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# A New Approach to Optimal Commodity Taxation

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**ABSTRACT:** This paper makes a fresh attempt at characterizing optimal commodity taxes. Under the usual assumptions, an extremely simple expression of second-best commodity taxes is derived, showing tax rates as functions of observable variables only, rather than as functions of unobservable variables such as compensated cross elasticities. The main formula is independent of special preferences, and independent of the number of commodities. It has a simple economic meaning and could be particularly useful for empirical research. Examples and remarks on the normalization problem are provided.

**KEYWORDS:** Optimal Commodity Taxation, Ramsey Rule.

**JEL-CLASSIFICATION:** H21

## 1. INTRODUCTION

For more than one and a half centuries, economists have been trying to characterize commodity taxes which minimize the excess burden of taxation. Ramsey (1927) surmounted the partial equilibrium framework of Dupuit (1844) and others, laying the foundation for the modern treatment. Research on this topic is still continuing, and if one takes to hand a contemporary textbook<sup>1</sup> or a recent survey<sup>2</sup> in order to pick up the fruits of these efforts, it turns out that the theory of public finance essentially offers three answers to the question as to how a policy maker should set commodity taxes optimally. However, each of the three answers has so severe shortcomings that they are almost useless for practical purposes. In the order of decreasing popularity<sup>3</sup>, the answers have become known as the Ramsey rule, the inverse elasticity rule, and the Corlett-Hague rule. Let us review these briefly.

- The *Ramsey rule* characterizes optimal changes in consumption, stating that optimal taxes diminish all demands in the same proportion. In fact, this interpretation holds only approximately in the neighbourhood of zero tax revenue.
- According to the *inverse elasticity rule*, optimal commodity tax rates are inversely proportional to elasticities of demand. This presupposes a very special form of the utility function and is far from a general result.
- Finally, the *Corlett-Hague rule* yields a substantive conclusion for any well-behaved preferences. It states that in a setting with two consumption goods and leisure, precisely that good which is more complementary with leisure should be taxed at a higher rate. But unfortunately, we live in a world with considerably more than two commodities.

This paper makes a fresh attempt at characterizing optimal commodity taxes. Concentrating on efficiency issues and thus adopting a framework with identical consumers<sup>4</sup>, a very simple rule for optimal commodity taxes will be derived. This rule holds independently of special assumptions on preferences, and independently of the number of commodities. It entails an explicit formula for optimal tax rates in terms of observable variables. Regarding empirical applications, an explicit solution in terms of observable variables seems important and useful; after all, the fact that most traditional results are formulated in terms of compensated

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1 E. g. Myles (1995).

2 E. g. Auerbach and Hines (2002).

3 To get an approximate measure of popularity, I performed an internet search which, at the time it was conducted, entailed over 1.000 hits for “Ramsey rule”, roughly 500 hits for “inverse elasticity rule” and less than 100 hits for “Corlett-Hague rule”.

4 For a recent treatment of distributional aspects of commodity taxation, see Coady and Drèze (2002).

elasticities is often considered as an eminent obstacle for putting the theory into practice.

The paper is organized as follows. Section 2 presents the standard approach in a nutshell, in order to facilitate comparison with the new approach. Section 3 contains our principal result. A general complementarity rule is derived in section 4. Some examples are provided in section 5. Section 6 considers optimal commodity taxes in the presence of an additional wage tax, and section 7 concludes. The appendix clarifies some minor points of rigour.

The common underlying framework is this one, where all symbols in bold face represent n-vectors: Consumers maximize utilities  $u(\mathbf{x}, y)$  by selecting a pair of consumption  $\mathbf{x}$  and effort  $y$  from a given consumption set  $C$ , subject to the budget constraint  $y \geq \mathbf{q}\mathbf{x}$  and at given consumer prices  $\mathbf{q}$ . Gross and net wage rates being normalized to unity,  $y$  represents earnings in monetary units as well as effort in hours. Forming a Lagrangean  $L = u(\mathbf{x}, y) + \mu(y - \mathbf{q}\mathbf{x})$  and differentiating it with respect to the choice variables gives

$$(1) \quad \frac{\partial u}{\partial \mathbf{x}} = \mathbf{m}\mathbf{q} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\mu.$$

## 2. CHARACTERIZING OPTIMAL QUANTITIES

The prevailing dual approach does not aim to describe optimal taxes, but seeks to characterize optimal quantities. It employs ordinary demand functions  $\mathbf{x}(\mathbf{q})$  which, under suitable assumptions on preferences, are defined for any  $\mathbf{q} \gg \mathbf{0}$ . The adding-up restriction defines an earnings function  $y(\mathbf{q}) = \mathbf{q}\mathbf{x}(\mathbf{q})$  and entails the Cournot aggregation  $\partial y / \partial \mathbf{q} = \mathbf{x} + \mathbf{q} \partial \mathbf{x} / \partial \mathbf{q}$ . Using consumer prices as controls, the government maximizes utilities subject to the revenue constraint  $\mathbf{t}\mathbf{x} \geq g$ , where  $g$  is an exogenous revenue requirement,  $\mathbf{t} = \mathbf{q} - \mathbf{p}$  are the commodity taxes, and  $\mathbf{p}$  represents given producer prices<sup>5</sup>. Differentiating  $L = u(\mathbf{x}(\mathbf{q}), y(\mathbf{q})) + \lambda(\mathbf{t}\mathbf{x}(\mathbf{q}) - g)$  with respect to the controls yields

$$(2) \quad \frac{\partial u}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{q}} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial \mathbf{q}} + \lambda \left( \mathbf{x} + \mathbf{t} \frac{\partial \mathbf{x}}{\partial \mathbf{q}} \right) = \mathbf{0},$$

since  $\partial \mathbf{t} / \partial \mathbf{q} = \mathbf{I}$ . This leads to the uncompensated version of the Ramsey rule, if we substitute from (1) and the Cournot aggregation:

$$(3) \quad \mathbf{t} \frac{\partial \mathbf{x}}{\partial \mathbf{q}} = -\frac{\mathbf{I} - \mathbf{m}}{\mathbf{I}} \mathbf{x}.$$

Ramsey (1927) assumed utility to be quasi-linear in effort. Then, as there are no income effects, the expression on the left involves a symmetric matrix and may be

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<sup>5</sup> It is well-known that all results remain unchanged in case of endogenous producer prices.

re-written as  $\partial \mathbf{x} / \partial \mathbf{q} t$ . According to the mean-value theorem, this equals the changes in demands,  $d\mathbf{x}$ , when taxes are increased from zero to  $d\mathbf{t}$ . Hence, all demands are diminished by a common factor.

In the presence of income effects, one can use the Slutsky decomposition to obtain the compensated version of the Ramsey rule, originally developed by Samuelson (1951). For some negative constant  $k$  and the Slutsky matrix  $S$ , this reads:

$$(4) \quad S\mathbf{t} = k\mathbf{x}.$$

Optimal taxes diminish the compensated demands by a common factor. Using Mirrlees' (1976) term "index of discouragement" for  $S_j S_{ij} t_j / x_i$ , the Ramsey rule asserts that the index of discouragement should be the same for all goods.

### 3. CHARACTERIZING OPTIMAL TAXES

The primal approach seeks to characterize optimal taxes rather than optimal quantities. It uses *inverse demand functions*  $\mathbf{q}(\mathbf{x})$  defined for all  $\mathbf{x} \in D$ , where  $D$  denotes the image of the demand function  $\mathbf{x}(\mathbf{q})$ . Now, the adding-up restriction defines an offer curve  $\mathbf{y}(\mathbf{x}) = \mathbf{q}(\mathbf{x})\mathbf{x}$  and entails the inverse Cournot aggregation  $\partial \mathbf{y} / \partial \mathbf{x} = \mathbf{x} \partial \mathbf{q} / \partial \mathbf{x} + \mathbf{q}$ . To understand what this means, observe that the government, by selecting appropriate prices, can induce consumers to choose any consumption plan  $\mathbf{x} \in D$ . In so doing, however, the government must ensure that consumers remain on their offer curves. In this sense, earnings are a function of consumption, and the derivative  $\partial \mathbf{y} / \partial x_i$  indicates the response of earnings to a small change in  $x_i$ . The taxes follow from the identity  $\mathbf{t}(\mathbf{x}) = \mathbf{q}(\mathbf{x}) - \mathbf{p}$  which implies  $\partial \mathbf{t} / \partial \mathbf{x} = \partial \mathbf{q} / \partial \mathbf{x}$ . Taking the quantities as controls, the government maximizes utilities subject to the revenue constraint. Differentiating the Lagrangean  $L = u(\mathbf{x}, \mathbf{y}(\mathbf{x})) + \lambda(\mathbf{t}(\mathbf{x})\mathbf{x} - g)$  with respect to the controls, we obtain:

$$(5) \quad \frac{\partial u}{\partial \mathbf{x}} + \frac{\partial u}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{x}} + \lambda \left( \mathbf{x} \frac{\partial \mathbf{q}}{\partial \mathbf{x}} + \mathbf{t} \right) = \mathbf{0}.$$

By using (1) and the inverse Cournot aggregation, this becomes

$$(6) \quad \mathbf{m}\mathbf{q} - \mathbf{m}\mathbf{x} \frac{\partial \mathbf{q}}{\partial \mathbf{x}} - \mathbf{m}\mathbf{q} + \lambda \left( \mathbf{x} \frac{\partial \mathbf{q}}{\partial \mathbf{x}} + \mathbf{t} \right) = \mathbf{0}.$$

Solving for  $\mathbf{t}$ , we obtain what could be called the inverse Ramsey rule:

$$(7) \quad \mathbf{t} = -\frac{\lambda - \mathbf{m}}{\lambda} \mathbf{x} \frac{\partial \mathbf{q}}{\partial \mathbf{x}}.$$

Comparison with (3) highlights the duality of the two approaches in a particularly nice fashion. Apart from this aesthetic virtue, the inverse Ramsey rule is as useless as the Ramsey rule itself. However, performing a simple operation that has no

analogue in the dual approach, one can substitute  $\partial y / \partial x - q$  for  $x \partial q / \partial x$ , using again the inverse Cournot aggregation. This yields our main result:

$$(8) \quad t = \frac{1 - m}{1} \left( q - \frac{\partial y}{\partial x} \right).$$

After a trivial re-calculation, an alternative version of the result reads:

$$(9) \quad t = \frac{1 - m}{m} \left( p - \frac{\partial y}{\partial x} \right).$$

The first version is more useful when it comes to applications, as will be shown below. From a theoretical point of view, the second version is more satisfactory because producer prices are exogenous. To get to grips with it, consider a marginal increase in some consumption  $x_i$ . This move costs  $p_i$  in a first-best context, but  $p_i - \partial y / \partial x_i$  in a second-best context, because this difference represents the derivative of  $t(x)x$  with respect to  $x_i$ . As the tax  $t_i$  itself can be interpreted as the marginal revenue from slightly increasing  $x_i$ , formula (9) equates marginal revenues and marginal costs multiplied by a uniform constant. The crucial elements in determining optimal taxes are the derivatives  $\partial y / \partial x_i$  which indicate the responses of earnings to changes in the respective consumptions. Owing to the minus sign, the rule asserts that commodities which stimulate earnings should be taxed at low rates, and vice versa.

Before concluding this section, some historical remarks are in order. The first economist adopting the primal approach to optimal commodity taxation was, of course, Ramsey himself, who worked in the Marshallian tradition of what we today call inverse demand. Indeed, Ramsey's central equation (3) is an archaic form of (7). Assuming  $\partial q / \partial x$  to be symmetric and taxes to be infinitesimal, he used it to derive his principal result "that optimal taxes diminish the production of all commodities in the same proportion" (Ramsey 1927, p. 54). Atkinson and Stiglitz (1972) also used a primal approach, treating earnings as an additional control and putting the condition (1) that consumers remain on their offer curves as an additional restriction into the Lagrangean. This facilitates describing optimal taxes in terms of the bordered Hessian of the utility function. Deaton (1979) essentially decomposed (7) into compensated prices changes and inverse income effects, using the distance function and the Antonelli matrix, which is a generalized inverse of the Slutsky matrix. Comparing these approaches in an elegant survey, Stern (1986) pointed out that the analyst has the choice between three different matrices: Slutsky, Hessian or Antonelli. The strategy followed above was not to add yet a fourth matrix type, but to let matrices completely disappear from the scene. This entails the simple vector equations (8) and (9) which are easy to interpret in economic terms, as we shall see.

#### 4. A PERFECTLY GENERAL COMPLEMENTARITY RULE

Equation (8) describes optimal taxes in terms of three observable variables, namely, consumer prices, earnings and consumption. To elucidate its economic meaning, let us define the *index of encouragement* for each commodity:

$$(10) \quad e_i = \frac{\partial y}{\partial x_i} \frac{1}{q_i}.$$

The index of encouragement indicates the response of earnings to an increase in the respective expenditure. It may be positive, negative or zero. Dividing some row in (8) by the consumer price, inserting (10), and dividing two such rows yields:

$$(11) \quad \frac{t_i/q_i}{t_j/q_j} = \frac{1-e_i}{1-e_j}.$$

This states *that commodities with a high index of encouragement should be taxed at low rates, and vice versa*. To illustrate this proposition, consider brandy and coffee. Brandy is likely to reduce earnings, at least if consumed in larger quantities. If this is true, brandy has a negative index of encouragement, and (11) suggests taxing it at a relatively high rate. By the same token, as long as coffee has a positive impact on earnings, the coffee tax should be low.

The economic logic behind these findings is as follows. Consumers have positive endowments of leisure which, by assumption, cannot be taxed directly. Commodities with a high index of encouragement induce consumers to supply large portions of their endowments in the market. A government restricted to taxing only market demands will stimulate the consumption of such goods since this broadens the tax base. Note, the impossibility of taxing leisure endowments is the driving force behind second-best taxes<sup>6</sup>. Without this restriction, the government could establish a first-best tax system consisting of a tax on the endowments only. The literature sometimes confounds untaxed endowments with untaxed market demands. Clearly, any market demand can be taken as untaxed, because only relative prices matter. For a clarification of this point, see Stern (1986, p. 298) and section 6 below.

Formula (11) is a general complementarity rule, reminiscent of the Corlett-Hague rule:

$$(12) \quad \frac{t_i/q_i}{t_j/q_j} = \frac{\mathbf{h}_{yi}^c - \mathbf{h}_{ii}^c - \mathbf{h}_{jj}^c}{\mathbf{h}_{yj}^c - \mathbf{h}_{ii}^c - \mathbf{h}_{jj}^c},$$

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<sup>6</sup> The implicit assumption that leisure is the only good with an untaxable endowment is not restrictive. If there were several such goods, these could be combined into a single Hicksian composite, since the government cannot change their relative prices by hypothesis.

The symbols on the right-hand side represent the compensated cross elasticities. Corlett and Hague (1953) named commodity  $i$  *stronger complementary with leisure* if  $\eta_{yi}^c > \eta_{yj}^c$ , i.e. if an increase in the  $i$ -th price induces a stronger compensated increase in earnings, or decrease in leisure, as compared with an increase in the  $j$ -th price. Equating (11) and (12) shows that commodity  $i$  is stronger complementary with leisure if and only if its index of encouragement is lower. In this sense, our approach generalizes the Corlett-Hague rule, which holds only in the presence of just two commodities, whereas (11) holds for arbitrarily many commodities. Moreover, the new rule involves observable variables instead of compensated cross elasticities that are hard to estimate, and is much easier to derive.

However, the most important advantage of the generalized complementarity rule lies in the economic intuition it provides. Let us consider three concrete commodities: cinema visits, watches and newspapers, in order to see what the traditional approach has to tell us regarding their optimal tax treatment. According to Sandmo (1987, p. 89), who worked out the deepest interpretations of the traditional approach, the “set of taxes should be chosen so as to make the role of substitution effects as small as possible”. This is in perfect accordance with prevailing intuition, but what does it mean, either mathematically or with respect to the three aforementioned commodities? In mathematical terms, the statement is empty because optimal taxes by no means minimize some norm of the Slutsky matrix. Practically, the statement is empty, too, except in the accustomed special cases of two goods or vanishing cross elasticities.

Using the present approach, one could argue as follows: Cinema visits or other time consuming hobbies are likely to reduce earnings. Therefore, they should be taxed at relatively high rates. On the other hand, it appears difficult to say something useful about the relationship between watches and earnings. According to the principle of sufficient reason, these should be taxed at the normal rate. Finally, reading newspapers has presumably a positive impact on earnings in a knowledge society, which suggests taxing them at a reduced rate. The latter recommendation holds also with respect to all kinds of work related expenditures. These are often deductible under the earnings tax, a feature equivalent to zero tax rates in a pure excise tax system. The preceding examples were chosen with an eye to actual tax provisions which, of course, are quite diverse among countries. Anyway, it appears perfectly sensible to tax commodities with a high index of encouragement at low rates, because these stimulate labour supply and are hence less costly in a second-best context.



## 5. EXAMPLES

In this section, we would like to provide some examples by considering concrete functional forms. All examples presume a leisure endowment equal to unity, so that  $1 - y$  denotes leisure.

*1. Cobb-Douglas Utility:* Firstly consider the economist's workhorse

$$(13) \quad u(\mathbf{x}, y) = (1 - y)^{a_0} x_1^{a_1} \cdot \dots \cdot x_n^{a_n},$$

where the numbers  $a_i$  sum up to unity. The textbook solutions read  $y = 1 - a_0$  and  $x_i = a_i/q_i$  for all  $i$ . Changes in any consumption do not affect earnings since all expenditure shares, including the share of leisure itself, remain constant. The derivatives  $\partial u/\partial x_i$  vanish for all goods, hence  $e_i = 0$ , and (8) implies taxation at the uniform rate  $(-\mu)/\mu$ .

*2. Inverse Elasticity Utility:* Next we derive the inverse elasticity rule:

$$(14) \quad u(\mathbf{x}, y) = 1 - y + u_1(x_1) + \dots + u_n(x_n).$$

Quasi-linearity in  $y$  excludes income effects, so that  $\mu = 1$ . Differentiating the left equation in (1) yields  $\partial q_i/\partial x_j = 0$  for all  $j \neq i$ , implying  $\partial y/\partial x_i = \partial q_i/\partial x_i x_i + q_i$ . Therefore, the index of encouragement equals

$$(15) \quad e_i = \frac{\partial q_i}{\partial x_i} \frac{x_i}{q_i} + 1,$$

from which the inverse elasticity rule  $t_i/q_i = -(-\mu)/\mu \cdot \partial q_i/\partial x_i \cdot x_i/q_i$  follows, written itself in a somewhat inverse fashion. Standard textbook wisdom declares that commodities whose demands react stronger to price changes should be taxed at lower rates because quantity reactions induce excess burdens. Though this is literally true under the very stringent assumption on preferences made here, such a view seems misleading. A much more convincing explanation, which already comes close to the intuition behind (11), was offered by Sandmo (1986, p. 92): If the compensated cross elasticities  $\partial_{ij}$  vanish for all  $i \neq j$ , the adding-up property implies  $\partial_{yi} = \partial_{ii}$ . Therefore, taxing a commodity with a high own elasticity of demand entails a strong reduction in earnings, and hence in tax revenue. From this perspective, the inverse elasticity rule only paraphrases the above contention that goods whose consumption stimulates earnings should be taxed at low rates. In fact, the impact on earnings is again the driving force behind the result, and the utility function (14) presents but a theoretical curiosity, implying a one-to-one correspondence between individual consumption expenditures on the one hand and earnings on the other.

*3. Weakly Separable Utility:* The following assumption covers a broader class of utility functions.

$$(16) \quad u(\mathbf{x}, y) = u(v(\mathbf{x}), y),$$

where  $v(\mathbf{x})$  is a linear-homogenous subutility function. The first order conditions (1) become:

$$(17) \quad \frac{\partial u(v(\mathbf{x}), y)}{\partial v} \frac{\partial v}{\partial \mathbf{x}} - \mathbf{m}\mathbf{q} = \mathbf{0} \quad \text{and} \quad \frac{\partial u(v(\mathbf{x}), y)}{\partial y} = -\mu.$$

Substituting the second equation into the first and multiplying by  $\mathbf{x}$  yields:

$$(18) \quad \frac{\partial u(v(\mathbf{x}), y)}{\partial v} v(\mathbf{x}) + \frac{\partial u(v(\mathbf{x}), y)}{\partial y} y = 0,$$

where Euler's equation  $v(\mathbf{x}) = \partial v / \partial \mathbf{x} \mathbf{x}$  and the identity  $y = \mathbf{q} \mathbf{x}$  have been exploited. The desired result now follows from differentiating this equation implicitly (demonstrating how neat some proofs become under the primal approach):

$$(19) \quad \frac{\partial y}{\partial x_i} = - \frac{\frac{\partial^2 u}{\partial v^2} \frac{\partial v}{\partial x_i} v + \frac{\partial u}{\partial v} \frac{\partial v}{\partial x_i} + \frac{\partial^2 u}{\partial y \partial v} \frac{\partial v}{\partial x_i} y}{\frac{\partial^2 u}{\partial v \partial y} v + \frac{\partial^2 u}{\partial v^2} y + \frac{\partial u}{\partial y}} = \mathbf{g} \frac{\partial v}{\partial x_i}.$$

The common factor  $\mathbf{g}$  is defined by the right-hand equation. As equation (17) asserts that  $\partial v / \partial x_i = -\partial u / \partial y / (\partial u / \partial v) q_i$ , the index of encouragement now amounts to  $e_i = -\partial u / \partial y / (\partial u / \partial v) \mathbf{g}$ . The index does not vanish in general, but is identical across commodities, implying uniform taxation. In the present case, the marginal rates of substitution of the consumption goods are independent of earnings, and *vice versa*. Therefore, distorting consumption along an indifference hyper-surface makes the consumption bundle more expensive and leaves earnings unchanged, which cannot be optimal.

*4. Stone-Geary Utility:* The last example regards utility functions of the form

$$(20) \quad u(\mathbf{x}, y) = (1 - y)^{a_0} (x_1 - \mathbf{b}_1)^{a_1} \cdots (x_n - \mathbf{b}_n)^{a_n},$$

the numbers  $a_i$  again summing up to unity. Using  $\hat{x}_i = x_i - \beta_i$ , we are back in the Cobb-Douglas case and obtain  $\hat{x}_i = a_i / q_i$  or  $q_i = a_i / (x_i - \beta_i)$ , the familiar linear expenditure system. As cross elasticities vanish,  $\partial y / \partial x_i = -a_i x_i / (x_i - \beta_i)^2 + a_i / (x_i - \beta_i)$ . Dividing the last expression by the consumer price yields the index of encouragement:

$$(21) \quad e_i = - \frac{\mathbf{b}_i q_i}{\mathbf{a}_i}.$$

As tax rates are proportional to  $1 - e_i$ , they are increasing in  $\beta_i$ . This is often taken to mean that taxes on basic needs such as food should be high. Considering the possibility of home production (Sandmo 1990) suggests a somewhat different interpretation: Tax theory focuses on *market demands* rather than on consumption

in the literal sense. If consumers can avoid taxes on specific market demands by home production, these demands should be taxed moderately. On the other hand, commodities like portables cannot be produced at home, and taxing them at high rates forces consumers to increase earnings. Following this interpretation, a high value of  $\beta_i$  indicates that consumers cannot easily dispense with the respective market demand. Again, the relationship between consumption and earnings is crucial for the optimal tax rates.

## 6. THE NORMALIZATION PROBLEM

The above analysis uses the classical framework of a pure excise tax system. This specific normalization of the consumer price vector, which assumes the absence of a wage tax, is not as innocuous as it appears, nor does it present a serious obstacle for developing a meaningful theory of optimal commodity taxes. In what follows, all producer prices are set equal to unity, which can always be done by an appropriate choice of units. The above analysis involves a vector  $(q_0, \mathbf{q})$  of consumer prices. As consumption demands and effort are homogenous of degree zero in  $(q_0, \mathbf{q})$ , the first component of this vector, the consumer price of leisure (or net wage rate), can also be chosen arbitrarily; and in the above analysis it was set to unity, implying that earnings remained untaxed. Thus, the variable  $y$  represented both earnings and effort, and we obtained concrete optimal tax rates – which may be the relevant one in case of a less developed country that raises most tax revenue through excises.

To generalize, let us now write earnings more explicitly as  $y = q_0 \ell$ , where  $\ell$  represents effort. Introducing a wage tax means reducing  $q_0$  somewhat. With total tax revenue given, this change does in no way influence the allocation. In the presence of a single commodity, for instance, it is immaterial whether we set  $q_1 = 2$  and  $q_0 = 1$  (a pure excise tax) or  $q_1 = 1$  and  $q_0 = 1/2$  (a pure wage tax). But the relative tax rates described by (11) are not invariant with respect to the normalization. This finding led Mirrlees (1976) to dismiss altogether the idea of characterizing optimal tax rates. However, this appears to push the point too far because, once we know the wage tax rate – and there is nothing to prevent us from doing so – we can infer the optimal commodity tax rates. These are uniquely defined, relative to the actual wage tax. The upshot is that calculating relative tax rates becomes possible once the wage tax is known. As the latter is public knowledge, an applied economist can easily use it. However, it is important to keep in mind – especially regarding Europe with its largely harmonized VAT regulations – that optimal commodity tax rates cannot be determined without regard to the actual mix of direct and indirect taxation<sup>7</sup>.

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<sup>7</sup> For a related study of the direct-indirect mix, see Smart (2002).

For a theorist not interested in concrete numerical solutions, there is yet an easier way to cope with the normalization problem. All producer prices having been normalized to unity, the inequality  $q_i - q_j > 0$  asserts unambiguously that commodity  $i$  bears a higher tax than commodity  $j$ . The normalization of commodity prices affects only the size of the difference, not its sign. Substituting  $t_i = q_i - 1$  into (9), subtracting two such equations and rearranging yields

$$(22) \quad q_i - q_j = -\frac{1 - m}{m}(e_i - e_j).$$

It remains true that goods are taxed the more heavily, the smaller their index of encouragement. Indeed, the index of encouragement itself is *invariant* with respect to the normalization of the commodity price vector, and so is the difference  $e_i - e_j$ , which provides an abstract measure of relative tax burdens. Normalizations influence the difference of consumer prices solely through their effects on the Lagrangean multipliers. Since scaling up the price vector does not affect the sign of  $q_i - q_j$ , a theorist interested in qualitative properties can forget about normalization.

## 7. CONCLUSION

In this paper, an alternative route to characterizing second-best commodity taxes has been developed. It has been shown that optimal tax rates depend directly on the impact of consumption on earnings. If a higher consumption of a specific good like coffee induces the consumer to work harder, then this good should be taxed only moderately. Conversely, time-consuming activities should be taxed heavily. The intuition behind this result is straight-forward: As the government cannot tax leisure endowments directly, it should stimulate consumption of precisely the same goods which induce consumers to work harder. This move increases earnings as the genuine tax base and reduces the average tax rate, since total taxes paid remain constant. Work-related expenses should be taxed moderately or be made deductible under a wage tax.

In essence, the new tax rule can be considered as a generalization of the Corlett-Hague rule. It differs from the latter in three respects: Firstly, it holds for arbitrarily many commodities, and not just in the two-commodities case. Secondly, it characterizes optimal tax rates in terms of directly observable variables rather than in terms of unobservable compensated elasticities. And finally, the primal approach in the form invented here allows deriving expressions of optimal tax rates in a few lines, whereas the standard method follows a pretty roundabout course: It starts with characterizing optimal quantities using the dual approach, obtains the Ramsey rule, and finally re-translates the result into primal terms in order to describe the optimal tax rates.

The last section has shown that optimal commodity tax rates can be determined without ambiguity once the wage tax is known. Since data on earnings, consumption, and prices are readily available, and since a researcher adopting this course does not need to know the utility function, it remains to be hoped that the approach proposed here will stimulate empirical research in the field.

## APPENDIX

In order to render the mathematics air-tight, it is most convenient to make the following two assumptions: The utility function is strictly quasi-concave, and  $\partial \mathbf{x} / \partial \mathbf{q}$  is negative definite<sup>8</sup>. Then, ordinary and inverse functions  $\mathbf{x}(\mathbf{q})$  and  $\mathbf{q}(\mathbf{x})$  exist. The excess demand function is the mapping  $(q_0, \mathbf{q}) \rightarrow [\ell(q_0, \mathbf{q}), \mathbf{x}(q_0, \mathbf{q})]$ , whose Jacobian determinant vanishes, of course.  $\partial \mathbf{x} / \partial \mathbf{q}$  is obtained from the Jacobian by deleting the first row and column. Note that we could also write effort in the more accustomed notation as  $-x_0$  without affecting any of the above results. The present notation has been adopted because it is more intuitive.

Negative definiteness of  $\partial \mathbf{x} / \partial \mathbf{q}$  – the famous “Law of Demand” which essentially rules out Giffen goods – is often assumed implicitly in optimal tax theory: Under the dual approach, post-multiplying equation (3) by the tax vector shows that precisely this assumption entails  $\partial > \mu$ , since  $\mathbf{t}\mathbf{x} > 0$  and  $\mu > 0$ . If the assumption were violated,  $\partial - \mu$  could become negative, implying that Ramsey taxes *increase*, rather than decrease, all uncompensated demands by a common factor. Therefore, the Law of Demand is the relevant assumption also in case of the dual approach. Under the primal approach, the relationship  $\partial > \mu$ , which has been used in the interpretations, follows from pre-multiplying equation (7) by the consumption vector. Moreover, the analysis assumes existence of an optimum, excludes corner solutions and relies on an appropriate constraint qualification in order to have necessary optimality conditions.

The offer curve  $y(\mathbf{x})$  is defined by the equation

$$(23) \quad \frac{\partial u(\mathbf{x}, y)}{\partial \mathbf{x}} \mathbf{x} + \frac{\partial u(\mathbf{x}, y)}{\partial y} y = 0,$$

which follows from combining equations (1), multiplying by  $\mathbf{x}$  and using  $y = \mathbf{q}\mathbf{x}$ . One can easily verify by non-vanishing Jacobian determinants that the offer curve exists if and only if the inverse demand functions exist, but owing to  $y(\mathbf{x}) = \mathbf{q}(\mathbf{x})\mathbf{x}$ , this is clear anyway. Maximizing consumers’ utilities subject to  $y - \mathbf{p}\mathbf{x} \geq g$  and (23)

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<sup>8</sup> Some would speak of negative quasi-definiteness, because  $\partial \mathbf{x} / \partial \mathbf{q}$  is not symmetric in general. Gale and Nikaido (1965), who proved global existence of inverse demand, would have called  $-\partial \mathbf{x} / \partial \mathbf{q}$  a P-matrix. If one only wants local invertibility, the assumption of a regular equilibrium suffices.

is an interesting way to state the optimal commodity tax problem, since taxes do not enter this formulation either directly or indirectly.

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