect to other policies. Therefore, the government may choose indirect ways to improve production in the presence of appropriation. Let us assume that a government cannot control property rights with respect to the consumption good, but is able to control the allocation of the basic resource. In this case, a utilitarian government could intervene by redistributing the whole resource to agent 1 in order to maximize the sum total of the individual utilities. This form of redistribution is qualitatively different from that analyzed before because it relies on compulsion and, therefore, on the existence of a central authority.

The allocation of the basic resource that maximized total utility, however, is not an actual Pareto improvement, given an initial allocation  $r^0$ , because agent 1 is worse off. If, however, in addition to the ability to control the allocation of the resource, the government is also able to control the final distribution of the consumption good (but still not able to enforce property rights), it can establish a voluntary exchange of resource for consumption goods by the implementation of a tax  $t$ , which agent 2 pays to agent 1 to compensate him for his lower utility at the social optimum  $r^* = 1$ . If the tax is lump-sum, the individual utilities after taxation amount to

$$v_1^N(r^*, t) = v_1^N(r^*, 0) - t; \quad v_2^N(r^*, t) = v_2^N(r^*, 0) + t, \quad (30)$$

where  $v_i^N(r^*, t)$ ,  $i = 1, 2$  are the Nash equilibrium utilities after optimization with respect to  $a_1$  and  $a_2$ .<sup>11</sup> To attain agreement by both agents the government has to choose a tax that falls below agent 2's utility gain and exceeds agent 1's utility loss, where gain and loss are measured as the differences in the Nash-equilibrium utilities at  $r^*$  and  $r^o$ .<sup>12</sup>

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$$v_2^N(r^*, 0) - v_2^N(r^o, 0) =: \bar{t}(r^*) \geq t \geq \underline{t}(r^*) := - (v_1^N(r^*, 0) - v_1^N(r^o, 0)), \quad (31)$$

<sup>11</sup>The utilities before optimization would be  $v_i(a_1, a_2, r^*, t)$ ,  $i = 1, 2$ .

<sup>12</sup>Agent 2 agrees to the taxation if  $v_2^N(r^*, t) - v_2^N(r^o, 0) \geq 0$ . Since the tax  $t$  is a lump-sum deduction, this is equivalent to  $v_2^N(r^*, 0) - t - v_2^N(r^o, 0) \geq 0$ . Transform this into  $v_2^N(r^*, 0) - v_2^N(r^o, 0) \geq t$  and define  $\bar{t} := v_2^N(r^*, 0) - v_2^N(r^o, 0)$ . The lower boundary  $\underline{t}$  is determined analogously.

where in our example  $r^* = 1$ . Note that it is not guaranteed that for all possible specifications of the production and conflict technologies such a tax actually exists. In this case, the government cannot successfully intervene, since, in our model, only Pareto-improving activities can be enforced by the government. (Recall that the government still is not able to enforce property rights.)

Our example clearly exhibits the social costs of conflict. Consider the situation where the entire resource is possessed by the first agent. Then, if both agents fully refrain from investing in conflict, total utility becomes  $v_1^N + v_2^N = x = 2$ . With conflict, however,  $x$  only reaches a maximum of 1.

The above discussion has shown that government redistribution can be used as a means to control the amount of conflict in an economy where property rights cannot be controlled directly. The reallocation of resources in this situation is a means to bring the economy closer to its production potential.

## 5 An Alternative Example

In this example, we choose specifications that balance the production abilities and the appropriative abilities of the agents. This balance leads to an interior solution: at the optimal allocation the first agent possesses  $0 < r^* < \bar{r}$  units of the basic resource. Therefore, neither the voluntary agreement of the agents nor a utilitarian government policy will opt for a boundary solution, where one agent owns all of the basic resource. This is in contrast to the examples of the preceding section.

We specify the production function as  $x = r^{1/2}\ell_1 + \ell_2$ , where  $r \in [0, 1/2]$ . Therefore, output increases if agent 1 possesses more of the basic resource,  $g' > 0$ , but he is less efficient with respect to his labor inputs, that is,  $\partial x / \partial \ell_1 < \partial x / \partial \ell_2$ . Moreover, we choose a Tullock-type of conflict function, which puts agent 1 at an absolute disadvantage, but at a marginal advantage. The absolute disadvantage is revealed by a comparison between  $p$  and  $1 - p$ :

$$p = 2r \frac{a_1}{a_1 + a_2} \quad 1 - p = (1 - 2r) \frac{a_1}{a_1 + a_2} + \frac{a_2}{a_1 + a_2}. \quad (32)$$

The marginal advantage shows in the derivatives  $\partial p / \partial r > 0, \partial p / \partial a_1 > 0, \partial^2 p / \partial r \partial a_1 > 0$ .

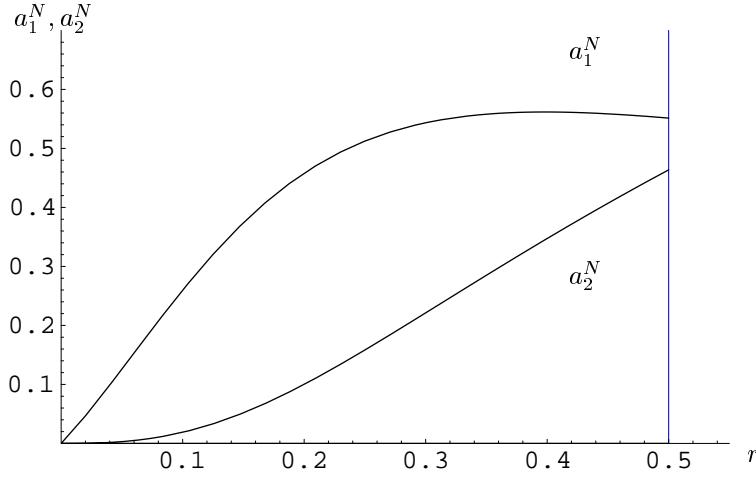


Figure 3: Conflict investments for varying allocations of the basic resource

Given the above specifications of conflict and production technologies, the agents maximize their utilities with respect to their time investments in conflict  $a_i$ . We obtain interior solutions that are characterized by the following first-order conditions:

$$\begin{aligned} r^{1/2}(1 - a_1) + (1 - a_2) &= \frac{r^{1/2}a_1(a_1 + a_2)}{a_2}, \\ r^{1/2}(1 - a_1) + (1 - a_2) &= \frac{(a_1 + a_2)^2 - 2ra_1(a_1 + a_2)}{2ra_1}. \end{aligned} \quad (33)$$

Solving for the Nash equilibrium values of  $a_1(r), a_2(r)$ , we obtain the individual conflict investments and utility levels.

Figures 3 and 4 present the conflict investments and the Nash-equilibrium utilities. In spite of his large investments in conflict, agent 1's utility continuously increases. If  $r$  increases, he gains from his share in the increased output and from the increased share in output – these two effects dominate a third effect that measures the feedback of the increase in  $r$  on the aggressiveness

of agent 2.<sup>13</sup>

For agent 2 an increase in  $r$  has quite different consequences than for agent 1. If  $r$  is very low, he gains if agent 1 owns more of the basic resource: giving more of the resource to agent 1 is also beneficial for agent 2. However, further increases in  $r$  decisively weaken agent 2's bargaining position and, therefore, agent 2's utility declines sharply if more and more of the resource is given to agent 1.<sup>14</sup>

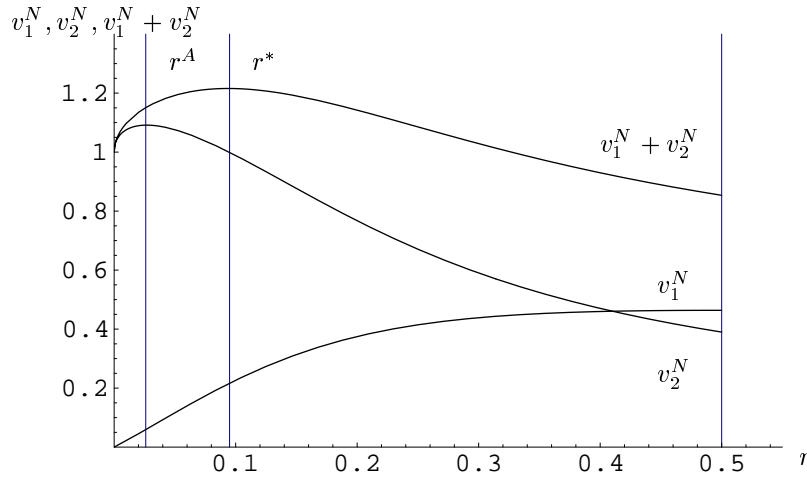


Figure 4: Nash utilities for varying allocations of the basic resource

Now consider an initial endowment of the basic resource  $r^o$  which is to the left of  $r^A$ . Both individual utilities increase if agent 2 transfers  $r^A - r^o$  units of

<sup>13</sup>Formally, a comparative-static analysis at the optimum yields

$$\frac{dv_1^N}{dr} = p \frac{\partial x}{\partial r} + \frac{\partial p}{\partial r} x - \frac{\partial x}{\partial a_2} \frac{\partial a_2}{\partial r},$$

where the right-hand side exhibits the three effects mentioned in the text. For the derivation of this formula see Bös and Kolmar (2002), Appendix 4, which will be sent to the reader on request.

<sup>14</sup>The comparative-static analysis for agent 2 yields:

$$\frac{dv_2^N}{dr} = (1 - p) \frac{\partial x}{\partial r} - \frac{\partial p}{\partial r} x - \frac{\partial x}{\partial a_1} \frac{\partial a_1}{\partial r}.$$

the resource to agent 1. Therefore, there is scope for voluntary redistribution of the resource, which is attained without any government intervention. This changes, however, if a further move from  $r^A$  to the optimal  $r^*$  is considered. This redistribution cannot be implemented without government intervention that enforces the optimal value  $r^*$  and introduces a tax  $t$  that makes the policy package acceptable for both players. As in the preceding section this tax must follow the requirement described in (31).

## 6 Conclusion

What is the economic role of redistribution in an economy where the property rights for some goods cannot be protected by the government? We analyzed this question for an economy with a common-pool contest and two types of potential distortions: the waste of resources in the contest and the dilution of incentives to produce as a result of the existence of externalities in the conflict equilibrium. We were able to show the following results.

In some situations Pareto-improving redistribution occurs voluntarily. This is in contrast to a world with complete markets and perfectly enforceable property rights, where redistribution would necessarily reduce the utility of at least one selfish individual. The reason for this result is that the appropriation of the consumption good takes the role of contractual compensations that would prevail in a world with perfect property rights. If the agent who gives away some of his resource is effective enough in the appropriation of the final good, he will participate in the overall increase in production.

However, it is well possible that the agent, who should give away parts of his resource in order to ensure Pareto-improving redistribution, is not strong enough to appropriate a sufficient amount of the consumption good in order to compensate his initial loss. In such a situation, compulsory redistribution may lead to Pareto improvements.

These findings show that in a situation of partial anarchy in some market, voluntary redistribution or redistributive policies enforced by the government in other markets may help to reduce the inefficiency of the anarchic market. As we have argued before, ‘partial anarchy’ is a rather common phenomenon,

under no condition an exotic exception. Hence, we think it is worthwhile to better understand the consequences of imperfect property rights for the optimal design of government policies. Our paper is a step in this direction.

The idea that redistributive policies are a means to control conflict in anarchic markets has potentially far-reaching consequences but, as with all second-best considerations, may be a delusive basis of policy recommendations. Technological innovations, like those in the internet, change the relative power of different groups in a society as well as the level of conflict. They may, therefore, create the need for modified government policies but they may also bring forth new room for unanimous political reforms that would have been impossible before the innovation. However, whether modified government policies or unanimous political reforms should be enacted, cannot generally be stated: In our model the extent and relationship between voluntary and compulsory redistribution depends in a complex way on the functional specifications. For this reason, our analysis is a good example for the delusiveness of second-best policy recommendations. Hence, we know that redistribution is an important indirect way to control conflict, but without detailed empirical studies it is impossible to learn anything about the direction and extent of redistribution in a given empirical situation.

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