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CORRUPT LOCAL GOVERNMENT AS RESOURCE
FARMERS: THE HELPING HAND AND THE
GRABBING HAND

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CORRUPT LOCAL GOVERNMENT AS RESOURCE FARMERS: THE HELPING HAND AND THE GRABBING HAND

Abstract

We study the role of tax share and transparency of governance on growth and stagnation. A local government maximizes its private benefits using two activities. The first one consists of providing local public goods that help local firms to increase profits, thus enlarging tax revenue. The second one consists of extortion. We show that there is a threshold level of local government tax share, and a threshold level of transparency. Below these thresholds, the economy will stagnate and above them, the economy will achieve perpetual growth.

JEL Code: H41.

Keywords: corruption, growth, local government, tax share.

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1 Introduction

Corruption can occur at several levels of government. Our interest is on factors that determine the pattern of corruption at the local government level. In this paper, we model a situation where a central government determines both the income tax rate and the share of tax revenue between the central government and the local governments, while the local governments seek to maximize their own private benefits, by engaging in two classes of activities. The first one consists of providing public goods to local private firms, because these public goods help the firms to make more profit, thus enlarging the tax base. The second one consists of extortionary activities, such as charging fees, protection money, etc., on local firms. We assume that either these extortionary activities escape the notice of the central government, or the latter is aware of them but does not have sufficient evidence to prosecute corrupt local government officials. The two classes of activities mentioned above may be called respectively the helping hand activities and the grabbing hand activities.

Olson (1993, 2000) distinguishes two types of bandits that correspond to the helping hand and the grabbing hand behaviour. A roving bandit seeks new preys and has no interest in the future development of the current preys. He wants to grab as much as possible. A stationary bandit, on the other hand, cares about the future prosperity of the current preys, and this is the reason for the helping hand behaviour to co-exist with the grabbing hand behaviour¹. Our view of corrupt local governments is that they exhibit both types of behaviour. Whether a local government puts more emphasis on the helping hand or on the grabbing hand depends on (a) the revenue share that the central government allows it to have, (b) the cost of hiding extortionary activities from the central government, and (c) the discount rate.

One may expect that when the central government raises its share of tax revenue, the local government may respond by reducing the level of its helping hand activities, and intensifies its grabbing hand activities. The long-term consequence of this shift of balance is a decrease in the tax base. Thus, the tax revenue of the central government, expressed as a function of its tax share, has the inverted U shaped. Such an outcome would be consistent with the view underlying the Laffer curve.

Our dynamic model contains several additional interesting results, especially those relating to growth and stagnation. First, there are two stationary equilibria. One of these (with a smaller steady state capital stock) has the saddlepoint stability property. The other stationary equilibrium (with a greater level of capital stock) is completely unstable. Second, a permanent increase in the central government's

¹A related phenomenon is the optimal feeding of slaves, see Rees et al. (2003) for theory and empirical evidence.

tax share will reduce the (stable) steady state capital stock, because it discourages the helping hand. The grabbing hand activity level increases initially, but eventually falls, because the new steady state stock is lower. Third, if the initial capital stock is below the unstable steady state stock, the optimal policy is to converge to the (lower) stable steady state stock, but if the initial capital stock exceeds the unstable steady state stock, it will be optimal to keep the grabbing activity level constant and increase the helping activity level steadily, causing the capital stock, and income, to increase without bound. The unstable steady state stock can be shown to be an increasing function of the central government's tax share imposed by the central government, and a decreasing function of the degree of transparency of local government activities. It follows that, given any initial capital stock, there is a corresponding threshold level of central government tax share, with the property that if the tax share is higher than this threshold level, the economy starting with this initial stock will stagnate, and if the tax share is below the threshold level, the economy will achieve positive growth for ever. Similarly, given the initial capital stock, there is a threshold level of transparency of governance, i.e., the difficulty of hiding corruption. If the transparency is below the threshold level, the economy will stagnate, and if the transparency is greater than the threshold level, the economy will achieve positive growth for ever.

There are static models that address issues relating to self-serving local governments. Keen and Kotsogiannis (2003) developed a model where the central government and the local governments compete in taxation. A section of their paper deals with the case where the central government is a Stackelberg leader. Chen (2003) examines the recentralization of tax in China, and argues that the central government of China did not take into account the possibility that an increase in central tax share may encourage the grabbing hand behaviour of local governments. However, unlike our dynamic model, those papers, being set in a static framework, cannot deal with issues such as perpetual growth versus stagnation.

The general topic of government corruption has been studied by many authors, but typically they use a static framework². Dynamic analyses of corruption include the work of Tornell and Velasco (1992), and Tornell and Lane (1999). These papers do not deal with taxation issues³, nor with revenue sharing

²The view that policy makers are revenue-maximizing Leviathans has been forcefully expressed by many authors, see in particular Brennan and Buchanan (1977, 1980). More recent works include Basu et al. (1992), Schleifer and Vishny (1993), Mauro (1995), Frye and Schleifer (1997), Acemoglu and Verdier (2002), Ades and Di Tella (2002), Ni and Pham (2003).

³There is a large literature that deals with taxation issues in a dynamic framework. See, for example, Kemp et al. (1993), Long and Shimomura (2002), and references cited therein. However, these authors typically assume that governments are benevolent bodies rather than Leviathans.

between the central government and the local government, nor with transparency of governance.

We begin our analysis with a static model, presented in Section 2. Our main contribution is the dynamic model, presented in Sections 3 to 7. Some concluding remarks are offered in Section 8.

2 A static model

In this section, a static model of tax sharing and local government corruption is presented. We assume there are no interactions among local governments, so that, without loss of generality, we focus on the case where there are only two governments: a central government and a local government. The static setting can be interpreted as the situation where the local government acts as a roving bandit. The local government has only one shot at extracting as much as possible from the firms, and it is not concerned with the capital accumulation and the associated future benefits.

Let the local firms have the aggregate production function $Y(h, K)$, where h is the flow of productivity-enhancing public goods provided by the local government, and $K > 0$ is the aggregate stock of capital.

We assume production function is strictly concave and increasing in each argument, with

$$Y_{hK} > 0, \text{ and } \lim_{h \rightarrow 0} Y_h(h, K) = \infty$$

The local government takes as given the tax rate firms face, which is denoted by v , and the revenue share between itself and the central government, $1 - \theta$ and θ , respectively. We assume $0 < v < 1$ and $0 \leq \theta \leq 1$.

The local government, in addition to providing useful public goods, can extort money from the local firms through a costly process. Let $g \geq 0$ denote the amount of money extorted. Let the local government's cost of extortion be represented by $C(g)$. This cost includes the effort cost as well as the cost of covering up to avoid audits by the central government. We assume that $C(0) = 0$, $C'(g) > 0$, $C'(0) < 1$ and $C''(g) > 0$. The assumption $0 \leq C'(0) < 1$ implies that the level of grabbing will always be positive.

The objective of the local government is to maximize its private net benefits:

$$\max_{g, h} B \equiv (1 - \theta)vY(h, K) - h + g - C(g) \tag{1}$$

subject to

$$(1 - v)Y(h, K) - g \geq 0 \tag{2}$$

$$h \geq 0 \text{ and } g \geq 0. \quad (3)$$

The constraint (2) states that the amount extorted cannot exceed the local firms' after-tax income. The inequalities (2) and (3) define a *feasible set* S which is a convex and non-empty subset of R^2 :

$$S = \{(h, g) \geq (0, 0) : g \leq (1 - v)Y(h, K)\}$$

If we represent the set S in the space $(h, g) \geq (0, 0)$, where h is measured along the horizontal axis, and g along the vertical axis, we can see that the upper boundary of this set is the curve

$$g = (1 - v)Y(h, K)$$

which has the concave shape. Since the set S has an interior, the Slater condition holds.

The Lagrangian function is:

$$L = (1 - \theta)vY(h, K) - h + g - C(g) + \lambda_1((1 - v)Y(h, K) - g) + \lambda_2g + \lambda_3h$$

The first order conditions are:

$$\frac{\partial L}{\partial h} = (1 - \theta)vY_h - 1 + \lambda_1(1 - v)Y_h + \lambda_3 = 0 \quad (4)$$

$$\frac{\partial L}{\partial g} = 1 - C'(g) - \lambda_1 + \lambda_2 = 0 \quad (5)$$

$$\lambda_1 \geq 0, (1 - v)Y(h, K) - g \geq 0 \text{ and } \lambda_1[(1 - v)Y(h, K) - g] = 0 \quad (6)$$

$$\lambda_2 \geq 0, g \geq 0 \text{ and } \lambda_2g = 0$$

$$\lambda_3 \geq 0, h \geq 0 \text{ and } \lambda_3h = 0.$$

Since the objective function is strictly concave, and the constraints are concave in (h, g) , the necessary conditions yield a unique global maximum. We now show that this unique solution (h^*, g^*) may be in the interior of the *feasible set* S , or may be at the upper boundary of this set, i.e., a point on the curve $g = (1 - v)Y(h, K)$.

First let us establish that, if $\theta < 1$ and $K > 0$, then at the optimum, $h^* > 0$ and $g^* > 0$. For suppose $h^* = 0$. Then equation (4) would yield

$$[(1 - \theta)v + \lambda_1(1 - v)]\infty + \lambda_3 = 1$$

which is not possible. It follows that h^* must be positive. Now suppose that $g^* = 0$. Then $(1 - v)Y(h^*, K) - g^* = (1 - v)Y(h^*, K) > 0$, and hence $\lambda_1 = 0$. Substituting this result into equation (5)

we get $1 - C'(0) + \lambda_2 = 0$ which yields $C'(0) = 1 + \lambda_2 \geq 1$. This is not possible because by assumption $C'(0) < 1$. It follows that g^* must be positive.

Lemma S1: At the optimum, $\lambda_2 = \lambda_3 = 0$ provided $\theta < 1$. If $\theta = 1$, then $h^* = 0$.

We conclude that if $\theta < 1$, the optimal point (h^*, g^*) must be either an interior point of the feasible set S , or a point on the boundary curve $g = (1 - v)Y(h, K)$. (Note that on this curve, all the after-tax profits of local firms are appropriated by the local government by extortion.)

Case 1: Interior solution.

At an interior solution, all the Lagrange multipliers are zero, and we get

$$(1 - \theta)vY_h(h, K) - 1 = 0$$

$$1 - C'(g) = 0$$

Let \hat{g} be the unique value of g that satisfies $1 - C'(\hat{g}) = 0$, that is

$$\hat{g} = (C')^{-1}(1)$$

Let $\hat{h}(K, v, \theta)$ be the unique solution of

$$Y_h(\hat{h}, K) = \frac{1}{(1 - \theta)v} \tag{7}$$

The pair (\hat{h}, \hat{g}) is the interior solution of problem (1) if and only if

$$(C')^{-1}(1) < (1 - v)Y(\hat{h}(K, v, \theta), K) \tag{8}$$

Comparative static results can be obtained by differentiating equation (7) totally:

$$Y_{hh}dh + Y_{hK}dK = -\frac{1}{(1 - \theta)v^2}dv + \frac{1}{(1 - \theta)^2v}d\theta$$

Thus

$$\begin{aligned} \frac{\partial \hat{h}(K, v, \theta)}{\partial K} &= -\frac{Y_{hK}}{Y_{hh}} > 0 \\ \frac{\partial \hat{h}(K, v, \theta)}{\partial v} &= -\frac{1}{(1 - \theta)v^2 Y_{hh}} > 0 \\ \frac{\partial \hat{h}(K, v, \theta)}{\partial \theta} &= \frac{1}{(1 - \theta)^2 v Y_{hh}} < 0 \end{aligned}$$

Thus we obtain the following proposition:

Proposition S1: *If $\theta < 1$ and condition (8) holds, the solution of the local government's optimization problem is an interior solution. In this case, small changes in the parameters v, θ and K do*

not affect the level of its grabbing activities. An increase in the central government's tax share θ will reduce the level of the helping activities, while an increase in the capital stock K or in the tax rate v will increase it.

It should be noted that since an increase in θ will reduce \hat{h} , a sufficiently large increase in θ will lead to a violation of condition (8) and move the optimal solution to the upper boundary of the feasible set S . (Similarly, a sufficiently large decrease in v or K may move the point $\hat{h}(K, v, \theta)$ to such an extent that the optimal solution will be on the upper boundary of the feasible set.)

Case 2: Boundary solution.

If condition (8) fails to hold, then the solution occurs at the upper boundary of the feasible set S . In this case, the solution must satisfy the following pair of equations

$$(1 - \theta)vY_h - 1 + [1 - C'(g)](1 - v)Y_h = 0 \quad (9)$$

$$g = (1 - v)Y(h, K) \quad (10)$$

Substituting (10) into (9) and rearranging we get,

$$\{(1 - \theta)v + (1 - v)[1 - C'((1 - v)Y(h, K))]\} Y_h(h, K) = 1$$

Thus

$$\frac{\partial h}{\partial \theta} = \frac{vY_h}{Y_{hh}[(1 - \theta)v - (1 - v)(1 - C'(g))] - Y_h^2(1 - v)^2 C''(g)} < 0$$

We differentiate equation (10) with respect to θ to get

$$\frac{\partial g}{\partial \theta} = (1 - v)Y_h \frac{\partial h}{\partial \theta} < 0$$

Proposition S2: *Assume that $\theta < 1$ and the central government tax share θ is sufficiently large, or K and v are sufficiently small, so that the local government's optimal solution is on the upper boundary of the feasible set. All the after-tax profits of local firms are appropriated by the local government by extortion. The level of helping hand activities declines with an increase in θ , and the amount to be extorted from the local firms also declines with θ .*

To summarize, the helping hand h is discouraged by decreases in the share of taxes received by the local government in both interior and corner solutions. The grabbing hand g is unaffected by changes

in θ in interior solutions. In the case of a boundary solution, the actual corruption is constrained by the total after-tax income of the firms, and an increase in θ (and the associated decline in h) decreases corruption in absolute value. But the proportion of after-tax income extracted illegally by the local government remains at 100 percent in this case.

3 A dynamic model of growth and stagnation

3.1 Assumptions and notation

A major drawback of the static model is that it cannot address issues such as perpetual growth and stagnation. In particular, static models cannot explain how a small change in parameter values (such as the central government's tax share, the local government's corruption ability, the level of government transparency) can turn a rapidly growing economy into a contracting economy. To deal with these issues, we develop and analyze a simple dynamic extension of the static model.

The local firms have an aggregate capital stock $K(t)$ at time t . This stock, together with the local government spending on public goods $h(t)$, yields a flow of taxable income

$$Y(h(t), K(t)) = h(t)^\alpha K(t)^\beta \text{ where } \alpha > 0, \beta > 0 \text{ and } \alpha + \beta \leq 1 \quad (11)$$

(In what follows, we often suppress the time argument for simplicity of notation.) The central government sets a constant tax rate $v < 1$. Tax revenue is $vh^\alpha K^\beta$. The central government's share of this revenue is θ , and $1 - \theta$ is the local government's share.

The local government can also charge extortionary fees (of which the central government is vaguely aware, but cannot provide evidence for prosecution). We denote by $g(t)$ the total of these off-budget revenues at time t . (Here, the symbols g and h stand for the **grabbing** hand and the **helping** hand, respectively). The net benefit received by the local government at time t is

$$R(t) = (1 - \theta)vh(t)^\alpha K(t)^\beta - h(t) + g(t) - C(g(t))$$

where $C(g(t))$ denote the effort cost of extortion.

For simplicity, assume that the effort cost of extortion is quadratic

$$C(g(t)) = \varepsilon g(t) + \frac{1}{2}\gamma g(t)^2 \text{ where } \gamma > 0 \text{ and } 1 > \varepsilon \geq 0$$

The parameters ε and γ are measures of the costliness of performing (and hiding the evidence of) grabbing activities. (They may serve as proxies for the degree of transparency of local government activities; for any increase in transparency would increase the costs of hiding extortion.)

The rate of growth of the capital stock is assumed to be

$$\dot{K}(t) = s(1-v)h(t)^\alpha K(t)^\beta - g(t) \quad (12)$$

where s is the proportion of profits that businessmen allocate to capital formation. We assume that the extortion yields revenue only if the capital stock is positive. (When the capital stock is zero, extortion is fruitless).

The local government chooses the time paths of $g(t)$ and $h(t)$ to maximize

$$\int_0^\infty R(t)e^{-rt} dt$$

subject to the differential equation (12), the non-negativity constraints $g(t) \geq 0$, $h(t) \geq 0$, the initial condition $K(0) = K_0$ and

$$\lim_{t \rightarrow \infty} K(t) \geq 0.$$

Here r is the rate of discount, assumed to be positive.

3.2 The Optimal Control Problem of the Local Government

The current-value Hamiltonian is

$$H = (1-\theta)vh(t)^\alpha K(t)^\beta - h(t) + (1-\varepsilon)g(t) - \frac{1}{2}\gamma g(t)^2 + \psi(t) [s(1-v)h(t)^\alpha K(t)^\beta - g(t)]$$

For $\psi(t) \geq 0$, the Hamiltonian is concave in the state and control variables. Thus any path (ψ, K, g, h) that satisfies the necessary conditions and leads to a steady state $(K_{ss}, \psi_{ss}) \geq (0, 0)$ is an optimal path, provided $\psi(t) \geq 0$ along that path⁴.

The necessary conditions consist of

(i) The maximality condition: the control variables must maximize H for given (K, ψ) :

$$\frac{\partial H}{\partial h} = \alpha h^{\alpha-1} K^\beta [v(1-\theta) + \psi s(1-v)] - 1 \leq 0, \quad h \geq 0 \quad \text{and} \quad h \frac{\partial H}{\partial h} = 0 \quad (13)$$

$$\frac{\partial H}{\partial g} = (1-\varepsilon - \psi) - \gamma g \leq 0, \quad g \geq 0 \quad \text{and} \quad g \frac{\partial H}{\partial g} = 0 \quad (14)$$

(ii) The adjoint equation:

$$\dot{\psi} = r\psi - \frac{\partial H}{\partial K} = r\psi - \beta h^\alpha K^{\beta-1} [v(1-\theta) + \psi s(1-v)] \quad (15)$$

⁴The relevant sufficiency theorems are in Chapter 9 of Leonard and Long (1992).

(iii) The transition equation:

$$\dot{K} = \frac{\partial H}{\partial \psi} = s(1-v)h^\alpha K^\beta - g \quad (16)$$

Let us define

$$Z = Z(\psi, \theta) \equiv v(1-\theta) + \psi s(1-v) \quad (17)$$

From (13) we can express h as a function of (K, ψ) and of the central government tax share parameter θ :

$$h = \left[\alpha K^\beta Z \right]^{1/(1-\alpha)} \text{ if } Z \geq 0 \quad (18)$$

(and $h = 0$ if $Z \leq 0$.) (We can expect that $\psi > 0$ even in the case $\theta = 1$, because the stock yields benefits to the local government: grabbing is not possible if the stock is zero.). Henceforth, we focus on the case $Z > 0$.

Substituting (18) into (15) we get

$$\dot{\psi} = r\psi - \beta(\alpha)^{\alpha/(1-\alpha)} Z^{1/(1-\alpha)} K^{-\delta/(1-\alpha)} \equiv M(K, \psi, \theta) \quad (19)$$

where

$$\delta = 1 - \alpha - \beta.$$

If $\delta = 0$, the income function (11) is said to exhibit constant returns to scale. If $\delta > 0$, we have decreasing returns to scale.

From (14) we can express g as a function of ψ and the effort cost parameter γ

$$g = \frac{1 - \varepsilon - \psi}{\gamma} \text{ if } \psi < 1 - \varepsilon \quad (20)$$

$$g = 0 \text{ if } \psi \leq 1 - \varepsilon$$

Substituting (20) and (18) into eq (16), we get

$$\dot{K} = s(1-v)K^{\beta/(1-\alpha)} (\alpha)^{\alpha/(1-\alpha)} Z^{\alpha/(1-\alpha)} - \max\left(0, \frac{1 - \varepsilon - \psi}{\gamma}\right) \equiv N(K, \psi, \theta) \quad (21)$$

Remark: The polar case where $\theta = 1$ deserves some mention. Note that if $\theta = 1$ then the local government has zero tax revenue. Unlike the static case, this does not mean that the local government will choose $h = 0$. If it were to set $h = 0$ always, then it would be in effect facing a non-renewable-resource exploitation problem, just like the classic Hotelling model of a mining firm. The shadow price would then be positive and would rise at the rate of interest (i.e., Hotelling Rule would apply). But then Z would be positive, and the helping hand would be positive, by (18).

4 The Case of Constant Returns to Scale

In this section, we analyze the case of constant returns to scale, i.e., $\delta = 0$. Then equation (15) becomes

$$\dot{\psi} = r\psi - \beta(\alpha)^{\alpha/(1-\alpha)} Z^{1/(1-\alpha)} \equiv M(K, \psi, \theta) \quad (22)$$

which is independent⁵ of K , and equation (21) becomes

$$\dot{K} = s(1-v)K(\alpha)^{\alpha/(1-\alpha)} Z^{\alpha/(1-\alpha)} - \max\left(0, \frac{1-\varepsilon-\psi}{\gamma}\right) \equiv N(K, \psi, \theta) \quad (23)$$

which is linear in K . Under this case, we consider two sub-cases: $\theta = 1$ and $\theta < 1$, and in particular, we search for steady states and their stability properties. (A steady state is a point (K_{ss}, ψ_{ss}) such that $M(K_{ss}, \psi_{ss}, \theta) = 0$ and $N(K_{ss}, \psi_{ss}, \theta) = 0$.)

4.1 The sub-case $\theta = 1$ under constant returns to scale

At first sight, it might seem that if the central government's tax share is $\theta = 1$, the local government might not have any incentive to extend the helping hand. Upon reflection, however, even though the local government's share of official tax revenue is zero, it knows that it can grab only as long as the capital stock remains positive. By extending the helping hand, it can reduce the rate of decline of the capital stock, and may even help maintain it at a steady-state level. So even if $\theta = 1$, the local government's optimal behavior is like that of a manager of a fishery: it is in his interest to conserve the fish stock.

Our analysis will rely on the phase diagram method. For the present sub-case, please refer to Figure 1. We begin our analysis by characterizing the curve $\dot{\psi} = 0$ in the space (K, ψ) . Let us look at the equation $M(K, \psi, \theta) = 0$.

Since we are dealing with the case $\theta = 1$, the equation $M(K, \psi, \theta) = 0$ reduces to

$$\frac{r\psi}{Q} K^{\delta/(1-\alpha)} = [s\psi(1-v)]^{1/(1-\alpha)} \quad (24)$$

where

$$Q = \beta(\alpha)^{\alpha/(1-\alpha)}$$

Now under constant returns to scale, $\delta = 0$, equation (24) becomes

$$\psi \left[\frac{r}{Q} - (s(1-v))^{1/(1-\alpha)} \psi^{\alpha/(1-\alpha)} \right] = 0$$

⁵Even though M is independent of case, there is no harm in writing $M(K, \psi, \theta)$.

This equation has two solutions, which we denote as ψ_a and ψ_b :

$$\begin{aligned}\psi_a &= 0 \\ \psi_b &= \left[\frac{r}{Q(s(1-v))^{1/(1-\alpha)}} \right]^{(1-\alpha)/\alpha} > \psi_a\end{aligned}\quad (25)$$

Thus $\dot{\psi} = 0$ along the horizontal line $\psi = 0$ and along the horizontal line $\psi = \psi_b$. Note that ψ_b can be greater than, or smaller than $1 - \varepsilon$. Clearly, if r is sufficiently small, then $\psi_b < 1 - \varepsilon$. Figure 1 depicts the case where $\psi_b < 1 - \varepsilon$.

Recall that with $\delta = 0$ and $\theta = 1$, the equation for $\dot{\psi}$ is

$$\dot{\psi} = r\psi - Q[s\psi(1-v)]^{1/(1-\alpha)} \quad (26)$$

The right-hand side of (26) is strictly concave in ψ , and is equal to zero at $\psi = \psi_a = 0$, and also at $\psi_b > \psi_a$. It follows that for $\psi < \psi_a$, we have $\dot{\psi} < 0$. If $\psi \in (\psi_a, \psi_b)$, we have $\dot{\psi} > 0$. If $\psi > \psi_b$, we have $\dot{\psi} < 0$.

Now we characterize the curve $\dot{K} = 0$ in the space (K, ψ) . Let us look at the equation $N(K, \psi, \theta) = 0$. To have $\dot{K} = 0$, we must have $g > 0$ when $K > 0$ and $\psi \geq 0$. Thus we infer that the curve $\dot{K} = 0$ must lie below the line $\psi = 1 - \varepsilon$. The equation $N(K, \psi, \theta) = 0$ can be written as

$$\gamma(1-v)^{1/(1-\alpha)}(\alpha s)^{\alpha/(1-\alpha)}K^{\beta/(1-\alpha)} = \frac{1-\psi-\varepsilon}{\psi^{\alpha/(1-\alpha)}} \quad (27)$$

Thus, along this curve, K is a function of ψ . Let us try to determine the shape of this curve. As $\psi \rightarrow (1 - \varepsilon)$, $K \rightarrow 0$, and as $\psi \rightarrow 0$, $K \rightarrow \infty$. We now show that the slope of the curve $\dot{K} = 0$ is negative. Differentiating (27) with respect to ψ , we get

$$\begin{aligned}\gamma(1-v)^{1/(1-\alpha)}(\alpha s)^{\alpha/(1-\alpha)} \left(\frac{\beta}{1-\alpha} \right) K^{-\delta/(1-\alpha)} \frac{dK}{d\psi} = \\ \frac{-\psi^{\alpha/(1-\alpha)} - (1-\psi-\varepsilon) \frac{\alpha}{1-\alpha} \psi^{(2\alpha-1)/(1-\alpha)}}{\psi^{\alpha/(1-\alpha)}}\end{aligned}$$

which is negative for $\psi < (1 - \varepsilon)$. Thus the curve $\dot{K} = 0$ is downward sloping, and never cuts the horizontal axis.

It follows that the curve $\dot{K} = 0$ intersects the curve $\dot{\psi} = 0$ exactly once provided that r is small enough to ensure that $\psi_b < (1 - \varepsilon)$. This intersection is the steady-state point (K_{ss}, ψ_b) , where

$$K_{ss} = \frac{1 - \psi_b - \varepsilon}{[\gamma(1-v)^{1/(1-\alpha)}(\alpha s)^{\alpha/(1-\alpha)}] \psi_b^{\alpha/(1-\alpha)}}$$

From the phase diagram, we know that this steady state is stable in the saddlepoint sense. Thus the optimal policy is as follows.

If $K_0 < K_{ss}$ then it is optimal to extend the helping hand to build up the stock K until it reaches the steady state K_{ss} . Along such a path, the grabbing activity may be zero over some initial time interval.

What happens if $K_0 > K_{ss}$? At first sight, one might be tempted to think that the unstable branch of the saddlepoint, i.e., the path with $\psi(t) = \psi_b$ for ever, and with K rising without bound, might be optimal. But upon reflection, this is not an optimal path, because along such a path, the pay-off to the local government is $(1 - \delta)g - (1/\gamma)g^2 - h$ where g is constant over time ($g = (1 - \varepsilon - \psi_b)/\gamma$) while h is increasing over time. Clearly, it would be better to follow the stable branch of the saddlepoint, and set $g > (1 - \varepsilon - \psi_b)/\gamma$ to ensure that K falls from K_0 to K_{ss} , and once K_{ss} is reached, choose $g = (1 - \varepsilon - \psi_b)/\gamma$ so as to stay at the steady state. In fact, it can be verified that the path of capital accumulation with $\psi(t) = \psi_b$ for ever and $K(t)$ increasing at a constant rate fails to satisfy the transversality condition

$$\lim_{t \rightarrow \infty} e^{-rt} \psi(t) K(t) = 0.$$

Figure 1 corresponds to the case where $\psi_b < 1 - \varepsilon$ (which holds if the discount rate r is low, i.e., if the local government is patient). If the rate of interest r is high, i.e., the local government is impatient, then ψ_b exceeds $1 - \varepsilon$, and it follows that there will not exist any steady state. Then the optimal policy is to run the stock down to zero.

Proposition D1: *Under constant returns to scale ($\delta = 0$), if the tax share of the central government is 100% ($\theta = 1$), the optimal policy for the local government depends on the magnitude of the rate of discount r .*

(a) *If r is sufficiently high to satisfy the following condition*

$$\left[\frac{r}{Q(s(1-v))^{1/(1-\alpha)}} \right]^{(1-\alpha)/\alpha} \geq 1 - \varepsilon \quad (28)$$

then there does not exist any interior steady state. The optimal policy for the local government is to let the capital stock fall to zero. Along such path, the grabbing activity $g(t)$ is positive and decreasing over time (in much the same way as the extraction of an exhaustible resource), while the helping activity $h(t)$ is always positive, until the stock is exhausted. If the condition (28) holds with equality, then the point $(K, \psi) = (0, 1 - \varepsilon)$ is a steady state, and the capital stock tends to zero as time tends to infinity.

(b) *If r is low, so that inequality (28) is reversed, then there is an interior steady state $(K_{ss}, \psi_b) > (0, 0)$, with the saddlepoint property. If the initial stock K_0 is below the steady state level K_{ss} , it is optimal to extend the helping hand to build up the stock K until it reaches the steady state K_{ss} . Along*

such a path, the grabbing activity may be zero over some initial time interval. If $K_0 > K_{ss}$, then grabbing dominates helping, and the stock will fall toward K_{ss} .

4.2 The sub-case $\theta < 1$ under constant returns to scale

In this sub-case, the local government has positive tax revenue as well as extortion revenue. The phase diagrams are Figures 2 and 3. The curve $\dot{\psi} = 0$ is given by

$$\left(\frac{r}{Q}\right)^{1-\alpha} K^\delta = \frac{Z(\psi, \theta)}{\psi^{1-\alpha}} \quad (29)$$

where

$$Z(\psi, \theta) = v(1 - \theta) + \psi s(1 - v)$$

Under constant returns to scale, $\delta = 0$, equation (29) becomes

$$\frac{r}{Q}\psi - [v(1 - \theta) + \psi s(1 - v)]^{1/(1-\alpha)} = 0 \quad (30)$$

Since the left-hand side of equation (30) is a concave function of ψ , this equation has at most two real roots, which we denote by ψ_1 and $\psi_2 \geq \psi_1$. For example, suppose $\alpha = 1/2$, then

$$\frac{r}{Q}\psi - (v(1 - \theta) + \psi s(1 - v))^2 = 0$$

i.e.,

$$(1 - v)^2 s^2 \psi^2 + \left[2sv(1 - v)(1 - \theta) - \frac{r}{Q}\right] \psi + v^2(1 - \theta)^2 = 0$$

This equation has two roots (both real, or both complex):

$$\psi_1 = \frac{\frac{r}{Q} - 2sv(1 - v)(1 - \theta) - \sqrt{\Delta}}{2(1 - v)^2 s^2} \quad (31)$$

$$\psi_2 = \frac{\frac{r}{Q} - 2sv(1 - v)(1 - \theta) + \sqrt{\Delta}}{2(1 - v)^2 s^2} \quad (32)$$

where

$$\Delta = \frac{r}{Q^2} [r - 4Qsv(1 - v)(1 - \theta)] \quad (33)$$

Consider the following Assumption:

Assumption 1: $r - 4Qsv(1 - v)(1 - \theta) \geq 0$.

If Assumption 1 is satisfied, we have $\Delta \geq 0$, thus we have two real roots. Note that both roots are positive. (This is because the product of the roots equals $v^2(1 - \theta)^2$ and the sum of the roots equals

$$\psi_1 + \psi_2 = \frac{\frac{r}{Q} - 2sv(1 - v)(1 - \theta)}{2(1 - v)^2 s^2}$$

which is positive, given Assumption 1.)

Recall that with $\delta = 0$ and $\theta < 1$, the equation for $\dot{\psi}$ is

$$\dot{\psi} = M(K, \psi, \theta) = r\psi - Q [v(1 - \theta) + s\psi(1 - v)]^{1/(1-\alpha)} \quad (34)$$

The right-hand side of (34) is concave in ψ . It follows that for $\psi < \psi_1$, we have $\dot{\psi} < 0$. If $\psi \in (\psi_1, \psi_2)$, we have $\dot{\psi} > 0$. If $\psi > \psi_2$, we have $\dot{\psi} < 0$.

In the space (K, ψ) , the line $\psi = \psi_1$ and the line $\psi = \psi_2$ divide the first quadrant $\{(K, \psi) : K \geq 0 \text{ and } \psi \geq 0\}$ into three regions. In the region A , defined as the set of points (K, ψ) with $K \geq 0$ and $\psi > \psi_2$, we have $M(K, \psi, \theta) < 0$ hence $\dot{\psi} < 0$. In the region C , defined as the set of points (K, ψ) with $K \geq 0$ and $\psi < \psi_1$, we also have $M(K, \psi, \theta) < 0$ hence $\dot{\psi} < 0$. In the region B defined as the set of points (K, ψ) with $K \geq 0$ and $\psi \in (\psi_1, \psi_2)$ (between the line $\psi = \psi_1$ and the line $\psi = \psi_2$), we have $\dot{\psi} > 0$. In other words

$$M(K, \psi, \theta) > 0 \text{ in region } B$$

$$M(K, \psi, \theta) < 0 \text{ in regions } A \text{ and } C.$$

Turning now to the curve $\dot{K} = 0$ i.e., $N(K, \psi, \theta) = 0$, we note that, for $\psi < 1 - \varepsilon$, this equation can be written as

$$K^{\beta/(1-\alpha)} = \frac{1 - \varepsilon - \psi}{\gamma s(1 - v)(\alpha)^{\alpha/(1-\alpha)} [v(1 - \theta) + \psi s(1 - v)]^{\alpha/(1-\alpha)}} \quad (35)$$

This curve has a negative slope in the space (K, ψ) . Along this curve, as $\psi \rightarrow 1 - \varepsilon$, we have $K \rightarrow 0$. The curve cuts the horizontal axis at a finite value of K . To see this, observe that as $\psi \rightarrow 0$, K tends to \tilde{K} where

$$\tilde{K} = \left[\frac{1 - \varepsilon}{\gamma s(1 - v)(\alpha)^{\alpha/(1-\alpha)} (v(1 - \theta))^{\alpha/(1-\alpha)}} \right]^{(1-\alpha)/\beta} \quad (36)$$

Along the curve (35), we have $N(K, \psi, \theta) = 0$ hence $\dot{K} = 0$. Above this curve, we have $\dot{K} > 0$ because

$$\frac{\partial N}{\partial \psi} > 0.$$

Below it, $\dot{K} < 0$.

We now determine the steady-state points. Please refer to Figures 2 and 3. In Figure 2, r is small enough so that $\psi_2 < 1 - \varepsilon$. Corresponding to ψ_2 and ψ_1 , we have from (35) two steady state levels for the capital stock:

$$K_2 = \left[\frac{1 - \varepsilon - \psi_2}{\gamma s(1 - v)(\alpha)^{\alpha/(1-\alpha)} [v(1 - \theta) + \psi_2 s(1 - v)]^{\alpha/(1-\alpha)}} \right]^{(1-\alpha)/\beta}$$

$$K_1 = \left[\frac{1 - \varepsilon - \psi_1}{\gamma s(1-v)(\alpha)^{\alpha/(1-\alpha)} [v(1-\theta) + \psi_1 s(1-v)]^{\alpha/(1-\alpha)}} \right]^{(1-\alpha)/\beta} > K_2$$

It is easy to see that the steady-state point (K_2, ψ_2) is stable in the saddlepoint sense. The other steady state point, (K_1, ψ_1) , is unstable. This is confirmed by linearization of the dynamic system around a steady state.

In Figure 3, r is so high that $\psi_2 > 1 - \varepsilon$, and thus there is only one steady state point.

Proposition D2: *Assume $\theta < 1$, and constant returns to scale ($\delta = 0$). If r is small enough so that $\psi_2 < 1 - \varepsilon$, then there exists two interior steady states. The one with a smaller steady-state stock (and a higher shadow price) is stable in the saddlepoint sense. The other steady state is unstable. If r is large enough so that $\psi_2 > 1 - \varepsilon$, then there exists only one interior steady state, and it is completely unstable.*

Remark: The above analysis tells us the following things about the optimal policy:

(a) if there are two steady states (as in Figure 2), and if $K_0 < K_1$, we simply take the stable branch of the saddlepoint and approach the steady state point (K_2, ψ_2) .

(b) if there is only one steady state (as in Figure 3), and if $K_0 < K_1$, we run the stock down to zero (approaching the point $(K, \psi) = (0, 1 - \varepsilon)$).

(c) if K_0 happens to be equal to K_1 , then the optimal policy is to set $\psi_0 = \psi_1$ and thus the system will remain at the unstable steady state point (K_1, ψ_1) for ever.

What is not obvious, however, is what one should do if K_0 is greater than K_1 ? There is no path that leads to an interior steady state, starting at $K_0 > K_1$. One is tempted to conjecture that, for any $K_0 > K_1$, the optimal policy is to choose $\psi_0 = \psi_1$ so that $\psi(t) = \psi_1$ for ever, and let the capital stock increase without bound. Such a path would be optimal if the following transversality condition is satisfied

$$\lim_{t \rightarrow \infty} e^{-rt} \psi(t) K(t) = 0 \quad (37)$$

Let us check to see if condition (37) is satisfied along the path of unbounded growth of the capital stock, with $\psi(t) = \psi_1$ for ever. Let us take the case where $\alpha = \beta = 1/2$. Under assumption 1, we have two real roots ψ_2 and ψ_1 . Using (31), we get

$$\psi_1 = \frac{2r - sv(1-v)(1-\theta) - \sqrt{4r(r - sv(1-v)(1-\theta))}}{(1-v)^2 s^2} \quad (38)$$

Let

$$b \equiv sv(1-v)(1-\theta)$$

then

$$(1-v)^2 s^2 \psi_1 = 2r - b - \sqrt{4r(r-b)} > 2r - b - \sqrt{(2r-b)^2} = 0 \quad (39)$$

thus $\psi_1 > 0$.

Next, from (13),

$$\frac{\partial H}{\partial h} = \frac{1}{2} h^{-1/2} K^{1/2} [v(1-\theta) + \psi s(1-v)] - 1 = 0$$

Thus

$$h^{1/2} = \frac{1}{2} K^{1/2} [v(1-\theta) + \psi s(1-v)] \quad (40)$$

On the other hand, from (16)

$$\dot{K} = \frac{\partial H}{\partial \psi} = s(1-v)h^{1/2}K^{1/2} - \frac{1-\psi-\varepsilon}{\gamma} \quad (41)$$

Substituting (40) into (41), we get

$$\dot{K} = \frac{1}{2} s(1-v)K [v(1-\theta) + \psi s(1-v)] - \frac{1-\psi-\varepsilon}{\gamma} \quad (42)$$

Setting $\psi = \psi_1$ in (42), we see that $\dot{K} = 0$ if and only if $K = K_1$, where

$$K_1 = \frac{2(1-\psi_1-\varepsilon)}{\gamma s(1-v) [v(1-\theta) + \psi_1 s(1-v)]} \quad (43)$$

Thus, given that $K_0 > K_1$, if we set $\psi(t) = \psi_1$ for all t , then $\dot{K} > 0$ for all $K(t) > K_1$, and eventually

$$\frac{\dot{K}}{K} \rightarrow \frac{1}{2} s(1-v) [v(1-\theta) + \psi_1 s(1-v)] \equiv \sigma$$

The transversality condition (37) is satisfied iff $-r + \sigma < 0$, i.e. iff

$$(1-v)^2 s^2 \psi_1 < 2r - sv(1-v)(1-\theta) = 2r - b$$

This condition is satisfied, in view of (39).

Thus we have established the following proposition:

Proposition D3: *Let (K_1, ψ_1) denote the unstable steady state. Then, if $K_0 > K_1$, the optimal policy is to set $\psi(t) = \psi_1$ and let $K(t)$ grow without bound. This path satisfies all the necessary conditions, and the transversality condition, and is therefore optimal⁶. Along the optimal path, grabbing is a constant, and local government tax revenue, net of expenditure on helping, is a constant proportion of income, which grows without bound.*

⁶Sufficiency is ensured by the concavity of the Hamiltonian with respect to the variables (K, h, g)

5 Implications: threshold levels of corruption and tax share

The analysis in the preceding section indicates that, for any initial level of capital stock, K_0 , there is a threshold level of corruption and a threshold level of central government tax share, below which the economy will grow without bound, and above which the economy will stagnate. As we have seen, if $K_0 > K_1$, the economy will grow without bound, and corruption (as measured by the ratio g/Y , i.e., the proportion of income that is grabbed by government officials) will fall steadily. This critical level K_1 is a function of the parameters ε, γ and θ . It is not difficult to show that K_1 is increasing in θ and decreasing in ε and γ . To prove this, consider equations (38) and (43). From equation (38), we see that ψ_1 is dependent on θ and independent of ε and γ . Differentiating (38), we get

$$(1-v)^2 s^2 \frac{d\psi_1}{d\theta} = sv(1-v) \left[1 - \frac{1}{\sqrt{r(r-sv(1-v)(1-\theta))}} \right] < 0 \quad (44)$$

where the inequality follows from $r < 1$.

Now, differentiating (43) with respect to θ , holding constant the parameters γ and ε :

$$\frac{\partial K_1}{\partial \theta} = \frac{-[v(1-\theta) + (1-\varepsilon)s(1-v)] \frac{d\psi_1}{d\theta} + v(1-\psi_1-\varepsilon)}{\frac{1}{2}\gamma s(1-v)[v(1-\theta) + \psi_1 s(1-v)]^2} > 0$$

Similarly, differentiating (43) with respect to γ , holding constant the parameters θ and ε :

$$\frac{\partial K_1}{\partial \gamma} = \frac{-2(1-\psi_1-\varepsilon)\gamma^{-2}}{s(1-v)[v(1-\theta) + \psi_1 s(1-v)]} < 0$$

and, differentiating (43) with respect to ε , holding constant the parameters θ and γ :

$$\frac{\partial K_1}{\partial \varepsilon} = \frac{-2(1-\psi_1-\varepsilon)}{\gamma s(1-v)[v(1-\theta) + \psi_1 s(1-v)]} < 0$$

It follows that, given the initial capital stock level K_0 , a change in the parameter θ can have qualitatively significant effects on the course of development of the economy. Suppose that initially θ is sufficiently low, so that $K_1(\theta, \varepsilon, \gamma) < K_0$. Then the local economy is due to embark on an expansion path with ever-rising capital and income. But if there is a policy shift by the central government that results in a higher θ (say from θ_L to $\theta_H > \theta_L$) such that $K_1(\theta_H, \varepsilon, \gamma) > K_0$, then the local economy will stagnate: the capital stock falls and converges to a low-level steady state.

Similarly, the parameters ε and γ indicate how difficult it is to carry out grabbing activities. An increase in ε or γ represents an increase in the level of transparency of governance at the local government level. An increase in transparency will reduce K_1 below K_0 thus enabling the economy to achieve perpetual growth.

Proposition D4: (i) *Given any initial capital stock, there exists a corresponding threshold level of central government tax share, below which the economy will achieve perpetual growth, and above which the economy will stagnate.*

(ii) *Given any initial capital stock, there exists a corresponding threshold level of transparency of governance, above which the economy will achieve perpetual growth, and below which the economy will stagnate.*

6 The Case of Decreasing Returns to Scale

We now consider the case $\delta > 0$

6.1 The sub-case $\theta = 1$ under decreasing returns to scale

We begin our analysis of this sub-case by characterizing the curve $\dot{\psi} = 0$ in the space (K, ψ) . Consider Figure 4. Let us look at the equation $M(K, \psi, \theta) = 0$. Since we are dealing with the case $\theta = 1$, that equation is reduced to

$$\frac{r\psi}{Q} K^{\delta/(1-\alpha)} = [s\psi(1-v)]^{1/(1-\alpha)}$$

where

$$Q = \beta(\alpha)^{\alpha/(1-\alpha)}$$

Since $\delta > 0$, the curve $\dot{\psi} = 0$ must satisfy the equation

$$\left(\frac{r}{Q}\right)^{1-\alpha} K^\delta = \psi^\alpha (s(1-v))$$

Thus the curve $\dot{\psi} = 0$ is upward-sloping in the space (K, ψ) . Since

$$\frac{\partial M(K, \psi, \theta)}{\partial K} > 0$$

we infer that $\dot{\psi} > 0$ to the right of the curve $\dot{\psi} = 0$ and $\dot{\psi} < 0$ to its left.

Now we try to characterize the curve $\dot{K} = 0$ in the space (K, ψ) . Let us look at the equation $N(K, \psi, \theta) = 0$. To have $\dot{K} = 0$, we must have $g > 0$ when $K > 0$ and $\psi \geq 0$. Thus we infer that the curve $\dot{K} = 0$ must lie below the line $\psi = 1 - \varepsilon$. The equation $N(K, \psi, \theta) = 0$ can be written as

$$\gamma(1-v)^{1/(1-\alpha)} (\alpha s)^{\alpha/(1-\alpha)} K^{\beta/(1-\alpha)} = \frac{1-\psi-\varepsilon}{\psi^{\alpha/(1-\alpha)}} \quad (45)$$

Thus, along this curve, K is a function of ψ . Let us try to determine the shape of this curve. As $\psi \rightarrow (1 - \varepsilon)$, $K \rightarrow 0$, and as $\psi \rightarrow 0$, $K \rightarrow \infty$. We now show that the curve $\dot{K} = 0$ has a negative slope. Differentiating (45) with respect to ψ , we get

$$\begin{aligned} \gamma(1-v)^{1/(1-\alpha)}(\alpha s)^{\alpha/(1-\alpha)} \left(\frac{\beta}{1-\alpha} \right) K^{-\delta/(1-\alpha)} \frac{dK}{d\psi} = \\ \frac{-\psi^{\alpha/(1-\alpha)} - (1-\psi-\varepsilon) \frac{\alpha}{1-\alpha} \psi^{(2\alpha-1)/(1-\alpha)}}{\psi^{\alpha/(1-\alpha)}} \end{aligned} \quad (46)$$

which is negative for $\psi < (1 - \varepsilon)$. Thus we can infer from (45) and (46) that the curve $\dot{K} = 0$ is downward sloping, and never cuts the horizontal axis.

It follows that the curve $\dot{K} = 0$ intersects the curve $\dot{\psi} = 0$ exactly once given that $\delta > 0$. This intersection is the steady-state point (K_{ss}, ψ_{ss}) . From the phase diagram (Figure 4), we know that this steady state is stable in the saddlepoint sense.

Proposition D5: *Assume $\theta = 1$. Under decreasing returns to scale, regardless of the magnitude of the interest rate, there exists a unique interior steady state point (K_{ss}, ψ_{ss}) . This unique steady state has the saddlepoint property. Starting from any $K_0 > 0$, the optimal path is the stable branch of the saddlepoint, leading to the interior steady state. For $K_0 < K_{ss}$, grabbing activities begin at a low level (possibly zero), and intensify as the capital stock grows toward K_{ss} . For $K_0 > K_{ss}$, grabbing activities begin at a high level, and diminish as the capital stock falls towards K_{ss} .*

Remark: Compare Figure 4 with Figure 1. The qualitative properties of the respective optimal paths are very similar. This is not surprising, because in both situations the tax share of the central government is 100 per cent.

6.2 The sub-case $\theta < 1$ under decreasing returns to scale

Figure 5 is the phase diagram for this sub-case. Equation (29) gives us the curve $\dot{\psi} = 0$, and along this curve, K can be expressed as a function of ψ . Differentiating both sides with respect to ψ , we get

$$\left(\frac{r}{Q} \right)^{1-\alpha} \delta K^{\delta-1} \frac{dK}{d\psi} = \frac{\psi^{-\alpha}}{\psi^{2/(1-\alpha)}} [s\alpha(1-v)\psi - v(1-\alpha)(1-v)]$$

Define

$$\hat{\psi} = \frac{v(1-\alpha)(1-v)}{s\alpha(1-v)}$$

Along the curve $\dot{\psi} = 0$, we have K as a decreasing function of ψ for $\psi \in (0, \hat{\psi})$ and K as an increasing function of ψ for $\psi > \hat{\psi}$. In the space (K, ψ) , the curve $\dot{\psi} = 0$ has the shape of the letter C . To the

right of this curve, we have $\dot{\psi} > 0$, because $\partial M/\partial K > 0$. At the turning point of the letter C -shaped curve, we have

$$\begin{aligned}\psi &= \hat{\psi} = \frac{v(1-\alpha)(1-v)}{s\alpha(1-v)} \\ K &= \hat{K} = \left[\frac{Z(\hat{\psi}, \theta)}{\hat{\psi}^{1-\alpha}} \left(\frac{Q}{r} \right)^{1-\alpha} \right]^{1/\delta}\end{aligned}\quad (47)$$

Now consider the curve $\dot{K} = 0$. We note that, for $\psi < 1 - \varepsilon$, this equation can be written as

$$K^{\beta/(1-\alpha)} = \frac{1 - \varepsilon - \psi}{\gamma s(1-v)(\alpha)^{\alpha/(1-\alpha)} [v(1-\theta) + \psi s(1-v)]^{\alpha/(1-\alpha)}}\quad (48)$$

This curve has a negative slope in the space (K, ψ) . Along this curve, as $\psi \rightarrow 1 - \varepsilon$, we have $K \rightarrow 0$. The curve cuts the horizontal axis at a finite value of K . To see this, observe that as $\psi \rightarrow 0$, K tends to \tilde{K} where

$$\tilde{K} = \left[\frac{1 - \varepsilon}{\gamma s(1-v)(\alpha)^{\alpha/(1-\alpha)} (v(1-\theta))^{\alpha/(1-\alpha)}} \right]^{(1-\alpha)/\beta}$$

Along the curve (48), we have $N(K, \psi, \theta) = 0$ hence $\dot{K} = 0$. Above this curve, we have $\dot{K} > 0$ because

$$\frac{\partial N}{\partial \psi} > 0.$$

Below it, $\dot{K} < 0$.

We now determine the steady-state points. Since the curve $\dot{\psi} = 0$ has the C shape, and the curve $\dot{K} = 0$ is downward sloping, there is the possibility of several intersections.

Proposition 6: *Assume $\theta < 1$ and $\delta > 0$ (decreasing returns to scale). There may exist several steady states. The one with the smallest capital stock is stable in the saddlepoint sense. It is also possible that there does not exist any interior steady state (for example, if $\hat{K} < \tilde{K}$). In the latter case, the optimal path is to run down the stock to zero in finite time.*

Remark: In Figure 5, we show two steady-state points, (K_1, ψ_1) and (K_2, ψ_2) . The point (K_2, ψ_2) is stable in the saddlepoint sense. For all $K_0 \in (0, K_1)$, the optimal policy is to follow the stable branch of the saddlepoint and approach (K_2, ψ_2) asymptotically. If $K_0 = K_1$, it is optimal to stay at K_1 for ever. If $K_0 > K_1$ it seems that the optimal policy is to let K grow for ever (in much the same way as the constant path $\psi(t) = \psi_1$ in Figure 2.)

7 Local stability analysis by linearization

In the preceding section, we relied on the phase-diagram approach in our analysis of stability. We now compliment that analysis by the method of linearization of the dynamic system around an interior steady state. This method also permits comparative dynamics (across steady states, as parameters of the system change).

7.1 Local stability analysis

We have the dynamic system

$$\begin{aligned}\dot{K} &= \frac{\partial H}{\partial \psi} \equiv N(K, \psi, \theta, \gamma) \\ \dot{\psi} &= r\psi - \frac{\partial H}{\partial K} \equiv M(K, \psi, \theta)\end{aligned}$$

Linearization around a steady state yields

$$\begin{bmatrix} \dot{K} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} H_{\psi K} & H_{\psi\psi} \\ -H_{KK} & r - H_{K\psi} \end{bmatrix} \begin{bmatrix} K - K_{ss} \\ \psi - \psi_{ss} \end{bmatrix}$$

This system has two characteristic roots, λ_1 and λ_2 . The sum of the roots is

$$\lambda_1 + \lambda_2 = r > 0$$

and the product is

$$\lambda_1\lambda_2 = -(H_{K\psi})^2 + rH_{K\psi} + H_{\psi\psi}H_{KK} = H_{K\psi}(r - H_{K\psi}) + H_{\psi\psi}H_{KK}$$

If the product $\lambda_1\lambda_2$ is negative, we have saddlepoint stability (i.e., one root has negative real part, and the other has positive real part). If the product is positive, the steady state is unstable. Now it is clear that $H_{\psi\psi} = N_{\psi} > 0$ and $H_{KK} = M_K = 0$. From (19) we have

$$H_{K\psi} > 0$$

Let us apply this analysis to the steady-state points (K_1, ψ_1) and (K_2, ψ_2) of the section on constant returns to scale, with $\theta < 1$. At the steady state (K_2, ψ_2) we have $r - H_{K\psi} < 0$, hence $\lambda_1\lambda_2 < 0$ thus it is stable in the saddlepoint sense. At the steady state (K_1, ψ_1) we have $r - H_{K\psi} > 0$, hence $\lambda_1\lambda_2 > 0$ thus it is unstable.

7.2 Comparative dynamics

We now study how an increase in the parameter θ affects the position of the stable steady state.

At the steady state, we have

$$N(K_{ss}, \psi_{ss}, \theta, \gamma) = 0$$

$$M(K_{ss}, \psi_{ss}, \theta) = 0$$

To see how the steady state changes with an increase in θ , we differentiate the system totally:

$$N_K dK_{ss} + N_\psi d\psi_{ss} + N_\theta d\theta = 0$$

$$M_K dK_{ss} + M_\psi d\psi_{ss} + M_\theta d\theta = 0$$

In matrix notation

$$\begin{bmatrix} N_K & N_\psi \\ M_K & M_\psi \end{bmatrix} \begin{bmatrix} \frac{dK_{ss}}{d\theta} \\ \frac{d\psi_{ss}}{d\theta} \end{bmatrix} = - \begin{bmatrix} N_\theta \\ M_\theta \end{bmatrix}$$

Using Cramer's rule, we get

$$\frac{dK_{ss}}{d\theta} = \frac{1}{\Delta} [M_\theta N_K - N_\theta M_\psi] \quad (49)$$

where, at the steady state,

$$\Delta = N_K M_\psi - M_K N_\psi = \lambda_1 \lambda_2 < 0$$

Let us apply this to the (saddlepoint stable) steady-state point (K_2, ψ_2) of the section on constant returns to scale, with $\theta < 1$. The numerator of the right-hand side of (49) can be computed as follows.

$$M_\theta = Zv > 0$$

$$N_K = \frac{(1-v)^2 K}{2} + \frac{1}{\gamma} > 0$$

$$N_\theta = -\frac{(1-v)Kv}{2} < 0$$

$$M_\psi = r - (1-v)Z$$

Thus

$$M_\theta N_K - N_\theta M_\psi = \frac{vZ}{\gamma} + \frac{r(1-v)Kv}{2} > 0$$

This establishes proposition D7 below.

Proposition D7: *Concerning the steady state that is stable in the saddlepoint sense, an increase in θ (the tax share of the central government) will lead to a fall in the steady-state capital stock, and a rise in the steady-state shadow price ψ .*

8 Concluding Remarks

We have shown that a corrupt local government may have long term interest in the health of the local economy. Thus its activities consist of helping as well as grabbing. Depending on the initial level of the capital stock being higher or lower than a certain threshold level, the outcome for the local economy may be perpetual growth at a positive rate (as in the AK model in the endogenous growth literature), or stagnation (a low level poverty trap). An important property of our results is that the threshold level of capital is a decreasing function of the central government tax share. Thus, the greater is the central government tax share, the more likely is the stagnation outcome. Another way of putting this is: given an initial capital stock K_0 , there is a threshold level of central government tax share, beyond which the economy will become stagnated, and below which perpetual growth will be achieved.

We also showed that the threshold level of capital is a decreasing function of the transparency of local governance (i.e., a decreasing function of the parameters ε and γ that reflect the difficulty of hiding extortion). Thus any increase in transparency will increase the likelihood that the economy can take off.

The model can be extended in several directions. First, the central government's choice of the tax rate and of its revenue share could be explicitly modelled. This would lead to a study of interesting interactions between the central government and the local government. Second, one could suppose that capital is mobile across several local regions, and thus there would be competition among various local governments. These extensions would involve an analysis of differential games⁷, possibly with a hierarchical structures. Finally, political uncertainty about the possibility of losing office can be added to the model, along the lines of Long (1975) and Konrad et al. (1994).

⁷For some examples of differential games in economics, see Benckroun and Long (1998, 2002). For a comprehensive treatment of differential games, including those with a hierarchical structures, see Docker et al. (2000).

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FIGURE 1
 $\theta=1, \delta=0$

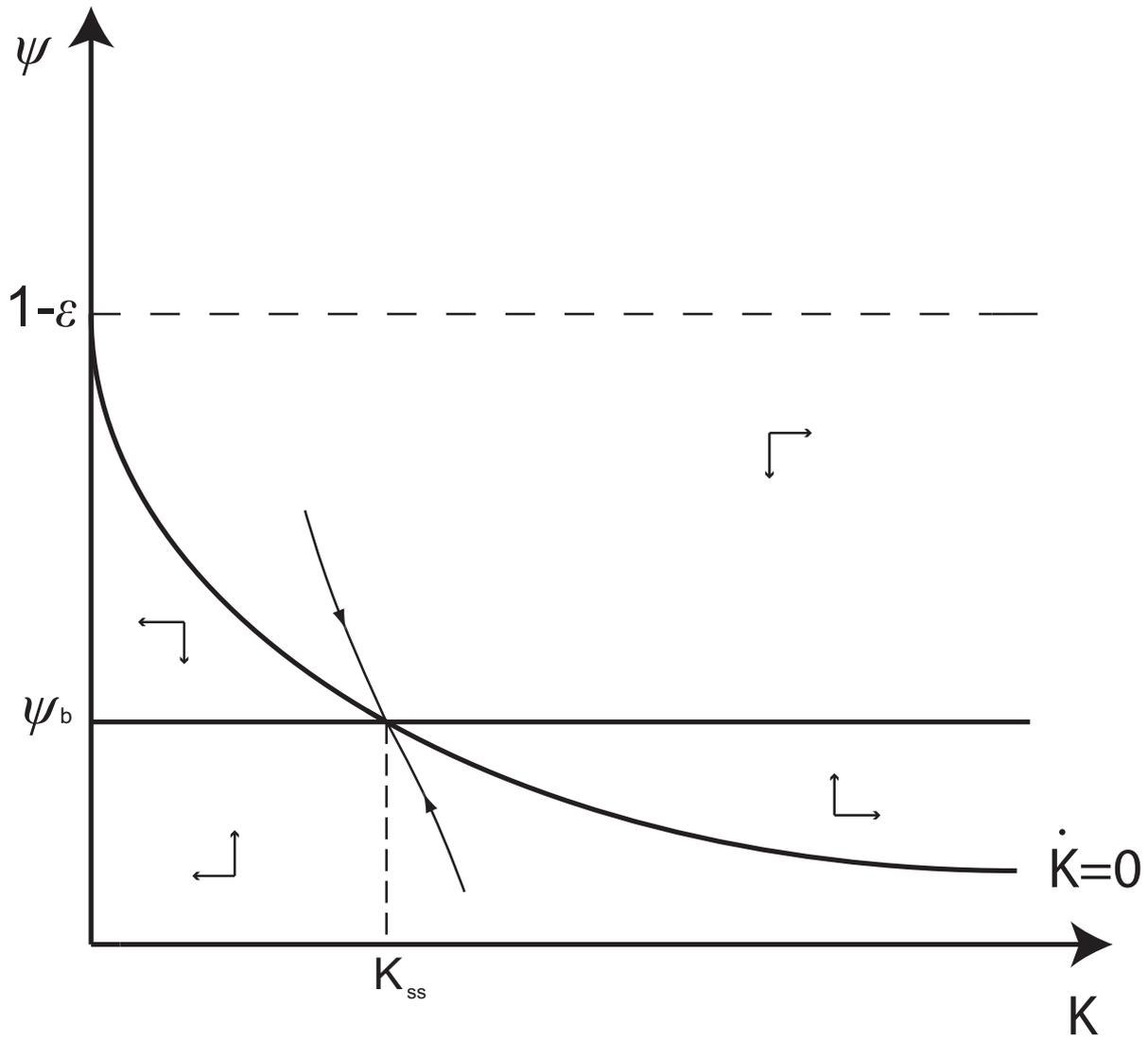


FIGURE 2

$$\theta < 1, \delta = 0, \psi_2 < 1 - \varepsilon$$

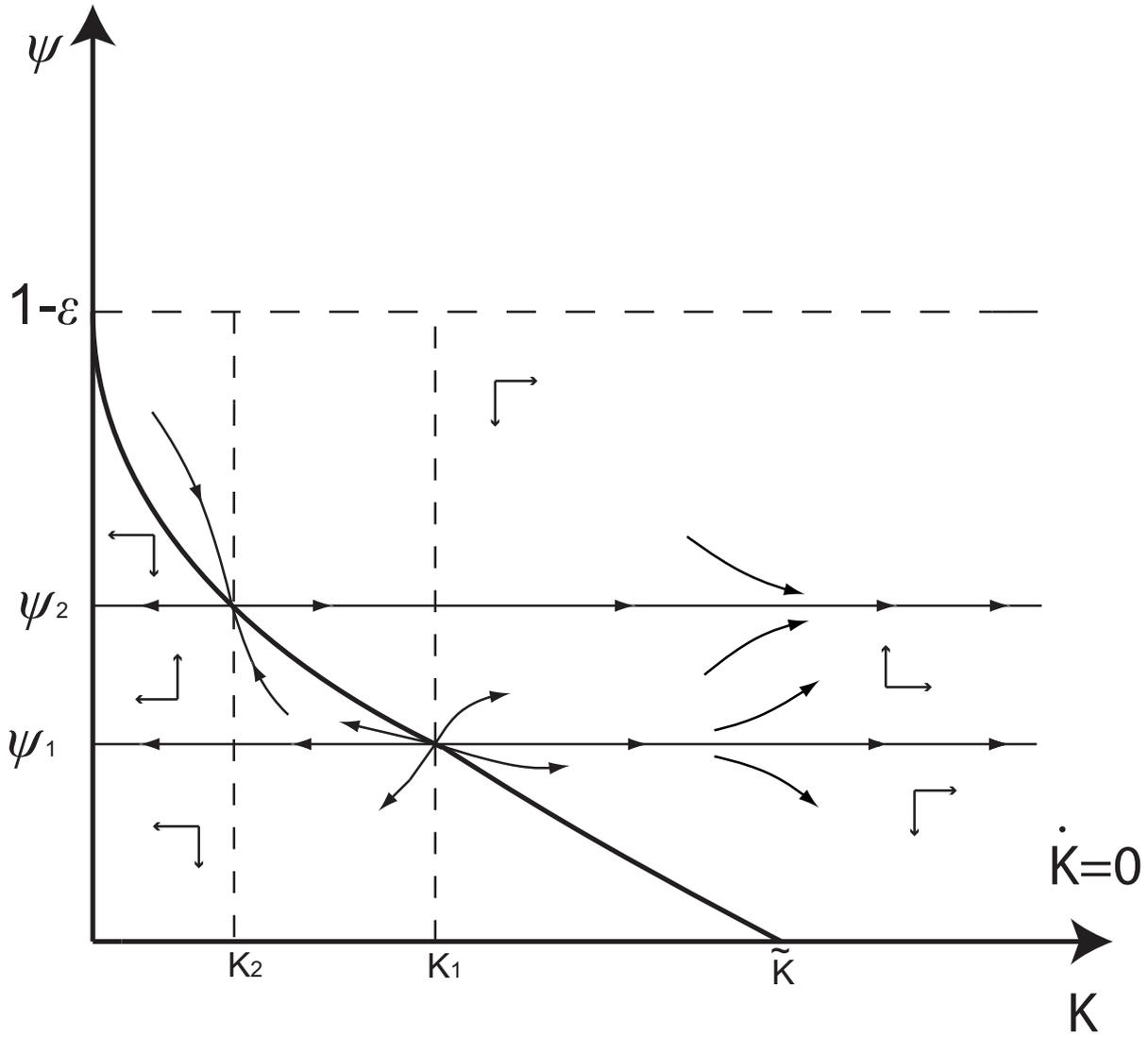


FIGURE 3

$$\theta < 1, \delta = 0, \psi_2 > 1 - \varepsilon$$

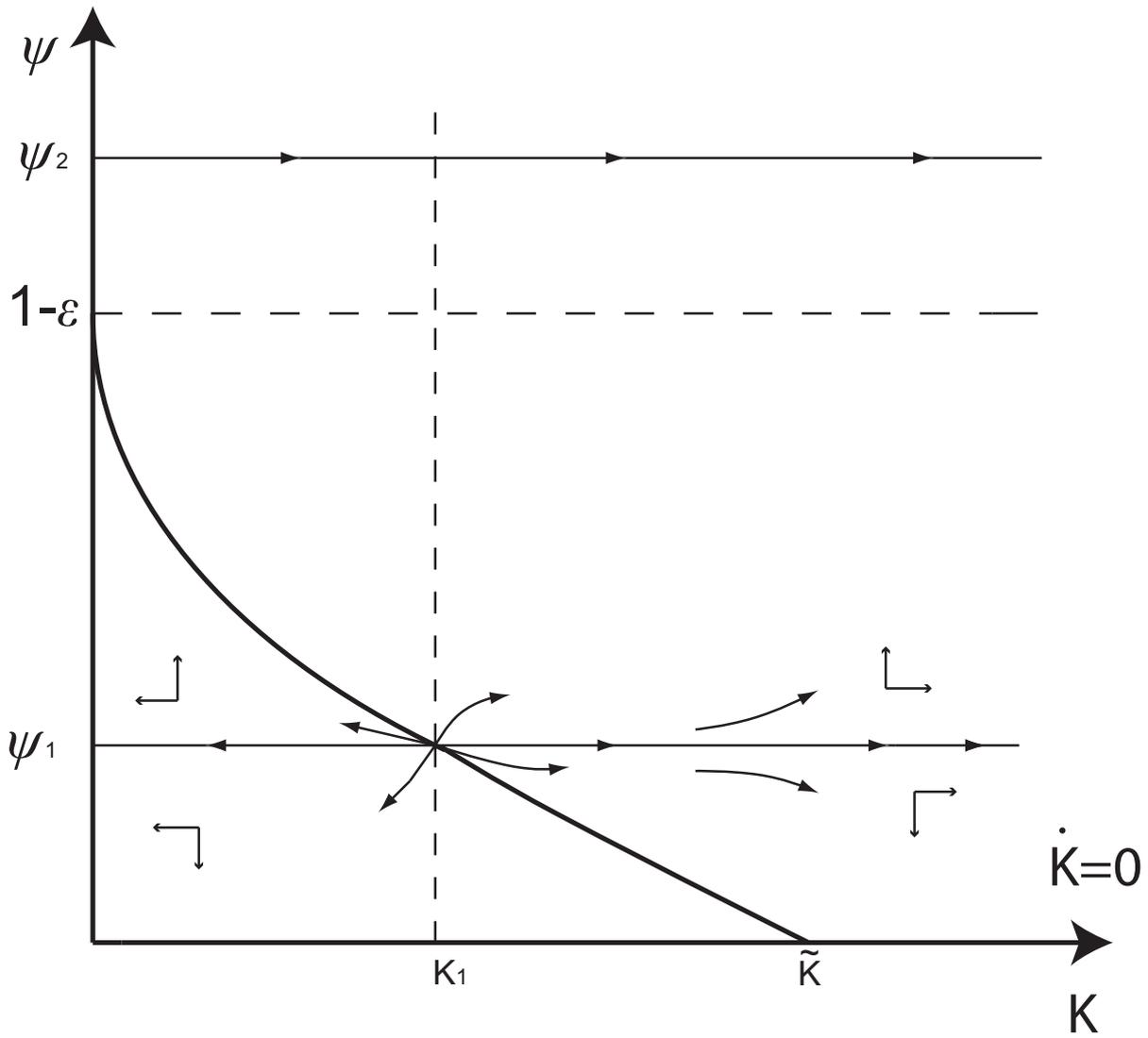


FIGURE 4
 $\theta=1, \delta>0$

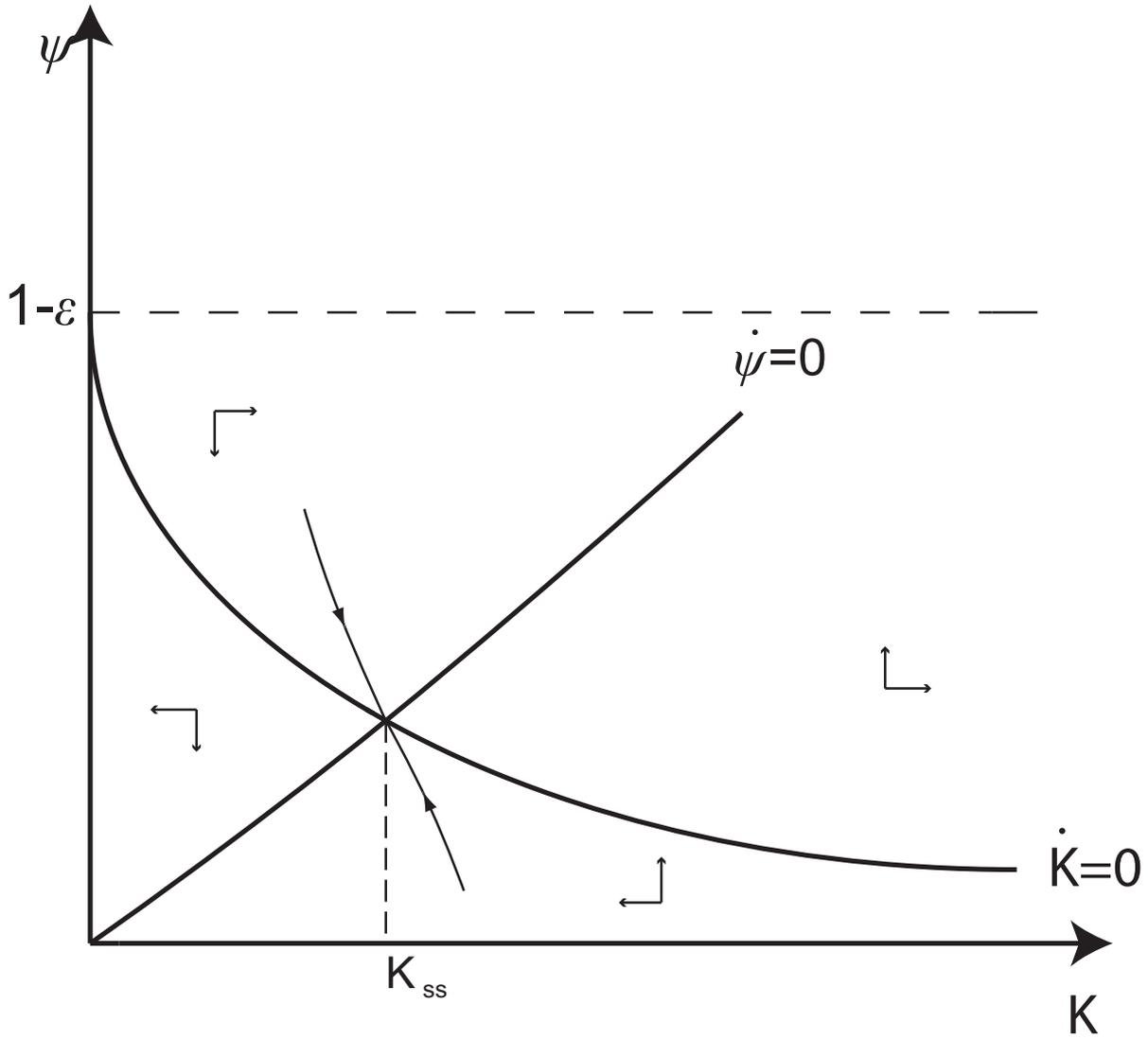
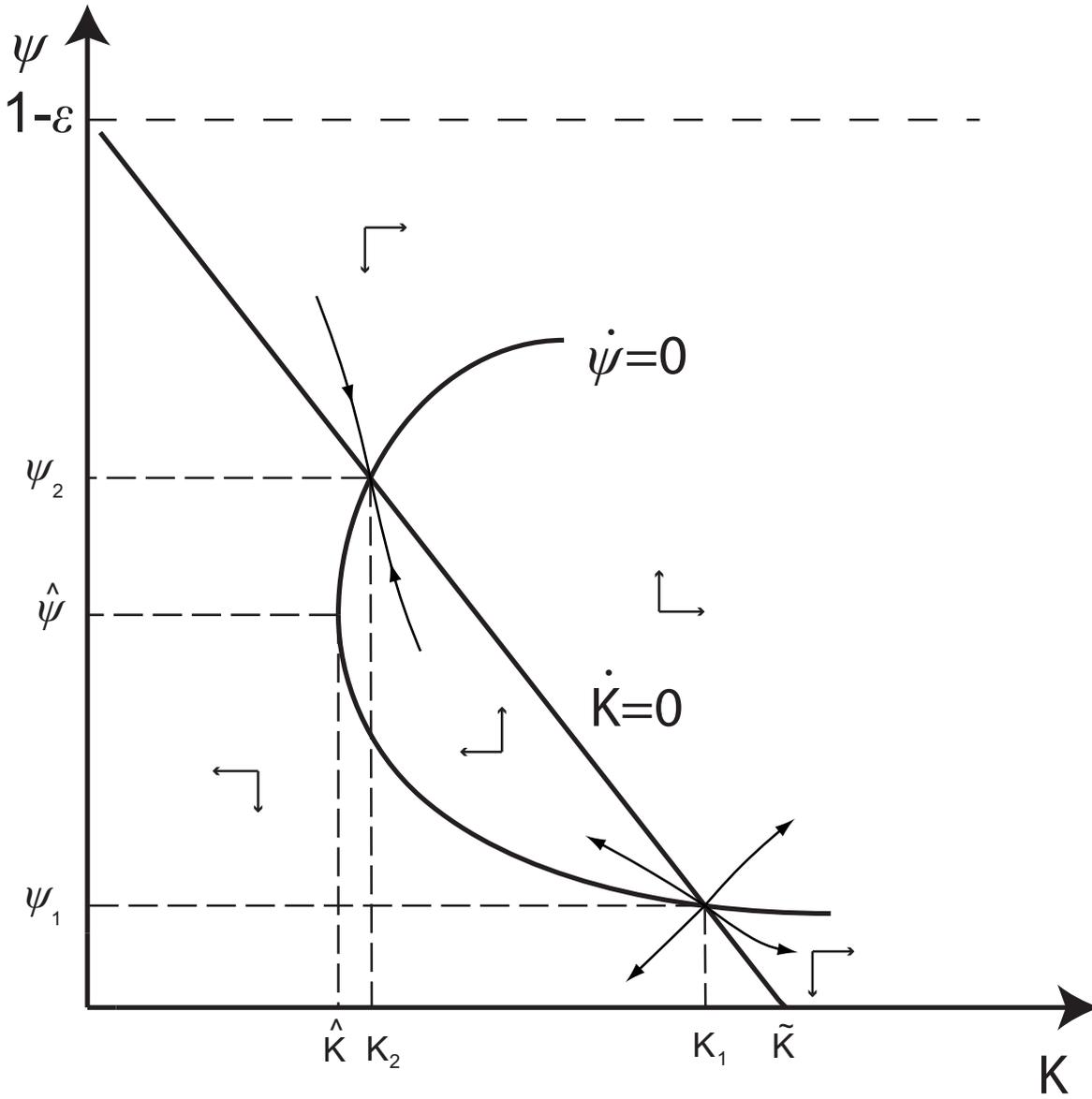


FIGURE 5
 $\theta < 1, \delta < 0$



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