

Wasmer, Etienne; Zenou, Yves

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Etienne Wasmer  
Yves Zenou

August 2000

# Space, Search and Efficiency

**Etienne Wasmer**

*ECARES-ULB, CEPR and IZA, Bonn*

**Yves Zenou**

*University of Southampton, CEPR and IZA, Bonn*

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IZA

P.O. Box 7240

D-53072 Bonn

Germany

Tel.: +49-228-3894-0

Fax: +49-228-3894-210

Email: [iza@iza.org](mailto:iza@iza.org)

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## ABSTRACT

### Space, Search and Efficiency\*

We investigate the role of spatial frictions in search equilibrium unemployment. For that, we develop a model of the labor market in which workers' location in an agglomeration depends on commuting costs, the endogenous price of land and the value of job search and employment. We first show that there exists a unique and stable market equilibrium in which both land and labor markets are solved for simultaneously. We then compare this decentralized equilibrium to a social planner's optimum and we find that distortions (subventions or imperfect competition in the transport market) modify the usual Hosios efficiency condition. Indeed, the social planner needs to adjust the transportation spending of the decentralized equilibrium. Given differences in commuting costs between the employed and the unemployed, this is realized by a change in the fraction of unemployed workers: the socially optimal number of unemployed workers depends both of matching externalities and on distortions in the transport market. In absence of these distortions and despite spatial terms in wages, the standard condition holds: a spatial efficient equilibrium may thus occur. We however show that space has still an important role on the interaction between land and labor markets, and decompose the equilibrium unemployment rate into two parts: a pure non-spatial one (which corresponds to the standard matching model) and a mixed of non-spatial and spatial elements, the first element amplifying the other one.

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Etienne Wasmer  
ECARES  
50 Avenue Franklin D. Roosevelt  
CP 114  
1050 Brussels  
Belgium  
Tel: +32 2 650 4212  
Fax: +32 2 650 4475  
Email: [ewasmer@ulb.ac.be](mailto:ewasmer@ulb.ac.be)

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## 1 Introduction

It has been recognized for a long time that distance interacts with the diffusion of information. In his seminal contribution to search, Stigler (1961) puts geographical dispersion as one of the four immediate determinants of price ignorance. The reason is simply that distance affects various costs associated with search. In most search models, say for example Diamond (1982), distance between agents or units implies a fixed cost of making another draw in the distribution. In other words, a spatial dispersion of agents creates more frictions and thus more unemployment. Conventional labor economics faces difficulties to think about these spatial differences because it is biased towards the notion of a spaceless marketplace ruled by the walrasian auctioneer.

This is a weakness of the analysis since empirical evidence support the idea of a clear *spatial dimension of the labor markets* (see for example the recent survey of Crampton, 1999). There are in fact several channels through which space affects the labor market. First, workers who live further away from jobs may have poorer labor market information and be less productive than those living closer to jobs (Seater, 1979). This is particularly true for younger and/or less-skilled workers who rely heavily on informal search methods for obtaining employment (Holzer, 1987).<sup>1</sup> The reliance on these informal methods of job search suggests that information on available job opportunities may decay rapidly with the distance from home (Ihlanfeldt and Sjoquist, 1990). Second, distance also implies higher commuting costs for the unemployed, which directly affects the search process (Van Ommeren et al., 1997). Third, workers residing too far away from jobs may quit their job more frequently because of too long commuting distances (Zax and Kain 1996). Finally, employers may discriminate against applicants living in remote areas (Zenou and Boccoard, 2000). As a result it is commonly observed that unemployment rates differ strongly across as well as within local labor markets (see e.g. Blanchflower and Oswald, 1994, Martson, 1985, Topa, 2000).

The interaction between space and labor markets is thus complex. The aim of our paper is to capture some of the phenomenons at work and, in particular, to account for the spatial

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<sup>1</sup>In Holzer (1988), it is shown that among 16-23 years old workers who reported job acceptance, 66% used informal search channels (30% direct application without referral and 36% friends/relatives), while only 11% using state agencies and 10% newspapers.

dimension of search. We have two questions in mind: does the efficiency results of decentralized search equilibria (Lucas and Prescott 1974, Moen 1997) still hold when the spatial dispersion of agents creates frictions? And does the search equilibrium strongly depend on these spatial terms? Our answers are yes and yes.

In our approach, the allocation of jobs and workers is a time-consuming process and the number of matches per unit of time between workers and open vacancies is represented by an aggregate matching function (à la Diamond-Mortensen-Pissarides). Even if firms pay workers their reservation wage, there is still a level of durable unemployment in the area (due to stochastic rationing not being eliminated by price adjustment). However, in this line of search models, the spatial dimension is often implicit. Here, we explicitly introduce it by considering that the distance between workers' residential locations and jobs plays an adverse role in the formation of a match. In this respect, our model can be viewed as a natural extension of the standard matching model. The land market will be kept rather simple in order to provide closed-form solutions. We consider a closed piece of land (that can be thought as an urban area, a city, an agglomeration or a region). This area is monocentric, i.e., firms are exogenously located in an employment center and workers consume inelastically one unit of space. In our analysis, local factors (rental price, distance to the employment center) and global factors (labor market tightness, wages) influence workers' location decisions, i.e. the land market equilibrium. Within this framework, we can have different land market equilibria. We only focus here on the equilibrium in which the unemployed reside further away from the employment center.

We then derive the labor market equilibrium in which spatial unemployment is due to frictions in the labor market. On the one hand, the land market equilibrium depends on aggregate variables (such as wages and labor market tightness) since these variables affect location choices of workers. On the other hand, the labor market equilibrium crucially depends on the land market equilibrium configuration. Indeed, the efficiency of aggregate matching depends on the average location of the unemployed.

In this context, we show that there exists a unique and stable market equilibrium in which both land and labor markets are solved for simultaneously. When comparing this decentralized equilibrium to a social planner's optimum, we find that distortions (subventions or imperfect competition in the transport market) modify the usual Hosios efficiency condition. Indeed, the social planner needs to adjust the transportation spending of the de-

centralized equilibrium. Given differences in commuting costs between the employed and the unemployed, this is realized by a change in the fraction of unemployed workers: the socially optimal number of unemployed workers depends both of matching externalities and on distortions in the transport market. In absence of these distortions and despite spatial terms in wages, the standard condition holds: a spatial efficient equilibrium à la Moen (1997) may thus occur.

We then show that space has still an important role on the interaction between land and labor markets. We notably decompose the equilibrium unemployment rate into two parts: a pure non-spatial one (which corresponds to the standard matching model) and a mixed of non-spatial and spatial elements, the first element amplifying the other one. In other words, space adds to search frictions in the labor market by making the access to jobs more difficult.

The remainder of the paper is organized as follows. The next section presents the model and its notations. In section 3, we derive the equilibrium land market configuration. The labor market equilibrium is then studied in section 4. Section 5 derives the welfare analysis. Section 6 shows the different roles of space in the determination of equilibrium unemployment. Finally, section 7 concludes.

## 2 The model and general notations

Firms and workers are all (*ex ante*) identical. A firm is a unit of production that can either be filled by a worker whose production is  $y$  units of output or be unfilled and thus unproductive. In order to find a worker, a firm posts a vacancy that can be filled according to a random Poisson process. Similarly, workers searching for a job will find one according to a random Poisson process. In aggregate, these processes imply that there is a number of contacts per unit of time between the two sides of the market that we assume to be determined by the following matching function:

$$x(\bar{s}U, V) \tag{1}$$

where  $\bar{s}$  is the average efficiency of search of the unemployed workers (a worker  $i$  has an efficiency of search equal to  $s_i$ ), and  $U$  and  $V$  denote the number of unemployed workers and vacancies respectively. Since  $\bar{s}$  represents the aggregate search frictions, it is also *an index of aggregate information about economic opportunities*.

We assume that  $x(\cdot)$  is increasing both in its arguments, concave and homogeneous of degree 1 (or equivalently has constant return to scale). In this context, the probability for a

vacancy to be filled per unit of time is  $\frac{x(\bar{s}U, V)}{V}$ . By constant return to scale, it can be written as:

$$x\left(\frac{1}{\theta}, 1\right) \equiv q(\theta)$$

where  $\theta = V/U\bar{s}$  is a measure of labor market tightness in efficiency units and  $q(\theta)$  is a Poisson intensity. By using the properties of  $x(\cdot)$ , it is easily verified that  $q'(\theta) \leq 0$ : the greater the labor market tightness, the lower the probability for a firm to fill a vacancy. We assume that firms have no impact on their own search efficiency and consider  $\bar{s}$ ,  $U$  and  $V$  as given. Similarly, for a worker  $i$  with efficiency  $s_i$ , the probability of obtaining a job per unit of time is:

$$\frac{x(\bar{s}U, V)}{U} \frac{s_i}{\bar{s}} \equiv \theta q(\theta) s_i \equiv p_i$$

where  $p_i$  is defined as the intensity of the exit rate from unemployment. In contrast to the standard model of job matching (Mortensen and Pissarides, 1999, Pissarides, 2000) where there is no spatial dimension, we make here the assumption that  $s_i$  depends on the location of the unemployed workers in the city: the closer the residential location to the workplace, the better the efficiency and the more likely is a contact ( $s_i = s(d_i)$  where  $d_i$  is the location of the worker with  $s'(d_i) < 0$ ).<sup>2</sup> Such a dependence of search on distance has several empirical supports. For instance, Barron and Gilley (1981) and Chirinko (1982) find evidences of diminishing returns to search when people live far away from jobs. Rogers (1997) also demonstrates that access to employment is a significant variable in explaining the probability of leaving unemployment.

For analytical simplicity, we assume that:

$$s_i(d_i) = s_0 - a d_i \tag{2}$$

with  $s_0 > 0$  and  $a > 0$ . In this formulation,  $s_0$  denotes the standard non-spatial search effort such as writing letters, buying newspapers ... while  $a$  represents the loss of information per unit of distance. In other words, when workers are further away from jobs, it is more difficult for them to have information about jobs than those who are located closer to jobs. When  $a = 0$ , we are back to the standard non-spatial search model.

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<sup>2</sup>In section 6.3, we relax this assumption but show that  $s_i$  and  $d_i$  are negatively related when search effort is affected by commuting costs.



Once the match is made, the wage is determined by the generalized Nash bargaining solution. There is also a probability  $\delta$  per unit of time that the match is destroyed. In order to determine the (general) equilibrium, we will proceed as follows. We first determine the partial land market equilibrium configuration. Then, depending on the location of workers and thus on the aggregate search efficiency  $\bar{s}$ , we determine the partial labor market equilibrium. Hereafter, by labor (respectively land market) equilibrium, it has to be understood the partial equilibrium. The general equilibrium involves two markets, and will be denominated a ‘market equilibrium’.

By denoting by  $R(d)$  the land market price at a distance  $d$  from the city-center, by  $w$  the wage earned by workers and by  $u$  the unemployment rate, we have the following definition.

**Definition 1** *A market equilibrium  $(R(d), w, \theta, u)$  is such that the land use equilibrium and the labor market equilibrium are solved for simultaneously.*

Thus, a market equilibrium requires solving simultaneously two problems:

- (i) a location and rental price outcome that determines  $R(d)$  (referred to as an urban land use equilibrium), and
- (ii) a (steady state) matching equilibrium that determines  $w, \theta$  and  $u$  (referred to as a labor market equilibrium).

We will give below more precise definitions of these two markets.

### 3 The equilibrium land market configuration

The area is monocentric, i.e., all firms are assumed to be exogenously located in the employment center. It is linear, closed and landlords are absent.<sup>3</sup> There is a continuum of workers uniformly distributed along the linear piece of land who decide their optimal residence between the employment center and the fringe. They all consume the same amount of land (normalized to 1) and the density of residential land parcels is taken to be unity so that there are exactly  $d$  units of housing within a distance  $d$  of the employment center.

The employed workers go to the employment center to work while the unemployed workers go to the employment center to be interviewed. Let us denote by  $t_e d$  and  $t_u d$  the transportation cost at a distance  $d$  from the employment center for respectively working activities and

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<sup>3</sup>All these assumptions are very standard in urban economics (see e.g. Brueckner, 1987 or Fujita, 1989). The extension to a circular symmetric area is straightforward since any ray through the center looks like any other ray. Examining a single ray is almost the same as looking at the whole area.

unemployed specific activities (interviews, registration), with  $t_e > 0$  and  $t_u > 0$ . All workers bear land rent costs at the market price  $R(d)$  and receive a wage  $w$  when employed and unemployment benefits  $b$  if unemployed. We denote by  $I_u$  and  $I_e$  the expected discounted lifetime net income of the unemployed and the employed respectively. We assume that location changes are costless. With the Poisson probabilities defined above, infinitely-lived workers have the following intertemporal utility functions:

$$rI_u(d) = b - t_u d - R(d) + p(d) \left[ \left( \max_{d'} I_e(d') \right) - I_u(d) \right] \quad (3)$$

$$rI_e(d) = w - t_e d - R(d) + \delta \left[ \left( \max_{d'} I_u(d') \right) - I_e(d) \right] \quad (4)$$

where  $r$  is the exogenous discount rate. Let us comment (3). When a worker is unemployed today, he/she resides in  $d$  and his/her net income is  $b - t_u d - R(d)$ . Then, he/she can get a job with a probability  $p(d)$  and if so, he/she relocates optimally in  $d'$  and obtains an increase in income of  $I_e(d') - I_u(d)$ . The interpretation of (4) is similar.

Due to the absence of relocation costs, the equilibrium is such that all the unemployed enjoy the same level of utility  $rI_u = r\bar{I}_u$  as well as the employed  $rI_e = r\bar{I}_e$ . Indeed, any utility differential within the area would lead to the relocation of some workers up to the point where differences in utility disappear. We thus need to formally derive the optimal location of all workers in the area. The standard way of doing it in urban economics is to use the concept of bid rents (Fujita, 1989) which are defined as the maximum land rent at a distance  $d$  that each type of worker is ready to pay in order to reach his/her respective equilibrium utility level. Therefore, the bid rents of the unemployed and employed are respectively given by:

$$\Psi_u(d, \bar{I}_u, \bar{I}_e) = b - t_u d + p(d)\bar{I}_e - [r + p(d)]\bar{I}_u \quad (5)$$

$$\Psi_e(d, \bar{I}_u, \bar{I}_e) = w - t_e d + \delta\bar{I}_u - (r + \delta)\bar{I}_e \quad (6)$$

If  $d_f$  denotes the area fringe, we have the following definition:

**Definition 2** *The land use equilibrium  $R(d)$  is the upper envelop of all workers' bid rents and of the agricultural land rent  $R_A$ , i.e.,*

$$R(d) = \max \{ \Psi_u(d, \bar{I}_u, \bar{I}_e), \Psi_e(d, \bar{I}_u, \bar{I}_e), R_A \} \quad \text{at each } d \in [0, d_f]$$

In order to analyze the location of the unemployed and employed workers in the area, we need to calculate the slopes of their bid rents. The linear formulation of  $s(d)$  given by (2) implies that  $p''(d) = 0$  and thus that the bid rents are linear, i.e.  $\frac{\partial^2 \Psi_u(d, \bar{I}_u, \bar{I}_e)}{\partial d^2} = \frac{\partial^2 \Psi_e(d, \bar{I}_u, \bar{I}_e)}{\partial d^2} = 0$ . In this context, a steeper bid rent implies a location closer to the employment center. We have:

$$\frac{\partial \Psi_u(d, \bar{I}_u, \bar{I}_e)}{\partial d} = -t_u + p'(d)(\bar{I}_e - \bar{I}_u) < 0 \quad (7)$$

$$\frac{\partial \Psi_e(d, \bar{I}_u, \bar{I}_e)}{\partial d} = -t_e < 0 \quad (8)$$

where  $p'(d) = \theta q(\theta) s'(d) < 0$ . These slopes (in absolute values) can be interpreted as the marginal cost that a worker is ready to pay in order to be marginally closer to the employment center.<sup>4</sup> On the one hand, the marginal cost of the unemployed is the sum of the marginal commuting cost,  $t_u$ , and the marginal probability of finding a job times the (intertemporal) surplus of being employed. On the other hand, the employed workers bear only marginal commuting costs equal to  $t_e$  since the probability of losing a job  $\delta$  is totally exogenous and does not depend on the location of workers.

Depending on the relative slopes, only two land market equilibria are possible: either the unemployed reside in the vicinity of the employment center and the employed at the outskirts of the area or the unemployed locate at the outskirts of the area and the employed close to the center. In this paper, we focus only on the equilibrium where the unemployed are far away from jobs: we want to show how the spatial access to jobs matters for the labor market outcomes of workers, without needing the exploration of all land market equilibria. The condition for this equilibrium occurs if that the slope of the bid rent of the employed exceeds that of the unemployed, that is:<sup>5</sup>

$$t_e - t_u > \theta q(\theta) a (\bar{I}_e - \bar{I}_u) \quad (9)$$

This is quite intuitive: for the employed to occupy the core of the area, it must be that they bid away the unemployed. This condition states that the differential in commuting costs

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<sup>4</sup>We assume that the intersection of the slopes of the two bid rents is of zero measure so that we exclude mixed configurations where both employed and unemployed are located in the same place. We only consider separated configurations where only one category of workers locate within the same segment.

<sup>5</sup>When this condition does not hold, we have another equilibrium in which the unemployed reside close to jobs. The properties of this equilibrium are easy to derive. They are similar to the one developed here in almost all respects.

between the employed and the unemployed must be higher than the expected return for the unemployed of being more efficient in search by being marginally closer to the center. So, the resulting equilibrium is determined by a trade-off between the difference in commuting costs,  $t_e - t_u$ , and the marginal probability of finding a job.

By using Definition 2 and (9), we are now able to define formally the land use equilibrium. For simplicity, we normalize the labor force  $L$  to 1, i.e.  $L = U + N = 1$  ( $N$  is the number of employed workers) so that the unemployment rate  $u$  is equal to the unemployment level  $U$ . If we denote the border between the unemployed and the employed by  $d_b$ , we have the following definition:

**Definition 3** *The land market equilibrium  $(\bar{I}_u, \bar{I}_e, d_b, d_f)$  is such that:*

$$d_b = 1 - u \quad (10)$$

$$d_f = 1 \quad (11)$$

$$\Psi_e(d_b, \bar{I}_u, \bar{I}_e) = \Psi_u(d_b, \bar{I}_u, \bar{I}_e) \quad (12)$$

$$\Psi_u(d_f, \bar{I}_u, \bar{I}_e) = R_A \quad (13)$$

This equilibrium is illustrated in Figure 1. The two first equations are the standard population constraints when density and housing consumption is equal to 1 everywhere in the area. Equations (12) and (13) guarantee that the land rent is continuous everywhere in the area. By using (5), (6), (10) and replacing them in (12) and (13), we obtain:

$$\bar{I}_e - \bar{I}_u = \frac{w - b - (t_e - t_u)d_b}{r + \delta + p(d_b)} \quad (14)$$

where  $w, u, \theta$  will be determined at the labor market equilibrium and  $p(d_b) = [s_0 - a(1 - u)]\theta q(\theta)$ .

The average efficiency intensity is equal to:

$$\bar{s} = s_0 - a\bar{d} = s_0 - a\left(1 - \frac{u}{2}\right) \quad (15)$$

[Insert Figure 1 here]

## 4 The market equilibrium

Given our land market equilibrium, we can now define the market equilibrium and then, solve the general problem. Let us first have the following definition of a labor market equilibrium:

**Definition 4** *A (steady-state) labor market equilibrium  $(w, \theta, u)$  is such that, given the matching technology defined by (1), all agents (workers and firms) maximize their respective objective function, i.e. this triple is determined by a free-entry condition for firms, a wage-setting mechanism and a steady-state condition.*

### 4.1 Free-entry condition and labor demand

Let us denote by  $I_J$  and  $I_V$  the intertemporal profit of a job and of a vacancy, respectively. If  $\gamma$  is the search cost for the firm per unit of time and  $y$  is the product of the match, then  $I_J$  and  $I_V$  can be written as:

$$rI_J = y - w + \delta(I_V - I_J) \quad (16)$$

$$rI_V = -\gamma + q(\theta)(I_J - I_V) \quad (17)$$

Following Pissarides (2000), we assume that firms post vacancies up to a point where:

$$I_V = 0 \quad (18)$$

which is a free entry condition. From this free entry condition, we have the following decreasing relation between labor market tightness and wages:

$$\frac{\gamma}{q(\theta)} = \frac{y - w}{r + \delta} \quad (19)$$

In words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of search for the firm.

Equation (19) defines in the space  $(\theta, w)$  a curve representing the supply of vacancies (which is also a labor demand curve  $L^d$ ). The  $L^d$  curve (19) is independent of  $d_b$  and is downward sloping in the plane  $(\theta, w)$ . Because wages are endogenous, the effect of distance on the equilibrium unemployment will notably appear through commuting costs that affect wages. Let us now determine the endogenous level of wages in the economy.

## 4.2 Wage determination

The usual assumption about wage determination is that, at each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between firms and workers. The total surplus is the sum of the surplus of the workers,  $\bar{I}_e - \bar{I}_u$ , and the surplus of the firms. At each period, the wage is determined by :

$$w = \text{Arg max}(\bar{I}_e - \bar{I}_u)^\beta (I_J - I_V)^{1-\beta}$$

Observe that  $\bar{I}_u$ , the threat point for the worker, does not depend on the current location of the worker since the latter relocates if there is a transition in his/her employment status. Since the wage is negotiated at each period,  $\bar{I}_u$  does not depend on the current  $w$  and so  $\frac{\partial \bar{I}_u}{\partial w} = 0$ . First order condition then yields:

$$(\bar{I}_e - \bar{I}_u)(w) = \frac{\beta}{1 - \beta} \frac{\gamma}{q(\theta)} \quad (20)$$

By using (4) and (18), we have:

$$w = (1 - \beta) [\bar{I}_u + (t_e - t_u)d_b] + \beta y \quad (21)$$

Now, using (20), we replace  $\bar{I}_u$  by its value and finally obtain:

$$w = (1 - \beta) [b + (t_e - t_u)d_b] + \beta [y + (s_0 - ad_b)\theta \gamma] \quad (22)$$

where  $d_b = 1 - u$ . We thus have an expression for wages that combines usual non-spatial effects with additional spatial effects. The former are given by  $(1 - \beta)b + \beta[y + s_0\theta \gamma]$  and correspond to the usual Nash-bargaining solution in Pissarides (1990): workers receive a share  $\beta$  of the surplus expressed as a weighted average of the marginal product of labor and of a term that partly reflects the outside option of workers. This outside option is higher when search efficiency is higher and the tightness of the labor market is higher. The additional (spatial) part contains first  $(1 - \beta)(t_e - t_u)d_b$ , i.e. a pure spatial term: this is what firms must pay to share the costs incurred by the workers. These costs appear as the difference between the employed and the unemployed of the cost of the marginal worker who is the furthest away from the employment center, i.e. located at  $d_b = 1 - u$  (at the point in space where the rent difference between an employed and an unemployed agent vanishes). This is referred to as the *compensation effect*.<sup>6</sup> The second part of the additional term is  $-\beta ad_b\theta \gamma$ . It involves both

<sup>6</sup>See Smith and Zenou (1997) for a similar effect in an efficiency wage framework.

spatial and labor elements. Indeed, when  $d_b$  increases, the unemployed worker who is the closest from jobs (the one situated at  $d_b = 1 - u$ ) is even less close to jobs (spatial element) and thus has a lower search efficiency (labor element). The employed workers' outside option then decrease, which implies a reduction in wages. This is referred to as the *spatial outside option effect*.

Using  $w$  and  $\theta$ , the condition (9) that ensures that the land market equilibrium always exists and is unique can be rewritten as (see Lemma A1 in the Appendix):<sup>7</sup>

$$\theta < \bar{\theta} = \frac{1 - \beta}{\gamma a \beta} (t_e - t_u) \quad (23)$$

The intuition of this condition is straightforward: when the difference in commuting costs,  $t_e - t_u$ , is large and/or when the loss of information per unit of distance,  $a$ , is small and/or the search cost of firms,  $\gamma$ , is small, and/or the workers' bargaining strength,  $\beta$ , is low, then the employed bid away the unemployed at the periphery of the city.

With this condition, we can comment further on the wage equation. One can notice that, at constant  $\theta$ ,

$$\frac{\partial w}{\partial u} = -(1 - \beta)(t_e - t_u) + \beta a \theta \gamma \quad (24)$$

The overall net effect is thus ambiguous. However, the condition (23) implies that  $\frac{\partial w}{\partial u} < 0$ . We thus have a negative relationship between local wages and unemployment. This is consistent with the empirical evidence on local wage curves by Blanchflower and Oswald (1994) and Topel (1986).

Finally, combining (19) and (22), one can eliminate the wage and obtain a relation between  $\theta$  and  $u$  as follows:

$$y - b = \frac{\gamma}{q(\theta)} \left[ \frac{\delta + r + \theta q(\theta) s(d_b) \beta}{1 - \beta} \right] + (t_e - t_u) d_b \quad (25)$$

Using (23), it is easy to check that this relation is positive. Furthermore, since the wage setting curve described by (22) is linear in  $\theta$  and positively sloped and the free entry condition (18) implies a decreasing relation between labor market tightness and wages, we thus have two unique relations  $\theta(u)$  and  $w(u)$ .

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<sup>7</sup>All lemmas are stated and proved in Appendix 1.

### 4.3 Steady-state labor market equilibrium and market equilibrium

We can now close the model by providing another relation between  $\theta$  and  $u$ . We have the following steady-state condition on flows:

$$\theta q(\theta) \bar{s} u = (1 - u) \delta \quad (26)$$

which is equivalent to:

$$u = \frac{\delta}{\delta + \theta q(\theta) \bar{s}} \quad (27)$$

This is a negative relation between  $\theta$  and  $u$ . Given equation (25), the existence and uniqueness of  $\theta$  and  $u$  is established in Lemma A2 (see figure 2 for an illustration). Denote by  $\theta^*$  and  $u^*$  these equilibrium values, and  $w^* = w(u^*)$ . To summarize:

**Proposition 1** *When the parameters are such that  $\theta^* < \bar{l}$ , then, there exists a unique and stable market equilibrium  $(R(d), \theta^*, u^*, w^*)$  in which the unemployed reside far away from jobs and the employed close to jobs.*

The main result here is that the equilibrium labor market tightness,  $\theta^*$  must be lower than a threshold value  $\bar{l}$  (defined as a function of parameters; see (23)) for the land market configuration described in Figure 1 to exist. Otherwise, we obtain the reverse configuration in which the unemployed are close to jobs and the employed far away.

[Insert Figure 2 here]

There is another more intuitive way of representing the equilibrium: in the  $u - V$  space. As stated above, equation (19) determines a value of  $\theta = V/(u\bar{s})$  that gives a relation between  $V$  and  $u$ . This is an upward sloping curve in the  $u - V$  space called the  $VS$  curve (see Lemma A3). Furthermore, equation (26) can be rewritten as:

$$\delta(1 - u) - V q(V/u\bar{s}) = 0 \quad (28)$$

we obtain the so-called *Beveridge curve*  $UV$ . Its properties are given in Lemma A4. The interesting feature of this Beveridge curve is that it is indexed by  $\bar{s}$ , which depends on the spatial dispersion of the unemployed: a lower  $\bar{s}$  is associated with an outward shift of Beveridge curve in the  $u - V$  space because more vacancies are needed to maintain the steady-state level of unemployment.



## 5 Welfare analysis

Having derived the equilibrium in each market, one can now proceed to the welfare analysis. For that, we need to define the structure of transport cost and to introduce a new agent: the transportation firm (that may be private or public). Let us denote by  $c_e$  and  $c_u$  the (exogenous) marginal cost of transportation of respectively the employed and the unemployed and by  $-1 < \sigma < +\infty$  the mark-up over the marginal cost. We thus have:

$$t_e = (1 + \sigma)c_e \text{ and } t_u = (1 + \sigma)c_u$$

For simplicity and to illustrate how the (in-)efficiency of this economy depends on  $\sigma$ , we do not model its determination and thus treat  $\sigma$  as a parameter. If transportation is subsidized, then  $\sigma < 0$  whereas if the transportation firm has some market power  $\sigma > 0$ . Finally, when prices are equal to their marginal cost so that profits are equal to zero,  $\sigma = 0$ . In this context, the social welfare for an infinitely-lived social planner is given by the sum of the utilities of workers, the profit of firms net of search costs, the land rent of landlords and the net profit of the transportation firm. We have:

$$\Omega = \int_0^{+\infty} e^{-rt} \left\{ \int_0^{1-u} (y - c_e x) dx + \int_{1-u}^1 (b - c_u x) dx - \gamma \theta \bar{s} u \right\} dt$$

Observe that the wage  $w$  as well as the land rent  $R(x)$  cancel out in the social welfare function because they are pure transfers. Observe also that  $b$  is considered here as the value of leisure for the unemployed. We obtain the following result.

**Proposition 2** *The private and the social outcomes coincide only when:*

$$\beta = \eta(\theta) + \Lambda \tag{29}$$

where

$$\Lambda = -(1 - \beta)(1 - \eta(\theta)) \frac{q(\theta)/\gamma}{\delta + r + \theta q(\theta)s(d_b)} \sigma (c_e - c_u) d_b \gtrless 0$$

which is the standard Hosios-Pissarides condition extended to a spatial framework.

**Proof.** See Appendix 2.

Observe that, in general, nothing guarantees that the private outcome is efficient. This is a standard result in search theory due to search externalities. In the literature, an *efficient search equilibrium* is a situation in which the standard Hosios condition  $\beta = \eta(\theta)$  holds (see

Hosios, 1990, Moen, 1997, Mortensen and Pissarides 1999, Pissarides, 2000, ch.8). Indeed, in the spaceless model, when  $\beta$  is larger than  $\eta(\theta)$ , there is too much unemployment, creating congestion in the matching process for the unemployed. When  $\beta$  is lower than  $\eta(\theta)$ , there is too little unemployment, creating congestion for firms.

Observe also that the existence of spatial terms modifies the standard Hosios condition. Here, the condition  $\beta = \eta(\theta)$  that guarantees that the market solution is efficient in a non-spatial matching framework is augmented by a term incorporating some spatial aspects. Hereafter, we call a *spatial efficient search equilibrium* a situation in which the efficiency condition (29) holds. To interpret these additional terms, let us first discuss the conditions on parameters that modify or validate the standard Hosios-Pissarides condition.

**Corollary 1** *Efficiency requires that:*

- (i)  $\beta < \eta(\theta)$  when transport prices are above their marginal cost, i.e.  $\sigma > 0$ ;
- (ii)  $\beta > \eta(\theta)$  when transport prices are below their marginal cost, i.e.  $\sigma < 0$ ;
- (iii)  $\beta = \eta(\theta)$  when transport prices equal their marginal cost, i.e.  $\sigma = 0$ .

The following comments are in order. First, the social planner cares about transport costs ( $c_e$  and  $c_u$ ) whereas the decentralized equilibrium only involves transport prices ( $t_e$  and  $t_u$  faced by workers). When  $\sigma = 0$  (case (iii)), both are equal and the social planner only cares about the internalization of search-matching externalities. This precisely corresponds to the efficient search equilibrium.<sup>8</sup>

Second, if  $\sigma \neq 0$ , transport spending in the decentralized equilibrium are not optimal. Notably, when  $\sigma > 0$ , the transport costs in the decentralized economy are below the optimum whereas when  $\sigma < 0$ , the transport costs in the decentralized economy are above the optimum. Since, given inequality (9) in our configuration, we necessarily have  $c_e > c_u$ , a larger number of employed workers in the economy is associated with larger commuting costs. Thus, a higher  $\beta$ , implying higher wages, reduces employment and thus reduces transports spending.

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<sup>8</sup>Note that case (iii) may also hold is a degenerate case, when the transport cost differential between the employed and the unemployed is zero i.e.  $c_e = c_u$ , which, given inequality (9), is possible only if  $a = 0$ . In such a case, the inequality (9) is binding and the slopes of bid rents of the employed and unemployed are equal: the location of the agents is thus undetermined. Note also that when the spatial ingredients of the model disappear,  $a = 0$ , and  $c_e = c_u = 0$ , one naturally obtains the standard efficiency condition  $\beta = \eta(\theta)$ .

Accordingly, efficiency in the case  $\sigma > 0$  requires a higher level of employment to increase transports spending, i.e. when  $\beta$  is lower than what is required to internalize the search-matching externalities. Similarly, efficiency in the case  $\sigma < 0$  requires a lower level of employment to reduce transports spending, i.e. when  $\beta$  is higher than what is required to internalize the search-matching externalities.

More generally, if  $\sigma \rightarrow 0$ , we obtain the spaceless Hosios-Pissarides efficiency condition even though spatial elements remain. Does it mean that space has no importance in the model? The answer is clearly no: if space does not always modify the efficiency result, it strongly affects the equilibrium. We could already guess this result from the fact that the Beveridge curve, defined by equation (28), depends on  $\bar{s}$ . We investigate the dependence of unemployment to the spatial dispersion of agents more systematically in the next section.

## 6 The role of space in the theory of unemployment

### 6.1 Interaction between land and labor markets

The interaction between land and labor market is partly due to the dependence of search efficiency on distance. To show that, we proceed *a contrario*: we assume first that wages are exogenous and  $a = 0$ , and in this case, both markets equilibria are independent. When we relax exogenous wages and keep  $a = 0$ , there is a one-way interaction between markets: the labor market does not depend on the land market. Finally, as soon as  $a > 0$ , one has a general equilibrium interaction between the markets.

One of the key assumption of our model is that the search efficiency  $s_i$  of each worker  $i$  depends on the distance between residence and the job-center, i.e.,  $s_i = s_i(d)$  with  $s'_i(d) < 0$ . This implies that the land and labor markets are interdependent. Indeed, on the one hand, the labor market strongly depends on the land market since the equilibrium values of  $u^*$ ,  $V^*$  and  $\theta^*$  depend on the value of  $\bar{s}$ . On the other hand, the land market strongly depends on the labor market since the inequality (9) determining the land market equilibrium configuration,  $\theta^* < \bar{l}$ , depends on the value of  $\theta^*$ .

To evaluate the implications of this relation  $s_i(d)$ , let us assume that  $s_i$  is independent of  $d$  ( $a = 0$  so that  $\bar{s} = s_0$ ) but workers still locate in the city and thus bear land rents and commuting costs. In this context, the inequality (9) reduces to  $t_e - t_u > 0$ . In other words, *the land market equilibrium is independent of the labor market equilibrium*. The location choices of the employed and the unemployed, which depend on the slopes of the bid rents, involve

only transportation costs. So, since  $t_u < t_e$ , the unemployed reside at the outskirts of the city, irrespective of the labor market equilibrium outcome. When wages are exogenous, we do not have anymore equation (22) but instead  $w = \bar{w}$ . So, the equilibrium is defined by two equations, (19) and (26) in which  $w = \bar{w}$ . Therefore, when wages are exogenous, the equilibrium unemployment and vacancy rates would be exactly the same as in the standard non-spatial matching models (see e.g. Pissarides, 1990) and  $\bar{s} = s_0$  is independent of the land use equilibrium. On the contrary, when the wage is a result of a bargaining between workers and firms, the main difference with the standard non-spatial matching model is that commuting costs affect wages. We can summarize our discussion by the following table.

Table 1: Interaction between land and labor markets

	Exogenous wages	Endogenous wages
$a = 0$	No Interaction	Partial Interaction ( $LME \rightarrow LE$ )
$a > 0$	Complete Interaction	Complete Interaction

( $LME \rightarrow LE$  means that the interaction is from the

Land Market Equilibrium to the Labor Equilibrium)

## 6.2 Decomposition of unemployment

We pursue our analysis of the importance of space in equilibrium unemployment by determining the part of unemployment only due to spatial frictions. Let us start with exogenous wages. In this case,  $\theta$  is constant and determined by (19). By using (27), the unemployment rate is given by:

$$u = \frac{\delta}{\delta + \theta q(\theta) [s_0 - a(1 - u/2)]} \quad (30)$$

Let us further define by:

$$u_0 = \frac{\delta}{\delta + \theta q(\theta) s_0} \quad (31)$$

the part of *unemployment that is independent of spatial frictions*, i.e. when  $a = 0$  so that  $\bar{s}_0 = s_0$ . By a Taylor first-order expansion for small  $a/s_0$ , we easily obtain:

$$u^* = u_0 \left[ 1 + \frac{a}{s_0} (1 - u_0) (1 - u_0/2) \right] = u_0 + u^s \quad (32)$$

where  $u^s \equiv u_0 [a (1 - u_0) (1 - u_0/2) / s_0]$  is the *unemployment that is only due to spatial frictions* and  $u_0$  is defined by (31). Observe that  $u^s$  is increasing in  $a/s_0$ , the parameter

representing the loss in information through distance and null when  $a = 0$ . Observe also that the pure frictional unemployment  $u_0$  affects  $u^s$  in the following way:

$$\text{If } u_0 < 1 - \frac{\sqrt{3}}{3} \approx 0.42, \text{ then } \frac{\partial u^s}{\partial u_0} > 0$$

In general  $u_0 < 0.42$  so that  $u_0$  affects positively  $u^s$ , showing the full interaction between land and labor markets. This is quite natural: higher ‘spaceless’ unemployment  $u_0$  affects positively frictions due to spatial heterogeneity (this is a side-effect of the dispersion of space on the unemployed themselves, which increases the average distance to jobs).

Under endogenous wage setting, a larger set of parameters determines the spatial component of unemployment. First, the endogenous wage  $w$  defined by (22) can be decomposed into three parts:

$$w = w_0 + w_t - w_a \tag{33}$$

where  $w_0 = (1 - \beta)b + \beta(y + s_0\theta\gamma)$  is the wage that would receive workers if all agents were located in the same point,  $w_t = (1 - \beta)(t_e - t_u)(1 - u)$  reflects the impact of distance on transportation costs and thus on wages, and  $w_a = \beta a (1 - u)\theta\gamma$  the fact that search efficiency varies with distance to jobs (this was called the ‘outside option effect’ of distance in the previous section). By using a Taylor expansion, one could also decompose  $\theta$  in different parts, thus further decomposing the spatial part of unemployment into three parts itself. This is a bit involving for just finding a decomposition looking exactly as the decomposition of wages.

The conclusion is that, compared to the non-spatial case, unemployment increases because of the loss of information due to spatial dispersion of agents and also because of the wage compensation of commuting costs. However, it also tends to decrease because of the outside option effect that reduces wages.

### 6.3 Long term unemployed and space

Finally, in this sub-section, we discuss the link between the location of workers to their labor market outcomes. In our model, the unemployed who live further away from jobs experience longer unemployment spells than those residing closer to jobs. This result stems from the assumption that search efficiency depends negatively on distance. Here, we derive this relation from more primitive assumptions about search effort. In addition, by doing so, we introduce two classes of unemployed workers: the long run and short run unemployed workers.

Each individual's search efficiency  $s_i$  now depends only on his/her effort  $e$ . We assume decreasing returns to scale to effort, i.e.,  $s'(e) > 0$  and  $s''(e) \leq 0$ . As above, each interview is carried out in the employment center and thus involves transport costs. We denote by  $C_u(e, d)$  the search costs associated with a level of effort  $e$  for a worker living at a distance  $d$  from the employment center.<sup>9</sup> We assume that the search commuting cost is an increasing and convex function of the effort level  $e$  devoted to job search, i.e.,  $\partial C_u / \partial e > 0$  and  $\partial^2 C_u / \partial^2 e \geq 0$ , and that, quite naturally,  $\partial^2 C_u / \partial e \partial d > 0$ : the search effort marginally costs more further away from jobs. There is therefore a trade-off between search costs and returns associated with a higher probability to exit from unemployment. It is then easy to show (see Appendix 3) that, when workers choose endogenously their effort, their search efficiency and thus their probability of obtaining a job decreases with the distance to jobs. Indeed, when choosing their optimal level of effort, the unemployed workers equalize their marginal gain (which is the probability generated by one more interview times the surplus when leaving unemployment) and their marginal loss (which is the marginal commuting cost of searching for a job). Then, because search effort marginally costs more further away from jobs, individuals search less in remote places and thus their probability to find a job decreases with distance to jobs.

It is then readily verified that the unemployed's bid rents are not anymore linear but convex since search efficiency  $s$  is now a non-linear function of distance. This implies that a new land market configuration can emerge in which the unemployed reside both at the vicinity of the employment center and at the outskirts of the city and the employed living in between the unemployed. In this case, even though all the unemployed enjoy the same utility level, the ones who reside close to the employment center experience short unemployment spells because their search efficiency is very high whereas those who live further away are long term unemployed since their probability to find a job is quite low. In fact, the trade off is quite clear. Either workers reside in remote areas, are long run unemployed, live on welfare but pay very low land rents or reside close to jobs, experience short unemployment spells but pay a very high land rent. Thus, *space (or location) makes workers heterogeneous in terms of access to employment: those who are further away from jobs experience longer unemployment spells* (see e.g. Rogers, 1997 for empirical results).

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<sup>9</sup>In the previous sections, the assumption of costs was  $C_u(e, d) = t_u d$ .

## 7 Conclusion

In this paper, we have modelled the general interaction between the spatial dispersion of economic agents and the imperfection in information about economic opportunities. We have first demonstrated that there exists a unique and stable market equilibrium in which both land and labor markets are solved for simultaneously. We have then shown that the market equilibrium is in general not efficient because of both search and spatial elements, but that without distortions on the transport market, a spatial efficient equilibrium may hold. Finally, we have investigated how space affects search by focussing on the interaction between land and labor markets.

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## Appendix 1: Lemmas

**Lemma A1:** *The condition (9) can be rewritten as:*

$$\theta^* < \bar{l} \equiv \frac{1}{\gamma} \frac{t_e - t_u}{a} \frac{1 - \beta}{\beta} \quad (34)$$

**Proof.**

Using (20), it is easy to verify that condition (9) rewrites (34).

**Lemma A2:** *There exists a unique  $\theta$  and  $u$  in equilibrium.*

**Proof.**

- First, we know from equation (25) that the relation between  $u$  and  $\theta$  is positive, with  $u(0) = 1 - \frac{y-b}{t_e-t_u}$ .
- Second, we know from (27) that

$$\frac{\partial \theta}{\partial u} = -\frac{V}{(u\bar{s})^2} \left( \bar{s} + u \frac{\partial \bar{s}}{\partial u} \right)$$

where  $\bar{s} = s_0 - a(1 - u/2)$  and  $\partial \bar{s} / \partial u = a/2 > 0$ . This implies that  $\frac{\partial \theta}{\partial u} < 0$ . The relation between  $u$  and  $\theta$  is thus negative, with  $u(0) = 1$ .

- Lemma A1 implies that  $t_e > t_u$ , otherwise the land market equilibrium focussed here does not hold. Thus,  $1 > 1 - \frac{y-b}{t_e-t_u}$  and there exists a unique  $\theta$  and  $w$  (see figure 2 for an illustration).

**Lemma A3:** *The following relations always hold along the (VS) curve:*

$$\frac{\partial \theta}{\partial V} > 0 \quad ; \quad \frac{\partial \theta}{\partial u} < 0 \quad ; \quad \frac{dV}{du} > 0$$

**Proof.**

- First, since  $\theta = V/(\bar{s}u)$ , we have:

$$\frac{\partial \theta}{\partial V} = \frac{1}{u\bar{s}} > 0$$

- Second,

$$\frac{\partial \theta}{\partial u} = -\frac{V}{(u\bar{s})^2} \left( \bar{s} + u \frac{\partial \bar{s}}{\partial u} \right)$$

where  $\bar{s} = s_0 - a(1 - u/2)$  and  $\partial \bar{s} / \partial u = a/2 > 0$ . This implies that:

$$\frac{\partial \theta}{\partial u} < 0$$

- Finally, using these two results, we have:

$$\frac{dV}{du} = -\frac{\partial\theta/\partial u}{\partial\theta/\partial V} > 0$$

This upward sloping curve in the  $u - V$  space is called the  $VS$  curve.

**Lemma A4:** *In the plane  $(V, u)$ , the Beveridge curve (26) is decreasing. It cuts the axes at  $u(V = 0) = +\infty$  and  $V(u = 0) = +\infty$ .*

**Proof.** By totally differentiating (28):

$$\frac{dV}{du}q(\theta) + Vq'(\theta)(\partial\theta/\partial u) = -\delta\bar{s}[1 + (1 - u)/\bar{s}\partial\bar{s}/\partial u]$$

Then,  $1 + (1 - u)/\bar{s}\partial\bar{s}/\partial u$  can be rewritten as  $1/\bar{s} : [s(1/2)] > 0$ . This implies that,

$$\frac{dV}{du} < 0$$

Next, by using (26), we have:

$$V(u = 0) = +\infty \quad \text{and} \quad u(V = 0) = +\infty$$

## Appendix 2: Proof of Proposition 2

The social planner chooses  $\theta$  and  $u$  maximizing  $\Omega$  under the constraint  $\dot{u} = \delta(1 - u) - \theta q(\theta)\bar{s}u$ . In this problem, the control variable is  $\theta$  and the state variable is  $u$ . Let  $\lambda$  be the co-state variable. The Hamiltonian is thus given by:

$$H = e^{-rt} \left\{ \int_0^{1-u} (y - c_e x) dx + \int_{1-u}^1 (b - c_u x) dx - \gamma\theta\bar{s}u \right\} + \lambda [\delta(1 - u) - \theta q(\theta)\bar{s}u]$$

The Euler conditions are  $\frac{\partial H}{\partial\theta} = 0$  and  $\frac{\partial H}{\partial u} = -\dot{\lambda}$ . They are thus equal to:

$$\gamma e^{-rt} + \lambda q(\theta) [1 - \eta(\theta)] = 0 \quad (35)$$

$$\left\{ y - b - (c_e - c_u)d_b + \gamma\theta \left[ \bar{s} + \frac{\partial\bar{s}}{\partial u}u \right] \right\} e^{-rt} + \lambda \left\{ \delta + \theta q(\theta) \left[ \bar{s} + \frac{\partial\bar{s}}{\partial u}u \right] \right\} = \dot{\lambda} \quad (36)$$

where  $\eta(\theta) = -q'(\theta)\theta/q(\theta)$ . Let us focus on the steady state equilibrium in which  $\dot{\theta} = 0$ . By differentiating (35), we easily obtain that  $\dot{\lambda} = -r\lambda$ . The transversality condition is given by:

$$\lim_{t \rightarrow +\infty} \lambda u = 0$$

and is obviously verified. By plugging this value and the value of  $\lambda$  from (35) in (36), and by observing that  $\bar{s} + \frac{\partial\bar{s}}{\partial u}u = s(d_b)$ , we finally obtain:

$$y - b = \frac{\gamma}{q(\theta)} \left[ \frac{\delta + r + \theta q(\theta)s(d_b)\eta(\theta)}{1 - \eta(\theta)} \right] + (c_e - c_u)d_b \quad (37)$$

In order to see if the private and social solutions coincide, we compare (25) and (37) and obtain the result of the proposition.

### Appendix 3: Endogenous search effort

When workers choose endogenously their effort level, the value of unemployment can be written as:

$$rI_u(d) = b - C_u(e, d) - R(d) + \theta q(\theta) s(e) \left[ \left( \max_{d'} I_e(d') \right) - I_u(d) \right] \quad (38)$$

while the value of employment is still given by (4). The unemployed worker located at a distance  $d$  from the employment center chooses  $e^*$  that maximizes his/her intertemporal utility (38). The first order condition on effort yields:

$$\theta q(\theta) s'(e^*) (\bar{I}_e - \bar{I}_u) = \partial C_u(e^*, d) / \partial e \quad (39)$$

By totally differentiating (39), we obtain:

$$\frac{\partial e^*}{\partial d} = \frac{\partial^2 C_u(e^*, d) / \partial e \partial d}{\theta q(\theta) (\bar{I}_e - \bar{I}_u) s''(e^*) - \partial^2 C_u(e^*, d) / \partial^2 e} < 0$$

and thus

$$\frac{\partial s}{\partial d} = s'(e) \frac{\partial e}{\partial d} < 0$$

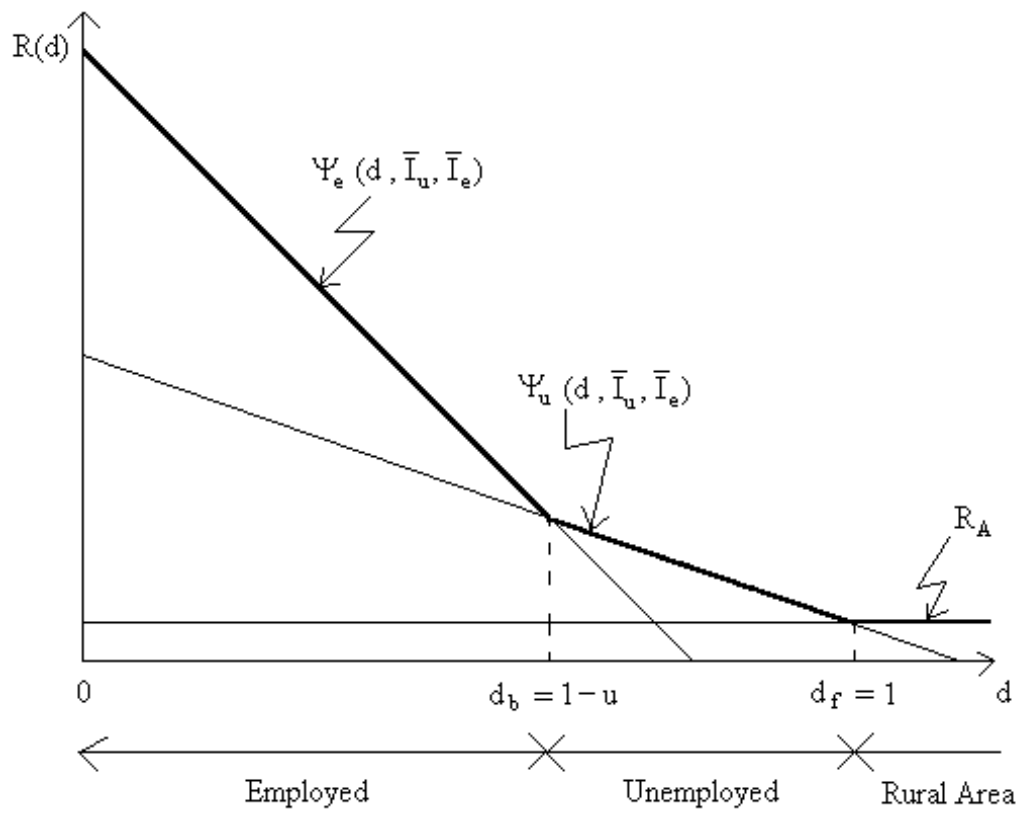


Figure 1 : Equilibrium Land Market

Figure 1:

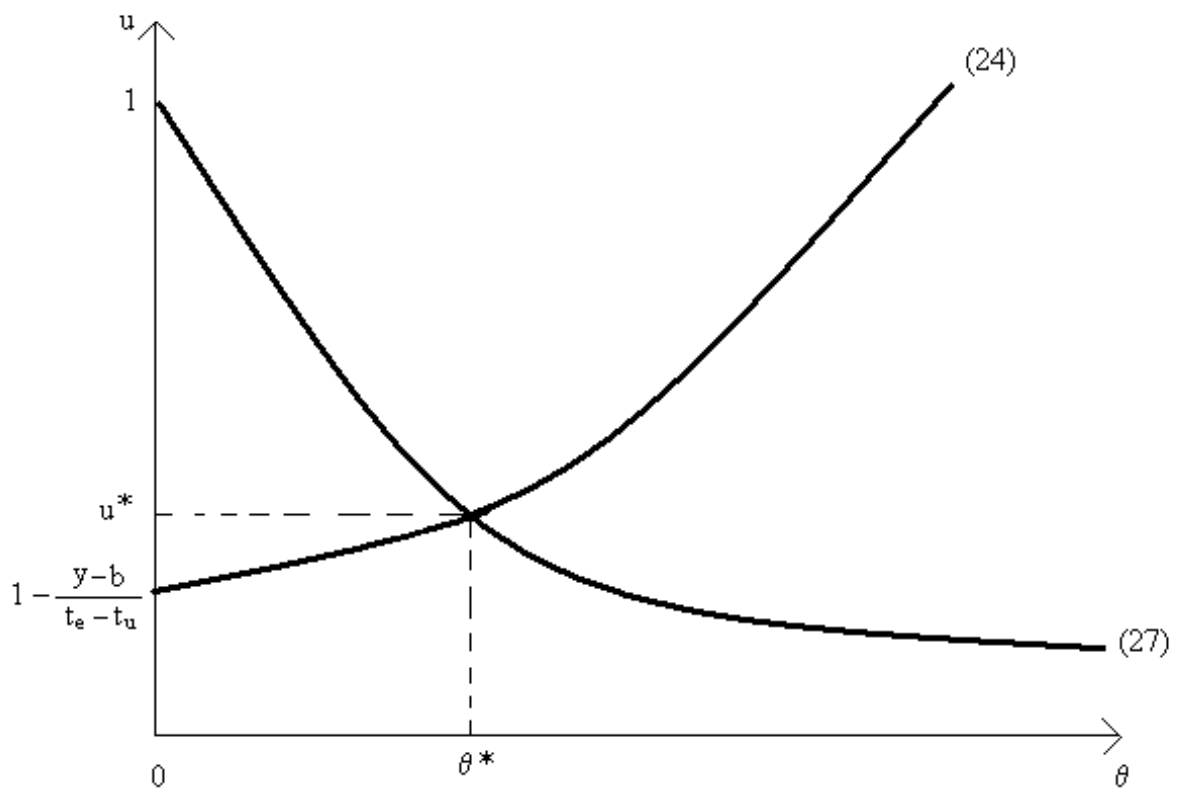


Figure 2 : Existence and Uniqueness of Equilibrium

Figure 2:

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