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Allocative Efficiency Measurement Revisited: Do We Really Need Input Prices?

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Discussion Papers

591

**Oleg Badunenko
Michael Fritsch
Andreas Stephan**

**Allocative Efficiency Measurement Revisited:
Do We Really Need Input Prices?**

Berlin, June 2006



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Discussion Papers 591

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Allocative Efficiency Measurement Revisited – Do We Really Need Input Prices?

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Berlin, June 2006

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Abstract

The traditional approach to measuring allocative efficiency is based on input prices, which are rarely known at the firm level. This paper proposes a new approach to measure allocative efficiency which is based on the output-oriented distance to the frontier in a profit – technical efficiency space – and which does not require information on input prices. To validate the new approach, we perform a Monte-Carlo experiment which provides evidence that the estimates of the new and the traditional approach are highly correlated. Finally, as an illustration, we apply the new approach to a sample of about 900 enterprises from the chemical industry in Germany.

Keywords: Allocative efficiency, data envelopment analysis, frontier analysis, technical efficiency, Monte-Carlo study, chemical industry.

JEL Classification: D61, L23, L25, L65

Zusammenfassung

“Ein neuer Ansatz zur Messung allokativer Effizienz – Sind Input-Preise wirklich erforderlich?“

Der traditionelle Ansatz zur Messung allokativer Effizienz erfordert Informationen über Input-Preise der Unternehmen, die allerdings nur selten vorliegen. In diesem Aufsatz schlagen wir eine neue Methode zur Bestimmung der allokativen Effizienz vor, der als wesentliche Information den Abstand eines Unternehmens von der Effizienz-Grenze nutzt und keine Information über Input-Preise erfordert. Ein Monte-Carlo Experiment zur Überprüfung der Tragfähigkeit dieses Ansatzes zeigt, dass die Schätzwerte nach der traditionellen Methode und dem von uns vorgeschlagenen Verfahren eng miteinander korreliert sind. Zur Illustration wenden wir den neuen Ansatz auf ein Sample von 900 Unternehmen der Chemischen Industrie in Deutschland an.

Schlagworte: Allokative Effizienz, Data Envelopment Analysis, Frontier Analysis, Technische Effizienz, Monte-Carlo Methode, Chemische Industrie.

JEL-Klassifikation: D61, L23, L25, L65

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1 Introduction

A significant number of empirical studies have investigated the extent and determinants of *technical* efficiency within and across industries (see Alvarez and Crespi (2003), Gumbau-Albert and Maudos (2002), Caves and Barton (1990), Green and Mayes (1991), Fritsch and Stephan (2004a)). Comprehensive literature reviews of the variety of empirical applications are made by Lovell (1993) and Seiford (1996, 1997). Compared to this literature, attempts to quantify the extent and distribution of *allocative* efficiency are relatively rare (for a survey, see Greene (1997)).¹ This is quite surprising since allocative efficiency has traditionally attracted the attention of economists: what is the optimal combination of inputs so that output is produced at minimal cost? How much could the profits be increased by simply reallocating resources? To what extent does competitive pressure reduce the heterogeneity of allocative inefficiency within industries?²

A firm is said to have realized allocative efficiency if it is operating with the optimal combination of inputs. The traditional approach to measuring allocative efficiency requires input prices (see Atkinson and Cornwell (1994), Green (1997), Kumbhakar (1991), Kumbhakar and Tsionas (2005), Oum and Zhang (1995)) which are hardly available in reality.³ This explains why empirical studies of allocative efficiency are highly concentrated on certain industries, particularly banking, because information on input price can be obtained for these industries.

This paper introduces a new approach to estimating allocative efficiency, which is solely based on quantities and profits and does not require information on input prices. An indicator for allocative efficiency is derived as the output-oriented distance to a frontier in a *profit-technical efficiency* space. What is, however, needed is an assessment of input-saving technical efficiency; i.e., how less input could be used to produce given outputs.

The paper proceeds as follows: section 2 theoretically derives a new method for estimating allocative efficiency and introduces a theoretical framework for activity analysis models. Section 3 presents the results of the Monte-Carlo experiment on comparison of allocative efficiency scores calculated using both traditional and new approaches. Section 4 provides a rationale and a simple illustration using the new approach; section 5 concludes.

¹ For studies in the financial sector, see the review by Berger and Humphrey (1997) and also Topuz et al. (2005), Färe et al. (2004), Isik and Hassan (2002). Some studies have been performed for the agricultural sector (e.g., Coelli et al., (2002), Chavas et al., (1993, 2005), Grazhdaninova (2005)). Studies for manufacturing sector are relatively rare (e.g., Burki (1997), Kim and Han (2001)).

² Moreover, allocative efficiency is also important for the analysis of the production process; e.g., to estimate the bias of (i) the cost function parameters, (ii) returns to scale, (iii) input price elasticities, and (iv) cost-inefficiency (Kumbhakar and Wang, forthcoming) or to validate the aggregation of productivity index (Raa (2005)).

³ This includes retrieving allocative efficiency using shadow prices (see Green (1997), Lovell (1993)).

2 Allocative efficiency measurement

2.1 Traditional approach to allocative efficiency measurement

A definition of technical and allocative efficiency was made by Farrell (1957). According to this definition, a firm is technically efficient if it uses the minimal possible combination of inputs for producing a certain output (input orientation). Allocative efficiency, or as Farrell called it price efficiency, refers to the ability of a firm to choose the optimal combination of inputs given input prices. If a firm has realized both technical and allocative efficiency, it is then cost efficient (overall efficient).

Figure 1
 Measurement and decomposition of cost efficiency

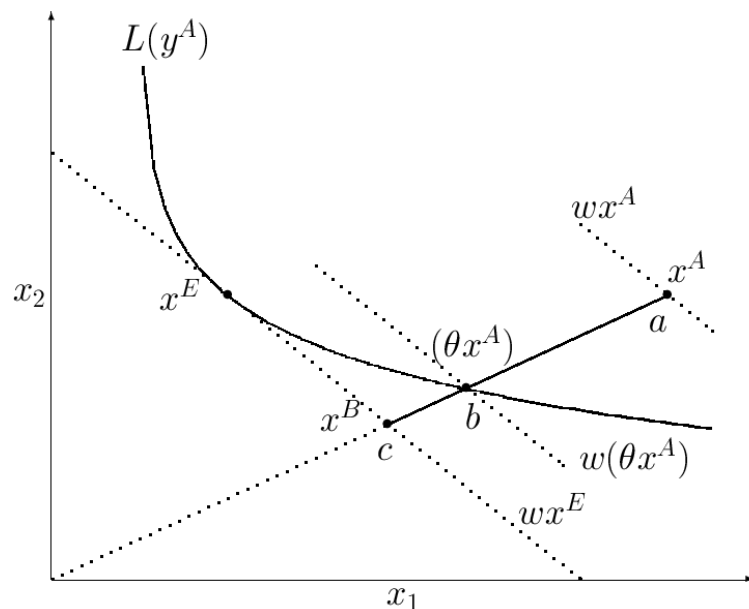


Figure 1, similarly to Kumbhakar and Lovell (2000), shows firm A producing output y^A represented by the isoquant $L(y^A)$. Dotted lines are the isocosts which show level of expenditures for a certain combination of inputs. The slope of the isocosts is equal to the ratio of input prices, $w(w_1, w_2)$. If the firm is producing output y^A with the factor combination x^A (a in Figure 1), it is operating technically inefficient. Potentially, it could produce the same output contracting both inputs x_1 and x_2 (available at prices w), proportionally (radial approach); the smallest possible contraction is in point b , representing (θx^A) a factor combination. Having reached this point, the firm is considered to be technically efficient. Formally, technical efficiency is measured by the ratio of the current input level to the lowest attainable input level for producing a given amount of output. In terms of Figure 1, technical inefficiency of unit x^A is given by

$$TE(y^A, x^A) = \theta = \frac{w(\theta x^A)}{wx^A} \quad (1)$$

or geometrically by ob/oa . The measure of cost inefficiency (overall efficiency) is given by the ratio of potentially minimal cost to actual cost:

$$CE(y^A, x^A, w^A) = \frac{wx^E}{wx^A} \quad (2)$$

or geometrically by oc/oa . Thus, cost inefficiency is the ratio of expenditures at x^E to expenditures at x^A while technical efficiency is the ratio of expenditures at (θx^A) to expenditures at x^A . The remaining portion of the cost efficiency is given by the ratio of expenditures at x^E to expenditures at (θx^A) . It is attributable to the misallocation of inputs given input prices and is known as allocative efficiency:

$$AE = \frac{CE}{TE} = \frac{wx^E / wx^A}{w(\theta x^A) / wx^A} \quad (3)$$

or in terms of Figure 1 is given by oc/ob .

2.2 A new approach to allocative efficiency measurement

When input prices are available, allocative efficiency in the pure Farrell sense can be calculated using, for example, a non-parametric frontier approach (Färe et al., 1994) or a parametric one (Greene (1997) among others). However, if input prices are not available these approaches are not applicable. In contrast to this, the new approach we propose allows measuring allocative efficiency without information on input prices. An estimate of allocative efficiency can be obtained with the new approach that is solely based on information on input and output quantities and on profits.

The first step of this new approach involves the estimation of technical efficiency; whereby, in the second step allocative efficiency is estimated as an output-oriented distance to the frontier in a *profit-technical efficiency* space.

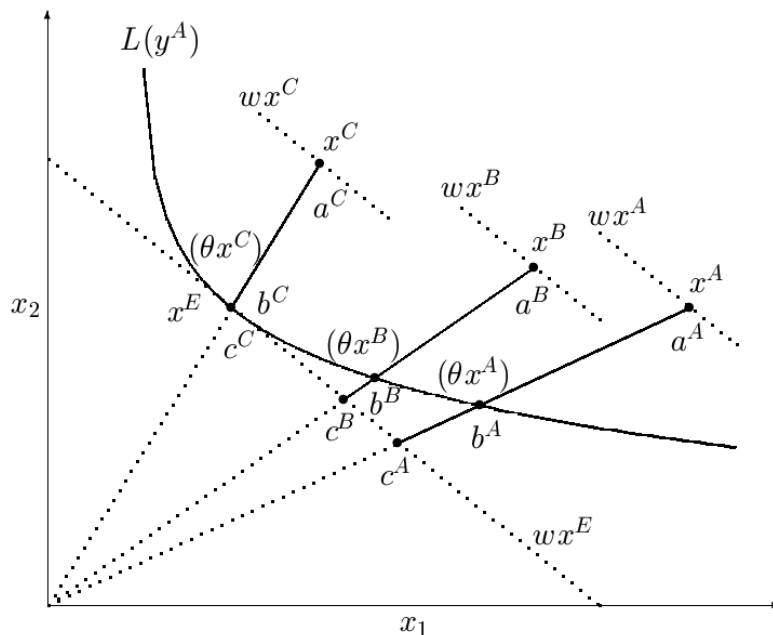
In Figure 2, three firms, A , B , and C using inputs x^A , x^B , and x^C , available at prices w ,⁴ produce output y^A , which is measured by the isoquant $L(y^A)$. For the sake of argument, firms A , B , and C are all equally technically efficient (the level of technical efficiency θ , however, is arbitrarily chosen) which is read from expenditure levels at (θx^A) , (θx^B) , and at (θx^C) , respectively. In geometrical terms $ob^A/oa^A = ob^B/oa^B = ob^C/oa^C$. The costs of these three firms are determined by wx^A , wx^B , and by wx^C . The isocost corresponding to expenditures at x^C is the closest possible to the origin o for this level of

⁴ Let us assume that the ratios of input prices are equal for each firm. This assumption is needed to have the isocosts parallel to each other.

technical efficiency and, therefore, implies the lowest level of cost. This is because x^C is the combination of inputs lying on the ray from origin and going through the tangent point of the isocost (corresponding to expenditure level of $w x^E$) to the isoquant $L(y^A)$. This implies that for θ -level of technical efficiency costs have a lower bound and using the fact that firms are producing the same output y^A , profits have an upper bound. Without loss of generality, for each level θ of technical efficiency there is a profit maximum, which proves the existence of a frontier in *profit—technical efficiency* space.

Proposition 1: *Existence of the frontier in profit-technical efficiency space.* A profit maximum exists for any level of technical efficiency.

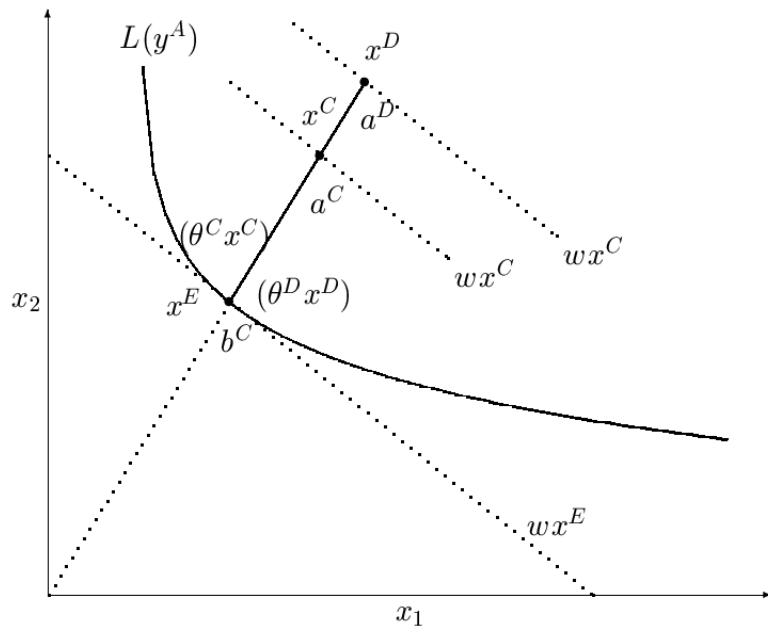
Figure 2
Bound of a profit



In Figure 3, two firms, C and D , use inputs x^C and x^D to produce output y^A , which is measured by the isoquant $L(y^A)$. Both firms are allocatively efficient because they lie on the same ray from the origin that goes through the tangent point x^E ; thus, in terms of proposition 1 we only look at the frontier points. These firms operate, however, at different levels of technical efficiency θ^C and θ^D , respectively. Since the isocost representing the level of expenditure $w x^C$ is closer to the origin than that of the expenditure level $w x^D$, costs of firm C are smaller than those of firm D and firm C is more profitable than firm D . Since $ob^C/oa^C > ob^D/oa^D$, $\theta^C > \theta^D$, larger technical efficiency is associated with larger profits for points forming the frontier in *profit-technical efficiency* space. This proves that such frontier is upward sloping.

Remark 1: Frontier in profit—technical efficiency space is sloped upwards.

Figure 3
 Relationship between technical efficiency and profit

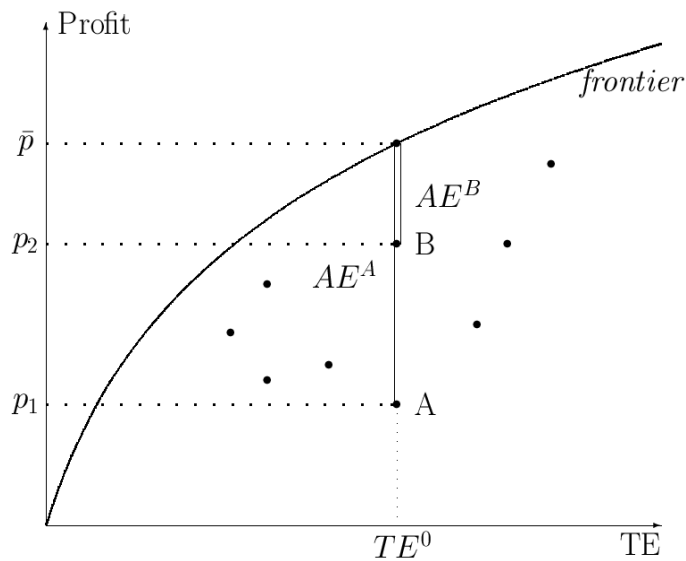


Proposition 2: *The higher the allocative efficiency the higher the profit.* For any arbitrarily chosen level of technical efficiency, the closer the input combination to the optimal one (i.e., the larger the allocative efficiency) the larger the profit will be.

Equation (3) suggests that in terms of Figure 2 (all three firms are equally technically efficient) expenditures solely depend on allocative efficiency. Moreover, the smaller the allocative efficiency the larger the expenditure. Keeping in mind that these firms produce the same output y^A , we conclude that for θ -level of technical efficiency (again chosen arbitrarily) the larger the allocative efficiency the lower the costs and the larger the profit is; as allocative efficiency reaches its maximum (for firm C), the maximal profit is also achieved. Without loss of generality, this statement is true for any level of technical efficiency.

Proposition 3: *Allocative efficiency in profit-technical efficiency space.* The Farrell output-oriented distance to the frontier in *profit-technical efficiency space* measures allocative efficiency.

Figure 4
Allocative efficiency in profit-technical efficiency space



In Figure 4 *frontier* is the locus of the maximum attainable profits as defined in Proposition 1. The firms *A*, *B*, and *C* have the same technical efficiency level TE^0 ; however, they have different profit levels: p_1 , p_2 , and \bar{p} , respectively. The potential level of profit which firms can reach is \bar{p} . The closer the observation is to the *frontier*, the larger the profit is. As we recall from Figure 2, the shift from firm *A* to firm *C* is only possible when the input-mix is changed; i.e., allocative efficiency is improved. Thus, in Figure 4 the shift from firm *A* to firm *B* means an increase in allocative efficiency (distance AE^A is larger than distance AE^B), and further increase in allocative efficiency within the same level of technical efficiency is only possible up to firm *C*'s observation, for which both profit and allocative efficiency are at the maximum. Thus, which is most remarkable, the distance from the observation to the frontier serves as a measure of the allocative efficiency.

To summarize, we have defined a new way of estimating allocative efficiency, specifically, this is the output-oriented distance to the frontier in *profit-technical efficiency* space.

3 Monte-Carlo simulation

To analyze whether our new approach to measuring allocative efficiency yields valid estimates, we conducted several Monte-Carlo experiments. According to a micro-economic theory, a firm which chooses such a combination of inputs, thus their ratio is equal to the ratio of output elasticities of the respective inputs will be most profitable. When we speak of optimal combination of inputs, the original notion of allocative efficiency comes into play, and we suggest that the closer the ratio of inputs to the ratio of elasticities the larger a firm's allocative efficiency will be.

3.1 Empirical implementation of the traditional approach

The traditional approach can be used when input prices are known. Under technology T such that

$$T = \left\{ (x, y) : x \text{ can produce } y \right\} \quad (4)$$

We measure input-oriented technical efficiency as the greatest proportion that the inputs can be reduced and still produce the same outputs:

$$F^i(y, x) = \inf \left\{ \lambda : \lambda x \text{ can still produce } y \right\} \quad (5)$$

We employ the Data Envelopment Analysis (DEA) all the way through the empirical estimation. For K observations, M outputs, and N inputs an estimate of the *Farrell Input-Saving Measure of Technical Efficiency* can be calculated by solving a linear programming problem for each observation j ($j = 1, \dots, K$):

$$\hat{TE}_j = \hat{F}_j^i(y, x | C) = \min \left\{ \lambda : \sum_{k=1}^K z_k \cdot y_{km} \geq y_j, \sum_{k=1}^K z_k \cdot x_{kn} \leq x_j \cdot \lambda, z_k \geq 0 \right\} \quad (6)$$

for $m = 1, \dots, M$ and $n = 1, \dots, N$. Note that superscript i stands for input orientation while C is the constant returns-to-scale. Other returns-to-scale are modeled adjusting process operating levels z_k s (see Färe et al., (1994) for details).

When input prices and quantities are given we can calculate the total costs and the minimum attainable cost (solve linear programming problem) and then compute an estimate of *cost efficiency* for each observation j ($j = 1, \dots, K$) as in equation (2):

$$\hat{C}_j^i(y, x, w | C) = \frac{\min \left\{ \sum_{n=1}^N w_{jn} \cdot x_{jn} : \sum_{k=1}^K z_k' \cdot y_{km} \geq y_j, \sum_{k=1}^K z_k' \cdot x_{kn} \leq x_j, z_k' \geq 0 \right\}}{\sum_{n=1}^N w_{jn} \cdot x_{jn}} \quad (7)$$

for $m = 1, \dots, M$ and $n = 1, \dots, N$. We refer to the residual of technical and cost efficiencies as *Input Allocative Efficiency*, which can be computed for each observation j ($j = 1, \dots, K$) as:

$$\hat{AE}_j^i(y, x, w | C) = \frac{\hat{C}_j^i(y, x, w | C)}{\hat{F}_j^i(y, x | C)}. \quad (8)$$

3.2 Empirical implementation of the new approach

As mentioned above, the main virtue of the new approach is that we do not necessarily need input prices for measuring allocative efficiency. Technically, we need output-oriented distances to the frontier in the profit-technical efficiency space. We take advantage of the technical efficiency estimates (denoted by TE) obtained as in equation (6) and profitability measure (denoted by Pr) to calculate (solve linear programming problem) allocative efficiency for each observation j ($j = 1, \dots, K$) as follows:

$$\hat{AE}_j = \hat{F}_j^o(Pr, TE | C) = \max \left\{ \theta : \sum_{k=1}^K z_k'' Pr_k \geq Pr_j \cdot \theta, \sum_{k=1}^K z_k'' TE_k \leq TE_j, z_k'' \geq 0 \right\}. \quad (9)$$

3.3 Design of the Monte-Carlo experiments

In each of the Monte-Carlo trials, we study a production process which uses two inputs to produce one output. Data for the i th observation in each Monte-Carlo experiment were generated using the following algorithm.

- (i) We chose output elasticities of two inputs to be 0.2 and 0.8; this ensures constant returns to scale. The optimal ratio of inputs, thus, is 4.
- (ii) Draw $x_1 \sim (\phi + \lambda \cdot \text{uniform})$; uniform on the interval (0;1).
- (iii) Draw $r \sim \text{uniform}$; uniform on the interval (0;8). This is meant to be an experimental ratio of used inputs.
- (iv) Set $x_2 = r \cdot x_1$.
- (v) Choose ε . In doing so, we allow the ratio of inputs in each Monte-Carlo trial to vary on the interval $[\varepsilon; 8 - \varepsilon]$ while keeping in mind that the optimal ratio is 4. Therefore, we

obtain enough variation of inefficient combinations of inputs, or in other words, enough variation of allocative inefficiency.

- (vi) Draw $u \sim N^+(0, \sigma_u^2)$ and set 'te_drawn' equal to $\exp(-u)$.
- (vii) Generate output data assuming trans-log production function, which will contain inefficiency component:⁵

$$y = 0.2 \cdot x_1 + 0.8 \cdot x_2 + \gamma_{11} \cdot x_1^2 + \gamma_{22} \cdot x_2^2 + \frac{1}{2} \gamma_{12} \cdot x_1 \cdot x_2 + te_drawn.$$

- (viii) Draw price of input x_1 : $w_1 \sim (\varphi + \psi \cdot uniform)$, uniform on the interval (0;1). The price of input x_2 is calculated as $w_2 = \theta \cdot w_1$ – we want to keep the ratio of input prices constant to have the isoquants parallel (recall Figure 2).
- (ix) Set profit as output (we set output price equal to 1) minus cost and this is divided by output.
- (x) DEA traditional allocative efficiency as in equation (8).
- (xi) DEA our measures of allocative efficiency using technical efficiency drawn in step (vi) as in equation (9).
- (xii) Solve for technical efficiency as in equation (6), and DEA our measure of allocative efficiency using these solved technical efficiency scores.
- (xiii) Calculate rank correlation coefficient between allocative efficiency estimates based on traditional and our approaches.
- (xiv) Repeat steps (i) through (xiii) L times.

In each of our experiments we set $\phi = 1$, $\lambda = 7$, $\varphi = 1$, $\psi = 0.05$, $\gamma_{11} = 0.01$, $\gamma_{22} = 0.01$ and $\gamma_{12} = -0.02$. In order to look at different variabilities of inappropriately chosen ratios of inputs, we set $\varepsilon = 0.5$, $\varepsilon = 1$, and $\varepsilon = 2$. With $\varepsilon = 2$, variability of allocative efficiency is expected to have been reduced considerably – range becomes (2;6); and vice versa, $\varepsilon = 0.5$ ensures very large variability – range increases to (0.5;7.5). We conduct three sets of experiments setting σ_u^2 to 0.0025, 0.025, and 0.25; this ensures covering a plausible range of standard deviations of technical efficiency.⁶ In each experiment we ran $L=500$ Monte-Carlo trials.⁷

⁵ Since the DEA is deterministic, we do not incorporate a stochastic term in the Monte-Carlo trials.

⁶ Using a different experiment, Greene (2005) obtains estimates of technical efficiency with standard deviations from 0.09 to 0.43.

⁷ The simulation is programmed in SAS 9.1.3; computationally, one run with $N=100$, $L=500$ takes about 7 hours on a Pentium IV processor running at 3GHz. Thus, we defined relatively few parameter constellations in the performed experiment.

3.4 Results

From tables 1 to 6 it is clearly seen that in all three cases the DEA estimates the drawn technical efficiency scores fairly accurately – the rank correlation coefficient (Corr4) is close to unity. This is an expected outcome since we do not assume a stochastic term in the production output generation (step (vii) of the experiment). The same argument applies to the rank correlation coefficient between allocative efficiency calculated in step (xi) and that calculated in step (xii) (Corr3). Thus, there is not much difference in using the true or the estimated technical efficiency in the new approach. However, what is of most interest to us are the rank correlation coefficients between allocative efficiency estimates from the traditional and our new approach (Corr1 and Corr2). Corr1 has been computed with the estimates of allocative efficiency based on ‘true’ technical efficiency while Corr2 has been computed with the estimates of allocative efficiency based on estimated values of technical efficiency. As previously mentioned, the rank correlation between these measures is quite high (Corr3). We argue that it is more appropriate to draw conclusions from Corr2 since we do not know the ‘true’ technical efficiency in practice.

Table 1
Rank correlations, $\varepsilon = 0.5$, $N = 100$

σ_u^2		0.0025			0.025			0.25		
		θ	0.75	1	1.25	0.75	1	1.25	0.75	1
Corr1	mean	0.8566	0.7375	0.6954	0.8608	0.7326	0.6942	0.8087	0.6879	0.6413
	st.d	0.0442	0.0625	0.0677	0.0434	0.0621	0.0686	0.0649	0.0760	0.0772
Corr2	mean	0.8642	0.7485	0.7038	0.8695	0.7526	0.7115	0.8712	0.7885	0.7365
	st.d	0.0416	0.0590	0.0663	0.0407	0.0589	0.0664	0.0469	0.0687	0.0818
Corr3	mean	0.9899	0.9880	0.9894	0.9915	0.9901	0.9895	0.9468	0.9419	0.9464
	st.d	0.0194	0.0212	0.0188	0.0148	0.0159	0.0168	0.0531	0.0492	0.0397
Corr4	mean	0.8928	0.8937	0.8893	0.9524	0.9528	0.9560	0.9830	0.9816	0.9825
	st.d	0.0409	0.0405	0.0423	0.0275	0.0268	0.0254	0.0124	0.0148	0.0141

Notes: Corr1 is the rank correlation between allocative efficiency calculated in step (x) and that calculated in step (xi). Corr2 is the rank correlation between allocative efficiency calculated in step (x) and that calculated in step (xii). Corr3 is the rank correlation between allocative efficiency calculated in step (xi) and that calculated in step (xii). Corr4 is the rank correlation between technical efficiency calculated in equation (6) and that drawn in step (vi).

Table 2
 Rank correlations, $\varepsilon = 1$, $N = 100$

		σ_u^2 0.0025			0.025			0.25		
		θ 0.75	1	1.25	0.75	1	1.25	0.75	1	1.25
Corr1	mean	0.8569	0.7043	0.6192	0.8519	0.6991	0.6053	0.7851	0.6381	0.5476
	st.d	0.0412	0.0653	0.0744	0.0429	0.0685	0.0779	0.0706	0.0803	0.0838
Corr2	mean	0.8611	0.7111	0.6264	0.8598	0.7197	0.6263	0.8470	0.7481	0.6709
	st.d	0.0393	0.0641	0.0722	0.0405	0.0654	0.0771	0.0480	0.0753	0.0944
Corr3	mean	0.9928	0.9922	0.9919	0.9912	0.9903	0.9889	0.9469	0.9356	0.9384
	st.d	0.0163	0.0152	0.0157	0.0149	0.0146	0.0170	0.0530	0.0542	0.0419
Corr4	mean	0.9183	0.9209	0.9196	0.9590	0.9633	0.9626	0.9874	0.9870	0.9869
	st.d	0.0341	0.0344	0.0353	0.0278	0.0248	0.0254	0.0111	0.0111	0.0113

Notes from Table 1 apply.

Table 3
 Rank correlations, $\varepsilon = 2$, $N = 100$

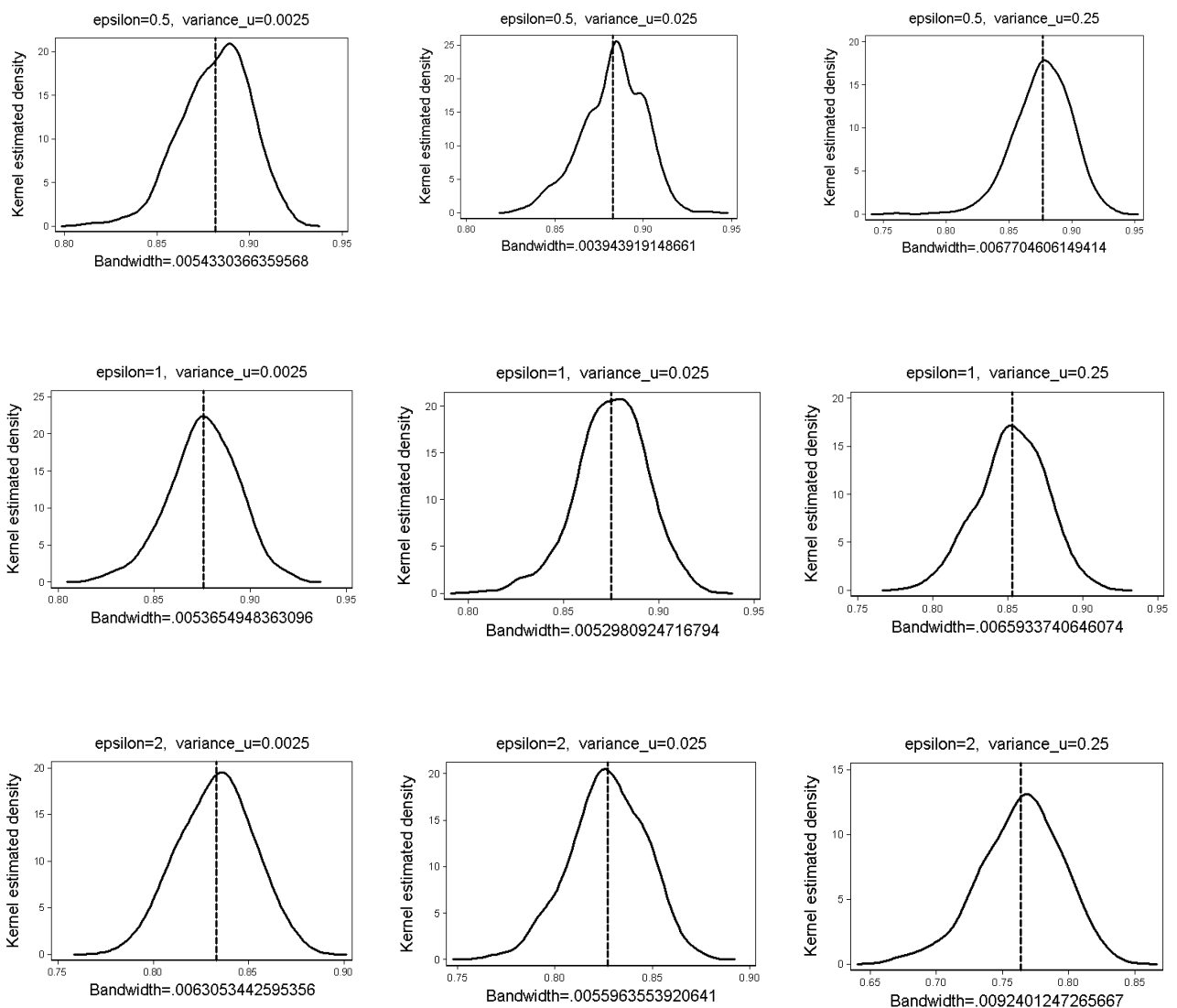
		σ_u^2 0.0025			0.025			0.25		
		θ 0.75	1	1.25	0.75	1	1.25	0.75	1	1.25
Corr1	mean	0.8140	0.5782	0.3386	0.8042	0.5561	0.3168	0.6841	0.4515	0.2602
	st.d	0.0453	0.0762	0.0835	0.0438	0.0794	0.0928	0.1020	0.1063	0.0984
Corr2	mean	0.8155	0.5837	0.3448	0.8091	0.5750	0.3498	0.7638	0.6048	0.4864
	st.d	0.0437	0.0738	0.0828	0.0425	0.0791	0.0937	0.0609	0.0992	0.1294
Corr3	mean	0.9939	0.9948	0.9938	0.9917	0.9904	0.9878	0.9265	0.9117	0.9049
	st.d	0.0144	0.0124	0.0130	0.0152	0.0156	0.0202	0.0765	0.0838	0.0652
Corr4	mean	0.9455	0.9449	0.9443	0.9749	0.9743	0.9731	0.9910	0.9908	0.9910
	st.d	0.0283	0.0300	0.0300	0.0202	0.0197	0.0206	0.0090	0.0089	0.0075

Notes from Table 1 apply.

The first observation worth mentioning is that when variability of sub-optimal ratios decreases (ε increases): our method is less successful in yielding similar estimates as the traditional one. Hence, our method deteriorates in terms of exactness when ‘true’ allocative efficiency is not very heterogeneous. Furthermore, the results show that our approach is robust with respect to variance of the drawn technical efficiency, σ_u^2 . Looking closely at correspondent ratios, one can notice that for the same θ 's Corr2 is increasing when σ_u^2 increases, whereas for other θ 's Corr2 decreases when we increase σ_u^2 ; however, the changes are minor. The same argument applies to the standard deviation of Corr2.

This implies that for different levels of σ_u^2 distributions of Corr2 are virtually the same. The skewness of the variable Corr2 is always negative and is about -0.6 which means that the distribution of Corr2 is skewed to the left and more values are clustered to the right of the mean. Kurtosis is about 0.6 , but it varies more than the skewness; it increases with increase of σ_u^2 . Kernel density estimates of Corr2 for the case $\theta = 0.75$ are shown in Figure 5. Note that we use the Gaussian kernel function and the Sheather and Jones (1991) rule to determine the “optimal” bandwidth.

Figure 5
 Estimates of Sampling Densities of Corr2 ($\theta = 0.75, L = 500, \varepsilon = 0.5, \varepsilon = 1$ and $\varepsilon = 2$)



Note: in each panel the vertical dashed line is the mean value of the corresponding density.

The results are better when the sample size is increased to 400 (Tables 4-6). However, the improvement does not change our main conclusions based on the experiments with sample size 100. As expected, standard deviations of rank coefficients are almost halved when the sample size is quadrupled.

Table 4
Rank correlations, $\varepsilon = 0.5$, $N = 400$

		$\sigma_u^2 = 0.0025$			$\sigma_u^2 = 0.025$			$\sigma_u^2 = 0.25$		
		$\theta = 0.75$	$\theta = 1$	$\theta = 1.25$	$\theta = 0.75$	$\theta = 1$	$\theta = 1.25$	$\theta = 0.75$	$\theta = 1$	$\theta = 1.25$
Corr1	mean	0.8812	0.7551	0.7132	0.8810	0.7543	0.7126	0.8585	0.7297	0.6750
	st.d	0.0182	0.0288	0.0311	0.0173	0.0286	0.0297	0.0232	0.0308	0.0334
Corr2	mean	0.8824	0.7567	0.7144	0.8828	0.7605	0.7173	0.8773	0.7675	0.7114
	st.d	0.0176	0.0287	0.0307	0.0171	0.0281	0.0295	0.0211	0.0418	0.0412
Corr3	mean	0.9987	0.9990	0.9987	0.9988	0.9985	0.9986	0.9887	0.9856	0.9870
	st.d	0.0035	0.0031	0.0036	0.0028	0.0030	0.0023	0.0122	0.0215	0.0095
Corr4	mean	0.9726	0.9730	0.9733	0.9909	0.9905	0.9904	0.9968	0.9969	0.9968
	st.d	0.0096	0.0106	0.0099	0.0053	0.0063	0.0060	0.0026	0.0025	0.0027

Notes from Table 1 apply.

Table 5
Rank correlations, $\varepsilon = 1$, $N = 400$

		$\sigma_u^2 = 0.0025$			$\sigma_u^2 = 0.025$			$\sigma_u^2 = 0.25$		
		$\theta = 0.75$	$\theta = 1$	$\theta = 1.25$	$\theta = 0.75$	$\theta = 1$	$\theta = 1.25$	$\theta = 0.75$	$\theta = 1$	$\theta = 1.25$
Corr1	mean	0.8760	0.7169	0.6362	0.8734	0.7185	0.6309	0.8363	0.6754	0.5798
	st.d	0.0178	0.0334	0.0350	0.0186	0.0316	0.0370	0.0240	0.0350	0.0402
Corr2	mean	0.8766	0.7185	0.6375	0.8748	0.7247	0.6370	0.8547	0.7185	0.6257
	st.d	0.0176	0.0333	0.0349	0.0185	0.0313	0.0371	0.0214	0.0395	0.0501
Corr3	mean	0.9992	0.9991	0.9992	0.9987	0.9984	0.9984	0.9882	0.9845	0.9853
	st.d	0.0026	0.0028	0.0025	0.0029	0.0031	0.0031	0.0139	0.0144	0.0104
Corr4	mean	0.9814	0.9809	0.9821	0.9930	0.9932	0.9931	0.9978	0.9978	0.9977
	st.d	0.0086	0.0086	0.0085	0.0049	0.0047	0.0049	0.0020	0.0019	0.0020

Notes from Table 1 apply.

Table 6
Rank correlations, $\varepsilon = 2$, $N = 400$

		σ_u^2 0.0025			0.025			0.25		
		θ 0.75	1	1.25	0.75	1	1.25	0.75	1	1.25
Corr1	mean	0.8337	0.5911	0.3410	0.8269	0.5692	0.3253	0.7463	0.4934	0.2858
	st.d	0.0195	0.0361	0.0455	0.0205	0.0395	0.0470	0.0359	0.0458	0.0476
Corr2	mean	0.8339	0.5924	0.3422	0.8271	0.5752	0.3353	0.7661	0.5512	0.3780
	st.d	0.0192	0.0362	0.0455	0.0206	0.0393	0.0470	0.0302	0.0485	0.0734
Corr3	mean	0.9994	0.9994	0.9995	0.9990	0.9986	0.9981	0.9840	0.9777	0.9754
	st.d	0.0025	0.0022	0.0017	0.0021	0.0028	0.0037	0.0175	0.0227	0.0195
Corr4	mean	0.9884	0.9882	0.9879	0.9955	0.9955	0.9957	0.9985	0.9985	0.9985
	st.d	0.0066	0.0071	0.0072	0.0037	0.0037	0.0033	0.0015	0.0014	0.0015

Notes from Table 1 apply.

Results of one run⁸ (sample size 500) are summarized in Figure 6; note optimal ratio of inputs is shown by the vertical-dashed line in each panel. Our methodology almost completely repeats the trend of the traditional approach for $\varepsilon = 0.5$ which is backed by a high correlation coefficient in Tables 1 and 4; as ε becomes larger Figure 6 suggests that our methodology is less able to predicts allocative efficiency. However, it is most remarkable that our methodology is in line with the traditional approach.

⁸ We repeated this experiment many times and the general picture was always similar; however, due to space constraints it is not possible to present all results here.

Figure 6
Allocative efficiency calculated using traditional and new approaches plotted against ratio of expenditure shares, w_2x_2 / w_1x_1 ($\theta = 0.75$, $N = 400$, $\varepsilon = 0.5$, $\varepsilon = 1$ and $\varepsilon = 2$)

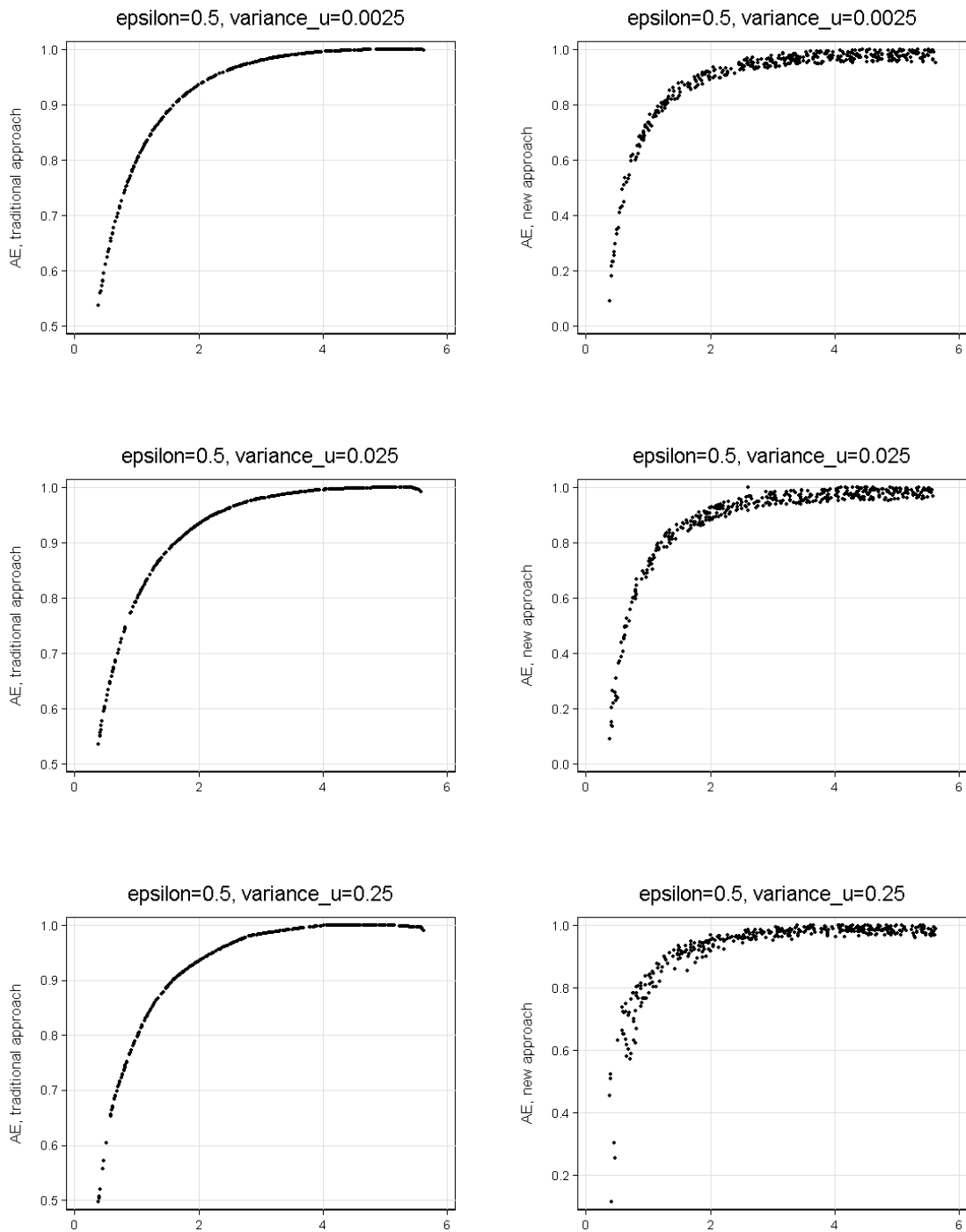


Figure 6 (continued)
Allocative efficiency calculated using traditional and new approaches plotted against ratio of expenditure shares, w_2x_2 / w_1x_1 ($\theta = 0.75$, $N = 400$, $\varepsilon = 0.5$, $\varepsilon = 1$ and $\varepsilon = 2$)

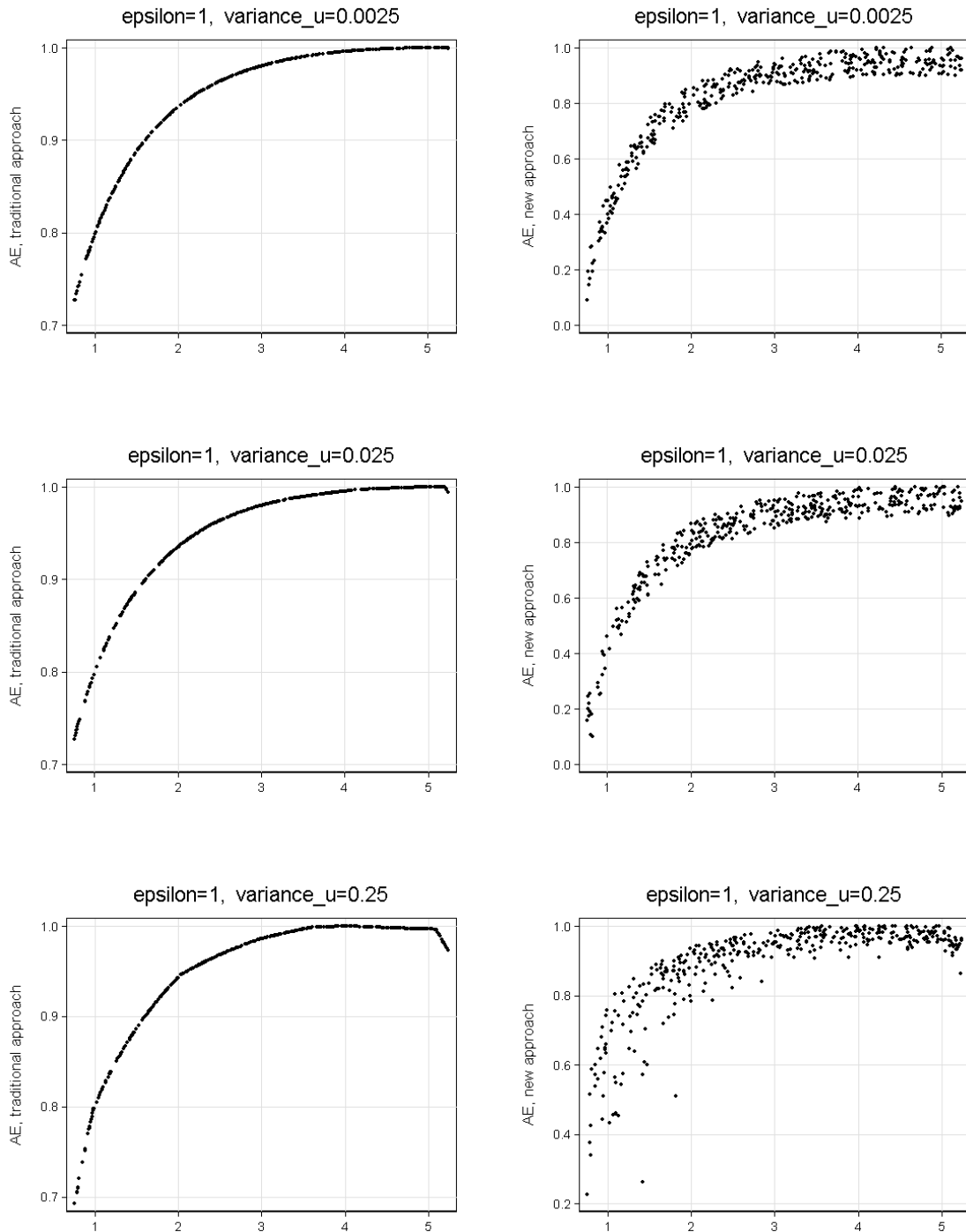
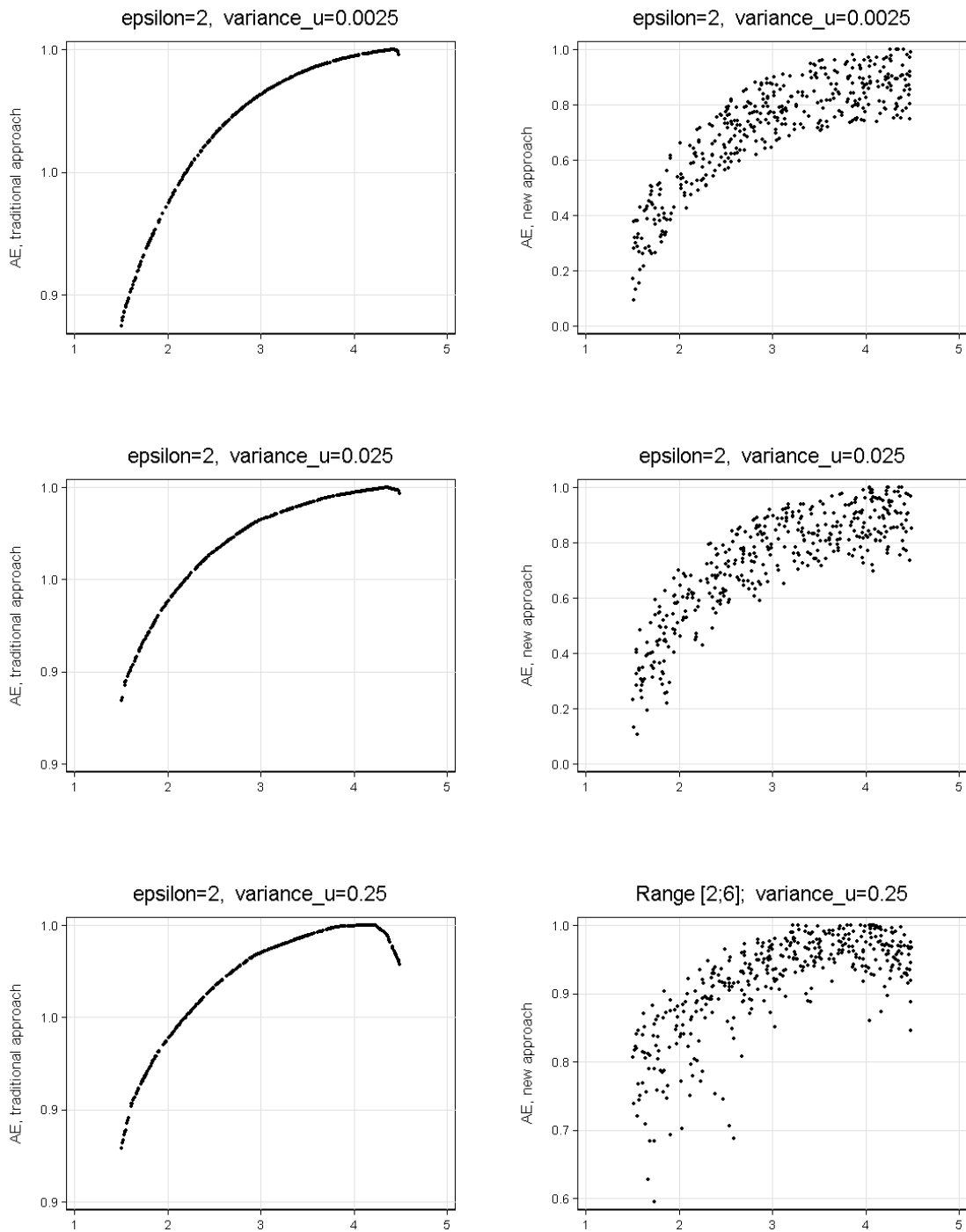


Figure 6 (continued)
Allocative efficiency calculated using traditional and new approaches plotted against ratio of expenditure shares, w_2x_2 / w_1x_1 ($\theta = 0.75$, $N = 400$, $\varepsilon = 0.5$, $\varepsilon = 1$ and $\varepsilon = 2$)



4 Empirical illustration of the new approach

4.1 Data

To illustrate the usefulness of the new approach for measuring allocative efficiency when input prices are not available, we apply it to micro-data from the German Cost Structure Census⁹ of manufacturing for the year 2003. Our sample comprises only enterprises from the chemical industry. The measure of output is gross production. This mainly consists of the turnover and the net-change of the stock of the final products.¹⁰

The Cost Structure Census contains information for a number of input categories.¹¹ These categories are payroll, employers' contribution to the social security system, fringe benefits, expenditure for material inputs, self-provided equipment, and goods for resale, for energy, for external wage-work, external maintenance and repair, tax depreciation of fixed assets, subsidies, rents and leases, insurance costs, sales tax, other taxes and public fees, interest on outside capital as well as "other" costs such as license fees, bank charges and postage, or expenses for marketing and transport.

Some of the cost categories which include expenditures for external wage-work and external maintenance and repair contain a relatively high share of reported zero values because many firms do not utilize these types of inputs. Such zeros make the firms incomparable and, thus, might bias the DEA results. In order to reduce the number of reported zero input quantities, we aggregated the inputs into the following categories: (i) material inputs (intermediate material consumption plus commodity inputs), (ii) labor compensation (salaries and wages plus employer's social insurance contributions), (iii) energy consumption, (iv) user cost of capital (depreciation plus rents and leases), (v) external services (e.g., repair costs and external wage-work), and (vi) "other" inputs related to production (e.g., transportation services, consulting, or marketing).

⁹ Aggregate figures are published annually in Fachserie 4, Reihe 4.3 of *Kostenstrukturerhebung im Verarbeitenden Gewerbe* (various years). The Cost Structure Census is gathered and compiled by the German Federal Statistical Office (Statistisches Bundesamt). Enterprises are legally obliged to respond to the Cost Structure Census; hence, missing observations due to non-response are precluded. The survey comprises all large German manufacturing enterprises which have 500 or more employees. Enterprises with 20-499 employees are included as a random sample that is representative for this size category in a particular industry. For more information about cost structure census surveys in Germany, we refer the reader to Fritsch et al., (2004).

¹⁰ We do not include turnover from activities that are classified as miscellaneous such as license fees, commissions, rents, leasing *etc.* because this kind of revenue cannot adequately be explained by the means of a production function.

¹¹ Though the production theory framework requires real quantities, using expenditures as proxies for inputs in the production function is quite common in the literature (see e.g., Paul et al., (2004), Paul and Nehring (2005)).

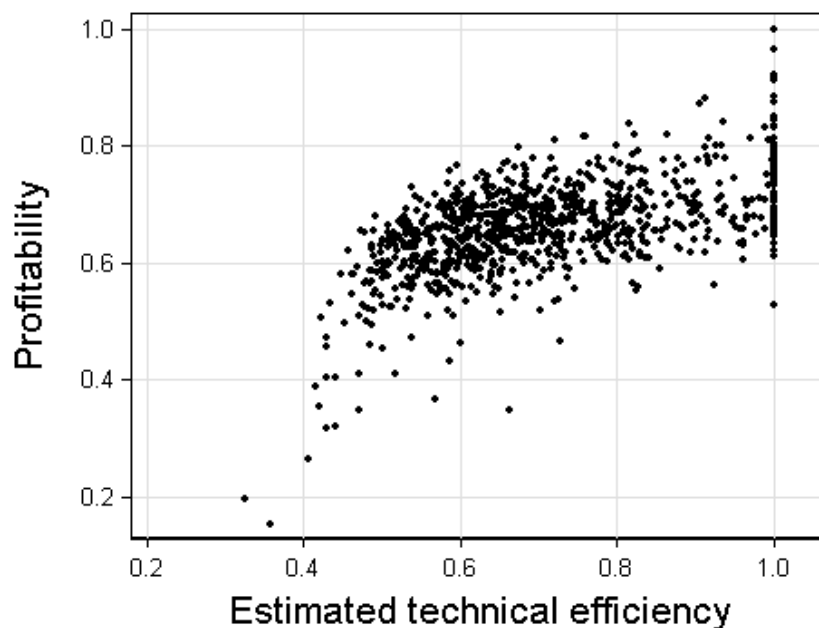
Profits are computed as one minus the total costs divided by the turnover. Since the DEA requires positive values, we standardize the profit measure to the interval (0,1) by adding the minimum profit and dividing this by the range of profits.

4.2 Results

Figure 7 shows profitability plotted against estimated technical efficiency. Remarkably, a frontier, as could be theoretically expected from Proposition 1, indeed exists. Another observation worth mentioning is that within a certain level of technical efficiency (i) profitability greatly varies suggesting variation in allocative efficiency (as firms A, B, and C in Proposition 3) and (ii) profits are bounded from above. Moreover, the frontier is positively sloped as was stated in the first theoretical part of this paper. Interestingly, Figure 7 suggests that even with 100 percent technical efficiency enterprises can be allocatively inefficient.

Figure 7

Profitability plotted against estimated technical efficiency scores for about 900 German enterprises from the chemical industry



We calculated technical efficiency scores as in equation (6). Table 7, which contains descriptive statistics of the estimated technical efficiencies, suggests that an average German chemical manufacturing enterprise is fairly inefficient. The median of technical efficiency implies that half of firms have an efficiency of 68 percent or less. The scores for allocative efficiency are obtained solving the linear programming problem as in equation (9). Descriptive statistics on allocative efficiency are also presented in Table 7. At a first glance, the mean and the variation of allocative efficiency appear to be strikingly similar to that of technical efficiency. However, the distribution of allocative

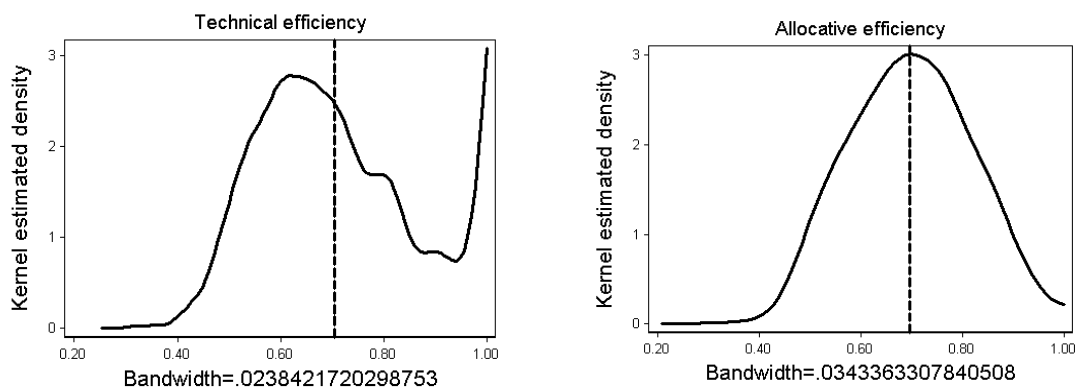
efficiency is more symmetric and has a lower variance compared to the technical efficiency distribution.

Table 7
Descriptive statistics of technical and allocative efficiency, N=905

Efficiency	mean	st.d.	coef. of var.	skew-ess	min	10 th perc.	25 th perc.	median	75 th perc.	90 th perc
Technical	0.6891	0.1507	0.2138	0.4399	0.3253	0.5287	0.5911	0.6817	0.8033	1.0000
Allocative	0.6963	0.1181	0.1696	-0.0018	0.3102	0.5360	0.6084	0.6974	0.7800	0.8523

Kernel estimated density of technical efficiency is shown in the left panel of Figure 8; we use Gaussian kernel function and the Sheather and Jones (1991) rule to determine the “optimal” bandwidth. Although the number of firms is quite large, we analyze the sensitivity of efficiency scores relative to the sampling variations of the estimated frontier in an additional step. Consequently, we perform the homogeneous bootstrap as described by Simar and Wilson (1998). The geometric mean of the bias-corrected efficiency scores is 0.6066, which is on average 0.0886 lower than that estimated via the DEA; the mean variance of bias is 0.0036. In comparison to other studies, however, the bias of estimates and its standard error are rather low, thereby indicating a robustness of the technical efficiency scores.

Figure 8
Estimates of sampling densities of technical and allocative efficiency scores



5 Conclusions

Allocative inefficiency, introduced in the seminal work by Farrell (1957), has important implications from the perspective of the firm. How much could firms increase their profits – given a certain output they produce – just by reallocating resources? On the other hand, the existing empirical evidence on the extent and determinants of allocative efficiency within and across industries is rather limited. The main reason is that the traditional approach to assessing allocative efficiency requires input prices. However, input prices are rarely accessible, which *per se*, precludes the analysis of the allocative efficiency with non-parametric approach.

In this paper, a new method is developed which enables calculating allocative efficiency without knowing input prices. This indicator is derived as the Farrell output-oriented distance to the frontier in *profit-technical efficiency* space. Thus, besides input and output quantities, only the profits of the firms are needed for calculating allocative efficiency. A simple Monte-Carlo experiment was performed to check the validity of the new methodology. We obtain high-rank correlation coefficients between allocative efficiency estimates based on both traditional and new approaches for different parameter constellations. Moreover, the new approach proved to be quite robust with respect to variance of true technical efficiency. Finally, we applied the new approach to a sample of about 900 enterprises in the German chemical industry. The results suggest a large variation of allocative efficiency even for technically efficient enterprises. Thus, the example highlights the usefulness of our method for obtaining allocative efficiency measures when input prices are not available.

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