

Berger, Helge; Mueller, Till

**Working Paper**

## How should large and small countries be represented in a currency union?

CESifo Working Paper, No. 1344

**Provided in Cooperation with:**

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Berger, Helge; Mueller, Till (2004) : How should large and small countries be represented in a currency union?, CESifo Working Paper, No. 1344, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/18707>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# HOW SHOULD LARGE AND SMALL COUNTRIES BE REPRESENTED IN A CURRENCY UNION?

HELGE BERGER  
TILL MUELLER

CESIFO WORKING PAPER NO. 1344  
CATEGORY 6: MONETARY POLICY AND INTERNATIONAL FINANCE  
NOVEMBER 2004

*An electronic version of the paper may be downloaded*

- *from the SSRN website:* [www.SSRN.com](http://www.SSRN.com)
- *from the CESifo website:* [www.CESifo.de](http://www.CESifo.de)

# HOW SHOULD LARGE AND SMALL COUNTRIES BE REPRESENTED IN A CURRENCY UNION?

## Abstract

The likely extension of the euro area has triggered a debate on the organization of the ECB, in particular on the apparent mismatch between relative economic size and voting rights in the Council. We present a simple model of optimal representation in a federal central bank addressing this question. Optimal voting weights reflect two opposing forces: the wish to insulate common monetary policy from changing preferences at the national level, and the attempt to avoid an overly active or passive reaction to idiosyncratic national economic shocks. A perfect match between economic size and voting rights is rarely optimal, and neither is the “one country, one vote principle”. Empirically, there are indications that the pattern of over- and under-representation of member countries in the ECB Council might be extreme by the standards of the US Fed and German Bundesbank and not always optimal.

JEL Code: D72, E52, E58, F33.

Keywords: Central Bank, Federal Central Bank, Currency Union, optimal representation, voting, ECB.

*Helge Berger*  
*Free University Berlin*  
*Department of Economics*  
*Boltzmannstr. 20*  
*14195 Berlin*  
*Germany*  
*hberger@wiwiss.fu-berlin.de*

*Till Mueller*  
*Free University Berlin*  
*Department of Economics*  
*Boltzmannstr. 20*  
*14195 Berlin*  
*Germany*  
*till.mueller@wiwiss.fu-berlin.de*

We thank seminar participants in Berlin and at the Bundesbank in Frankfurt for helpful comments. Anne-Kristin Koch provided able research assistance.

[T]he co-responsibility and active involvement of the Länder, in the form of participation in the appointment of the Land Central Bank (...), are an important element in the Bundesbank's structure and independence.

Bundesbank (1992, p. 49-50)

Whether within the Convention or in bodies such as the European Central Bank, representatives of the large countries believe they are under threat of being tied up by a gang of small countries, which are by definition irresponsible and which, following enlargement, will form the majority within the Council in terms of numbers.

R. Goebbels, MEP Luxembourg (European Parliament 2003)

## 1 Introduction

The likely extension of the Economic and Monetary Union (EMU) has triggered a lively debate on the organization of monetary policy in the euro area. Following a suggestion by the European Central Bank (ECB), current EMU member governments have agreed on a plan to reform the ECB's organizational structure with a view to (*i*) better match the economic and political weights of member countries in the ECB Council and (*ii*) limit the overall size of the Council.<sup>1</sup>

While there is variation in detail, it is probably fair to say that most academic papers discussing the merits of the reform (or its necessity) agree that limiting the overall size of the ECB Council is a crucial step to ensure efficiency in monetary policymaking in the euro area.<sup>2</sup> Already today the ECB Council is exceptionally large in terms of members and—even after the reform—euro area enlargement might leave the ECB with “too many [members] to decide on where to go to dinner,” as Baldwin (2001) remarked.

There is less agreement, however, on whether—or to what degree—correcting the existing lack of correlation between the member countries' economic size and their voting power is sound policy. The current ECB structure, by following the “one country, one vote” principle, gives economically smaller countries a disproportional large vote. EMU enlargement is likely to amplify this problem, even with ECB reform. Most (if not all) prospective new members are small enough to be over-represented even after the reform; they

---

<sup>1</sup>The reform has been agreed to at the government level, but formal ratification by current member states is pending.

<sup>2</sup>Studies discussing these and related arguments include, among others, Baldwin et al. (2001), Hefeker (2002), Gros et al. (2002), Fitoussi and Creel (2002), de Grauwe (2003), and Meade (2003).

also show stark differences in economic development compared to current members (de Haan et al. 2004).

The over-representation of smaller member states could introduce a bias into the ECB's decision-making. The Maastricht treaty would have the ECB stabilize inflation within the euro area using the Harmonized Consumption Price Index.<sup>3</sup> This encourages the ECB to take a European perspective by evaluating the potential impact of national economic developments on euro area inflation based on the respective relative economic size of a member country. If national central bank governors put at least some weight on national economic developments, their over-representation could distort this perspective by directing monetary policy toward national issues.

So does the plan for ECB reform fall short? The answer is far from clear. While reducing the degree of over-representation will ensure that the decision-making process within the ECB Council will be more likely to resemble the perspective of a benevolent European social planner, there are a number of arguments that caution against a too ambitious reform. For instance, Gros and Hefeker (2002) and Benigno (2004) point out that over- and under-representation of member countries in the planner's target function or, equivalently, in the distribution of voting rights within the actual ECB Council, could be optimal if transmission mechanisms differ. How important these differences might be is, however, mostly an empirical question.<sup>4</sup> Another argument is made by Casella (1992), who points out that over-representation could be a necessary condition for smaller countries to join a currency union.

The present paper adds central bank independence as a potentially crucial argument to this debate.<sup>5</sup> We focus on the need for federal central banks such as the ECB to strive for both political independence from, and fair representation of, member states on their policymaking bodies. We show that the interplay between two opposing forces—(i) the wish to reduce the impact of national preference shocks on union-wide policymaking, and (ii) the attempt to minimize misrepresentation of any one country's relative economic size so as to avoid over- or under-reactions to national economic shocks—determines the optimal representation of national interest on the Council. Calibrating Council representation to moderate the impact of preference shocks helps to

---

<sup>3</sup>Euro area inflation is computed by Eurostat using a weighted average of (harmonized) current euro area member inflation rates, where the weights are based on relative expenditure on final private domestic consumption. The distribution of relative consumption very closely mirrors that of relative GDP.

<sup>4</sup>Ciccarelli and Rebucci (2002) present evidence that suggests that transmission mechanisms have become fairly similar among current EMU members already during the 1990s.

<sup>5</sup>Advantages of the “one country, one vote” principle based on considerations of political economy are also discussed in Berger (2002) and Berger et al. (2004).

insulate common monetary policy from unwanted volatility when national targets might deviate from the common policy goal. Matching Council representation and economic weight, on the other hand, ensures that actual monetary policy stabilizes national economic shocks in line with the union ideal.

Optimal representation will, as a rule, weigh both arguments, reflecting economic size as well as the stochastic properties of economic and preference shocks. As a consequence, whether a country will be optimally over- or under-represented compared to its relative economic size depends on all these determinants. While one might expect that most small countries would be over-represented and most large countries would be under-represented (as is the case in the ECB today), this does not always hold true. For instance, it might be optimal to over-represent a large country if its policy preferences are very stable relative to other union members.

In what follows, Section 2 will briefly review recent related literature. Section 3 describes the model, the first-best benchmark policy, and derives the conditions for optimal representation of national interests within a currency union in the presence of economic and preference shocks. Section 4 allows for dependencies between shocks and hints at the impact of continued integration in the economic and preference domain on optimal representation. Section 5 provides a robustness check by allowing for alternative sources of national preference shocks. Section 6 attempts to put everything into perspective by comparing the degree of over- and under-representation relative to economic size for the ECB, the US Federal Reserve, and the German Bundesbank. In addition, determinants of optimal representation identified by the theoretical model are compared with actual misrepresentation within the ECB. Finally, Section 7 draws some conclusions.

## 2 Relation to Recent Literature

Our contribution is related to three intertwined strands in the literature on central bank design. One, including von Hagen and Stüppel (1994) and Lohmann (1997, 1998), asks whether a central bank with a centralized or a decentralized structure is better suited to cope with partisan policy making at the national level. The argument is involved, but in general strong national representation in the joint central bank Council often leads to inefficiencies at the union level. For instance, in the Lohmann (1997) model, a more decentralized central bank organization increases the frequency at which the Council's median voter (and, thus, central bank policy) changes, resulting in unwanted volatility in monetary policy.

This contrasts with a somewhat more recent body of papers discussing the efficiency of alternative decision-making structures (see Gerling et al. (2003) for a survey). Gerlach-Kristen (2002), for instance, argues that committees with multiple members might be better suited than single individuals to process information, fostering efficient decision-making—a theoretical result supported by experimental evidence produced by Blinder and Morgan (2002).<sup>6</sup> Since much of the information that federal central banks are processing is regional, this can be taken to suggest that regional or national representation in the Council has advantages (Maier et al. 2003).<sup>7</sup> Thus, full centralization would not be optimal.

A third group of papers takes the influence of national interest on central bank Council policies as given—either because full centralization might not be optimal or because national representation is too deeply ingrained into the political setup of the currency union to be abandoned any time soon. The question is then how to deal with shocks to national preferences within a federal central bank system. The best-known contributions addressing this question include Waller and Walsh (1996), who suggest long and overlapping contracts for Council members as a device to moderate the impact of national preference shocks (see also Lindner 2000), an idea already reflected in the actual term structure of ECB Council members.

Other recent proposals remain largely theoretical to date. The first such proposal would institute flexible majority rules for Council decisions (Gersbach and Pacht 2004). These rules would attempt to moderate demands for policy changes based on idiosyncratic national economic shocks by raising the majority requirements in line with the size of the desired interest rate change. A second proposal, from Heisenberg (2003), argues that increasing the transparency of Council decision-making would diminish national influences on policies, helping to constrain the problem of national preference shocks at its source. Finally, Bullard and Waller (2004) discuss the advantages of alternative decision-making arrangements, including simple majority voting, bargaining, and a supermajority design, in a general equilibrium framework.

The present paper adds to this small but growing literature. As we will argue below, optimizing over- or under-representation of national representatives on the federal central bank Council compared to the relative economic size of their respective countries is another tool that can be used to moderate the impact of national preference shocks on the common monetary policy.

---

<sup>6</sup>Gersbach and Hahn (2001) explore similar issues from a transparency perspective.

<sup>7</sup>Also see Goodfriend (2000). Alan Greenspan frequently stresses that the information provided by the presidents of the regional Federal Reserve Banks “contribute[s] vitally to the formulation of monetary policy” (Greenspan 2000, p. 2) in the case of the U.S. Federal Reserve System.

Whether the observed misrepresentation of economic size in the ECB Council can be reconciled with the theoretical argument is, of course, another, ultimately empirical, question. We shall revisit this issue toward the end of the paper.

### 3 The Model

#### 3.1 The Economy and the First-Best Policy

The output gap in each member country of the currency union  $i$ , defined as the percentage deviation of the actual output level from the level of natural output  $y_i^n$ , is given by a standard Lucas supply function

$$y_i = \pi - \pi^e + \theta_i; \quad \theta_i \sim (0, \sigma_{\theta_i}^2). \quad (1)$$

In what follows, we will assume that decision-makers are well aware of the limits the Lucas function puts on real activity in the long run. Inflation,  $\pi$ , is assumed to be similar across the currency union, that is,  $\pi = \pi_i = \pi_{\neq i}$ , and under the full control of the common central bank. Inflation expectations, denoted by  $\pi^e$ , are set rationally, so that  $\pi^e = E\pi$ . The last term in equation (1),  $\theta_i$ , is a country-specific economic shock with zero mean and known (positive) variance.

A reasonable assumption—one that seems to be broadly in line with the spirit of the Maastricht treaty in the example of the ECB or the policy targets pursued by the US Federal Reserve—is that the first-best policy minimizes a standard quadratic loss function based on the deviations of inflation and the aggregate output gap,  $y$ , from their commonly (currency union-wide) agreed target levels:

$$L^* = (\pi - \pi^*)^2 + \lambda y^2.$$

The term  $\pi^* > 0$  is an exogenous inflation target, say 2 percent, and  $\lambda$  is a coefficient measuring the relative weight attached to output stabilization. We assume that the latter fulfills  $0 < \lambda < \infty$ . The target level for the aggregate output gap has been set to zero, ensuring that the first-best policy does not suffer from a time inconsistency problem. The aggregate output gap is the weighted sum of the respective national output gaps, that is,  $y = \sum \chi_i y_i$ , where we can define the economic weights of each country as the expected share in aggregate union output:  $\chi_i \equiv y_i^n / \sum y_i^n$ . This allows us to express  $L^*$  as

$$L^* = (\pi - \pi^*)^2 + \lambda \left( \sum \chi_i y_i \right)^2,$$



or, in the two-country case,

$$L^* = (\pi - \pi^*)^2 + \lambda(\chi y_1 + (1 - \chi)y_2)^2, \quad (2)$$

where  $\chi$  and  $(1 - \chi)$  denote the relative economic weight of country 1 and country 2, respectively. In what follows, we will focus on the two-country case for simplicity.

The social planner sets inflation by minimizing (2), taking into account (1), yielding the benchmark reaction function

$$\pi = \frac{1}{1 + \lambda} \pi^* + \frac{\lambda}{1 + \lambda} (\pi^e - \chi \theta_1 - (1 - \chi) \theta_2).$$

Under rational expectations, equilibrium inflation in the first-best will be

$$\pi = \pi^* - \frac{\lambda}{1 + \lambda} (\chi \theta_1 + (1 - \chi) \theta_2) \equiv \pi_{FB},$$

which, using (1), implies actual output in the two member countries will be

$$\begin{aligned} y_1 &= \frac{1}{1 + \lambda} (-\lambda(1 - \chi)\theta_2 + (1 + \lambda(1 - \chi))\theta_1) \equiv y_{1FB}, \\ y_2 &= \frac{1}{1 + \lambda} (-\lambda\chi\theta_1 + (1 + \lambda\chi)\theta_2) \equiv y_{2FB}. \end{aligned}$$

Thus, the expected welfare (loss) under a first-best policy is

$$EL^*(\pi_{FB}, y_{1FB}, y_{2FB}) = \frac{\lambda(\chi^2 \sigma_{\theta_1}^2 + (1 - \chi)^2 \sigma_{\theta_2}^2 + 2\chi(1 - \chi)\varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2})}{1 + \lambda},$$

where  $\varphi_{\theta_1, \theta_2}$  marks the coefficient of correlation (and  $\varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}$  the covariance) between economic shocks in countries 1 and 2. The covariance term appears in  $EL^*$  because, while the central bank “leans against the wind” with its stabilization policy, it never fully compensates economic shocks in either country as long as the relative weight of the real term in the underlying loss function  $L^*$  is not infinitely high.

### 3.2 Actual Decision-Making

The purely union-wide perspective employed to derive the benchmark solution might not be a good description of actual decision-making in a federal central bank. While, for instance, the ECB (1999, p. 55) rightfully stresses that “members of the [Council] do not act as national representatives, but

in a fully independent personal capacity,” there is reason to assume that national economic considerations play at least some role in the voting behavior of governors in the Council.<sup>8</sup> This assumption is supported by evidence of national (or regional) influences in other federal central bank systems. Berger and de Haan (2002) show that regional differences in growth and inflation influenced voting behavior in the pre-1999 Bundesbank Council; Meade and Sheets (2002) find that Federal Reserve FOMC members do take into account developments in regional unemployment when deciding monetary policy; and Heinemann and Huefner (2004) and Meade and Sheets (2002) argue that there might even be indications of regional voting behavior in actual ECB policy.

A simple, yet plausible, description of actual decision-making within the common central bank is a weighted voting approach or a form of Nash-bargaining in which voting weights are the fall-back positions.<sup>9</sup> In this case, decisions will be based on a loss function of the form

$$L_A = \sum \alpha_i L_i, \quad (3)$$

where  $\alpha_i$  denotes the political weight of country  $i$ ’s representative or governor in the Council, with  $\sum \alpha_i = 1$ . In other words, the loss function underlying actual central bank decisions is seen as a weighted sum of the individual loss functions of the member countries,  $L_i$ , where the political weights can differ from the economic weights, that is,  $\alpha_i \not\equiv \chi_i$ .

Before moving on, note that the description of actual decision-making in the currency union’s central bank Council ignores the role of a Board. In the current ECB, the Board casts 6 out of 18 votes in the Council; in the US Federal Reserves’ FOMC the Board holds 7 out of 12 votes. Not taking into account the Board can be justified by the notion that the Board is likely to target a loss function similar to the social planner’s described in (2). In the case of the ECB, for instance, the EU Treaty specifies that the Board is appointed by “common accord of the governments of the member states at the level of Heads of State or Government, on a recommendation from the Council, after it has consulted the European Parliament and the Governing Council of the ECB” (EU 1997, Article 112 2. (b)). Arguably, this political

---

<sup>8</sup>The assumption that national interests play at least some role also is fairly wide-spread in the academic literature. See, among others, the contributions by von Hagen and Stüppel (1994), Lindner (2000), Aksoy et al. (2002), Gros and Hefeker (2002, 2003), Gersbach and Pacht (2004), and Frey (2004).

<sup>9</sup>This representation of decision-making abstracts from possible strategic interaction between Council members. For an analysis of coalition forming in the Council in light of EMU enlargement see, for instance, Baldwin et al. (2001).

process, which is highly centralized at the European level, tends to select Board members with a euro area-wide perspective. With the Board following a first best or Maastricht policy, however, analysis of possible deviations of ECB behavior from the Maastricht norm should focus (without loss of generality) on the behavior of national representatives.<sup>10</sup>

But how will national central bank governors act in the Council? As already discussed, we assume that they base their decisions on a loss function thought to measure country  $i$ 's welfare:

$$L_i = (\pi - \pi_i^*)^2 + \lambda y_i^2, \quad (4)$$

where  $\pi_i^*$  (to which we will return in a moment) is the target level for inflation.

This specification resembles that of the social planner with respect to the absence of an inflationary bias. As in (2), the loss function of country  $i$  includes a real target compatible with the level of natural output in country  $i$ , and we assume that the relative weight of the real argument in (4),  $\lambda$ , is the same as in the first-best scenario.

There is, however, a difference with respect to the inflation bliss point. The national inflation target is defined as

$$\pi_i^* = \pi^* + \varepsilon_i; \quad \varepsilon_i \sim (0, \sigma_{\varepsilon_i}^2) \quad (5)$$

that is, country  $i$ 's inflation target might deviate from the common target,  $\pi^*$ , by a preference shock  $\varepsilon_i$  with zero mean and known variance  $\sigma_{\varepsilon_i}^2$ .

Preference shocks can occur for various reasons, but the most natural explanation ties them to changing (partisan) government preferences concerning inflation. For instance, Hibbs (1977) and Alesina (1987) argue that policymakers—and thus the governments selecting the national governors in the currency union's central bank Council—have different objective functions, including (but not necessarily restricted to) the inflation target. As a consequence, shocks to the composition of government can lead to unexpected changes in national preferences concerning inflation. Alesina and Rosenthal (1995) discuss empirical evidence to support this view. A related theory, put forth by Bullard and Waller (2004), argues that changing preferences concerning inflation might reflect random changes in the political dominance of agents losing (savers, for instance) and gaining (such as borrowers and wage earners) from high inflation. This could influence the selection of national

---

<sup>10</sup>Frey (2004) takes a comparable view of the Board's perspective. He concludes that, as a consequence, in an OCA-type model, larger member countries prefer a more important role for the Board than smaller members.

central bank governors for the common Council.<sup>11</sup>

An alternative, non political-economic, approach would interpret the shocks,  $\varepsilon_i$ , as country-specific deviations from the broader trend of structural inflation (along the lines of Balassa-Samuelson).<sup>12</sup>

While targeted inflation is a plausible explanation for the discord between different national Council members, it is not, of course, the only possible channel through which national preference shocks could influence common monetary policy. In particular, there could be partisan shocks to the preferred output gap or to the relative weight of the real argument in (4). However, allowing the output target instead of the inflation target to fluctuate around zero at the national level has little impact on the analysis. The same holds, broadly speaking, for preference shocks to  $\lambda$ . We shall return to this issue further below.

To compute actual central bank policy in the two-country case, we substitute (4) into (3) to get

$$L_A = \alpha ((\pi - \pi_1^*)^2 + \lambda y_1^2) + (1 - \alpha) ((\pi - \pi_2^*)^2 + \lambda y_2^2). \quad (6)$$

Minimizing (6) with regard to inflation while taking into account (1) yields the reaction function

$$\pi = \frac{1}{1 + \lambda} (\alpha \pi_1^* + (1 - \alpha) \pi_2^*) + \frac{\lambda}{1 + \lambda} (\pi^e - \alpha \theta_1 - (1 - \alpha) \theta_2), \quad (7)$$

which, under rational expectations, implies the following actual equilibrium values for inflation and output:

$$\begin{aligned} \pi &= \pi^* + \frac{1}{1 + \lambda} [\alpha \varepsilon_1 + (1 - \alpha) \varepsilon_2 - \lambda (\alpha \theta_1 + (1 - \alpha) \theta_2)] \equiv \pi_A, \\ y_1 &= \frac{1}{1 + \lambda} [(\alpha \varepsilon_1 + (1 - \alpha) \varepsilon_2) - \lambda (1 - \alpha) \theta_2 + (1 + \lambda (1 - \alpha)) \theta_1] \equiv y_{1A}, \\ y_2 &= \frac{1}{1 + \lambda} [(\alpha \varepsilon_1 + (1 - \alpha) \varepsilon_2 - \lambda \alpha \theta_1 + (1 + \lambda \alpha) \theta_2)] \equiv y_{2A}. \end{aligned}$$

---

<sup>11</sup>A related interpretation would be to assume that preference shocks reflect exogenous changes in fiscal fortitude, where, for example, governments with unexpectedly high deficits will adjust their inflation preferences upward.

<sup>12</sup>It has been argued, for instance, that the ECB's inflation target of (less than) 2 percent does not adequately reflect the Balassa-Samuelson effect, especially for the EU accession countries still on a real convergence path to the EMU core (Berger et. al 2004). In this case equation (5) should be seen as a special case of the more general form  $\pi_i^* = \pi^* + \bar{\pi}_i + \varepsilon_i$ , with the (somewhat unrealistic) assumption that the contribution of trend structural inflation to the national inflation target,  $\bar{\pi}_i$ , will be zero across countries. Note, however, that allowing  $\bar{\pi}_i > 0$ , while introducing an additional dimension in the discussion of optimal representation, would not alter the thrust of the results of the analysis.

## 4 Optimal Representation

Substituting  $\pi_A$ ,  $y_{1A}$ , and  $y_{2A}$  in (2) and taking expectations, we can compute the expected welfare loss associated with the actual monetary policy,  $EL^*(\pi_A, y_{1A}, y_{2A})$  (see Appendix 1). The optimal representation of country 1,  $\alpha^*$ , is simply the value of  $\alpha$  that minimizes the difference between  $EL^*(\pi_A, y_{1A}, y_{2A})$  and expected welfare under the first-best policy, that is

$$\alpha^* \Leftrightarrow \arg \min [EL^*(\pi_A, y_{1A}, y_{2A}) - EL^*(\pi_{FB}, y_{1FB}, y_{2FB})].$$

Country 2's optimal weight is, equivalently,  $1 - \alpha^*$ .

Optimal representation will depend not only on the weight of the real argument in the loss function,  $\lambda$ , and the economic weight,  $\chi$ , but also on both countries' economic and preference shocks and their possible interaction terms (see Appendix 1).

In the next Section we will take a closer look at what defines optimal representation, with a focus on its relation to a country's economic weight. To facilitate the analysis, we will start with the assumption that all shocks are independent. Analysis in subsequent Sections will allow for correlated shocks across and within countries.

### 4.1 The Baseline Case with Independent Shocks

Assuming that  $\varphi_{\varepsilon_1, \varepsilon_2} = \varphi_{\theta_1, \theta_2} = \varphi_{\varepsilon_i, \theta_i} = \varphi_{\varepsilon_i, \theta_{\neq i}} = 0$  for  $i = 1, 2$ , the optimal weight for country 1 becomes

$$\alpha^* = \frac{\sigma_{\varepsilon_2}^2 + \chi \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)}{\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)}, \quad (8)$$

which obviously satisfies  $0 < \alpha^* < 1$  because  $\chi < 1$  and  $\sigma_{\varepsilon_1}^2 > 0$ .

#### 4.1.1 Over- and under-representation

Equation (8) has a straightforward implication for the relation between economic size and a country's optimal voting weight. In particular, we find that

$$\alpha^* \gtrless \chi \Leftrightarrow \chi \sigma_{\varepsilon_1}^2 \lesseqgtr (1 - \chi) \sigma_{\varepsilon_2}^2. \quad (9)$$

Broadly speaking, equation (9) states that over-representation in the Council in relation to a country's economic size is more likely to be optimal for smaller countries with relatively stable preferences. Under-representation, on the other hand, is more likely to be optimal for larger countries with rel-

atively volatile preferences. This becomes even clearer if we rewrite (9) to highlight the tension between economic size and relative preference stability:

$$\alpha^* \begin{matrix} \geq \\ \leq \end{matrix} \chi \Leftrightarrow \chi \begin{matrix} \leq \\ \geq \end{matrix} \frac{\sigma_{\varepsilon_2}^2}{\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2} \equiv \alpha_P. \quad (10)$$

Obviously, over-representation is optimal if a country's share in the currency union's GDP is lower than a critical threshold value,  $\alpha_P$ , measuring the other country's relative contribution to overall preference volatility. Vice versa, under-representation is optimal when a country is large relative to the other currency union member's contribution to preference volatility.

The threshold value  $\alpha_P$  has an interesting interpretation. Note that according to (8) and (10),  $\alpha^* \rightarrow \alpha_P$  as  $\sigma_{\theta_1}^2, \sigma_{\theta_2}^2 \rightarrow 0$ , that is,  $\alpha_P$  can be interpreted as the optimal political voting weight that results purely from trading off differences in the volatility of preferences between countries in the absence of economic shocks.

Equivalently, in the absence of preference shocks, the optimal political weight,  $\alpha^*$ , converges with a country's relative economic weight,  $\chi$ , which, according to equation (2), is the weight it should receive under the first-best scenario:  $\alpha^* \rightarrow \chi$  as  $\sigma_{\varepsilon_1}^2, \sigma_{\varepsilon_2}^2 \rightarrow 0$ .

This suggests the following observation.

**Remark 1** *In general, optimal representation balances two opposing forces: the wish to reduce the impact of preference shocks on monetary policy (by bringing  $\alpha^*$  as close as possible to  $\alpha_P$ ), and the attempt to limit misrepresentation of a country's relative economic size to avoid an overly active or passive reaction to national economic shocks (by keeping  $\alpha^*$  as closely as possible to  $\chi$ ).*

As a consequence, a country's optimal representation in the Council,  $\alpha^*$ , will always be in an interval defined by  $\chi$  on the one hand and  $\alpha_P$  on the other. Thus, whether a country will be over- or under-represented depends on the relative size of the country and the characteristics of both countries' preference shocks. Figure 1 illustrates both scenarios.

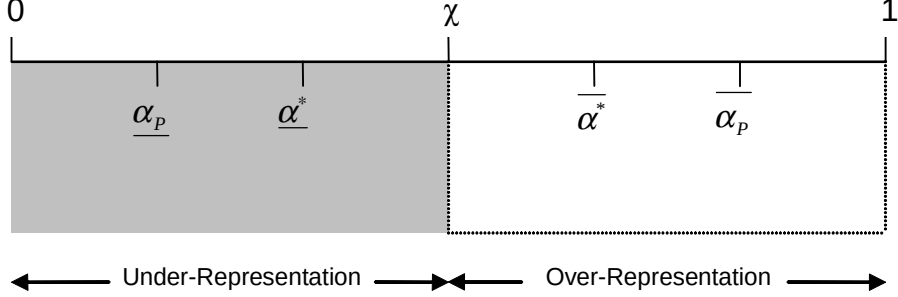


Figure 1. An Illustration of the Baseline Model.

Intuitively, we would expect small countries to be over-represented and large countries to be under-represented, but this is not necessarily the case. However, the intuitive scenario is the outcome if preferences are similar across the currency union:<sup>13</sup>

**Remark 2** *If preference shocks were sufficiently similar, over-representation would always be optimal for small countries and under-representation would always be optimal for large countries.*

On the other hand, if differences in preference shocks are stark, there is room for a counterintuitive result:

**Remark 3** *Under-representation of a small country can be optimal if its inflation preferences are relatively volatile. By the same token, over-representation of a large country can be optimal if its inflation preferences are stable in comparison. Size continues to be important, however, as these outcomes are less likely for very small or very large countries, respectively.*

**Proof.** (10) requires  $\chi < \sigma_{\varepsilon_2}^2 / (\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2)$  for  $\alpha^* > \chi$ . Thus, a large country with  $\chi > 1/2$  can only be over-represented if  $\sigma_{\varepsilon_1}^2 < \sigma_{\varepsilon_2}^2$ . By the same logic,  $\alpha^* < \chi$  requires  $\sigma_{\varepsilon_1}^2 > \sigma_{\varepsilon_2}^2$  for a small country with  $\chi < 1/2$ . The inequalities for over- and under-representation are both more likely to be fulfilled if  $|\chi - 1/2| \rightarrow 0$ . ■

Figure 2 depicts the two scenarios discussed in the Remark.

<sup>13</sup>Equation (9) reduces to  $\alpha^* \gtrless \chi \Leftrightarrow \chi \lesseqgtr (1 - \chi)$  when  $\sigma_{\varepsilon_1}^2 = \sigma_{\varepsilon_2}^2$ . Obviously, similar outcomes can be found for asymmetrical preference shocks as long as the differences in preferences remain small compared to the differences in economic size.

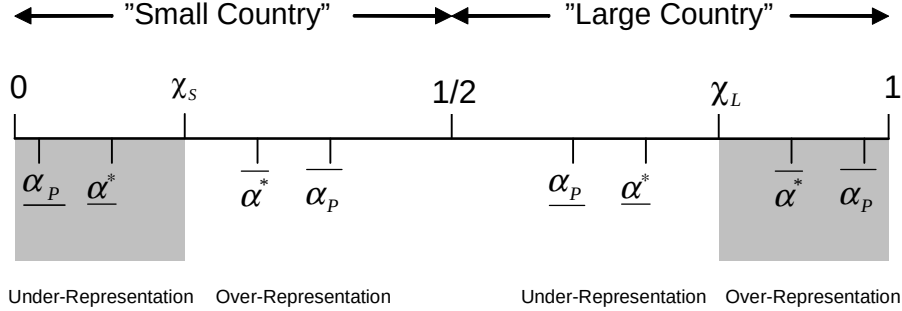


Figure 2. Under- (Over-) Represented Small (Large) Countries

The above analysis suggests that the counterintuitive case of, for instance, a large country being over-represented, is most relevant when the actual overall difference in country sizes within the union is small. In the extreme case of a monetary union of economic equals (i.e., when  $\chi = (1 - \chi) = 1/2$ ), asymmetry in representation would always be optimal if there were asymmetries in preference shocks. As Figure 1 illustrates, in such a case countries with relatively more volatile inflation preferences would receive less than  $\chi = 1/2$  (and countries with relatively less volatile inflation preferences would receive more than  $\chi = 1/2$ ) of the voting rights in the joint central bank Council.

Finally, equation (9) sheds light on the “one country, one vote” principle featured so prominently in the debate on ECB reform. It shows that:

**Remark 4** *In the absence of economic shocks and if preference shocks are symmetrical, optimal representation in the Council follows the “one country, one vote” principle no matter the distribution of economic size.*

While this Remark does not quite rule out “one country, one vote” as an optimal solution, it marks it as a rather special case. Equations (8) and (10) imply  $\alpha^* = 1 - \alpha^* = \alpha_P = 1/2$  when economic shocks are absent (i.e.  $\sigma_{\theta_1}^2, \sigma_{\theta_2}^2 = 0$ ) and when preference shocks are perfectly symmetrical (and uncorrelated)  $\sigma_{\varepsilon_1}^2 = \sigma_{\varepsilon_2}^2$ .

#### 4.1.2 Comparative Statics

How does optimal representation change with the characteristics of economic and preference shocks? As one would expect, inspection of (8) reveals a clear-cut relation between representation and preference stability.



**Remark 5** *An increase in the volatility of preference shocks unconditionally reduces the optimal weight a country receives in the Council.*

This should not come as a surprise. If optimal representation indeed balances the attempt to correctly mirror a country's relative economic size with the need to reduce the impact of national preference shocks on the Council, a country that suffers a decrease in preferences stability will see its optimal representation in the currency union's central bank being reduced. The finding is independent of the initial degree of over- or under-representation.

In contrast, the impact of a marginal increase in economic volatility depends on a country's initial status.

**Remark 6** *An increase in economic volatility in a country reduces the gap between economic weight and optimal representation. Over-represented countries will see their optimal voting weight reduced, while under-represented countries will see their optimal voting right increased.*

**Proof.** Taking the partial derivative of (8) and rearranging yields  $\partial\alpha^*/\partial\sigma_{\theta_1} \gtrless 0 \Leftrightarrow \chi\sigma_{\varepsilon_1}^2 \gtrless (1-\chi)\sigma_{\varepsilon_2}^2$ , which, by (9), implies the result. ■

For given country size, neglecting economic shocks in favor of moderating preference shocks becomes more expensive (in welfare terms) as economic volatility increases. As a consequence, a country which is burdened with a more volatile economy should see its optimal voting right increase. Figure 3 illustrates the result.

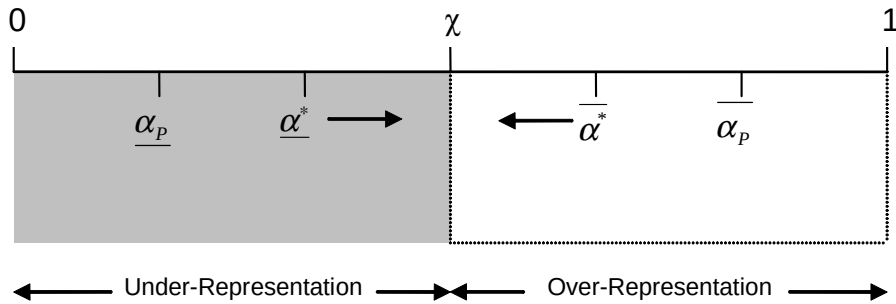


Figure 3. An Increase in Economic Volatility in Country 1.

## 4.2 Optimal Representation With Correlated Shocks

Allowing for dependencies between economic and political shocks across countries and within a country is interesting on at least on two counts. First, introducing non-zero correlated shocks allows us to shed some light on the question of how closer integration of currency union member countries influences optimal representation. There is, for instance, the question of whether closer synchronization of business cycles will change the trade-off underlying optimal representation in favor of economic size or preference stability. In addition, allowing for dependencies between shocks serves as a robustness check for the baseline results built on the assumption of zero correlations.

### 4.2.1 Correlated Economic Shocks

Economic shocks is one obvious area where there could be cross-country dependencies. As discussed above, even under the first-best scenario, the central bank would only “lean against the wind” and not fully compensate for an economic shock impacting all member countries of the currency union. This could mean that the “residual” country shocks will continue to influence all member countries in a correlated fashion. The correlation could be positive or negative. An example for a common shock with positive correlation would be an unanticipated change in oil prices. On the other hand, a surprise depreciation of the common currency, for instance, might help members that are net-exporters but hurt others that are net-importers, resulting in a negative correlation of economic shocks across countries.

The first insight from the model is that the baseline results on optimal representation hardly change when we allow for cross-country correlation of economic shocks. Allowing for  $\varphi_{\theta_1, \theta_2} \neq 0$  but otherwise following the same steps as before, we find that the optimal weight for country 1 resembles (8), except for additional (additive) terms involving the cross-country covariance of economic shocks,  $\varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}$ . In fact, the condition determining whether a country will be optimally over- or under-represented compared to its economic weight is identical to (9) in the no-correlation case. This generalizes and strengthens the baseline findings. Appendix 2 lays out the formal results in some detail.

A second finding can be summarized as follows:

**Remark 7** *As the currency union’s economies become more similar in terms of their economic shocks, countries with relatively stable preferences are likely to see their optimal voting weight increase.*

The rationale is—in line with the discussion of equation (10) earlier—that increased business cycle synchronization reduces the cost of moderating

the impact of preference shocks on monetary policy, because a possible misrepresentation of economic size is now less likely to lead to a deviation of stabilization policy from its first-best benchmark.

Two comparative-static results lead to this conclusion (see Appendix 2). First, when business cycles are positively correlated across the currency union, and when country 1's economy is less volatile than country 2's to start with, then an increase in economic volatility in country 1,  $\sigma_{\theta_1}$ , will make the two economies more similar. In this case, it becomes less costly to offset preference shocks by allowing voting rights to deviate from relative economic size. As a consequence, optimal representation requires that the country's optimal weight in monetary policy decisions should be based more on preference shock considerations, and the spread between economic and political weights in the currency union grows.<sup>14</sup> The second relevant result is that a higher coefficient of correlation between economic shocks will lead to higher optimal representation for the country initially over-represented and vice versa. Thus, once again, as economic shocks become more similar, the optimal spread between economic and political weights in the currency union increases.

This analysis suggests that currency unions should optimally pay more attention to relative preference stability considerations as their joint economy "matures" and becomes increasingly more integrated. If integration implies an increasing likeness of economic shocks, preference stability considerations should eventually dominate the calculation of members' optimal Council representation. In somewhat more formal terms: we find that  $\alpha^*$  approaches the optimal weight in the absence of economic shocks,  $\alpha_P$ , as the correlation between national economic shocks approaches positive unity and the difference between the volatility of economic shocks reduces to zero.<sup>15</sup>

#### 4.2.2 Correlated preference shocks

Next, we allow for correlation between preference shocks while assuming zero correlation between all other shocks. The idea is that surprise changes in inflation preferences might well take the form of union-wide "mood swings" that simultaneously affect all member countries and their representatives in the common central bank. Alternatively, one might speculate that preference changes are negatively correlated across countries.

While the optimal weight in this scenario once again resembles (8) in the

---

<sup>14</sup>To be precise, country 1's optimal representation in the Council increases if it was initially over-represented relative to its economic weight, and it decreases if it was initially under-represented. See Appendix 2.

<sup>15</sup>Note that  $\alpha_P$  remains unchanged from the baseline scenario (see Appendix 2).

case with independent shocks, allowing for  $\varphi_{\varepsilon_1, \varepsilon_2} \neq 0$  influences the condition determining whether a country will be over- or under-represented compared to its economic size (see Appendix 3). In particular, if preference shocks are positively correlated, it might be optimal to over-represent (or under-represent) member countries with very stable (or very unstable) preferences irrespective of their economic size. The intuition is that a positive correlation of preference shocks across countries reduces the chance that national preference shocks will neutralize each other within the Council. Increasing the voting weight of countries with very stable preference can be optimal to minimize the resulting unwanted volatility in joint monetary policy.

In addition, we can make a statement that parallels the similarity result in the case with correlated economic shocks:<sup>16</sup>

**Remark 8** *As currency union member countries become more similar in terms of their preference shocks, economically large countries are likely to see their optimal voting weight increase and small countries are likely to see their optimal voting weights decrease.*

The rationale behind this rests on the implied change in the balance of forces driving optimal representation. In this case, greater likeness of preference shocks reduces the potential gains from moderating these shocks by letting optimal voting weights deviate from economic size, thereby allowing preference shocks to compensate each other in the Council. As a consequence, large countries (which are more likely to be under-represented when shocks become more similar) should see their voting weights being increased and small (probably over-represented) countries should see them reduced.

Two comparative-static results support this conclusion (see Appendix 3). First, a rise in the correlation of preference shocks will increase a country's optimal voting weight if it is large in economic terms and its preferences are relatively stable. Second, an increase in country 1's preference volatility that brings its volatility level closer to country 2's will lead to a decrease in its optimal representation. Since, in this case, country 1 was blessed with more stable preferences at the outset, it was also over-represented before the change. As a consequence, the decrease in optimal representation brings its voting weight closer to its economic weight.

The above analysis implies that increasing likeness of preference shocks—arguably a possibility within an ever more integrated currency union such as the U.S. or the European Union—should prompt the currency union to better tailor Council voting weights to members' economic size. It is straightforward

---

<sup>16</sup>See Appendix 3. The calculations assume that the starting point for  $\alpha^*$  is not too extreme, that is, that we start from an interior solution.

to show that  $\alpha^*$  approaches  $\chi$  as the correlation between national preference shocks approaches positive unity, and the difference between the volatility of these shocks reduces to zero. Note that this possible “integration effect” runs counter to the implications of increasing likeness of economic shocks discussed in the previous Section.

### 4.2.3 Correlated Economic and Preference Shocks

If preference shocks are, at least in part, a consequence of changes in government, and if changes in government are influenced by economic conditions, preference and economic shocks might not be independent from each other. In fact, there is room for something akin to a political business cycle. For instance, one can imagine that voters elect a government that is more tolerant to inflation when economic activity is in decline, giving rise to a negative correlation between economic and preference shocks.

As with cross-country correlations, allowing preference shocks to be correlated with economic shocks within country 1 (i.e. ,  $\varphi_{\varepsilon_1, \theta_1} \neq 0$ ) changes optimal representation and the conditions for over- or under-representations compared to the baseline (see Appendix 4)—albeit not fundamentally. Interestingly, however, under certain conditions a strong political business cycle in the sense just discussed might make it optimal to decrease a country’s voting weight below its relative economic size.

**Remark 9** *A negative correlation between preference shocks and economic shocks (a “political business cycle”) amplifies country 1’s policy demands in the Council after economic shocks—making optimal under-representation more likely.*

The rationale behind this finding is that a negative correlation of output and preference shocks in country 1 increases the cost of over-representing a country in the Council. To see this, note that, according to equation (7), the preferred inflation rate of country 1 is:<sup>17</sup>

$$\pi_1 = \frac{1}{1 + \lambda} (\pi^* + \varepsilon_1) + \frac{\lambda}{1 + \lambda} (\pi^e - \theta_1) .$$

Over-representation of country 1 would mean that, for instance, a negative shock to the output gap ( $\theta_1 < 0$ ) would trigger a too expansionary monetary policy at the union level, as country 1’s preferred policy reaction ( $-\theta_1 \lambda / (1 + \lambda) > 0$ ) would receive greater influence on Council decisions than suggested by its economic weight. This policy request would be further amplified if

---

<sup>17</sup>Set  $\alpha = 1$  in equation (7) to arrive at the expression shown here.

country 1 is, in addition, subject to an inflation preference shock pointed in the opposite direction as the output shock ( $\varepsilon_1 > 0$  in this example), increasing the distance to the first-best policy.

The above analysis is reinforced by the comparative statics for optimal representation  $\alpha^*$  (see Appendix 4 for details).<sup>18</sup> As one would expect, the optimal voting weight increases if the correlation between preference and economic shocks rises in circumstances in which the country's inflation preferences are relatively stable and economic volatility is high across the currency union. Moreover, a country will see its optimal representation in the Council increase if its economic shocks become more volatile and the correlation between economic and preference shocks is sufficiently positive and large. That is, unlike in the previous scenarios, the impact of higher economic volatility does not depend on whether a country is initially over- or under-represented. As a consequence, the gap between economic weight and optimal representation might not be reduced. A related result is that a country might see its optimal voting weight increase after a marginal rise in preference volatility. This, too, is in strict contrast with all previous findings. The intuition is that the “blessings” hidden in a higher and positive correlation of preference and economic shocks depend on a certain balance between the two. For exceedingly volatile preferences, their potentially moderating impact becomes mute.

## 5 Relative-Weight Preference Shocks

The principle results of the baseline model with uncertain national inflation preferences are robust with regard to alternate sources of preference uncertainty. In particular, optimal representation continues to depend not only on relative economic size but also on the relative characteristics of economic and preference shocks. As a consequence, over-representation of large and under-representation of small countries remain a theoretical possibility.

Following Beetsma and Jensen (1998), we allow for preference shocks regarding the *weight on the relative real target*. In this case the individual loss functions of member countries become

$$L_i = (1 + \varepsilon_i)(\pi - \pi^*)^2 + (\lambda - \varepsilon_i)y_i^2, \quad (11)$$

where  $\varepsilon_i \sim (0, \sigma_{\varepsilon_i}^2)$  and  $\lambda > 0$  in line with the first-best policy. Again focusing on the two country case, actual central bank policy can be calculated based on the Lucas supply function (1) and the representation-weighted sum of

---

<sup>18</sup>All results assume an interior solution, i.e., that  $0 < \alpha^* < 1$ .

national loss functions

$$L_A = \alpha [(1 + \varepsilon_1)(\pi - \pi^*)^2 + (\lambda - \varepsilon_1)y_1^2] \\ + (1 - \alpha) [(1 + \varepsilon_2)(\pi - \pi^*)^2 + (\lambda - \varepsilon_2)y_2^2].$$

Based on the resulting equilibrium values for inflation and output, we can compute the expected welfare loss. Minimizing the latter with regard to country 1's voting weight in the Council, we find that optimal representation satisfies

$$\alpha_\lambda^* = \frac{\sigma_{\theta_2}^2 \sigma_{\varepsilon_2}^2 + \chi \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)}{\sigma_{\theta_1}^2 \sigma_{\varepsilon_1}^2 + \sigma_{\theta_2}^2 \sigma_{\varepsilon_2}^2 + \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)}, \quad (12)$$

where the subscript “ $\lambda$ ” marks the relative-weight preference shock scenario. Note that  $0 < \alpha_\lambda^* < 1$ .

Equation (12) reveals similarities and some differences with the inflation preference shock case depicted in (8). Comparing  $\alpha_\lambda^*$  and  $\alpha^*$ , one notes that the terms involving economic volatility alone are similar. As opposed to what we saw in (8), however, the volatility of preference shocks in (12) does not appear independently. Instead, all  $\sigma_{\varepsilon_i}^2$  terms are weighted by the variance of economic shocks,  $\sigma_{\theta_i}^2$ . This is due to the fact that—in a model without inflationary bias—relative-weight preference shocks only impact actual policy if there is a shock to output. In the absence of economic shocks, that is if  $y_i = 0$ , the Council will set inflation equal to the inflation target (which in this case is constant).

In general, however, optimal representation continues to depend on relative economic size as well as on the relative characteristics of economic and preference shocks, just as in the baseline model.

Reflecting the similarities in optimal representation, the condition for over- and under-representation with relative-weight uncertainty resembles the condition in the baseline case and over-representation of large and under-representation of small countries remain a theoretical possibility. The condition for over- and under-representation becomes

$$\alpha_\lambda^* \gtrless \chi \Leftrightarrow \chi \sigma_{\theta_1}^2 \sigma_{\varepsilon_1}^2 \lesseqgtr (1 - \chi) \sigma_{\theta_2}^2 \sigma_{\varepsilon_2}^2, \quad (13)$$

which, except for the weighting of preference volatility, is identical with (9) in the baseline model. As before, given economic volatility, over-representation is more likely to be optimal for small and politically relatively stable countries. If economic shocks were symmetrical, equations (9) and (13) would be identical.

Rearranging, we find that the critical value indicating the maximum coun-

try size for a country to be over-represented,  $\alpha_{P_\lambda}$ , is implied by

$$\alpha_\lambda^* \gtrless \chi \Leftrightarrow \chi \gtrless \frac{\sigma_{\theta_2}^2 \sigma_{\varepsilon_2}^2}{\sigma_{\theta_1}^2 \sigma_{\varepsilon_1}^2 + \sigma_{\theta_2}^2 \sigma_{\varepsilon_2}^2} \equiv \alpha_{P_\lambda}. \quad (14)$$

Similar to  $\alpha_P$  in equation (10) in the case with inflation preference shocks,  $\alpha_{P_\lambda}$  measures the other country's relative contribution to overall preference volatility. As before, optimal representation can be interpreted as balancing preference shock moderation (by bringing  $\alpha_\lambda^*$  closer to preference-uncertainty based optimal weight  $\alpha_{P_\lambda}$ ) and minimizing misrepresentation of economic size (by letting  $\alpha_\lambda^*$  approach the first-best economic weight  $\chi$ ). As a consequence, Remarks 1-4 in the baseline case also apply to the case of uncertain relative-weight preferences.

One difference between the baseline and the present model is the influence of economic volatility on optimal representation. While optimal representation continues to be decreasing in the volatility of preference shocks in the present model, an increase in economic volatility now unconditionally reduces a country's optimal weight in the Council.

When preference uncertainty shifts from the inflation target to the relative weight, even under-represented countries will have their optimal representation reduced. The reason is that an increase in economic volatility amplifies the unwanted impact of preference shocks on monetary policy, leading to a decrease in  $\alpha_{\lambda_P}^*$ . Because optimal representation weighs both  $\chi$  and  $\alpha_{\lambda_P}^*$ , the reduction in  $\alpha_{\lambda_P}^*$  "pulls"  $\alpha_\lambda^*$  downward. While the forces that helped create an increase in optimal representation in the baseline model are still present, the negative effect stemming from higher preference-uncertainty always dominates (see Appendix 5).

## 6 Some Empirical Observations

How does the distribution of  $\alpha_i$  and  $\chi_i$  look in the example of the ECB? Under the current "one country, one vote" rule, the relative voting rights of national governors are strictly symmetrical, that is,  $\alpha_i = 1/n$ , if we disregard the votes allocated to the Board. As Figure 4 illustrates, for a hypothetical EMU with 24 members (assuming that the ten EU accession countries as well as Romania and Bulgaria, have joined the euro area), this will change if the ECB reform is implemented and EMU enlargement proceeds, giving way to a (somewhat) more asymmetric voting scheme, in which  $\alpha_i$  will be



adjusted to better reflect economic size.<sup>19</sup>

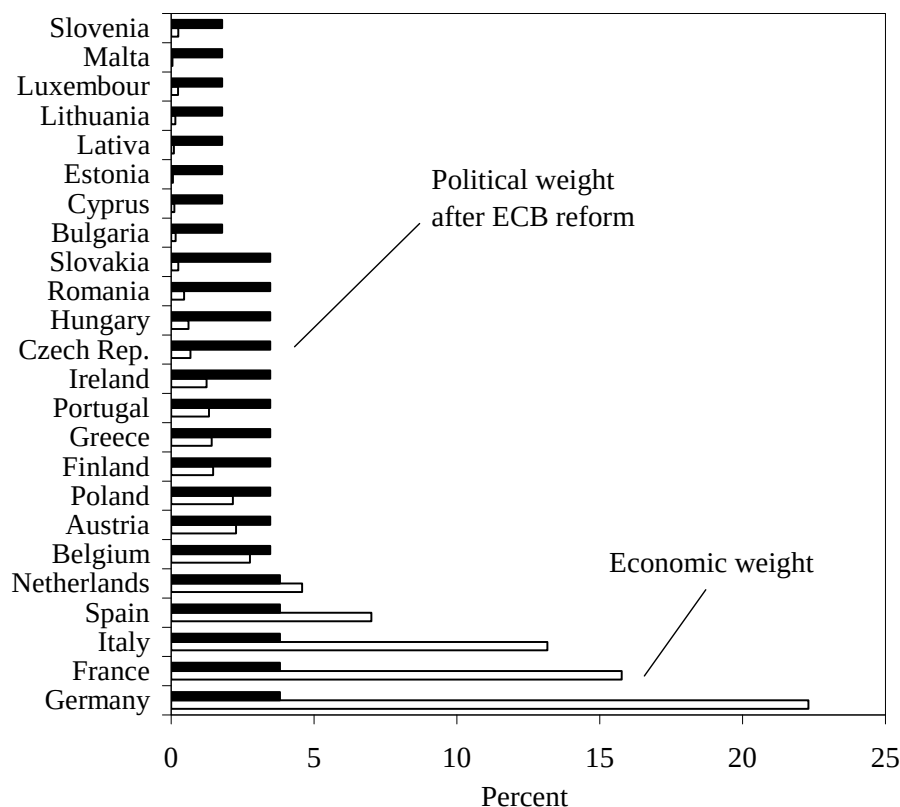


Figure 4. Political and Economic Weights in the EMU24 Council

To provide some perspective, it is interesting to relate the degree of misrepresentation in today's ECB, as well as in the hypothetical ECB with 24 members depicted in Figure 4, to the example of other federal central banks—namely the US Federal Reserve System and the German Bundesbank before the advent of the euro. Table 1 shows two indicators of misrepresentation based on the sum of the squared deviation of the voting rights held by a country or region in the decision-making Council or committee and its GDP share.

Strikingly, the misrepresentation of economic size in the ECB Council is about an order of magnitude more severe than in the Fed or, after the post-unification reform of 1992, the Bundesbank. This is true if the misrepresentation indicator ignores the role of the Board, if we look at an EMU

<sup>19</sup>Political voting rights assume that the Board does not vote in line with country interests. For sources and notes see Tables 1 and 2 below.

Table 1. Indicators of Misrepresentation in Federal Central Banks, 1959-2001

|           | US Federal Reserve |      | German Bundesbank |      | ECB before & after Enlargement |                  |                   |
|-----------|--------------------|------|-------------------|------|--------------------------------|------------------|-------------------|
|           | 1977               | 2001 | 1959              | 1992 | 2001                           | EMU24 w/o Reform | EMU24 with Reform |
| STD       | 4.46               | 5.54 | 8.48              | 5.70 | 11.26                          | 12.47            | 11.34             |
| STD_Board | 0.41               | 0.66 | 2.65              | 0.91 | 4.60                           | 7.88             | 5.99              |

Sources: Bureau of Economic Analysis; US Federal Reserve; Statistisches Bundesamt; IFS; and own calculations.

Notes: “STD” measures the sum of the squared difference between national or regional vote shares in the overall decision-making committee and the relevant GDP or GSP share of the given year. It is assumed that the Board does not vote in the interest of the respective country. “STD\_Board” assumes that the Board votes with each region or country in line with its GDP or GSP share. This increases (decreases) the political clout of large (small) member countries.<sup>a</sup> The data for the US Feds has been calculated by allocating state GSP data to Fed districts on a county-by-county basis. Rotation schemes have been taken into account in the calculation of Fed and ECB post-reform voting rights. In case of the Fed, with no major structural reform of representation in the post-war period, the years selected mark the time-span of the available data. In the case of the Bundesbank, 1959 and 1992 mark years with major structural reforms that helped to reduce misrepresentation.

<sup>a</sup>Berger (2002, Appendix I) discusses the effect of EMU enlargement on misrepresentation indicators under different assumptions on Board behavior.

with 12 or 24 members, or if we look at the situation before or after the planned ECB reform. In other words, economic size plays a significantly smaller role in the distribution of voting rights within the ECB than in other federal central banks.

Can the comparatively stark pattern of misrepresentation of size in the ECB be explained by some of the determinants of optimal representation identified by the theoretical model? To answer this question, we need to find proxies for the latter—certainly not an easy feat. While it is straightforward to measure relative size (by GDP share, for instance), indicators of preference stability are less easily identified. One plausible assumption is that frequent changes of government are indicative of (or a prerequisite for) changing preferences. Moreover, such changes might be more significant when governments show a high degree of cohesion. Table 2 gives information on the degree of government cohesion, a measure of government change, and relative economic size for the existing 12 members of EMU. Table 3 does the same for the hypothetical EMU with 24 members.

Given the approximate nature of the indicators and, in the case of Table 3, the short time-span covered, any interpretation of the data should be taken with a grain of salt. Nevertheless, a cautious interpretation of Tables 2 and 3 suggests that not all over- and under-representation of national representatives on the ECB Council could be easily justified. In only 5 out of 12 countries in Table 2, and in only 10 out of 24 countries in Table 3, do GDP share, change, and cohesion point in the same direction. Moreover, it would seem that actual over- or under-representation of very large and very small countries is least in line with the suggestions of the model.

## 7 Concluding Remarks

The question of optimal representation of regional interests within a federal central bank has received much attention recently. The likely extension of the euro area has highlighted problems—such as a possible mismatch between relative economic size and voting rights in the decision-making committee—with the “one country, one vote principle” in a currency union such as EMU.

The present paper adds central bank independence as a potentially crucial argument to this discussion. We present a simple model of optimal representation in a federal central bank in which optimal voting weights reflect two opposing forces: the wish to insulate common monetary policy from changing preferences at the national level, and the attempt to avoid an overly active or passive reaction to idiosyncratic national economic shocks. Adjusting representation in the decision-making committee to moderate preference shocks

Table 2. EMU12: Indicators of Political Stability, GDP Share, and ECB Voting Rights, 1975-2000

|             | (1)         | (1) < Avg. | (2)         | (1) < Avg. | (3)         | (3) < Avg. | Consensus?<br>(1) = (2) = (3) | Political Weight<br>in excess of (3) |
|-------------|-------------|------------|-------------|------------|-------------|------------|-------------------------------|--------------------------------------|
|             | Cohesion    |            | Change      |            | GDP Share   |            |                               |                                      |
| Germany     | 1.57        | -          | 0.07        | +          | 30.33       | -          |                               | -22.00                               |
| France      | 1.61        | -          | 0.22        | -          | 21.44       | -          | -                             | -13.11                               |
| Italy       | 1.87        | -          | 0.30        | -          | 17.90       | -          | -                             | -9.57                                |
| Spain       | 1.26        | +          | 0.12        | +          | 9.52        | -          |                               | -1.19                                |
| Netherlands | 1.30        | +          | 0.09        | +          | 6.24        | +          | +                             | 2.10                                 |
| Belgium     | 2.26        | -          | 0.15        | +          | 3.75        | +          |                               | 4.58                                 |
| Austria     | 0.61        | +          | 0.06        | +          | 3.09        | +          | +                             | 5.24                                 |
| Finland     | 1.96        | -          | 0.20        | -          | 2.00        | +          |                               | 6.34                                 |
| Greece      | 0.61        | +          | 0.19        | -          | 1.93        | +          |                               | 6.41                                 |
| Portugal    | 0.87        | +          | 0.22        | -          | 1.80        | +          |                               | 6.54                                 |
| Ireland     | 1.26        | +          | 0.18        | -          | 1.69        | +          |                               | 6.65                                 |
| Luxembourg  | 1.00        | +          | 0.06        | +          | 0.32        | +          | +                             | 8.01                                 |
| <i>Avg.</i> | <i>1.35</i> |            | <i>0.16</i> |            | <i>8.33</i> |            |                               |                                      |
| <i>Std.</i> | <i>0.53</i> |            | <i>0.08</i> |            | <i>9.69</i> |            |                               |                                      |

Sources: IFS; Worldbank Database of Political Institutions based on Beck et al. (2001); Sturm et al. (2004); and own calculations.

Notes: Political data are 1975-1997/2000 averages; GDP data as of 2001. "Cohesion" (IPCOH in Worldbank parlance, available until 1997) measures the degree of political cohesion of the national government for a given year, with 0 indicating, for instance, a one-party majority parliamentary government, and 3 a minority government. The measure is based on (but not identical to) the Roubini and Sachs (1989) index of cohesion. "Change" (STABS in Worldbank parlance, available until 2000) indicates the number of veto players dropping from government within a year, meaning political parties within a parliamentary system and the president, or the largest party, in a presidential system. The index varies between 0 and 1. A high indicator value indicates less government stability. "GDP Share" measures the respective country's share in aggregate EMU12 or EMU24 GDP based on 2001 data (and annual average market exchange rates where applicable). "Political Weight in Excess of (3)" shows the difference between a country's voting power within the ECB Council (excluding the Board) and its GDP share. A positive sign indicates over-representation, a negative sign under-representation.

Table 3. EMU24: Indicators of Political Stability, GDP Share, and ECB Voting Rights (*post* reform), 1991-2000

|             | (1)<br>Cohesion | (1) < Avg. | (2)<br>Change | (1) < Avg. | (3)<br>GDP Share | (3) < Avg. | Consensus?<br>(1) = (2) = (3) | Political Weight<br>in Excess of (3) |
|-------------|-----------------|------------|---------------|------------|------------------|------------|-------------------------------|--------------------------------------|
| Germany     | 2.00            | —          | 0.12          | +          | 28.40            | —          | —                             | -23.07                               |
| France      | 1.57            | —          | 0.23          | —          | 20.08            | —          | —                             | -14.74                               |
| Italy       | 1.57            | —          | 0.41          | —          | 16.76            | —          | —                             | -11.43                               |
| Spain       | 1.57            | —          | 0.07          | +          | 8.92             | —          | —                             | -3.59                                |
| Netherlands | 1.43            | —          | 0.07          | +          | 5.84             | —          | —                             | -0.51                                |
| Belgium     | 2.00            | —          | 0.12          | +          | 3.51             | +          | —                             | 1.33                                 |
| Austria     | 1.00            | +          | 0.05          | +          | 2.89             | +          | +                             | 1.96                                 |
| Poland      | 1.00            | +          | 0.07          | +          | 2.75             | +          | +                             | 2.10                                 |
| Slovenia    | 1.67            | —          | 0.11          | +          | 0.32             | +          | —                             | 2.18                                 |
| Luxembourg  | 1.00            | +          | 0.05          | +          | 0.30             | +          | +                             | 2.20                                 |
| Bulgaria    | 0.57            | +          | 0.30          | —          | 0.19             | +          | —                             | 2.31                                 |
| Lithuania   | 0.17            | +          | 0.10          | +          | 0.18             | +          | +                             | 2.32                                 |
| Cyprus      | 1.00            | +          | 0.05          | +          | 0.14             | +          | +                             | 2.36                                 |
| Latvia      | 1.67            | —          | 0.33          | —          | 0.12             | +          | —                             | 2.38                                 |
| Estonia     | 1.00            | +          | 0.10          | +          | 0.08             | +          | +                             | 2.42                                 |
| Malta       | 0.00            | +          | 0.15          | +          | 0.06             | +          | +                             | 2.44                                 |
| Finland     | 2.00            | —          | 0.15          | +          | 1.87             | +          | —                             | 2.98                                 |
| Greece      | 0.43            | +          | 0.25          | —          | 1.80             | +          | —                             | 3.04                                 |
| Portugal    | 0.86            | +          | 0.10          | +          | 1.68             | +          | +                             | 3.17                                 |
| Ireland     | 1.86            | —          | 0.18          | —          | 1.58             | +          | —                             | 3.27                                 |
| Czech Rep.  | 1.57            | —          | 0.30          | —          | 0.85             | +          | —                             | 4.00                                 |
| Hungary     | 1.57            | —          | 0.33          | —          | 0.77             | +          | —                             | 4.08                                 |
| Romania     | 1.57            | —          | 0.23          | —          | 0.58             | +          | —                             | 4.27                                 |
| Slovakia    | 1.60            | —          | 0.17          | +          | 0.31             | +          | —                             | 4.53                                 |
| <i>Avg.</i> | <i>1.28</i>     |            | <i>0.22</i>   |            | <i>4.17</i>      |            |                               |                                      |
| <i>Std.</i> | <i>0.57</i>     |            | <i>0.16</i>   |            | <i>7.31</i>      |            |                               |                                      |

Sources: IFS; Worldbank Database of Political Institutions based on Beck et al. (2001); Sturm et al. (2004); and own calculations.

Notes: “Change” based on 1991-2000 averages, “Cohesion” 1991-1997; GDP share as of 2001. See notes to Table 1 for a detailed explanation of the variables.

insulates joint monetary policy from unwanted volatility when national or regional policy targets deviate from common goals. Basing representation on economic weight, on the other hand, helps to prevent national or regional economic shocks from undermining the common goal. Optimal representation weighs both arguments, reflecting economic size as well as the stochastic properties of economic and preference shocks.

An important theoretical result is that a perfect match between economic size and voting rights is rarely optimal, and neither is the “once country, one vote principle”. Consequently, whether a country should be over- or under-represented compared to its relative economic size depends on a number of different forces, including relative size, the relative weight of the real target, and the stochastic properties of economic and preference shocks. Some might intuitively expect small countries to be over-represented and large countries to be under-represented. But there is room for a counter-intuitive result: for instance, it might be optimal to over-represent a large country if its policy preferences are very stable relative to other union members.

Taking a closer look at the possible interaction of economic and preference shocks, we find that continued integration in the form of better business cycle synchronization and more similar preferences can have opposing effects on optimal representation. Increasing likeness of preference shocks gives an incentive to tailor committee voting weights closer to economic size. Increasing likeness of economic shocks has the opposite effect: relative preference stability considerations gain in importance as economic shocks become more similar. Finally, allowing for political business cycle in the sense of a positive correlation between preference shocks and economic shocks within a country might help to moderate country 1’s policy demands in the Council after economic shocks.

The basic results of optimal over- or under-representation are fairly robust with regard to alternative assumptions on shock correlations. Moreover, the principle findings seem to be independent of the source of preferences uncertainty. The baseline model assumes uncertain inflation preferences, but optimal representation continues to follow similar determinants when preference shocks are tied, instead, to the relative weight of the real argument in the national welfare functions. As a consequence, for instance, over-representation of large and under-representation of small countries remains a possibility.

Empirically, there are indications that misrepresentation of member countries in the ECB Council might be extreme and not always optimal. A comparison of deviations of actual representation from relative economic size in the ECB Council with the US Federal Reserve’s FOMC and the pre-euro Bundesbank Council reveals that misrepresentation of economic size in the ECB is about an order of magnitude more severe. The theoretical model

suggest two possible explanations. First, preference homogeneity within the German and the US currency areas might be higher than in today's euro area or, somewhat less likely, business cycle synchronization could be better within the euro area. In both cases the model implies that optimal representation within the ECB Council (relative to the two other federal central banks) should focus more on preferences than on economic size. Alternatively, of course, representation within the ECB Council might not be optimal in the first place. Indeed, even though the empirical proxies used need to be treated with caution, the ECB pattern of misrepresentation of economic size is difficult to explain with theoretically identified determinants of optimal representation alone. This suggests further room for discussion, even after the ECB reform.

## References

- [1] Aksoy, Yunus, Paul de Grauwe, and Hans Dewachter (2002), Do Asymmetries Matter for European Monetary Policy?, *European Economic Review*, 46(3), 443-69.
- [2] Alesina, Alberto (1987), Macroeconomic Policy in a Two-Party System as a Repeated Game, *Quarterly Journal of Economics*, 102, 651-78.
- [3] Alesina, Alberto and Howard Rosenthal (1995), *Partisan Politics, Divided Government, and the Economy*, Cambridge University Press: Cambridge,
- [4] Baldwin, Richard (2001), The ECB's Number Problems, *Financial Times*, London, December 4.
- [5] Baldwin, Richar, Erik Berglöf, Francesco Giavazzi, and Mika Widgren (2001), Preparing the ECB for Enlargement, *CEPR Policy Paper*, 6, CEPR: London.
- [6] Beck, Thorsten, George Clarke, Alberto Groff, Philip Keefer, and Patrick Walsh (2001), New Tools in Comparative Political Economy: The Database of Political Institutions, *World Bank Economic Review*, 15(1), 165-76.
- [7] Beetsma, Roel and Henrik Jensen (1998), Inflation Targets and Contracts with Uncertain Central Bank Preferences, *Journal of Money, Credit, and Banking*, 30(3), 384-403.
- [8] Benigno, Pierpaolo (2004), Optimal Monetary Policy in a Currency Area, *Journal of International Economics*, 63(2), 293-320.
- [9] Berger, H., 2002, The ECB and Euro-Area Enlargement, *IMF Working Paper*, 02/175.
- [10] Berger, H. and J. de Haan (1999), A State Within a State? An Event Study on the Bundesbank, *Scottish Journal of Political Economy*, 46(1), 17-39.
- [11] Berger, H. and J. de Haan (2002), Are Small Countries too Powerful Within the ECB?, *Atlantic Economic Journal*, 30 (3), 1-20.
- [12] Berger, Helge, Jakob de Haan, and Robert Inklaar (2004), Is the ECB Too Decentralized?, in: Hans-Werner Sinn, Mika Widgren, and Marko



- Köthenburger (eds.), *European Monetary Integration*, Cambridge: The MIT Press, 71-97.
- [13] Bindseil, U. (2001), A Coalition-Form Analysis of the “One Country-One Vote” Rule in the Governing Council of the European Central Bank”, *International Economic Journal*, 15(1), 141-64.
  - [14] Blinder, Alan and John Morgan (2002), Are Two Heads Better Than One? Monetary Policy by Committee, *Journal of Money, Credit and Banking*, forthcoming.
  - [15] Bullard, James and Christopher Waller (2004), Central Bank Design in General Equilibrium, *Journal of Money, Credit, and Banking*, 36(1), 95-113.
  - [16] Bundesbank (1992), *Monthly Report*, August, Deutsche Bundesbank: Frankfurt.
  - [17] Casella, Alessandra (1992), Participation in a Currency Union, *American Economic Review*, 82(4), 847-63.
  - [18] Ciccarelli, Matteo and Alessandro Rebucci (2002), The Transmission Mechanism of European Monetary Policy: Is There Heterogeneity? Is it Changing over Time?, *IMF Working Paper*, 02/54.
  - [19] Eichengreen, Barry (1992), Designing a Central Bank for Europe: A Cautionary Tale from the Early Years of the Federal Reserve System, in: M. Canzoneri, V. Grilli, and R. Masson (eds.), *Establishing a Central Bank in Europe and Lessons from the US*, Cambridge: Cambridge University Press, 13-40.
  - [20] European Union (1997), Consolidated Version of the Treaty Establishing the European Community, *Official Journal of the European Communities*, C 340, November 10.
  - [21] European Parliament (2003), *Debates of the European Parliament*, Sitzings of 11 March 2003, ECB Statute, available on the internet: <http://www.europarl.eu.int>.
  - [22] European Central Bank (1999), The Institutional Framework of the European System of Central Banks, *Monthly Bulletin*, July, 55-63.
  - [23] Fitoussi, Jean-Paul and Jerome Creel (2002), *How To Reform the European Central Bank*, CER: London.

- [24] Frey, Rainer (2004), Welfare Implications of the Design of a Currency Union in Case of Member Countries of Different Sizes and Output Persistence, *Deutsche Bundesbank Discussion Paper*, Series 1, 15/2004 (forthcoming in: *Kredit und Kapital*).
- [25] Gerlach-Kristen, Petra (2002), Monetary Policy Committees and Interest-Rate Setting, *Mimeo*, University of Basel.
- [26] Gerling, Kerstin, Hans Peter Grüner, Alexandra Kiel, and Elisabeth Schulte (2003), Information Acquisition and Decision Making in Committees: A Survey, *ECB Working Paper*, 256.
- [27] Gersbach, Hans and Volker Hahn (2001), Voting Transparency and Conflicting Interests in Central-Bank Councils, *Mimeo*, University of Heidelberg.
- [28] Gersbach, Hans and Bernhard Pacht (2004), Flexible Rules for Central Banks, *Mimeo*, University of Heidelberg.
- [29] Goodfriend, Marvin (2000), The Role of a Regional Bank in a System of Central Banks, *Federal Reserve Bank of Richmond Economic Quarterly*, Winter, 7-25.
- [30] Grauwe, Paul de (2003), The Monetary Union in an Enlarged Europe, *CESifo Economic Studies*, 49(1), 103-21.
- [31] Greenspan, Alan (2000), Productivity and Efficiency in the Federal Reserve System, *Speech* delivered at the dedication of the new Birmingham Branch Building of the Federal Reserve Bank of Atlanta, December 8. Available via the internet: [www.federalreserve.gov](http://www.federalreserve.gov).
- [32] Gros, Daniel [Chairman] and others (2002), *The Euro at 25*, Special Report of the CEPS Macroeconomic Policy Group, CEPS: Brussels.
- [33] Gros, Daniel and Carsten Hefeker (2002), One Size Must Fit All: National Divergences in a Monetary Union, *German Economic Review*, 3 (3), 247-62.
- [34] Haan, Jakob de, Helge Berger, and Robert Inklaar (2004), Restructuring the ECB, in: Helge Berger and Thomas Moutos (eds.), *Managing EU Enlargement*, Cambridge: The MIT Press, 29-66.
- [35] Hagen, Jürgen von and J. Süppel (1994), Central Bank Constitutions for Federal Monetary Unions, *European Economic Review*, 38, 774-82.

- [36] Hefeker, Carsten (2002), Monetary Policy in a Union of 27: Enlargement and Reform Options, *Intereconomics*, November/December, 315-20.
- [37] Hefeker, Carsten (2003), Federal Monetary Policy, *Scandinavian Journal of Economics*, 105, 641-59.
- [38] Heinemann, Friedrich and Felix Huefner (2004), Is the View from the Eurotower Purely European? National Divergence and ECB Interest Rate Policy, *Scottish Journal of Political Economy*, 51(4), 544-58.
- [39] Heisenberg, Dorthée (2003), Cutting the bank Down to Size: Efficient and Legitimate Decision-Making in the European Central Bank After Enlargement, *Journal of Common Market Studies*, 41(3), 397-420.
- [40] Hibbs, Douglas (1977), Political Parties and Macroeconomic Policy, *American Political Science Review*, 71, 1467-87.
- [41] Lindner, Axel (2000), Long-Term Appointment of Central Bankers: Cost and Benefits, *European Journal of Political Economy*, 16, 639-54.
- [42] Lohmann, Susanne (1997), Partisan Control of the Money Supply and Decentralized Appointment Powers, *European Journal of Political Economy*, 13, 225-46.
- [43] Lohmann, Susanne (1998), Federalism and Central Bank Independence: The Politics of German Monetary Policy, 1957-1992, *World Politics*, 50, 401-46.
- [44] Maier, Philipp, Beata Bierut, and Robert-Paul Berben (2003), The Role of Regional Information in the Optimal composition of a Committee, *MEB Discussion Paper* (De Nederlandsche Bank), 2003-08.
- [45] Meade, Ellen (2003), A (Critical) Appraisal of the ECB's Voting Reform, *Intereconomics*, forthcoming.
- [46] Meade, Ellen and D.N. Sheets (2002), Regional Influences on U.S. Monetary Policy: Some Implications for Europe, *Centre for Economic Performance Discussion Paper*, No. 523.
- [47] Waller, C. and Carl Walsh (1996), Central Bank Independence, Economic Behavior, and Optimal Term Length, *American Economic Review*, 85(5), 1139-54.

## Appendices

### Appendix 1 (Expected welfare under actual policy)

Expected welfare under the actual policy is:

$$\begin{aligned}
EL^*(\pi_A, y_{1A}, y_{2A}) &= \frac{1}{1+\lambda} \alpha^2 \sigma_{\varepsilon_1}^2 + \frac{1}{1+\lambda} (1-\alpha)^2 \sigma_{\varepsilon_2}^2 \\
&+ \left( \frac{\lambda^2}{1+\lambda} \alpha(\alpha - 2\chi) + \lambda\chi^2 \right) \sigma_{\theta_1}^2 + \left( \frac{\lambda^2}{1+\lambda} (1-\alpha)((1-\alpha) - 2(1-\chi)) + \lambda(1-\chi)^2 \right) \sigma_{\theta_2}^2 \\
&\quad + \left( \frac{1}{1+\lambda} 2\alpha(1-\alpha) \right) \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2} \\
&\quad + \left( 2 \frac{\lambda}{1+\lambda} \alpha(\chi - \alpha) \right) \varphi_{\varepsilon_1, \theta_1} \sigma_{\varepsilon_1} \sigma_{\theta_1} \\
&\quad + \left( 2 \frac{\lambda}{1+\lambda} \alpha((1-\chi) - (1-\alpha)) \right) \varphi_{\varepsilon_1, \theta_2} \sigma_{\varepsilon_1} \sigma_{\theta_2} \\
&\quad + \left( 2 \frac{\lambda}{1+\lambda} (1-\alpha)(\chi - \alpha) \right) \varphi_{\varepsilon_2, \theta_1} \sigma_{\varepsilon_2} \sigma_{\theta_1} \\
&\quad + \left( 2 \frac{\lambda}{1+\lambda} (1-\alpha)((1-\chi) - (1-\alpha)) \right) \varphi_{\varepsilon_2, \theta_2} \sigma_{\varepsilon_2} \sigma_{\theta_2} \\
&+ \left( 2 \frac{\lambda^2}{1+\lambda} (\alpha(1-\alpha) - \alpha(1-\chi) - \chi(1-\alpha)) + 2\lambda\chi(1-\chi) \right) \varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}
\end{aligned}$$

The optimal representation of country one is:

$$\begin{aligned}
&\sigma_{\varepsilon_2}^2 + \chi\lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2} - 2\chi\lambda^2 \varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2} \\
&\quad - \lambda(\chi \varphi_{\varepsilon_1, \theta_1} \sigma_{\varepsilon_1} \sigma_{\theta_1} + (1+\chi) \varphi_{\varepsilon_2, \theta_2} \sigma_{\varepsilon_2} \sigma_{\theta_2}) \\
&\quad + \lambda(\chi \varphi_{\varepsilon_1, \theta_2} \sigma_{\varepsilon_1} \sigma_{\theta_2} + (1+\chi) \varphi_{\varepsilon_2, \theta_1} \sigma_{\varepsilon_2} \sigma_{\theta_1}) \\
\alpha^* &= \frac{\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - 2(\varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2} \\
&\quad + \lambda^2 \varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}) - 2\lambda(\varphi_{\varepsilon_1, \theta_1} \sigma_{\varepsilon_1} \sigma_{\theta_1} + \varphi_{\varepsilon_2, \theta_2} \sigma_{\varepsilon_2} \sigma_{\theta_2})}{+ 2\lambda(\varphi_{\varepsilon_1, \theta_2} \sigma_{\varepsilon_1} \sigma_{\theta_2} + \varphi_{\varepsilon_2, \theta_1} \sigma_{\varepsilon_2} \sigma_{\theta_1})}.
\end{aligned}$$

### Appendix 2: The case with correlated economic shocks ( $\varphi_{\theta_1, \theta_2} \neq 0$ )

The optimal weight for country 1 is

$$\alpha^* = \frac{\sigma_{\varepsilon_2}^2 + \chi\lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - 2\chi\lambda^2 \varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}}{\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - 2\lambda^2 \varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}}, \quad (15)$$

which is always within the permissible range  $0 < \alpha^* < 1$  (see below).

### Conditions for $0 < \alpha^* < 1$

Since  $\sigma_{\varepsilon_2}^2, \sigma_{\varepsilon_1}^2 > 0$ , a sufficient condition for  $\alpha^* > 0$  is that

$$\varphi_{\theta_1, \theta_2} \leq (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) / 2\sigma_{\theta_1}\sigma_{\theta_2}.$$

Because, by definition,  $\varphi_{\theta_1, \theta_2} \leq 1$ , this condition always holds if  $(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) / 2\chi\sigma_{\theta_1}\sigma_{\theta_2} \geq 1$  or  $(\sigma_{\theta_1} - \sigma_{\theta_2})^2 \geq 0$ —which is always fulfilled. The condition for  $\alpha^* < 1$  is

$$\sigma_{\varepsilon_1}^2 > (\chi - 1) \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2 - 2\varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}).$$

Because  $\sigma_{\varepsilon_1}^2 > 0$  and  $\chi < 1$ , a sufficient condition for the inequality to hold is that the last bracket on the RHS be positive or zero, that is,  $\varphi_{\theta_1, \theta_2} \leq (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) / 2\sigma_{\theta_1}\sigma_{\theta_2}$ . This is always fulfilled.

### Conditions for $\alpha^* \gtrless \chi$

As in the baseline model, it holds that

$$\alpha^* \gtrless \chi \Leftrightarrow \chi \sigma_{\varepsilon_1}^2 \lesseqgtr (1 - \chi) \sigma_{\varepsilon_2}^2.$$

**Proof.** Going through the same movements as before, we get

$$\alpha^* \gtrless \chi \Leftrightarrow \chi \sigma_{\varepsilon_1}^2 \lesseqgtr (1 - \chi) \sigma_{\varepsilon_2}^2 \text{ when } \varphi_{\theta_1, \theta_2} < \frac{\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)}{2\lambda^2 \sigma_{\theta_1} \sigma_{\theta_2}},$$

where the last inequality is always fulfilled. To see this, reformulate as  $1 < (\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)) / 2\lambda^2 \varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}$  or, equivalently,  $\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2 - 2\varphi_{\theta_1, \theta_2} \sigma_{\theta_1} \sigma_{\theta_2}) > 0$ . As  $\sigma_{\varepsilon_1}^2, \sigma_{\varepsilon_2}^2 > 0$ , this must be true if  $\varphi_{\theta_1, \theta_2} \leq (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) / 2\sigma_{\theta_1}\sigma_{\theta_2}$ . This inequality always holds as  $\varphi_{\theta_1, \theta_2} \leq 1$  and  $(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) / 2\sigma_{\theta_1}\sigma_{\theta_2} \geq 1$  because  $(\sigma_{\theta_1} - \sigma_{\theta_2})^2 \geq 0$ . ■

### Comparative statics

Taking the derivative of (15) with regard to **economic volatility**, rearranging, and consulting equation (9) we find that

$$(a) \quad \frac{\partial \alpha^*}{\partial \sigma_{\theta_1}} \gtrless 0 \Leftrightarrow \alpha^* \lesseqgtr \chi \text{ when } \varphi_{\theta_1, \theta_2} < \frac{\sigma_{\theta_1}}{\sigma_{\theta_2}}, \quad (16)$$

$$(b) \quad \frac{\partial \alpha^*}{\partial \sigma_{\theta_1}} \lesseqgtr 0 \Leftrightarrow \alpha^* \lesseqgtr \chi \text{ when } \varphi_{\theta_1, \theta_2} > \frac{\sigma_{\theta_1}}{\sigma_{\theta_2}}. \quad (17)$$

Also note that  $\partial\alpha^*/\partial\sigma_{\theta_1} = 0$  when economic shocks are symmetrical in the sense that  $\sigma_{\theta_1} = \sigma_{\theta_2}$ .

In *scenario (a)*, business cycles are unsynchronized across countries, and a further increase in economic volatility in country 1 is likely to drive the economies even more apart. To see this, note that  $\varphi_{\theta_1, \theta_2} < \sigma_{\theta_1}/\sigma_{\theta_2}$  implies either a negative correlation between economic shocks or, when the correlation is positive, that country 1's economy is more (or at least not significantly less) volatile than country 2's. In this case, equation (16) demands that the weight of country 1 in the Council should be increasing if it was initially under-represented relative to its economic weight; and it should be decreasing if it was initially over-represented. This helps to reduce the spread between economic and political weights in the currency union.

The alternative *scenario (b)* depicts a currency union with positively correlated business cycles in which country 1's economy is less volatile than country 2's—thus, somewhat counterintuitively, the economies actually become more similar as  $\sigma_{\theta_1}$  increases.<sup>20</sup> In this case, it becomes less costly to offset inflation preference shocks by allowing voting rights to deviate from the proportional representation of economic size. As a consequence, equation (17) requires that the country's optimal weight in monetary policy decisions be based more on preference shock considerations. To be precise, country 1's optimal representation in the Council increases if it was initially over-represented relative to its economic weight; and it decreases if it was initially under-represented. That is, the spread between economic and political weights in the currency union grows.

Taking the derivative of (15) with regard to the **coefficient of correlation**, rearranging, and consulting equation (9) yields:

$$\frac{\partial\alpha^*}{\partial\varphi_{\theta_1, \theta_2}} \begin{matrix} \geq \\ \leq \end{matrix} 0 \Leftrightarrow \alpha^* \begin{matrix} \geq \\ \leq \end{matrix} \chi.$$

Finally, it is straightforward to show that higher **preference volatility** results in a reduction in optimal representation as in the no-correlation case.

---

<sup>20</sup>Note that the identifying inequality for scenario (b),  $\varphi_{\theta_1, \theta_2} > \sigma_{\theta_1}/\sigma_{\theta_2}$ , requires  $\varphi_{\theta_1, \theta_2} > 0$  and  $\sigma_{\theta_1} < \sigma_{\theta_2}$  since  $\varphi_{\theta_1, \theta_2} < 1$ .

### Appendix 3: The case with correlated preference shocks ( $\varphi_{\varepsilon_1, \varepsilon_2} \neq 0$ )

In this case the optimal weight for country 1 is

$$\alpha^* = \frac{\sigma_{\varepsilon_2}^2 + \chi \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2}}{\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - 2\varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2}}. \quad (18)$$

**Conditions for  $0 < \alpha^* < 1$**

The ratio (18) has a positive denominator.<sup>21</sup> But the nominator might be either positive or negative depending on the coefficient of correlation between preference shocks ( $\varphi_{\varepsilon_1, \varepsilon_2}$ ), and the relative size of preference instability and economic volatility. In particular,  $\alpha^* > 0$  requires

$$\varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2} < \sigma_{\varepsilon_2}^2 + \chi \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)$$

and the condition

$$\varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2} < \sigma_{\varepsilon_1}^2 + (1 - \chi) \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)$$

secures that  $\alpha^* < 1$ . The RHS-terms in both inequalities reflect, in turn, the volatility of preference shocks, country size, and the welfare costs of output volatility. The LHS of both conditions consists of the covariance of preference shocks. Thus, in general, an internal solution for optimal representation requires that the welfare costs induced by the variance of economic shocks be large compared to the variance of inflation preferences. Note that a sufficient condition for an internal solution for optimal representation is that national shocks to inflation preference differ (only) moderately in terms of their volatility and correlation. To be more precise:

$$0 < \alpha^* < 1 \Leftrightarrow \varphi_{\varepsilon_1, \varepsilon_2} \leq \min \left( \frac{\sigma_{\varepsilon_1}}{\sigma_{\varepsilon_2}}, \frac{\sigma_{\varepsilon_2}}{\sigma_{\varepsilon_1}} \right).$$

**Conditions for  $\alpha^* \gtrless \chi$**

We find that

$$\alpha^* \gtrless \chi \Leftrightarrow \chi (\sigma_{\varepsilon_1}^2 - \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2}) \lesseqgtr (1 - \chi) (\sigma_{\varepsilon_2}^2 - \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2}). \quad (19)$$

---

<sup>21</sup>The argument is the familiar one: a sufficient condition for a positive denominator is  $\varphi_{\varepsilon_1, \varepsilon_2} \leq (\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2) / 2\sigma_{\varepsilon_1} \sigma_{\varepsilon_2}$ , which is fulfilled because  $\varphi_{\varepsilon_1, \varepsilon_2} \leq 1$ .

This implies that over- (under-)representation will always be optimal if a country's preferences are very stable (very volatile) in relative terms and preference shocks are sufficiently positively correlated across countries. Formally:

$$\begin{aligned}\alpha^* &> \chi \Leftrightarrow \sigma_{\varepsilon_1} < \sigma_{\varepsilon_2} \wedge \varphi_{\varepsilon_1, \varepsilon_2} > \sigma_{\varepsilon_1} / \sigma_{\varepsilon_2}, \\ \alpha^* &< \chi \Leftrightarrow \sigma_{\varepsilon_2} < \sigma_{\varepsilon_1} \wedge \varphi_{\varepsilon_1, \varepsilon_2} > \sigma_{\varepsilon_2} / \sigma_{\varepsilon_1}.\end{aligned}$$

**Proof.** According to (19), over-representation requires  $\chi (\sigma_{\varepsilon_1}^2 - \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2}) < (1 - \chi) (\sigma_{\varepsilon_2}^2 - \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2})$ , which is always fulfilled for  $\sigma_{\varepsilon_1}^2 < \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2} < \sigma_{\varepsilon_2}^2$ , or, equivalently,  $\sigma_{\varepsilon_1} / \sigma_{\varepsilon_2} < \varphi_{\varepsilon_1, \varepsilon_2} < \sigma_{\varepsilon_2} / \sigma_{\varepsilon_1}$ . Since, by definition,  $\varphi_{\varepsilon_1, \varepsilon_2} \leq 1$ , for  $\sigma_{\varepsilon_1} < \sigma_{\varepsilon_2}$  this reduces to  $\varphi_{\varepsilon_1, \varepsilon_2} > \sigma_{\varepsilon_1} / \sigma_{\varepsilon_2}$ . Equivalently, under-representation requires  $\varphi_{\varepsilon_1, \varepsilon_2} > \sigma_{\varepsilon_2} / \sigma_{\varepsilon_1}$  if  $\sigma_{\varepsilon_2} < \sigma_{\varepsilon_1}$ . ■

### Comparative statics

Starting from an interior solution, an increase in the **volatility of preference shocks** reduces the optimal weight a country holds in the Council.

**Proof.** Taking the derivative of equation (18) with regard to  $\sigma_{\varepsilon_1}$ , one finds that

$$\frac{\partial \alpha^*}{\partial \sigma_{\varepsilon_1}} < 0 \Leftrightarrow \varphi_{\varepsilon_1, \varepsilon_2} \leq \min \left( \frac{\sigma_{\varepsilon_1}}{\sigma_{\varepsilon_2}}, \frac{\sigma_{\varepsilon_2}}{\sigma_{\varepsilon_1}} \right).$$

As shown above, if the RHS-inequality is binding, we also have  $0 < \alpha^* < 1$ . ■

An increase in **economic volatility** suggests a higher optimal voting right in the Council, if the country was initially under-represented and vice versa. More formally:

$$\frac{\partial \alpha^*}{\partial \sigma_{\theta_1}} \gtrless 0 \Leftrightarrow \alpha^* \lesseqgtr \chi.$$

**Proof.** Taking the derivative of (18) with regard to  $\sigma_{\theta_1}$  leads to the condition

$$\frac{\partial \alpha^*}{\partial \sigma_{\theta_1}} \gtrless 0 \Leftrightarrow \chi (\sigma_{\varepsilon_1}^2 - \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2}) \gtrless (1 - \chi) (\sigma_{\varepsilon_2}^2 - \varphi_{\varepsilon_1, \varepsilon_2} \sigma_{\varepsilon_1} \sigma_{\varepsilon_2}),$$

which by equation (19) implies the above. ■

If the **correlation between preference shocks** across countries rises, a country is more likely to see its optimal voting weight increase, if its preferences are relatively stable and it is large in economic terms. To be precise:

$$\frac{\partial \alpha^*}{\partial \varphi_{\varepsilon_1, \varepsilon_2}} \gtrless 0 \Leftrightarrow \sigma_{\varepsilon_1}^2 - \chi \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) \lesseqgtr \sigma_{\varepsilon_2}^2 - (1 - \chi) \lambda^2 (\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2).$$



**Proof.** Taking the derivative of (18) with regard to  $\varphi_{\varepsilon_1, \varepsilon_2}$  yields the above term, where the RHS of the last inequality is increasing in country 1's relative economic size,  $\chi$ , making the case  $\partial\alpha^*/\partial\varphi_{\varepsilon_1, \varepsilon_2} > 0$  more probable to hold. This, trivially, is also true for a higher  $\sigma_{\varepsilon_2}^2$  or a lower  $\sigma_{\varepsilon_1}^2$ . ■

#### Appendix 4: The case with correlated economic and preference shocks ( $\varphi_{\varepsilon_1, \theta_1} \neq 0$ )

Assuming that all cross-country shocks are independent but allowing economic and shocks to inflation preferences to be correlated within country 1 (i.e. ,  $\varphi_{\varepsilon_1, \theta_1} \neq 0$ ), we find that the optimal weight for country 1 becomes

$$\alpha^* = \frac{\sigma_{\varepsilon_2}^2 + \chi\lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - \chi\lambda\varphi_{\varepsilon_1, \theta_1}\sigma_{\varepsilon_1}\sigma_{\theta_1}}{\sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - 2\lambda\varphi_{\varepsilon_1, \theta_1}\sigma_{\varepsilon_1}\sigma_{\theta_1}}. \quad (20)$$

While extreme values for  $\alpha^*$  cannot be excluded in general in this case, the optimal voting weight is likely to fall into the permissible range  $0 < \alpha^* < 1$  for a wide range of parameters (see below).

##### Conditions for $0 < \alpha^* < 1$

The denominator of (20) is positive.<sup>22</sup> Thus  $\alpha^* > 0$  requires the nominator to be positive, too. Obviously, this is more likely to be the case if preference shocks and economic shocks in country 2 are volatile. As far as country 1 is concerned, inspection of (20) reveals that, because  $\sigma_{\varepsilon_2}^2, \sigma_{\theta_2}^2 > 0$ , a sufficient condition for  $\alpha^* > 0$  is

$$\varphi_{\varepsilon_1, \theta_1} \leq \frac{\lambda\sigma_{\theta_1}}{\sigma_{\varepsilon_1}},$$

which is always fulfilled if the welfare costs associated with economic volatility exceed the volatility of preferences in country 1 or the correlation between preference and economic shocks in country 1 is non-positive, i.e. if  $\varphi_{\varepsilon_1, \theta_1} \leq 0$ .

To ensure that  $\alpha^* < 1$ , we need that

$$\varphi_{\varepsilon_1, \theta_1}\sigma_{\varepsilon_1}\sigma_{\theta_1} < \frac{1}{(2-\chi)}\sigma_{\varepsilon_1}^2 + \frac{(1-\chi)\lambda}{(2-\chi)}(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2),$$

which is likely to be fulfilled unless the “political-economic” covariance between preference and economic shocks within country 1, the LHS of the

---

<sup>22</sup>The denominator is positive if  $\sigma_{\varepsilon_2}^2 + \lambda^2\sigma_{\theta_2}^2 > 2\lambda\varphi_{\varepsilon_1, \theta_1}\sigma_{\varepsilon_1}\sigma_{\theta_1} - \sigma_{\varepsilon_1}^2 - \lambda^2\sigma_{\theta_1}^2$ . The RHS is always non-positive and the inequality always holds because  $\sigma_{\varepsilon_1}^2 > 0$  and  $\varphi_{\varepsilon_1, \theta_1} \leq 1$ .

inequality, takes on extreme values that dominate the RHS, which unambiguously increases in the sum of the variances of the economic shocks in countries 1 and 2 *and* the preference shock in country 1. Note that this inequality, too, will always be fulfilled for non-positive values of  $\varphi_{\varepsilon_1, \theta_1}$ .

We conclude that an interior solution with  $0 < \alpha^* < 1$  is more likely the smaller the coefficient of correlation (with a negative correlation always implying an interior solution) and the larger the welfare costs of economic volatility in the currency union.

### Conditions for $\alpha^* \gtrless \chi$

We find:

$$\alpha^* \gtrless \chi \Leftrightarrow \chi (\sigma_{\varepsilon_1}^2 - \lambda \varphi_{\varepsilon_1, \theta_1} \sigma_{\varepsilon_1} \sigma_{\theta_1}) \lesseqgtr (1 - \chi) \sigma_{\varepsilon_2}^2. \quad (21)$$

### Comparative statics

Taking the derivative of optimal representation (20) with regard to **preference volatility** yields

$$\frac{\partial \alpha^*}{\partial \sigma_{\varepsilon_1}} \gtrless 0 \Leftrightarrow \varphi_{\varepsilon_1, \theta_1} \gtrless \frac{2\sigma_{\varepsilon_1}}{(2 - \chi)\lambda\sigma_{\theta_1}}.$$

Moreover, note that if preference shocks are symmetrical or more volatile than economic shocks in welfare terms, i.e.  $\sigma_{\varepsilon_1} \geq \lambda\sigma_{\theta_1}$ , the optimal voting weight will always decrease as the preferences become less stable.

**Proof.** Define  $\Gamma = \sigma_{\varepsilon_2}^2 + \chi\lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - \chi\lambda\varphi_{\varepsilon_1, \theta_1}\sigma_{\varepsilon_1}\sigma_{\theta_1}$  and  $\Delta = \sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - 2\lambda\varphi_{\varepsilon_1, \theta_1}\sigma_{\varepsilon_1}\sigma_{\theta_1}$  so that  $\alpha^* = \Gamma/\Delta$ . Then, taking the derivative of equation (20) with regard to  $\sigma_{\varepsilon_1}$ , one finds that

$$\frac{\partial \alpha^*}{\partial \sigma_{\varepsilon_1}} \gtrless 0 \Leftrightarrow -\Delta (\lambda\chi\varphi_{\varepsilon_1, \theta_1}\sigma_{\theta_1}) - \Gamma 2(\sigma_{\varepsilon_1} - \lambda\varphi_{\varepsilon_1, \theta_1}\sigma_{\theta_1}) \gtrless 0.$$

In an interior solution:  $0 < \alpha^* < 1$  and, thus,  $0 < \Gamma < \Delta$ . This leads to the sufficient condition noted above. Note that  $0 < \chi < 1$  implies  $2\sigma_{\varepsilon_1}/(2 - \chi)\lambda\sigma_{\theta_1} > 1$  when  $\lambda\sigma_{\theta_1} \geq \sigma_{\varepsilon_1}$ . ■

Taking the derivative of optimal representation with regard to **economic volatility** gives

$$\frac{\partial \alpha^*}{\partial \sigma_{\theta_1}} \gtrless 0 \Leftrightarrow \varphi_{\varepsilon_1, \theta_1} \gtrless \frac{2(1 - \chi)\lambda\sigma_{\theta_1}}{(2 - \chi)\sigma_{\varepsilon_1}}.$$

**Proof.** Define  $\Gamma = \sigma_{\varepsilon_2}^2 + \chi\lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - \chi\lambda\varphi_{\varepsilon_1, \theta_1}\sigma_{\varepsilon_1}\sigma_{\theta_1}$  and  $\Delta = \sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 + \lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) - 2\lambda\varphi_{\varepsilon_1, \theta_1}\sigma_{\varepsilon_1}\sigma_{\theta_1}$  so that  $\alpha^* = \frac{\Gamma}{\Delta}$ . Taking the derivative of

equation (20) with regard to  $\sigma_{\varepsilon_1}$ , one finds that

$$\frac{\partial \alpha^*}{\partial \sigma_{\theta_1}} \gtrless 0 \iff \Delta (2\sigma_{\theta_1}\chi\lambda^2 - \lambda\chi\varphi_{\varepsilon_1,\theta_1}\sigma_{\varepsilon_1}) - \Gamma 2(\lambda^2\sigma_{\theta_1} - \lambda\varphi_{\varepsilon_1,\theta_1}\sigma_{\varepsilon_1}) \gtrless 0.$$

In an interior solution:  $0 < \alpha^* < 1$  and, thus,  $0 < \Gamma < \Delta$ . This implies the sufficient condition stated in the Result. This scenario is especially plausible if the initial level of economic volatility (in welfare terms) is small compared to preference volatility.<sup>23</sup> ■

Taking the derivative of (20) with regard to the **coefficient of correlation**  $\varphi_{\varepsilon_1,\theta_1}$  yields

$$\frac{\partial \alpha^*}{\partial \varphi_{\varepsilon_1,\theta_1}} \gtrless 0 \Leftrightarrow \chi\sigma_{\varepsilon_1}^2 \lesseqgtr (2 - \chi)\sigma_{\varepsilon_2}^2 + \chi\lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2).$$

The RHS of the last inequality unambiguously increases in  $\sigma_{\varepsilon_2}^2$ ,  $\sigma_{\theta_1}^2$ , and  $\sigma_{\theta_2}^2$ .

## Appendix 5: Comparative statics with relative-weight uncertainty

On the one hand, an increase in **economic volatility** strengthens the economic argument underlying the calculation of  $\alpha_\lambda^*$ , suggesting higher representation for initially under-represented countries. On the other, a higher  $\sigma_{\theta_1}$  decreases  $\alpha_{\lambda_P}^*$ , which tends to reduce  $\alpha_\lambda^*$ . This latter effect always dominates. To see this, note the partial derivative

$$\frac{\partial \alpha_\lambda^*}{\partial \sigma_{\theta_1}} \gtrless 0 \Leftrightarrow \chi\sigma_{\theta_1}^2\sigma_{\varepsilon_1}^2 \gtrless (1 - \chi)\sigma_{\theta_2}^2\sigma_{\varepsilon_2}^2 + \chi\sigma_{\varepsilon_1}^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) + \frac{1}{\lambda^2}\sigma_{\theta_2}^2\sigma_{\varepsilon_1}^2\sigma_{\varepsilon_2}^2. \quad (22)$$

The term  $\chi\sigma_{\theta_1}^2\sigma_{\varepsilon_1}^2 \gtrless (1 - \chi)\sigma_{\theta_2}^2\sigma_{\varepsilon_2}^2$  in (22) represents the condition for over- and under-representation in (13). That is, in principle, under-representation works toward a positive impact of  $\sigma_{\theta_1}$  on optimal representation just as in the baseline model with preference uncertainty regarding the inflation target. However, the additional terms  $\chi\sigma_{\varepsilon_1}^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) + \frac{1}{\lambda^2}\sigma_{\theta_2}^2\sigma_{\varepsilon_1}^2\sigma_{\varepsilon_2}^2$  on the RHS inequality in (22) will always over-compensate that effect. This can be shown by rearranging (22):

$$\frac{\partial \alpha_\lambda^*}{\partial \sigma_{\theta_1}} = -(1 - \chi)\lambda^2\sigma_{\theta_2}^2\sigma_{\varepsilon_2}^2 - \chi\lambda^2\sigma_{\theta_2}^2\sigma_{\varepsilon_1}^2 - \sigma_{\theta_2}^2\sigma_{\varepsilon_1}^2\sigma_{\varepsilon_2}^2 < 0.$$

---

<sup>23</sup>The proof suggests that a necessary condition for  $\partial \alpha^*/\partial \sigma_{\theta_1}$  to be positive is that  $\lambda\sigma_{\theta_1} < \sigma_{\varepsilon_1}$ . Note that  $\partial \alpha^*/\partial \sigma_{\theta_1}$  is always negative if  $\lambda\sigma_{\theta_1}$  is already very large compared to  $\sigma_{\varepsilon_1}$  in the sense that  $\lambda\sigma_{\theta_1}/\sigma_{\varepsilon_1} > (2 - \chi)/2(1 - \chi) > 1$ .

In the case of under-representation, we have  $\alpha_{\lambda_P}^* < \alpha_\lambda^* < \chi$ . In contrast to the result of the baseline model where  $\alpha^*$  approaches  $\chi$  as  $\sigma_{\theta_1}$  rises when a country was initially under-represented, here, the distance between  $\alpha_\lambda^*$  and  $\chi$  grows. However, for plausible parameter values  $\alpha_\lambda^*$  does not approach  $\alpha_{\lambda_P}^*$ , either as this moves down even further with  $\sigma_{\theta_1}$  increasing. Technically we need

$$\left| \frac{\partial \alpha_{\lambda_P}^*}{\partial \sigma_{\theta_1}} \right| > \left| \frac{\partial \alpha_\lambda^*}{\partial \sigma_{\theta_1}} \right|,$$

which is true if

$$\frac{2\sigma_{\theta_1}\sigma_{\varepsilon_1}^2\sigma_{\varepsilon_2}^2}{(\sigma_{\theta_1}^2\sigma_{\varepsilon_1}^2 + \sigma_{\theta_2}^2\sigma_{\varepsilon_2}^2)^2} > \frac{(1-\chi)\lambda^2\sigma_{\varepsilon_2}^2 + \chi\lambda^2\sigma_{\theta_1}^2 + \sigma_{\varepsilon_1}^2\sigma_{\varepsilon_2}^2}{(\lambda^2(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2) + \sigma_{\theta_1}^2\sigma_{\varepsilon_1}^2 + \sigma_{\theta_2}^2\sigma_{\varepsilon_2}^2)^2}.$$

Again, this is fulfilled for plausible parameter values (numerical results available on request).

# CESifo Working Paper Series

(for full list see [www.cesifo.de](http://www.cesifo.de))

---

- 1281 Ayal Kimhi, Growth, Inequality and Labor Markets in LDCs: A Survey, September 2004
- 1282 Robert Dur and Amihai Glazer, Optimal Incentive Contracts for a Worker who Envis his Boss, September 2004
- 1283 Klaus Abberger, Nonparametric Regression and the Detection of Turning Points in the Ifo Business Climate, September 2004
- 1284 Werner Güth and Rupert Sausgruber, Tax Morale and Optimal Taxation, September 2004
- 1285 Luis H. R. Alvarez and Erkki Koskela, Does Risk Aversion Accelerate Optimal Forest Rotation under Uncertainty?, September 2004
- 1286 Giorgio Brunello and Maria De Paola, Market Failures and the Under-Provision of Training, September 2004
- 1287 Sanjeev Goyal, Marco van der Leij and José Luis Moraga-González, Economics: An Emerging Small World?, September 2004
- 1288 Sandro Maffei, Nikolai Raabe and Heinrich W. Ursprung, Political Repression and Child Labor: Theory and Empirical Evidence, September 2004
- 1289 Georg Götz and Klaus Gugler, Market Concentration and Product Variety under Spatial Competition: Evidence from Retail Gasoline, September 2004
- 1290 Jonathan Temple and Ludger Wößmann, Dualism and Cross-Country Growth Regressions, September 2004
- 1291 Ravi Kanbur, Jukka Pirttilä and Matti Tuomala, Non-Welfarist Optimal Taxation and Behavioral Public Economics, October 2004
- 1292 Maarten C. W. Janssen, José Luis Moraga-González and Matthijs R. Wildenbeest, Consumer Search and Oligopolistic Pricing: An Empirical Investigation, October 2004
- 1293 Kira Börner and Christa Hainz, The Political Economy of Corruption and the Role of Financial Institutions, October 2004
- 1294 Christoph A. Schaltegger and Lars P. Feld, Do Large Cabinets Favor Large Governments? Evidence from Swiss Sub-Federal Jurisdictions, October 2004
- 1295 Marc-Andreas Mündler, The Existence of Informationally Efficient Markets When Individuals Are Rational, October 2004

- 1296 Hendrik Jürges, Wolfram F. Richter and Kerstin Schneider, Teacher Quality and Incentives: Theoretical and Empirical Effects of Standards on Teacher Quality, October 2004
- 1297 David S. Evans and Michael Salinger, An Empirical Analysis of Bundling and Tying: Over-the-Counter Pain Relief and Cold Medicines, October 2004
- 1298 Gershon Ben-Shakhar, Gary Bornstein, Astrid Hopfensitz and Frans van Winden, Reciprocity and Emotions: Arousal, Self-Reports, and Expectations, October 2004
- 1299 B. Zorina Khan and Kenneth L. Sokoloff, Institutions and Technological Innovation During Early Economic Growth: Evidence from the Great Inventors of the United States, 1790 – 1930, October 2004
- 1300 Piero Gottardi and Roberto Serrano, Market Power and Information Revelation in Dynamic Trading, October 2004
- 1301 Alan V. Deardorff, Who Makes the Rules of Globalization?, October 2004
- 1302 Sheilagh Ogilvie, The Use and Abuse of Trust: Social Capital and its Deployment by Early Modern Guilds, October 2004
- 1303 Mario Jametti and Thomas von Ungern-Sternberg, Disaster Insurance or a Disastrous Insurance – Natural Disaster Insurance in France, October 2004
- 1304 Pieter A. Gautier and José Luis Moraga-González, Strategic Wage Setting and Coordination Frictions with Multiple Applications, October 2004
- 1305 Julia Darby, Anton Muscatelli and Graeme Roy, Fiscal Federalism, Fiscal Consolidations and Cuts in Central Government Grants: Evidence from an Event Study, October 2004
- 1306 Michael Waldman, Antitrust Perspectives for Durable-Goods Markets, October 2004
- 1307 Josef Honerkamp, Stefan Moog and Bernd Raffelhüschen, Earlier or Later: A General Equilibrium Analysis of Bringing Forward an Already Announced Tax Reform, October 2004
- 1308 M. Hashem Pesaran, A Pair-Wise Approach to Testing for Output and Growth Convergence, October 2004
- 1309 John Bishop and Ferran Mane, Educational Reform and Disadvantaged Students: Are They Better Off or Worse Off?, October 2004
- 1310 Alfredo Schclarek, Consumption and Keynesian Fiscal Policy, October 2004
- 1311 Wolfram F. Richter, Efficiency Effects of Tax Deductions for Work-Related Expenses, October 2004

- 1312 Franco Mariuzzo, Patrick Paul Walsh and Ciara Whelan, EU Merger Control in Differentiated Product Industries, October 2004
- 1313 Kurt Schmidheiny, Income Segregation and Local Progressive Taxation: Empirical Evidence from Switzerland, October 2004
- 1314 David S. Evans, Andrei Hagiu and Richard Schmalensee, A Survey of the Economic Role of Software Platforms in Computer-Based Industries, October 2004
- 1315 Frank Riedel and Elmar Wolfstetter, Immediate Demand Reduction in Simultaneous Ascending Bid Auctions, October 2004
- 1316 Patricia Crifo and Jean-Louis Rullière, Incentives and Anonymity Principle: Crowding Out Toward Users, October 2004
- 1317 Attila Ambrus and Rossella Argenziano, Network Markets and Consumers Coordination, October 2004
- 1318 Margarita Katsimi and Thomas Moutos, Monopoly, Inequality and Redistribution Via the Public Provision of Private Goods, October 2004
- 1319 Jens Josephson and Karl Wärneryd, Long-Run Selection and the Work Ethic, October 2004
- 1320 Jan K. Brueckner and Oleg Smirnov, Workings of the Melting Pot: Social Networks and the Evolution of Population Attributes, October 2004
- 1321 Thomas Fuchs and Ludger Wößmann, Computers and Student Learning: Bivariate and Multivariate Evidence on the Availability and Use of Computers at Home and at School, November 2004
- 1322 Alberto Bisin, Piero Gottardi and Adriano A. Rampini, Managerial Hedging and Portfolio Monitoring, November 2004
- 1323 Cecilia García-Peñalosa and Jean-François Wen, Redistribution and Occupational Choice in a Schumpeterian Growth Model, November 2004
- 1324 William Martin and Robert Rowthorn, Will Stability Last?, November 2004
- 1325 Jianpei Li and Elmar Wolfstetter, Partnership Dissolution, Complementarity, and Investment Incentives, November 2004
- 1326 Hans Fehr, Sabine Jokisch and Laurence J. Kotlikoff, Fertility, Mortality, and the Developed World's Demographic Transition, November 2004
- 1327 Adam Elbourne and Jakob de Haan, Asymmetric Monetary Transmission in EMU: The Robustness of VAR Conclusions and Cecchetti's Legal Family Theory, November 2004
- 1328 Karel-Jan Alsem, Steven Brakman, Lex Hoogduin and Gerard Kuper, The Impact of Newspapers on Consumer Confidence: Does Spin Bias Exist?, November 2004

- 1329 Chiona Balfoussia and Mike Wickens, Macroeconomic Sources of Risk in the Term Structure, November 2004
- 1330 Ludger Wößmann, The Effect Heterogeneity of Central Exams: Evidence from TIMSS, TIMSS-Repeat and PISA, November 2004
- 1331 M. Hashem Pesaran, Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure, November 2004
- 1332 Maarten C. W. Janssen, José Luis Moraga-González and Matthijs R. Wildenbeest, A Note on Costly Sequential Search and Oligopoly Pricing, November 2004
- 1333 Martin Peitz and Patrick Waelbroeck, An Economist's Guide to Digital Music, November 2004
- 1334 Biswa N. Bhattacharyay and Prabir De, Promotion of Trade, Investment and Infrastructure Development between China and India: The Case of Southwest China and East and Northeast India, November 2004
- 1335 Lutz Hendricks, Why Does Educational Attainment Differ Across U.S. States?, November 2004
- 1336 Jay Pil Choi, Antitrust Analysis of Tying Arrangements, November 2004
- 1337 Rafael Lalive, Jan C. van Ours and Josef Zweimueller, How Changes in Financial Incentives Affect the Duration of Unemployment, November 2004
- 1338 Robert Woods, Fiscal Stabilisation and EMU, November 2004
- 1339 Rainald Borck and Matthias Wrede, Political Economy of Commuting Subsidies, November 2004
- 1340 Marcel Gérard, Combining Dutch Presumptive Capital Income Tax and US Qualified Intermediaries to Set Forth a New System of International Savings Taxation, November 2004
- 1341 Bruno S. Frey, Simon Luechinger and Alois Stutzer, Calculating Tragedy: Assessing the Costs of Terrorism, November 2004
- 1342 Johannes Becker and Clemens Fuest, A Backward Looking Measure of the Effective Marginal Tax Burden on Investment, November 2004
- 1343 Heikki Kauppi, Erkki Koskela and Rune Stenbacka, Equilibrium Unemployment and Capital Intensity Under Product and Labor Market Imperfections, November 2004
- 1344 Helge Berger and Till Müller, How Should Large and Small Countries Be Represented in a Currency Union?, November 2004