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## Diploma No Problem : Can Private Schools Be of Lower Quality than Public Schools?

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October 2004

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## ABSTRACT

### **Diploma no Problem: Can Private Schools Be of Lower Quality than Public Schools?\***

Motivated by anecdotal as well as econometric evidence from Italy, we ask whether private schools can provide lower quality than public schools. Using a stylized model of the education market with sequential entry of a public and a private school, we show that, depending on the underlying parameters of the model, a market structure with the private school offering at a positive price lower quality than the public school can be an equilibrium. The calibrated parameters for Italy suggest the existence of such an equilibrium in the Italian market for education.

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# 1 Introduction

Do private schools always provide better service than public schools? The answer is apparently straightforward: since private schools charge a positive price (tuition), they can only attract students by providing better service than public schools, which are funded by the taxpayer (see De Fraja, 2004). Yet quality is not the only service that private schools can provide. In a recent scandal, Italian prosecutors have found that some private schools in the country used to sell high school diplomas at a price. The so called "Diploma no problem" organization provided "good service" to its customers: answers were supplied in advance for written and oral exams, and attendance records were fixed. The national exam for the leaving high school certificate was also by-passed by having customers take the exams in places where the outcome was assured (*The Economist*, June 12th, 2004, p.31).

In this admittedly extreme example, private schools can charge a fee by allowing customers to grab the degree with little effort: the service offered is not quality but leisure. Less extreme perhaps is the evidence discussed by Figlio and Stone, 1997, that religious private schools in the US provide lower quality in mathematics and science than public schools. In contrast, nonreligious private schools are found to offer in these fields higher quality than public schools. This evidence suggests that private schools are heterogeneous, with some offering poorer academic quality and some others offering better quality than public schools<sup>1</sup>. Why do US households pay to send their offspring to school of lower academic quality? Figlio and Stone argue that parents may care for other outcomes, such as discipline, extracurricular activities, religious matters and the opportunity to interact with a certain peer group.

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<sup>1</sup>Vandenberghe and Robin, 2004, use the PISA dataset for an heterogeneous group of countries to examine the effect of private versus public education on pupils' achievement and show that private education does not generate systematic benefits. De Fraja, 2004, reports additional evidence on the UK by Marks and co-authors, who find that there is considerable variation in the quality of UK religious - and private - schools: some are very good but others are very poor. He also quotes evidence by Feinstein and Symons, 1999, who find that attendance of private schools does not affect on average individual performance in the UK.

In spite of this evidence, the theoretical literature does not allow for the possibility that private schools are of lower academic quality than public schools. An important example is Epple and Romano, 1998, who model the education market as a stratified hierarchy of school qualities, with private schools doing systematically better than public schools. Schools in their model are clubs of students who differ in their academic ability, and school quality is simply the average quality of enrolled pupils. The essential reason for the existence of a hierarchy with public schools dominated by private institutions is that all private schools must be of higher peer quality than schools in the public sector, otherwise no student would be willing to pay to attend a private school. In their model, state schools act as residual repositories, taking in all those students who do not enrol in private schools, because of their free admission policy.

The assumptions that private schools can only offer quality for a price and that state schools act as residual repositories are questionable. First, private schools can charge a positive price for leisure, access to networks or for religious education. Second, the assumption that public schools are of the poorest quality is both not always consistent with the stylized facts, at least in Europe and in Asia, and not derived from a policy decision rule, be it the maximization of a social welfare function or a political equilibrium based on some sort of majority rule.

This paper studies the implications of removing these two assumptions. We consider a very simplified market for education with a sequential structure. In the first stage of the sequential game, the government decides the quality standard of the public school, which charges no admission fees. The criterion used in the decision is majority voting. We believe that this is an appealing and intuitive criterion in a democracy, but we are also aware that there are other possible decision rules, such as the maximization of a welfare function (see De Fraja, 2002)<sup>2</sup>. In the second stage, a private school enters in the market and chooses both the positive price to charge its pupils and the quality standard,

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<sup>2</sup>Our characterization of the equilibrium via majority voting produces the same results as a political equilibrium based on the median voter.

which could be above or below the standard set by the public school. The private school maximizes profits by taking into account that its choice of price and standard affects the demand for its services. By restricting entry to a single private firm, we focus on the relative quality of public and private schools at the cost of overlooking the heterogeneity of private schools. We feel that the treatment of this heterogeneity is important but would require a separate paper.

Define an equilibrium in this market as the combination of public and private school standards with a nonnegative tuition fee set by the private school, which satisfy both majority voting and profit maximization. We show that, conditional on the standard chosen by the public school, there are three possible equilibria: in the first regime, public school quality is strictly higher than a threshold value and the private school selects the lowest available quality standard. In the second regime, public school quality is below the threshold and the private school selects a higher quality than the public sector. In the last regime, the private school does not enter in the education market.

Conditional on our assumptions, we also show that majority voting by the relevant population of parents selects either the lowest or the highest available quality standard for the public school. While the former case is consistent with Epple and Romano's story, the latter case is not and produces a hierarchy with private schools providing lower quality than public schools.

The possibility that an equilibrium exists with the private school offering lower quality than public schools has important implications for education policy. Take vouchers, for instance. A voucher paid out of taxpayer money by the national or local government to households enrolling their offspring in private schools can be justified on efficiency grounds if these schools provide better quality to individuals, who could not afford the price in the absence of the subsidy. Justification is harder, however, if the taxpayer's money is spent to finance vouchers paid out to households who enrol their children in private schools of low quality.

The paper is organized as follows. Section 2 presents empirical evidence on the relatively poor quality of Italian private schools. Section 3 introduces

the model with the choice of the academic standard and the tuition fee by the private school and Section 4 describes the choice of the standard by the public school. The model is applied to Italy in Section 5. Section 6 concludes.

## 2 Evidence from Italy

Recent empirical research in Italy questions the relative quality of Italian private schools. Bertola and Checchi, 2004, for instance, examine the correlation between the type of upper secondary school - private or public - and the academic performance of students enrolled at the University of Milan, a large public university, and conclude that "...private schooling appears to be associated with poorer university performance regardless of whether or not the talent proxy is controlled for, and this casts considerable doubt on the notion that private schools are unequivocally better..." (p.91). They argue that in Italy private schools appear to play a remedial role. On average, they increase the performance of students from rich families, but their value added seems to be the recovery of less brilliant students rather than across-the-board high quality education. Cappellari, 2004, confirm their results in a study of a cohort of Italian high school graduates.

Evidence that Italian private schools can be of lower quality than public schools can be obtained from the Italian survey on High School and Beyond. Table 1 shows the estimates of the probability of enrolment in a public rather than private upper secondary school for a sample of 21922 high school graduates who completed their degree in 1998. The probit model suggests that the probability of enrolment in a public school is higher in the South and lower for individuals with "better" family background, measured by the education and occupation of both parents. More interesting for the purposes of this paper is the finding that enrolment in an Italian public school is more likely among individuals with better marks in junior high school. This result is partially in contrast with Epple, Figlio and Romano, 2004, who find instead that enrolment in US private high schools increases both with household income and with



Table 1. Probit estimate of the probability of enrolment in a public upper secondary school. Italy 2001. Dependent variable: a dummy equal to 1 in the event of enrolment in a public school and to 0 otherwise. Marginal effects.

	Coefficient	Standard error
South dummy	0.011***	0.003
Father with higher education	-0.028***	0.004
Mother with higher education	-0.051***	0.004
Male	0.034***	0.003
Father top ranked employee	-0.079***	0.011
Mother top ranked employee	0.005	0.007
Father top ranked professional	-0.063***	0.006
Mother top ranked professional	-0.041***	0.009
Marks in junior high school	0.004**	0.001
Nobs	21922	
Pseudo R Squared	0.074	

Note: robust standard errors. \*\*\*, \*\* and \* when the coefficient is statistically significant at the 1, 5 and 10 percent level of confidence respectively.

measured ability.

Table 2 looks at the probability of enrolling in college after completing high school. Again, family background and the area of residence during high school matter, as expected. Enrolment is also higher among individuals with higher high school marks who graduated from the generalist track of upper education (lyceum). Conditional on these controls, students graduating from a public school are significantly more likely to continue into college than students from private high schools.

In Italy, students who have been failed during high school can try to recover the time lost by enrolling in schools with special programs tailored to catching up. These are usually private and less demanding institutions. As discussed in the opening remarks, there have been ways in Italy to get around the formally national graduation exam, for instance by taking the exam in places with lower passing standards<sup>3</sup>. Table 3 shows the estimates of the probability of enrolment in such institutions. It turns out that this probability is higher for those with lower junior high school marks who are enrolled in a private school and have a

<sup>3</sup>In an effort to reduce this opportunistic behavior, the government has recently restricted the possibility of taking the graduation national exam in a different area of the country.

Table 2. Probit estimate of the probability of enrolment in college, conditional on attained upper secondary education. Italy 2001. Dependent variable: a dummy equal to 1 in the event of enrolment in college and to 0 otherwise. Marginal effects.

	Coefficient	Standard error
South dummy	0.059***	0.006
Public secondary school	0.028**	0.013
Lyceum	0.492***	0.008
Father with higher education	0.089***	0.008
Mother with higher education	0.118***	0.008
Male	-0.036***	0.007
Father top ranked employee	0.093***	0.019
Mother top ranked employee	-0.008	0.014
Father top ranked professional	0.068***	0.009
Mother top ranked professional	0.014	0.017
Marks in high school	0.128***	0.003
Nobs	23082	
Pseudo R Squared	0.243	

Note: robust standard errors. \*\*\*, \*\* and \* when the coefficient is statistically significant at the 1, 5 and 10 percent level of confidence respectively.

Table 3. Probit estimate of the probability of enrolment in school to catch up on failed years. Italy 2001. Dependent variable: a dummy equal to 1 in the event of enrolment in catching up programs and to 0 otherwise. Marginal effects.

	Coefficient	Standard error
South dummy	0.004**	0.002
Public secondary school	-0.087***	0.008
Lyceum	-0.003	0.003
Male	0.009***	0.002
Father top ranked employee	0.011**	0.006
Mother top ranked employee	0.003	0.004
Marks in junior high school	-0.013***	0.001
Nobs	21922	
Pseudo R Squared	0.062	

Note: robust standard errors. \*\*\*, \*\* and \* when the coefficient is statistically significant at the 1, 5 and 10 percent level of confidence respectively.

father with a high ranked occupation.

We believe that the evidence assembled in this section provides little support to the view that Italian private schools are of better quality than public schools. Private schools do not attract students with higher graduating marks from junior high school. Moreover, students from these schools not only use more frequently remedial education, but are also less likely to continue their education by enrolling in college.

### 3 The Choice of the Standard by the Private School

Consider a market for the provision of upper secondary education. Entry in the market is sequential: in the first stage the government enters by setting up a public school, which offers the academic standard  $s_G \in [1, 2]$  at a positive cost. Graduation requires that pupils attain the standard. The cost of setting the standard  $s_G$  is funded by a lump sum tax paid by all households, and there is no opting out. We find it convenient to normalize the cost and the tax to zero. Since attainment requires effort, as in Costrell, 1984, not all the pupils in this economy enrol in high school.

In the second stage a private school can enter in this market by choosing the standard  $s_P$  and the price  $p$  to maximize (expected) profits. We characterize the model and its solutions by backward induction, starting from the final stage. The profit function of the private school is

$$\pi = pD - cD - ks_P \tag{1}$$

where  $D$  is the demand faced by the private school,  $c$  is the constant marginal cost to serve an additional student and  $k$  is the cost of setting up the standard  $s_P$ . Profits are positive in the event of entry and equal to zero in the event of no entry.

The demand side is composed of a finite number of students, who differ in their ability and family background. Individual differences are described by the pair  $(\theta, y)$  where  $\theta \in [0, 1]$  is an inverse measure of ability, lower for higher academic ability, and  $y \in [0, 1]$  is a measure of family background. Each household in this economy consists of a mother and a daughter. The household utility function is linear in the net income of the mother and concave in the income of the daughter, a useful simplification in line with the relevant literature - see De Fraja, 2002<sup>4</sup>. The mother has endowed income  $y$ , which can be used to fund the daughter's costs of schooling. We rule out liquidity constraints by assuming that each household can freely borrow against the future income of the offspring  $w$ . Household utility  $U$  is

$$U = y - p + \ln w(s, y) - \theta s \tag{2}$$

where  $s = s_G, s_P$ ,  $p$  is the tuition fee - equal to zero in the public school and positive in the private school - and  $\theta s$  is the cost of attaining the academic standard  $s$ , a decreasing linear function of individual ability.

A key finding of the empirical labor economics literature since Mincer is that earnings are a log-linear function of individual characteristics, which include

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<sup>4</sup>In De Fraja the household utility function is concave in the mother's consumption and linear in the daughter's income.

years of schooling. Drawing from this literature, we specify the daughter's earnings as follows

$$\ln w(\theta, y) = \lambda_0 + \lambda_1 s + \Omega sy \tag{3}$$

where the constant term  $\lambda_0$  captures the gains associated to the attained upper secondary degree,  $\lambda_1$  measures the labor market returns to school quality and  $\Omega sy$  captures the effects of the interaction between school quality and family background. If  $\Omega$  is positive, a given level of earnings can be attained either by combining good school quality with poor family background or with a combination of poor school quality and good family background<sup>5</sup>.

### 3.1 Entry by the Private School

The private school chooses both the tuition fee and the academic standard after observing the standard selected by the public school. This choice is influenced by the expected demand for the service. Students are indifferent between the private and the public school if

$$U(s_P) = U(s_G) \tag{4}$$

which yields the following separating condition

$$\theta - \Omega y = \lambda_1 - \frac{p}{s_P - s_G} \tag{5}$$

This condition says that the private school has no incentive to set the standard at the same level of the public standard, because with a positive price it would attract no student.

Students participate to the upper secondary school market only if the attained utility is higher than the reservation utility  $\bar{U} = y$ . No participation eliminates the monetary and non-monetary costs of schooling but reduce log earnings

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<sup>5</sup>In Italy an upper secondary school diploma has a legal value, and should therefore produce similar labor market returns across schools, unless firms can see through the veil of the diploma, as we assume in the paper.

to zero, a useful normalization. If the student is in the public school, the condition  $U(s_G) \geq \bar{U}$  is required. If she is in the private school,  $U(s) > U(s_G)$  and  $U(s) \geq \bar{U}$ . Therefore, the sufficient condition for participation is  $U(s_G) \geq \bar{U}$ , which can be re-written as

$$\theta - \Omega y \leq \frac{\lambda_0}{s_G} + \lambda_1 \quad (6)$$

Notice that the separating condition (5) changes according to whether  $s_P$  is higher or lower than  $s_G$ . There are two regimes, and we characterize each regime in turn.

### 3.1.1 Entry with a standard higher than $s_G$

Let the standard  $s_P$  be higher than  $s_G$ . By combining the separating and participation conditions we obtain

$$\begin{aligned} \theta - \Omega y &\leq \lambda_1 - \frac{p}{s - s_G} \\ \theta - \Omega y &\leq \lambda_1 + \frac{\lambda_0}{s_G} \end{aligned}$$

so that the second condition is not binding. Let the random variable  $\theta - \Omega y$  be distributed in the population of students according to the function  $F$ . Then the demand faced by the private school is

$$D = F\left(\lambda_1 - \frac{p}{s - s_G}\right) \quad (7)$$

### 3.1.2 Entry with a standard lower than $s_G$

Let the standard  $s_P$  be lower than  $s_G$ . In this case the separating condition becomes

$$\theta - \Omega y \geq \lambda_1 + \frac{p}{s_G - s_P}$$

and the combination of this and the participation condition yields the following demand for the services of the private school

$$D = F\left(\frac{\lambda_0}{s_G} + \lambda_1\right) - F\left(\lambda_1 + \frac{p}{s_G - s}\right) \quad (8)$$

A necessary condition for a positive demand is<sup>6</sup>

$$p < \lambda_0 \frac{s_G - s_P}{s_G}$$

Hence, the tuition price  $p$  increases in the academic standard of the public sector  $s_G$  and decreases in the standard set by the private school  $s_P$ .

### 3.2 The distribution of $\theta - \Omega y$

We assume that the random variable  $\theta - \Omega y$  is uniformly distributed in the support  $\theta - \Omega y \in [-\Omega, 1]$ , with density  $\frac{1}{1+\Omega}$ . This assumption has the great advantage of simplifying the algebra considerably, but comes at the price of lack of realism. Therefore, we present in the Appendix a generalization of some of our results to the case of general distributions. To illustrate the implications of assuming a uniform distribution, consider the following discrete case:  $\theta \in (\theta_L, \theta_H)$ , with  $\theta_H > \theta_L$ , and  $y \in (y_L, y_H)$ , with  $y_H > y_L$ , and let the probability of each case by  $\frac{1}{2}$ . Then the uniform distribution implies either

$$\theta_L - \Omega y_H < \theta_L - \Omega y_L < \theta_H - \Omega y_H < \theta_H - \Omega y_L$$

or

$$\theta_L - \Omega y_H < \theta_H - \Omega y_H < \theta_L - \Omega y_L < \theta_H - \Omega y_L$$

with each event occurring with probability equal to  $\frac{1}{4}$ .

When the distribution is uniform, the demand faced by the private school is

$$D_{top} = \frac{\lambda_1 - \frac{p}{s_P - s_G} + \Omega}{1 + \Omega} \quad (9)$$

if  $s_P > s_G$ , which is positive for

$$p < (\lambda_1 + \Omega)(s_P - s_G)$$

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<sup>6</sup>The demand for low quality private schools is higher if firms can only observe whether individuals have a degree or not.

and

$$D_{bottom} = \left[ \frac{\frac{\lambda_0}{s_G} - \frac{p}{s_G - s_P}}{1 + \Omega} \right] \quad (10)$$

in the case of  $s_P < s_G$ .

### 3.3 The choice of the price and the standard by the private school

Since the demand for the service varies depending on whether the private school selects a standard higher or lower than  $s_G$ , we distinguish two separate cases. Consider first the case  $s_P > s_G$  and let profits be  $\pi_H = (p - c)D_{top} - ks_P$ . Profit maximization yields two first order conditions

$$\frac{\partial \pi_H}{\partial s_P} = (p - c) \frac{p}{(s_P - s_G)^2 (1 + \Omega)} - k = 0 \quad (11)$$

and

$$\frac{\partial \pi_H}{\partial p} = \frac{\Omega + \lambda_1 - \frac{p}{(s_P - s_G)}}{\Omega + 1} - \frac{(p - c)}{(s_P - s_G)(1 + \Omega)} = 0 \quad (12)$$

from which we get

$$p = \frac{(s_P - s_G)(\Omega + \lambda_1) + c}{2} \quad (13)$$

The optimal price increases both with the marginal cost and with the academic standard  $s_P$  and decreases with the academic standard set by the public school. By substituting equation (13) into condition (11), we get

$$s_P = \min[s_G + \phi, 2] \quad (14)$$

where

$$\phi = \frac{c}{\sqrt{(\lambda_1 + \Omega)^2 - 4k(1 + \Omega)}}$$

which requires

$$(\lambda_1 + \Omega)^2 - 4k(1 + \Omega) > 0$$

Notice that  $s_P$  increases in the cost of setting up the academic standard  $k$ : when the cost of quality  $s_P$  increases, the private school needs to raise prices to



meet the higher costs. As shown by (13), this can only be done by raising the standard. The demand positivity condition can now be written as

$$c < (s_P - s_G)(\Omega + \lambda_1) \quad (15)$$

which is always verified by the optimal value  $s_P = \min[s_G + \phi, 2]$ . Upon substitution, the optimal profit is

$$\pi_H = \frac{[(\lambda_1 + \Omega)(s_P - s_G) - c]^2}{4(s_P - s_G)(1 + \Omega)} - ks_P \quad (16)$$

Notice that the condition for a positive profit  $\pi_H$  is stronger than the demand positivity condition. Since  $\frac{\partial \pi_H}{\partial s_G} < 0$  when (14) holds, we define  $\bar{s}_G$  as the maximum value of  $s_G$  such that  $\pi_H \geq 0$ . For  $s_G > \bar{s}_G$  we have  $\pi_H \equiv 0$  and no entry<sup>7</sup>.

Next consider the other case  $s_P < s_G$ . Here profit maximization yields

$$\frac{\partial \pi_L}{\partial s} = -\frac{p(p-c)}{(s_P - s_G)^2(1 + \Omega)} - k < 0$$

and

$$\frac{\partial \pi_L}{\partial p} = \frac{\frac{\lambda_0}{s_G} - \frac{p}{s_G - s_P}}{\Omega + 1} - \frac{(p-c)}{(s_G - s_P)(1 + \Omega)}$$

from which we get

$$s_P = 1 \quad \text{and} \quad p = \frac{1}{2} \left( \frac{\lambda_0(s_G - 1)}{s_G} + c \right) \quad (17)$$

The necessary condition for a positive demand of private school services turns out to be

$$s_G > \frac{\lambda_0}{\lambda_0 - c} \quad (18)$$

Therefore the government can stop a low-quality private school from entering the education market by setting the value of the public school standard,  $s_G$ , at a sufficiently low level. This is the case considered by Epple and Romano, 1998, where public schools are of the lowest standard and private schools are stratified by (higher) quality. The optimal profit of the private school is

<sup>7</sup>Profits cannot be negative, because in this case the private school prefers no entry.

$$\pi_L = \frac{(\lambda_0(s_G - 1) - cs_G)^2}{4s_G^2(s_G - 1)(1 + \Omega)} - k \quad (19)$$

Again, the condition for a positive profit  $\pi_L$  is stronger than the demand positivity condition. Since  $\frac{\partial \pi_L}{\partial s_G} > 0$  when (18) holds, we define  $\underline{s}_G$  as the minimum value of  $s_G$  such that  $\pi_L \geq 0$ . For  $s_G < \underline{s}_G$  we get  $\pi_L \equiv 0$  and no entry.

### 3.4 Choosing the standard above or below $s_G$

In the previous sub-sections we have derived the optimal tuition fee and the optimal standard in two separate cases: in the former case the private school chooses a higher standard than the public school and in the latter case the opposite occurs. How does the private school choose between these two cases? At the optimal pricing policy, both profit functions (16) and (19) depend on the standard set by the government for public schools,  $s_G$ . We establish the following Propositions

**Proposition 1** *Suppose that  $\underline{s}_G < \bar{s}_G$ . Then there exists a unique value  $\underline{s}_G < s_G^* < \bar{s}_G$  such that the private school chooses a standard below  $s_G$  for any  $s_G > s_G^*$  and a standard above  $s_G$  for any  $s_G < s_G^*$ .*

**Proof.** First recall that  $\pi_H \equiv 0$  for  $s_G > \bar{s}_G$  and  $\pi_L \equiv 0$  for  $s_G < \underline{s}_G$ . Next define  $\Delta = \pi_L - \pi_H$ : clearly, when  $\Delta > 0$  the private school will choose to provide a quality lower than the public school, and viceversa when  $\Delta < 0$ . To study the sign of  $\Delta$ , consider its derivative with respect to  $s_G$ , i.e.  $\frac{\partial \Delta}{\partial s_G} = \frac{\partial \pi_L}{\partial s_G} - \frac{\partial \pi_H}{\partial s_G}$ . Notice that

$$\frac{\partial \pi_H}{\partial s_G} \begin{cases} < 0 & \text{for } s_G < \bar{s}_G \\ = 0 & \text{otherwise} \end{cases}$$

Next,

$$\frac{\partial \pi_L}{\partial s_G} = \begin{cases} \frac{(\lambda_0(s_G - 1) - cs_G)(s_G^2(c - \lambda_0) + \lambda_0(3s_G - 2))}{4s_G^3(s_G - 1)^2(1 + \Omega)} & \text{for } s_G > \underline{s}_G \\ 0 & \text{otherwise} \end{cases}$$

The derivative  $\frac{\partial \pi_L}{\partial s_G}$  for  $s_G > \underline{s}_G$  is positive only if

$$s_G^2(c - \lambda_0) + \lambda_0(3s_G - 2) > 0 \quad (20)$$

Given that  $c < \frac{\lambda_0(s_G - 1)}{s_G}$  (by the demand positivity condition), it must be that  $c < \lambda_0$ . Then condition (20) is decreasing in  $s_G$  within a range which strictly contains the feasible set  $[1, 2]$ . Using this result, since at  $s_G = 2$  inequality (20) is verified, then it will also be verified for all  $s_G \in [1, 2]$ . Hence  $\frac{\partial \pi_L}{\partial s_G} > 0$  for  $s_G > \underline{s}_G$ . Summing up,  $\Delta = \pi_L - \pi_H$  is strictly increasing in  $s_G$ . Moreover,  $\Delta < 0$  for  $s_G < \underline{s}_G$  because  $\pi_L \equiv 0$  and  $\pi_H > 0$ ; symmetrically,  $\Delta > 0$  for  $s_G > \bar{s}_G$  because  $\pi_L > 0$  and  $\pi_H \equiv 0$ . Therefore, being  $\Delta$  strictly increasing and continuous, there exists a unique value  $\underline{s}_G < s_G^* < \bar{s}_G$  such that for any  $s_G > s_G^*$  the private school chooses a standard below  $s_G$  and for any  $s_G < s_G^*$  the private school chooses a standard above  $s_G$ . ■

Figure 1 illustrates the intersection of the profit functions  $\pi_H$  and  $\pi_L$ .

**Proposition 2** *Suppose that  $\underline{s}_G > \bar{s}_G$ . Then for  $\bar{s}_G < s_G < \underline{s}_G$  the private school always makes negative profits and refrains from entering the education market. For  $s_G < \bar{s}_G$  high quality is preferred and for  $s_G > \underline{s}_G$  low quality is preferred.*

**Proof.** The derivatives  $\frac{\partial \pi_L}{\partial s_G}$  and  $\frac{\partial \pi_H}{\partial s_G}$  look as shown in the proof of Proposition 1. Once more,  $\pi_H \equiv 0$  for  $s_G > \bar{s}_G$  and  $\pi_L \equiv 0$  for  $s_G < \underline{s}_G$  (i.e. no entry conditions). Nevertheless, in this case the difference  $\Delta = \pi_L - \pi_H$  is only weakly increasing in  $s_G$ . First,  $\Delta < 0$  for  $s_G < \bar{s}_G < \underline{s}_G$  because  $\pi_L \equiv 0$  and  $\pi_H > 0$ , which makes high quality preferable by the entrant private school; second,  $\Delta > 0$  for  $\bar{s}_G < \underline{s}_G < s_G$  because  $\pi_L > 0$  and  $\pi_H \equiv 0$ , so low quality is preferred. Finally  $\Delta \equiv 0$  for  $\bar{s}_G < s_G < \underline{s}_G$  because simultaneously both  $\pi_L \equiv 0$  and  $\pi_H \equiv 0$ . Indeed there exists a region  $\bar{s}_G < s_G < \underline{s}_G$  such that entry is never profitable. ■

Figure 2 illustrates the case of Proposition 2. The two propositions show that, if the private school enters in the education market, it chooses either

a higher or a lower standard than the public school. If it chooses a higher standard, it sets a premium over the public school standard equal at most to  $\phi$ . If it chooses a lower standard, it sets it to the minimum feasible value. The former case corresponds to the Epple-Romano hierarchical model with public schools of lower quality than private schools. The latter case corresponds to the "Diploma no problem" type of school prosecuted by the Italian authorities and reported by *The Economist*. In both cases, tuition fees paid to private schools are positive. In the Appendix we discuss in an intuitive way how these propositions are affected by assuming a general distribution for  $\theta - \Omega y$ .

The government chooses the standard of the public school by taking into account the subsequent entry of the private school. We turn to this decision in the next section of the paper.

## 4 The choice of the standard for the public school

In the characterization of government choice, we need to distinguish between equilibria where  $\underline{s}_G < \bar{s}_G$  and equilibria where  $\underline{s}_G > \bar{s}_G$ . In the following subsections we examine government choice when  $\underline{s}_G < \bar{s}_G$ , and leave the discussion of the case  $\underline{s}_G > \bar{s}_G$ , which is very similar, to the Appendix.

We assume that the choice of the public school standard  $s_G$  is based on majority voting. Unfortunately, the analysis is rather tedious because the indirect utility function of some voters is discontinuous at  $s_G = s_G^*$ . In particular, there are individuals who shift from the private to the public school as  $s_G$  increases above  $s_G^*$ , because the structure of the education market changes. This is illustrated in Figure 3, where individual corrected ability  $z = \theta - \Omega y$  is on the horizontal axis and the public school quality standard lies on the vertical axis. The darker area indicates the individuals who choose the private school, the shadowed area the voters who prefer the public school, and the empty area those who do not enrol in an upper secondary school. Take for instance the individuals endowed with  $z < \lambda_1 - \frac{p}{s - s_G}$ : as  $s_G$  moves upwards, they switch from the private to the public school and the shape of their indirect utility with

respect to  $s_G$  changes.

We deal with this discontinuity by studying the majority voting outcome separately for the two alternative market structures, which are characterized respectively by: 1) the public school with a high standard and the private school with a low standard (the good public school case); 2) the public school with a low standard and the private school with a high standard (the bad public school case). We next compare the two outcomes and select the one preferred by the majority of voters.

#### 4.1 Case 1: good public school ( $s_G > s_G^*$ )

When the public school has a relatively high standard, and  $s_G > s_G^*$ , the private school chooses  $s_P = 1$  and  $p = \frac{1}{2} \left[ \frac{\lambda_0(s_G-1)}{s_G} + c \right]$ . Positive profits require  $s_G > \underline{s}_G$ . Moreover, since  $\underline{s}_G < s_G^*$  because of Proposition 1, the minimum value of  $s_G$  is  $s_G^*$ . Using the optimal values of prices and standards in the individual utility function, we obtain

$$\frac{\partial U}{\partial s_G} < 0 \quad \rightarrow \quad s_G = s_G^*$$

if the individual is enrolled in the private school, and

$$\frac{\partial U}{\partial s_G} \begin{cases} > 0 & \text{if } -z + \lambda_1 > 0 \quad \rightarrow \quad s_G = 2 \\ < 0 & \text{if } -z + \lambda_1 < 0 \quad \rightarrow \quad s_G = s_G^*, \end{cases}$$

if individual is enrolled in the public school, where  $z = \theta - \Omega y$ . Since individuals are distributed in the interval  $[-\Omega, 1]$ , we need to examine how they vote. We establish the following:

- The individuals with adjusted ability  $z \in [-\Omega, \lambda_1 + \frac{p}{s_G - s_P}]$  choose to enrol in the public school. If they do so, their utility is equal to  $U = y + s_G(\lambda_1 - z) + \lambda_0$ . Therefore, all the individuals with  $z < \lambda_1$  vote for  $s_G = 2$ , and all the individuals with  $\lambda_1 < z < \lambda_1 + \frac{p}{s_G - s_P}$  vote for  $s_G = s_G^*$ . The marginal individual with  $z = \lambda_1$  is indifferent.

- The individuals belonging to  $z \in [\lambda_1 + \frac{p}{s_G - s_P}, \lambda_1 + \frac{\lambda_0}{s_G}]$  prefer the private school and vote for  $s_G = s_G^*$ . Consider now the individual  $z = \lambda_1 + \frac{\lambda_0}{s_G}$ . Since her utility is equal to  $y$ , she is indifferent between participating to upper secondary education and not participating.
- The individuals with  $z \in [\lambda_1 + \frac{\lambda_0}{s_G}, 1]$  do not participate and are indifferent to the choice of  $s_G$ . We assume that they do not participate to the voting<sup>8</sup>. It follows that the relevant voting population is  $\frac{\lambda_1 + \frac{\lambda_0}{s_G} + \Omega}{1 + \Omega}$ .

Therefore, if the group of individuals in the range  $z \in [-\Omega, \lambda_1]$  is the majority, the optimal choice is  $s_G = 2$ ,  $s_P = 1$  and  $p = \frac{1}{2} (\frac{1}{2}\lambda_0 + c)$ . If instead the group  $z \in [\lambda_1, \lambda_1 + \frac{\lambda_0}{s_G}]$  is the majority, the optimal choice is  $s_G = s_G^*$ ,  $s_P = 1$  and  $p = \frac{1}{2} \left( \frac{\lambda_0(s_G^* - 1)}{s_G^*} + c \right)$ .

## 4.2 Case 2: bad public school ( $s_G < s_G^*$ )

When the public school has a relatively high standard, and  $s_G < s_G^*$ , the private school selects  $s_P = \min[s_G + \phi, 2]$  and  $p = \frac{(s_P - s_G)(\Omega + \lambda_1) + c}{2}$ . Positive profits require  $s_G < \bar{s}_G$ . Since we know that  $s_G^* < \bar{s}_G$ , the maximum attainable value of  $s_G$  is  $s_G^*$ . Using the optimal values of prices and standards in the individual utility function, we obtain

$$\frac{\partial U}{\partial s_G} = (-z + \lambda_1) \frac{\partial s_P}{\partial s_G} - \frac{\partial p}{\partial s_G} > 0 \text{ for } (-z + \lambda_1) > 0 \rightarrow s_G = s_G^*$$

if the individual prefers the private school, where

$$\frac{\partial s_P}{\partial s_G} = \begin{cases} 1 & \text{if } s_G + \phi < 2 \\ 0 & \text{if } s_G + \phi > 2 \end{cases} \quad \frac{\partial p}{\partial s_G} = \begin{cases} 0 & \text{if } s_G + \phi < 2 \\ -\frac{(\Omega + \lambda_1)}{2} & \text{if } s_G + \phi > 2 \end{cases}$$

and

$$\frac{\partial U}{\partial s_G} \begin{cases} > 0 & \text{if } -z + \lambda_1 > 0 \\ < 0 & \text{if } -z + \lambda_1 < 0 \end{cases} \rightarrow \begin{cases} s_G = s_G^* \\ s_G = 1 \end{cases}$$

if the individual is in the public school. Then we have:

<sup>8</sup>In principle, we cannot exclude that the vote of these individuals can be purchased by other groups via money transfers. Here, we do not consider this possibility.

- The group  $z \in [-\Omega, \lambda_1 - \frac{p}{s_P - s_G}]$  chooses the private school and votes  $s_G = s_G^*$ , because utility is strictly increasing in  $s_G$ .
- The group  $z \in [\lambda_1 - \frac{p}{s_P - s_G}, \lambda_1 + \frac{\lambda_0}{s_G}]$  chooses the public school. All individuals with  $\lambda_1 - \frac{p}{s_P - s_G} < z < \lambda_1$  vote  $s_G = s_G^*$ , and all individuals  $\lambda_1 < z < \lambda_1 + \lambda_0$  vote  $s_G = 1$ . Clearly, the marginal individual  $z \in \lambda_1$  is indifferent.
- Finally, the group  $z \in [\lambda_1 + \frac{\lambda_0}{s_G}, 1]$  does not participate to upper secondary school and does not vote.

To summarize, the group  $[-\Omega, \lambda_1]$  votes  $s_G = s_G^*$  and the private school chooses  $s_P = \min[s_G^* + \phi, 2]$  and  $p = \frac{(s_P - s_G)(\Omega + \lambda_1) + c}{2}$ . On the other hand, the group  $[\lambda_1, \lambda_1 + \frac{\lambda_0}{s_G}]$  votes for  $s_G = 1$ , and  $s_P = \min[1 + \phi, 2]$  and  $p = \frac{(s_P - s_G)(\Omega + \lambda_1) + c}{2}$ .

### 4.3 The choice between regimes

In the previous sub-sections we have shown that, if the group  $[-\Omega, \lambda_1]$  is the majority of the relevant population, the optimal choice is  $s_G = 2$  if  $s_G > s_G^*$  and  $s_G = s_G^*$  if  $s_G < s_G^*$ . The marginal individual  $z = \lambda_1$  is indifferent and her utility is  $y + \lambda_0$ . Consider now another individual,  $z = \lambda_1 - \varepsilon$ , with  $\varepsilon$  small. If  $s_G > s_G^*$  this individual is in the public school and obtains utility  $U = y + \lambda_0 + 2\varepsilon$ . If  $s_G < s_G^*$ , she is still in the public school and obtains  $U = y + \lambda_0 + s_G^* \varepsilon$ . Therefore, this individual votes for the regime  $s_G > s_G^*$ . The same choice is taken by individuals with  $z < \lambda_1$  who select the public school in both regimes. When  $\varepsilon$  is large enough, however, the individual enrolls in the private school if  $s_G < s_G^*$  and in the public school if  $s_G > s_G^*$ . In the private school her utility is  $U = (s_G + \phi)\varepsilon + \lambda_0 + y - p$ , which is certainly lower than the utility attainable in the regime  $s_G > s_G^*$ . Therefore, this individual also chooses the latter regime. We conclude that, if the group  $[-\Omega, \lambda_1]$  is the majority, the optimal choice is  $s_G = 2$ .

If the group  $[-\Omega, \lambda_1]$  is not the majority, the regime  $s_G > s_G^*$  implies  $s_G = s_G^*$  and the regime  $s_G < s_G^*$  implies  $s_G = 1$ . As above, consider the individual

$z = \lambda_1 + \varepsilon$ . If she is in the public school in both regimes, her utility is  $U_{\lambda_1 + \varepsilon} = y + \lambda_0 - s_G^* \varepsilon$  if  $s_G > s_G^*$  and  $U_{\lambda_1 + \varepsilon} = y + \lambda_0 - \varepsilon$  if  $s_G < s_G^*$ . Since  $s_G^* > 1$  the second alternative is selected and  $s_G = 1$ . As in the previous case, the ordering does not change if the individual belongs to the private school in one regime and to the public school in the other regime. Therefore, when the majority is with the complementary group  $z \in \left[ \lambda_1, \lambda_1 + \frac{\lambda_0}{s_G} \right]$ , the optimal choice is  $s_G = 1$ .

The group  $[-\Omega, \lambda_1]$  is the majority when

$$\frac{\lambda_1 + \Omega}{1 + \Omega} > \frac{1}{2} \frac{\lambda_1 + \frac{\lambda_0}{2} + \Omega}{1 + \Omega}$$

where the right hand side is the relevant voting population when  $s_G = 2$ , or

$$\lambda_1 + \Omega > \frac{\lambda_0}{2} \tag{21}$$

The above condition is more likely to be satisfied the higher the values assumed by parameters  $\lambda_1$  and  $\Omega$  relative to  $\lambda_0$ , the return to the "quantity" of education. These parameters measure the impact of the academic standard and of family background, conditional on the academic standard, on individual earnings.

When school quality and family background matter for earnings relatively to the quantity of education, our model suggests that the quality of the public school is high and the quality of the private school is low. The intuitive reason is that the majority going to the public school is composed of individuals with relatively high ability, who can pass the high standard in the public school with relatively little effort. Another reason is that the poor performance of the private school can be compensated by the informal networks afforded by family background. On the other hand, when the effects of the academic standard and family background on earnings are small relative to the effects of school quantity, the quality standards of the public and private school are low and high respectively. In this case the majority still goes to the public school but consists of individuals with relatively low ability - and high cost of effort - and limited informal networks.



## 5 An Application to Italy

Is condition (21) likely to be satisfied? By construction, the parameters affecting this condition are those which influence individual earnings after upper secondary education, as shown by equation (3). Since it is natural to interpret  $s$  as a measure of school quality, this equation suggests that earnings depend on the quantity and quality of education, as well as on the interaction of quality with family background.

One way to capture the effects of school quality on earnings is to augment the standard Mincerian earnings function with measures of cognitive achievement. Hanushek, 2002, reviews this literature and concludes that the uncovered effects of quality are modest when compared to the returns to quantity. Cognitive achievement, however, mixes individual ability with the contribution of schooling institutions. In principle, we cannot exclude that individuals with high talent have high cognitive achievement in spite of the poor quality of the attended school.

A better measure of school quality for our purposes is the pupil - teacher ratio, used by Card and Krueger, 1992, in their path-breaking investigation of the effects of school quality on labor market returns. They argue that, *coeteris paribus*, schools with a lower pupil-teacher ratio invest more resources in their students and supply as a consequence better quality than less equipped schools. Card and Krueger find significant effects of school quality on earnings, but fail to include in their regressions controls for family background - see the discussion in Hanushek, 2002.

We estimate equation (3) on longitudinal Italian data drawn from the *Survey on the Income and Wealth of Italian Households* (SHIW) for the period 1991-2000. This survey contains individual information on earnings, education and family background but lacks data on the measure of school quality we are interested in, the pupil-teacher ratio. In order to avoid aggregation bias, it would be desirable to have this information for each school involved in the education of the individuals in our sample. This is not possible, however, because the Sur-

Table 4. OLS estimate of equation (3). Italy 1991-2000.

	Coefficient	Standard error
$\lambda_0$	0.228***	0.013
$\lambda_1$	0.115	0.131
$\Omega$	0.041***	0.007
Nobs	15171	
R Squared	0.619	

Note: robust standard errors. \*\*\*, \*\* and \* when the coefficient is statistically significant at the 1, 5 and 10 percent level of confidence respectively.

vey does not provide information on the schools experienced by the interviewed individuals. Therefore, we follow Card and Krueger [1992] and use aggregate measures of quality based on the region and the cohort of birth. We collect regional data on the pupil – teacher ratio for different types of schools, ranging from kindergarten to upper secondary education, every two years, and match school quality to individuals in the sample by attributing to each individual the pupil – teacher ratio in the region of birth during the period when she went to school<sup>9</sup>.

We consider only individuals with at most upper secondary education and define  $H$  as a dummy taking the value 1 if the individual has at most an upper secondary degree and 0 otherwise. Family background  $FB$  is measured as a dummy equal to 0 if the occupation of the father is blue collar, self-employed or unemployed and to 1 otherwise. Finally, school quality is defined as the reciprocal of the pupil-teacher ratio. We normalize this number in the range  $[1, 2]$  by using a simple transformation.

The empirical specification adds a set of individual controls<sup>10</sup> to equation (3). The clustered - adjusted OLS estimates of the key parameters are shown in Table 4. It turns out that the estimated coefficients attract a positive sign and are statistically significant, with the exception of  $\lambda_1$ .

<sup>9</sup>These data were originally collected by Brunello and Checchi, 2004. We refer the reader to their paper for further details.

<sup>10</sup>Gender, marital status, age, age squared, size of the town of residence dummies, year dummies and type of job.

We use these estimates to check whether condition (21) is satisfied and obtain

$$0.115 + 0.041 > \frac{0.228}{2}$$

which suggests that the voting majority in Italy is likely to be in favor of a relatively high quality public school. Conditional on this choice, our model suggests that it is optimal for a private school to select a lower quality standard than the public school, in line with the empirical evidence presented in Section 2 of the paper.

## 6 Conclusions

Motivated by anecdotal as well as econometric evidence from Italy, we have started this paper from the provocative question "Can private schools provide lower quality than public schools?". The answer to this question is only apparently trivial, because private schools can offer alternatives to quality in exchange for a positive price. The empirical evidence from Italy suggests that they offer leisure, but this is obviously only one of the possible alternatives.

We have used a simple stylized model of the education market with sequential entry of a public and a private school, and defined an equilibrium in this market as a configuration of academic standards and tuition fees levied by the private school which satisfy both majority voting and profit maximization. We have shown that, depending on the underlying parameters of the model, a market structure with the private school offering at a positive price lower quality than the public school can be an equilibrium if the returns to school quality and the effects of family background on individuals earnings are large relative to the returns to school quality. We have calibrated the model by using longitudinal data for Italy and found that the calibrated parameters confirm the existence of such an equilibrium in the Italian labor market.

These results have been obtained at the price of important simplifications. First, we have assumed away the public finance side of the story by choosing to have only lump sum taxes. When taxation is progressive and individuals can opt

out of the public school system, the relative incentive to vote for a high quality public school system changes across the distribution of income, with potentially interesting implications on the voting majority equilibrium<sup>11</sup>. Second, we have assumed that the distribution of income adjusted ability is uniform. This is very handy but unrealistic. A complete characterization of the model with a more realistic distribution of income - adjusted ability would be useful to verify the robustness of our main results to distributional assumptions. This is only partly done in the Appendix to this paper. Last but not least, we have restricted the market to a single private school. This has allowed us to focus in a sharp way on the relative quality of private and public schools, at the price of ignoring the heterogeneity of private schools. Future work in this area is clearly desirable.

## 7 Appendix

Let the distribution of income - adjusted ability be a standard normal  $F(\cdot)$  with density  $f(\cdot)$ . Consider first the case  $s_P > s_G$  and assume that profit maximization yields  $p^*(s_G)$  and  $s^*(s_G)$ . Then by the envelope theorem  $\frac{d\pi_H^*}{ds_G} = \frac{\partial \pi_H}{\partial s_G}$ , where  $\pi_H^*$  is the value function, and

$$\frac{d\pi_H^*}{ds_G} = (p^* - c) \times f\left(\lambda_1 - \frac{p^*}{s_P^* - s_G}\right) \times \left(-\frac{p^*}{(s_P^* - s_G)^2}\right) < 0$$

for  $s_G < \bar{s}_G$ , as in the case of a uniform distribution.

Next consider the case  $s_P < s_G$ . We have

$$\frac{\partial \pi_L}{\partial s_P} = (p - c) \times \left[-f\left(\lambda_1 + \frac{p}{s_G - s_P}\right)\right] \frac{p}{(s_G - s_P)^2} - k < 0$$

implying that  $s_P^* = 1$ . Therefore we can re-write  $\pi_L$  as a function independent of  $s_P$ . The optimal price is given by

$$\frac{\partial \pi_L}{\partial p} = D_{bottom} + (p - c) \left[-f\left(\lambda_1 + \frac{p}{s_G - 1}\right)\right] \times \left(\frac{1}{s_G - 1}\right) = 0$$

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<sup>11</sup> See Besley and Coates, 1991, for an interesting discussion of redistribution when there is public provision of private goods.

which in implicit form can be written as  $p^*(s_G)$ . Applying again the envelope theorem we have  $\frac{d\pi_L^*}{ds_G} = \frac{\partial\pi_L}{\partial s_G}$ , and

$$\frac{\partial\pi_L}{\partial s_G} = (p^* - c) \frac{\partial D_{bottom}}{\partial s_G}$$

where

$$\frac{\partial D_{bottom}}{\partial s_G} = \left[ -f\left(\frac{\lambda_0}{s_G} + \lambda_1\right) \frac{\lambda_0}{s_G^2} + f\left(\lambda_1 + \frac{p^*}{s_G - 1}\right) \times \frac{p^*}{(s_G - 1)^2} \right]$$

Compared to the case of the uniform distribution, the slope of the demand function is positive only when a higher public school standard reduces participation to upper secondary school less than it increases the shift from the public to the private school. If this condition holds, we are back to Figures 1 and 2 in the paper.

### 7.1 The case $\underline{s}_G > \bar{s}_G$

We need to examine the remaining case  $\underline{s}_G > \bar{s}_G$ . When this happens, Proposition 2 above states that no private school enters the market if  $\bar{s}_G < s_G < \underline{s}_G$ . When  $s_G < \bar{s}_G$  the private school selects  $s = \min[s_G + \phi, 2]$ , while when  $s_G > \underline{s}_G$  it chooses  $s = 1$ .

The social choice between regimes is identical to the one described in the text, with the exception that individuals need to choose between three rather than two alternatives: 1) a good public school and a bad private school; 2) a bad public school and good private school; 3) only the public school.

The former two alternatives can be handled as in the text, after replacing  $s_G^*$  with  $\underline{s}_G$  in the first case and  $s_G^*$  with  $\bar{s}_G$  in the second case. In the third case, there are no private schools and all individuals are enrolled in the public school. Their utility varies with the public school standard as follows

$$\frac{\partial U}{\partial s_G} \begin{cases} > 0 & \text{if } -z + \lambda_1 > 0 & \rightarrow & s_G = \underline{s}_G \\ < 0 & \text{if } -z + \lambda_1 < 0 & \rightarrow & s_G = \bar{s}_G \end{cases}$$

Therefore

- individuals in the set  $z \in [-\Omega, \lambda_1]$  choose  $s_G = \underline{s}_G$ ;

- individuals in the set  $z \in [\lambda_1, \lambda_1 + \frac{\lambda_0}{s_G}]$  choose  $s_G = \bar{s}_G$ ;
- individuals in the set  $z \in [\lambda_1 + \frac{\lambda_0}{s_G}, 1]$  do not enrol in upper secondary schools and do not vote.

Assume that  $[-\Omega, \lambda_1]$  is the majority and consider individual  $z = \lambda_1 - \varepsilon$ .

Then

$$U_{\lambda_1 - \varepsilon} = y + \lambda_0 + 2\varepsilon \quad \text{if } s_G > \underline{s}_G$$

$$U_{\lambda_1 - \varepsilon} = y + \lambda_0 + \bar{s}_G \varepsilon \quad \text{if } s_G < \bar{s}_G$$

and

$$U_{\lambda_1 - \varepsilon} = y + \lambda_0 + \underline{s}_G \varepsilon \quad \text{if } \bar{s}_G < s_G < \underline{s}_G$$

The optimal choice is  $s_G = 2$ , as in the text.

If the set  $[-\Omega, \lambda_1]$  is not the majority, consider individual  $z = \lambda_1 + \varepsilon$ . Then:

$$U_{\lambda_1 + \varepsilon} = y + \lambda_0 - \underline{s}_G \varepsilon \quad \text{if } s_G > \underline{s}_G$$

$$U_{\lambda_1 + \varepsilon} = y + \lambda_0 - \varepsilon \quad \text{if } s_G < \bar{s}_G$$

and

$$U_{\lambda_1 + \varepsilon} = y + \lambda_0 - \bar{s}_G \varepsilon \quad \text{if } \bar{s}_G < s_G < \underline{s}_G$$

As in the case discussed in the text, the optimal choice is  $s_G = 1$ .

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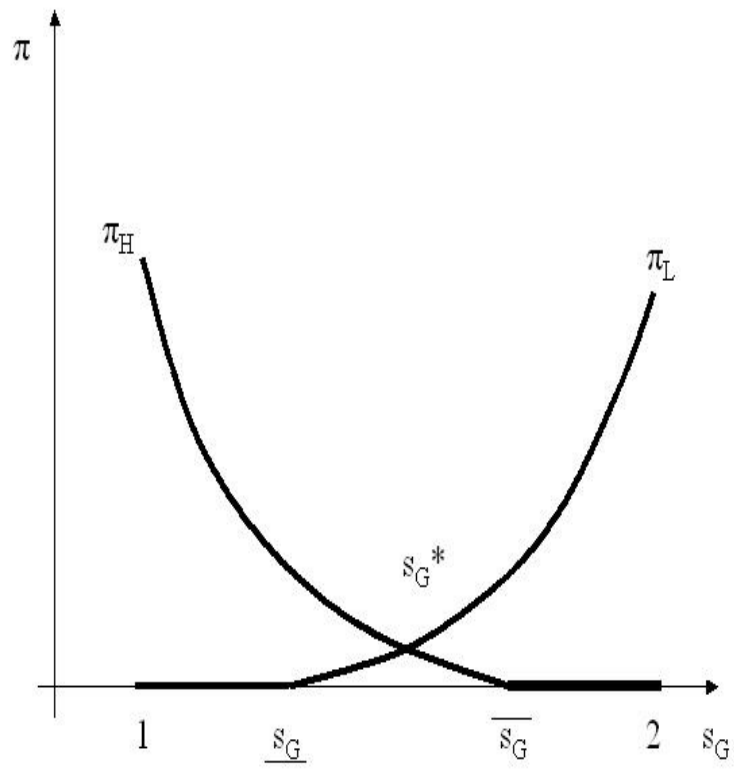


Figure 1: Proposition 1

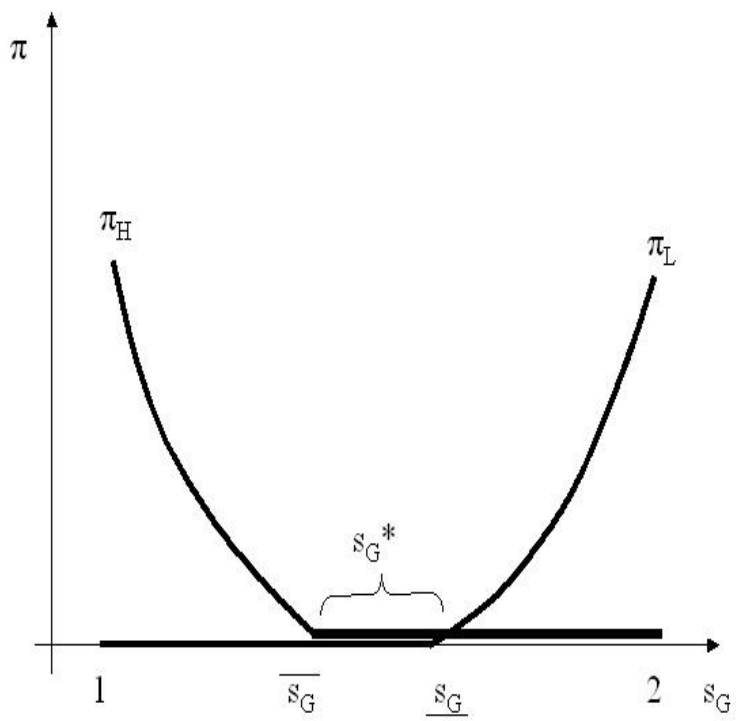


Figure 2: Proposition 2

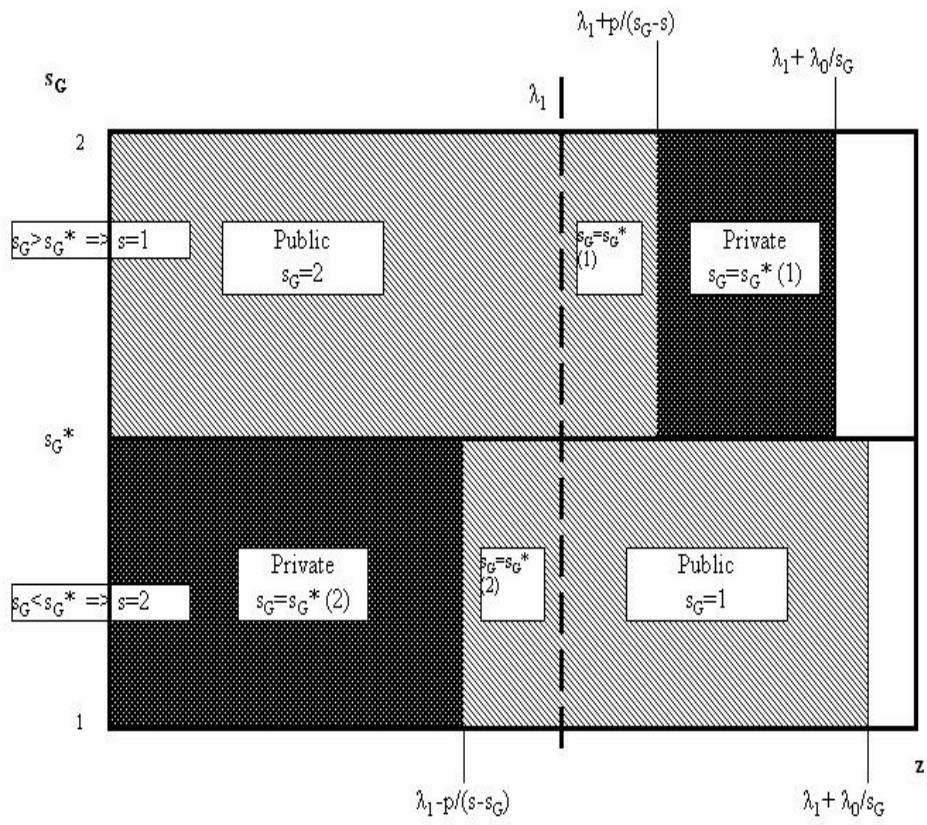


Figure 3: Household preferences