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## Dynamic poverty decomposition analysis: An application to the Philippines

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**Dynamic Poverty  
Decomposition Analysis:  
An Application to the Philippines**

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**Abstract**

In this paper, we propose a new method of poverty decomposition. Our method remedies the shortcomings of existing methods and has some desirable properties such as time-reversion consistency and subperiod additivity. It integrates the existing methods of growth-redistribution decomposition and sector-based decomposition, because it allows us to decompose the change in poverty into growth and redistribution components for each group (e.g., regions or sectors) in the economy. We extend our method to include six components and provide an empirical application to the Philippines for the period 1985–2009.

**JEL Classification:** I32, O10

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## 1. INTRODUCTION

Poverty statistics are the most basic piece of information for assessing the poverty situation of a country and for formulating antipoverty policies. With broader recognition of their importance, the availability of poverty statistics has significantly improved over the last 4 decades. The World Bank's Living Standards Measurement Study (LSMS) website alone lists 38 countries with household surveys,<sup>1</sup> and many other countries not in the list also routinely conduct surveys and publish national poverty statistics without much external assistance.

The quality of poverty statistics has also improved with the accumulation of knowledge and experience. Better survey designs have helped make the measurement of standards of living more accurate and more readily comparable across regions, within a country, and over years. As a result, we have a better understanding of the profile of the poor and its transition over time.

However, in the standard poverty profile approach, it is often unclear what has caused the observed change in poverty. Adding to this problem, the methodology used to derive national poverty statistics is not always uniform, making the poverty statistics incomparable across regions or over time. To address these issues, we offer a new methodology of poverty decomposition in this paper.<sup>2</sup>

Our method is highly flexible and allows us to decompose the poverty change into several components (e.g., growth and redistribution components) for each region or each sector in a country in a coherent manner. Such decomposition is useful for choosing the appropriate policies to fight poverty. For example, in regions where economic growth is pro-poor but slow, policies to enhance regional economic growth (e.g., investment in infrastructure) may be an appropriate poverty reduction policy. On the other hand, in regions with high but anti-poor economic growth, distribution-improving policies (e.g., cash transfers) may be more appropriate. Our method is easy to implement, especially when a set of simplifying (but reasonable) assumptions are made. Our method produces a neat decomposition result that does not have an interaction term or residual, which is difficult to interpret. Further, as discussed in section 2, it satisfies two desirable properties of time-reversion consistency and subperiod additivity unlike the existing decomposition methods.

We apply our method to the Philippines for three reasons. First, the poverty reduction process in the Philippines has been slower than that of most other countries in Southeast Asia. It is therefore useful to identify the sources of slow progress in the Philippines. To this end, we decompose the poverty change in each region in the Philippines into six components: population shift (PS), within-region redistribution (WR), between-region redistribution (BR), nominal growth (NG), inflation (IF), and methodological change (MC). Our decomposition shows that most of the poverty reduction achieved by nominal growth is offset by inflation and worsening distribution within each region when we look at overall poverty change in the Philippines. Our regional disaggregation results show that the sources of poverty change are heterogeneous across regions and thus the suitable poverty reduction policies also vary across regions. For example, we find that growth-enhancing policies are desirable for poverty reduction in the Autonomous Region in Muslim Mindanao (ARMM), whereas distribution-improving policies are also important in Region VIII (Eastern Visayas).

Second, the official poverty statistics in the Philippines are calculated with poverty lines that are specific to a region or a province. Therefore, the changes in the national statistics reflect

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<sup>1</sup> See <http://go.worldbank.org/PDHZFQZ6L0> (accessed 1 October 2013).

<sup>2</sup> Here, we are concerned with the case where the reference standards of living at the poverty line are not comparable across time. However, incomparability can occur for other reasons, such as the variations in survey design over time. See, for example, Lanjouw and Lanjouw (2001). Deaton and Kozel (2005) provide an overview of the related debate in India.

not only the real changes in poverty but also the superficial changes due to the way official poverty lines are adjusted over time. By applying our method to the Philippines, we can separate the superficial changes from the observed changes. We find that the slow progress in the reduction of official poverty in the Philippines is partly driven by the superficial changes due to the change in methodology.

Finally, the Philippines has collected household income data once every 3 years since 1985. This allows us to see the poverty change over a relatively long period of time. Therefore, it is possible to see whether the driving force of poverty change has altered over time. We find that worsening distribution severely crippled the progress in poverty reduction in the two periods 1988–1991 and 1994–1997. In other years, the slow progress in poverty reduction was mainly explained by the lack of high real economic growth.

This paper is organized as follows. In the next section, we briefly review existing methodologies of poverty decomposition and develop a new method of dynamic poverty decomposition. In section 3, we describe the data and discuss some measurement issues. In section 4, we present the decomposition results in the Philippines. Section 5 provides some discussion.

## 2. METHODOLOGY

In this section, we develop a new method of dynamic poverty decomposition. To highlight the novelty of our method, we first introduce the notations and review the existing methods. We then present our general decomposition method. This method requires that we know the path of the changes in the components of interest (e.g., mean and distribution of income). However, this requirement is typically not fulfilled in a practical application. Therefore, we will consider two sets of assumptions that allow us to implement the method in a straightforward manner.

In section 2.3, we consider a simple linear approximation, in which the relative poverty line (poverty line relative to the mean income) and the cumulative distribution function of the relative income (individual income relative to the mean income) change linearly. This assumption leads to a very simple expression when the poverty measure of interest is the poverty rate.

In section 2.4, we consider a log-linear approximation as an alternative to the simple linear approximation. We use a linear approximation for the *logarithmic* relative poverty line and distribution of the *logarithmic* relative income. This approach also has some attractions as it has some connections with pro-poor growth literature. With either the linear or log-linear assumption, the poverty decomposition can be implemented easily. We then vary the speed of change in the mean income relative to that of the income distribution to check the robustness of the linear approximation.

In section 2.6, we consider an extension of the method with six components. Each component can be further divided by groups such as regions or sectors. This extension helps researchers and policy makers decide what poverty reduction policies are suitable for each group. Finally, we discuss some implementation issues.

### 2.1 Notations and Existing Methods

We assume that the individual-level poverty measure is determined by the individual income and poverty line. The nominal income per capita  $y$  has a positive infimum,<sup>3</sup> which is denoted by  $\varepsilon$ , and the income distribution at time  $t$  for the population of interest is given by the

<sup>3</sup> For most of our discussion, we only require that the nominal income is non-negative. Positive infimum is only necessary in section 2.4.

probability density function  $f(y, t)$ . The corresponding cumulative distribution function is denoted by  $F(y, t)$ , which satisfies  $F(0, t) = 0$  for all  $t$  because of the positive infimum. The poverty line at time  $t$ , or the threshold income level below which the individual is deemed poor, is denoted by  $z(t) (> 0)$ .

With some slight abuse of notation, we consider a class of poverty measures  $M$  that has the following form:

$$M(t) \equiv M(F(\cdot, t), z(t)) \equiv \int_0^{z(t)} g(y/z(t))f(y, t)dy, \quad (1)$$

where the function  $g(\cdot)$  represents the individual-level poverty measure, which we assume is differentiable at any point on the unit interval except for 0. The class of poverty measures defined in equation (1) is additively decomposable. That is, the poverty measure for any group can be expressed as the mean of subgroup poverty measures weighted by the subgroups' population shares. This is a useful property for poverty analysis, because it allows us to identify the major contributing groups to poverty. Further, additive decomposability is not a restrictive requirement, because any poverty measure that satisfies the subgroup consistency—a property that requires the group poverty measure to increase whenever the poverty measure for any of its subgroups increases—can be expressed as a monotonic transformation of an additively decomposable measure (Foster and Shorrocks 1991).

The Foster–Greer–Thorbecke (FGT) measure (Foster, Greer, and Thorbecke 1984), which is the most popular measure in the recent poverty literature, is a special case of equation (1) with  $g(\tilde{y}) = (1 - \tilde{y})^\alpha$ , where  $\tilde{y} \equiv y/z$  is the income normalized by the poverty line and  $\alpha (\geq 0)$  is a parameter. The Watts measure (Watts 1968) is also a special case of equation (1) with  $g(\tilde{y}) = -\ln \tilde{y}$ . While the Watts measure is not as widely used in applied research as the FGT measure, it follows from a set of reasonable axioms (Zheng 1993; Tsui 1996) and is closely related to our decomposition analysis as shown in section 2.4. The Chakravarty measure (Chakravarty 1983) can also be obtained as a special case of equation (1) by letting  $g(\tilde{y}) = 1 - \tilde{y}^\beta$ , where  $\beta$  is a parameter.

Because  $g$  is independent of  $F$  in equation (1), a number of other poverty indices are excluded from consideration, including those proposed by Sen (1976), Kakwani (1980), Takayama (1979), and Clark, Hemming, and Ulph (1981). While it is possible to modify our analysis to let  $g$  depend on  $F$ , we maintain the independence for the sake of simplicity of presentation.

In what follows, we focus on the FGT and Watts measures, which are denoted by  $P_\alpha$  and  $W$  with the following definitions, respectively:

$$P_\alpha(F, z) \equiv \int_0^z \left(1 - \frac{y}{z}\right)^\alpha dF \quad (2)$$

$$W(F, z) \equiv \int_0^z \ln \frac{z}{y} dF. \quad (3)$$

We shall refer to the FGT measure with parameter 0, 1, and 2 as poverty rate ( $P_0$ ), poverty gap ( $P_1$ ), and poverty severity ( $P_2$ ), respectively.

To conduct poverty decomposition, it is useful to introduce a few additional notations. We denote the mean income at time  $t$  by  $\mu(t) \equiv \int_0^\infty yf(y, t)dy$ , the relative income by  $\tilde{y} \equiv y/\mu(t)$ , and the relative poverty line by  $\tilde{z} \equiv z/\mu(t)$ . Here, the tilde notations ( $\tilde{\cdot}$ ) are used to emphasize that the quantity is relative to the population mean. The probability density function of the relative income is  $\tilde{f}(\tilde{y}, t)$ , which satisfies  $\tilde{f}(\tilde{y}, t) = \mu(t)f(y, t)$  for all  $t$  and  $y$ , and the corresponding cumulative distribution function is  $\tilde{F}$ . It is straightforward to show  $M(F(\cdot, t), z(t)) = M(\tilde{F}(\cdot, t), \tilde{z}(t))$ .



The purpose of poverty decomposition is to attribute the actual poverty change  $\Delta M(t_0, t_1) \equiv M(t_1) - M(t_0)$  to the components of interest, such as the growth and redistribution components. Formally, we define poverty decomposition as follows:

**Definition 1.** Let  $C$  be the index set for the components of interest and  $\Delta^c M$  be the contribution of component  $c \in C$  to the poverty change. The pair  $(C, \{\Delta^c M(t_0, t_1)\}_{c \in C})$  is called a **poverty decomposition** for the poverty change between  $t = t_0$  and  $t = t_1$  when  $\Delta M(t_0, t_1) = \sum_c \Delta^c M(t_0, t_1)$ .

One of the most popular decomposition methods was proposed by Datt and Ravallion (1992), which has been used in a number of studies, including Ravallion and Huppi (1991), Grootaert (1995), and Sahn and Stifel (2000). The Datt–Ravallion (DR) decomposition uses the initial time point  $t_0$  as the reference time point. In their study, the poverty line,  $z$ , is fixed. Therefore, the change in the relative poverty line,  $\tilde{z}$ , is driven only by the change in mean income (i.e., growth). By fixing either the relative poverty line or relative income distribution and letting the other change, we can decompose the poverty change into the growth component  $\Delta_{DR}^{GR}$  and redistribution component  $\Delta_{DR}^{RD}$  in the following manner with the notations introduced above:<sup>4</sup>

$$\begin{aligned}\Delta_{DR}^{GR} M(t_0, t_1) &= M(\tilde{F}_0, \tilde{z}_1) - M(\tilde{F}_0, \tilde{z}_0) \\ \Delta_{DR}^{RD} M(t_0, t_1) &= M(\tilde{F}_1, \tilde{z}_0) - M(\tilde{F}_0, \tilde{z}_0) \\ \Delta_{DR}^{RS} M(t_0, t_1) &= \Delta M(t_0, t_1) - \Delta_{DR}^{GR} M(t_0, t_1) - \Delta_{DR}^{RD} M(t_0, t_1),\end{aligned}$$

where  $\tilde{F}_a(\cdot) \equiv \tilde{F}(\cdot, t_a)$  and  $\tilde{z}(t_a) \equiv \tilde{z}_a$  for  $a \in \{0, 1\}$  are the (relative) income distribution and poverty line at time  $t_a$ , respectively.

The residual term  $\Delta_{DR}^{RS}$  captures the poverty change not explained by the growth and redistribution components. It captures the interaction between growth and redistribution components and can be interpreted as the difference between the growth [redistribution] components evaluated under the terminal and initial relative income distributions [mean incomes] (Datt and Ravallion 1992).

While setting the reference time point at the initial point is a natural choice, the presence of the residual term undermines the usefulness of the decomposition analysis. This is particularly true when the residual term is large in absolute value. As Baye (2006) argues, knowledge of how much of observed changes in poverty are due to changes in the redistribution as distinguished from growth in average incomes is critical for public policy and debate. Thus, if most of the poverty change is inexplicable, the decomposition results do not give much useful information to the policy makers.

We can easily avoid this problem if we are willing to assume that the change in mean income and distribution occurs in a certain sequence. In this case, we attribute the residual term to either the growth or redistribution component in effect. For example, Kakwani and Subbarao (1990) implicitly assume that the growth takes place first and the redistribution second, and thus  $\Delta_{DR}^{RS}$  is attributed to the redistribution component in their decomposition. On the other hand, Jain and Tendulkar (1990) consider a decomposition in which redistribution takes place first and growth second. Formally, the Kakwani–Subbarao (KS) and Jain–Tendulkar (JT) decompositions are defined as follows:

<sup>4</sup> Datt and Ravallion (1992) use a discrete time model. On the other hand, our presentation is based on a continuous time model. However, this distinction makes no essential difference. The same remark applies to other decomposition methods discussed in this subsection.

$$\begin{aligned}
\Delta_{KS}^{GR}M(t_0, t_1) &= M(\tilde{F}_0, \tilde{z}_1) - M(\tilde{F}_0, \tilde{z}_0) & (= \Delta_{DR}^{GR}M(t_0, t_1)) \\
\Delta_{KS}^{RD}M(t_0, t_1) &= M(\tilde{F}_1, \tilde{z}_1) - M(\tilde{F}_0, \tilde{z}_1) & (= \Delta_{DR}^{RD}M(t_0, t_1) + \Delta_{DR}^{RS}M(t_0, t_1)) \\
\Delta_{JT}^{GR}M(t_0, t_1) &= M(\tilde{F}_1, \tilde{z}_1) - M(\tilde{F}_1, \tilde{z}_0) & (= \Delta_{DR}^{GR}M(t_0, t_1) + \Delta_{DR}^{RS}M(t_0, t_1)) \\
\Delta_{JT}^{RD}M(t_0, t_1) &= M(\tilde{F}_1, \tilde{z}_0) - M(\tilde{F}_0, \tilde{z}_0) & (= \Delta_{DR}^{RD}M(t_0, t_1)).
\end{aligned}$$

However, the KS and JT decompositions are also unsatisfactory because the assumptions about the sequence of change are arbitrary. Furthermore, neither the DR, the KS, nor the JT decomposition satisfies the *time-reversion consistency* defined below:

**Definition 2.** *The decomposition  $(C, \{\Delta^c M(t_0, t_1)\}_{c \in C})$  is **time-reversion consistent** when  $\Delta^c M(t_0, t_1) + \Delta^c M(t_1, t_0) = 0$  for all  $c$ ,  $t_0$ , and  $t_1$ .*

The time-reversion consistency requires that when the poverty line and income distribution revert from the terminal state  $(\tilde{z}_1, \tilde{F}_1)$  to the original state  $(\tilde{z}_0, \tilde{F}_0)$ , the reverse decomposition yields the same decomposition result except that each component has the opposite sign.

To see why the time-reversion consistency is a reasonable requirement, imagine that you are a time traveler. You start the travel at  $t = t_0$  and end at  $t = t_1$ . You observe all the changes between  $t = t_0$  and  $t = t_1$ , and conduct the poverty decomposition. Now, you return from  $t = t_0$  and  $t = t_1$  along the same path of change such that you experience all the changes backwards. If the time-reversion consistency is not satisfied, some components contribute either positively or negatively to the poverty measure during the entire time travel, even though all the changes that you have experienced during the “outgoing” travel have been cancelled during the “return” travel.

One way to obtain a time-reversion consistent decomposition is to take the average of KS and JT decompositions. This is the average of all possible sequences (i.e., growth-redistribution and redistribution-growth in the standard two-way decomposition). Because this decomposition is essentially based on the average of the marginal contributions of each component in all the possible sequences, it is similar to the Shapley solution in cooperative games and thus called the Shapley decomposition (Kolenikov and Shorrocks 2005; Maasoumi and Mahmoudi 2013). Formally, each component in the Shapley decomposition is defined as follows:

$$\Delta_S^c M(t_0, t_1) \equiv (\Delta_{KS}^c M(t_0, t_1) + \Delta_{JT}^c M(t_0, t_1))/2,$$

where  $c \in \{GR, RD\}$ .

It is straightforward to verify that the Shapley decomposition is a time-reversion consistent decomposition (see also Kakwani 2000). Unlike KS and JT decompositions, the Shapley decomposition can also be extended to the case of multiple components. Son (2003) proposes a four-component Shapley-type decomposition method applied to the *rate* of poverty change for a general poverty measure.

These features of the Shapley decomposition are attractive. However, as with DR, KS, and JT decompositions, it does not satisfy the *subperiod additivity* defined below:

**Definition 3.** *Assume that we have observations of the poverty measure and other relevant parameters at time  $t = s_0, s_1, \dots, s_D$  in the time period between  $t = t_0$  and  $t = t_1$  with  $t_0 = s_0 < s_1 < \dots < s_D = t_1$ . Then, the decomposition  $(C, \{\Delta^c M(t_0, t_1)\}_{c \in C})$  is **subperiod additive** when the following equation is satisfied for all  $c \in C$ :*

$$\Delta^c M(t_0, t_1) = \sum_{d=1}^D \Delta^c M(s_{d-1}, s_d). \quad (4)$$

The subperiod consistency requires that the poverty change due to a particular component for two contiguous subperiods is equal to the sum of the poverty change due to that component in each subperiod.<sup>5</sup>

Datt and Ravallion (1992) propose to address this problem by fixing the reference time point  $r \in [t_0, t_1]$  for the decomposition of *all* subperiods. This approach, however, is not ideal because the reference period lies outside most of the subperiods. Kakwani (2000) proposes another method to address this issue but his method is also unsatisfactory because an additional observation changes the decomposition results for *all* subperiods. In the next subsection, therefore, we propose a simple decomposition method that addresses all the issues mentioned above.

## 2.2 New Method of Dynamic Poverty Decomposition

To derive a decomposition method that is residual-free, time-reversion consistent, and subperiod additive, we can first consider an infinitesimal change of  $M(t)$  with respect to time  $t$  and find growth and redistribution components for this change. This allows us to ignore the (second-order) interaction effect so that the results are residual-free. By integrating each component over the time interval of interest, we obtain the growth and redistribution components. Because the reference time point is already built-in in this decomposition method, our method is clearly time-reversion consistent. The subperiod additivity follows from the property of integration. Despite this simplicity of the method, this is the first paper to employ time derivative and integration to poverty decomposition.<sup>6</sup>

Using the notations introduced in Section 2.1, we can obtain the following results:<sup>7</sup>

**Proposition 1.** *Let  $C \equiv \{RD, GR\}$  and define the following:*

$$\Delta_*^{RD} M(t_0, t_1) \equiv \int_{t_0}^{t_1} \left[ \int_0^{\tilde{z}} g \left( \frac{\tilde{y}}{\tilde{z}} \right) \frac{\partial \tilde{f}(\tilde{y}, t)}{\partial t} d\tilde{y} \right] dt \quad (5)$$

$$\Delta_*^{GR} M(t_0, t_1) \equiv \int_{t_0}^{t_1} \left[ g(1) \tilde{f}(\tilde{z}, t) - \int_0^{\tilde{z}} g' \left( \frac{\tilde{y}}{\tilde{z}} \right) \frac{\tilde{y}}{\tilde{z}^2} \tilde{f}(\tilde{y}, t) d\tilde{y} \right] \frac{d\tilde{z}}{dt} dt. \quad (6)$$

*Then, the pair  $(C, \{\Delta^C M(t_0, t_1)\}_{C \in C})$  is a time-reversion consistent and subperiod-additive poverty decomposition.*

Four points are in order. First, we chose to use the cumulative distribution function of the relative income  $\tilde{F}$  to represent our decomposition. In a number of previous studies of poverty decomposition, however, the Lorenz curve has been used. Because the Lorenz curve and  $\tilde{F}$  carry the same information, we can rewrite equations (5) and (6) using the Lorenz curve. However, we choose to use  $\tilde{F}$  for simplicity of presentation.

Second, it is straightforward to verify that  $\Delta_*^{RD} M(t_0, t_1) = 0$  holds when  $\tilde{F}(\tilde{y}, t)$  is constant over  $t \in [t_0, t_1]$  for given  $\tilde{y}$ . In other words,  $\Delta_*^{RD} M$  is driven by the changes in the distribution and thus we call it the redistribution component. Similarly, we have  $\Delta_*^{GR} M(t_0, t_1) = 0$  if  $\tilde{z}$  is constant over  $t$ . In line with the previous studies, we call  $\Delta_*^{GR} M$  the growth component, even though it is driven by the changes in both  $z$  and  $\mu$ . The reason that we do so is that all the

<sup>5</sup> If we allow  $s$  to be outside the interval between  $t_0$  and  $t_1$  and  $\Delta^c M(t_0, t_1)$  is continuous, then time-reversion consistency follows from subperiod additivity. To see this, first let  $t_1 \downarrow t_0$ . Then, we must have  $\Delta^c M(t_0, t_0) = 0$  by eq. (4). By letting  $t_0 = t_1 = \tau_0$  and  $s_1 = \tau_1$ , we have  $0 = \Delta^c M(t_0, t_1) + \Delta^c M(\tau_1, \tau_0)$ , proving the claim.

<sup>6</sup> In the context of source decomposition of changes in inequality, Okamoto (2011) proposes an integration-based approach to justify Shapley-type decomposition.

<sup>7</sup> All the proofs are provided in Appendix B.

changes are due to growth (change in the mean income) once the poverty line is fixed, which is what is assumed in most previous studies on poverty decomposition.

Third, the first [second] term in the integral in  $\Delta_*^{RD}M$  in equation (5) represents the change in poverty in the extensive [intensive] margin of poverty. The second term is 0 for the poverty rate measure and the first term is 0 for the Watts measure and the FGT measure with  $\alpha > 0$ . Therefore, in our applications, only one of these two terms matters.

Fourth, equations (5) and (6) show that  $\Delta_*^{RD}M$  and  $\Delta_*^{GR}M$  include both  $\tilde{f}$  and  $\tilde{z}$  in their integrations and thus depend on the way  $\tilde{f}$  and  $\tilde{z}$  vary between  $t = t_0$  and  $t = t_1$ . This means that the decomposition is path-dependent. Therefore, to implement equations (5) and (6), we need the observation of  $\tilde{z}$  and  $\tilde{F}$  over  $t \in [t_0, t_1]$  in general.

In a typical application, however, we observe them only at the beginning and end of the time interval (i.e.,  $t = t_0$  and  $t = t_1$ ) and possibly a few other time points in between. Therefore, we need to make some assumptions about the path to make the decomposition operational. Once the assumptions are made, we can calculate the integrals in equations (5) and (6) by numerical integration for a general form of  $\tilde{f}$  and  $\tilde{g}$ .

In each of the next two subsections, we make a set of specific assumptions about the path of  $\tilde{z}$  and  $\tilde{F}$ . In each case, they simultaneously and smoothly change over time, which is more realistic than the sequential changes (implicitly) assumed in the DR, KS, JT, and Shapley decompositions. We show that our assumptions lead to a convenient expression that does not require numerical integration.

## 2.3 Poverty Rate Decomposition under Linear Approximation

In this subsection, we assume that both  $\tilde{z}$  and  $\tilde{F}$  vary linearly between  $t = t_0$  and  $t = t_1$ . This assumption is not very restrictive, because it can be interpreted as taking a first-order approximation to an unknown functional form of  $\tilde{z}$  and  $\tilde{F}$  with respect to  $t$ . We define  $\tilde{z}_a \equiv \tilde{z}(t_a)$  and  $\tilde{F}_a(\cdot) \equiv \tilde{F}(\cdot, t_a)$  for  $a \in \{0,1\}$  to simplify the expressions below. For example,  $\tilde{z}_0$  and  $\tilde{z}_1$  are the relative poverty lines at the initial and terminal time points, respectively. Using these notations, our linearity assumption is as follows:

**Assumption 1.** For  $t \in [t_0, t_1]$ ,  $\tilde{z}$  and  $\tilde{F}$  respectively satisfy the following equations:

$$\tilde{F}(\tilde{y}, t) = (1 - \tau)\tilde{F}_0(\tilde{y}) + \tau\tilde{F}_1(\tilde{y}) \quad (7)$$

$$\tilde{z}(t) = (1 - \tau)\tilde{z}_0 + \tau\tilde{z}_1, \quad (8)$$

where  $\tau \equiv \frac{t-t_0}{t_1-t_0}$ .

We focus on the poverty rate measure  $P_0$  because it is the most frequently used measure of poverty in the literature and leads to a final expression that is simple and easy to implement, as shown in the following proposition:

**Proposition 2.** Suppose that Assumption 1,  $\tilde{z}_0 \neq \tilde{z}_1$ , and  $M = P_0$  hold. Then, the poverty decomposition given in Proposition 1 can be written as follows:

$$\Delta_{l*}^{RD}M(t_0, t_1) = \frac{\tilde{z}_1 P_1(\tilde{F}_1, \tilde{z}_1) - \tilde{z}_0 P_1(\tilde{F}_1, \tilde{z}_0) - \tilde{z}_1 P_1(\tilde{F}_0, \tilde{z}_1) + \tilde{z}_0 P_1(\tilde{F}_0, \tilde{z}_0)}{\tilde{z}_1 - \tilde{z}_0} \quad (9)$$

$$\Delta_{l*}^{GR}M(t_0, t_1) = P_0(\tilde{F}_1, \tilde{z}_1) - P_0(\tilde{F}_0, \tilde{z}_0) - \Delta_*^{RD}M(t_0, t_1). \quad (10)$$

We added the subscript  $l$  to the left-hand-side variable to emphasize that this is the linear approximation. In equation (9),  $P_1(\tilde{F}_a, \tilde{z}_b)$  for  $a \in \{0,1\}$  and  $b \in \{0,1\}$  is simply the poverty gap calculated with the relative income distribution for  $t = t_a$  and relative poverty line for  $t = t_b$ . In equation (10),  $P_0(\tilde{F}_a, \tilde{z}_a)$  for  $a \in \{0,1\}$  is just the poverty rate at  $t = t_a$ . Therefore,

equations (9) and (10) can be implemented without any special software package and without numerical integration.

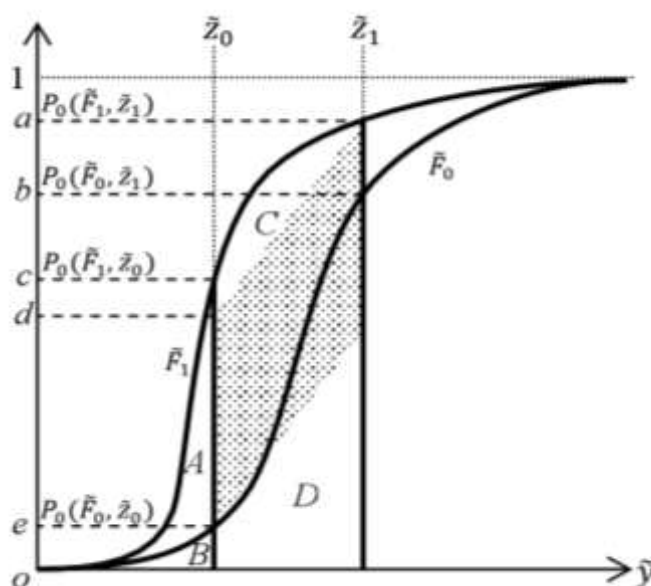
It should be noted that equations (9) and (10) do not satisfy the subperiod additivity. That is, we have  $\Delta_{l^*}^c M(t_0, t_2) \neq \Delta_{l^*}^c M(t_0, t_1) + \Delta_{l^*}^c M(t_1, t_2)$  in general. The reason for this is that the left-hand side of this equation is based on the assumption that  $\tilde{F}$  and  $\tilde{z}$  change linearly between  $t_0$  and  $t_2$ , whereas the right-hand side is based on the assumption that they change linearly piecewise between  $t_0$  and  $t_1$  and between  $t_1$  and  $t_2$ .

This breakdown of the subperiod additivity is *not* an undesirable property. The discussion above shows that we should generally prefer  $\Delta_{l^*}^c M(t_0, t_1) + \Delta_{l^*}^c M(t_1, t_2)$  over  $\Delta_{l^*}^c M(t_0, t_2)$  in the absence of other information, because the piecewise linear approximation is likely to produce more accurate approximation to the underlying path of change than the naïve linear approximation between  $t_0$  and  $t_2$ . This argument does not immediately apply to other decompositions. That is, for  $d \in \{KS, JT, DR, S\}$ , it is not immediately apparent whether we should favor  $\Delta_d^c M(t_0, t_1) + \Delta_d^c M(t_1, t_2)$  over  $\Delta_d^c M(t_0, t_2)$  because the former entails an implicit change in the reference period or the sequence of change. For example, in the case of *DR*, the former uses two different reference time periods;  $t_0$  for the poverty change between  $t_0$  and  $t_1$  and  $t_1$  for the change between  $t_1$  and  $t_2$ .

In Proposition 2, we excluded the possibility of  $\tilde{z}_0 = \tilde{z}_1$ . If equation (8) and  $\tilde{z}_0 = \tilde{z}_1$  hold, we have  $d\tilde{z}(t)/dt = 0$  for  $t \in [t_0, t_1]$  and thus  $\Delta_{l^*}^{GR} M = 0$  and  $\Delta_{l^*}^{RD} M = \Delta M$ . Since this is not an interesting case, we excluded this possibility in Proposition 2. However, it should be noted that  $\tilde{z}_0 = \tilde{z}_1$  does not imply  $\Delta_{l^*}^{GR} M = 0$  in general without the linearity assumption.

Figure 1 is useful for interpreting the decomposition results given in equations (9) and (10) and for comparing our method with previously proposed methods. The figure provides a graphical representation of the cumulative distribution function of relative income and the relative poverty line at  $t = t_0$  and  $t = t_1$ . The lengths of line segments *oe* and *oa* respectively represent the poverty rate at  $t = t_0$  and  $t = t_1$ , or  $P_0(\tilde{F}_0, \tilde{z}_0)$  and  $P_1(\tilde{F}_1, \tilde{z}_1)$ . Therefore, poverty has worsened between these two time periods in this figure. The capital letters *A* to *D* are used to represent an area defined by bold lines. Note that areas *C* and *D* include some parts of the shaded areas.

**Figure 1: Graphical Representation of Poverty Rate Decomposition under Assumption 1**



Source: Author's own illustration.

The goal of a conventional two-way poverty decomposition is to split the line segment  $ea$  into the growth and redistribution components. If the changes in the relative poverty line and the relative income distribution take place sequentially, this decomposition is straightforward because we only need to look at one component at a time. In this case, the redistribution [growth] component measures the effect of the change in the relative income distribution [the relative poverty line]. Graphically, the redistribution component is the vertical distance between the two cumulative distributions  $\tilde{F}_0$  and  $\tilde{F}_1$  at a particular  $\tilde{z}$ , whereas the growth component is the difference in a particular cumulative distribution between  $\tilde{z}_0$  and  $\tilde{z}_1$ .

If we assume that the change in the relative poverty line precedes [follows] the change in the relative income distribution, we obtain the KS [JT] decomposition. The growth and redistribution components are the lengths of line segments  $eb$  [ $ca$ ] and  $ba$  [ $ec$ ], respectively, in Figure 1. In the case of the DR decomposition, the growth and redistribution component are  $ec$  and  $eb$ , respectively, and the residual component is what is not explained by these terms, which is  $ea - ec - eb$ . The Shapley decomposition is simply the average of the KS and JT decompositions, respectively.

To interpret equation (9), first note that  $\tilde{z}_a P_1(\tilde{F}_b, \tilde{z}_a)$  represents the average shortfall per person from the poverty line relative to the mean income. Therefore,  $\tilde{z}_a P_1(\tilde{F}_b, \tilde{z}_a)$  is the area below the cumulative distribution function  $\tilde{F}_b$  and to the left of  $\tilde{z}_a$  (see also equation (B2) in Appendix B). For example,  $\tilde{z}_0 P_1(\tilde{F}_1, \tilde{z}_0)$  is the area of  $A$  and  $B$  combined. It is straightforward to verify that the numerator of the right-hand side of equation (9) is area  $C$ . By dividing this area by  $\tilde{z}_1 - \tilde{z}_0$ , we see that the redistribution component is represented by the vertical distance between  $\tilde{F}_0$  and  $\tilde{F}_1$  averaged over  $\tilde{z} \in \{\tilde{z}_0, \tilde{z}_1\}$ . Suppose now that the redistribution component is  $ed$  in Figure 1. Then, the shaded parallelogram has the same area as  $C$ . Equation (10) shows that the growth component is the part of poverty change not accounted for by the redistribution component, which is  $da$  in Figure 1.

Figure 1 also allows us to show the relationship between the Shapley decomposition and our decomposition. While taking the average of possible sequential changes appears arbitrary, our results show that it is not completely unreasonable. To see this, first note that we obtain the Shapley redistribution component if we replace the numerator of equation (9) by the area of the trapezoid (not explicitly drawn) with two bases  $ec$  and  $ba$ . Therefore, we can consider the Shapley decomposition as a way to approximate the area  $C$  by this trapezoid. In particular, if the cumulative distribution functions  $\tilde{F}_0$  and  $\tilde{F}_1$  are linear between  $\tilde{z} = \tilde{z}_0$  and  $\tilde{z} = \tilde{z}_1$ , the Shapley decomposition is exactly equal to our linear approximation in Proposition 2.

## 2.4 Poverty Rate Decomposition under Log-Linear Approximation

Assumption 1 may be reasonable when the change in  $\tilde{z}$  is relatively small. However, when the economy is experiencing rapid economic growth for a long period of time, the linearity assumption in Assumption 1 may not be appropriate. In such a case, we may be able to obtain a better approximation by making a linear approximation with respect to the logarithmic relative income  $\tilde{\eta} \equiv \ln \tilde{y}$  and logarithmic relative poverty line  $\tilde{\zeta} \equiv \ln \tilde{z}$ . Therefore, we make the following assumption in this subsection:

**Assumption 2.** Let the cumulative distribution function of  $\tilde{\eta}$  at time  $t$  be  $\tilde{\Phi}(\tilde{\eta}, t)$ . For  $t \in [t_0, t_1]$ ,  $\tilde{\Phi}$  and  $\tilde{\zeta}$  respectively satisfy the following equations:

$$\tilde{\Phi}(\tilde{\eta}, t) = (1 - \tau)\tilde{\Phi}_0(\tilde{\eta}) + \tau\tilde{\Phi}_1(\tilde{\eta}) \quad (11)$$

$$\tilde{\zeta}(t) = (1 - \tau)\tilde{\zeta}_0 + \tau\tilde{\zeta}_1, \quad (12)$$

where  $\tau \equiv \frac{t-t_0}{t_1-t_0}$ ,  $\tilde{\Phi}_a(\tilde{\eta}) \equiv \tilde{\Phi}(\tilde{\eta}, t_a)$ , and  $\tilde{\zeta}_a \equiv \tilde{\zeta}(t_a)$  for  $a \in \{0, 1\}$ .

Assumption 2 is identical to Assumption 1 except that relative income distribution and relative poverty line are expressed in logarithmic form. As with Section 1, we focus on the poverty rate measure in this subsection. Under Assumption 2, we have the following results:

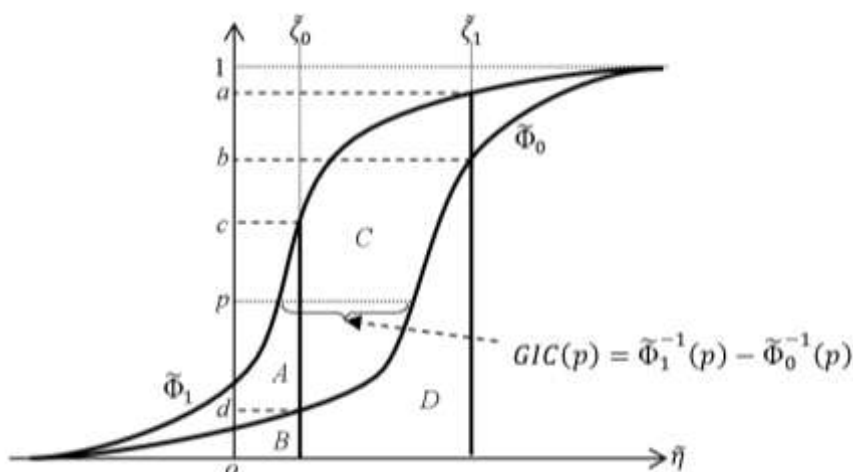
**Proposition 3.** Suppose that Assumption 2 and  $M = P_0$  hold. Then, the poverty decomposition given in Proposition 1 can be written as follows:

$$\Delta_{H^*}^{RD} M(t_0, t_1) = \frac{W(\tilde{F}_1, \tilde{z}_1) - W(\tilde{F}_1, \tilde{z}_0) - W(\tilde{F}_0, \tilde{z}_1) + W(\tilde{F}_0, \tilde{z}_0)}{\tilde{\zeta}_1 - \tilde{\zeta}_0} \quad (13)$$

$$\Delta_{H^*}^{GR} M(t_0, t_1) = P_0(\tilde{F}_1, \tilde{z}_1) - P_0(\tilde{F}_0, \tilde{z}_0) - \Delta_{H^*}^{RD} M(t_0, t_1). \quad (14)$$

In Figure 2, we provide a graphical representation of the poverty decomposition under Assumption 2, which is similar to Figure 1. One important difference is, however, that the area below  $\tilde{\Phi}_a$  and to the right of  $\tilde{\zeta}_b$  represents the Watts measure  $W(\tilde{\Phi}_a, \tilde{\zeta}_b)$ , because the Watts measure is the cumulative gap between the logarithmic poverty line and the logarithmic income (see also equation (20) in Appendix B). Therefore, the numerator of the right-hand side of equation (13) is area  $C$  and  $\Delta_{H^*}^{RD} M$  is the vertical distance between  $\tilde{\Phi}_0$  and  $\tilde{\Phi}_1$  averaged over  $\tilde{\zeta} \in [\tilde{\zeta}_0, \tilde{\zeta}_1]$ .

**Figure 2: Graphical Representation of Poverty Rate Decomposition under Assumption 2**



Source: Author's own illustration

Figure 2 also helps us to understand the relationship between our poverty decomposition analysis and pro-poor growth. Notice first that  $\tilde{\Phi}_a^{-1}(p)$  for  $a \in \{0,1\}$  is the 100p-percentile logarithmic income. Therefore, the growth incidence curve proposed by Ravallion and Chen (2003), which is the growth of income for each quantile, appears as the horizontal distance between  $\tilde{\Phi}_0^{-1}(p)$  and  $\tilde{\Phi}_1^{-1}(p)$  in Figure 2.<sup>8</sup> In the graph,  $GIC(p)$  is negative because  $\tilde{\Phi}_1^{-1}(p)$  is to the left of (i.e., smaller than)  $\tilde{\Phi}_0^{-1}(p)$ . An overall measure of pro-poor growth proposed by Son and Kakwani (2008, equation (13)) is the integral of  $GIC(p)$  over the unit interval. This simply appears as the net area between  $\tilde{\Phi}_0$  and  $\tilde{\Phi}_1$ , which is negative in Figure 2.

<sup>8</sup> Ravallion and Chen (2003) uses a discrete time model and their growth rate is the growth per period. The growth in Figure 2 also refers to the growth in the time period between  $t = t_0$  and  $t = t_1$  and is expressed as a difference in the logarithmic income per capita.

## 2.5 Robustness Check with a Speed of Change Parameter

The KS and JT decompositions implicitly assume that the mean and distribution of income change sequentially. The RD and Shapley decompositions do not impose a particular sequence, but their calculations are also based on some sequential changes. Our results presented in sections 2.3 and 2.4, on the other hand, are based on the assumption that both change simultaneously and smoothly, which is more realistic.

However, one could argue that Assumption 1 is strong because both relative income distribution and relative poverty line are assumed to change at the “same speed.” Therefore, we relax Assumption 1 and replace equation (8) with the following equation:

$$\tilde{z}(t) = (1 - \tau^\gamma) \tilde{z}_0 + \tau^\gamma \tilde{z}_1, \quad (15)$$

where  $\gamma (> 0)$  is the parameter that describes the speed of change for  $\tilde{F}$  relative to  $\tilde{z}$ . When  $\gamma$  is large, most of the changes in  $\tilde{z}$  occur when  $\tilde{F}$  is already close to  $\tilde{F}_1$ . In fact, when we let  $\gamma \rightarrow \infty$ , the decomposition converges to the JT decomposition. On the other hand, when we let  $\gamma \downarrow 0$ , the decomposition converges to the KS decomposition. Therefore, by varying  $\gamma$ , we can check the robustness of the results in Proposition 2. It is also possible to do a similar robustness check for Proposition 3 by replacing  $\tau$  with  $\tau^\gamma$  in the right-hand side of equation (12).

## 2.6 Extension to Six-Way Decomposition

In this subsection, we consider a more detailed decomposition, in which the poverty change in each group in the population is decomposed into six components. While each group represents a region in our application, it may represent other household characteristics such as the household size and the sector in which the household head works. Our decomposition is useful because researchers and policy makers are often interested in finding which group is contributing to national poverty change and why. While we consider a particular six-component decomposition, our decomposition can be easily modified to have more or fewer components.

It should also be noted that our decomposition presented in Proposition 4 below can be considered as an integration of the sector-based decomposition proposed by Ravallion and Huppi (1991) and the growth-redistribution decomposition discussed earlier. Unlike Ravallion and Huppi (1991), however, our decomposition does not have an interaction term, whose interpretation is not straightforward. Therefore, our results allow researchers and policy makers to identify the source of poverty change more easily and more clearly.

We hereafter assume that there are  $G$  groups (e.g., regions or sectors) in the country and each group  $g$  has a group-specific poverty line  $z^g(t)$  at time  $t$ . We further assume that the group-specific poverty lines satisfy  $z^g = \sum_{j=1}^J p_j^g(t) q_j^g(t)$ , where  $p_j^g(t)$  and  $q_j^g(t)$  are the price and quantity of good  $j \in \{1, \dots, J\}$  consumed by a typical household near the poverty line in group  $g$ . Therefore, the poverty lines may change not only by the changes in prices but also by the changes in the underlying bundle of goods.

We denote the population share of group  $g$  by  $w^g$ . The income distribution of group  $g$  has the probability density function  $f^g$  and cumulative distribution function  $F^g$ . Therefore, we have:

$$f(y, t) = \sum_g w^g(t) f^g(y, t) \quad \text{and} \quad F(y, t) = \sum_g w^g(t) F^g(y, t) \quad (16)$$

for all  $t$  and  $y$ . We denote the mean income for group  $g$  at time  $t$  by  $\mu^g(t) \equiv \int_0^\infty y f^g(y, t) dy$  and its ratio to the population mean by  $\hat{\mu}^g(t) \equiv \mu^g(t) / \mu(t)$ . We denote the income relative to the group mean by  $\hat{y} \equiv y / \mu^g$  and the poverty line relative to the group mean by  $\hat{z}^g \equiv z^g / \mu^g = z^g / \mu \hat{\mu}^g$ . The relative income distribution for group  $g$  is characterized by the



probability density function  $\hat{f}^g$ , which satisfies  $\hat{f}^g(\hat{y}, t) = \mu^g f^g(y, t)$  for all  $g$ . We use hat notations ( $\hat{\cdot}$ ) here to emphasize that the relative income is relative to the group mean. Using these notations, we can construct the following six-way decomposition:

**Proposition 4.** Let  $C \equiv \{PS, WR, BR, NG, IF, MC\}$  and define the following terms:

$$\begin{aligned} \Delta_{**}^{PS} M^g(t_0, t_1) &\equiv \int_{t_0}^{t_1} \left[ \frac{dw^g}{dt} \int_0^{\hat{z}^g} g\left(\frac{\hat{y}}{\hat{z}^g}\right) \hat{f}^g d\hat{y} \right] dt \\ \Delta_{**}^{WR} M^g(t_0, t_1) &\equiv \int_{t_0}^{t_1} \left[ w^g \int_0^{\hat{z}^g} g\left(\frac{\hat{y}}{\hat{z}^g}\right) \frac{\partial \hat{f}^g}{\partial t} d\hat{y} \right] dt \\ \Delta_{**}^{BR} M^g(t_0, t_1) &\equiv \int_{t_0}^{t_1} \left[ w^g \left[ g(1) \hat{f}^g(\hat{z}^g, t) - \int_0^{\hat{z}^g} g'\left(\frac{\hat{y}}{\hat{z}^g}\right) \frac{\hat{y}}{(\hat{z}^g)^2} \hat{f}^g d\hat{y} \right] \frac{z^g}{\mu(\hat{\mu}^g)^2} \frac{d\hat{\mu}^g}{dt} \right] dt \\ \Delta_{**}^{NG} M^g(t_0, t_1) &\equiv \int_{t_0}^{t_1} \left[ w^g \left[ g(1) \hat{f}^g(\hat{z}^g, t) - \int_0^{\hat{z}^g} g'\left(\frac{\hat{y}}{\hat{z}^g}\right) \frac{\hat{y}}{(\hat{z}^g)^2} \hat{f}^g d\hat{y} \right] \frac{z^g}{\mu^2 \hat{\mu}^g} \frac{d\mu}{dt} \right] dt \\ \Delta_{**}^{IF} M^g(t_0, t_1) &\equiv \int_{t_0}^{t_1} \left[ w^g \left[ g(1) \hat{f}^g(\hat{z}^g, t) - \int_0^{\hat{z}^g} g'\left(\frac{\hat{y}}{\hat{z}^g}\right) \frac{\hat{y}}{(\hat{z}^g)^2} \hat{f}^g d\hat{y} \right] \frac{1}{\mu \hat{\mu}^g} \sum_j \frac{q_j^g dp_j^g}{dt} \right] dt \\ \Delta_{**}^{MC} M^g(t_0, t_1) &\equiv \int_{t_0}^{t_1} \left[ w^g \left[ g(1) \hat{f}^g(\hat{z}^g, t) - \int_0^{\hat{z}^g} g'\left(\frac{\hat{y}}{\hat{z}^g}\right) \frac{\hat{y}}{(\hat{z}^g)^2} \hat{f}^g d\hat{y} \right] \frac{1}{\mu \hat{\mu}^g} \sum_j \frac{p_j^g dq_j^g}{dt} \right] dt, \end{aligned}$$

where  $\Delta_{**}^c M^g$  for  $c \in C$  is the contribution of group  $g$  to component  $c$ , which is denoted by  $\Delta_{**}^c M \equiv \sum_g \Delta_{**}^c M^g$ . Then, the pair  $(C, \{\Delta_{**}^c M(t_0, t_1)\}_{c \in C})$  is a time-reversion consistent and subperiod-additive poverty decomposition.

The first component,  $\Delta_{**}^{PS} M$ , is the population-shift component because it accounts for the poverty changes due to the changes in the relative size of each group. The population-shift component represents both inter-group migration and differences in the mortality and fertility across groups. When  $M = P_0$ , the population-shift component positively contributes to poverty reduction when the size of wealthier groups grows faster than the size of poorer groups. For example, if we define groups to be urban and rural areas, we can measure the demographic effect of urbanization on poverty by the population-shift component.

The second component,  $\Delta_{**}^{WR} M$ , is the within-group redistribution component, which accounts for the poverty change due to the change in the relative income distribution in each group. The third component,  $\Delta_{**}^{BR} M$ , is the between-group redistribution component, because it is driven by the change in the ratio of the group-level mean income to the population mean.

The fourth component,  $\Delta_{**}^{NG} M$ , can be called the nominal growth component because it represents the change in poverty due to the change in the nominal mean income. The fifth component,  $\Delta_{**}^{IF} M$ , can be considered the inflation component, because it represents the poverty change due to the changes in the price of the bundle of goods for the poverty line. The sixth component,  $\Delta_{**}^{MC} M$ , can be called the methodological change component, because this is the poverty change due to the quantity changes in the underlying bundle of goods for the poverty line. The fifth and sixth components combined represent the changes in poverty due to the shift in the nominal poverty lines.

As with the previous cases, we need to make some assumptions about the path of change to implement the decomposition in Proposition 4. Therefore, we simply assume that  $w^g$ ,  $\hat{f}^g$ ,  $\mu$ ,  $\hat{\mu}^g$ ,  $p_j^g$ , and  $q_j^g$  change linearly. That is, we first estimate  $\hat{f}^g$  by kernel density estimation and calculate  $w^g$ ,  $\mu$ ,  $\hat{\mu}^g$ ,  $p_j^g$ , and  $q_j^g$  for all  $j$  at  $t = t_0$  and  $t = t_1$ . Then, we take the linear

interpolation. In case of  $w^g$ , for example, we assume  $w^g(t) = (1 - \tau)w^g(t_0) + \tau w^g(t_1)$  for  $\tau \equiv (t - t_0)/(t_1 - t_0)$ . We make a similar assumption for  $\mu, \hat{\mu}^g, p_j^g$ , and  $q_j^g$ . Note that we are unable to obtain simple closed-form results, because  $\tilde{f}$  is multiplied with another time-varying variable in the integration.

## 2.7 Some Implementation Issues

To implement the decomposition in Proposition 1 in its general form, we need to estimate  $\tilde{f}$  in a typical empirical setup. Therefore, the choice of kernel density function and bandwidth used in the estimation of  $\tilde{f}$  affects the results. Following the standard choice in the literature, we use the Epanechnikov kernel density function. Typically, the choice of the kernel density function is not particularly important.<sup>9</sup>

However, the choice of the bandwidth is important and can affect the decomposition results in a non-negligible manner. If we use a small bandwidth, the resulting poverty estimates are closer to those directly calculated from the observed data. However, the graph of the estimated density function is likely to be more spiky. On the other hand, if we use a large bandwidth, the estimate of  $\tilde{f}$  is likely to become inaccurate.

This issue is particularly important for the decomposition of  $P_0$ . If the bandwidth is too small,  $\tilde{f}$  takes a very high value near observed income levels and 0 for all other values. In this case, we cannot estimate  $\tilde{f}$  very accurately, which makes it difficult to perform the numerical integration required for decomposition.

Given the considerations mentioned above, we set the half-width of kernel at  $b = 0.01$  (i.e., one percentage point in the relative income), which is small enough to reproduce the poverty statistics that are very close to poverty statistics derived directly from the original sample but large enough to eliminate the spikes in the density estimate from our data. All the empirical results presented in section 4 that rely on kernel density estimation are based on this choice of bandwidth.<sup>10</sup>

To implement the numerical integration, we adopted the following procedure. First, we estimate  $\tilde{f}(\cdot, t_0)$  and  $\tilde{f}(\cdot, t_1)$  on a set of fixed evaluation points  $\{a_1, a_2, \dots, a_M\}$ , where  $M$  is the number of evaluation points,  $a_1 = 0$ , and  $a_M \geq \bar{z}(t)$  for all  $t \in [t_0, t_1]$ . Second, following the interpolation rule specified in the assumption (e.g., equation (7)), we derive  $\tilde{f}(\cdot, t)$  on each evaluation point at time  $\{b_1, b_2, \dots, b_N\}$ , where  $b_1 = t_0$ ,  $b_N = t_1$ , and  $N$  is the number of evaluation points for the outer integral in Proposition 1. Third, using the estimate of  $\tilde{f}(\cdot, t)$ , we evaluate the inner integral using a numerical integration method. Once we obtain the inner integral, we evaluate the outer integral in a similar manner.<sup>11</sup> To obtain sufficiently accurate results, we set  $N = M = 20,000$  in our empirical application.<sup>12</sup>

<sup>9</sup> Note here that we only need the kernel density estimates for the lower tail for our analysis. By focusing on the lower tail, we can reduce the memory usage.

<sup>10</sup> In Appendix D, we present the results with 50% larger and smaller bandwidth to verify the robustness of our results.

<sup>11</sup> We implemented this with a quadratic interpolation, which is essentially Simpson's rule. We make some adjustments, because the upper end of the integral,  $\bar{z}$ , varies over time and does not coincide with an evaluation point in general. Also, because  $d\bar{z}/dt$  diverges to infinity at  $t = t_0$  when  $\gamma < 1$  under the assumption of eq. (15), we use a linear approximation of the expression inside the square bracket in eq. (6) in this case to calculate the integral over the first interval (i.e.,  $[b_1, b_2]$ ). The details of this treatment are given in Appendix C.

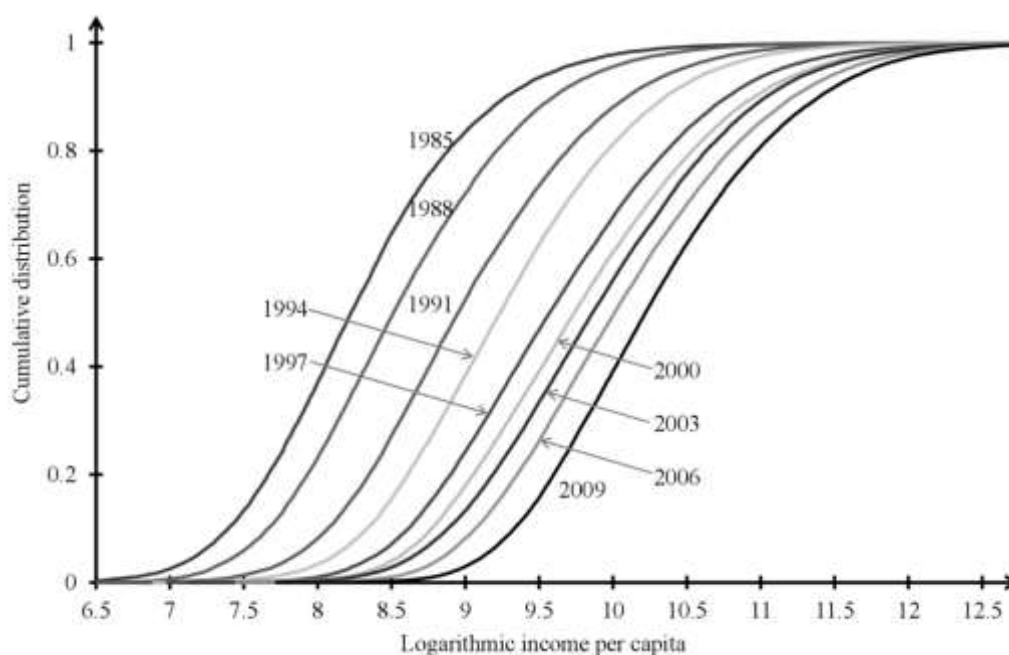
<sup>12</sup> Comparison of the numerical integration results under linear assumption with the analytical results presented in Section 2.3 indicates that the margin of the error is at most 0.004 percentage points for two-way decomposition analysis. For other results, we cannot directly evaluate the accuracy of our numerical results. However, the comparison between the sum of each component in the decomposition analysis and the observed change provides some guidance. According to this criterion, the six-way decomposition in Section 4 is slightly less

### 3. DATA AND POVERTY MEASUREMENT

We use the public user files for the following nine rounds of the Family Income and Expenditure Survey (FIES): 1985, 1988, 1991, 1994, 1997, 2000, 2003, 2006, and 2009. The FIES was collected by the National Statistics Office (NSO). The FIES data include income, expenditure, and various other household information. They are used for calculating official poverty statistics published by the National Statistical Coordination Board (NSCB). For the six-way decomposition in Proposition 4, we also use the price data taken from the consumer price index (CPI), also collected by the NSO. As noted earlier, we take each region as a group in the six-way decomposition, but the definition of regions in the Philippines has changed over time. Thus, we choose to adopt the latest definition, which has 17 regions, and constructed the region variable under this definition for earlier rounds of FIES from the province variable in the data.

The distribution of the logarithmic nominal annual income per capita in the Philippines is presented in Figure 3. The figure shows that the distribution in each FIES round after 1985 first-order stochastically dominates the previous round, implying that the nominal income has increased for both the rich and the poor in the Philippines. However, this figure ignores the inflation and heterogeneity across regions, and thus does not provide a clear picture about the sources of poverty change in the Philippines. Therefore, the poverty decomposition methods developed in the previous section are useful.

**Figure 3: Cumulative Distribution Function of the Logarithmic Income per Capita per Year in Philippine Pesos**



Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

To implement the decomposition, we first need to set the poverty lines. A natural choice would be the NSCB's official poverty lines, because the official poverty statistics are widely used by the government and are one of the most important statistics for the formulation of poverty reduction policies in the Philippines.

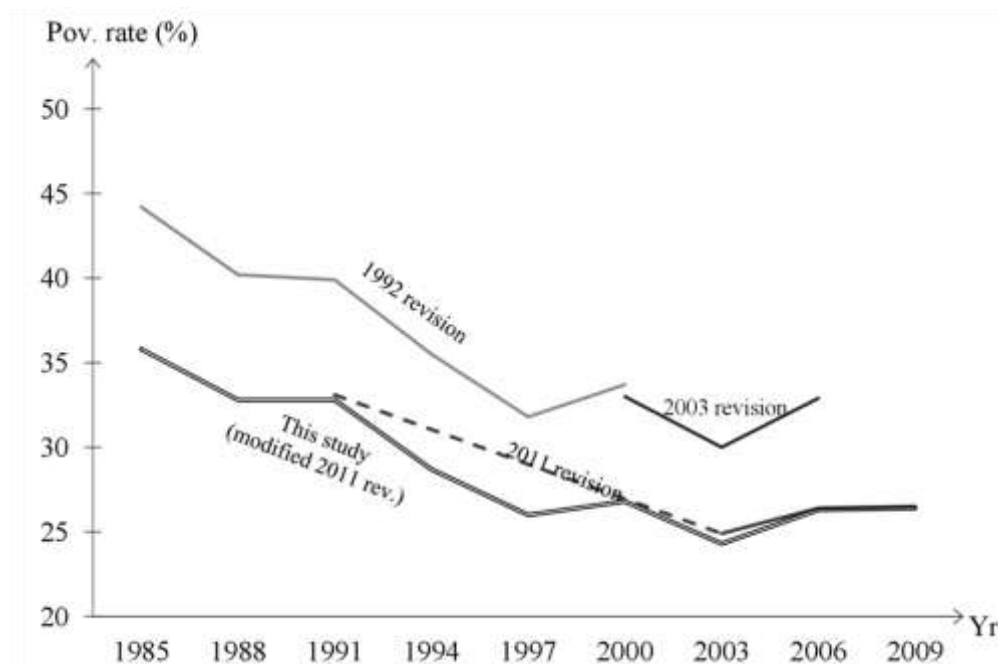
However, the official methodology for setting the poverty lines has been revised three times. As clearly seen in Figure 4, the estimates based on different revisions of methodology are

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accurate. However, our estimates (in percentage points) are accurate at least up to the first decimal point and up to the second decimal point in most cases. Detailed results are provided in Appendix D.

not directly comparable because poverty estimates for a given year may vary substantially with the methodology employed. Moreover, no revision of the official methodology covers the entire nine rounds of FIES, making it difficult to understand the nature of the long-term poverty changes in the Philippines. Furthermore, even when the same revision of methodology is used, the comparability of official poverty statistics over time and across regions has been disputed (Balisacan 2003; Bernales 2009) and multiple versions of “official” estimates appear to exist for some years.

**Figure 4: Comparison between Official Poverty Statistics and Our Poverty Statistics**



Note: The horizontal and vertical axis indicate the calendar year and the poverty rate in percentage, respectively.

Source: Official figures are compiled from Asian Development Bank (2005) and [http://www.nscb.gov.ph/poverty/2009/table\\_2.asp](http://www.nscb.gov.ph/poverty/2009/table_2.asp).

Hence, we chose to adopt a modified version of the 2011 revision of the official methodology and produced our own back estimates. For the years in which official poverty estimates based on the 2011 revision of the methodology are available, our poverty statistics are very similar to the official poverty statistics as shown in Figure 4. They also have a trend very similar to official poverty statistics for other years. Therefore, our poverty statistics capture well the changes in the official poverty statistics over time. Further details on the data and poverty measurement for this study are provided in Appendix A.

## 4. RESULTS

In this section, we present various decomposition results. We start with the two-way decomposition under the linear approximation discussed in section 2.3, the results of which are given in Table 1. While convenient analytical results under linear approximation are only available for the poverty rate  $P_0$ , we have also carried out the decomposition for poverty gap  $P_1$ , poverty severity  $P_2$ , and the Watts measure  $W$  by numerical integration.<sup>13</sup> The first two columns in Table 1 provide the initial year  $t_0$  and the terminal year  $t_1$ . For each poverty

<sup>13</sup> The decomposition result for  $P_0$  reported in Table 1 is based on Proposition 2 and thus derived without kernel density estimation. For all other decomposition results, kernel density estimation is used. In the case of two-way decomposition,  $\hat{f}$  is estimated for the Philippines. In the case of six-way decomposition,  $\hat{f}^g$  is estimated for each region of the Philippines. This leads to a small discrepancy in  $\Delta$  across tables.

measure, we report the initial level of poverty  $M(t_0)$ , the change  $\Delta$  in the poverty measure between  $t_0$  and  $t_1$ , the growth component  $GR$ , and the redistribution component  $RD$ . For example, the growth component of the change in poverty gap between 1994 and 1997 is -3.40 percentage points. The last row (all periods) is the sum of all the changes in the eight 3-year periods.

Table 1 shows that the poverty changes in the Philippines have been largely driven by the growth component. Notice here that the growth component in this analysis refers to the change in poverty due to the relative poverty line, or the poverty line over the mean income. Therefore, the effect of inflation at the poverty line is already accounted for in the growth component.

Table 1 also shows that the patterns of poverty change are similar across all the poverty measures considered here. Over the periods between 1985 and 2009, about 30% of the poverty reduction achieved by economic growth has been offset by worsened income inequality, regardless of the poverty measure used. Most of the effects of worsening income inequality took place in the two periods 1988–1991 and 1994–1997.

One concern about this analysis is that the results may be driven by our linearity assumption. Therefore, we have carried out a robustness check assuming equations (7) and (15) as described in section 2.5. Table 2 shows the decomposition results for the poverty rate using various values of  $\gamma$  and a few other methods described in section 2. The third column,  $\Delta$ , is the change in poverty rate between  $t_0$  and  $t_1$ . The fourth column,  $\Delta_{KS}^{GR}$ , is the growth component in the KS decomposition, which corresponds to  $\gamma \downarrow 0$ . The fifth column,  $\Delta_{*,\gamma=1/4}^{GR}$ , is the growth component for  $\gamma = 1/4$ . The results in the seventh column,  $\Delta_{I*}^{GR}$ , are the same as those presented in the fifth column of Table 1. The tenth column,  $\Delta_{JT}^{GR}$ , is the growth component for the JT decompositions, which corresponds to  $\gamma \rightarrow \infty$ . As shown in the fifth to tenth columns of Table 2, the decomposition results are quite stable.

The eleventh column,  $\Delta_S^{GR}$ , is the growth component in the Shapley decomposition. The twelfth column,  $\Delta_{ll*}^{GR}$ , is the growth component under the log-linear approximation discussed in Section 4. These decomposition results are also similar to the results presented in Table 1. The last column,  $\Delta_{DR}^{GR}$ , is the residual component in the DR decomposition, which turns out to be small. Thus, our decomposition results and the decomposition results based on the existing methods are generally close in the Philippines for the time periods we have considered.

However, our finding does not imply that the choice of decomposition method does not matter. To see how much the choice of method may matter, we carry out an experiment for poverty rate decomposition. We assume that the linearity assumption is satisfied piecewise for all eight 3-year periods from 1985 to 2009 and treat the decomposition under this assumption as the benchmark decomposition result. We then drop from the data some years in between and calculate the growth component of poverty change for each period in the data (e.g., if years 1988, 1991, 1994, 2003, and 2006 are dropped from the data, there are three periods: 1985–1997, 1997–2000, and 2000–2009) and add the growth component for these periods to arrive at an estimate of the growth component for the entire period 1985–2009. We do this for all the possible combinations for each number of observations dropped. We then calculate the maximum and mean absolute deviation of the estimated growth component from the benchmark growth component. Note here that whether we use the growth component or redistribution component makes no difference in the two-way decomposition because the change in poverty between 1985 and 2009 is fixed and thus the absolute deviations are the same for growth and redistribution components.

**Table 1: Comparison of Two-Way Decomposition Results across Various Poverty Measures**

$t_0$	$t_1$	Poverty rate ( $P_0$ )				Poverty gap ( $P_1$ )				Poverty severity ( $P_2$ )				Watts measure ( $W$ )			
		$M(t_0)$	$\Delta$	$GR$	$RD$	$M(t_0)$	$\Delta$	$GR$	$RD$	$M(t_0)$	$\Delta$	$GR$	$RD$	$M(t_0)$	$\Delta$	$GR$	$RD$
1985	1988	35.83	-3.07	-2.82	-0.25	11.13	-1.46	-1.12	-0.34	4.82	-0.85	-0.56	-0.29	15.08	-2.28	-1.61	-0.67
1988	1991	32.76	0.08	-3.15	3.23	9.66	0.24	-1.33	1.57	3.97	0.21	-0.66	0.87	12.80	0.48	-1.90	2.39
1991	1994	32.84	-4.21	-2.49	-1.72	9.90	-1.45	-1.01	-0.44	4.18	-0.66	-0.50	-0.16	13.28	-2.01	-1.44	-0.57
1994	1997	28.63	-2.68	-8.47	5.79	8.45	-0.84	-3.40	2.56	3.52	-0.39	-1.67	1.28	11.27	-1.19	-4.83	3.64
1997	2000	25.95	0.86	0.98	-0.12	7.61	0.22	0.39	-0.17	3.13	0.06	0.19	-0.13	10.08	0.26	0.55	-0.29
2000	2003	26.80	-2.52	-1.01	-1.51	7.83	-0.75	-0.39	-0.36	3.20	-0.28	-0.19	-0.09	10.34	-0.96	-0.55	-0.41
2003	2006	24.29	1.99	1.71	0.27	7.08	0.52	0.68	-0.16	2.91	0.14	0.33	-0.19	9.39	0.58	0.96	-0.38
2006	2009	26.27	0.09	1.07	-0.98	7.60	-0.34	0.42	-0.77	3.05	-0.23	0.20	-0.43	9.97	-0.55	0.59	-1.14
All periods			-9.47	-14.17	4.70		-3.87	-5.75	1.88		-2.00	-2.86	0.87		-5.66	-8.23	2.57

Note:  $M(t_0)$  is in percentage.  $\Delta$ ,  $GR$ , and  $RD$  are all in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

**Table 2: Decomposition of Poverty Rate with Various Methods and Values of  $\gamma$**

$t_0$	$t_1$	$\Delta$	$\Delta_{KS}^{GR}$	$\Delta_{*,\gamma=1/4}^{GR}$	$\Delta_{*,\gamma=1/2}^{GR}$	$\Delta_{l*}^{GR}$	$\Delta_{*,\gamma=2}^{GR}$	$\Delta_{*,\gamma=4}^{GR}$	$\Delta_{JT}^{GR}$	$\Delta_S^{GR}$	$\Delta_{ll*}^{GR}$	$\Delta_{DR}^{RS}$
1985	1988	-3.07	-2.92	-2.91	-2.87	-2.82	-2.77	-2.73	-2.67	-2.80	-2.82	0.25
1988	1991	0.08	-3.19	-3.19	-3.16	-3.15	-3.14	-3.14	-3.12	-3.16	-3.15	0.06
1991	1994	-4.21	-2.49	-2.51	-2.49	-2.49	-2.49	-2.50	-2.50	-2.49	-2.49	-0.01
1994	1997	-2.68	-8.38	-8.48	-8.45	-8.47	-8.48	-8.48	-8.46	-8.42	-8.47	-0.08
1997	2000	0.85	0.95	0.97	0.97	0.98	0.99	0.99	1.00	0.97	0.98	0.05
2000	2003	-2.51	-1.03	-1.03	-1.02	-1.01	-1.00	-0.99	-0.98	-1.00	-1.00	0.05
2003	2006	1.98	1.76	1.75	1.73	1.71	1.70	1.69	1.68	1.72	1.71	-0.08
2006	2009	0.09	1.03	1.05	1.06	1.07	1.09	1.11	1.13	1.08	1.08	0.10

Note: All the figures for poverty decomposition are in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

Table 3 shows the results of this experiment. The KS, JT, and S columns respectively show the maximum and mean absolute deviations of the growth component in the KS, JT and Shapley decompositions from the benchmark growth component, whereas the equation (10) and equation (14) columns respectively show the corresponding statistics under the linear and log-linear approximations.

**Table 3: The Mean and Maximum Absolute Deviations from the Benchmark Decomposition Results**  
(percentage points)

# dropped obs.	# comb	stat	KS	JT	S	eq. (10)	eq. (14)
7	1	Max	<b>0.25</b>	<b>0.34</b>	<b>0.04</b>	<b>0.22</b>	<b>0.23</b>
		Mean	0.25	0.34	0.04	0.22	0.23
6	7	Max	<b>0.50</b>	<b>0.98</b>	<b>0.45</b>	<b>0.36</b>	<b>0.37</b>
		Mean	0.22	0.53	0.25	0.19	0.19
5	21	Max	<b>0.96</b>	<b>0.97</b>	<b>0.47</b>	<b>0.41</b>	<b>0.41</b>
		Mean	0.34	0.54	0.23	0.16	0.17
4	35	Max	<b>1.01</b>	<b>1.10</b>	<b>0.49</b>	<b>0.38</b>	<b>0.40</b>
		Mean	0.39	0.50	0.21	0.15	0.16
3	35	Max	<b>1.05</b>	<b>0.98</b>	<b>0.45</b>	<b>0.45</b>	<b>0.47</b>
		Mean	0.40	0.43	0.18	0.15	0.15
2	21	Max	<b>1.05</b>	<b>0.68</b>	<b>0.41</b>	<b>0.41</b>	<b>0.42</b>
		Mean	0.36	0.34	0.16	0.14	0.14
1	7	Max	<b>1.09</b>	<b>0.49</b>	<b>0.36</b>	<b>0.25</b>	<b>0.24</b>
		Mean	0.31	0.27	0.12	0.12	0.12
0	1	Max	<b>0.25</b>	<b>0.18</b>	<b>0.03</b>	<b>0.00</b>	<b>0.00</b>
		Mean	0.25	0.18	0.03	0.00	0.00

Note: All the decomposition results are derived directly from the data without density estimation.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

The first row shows the case in which all seven observations strictly between 1985 and 2009 are dropped (i.e., only years 1985 and 2009 are used). Because there is only one combination in this case, the maximum and mean are identical. For example, Table 3 shows that the growth component under the linearity assumption between 1985 and 2009 is different from that in the benchmark case of piecewise linearity assumption by 0.22 percentage points. In the second row, we consider the case in which we drop six observations. Because we keep only one of the seven observations strictly between 1985 and 2009 in this case, there are seven possible combinations as shown in the second column. The benchmark case corresponds to equation (10) with no observation dropped and hence the last row for the equation (10) column is 0 by construction.

We see that generally the last three columns perform better than the KS and JT columns. This is not surprising because both the KS and JT decompositions rely on the assumption that growth and redistribution change sequentially. While the Shapley decomposition is simply the average of these two decompositions, it is close to the benchmark case because it can approximate the linear assumption reasonably well, as argued in Section 2.3. Equation (10) is close to the benchmark by construction, because the assumed path of change is the same as the benchmark case except for the periods that involve the dropped observations. Equation (14) is also close to the benchmark, because the underlying path of changes is similar to equation (10). If the piecewise linearity assumption is an accurate

approximation to the actual change, Table 3 shows that our method is generally better than other methods.

**Table 4: Six-Way Decomposition of  $P_0$  by Time Period**

$t_0$	$t_1$	PS	WR	BR	NG	IF	MC	$\Delta P_0$
1985	1988	0.23	0.41	0.83	-17.92	9.65	3.74	-3.06
1988	1991	-0.29	1.40	2.21	-26.25	21.05	1.95	0.07
1991	1994	0.24	-1.97	-0.23	-12.83	12.74	-2.14	-4.19
1994	1997	-0.13	4.64	1.47	-20.29	9.18	2.43	-2.70
1997	2000	-0.38	0.40	0.14	-7.61	7.84	0.46	0.86
2000	2003	-0.21	0.38	-2.26	-3.71	5.42	-2.13	-2.52
2003	2006	0.20	-0.25	-0.13	-7.00	8.21	0.96	1.99
2006	2009	0.06	-0.55	-1.35	-9.48	8.64	2.77	0.10
All periods		-0.28	4.46	0.67	-105.09	82.73	8.05	-9.45

Note: All the figures for poverty decomposition are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

Another important advantage of our method is that it allows for more detailed decompositions. Unlike the KS, JK, and RD decompositions, we can neatly decompose the poverty change into population shift, within-group redistribution, between-group redistribution, nominal growth, inflation, and methodological change components for each group in the population of interest. While the Shapley decomposition also allows us to produce residual-free decomposition results, it is computationally infeasible to carry out six-way decomposition with regional disaggregation because the number of possible sequences of change is equal to the factorial of the number of components.

In Table 4, we report the results of six-way decomposition of poverty rate described in section 2.6. The last row is the sum of the eight 3-year periods and the last column, which represents the change in poverty rate, is equal to the sum of all six components. There are three important points to note in this table.

First, Table 4 shows that the nominal growth has contributed to a more than 100-percentage-point reduction in poverty rate between 1985 and 2009. This is possible because nominal growth can eliminate poverty created by other factors such as inflation. In fact, Table 4 shows that much of the poverty reduction by nominal growth has been offset by inflation. If we define the effect of real growth as the combined effects of nominal growth and inflation components, we see that the effect of real growth has contributed to the reduction of poverty by more than 20 percentage points between 1985 and 2009 in the Philippines. This overwhelming effect of real growth would not be surprising given that countries with higher growth have tended to reduce poverty at a faster rate. It is also worth noting that much of the real-growth effect has taken place before 1997 and real growth has not contributed to poverty reduction since then.

Second, Table 4 also shows that poverty has increased due to both within-group redistribution and between-group redistribution effects, but the former effect is much larger than the latter. It also shows that their relative importance has changed over time. For example, the main driver of poverty increase due to worsening distribution was between-region inequality for 1988–1991, but it was within-region inequality for 1994–1997. We also see from Table 4 that the between-region inequality has changed favorably for poverty reduction since 2000. We also see that the population shift did not have much impact on the national poverty rate in the Philippines.

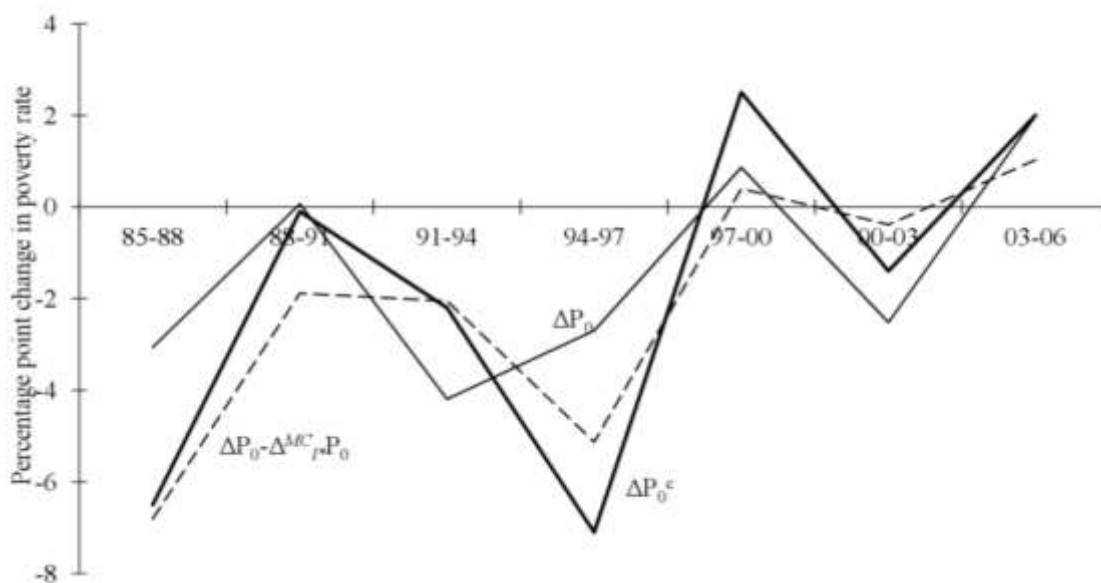
Third, the methodological change component is not negligible. Poverty has increased by as much as 8 percentage points due to this component. The interpretation of the methodological



component is slightly tricky. The methodological component reflects the changes in the quantity of goods at the level of the poverty line. Therefore, if the standards of living at the poverty line go up over time (e.g., because of the inconsistency in the way poverty lines are drawn), poverty goes up even when there is no change in the mean and distribution of income in the Philippines. Hence, it is possible that the methodological change component reflects a spurious change due to methodological inconsistency. It is also possible, however, that the poor have systematically increased consumption of goods that are getting expensive.

The latter possibility, however, is unlikely to be true in the Philippines because the poor typically tend to shift away from goods that are getting rapidly expensive (Fujii 2013). The comparison of the changes in consumption poverty ( $\Delta P_0^C$ ) calculated by Professor Balisacan (see Balisacan [2003] and ADB [2009]) against the “raw” change in poverty rate ( $\Delta P_0$ ) and the one adjusted for the methodological change ( $\Delta P_0 - \Delta_{I^*}^{MC} P_0$ ) also suggest that the methodological change component is indeed spurious. As seen in Figure 5, the changes in our poverty measure with the adjustment for the methodological component (dashed line) are closer to the changes in Balisacan’s consumption poverty rate (bold line) than those without the adjustment (solid line). Because Balisacan uses consumption poverty lines that are supposed to be comparable over time, our results indicate that the slow progress in poverty reduction in our poverty measure is partly because of the increases in the standards of living at the poverty line. Therefore, actual poverty reduction may have been faster than what Figure 4 suggests, once we fix the standards of living at the poverty line.

**Figure 5: Comparison with Balisacan’s Estimate of Consumption Poverty**



Sources: Author’s calculation based on Family Income and Expenditure Survey (FIES) data; Balisacan (2003); ADB (2009).

As discussed in section 2.6, it is possible to decompose the poverty change into six components for each region so that we can pin down the important sources of poverty change. Table 5 shows the regional decomposition results for poverty rate for 1985–2009, which are calculated as the sum of the decomposition results for the eight 3-year periods. By construction, the last row is the same as that in Table 4. The last column of Table 5 is the sum of all six components, which can be interpreted as each region’s contribution to the change in the national poverty rate. Therefore, most regions have made some contribution to the national poverty reduction, with Region VI and Region VII making particularly large contributions.<sup>14</sup> However, some other regions, such as ARMM, have negated some of these

<sup>14</sup> See Table A1 in the Appendix for a description of the regions.

reductions. This observation is still true when we adjust for the MC component. It should also be noted that everything else being equal, each component tends to be larger in absolute value when the region is large. However, our results are not driven by the size of the regions, because Region VI, Region VII, and ARMM are not particularly large regions.

Table 5 also shows that there is substantial heterogeneity in the way each region has contributed to poverty change in the Philippines. Although nominal growth and inflation are the largest components in absolute value, the impact of real growth (NG+IF) on national poverty varies quite substantially, ranging from -2.58 in Region VI to -0.07 in ARMM. The magnitude of within-group and between-group inequality also varies over regions. We find that the within-group redistribution component has contributed to an increase in the poverty rate in most regions, whereas the impact of the between-group redistribution component is quite diverse. While we only discussed the six-way decomposition results for the poverty rate, the results for the poverty gap, poverty severity, and Watts measure, which are reported in Appendix D, are qualitatively similar.

**Table 5: Six-Way Decomposition of  $P_0$  by Region**

Region	PS	WR	BR	NG	IF	MC	$\Delta P_0$
NCR	-0.06	-0.02	-0.53	-6.15	5.21	0.37	-1.17
CAR	-0.06	0.21	0.14	-1.75	1.32	0.20	0.05
Region I	-0.19	0.22	0.29	-6.93	5.23	0.84	-0.55
Region II	-0.17	0.36	-0.04	-4.85	3.73	0.32	-0.66
Region III	0.17	-0.27	0.66	-9.98	7.69	1.20	-0.53
Region IV-A	0.50	0.40	-0.42	-8.84	7.06	0.66	-0.64
Region IV-B	0.12	-0.12	0.23	-3.46	2.59	0.60	-0.04
Region V	-0.27	0.68	-0.18	-9.01	7.22	0.45	-1.12
Region VI	-0.14	0.32	-0.22	-10.82	8.23	0.64	-1.98
Region VII	0.16	0.22	-0.76	-8.31	6.79	0.19	-1.71
Region VIII	-0.22	0.89	-0.68	-6.53	5.12	0.53	-0.89
Region IX	-0.07	0.24	0.28	-4.03	3.09	0.23	-0.26
Region X	-0.10	0.26	0.22	-5.15	3.94	0.45	-0.39
Region XI	-0.07	0.29	0.25	-5.62	4.17	0.61	-0.36
Region XII	0.30	0.58	0.07	-4.73	3.23	0.79	0.23
ARMM	-0.09	-0.34	1.33	-5.27	5.20	-0.27	0.57
Caraga	-0.11	0.53	0.05	-3.66	2.92	0.24	-0.03
Philippines	-0.28	4.46	0.67	-105.09	82.73	8.05	-9.45

Note: All the figures for poverty decomposition are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

## 5. DISCUSSION

In this paper, we have proposed a method of dynamic poverty decomposition that is subperiod-additive and time-reversion consistent. Our decomposition analysis consistently integrates the conventional dynamic poverty decomposition such as Datt and Ravallion (1992) and group-based decomposition such as Ravallion and Huppi (1991). Our method has an additional advantage in that there are no residual or interaction terms. While our method requires the specification of the path of change, we have provided a practical way to implement the decomposition under a set of reasonable assumptions.

In our empirical application to the Philippines, we considered a six-way decomposition in which the national poverty change is decomposed into population shift, within-group redistribution, between-group redistribution, nominal growth, inflation, and methodological change components for each of the 17 regions in the Philippines.

We find that nominal growth and inflation are by far the largest components in absolute value in each region and that the impacts of other components are heterogeneous, which indicates that the appropriate poverty reduction policies may vary from region to region. For example, the results reported in Table 5 suggest that some regions, such as ARMM, would require growth-enhancing policies to reduce poverty effectively, whereas other regions, such as Region VIII, may need policies to improve the income distribution within the region. We also find that poverty reduction in the Philippines has been slowed substantially by worsening inequality for the periods 1988–1991 and 1994–1997. For other periods, the apparent slow progress in poverty reduction was mostly because of the lack of real economic growth but also partly because of the methodological change.

In this study, we chose regions as a unit of the group for empirical illustration, because the Philippines is spatially heterogeneous in terms of consumption patterns, growth rate, and inflation rate. However, our analysis can also be applied to a number of other issues by using other variables as a unit of group, such as the ethnic groups, the education of household head, the employment status or sector of the household head, and the household size. Using these variables, we can expand the scope of the standard poverty profile approach. That is, instead of simply comparing the poverty rate, poverty gap, and poverty severity across different groups for various years, as is done in the standard poverty profile approach, using our method, we can decompose the change in national poverty into various components for each group in the population. Hence, our decomposition method can complement and enhance the usefulness of the poverty profile approach.

While we chose to apply our method to a developing country, it is also applicable to poverty analysis in developed countries, where poverty lines are typically drawn separately for each household category and defined as a fraction of mean or median income. Therefore, even if everyone's income is rising, poverty may still increase if the income at the center of the distribution rises faster than incomes in the lower tail of the distribution. Using our method (with a slight modification), it is possible to separate the effects of increasing income at the bottom and middle on poverty.

It should also be noted that our method does not necessarily require all the components of interest to independently vary over time. For example, if inequality determines growth through the process of human and physical capital accumulation (e.g., Galor and Moav 2004) while  $z$  is held constant, we would have the following relationship instead of, say, equation (8) in Assumption 8:  $d\tilde{z}(t)/dt = J(\tilde{F}(\tilde{y}, t))$  for some function  $J(\cdot)$  with its definite integral from  $t = t_0$  to  $t = t_1$  being  $\tilde{z}_1 - \tilde{z}_0$ . In a case like this, poverty change is completely determined by the change in income distribution. However, it is still possible to identify the growth and redistribution components in our approach, because Proposition 1 still holds and thus the growth component can be computed by simply replacing  $d\tilde{z}(t)/dt$  by  $J(\tilde{F}(\tilde{y}, t))$  in equation (6). Similarly, it is also possible to consider the effects of demographic dividend by explicitly modeling the relationship between demographic characteristics of the population and economic growth.

As with other decomposition methods, our decomposition is descriptive and essentially an accounting exercise. Therefore, it does not in general describe the causal relationship between poverty and the components of interest. However, the discussion in the preceding paragraph means that we can potentially make a causal inference if the underlying relationship between components is known.<sup>15</sup> Even when no such relationship is known, our

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<sup>15</sup> Note, though, that we are still agnostic about the driving force of the change in  $\tilde{F}(\tilde{y}, t)$ .

decomposition still provides researchers and policy makers with information that is useful for understanding the source of poverty change and formulating poverty reduction policies.

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## APPENDIX

### A. Details of Data and Poverty Measurement

In the official methodology, the poverty lines are set at the level where one can satisfy some basic nutritional requirements and meet some non-food needs. The methodology to calculate the poverty line, however, has been revised three times: in 1992, 2003, and 2011.<sup>1</sup> The official poverty statistics based on the 1992 and 2003 revisions are available for 1985–2000 and 2000–2006, respectively. Those based on the 2011 revision are currently available for 1991, 2003, 2006, and 2009.

We adopt the 2011 revision with some modifications. Because poverty lines based on the 1992 revision are available only at the regional level, we use the regional (and not provincial) poverty line for all years. Further, we recalculate the aggregate income for all rounds of the Family Income and Expenditure Survey (FIES) because there was a minor inconsistency in the definition of incomes. As a result, the poverty statistics used in our study are not exactly the same as the official statistics. However, these modifications have little impact on the resulting poverty statistics.

To obtain the poverty lines for earlier years, we first calculate the rate of change in poverty line between two contiguous survey rounds for each region. When more than one rate is available for a given period, we use the harmonic mean for that period. Because some regions did not exist in earlier years, we instead use the rate for the island group to which the region belongs. Using these rules, we extrapolate poverty lines for earlier years, which are presented in Table A1.

To carry out the six-way decomposition, we need to define the set of goods used in the calculation of poverty lines. Based on the availability of data, we set  $J = 10$  and use the following 10 goods: (i) food, (ii) alcohol and tobacco, (iii) clothing and footwear, (iv) housing and utility, (v) furniture, household equipment, and operation, (vi) medical care, (vii) transportation and communication, (viii) recreation, (ix) education, and (x) miscellaneous goods and services. The consumption expenditure data in FIES are aggregated up to these 10 goods. The consumer price index (CPI) data are also aggregated up to this level using the CPI weights.

For all the years except 2009, the reference year for the CPI data is year 2000. Because we could only obtain base-2006 CPI data for 2009, we simply multiply base-2000 CPI figures for 2006 by base-2006 CPI figures for 2009 to obtain an estimate of base-2000 CPI figures for 2009. For years before 1988, the CPI has only six types of goods. For example, we have the CPI data for “food, alcohol, and tobacco,” without the breakdown for “food” and “alcohol and tobacco.” Therefore, we assume that the CPI change is the same for these two categories. We apply a similar rule for other aggregate categories, too.

The price  $p_j^g$  of good  $j$  in region  $g$  is taken from the annual average CPI for good  $j$  in region  $g$ . Because the bundles of goods underlying the poverty lines are not available, we first take the average expenditure share for those households within 10% of the poverty line in region  $g$  for each survey year. Then, multiplying it by the poverty line and dividing it by  $p_j^g$ , we obtain the quantity  $q_j^g$  of good  $j$  in region  $g$  at the poverty line for each survey year.

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<sup>1</sup> See Asian Development Bank (2005) and National Statistical Coordination Board (2011) for the details of these changes.

**Table A1: Income Poverty Lines in the Philippines Used for This Study**

Region	Description of Region	1985	1988	1991	1994	1997	2000	2003	2006	2009
NCR	National Capital Region	3,360	4,881	6,892	8,335	10,613	12,873	13,704	16,487	19,802
CAR	Cordillera Administrative Region	2,669	3,456	5,629	7,333	8,672	10,356	10,841	12,976	16,122
Region I	Ilocos Region	2,999	3,920	6,404	7,962	9,514	11,536	11,835	14,350	17,768
Region II	Cagayan Valley	2,765	3,667	5,641	6,668	7,922	9,809	10,250	12,212	15,306
Region III	Central Luzon	3,126	4,207	6,559	7,830	9,501	11,846	12,756	15,374	18,981
Region IV-A	CALARARZON	3,002	3,823	6,389	7,546	9,852	11,798	12,180	14,284	17,779
Region IV-B	MIMAROPA	2,431	3,096	5,174	6,110	7,978	9,554	10,397	12,610	15,769
Region V	Bicol Region	2,928	3,533	5,444	7,093	8,849	10,640	11,559	13,645	17,146
Region VI	Western Visayas	2,876	3,399	5,010	6,414	8,263	9,653	9,799	12,432	16,036
Region VII	Central Visayas	3,061	3,437	5,172	5,950	8,073	10,133	11,488	14,468	17,848
Region VIII	Eastern Visayas	2,650	3,081	4,147	5,201	7,043	8,420	9,600	11,885	15,910
Region IX	Zamboanga Peninsula	2,774	2,988	5,003	5,573	7,667	8,736	9,647	11,810	15,160
Region X	Northern Mindanao	2,770	3,533	5,026	6,201	8,156	9,197	10,200	12,987	16,568
Region XI	Davao Region	2,965	3,966	5,322	6,670	8,542	9,907	10,731	13,469	17,040
Region XII	SOCCSKSARGEN	2,850	3,218	5,681	6,961	8,628	9,657	10,368	12,530	15,762
Caraga	Caraga Region Autonomous Region in Muslim	2,938	3,585	5,468	6,637	8,595	10,039	10,478	12,935	16,858
ARMM	Mindanao	2,566	3,130	4,775	5,697	7,136	8,810	9,693	12,358	16,334

Note: Poverty lines are expressed as the annual household income per capita in Philippine pesos.

Source: Author's calculation based on poverty lines published in National Statistics Office (NSO) webpages and publications.

## B. Proofs of Propositions

**Proof of Proposition 1:** Note first that  $y/z = \tilde{y}/\tilde{z}$  holds. Therefore, using the change of variables, we have  $M(\tilde{F}(\tilde{y}, t), \tilde{z}(t)) = M(F(y, t), z(t))$  for all  $t$ . Using this and because the poverty change between  $t_0$  and  $t_1$  can be written as the integral of the time derivative of  $M(t)$ , we can make the following transformation:

$$\begin{aligned}
\Delta M(t_0, t_1) &= \int_{t_0}^{t_1} \frac{dM(t)}{dt} dt = \int_{t_0}^{t_1} \frac{d}{dt} \left[ \int_0^{\tilde{z}} g(\tilde{y}/\tilde{z}) \tilde{f}(\tilde{y}, t) d\tilde{y} \right] dt \\
&= \int_{t_0}^{t_1} \left[ \int_0^{\tilde{z}} g\left(\frac{\tilde{y}}{\tilde{z}}\right) \frac{\partial \tilde{f}(\tilde{y}, t)}{\partial t} d\tilde{y} \right] dt + \int_{t_0}^{t_1} \left[ \frac{\partial}{\partial \tilde{z}} \left[ \int_0^{\tilde{z}} g(\tilde{y}/\tilde{z}) \tilde{f}(\tilde{y}, t) d\tilde{y} \right] \frac{d\tilde{z}}{dt} \right] dt \\
&= \Delta_*^{RD} M(t_0, t_1) + \Delta_*^{GR} M(t_0, t_1), \tag{B1}
\end{aligned}$$

where the third line follows from the chain rule. It is clear from equation (B1) that  $(C, \{\Delta_*^c M(t_0, t_1)\}_{c \in C})$  is a poverty decomposition. The time-reversion consistency and subperiod additivity follow immediately from the basic properties of integrals.

**Proof of Proposition 2:** By setting  $\alpha = 1$  and using integration by parts in equation (2), we have:

$$P_1(\tilde{F}(\cdot), \tilde{z}) = \tilde{z}^{-1} \int_0^{\tilde{z}} \tilde{F}(\tilde{y}) d\tilde{y}. \tag{B2}$$



Let us write the probability density function for  $\tilde{F}_a$  by  $\tilde{f}_a$ . Then, noting that we have  $g(\cdot) = 1$  for the poverty rate measure and substituting equations (7) and (8) in equation (5), we have:

$$\begin{aligned} \Delta_{l^*}^{RD} M(t_0, t_1) &= \int_{t_0}^{t_1} \left[ \int_0^{\tilde{z}} \frac{\tilde{f}(\tilde{y}, t_1) - \tilde{f}(\tilde{y}, t_0)}{t_1 - t_0} d\tilde{y} \right] dt = \int_{\tilde{z}_0}^{\tilde{z}_1} \left[ \frac{\tilde{F}(\tilde{z}, t_1) - \tilde{F}(\tilde{z}, t_0)}{\tilde{z}_1 - \tilde{z}_0} \right] d\tilde{z} \\ &= \frac{\tilde{z}_1 P_1(\tilde{F}_1, \tilde{z}_1) - \tilde{z}_0 P_1(\tilde{F}_1, \tilde{z}_0) - \tilde{z}_1 P_1(\tilde{F}_0, \tilde{z}_1) + \tilde{z}_0 P_1(\tilde{F}_0, \tilde{z}_0)}{\tilde{z}_1 - \tilde{z}_0}, \end{aligned} \quad (\text{B3})$$

where the second and third lines follow from the change of variables and equation (B2), respectively. The result for  $\Delta_{l^*}^{GR} M(t_0, t_1)$  follows immediately from this.

**Proof of Proposition 3:** The proof is similar to that of Proposition 2. Let  $\tilde{\phi}(\tilde{\eta}, t)$  be the probability density function of  $\tilde{\eta}$  at time  $t$ , which satisfies  $\tilde{\phi}(\tilde{\eta}, t) = \tilde{f}(\tilde{y}, t)\tilde{y}$ . Therefore, the Watts poverty measure satisfies the following relationship:

$$\begin{aligned} W(\tilde{F}(\cdot, t), \tilde{z}) &= \int_0^{\tilde{z}} (\ln \tilde{z} - \ln \tilde{y}) \tilde{f}(\tilde{y}, t) d\tilde{y} = \int_{-\infty}^{\tilde{z}} (\tilde{z} - \tilde{\eta}) \tilde{\phi}(\tilde{\eta}, t) d\tilde{\eta} \\ &= \left[ (\tilde{z} - \tilde{\eta}) \tilde{\Phi}(\tilde{\eta}, t) \right]_{\tilde{\eta}=-\infty}^{\tilde{\eta}=\tilde{z}} + \int_{-\infty}^{\tilde{z}} \tilde{\Phi}(\tilde{\eta}, t) d\tilde{\eta} = \int_{-\infty}^{\tilde{z}} \tilde{\Phi}(\tilde{\eta}, t) d\tilde{\eta}, \end{aligned} \quad (\text{B4})$$

where the first term in the third line drops because  $\tilde{\phi}(\tilde{\eta}, t) = 0$  for all  $\tilde{\eta} < \ln \varepsilon$  and  $t$ .

Differentiating equation (11) by  $\tilde{\eta}$  and  $t$  and using  $\tilde{\phi}(\tilde{\eta}, t) = \tilde{f}(\tilde{y}, t)\tilde{y}$ , we have:

$$\frac{d\tilde{f}(\tilde{y}, t)}{dt} = \frac{\tilde{\phi}_1(\tilde{\eta}) - \tilde{\phi}_0(\tilde{\eta})}{t_1 - t_0} e^{-\tilde{\eta}} \quad (\text{B5})$$

Therefore, by  $g(\cdot) = 1$ , equations (B4) and (B5), and the change of variables, we have the following result:

$$\begin{aligned} \Delta_{l^*}^{RD} M(t_0, t_1) &= \int_{t_0}^{t_1} \left[ \int_0^{\tilde{z}(t)} \frac{d\tilde{f}(\tilde{y}, t)}{dt} d\tilde{y} \right] dt = \int_{t_0}^{t_1} \left[ \int_{-\infty}^{\tilde{z}(t)} \frac{\tilde{\phi}_1(\tilde{\eta}) - \tilde{\phi}_0(\tilde{\eta})}{t_1 - t_0} d\tilde{\eta} \right] dt \\ &= \int_{t_0}^{t_1} \frac{\tilde{\Phi}_1(\tilde{\zeta}(t)) - \tilde{\Phi}_0(\tilde{\zeta}(t))}{t_1 - t_0} dt = \int_{\tilde{\zeta}_0}^{\tilde{\zeta}_1} \frac{\tilde{\Phi}_1(\tilde{\zeta}) - \tilde{\Phi}_0(\tilde{\zeta})}{\tilde{\zeta}_1 - \tilde{\zeta}_0} d\tilde{\zeta} \\ &= \frac{W(\tilde{F}_1, \tilde{z}_1) - W(\tilde{F}_1, \tilde{z}_0) - W(\tilde{F}_0, \tilde{z}_1) + W(\tilde{F}_0, \tilde{z}_0)}{\tilde{\zeta}_1 - \tilde{\zeta}_0}, \end{aligned} \quad (\text{B6})$$

proving equation (13). Equation (14) immediately follows from this.

**Proof of Proposition 4:** The proof is similar to that of Proposition 1. First, by  $\hat{z}^g = \sum_j p_j^g q_j^g / \mu \hat{\mu}^g$ , we have the following relationship:

$$\frac{d\hat{z}^g}{dt} = -\frac{\hat{z}^g}{\hat{\mu}^g} \frac{d\hat{\mu}^g}{dt} - \frac{\hat{z}^g}{\mu} \frac{d\mu}{dt} + \frac{1}{\mu \hat{\mu}^g} \sum_{j=1}^J \frac{q_j^g dp_j^g}{dt} + \frac{1}{\mu \hat{\mu}^g} \sum_{j=1}^J \frac{p_j^g dq_j^g}{dt}. \quad (\text{B7})$$

Now, consider the time-derivative of  $M(t)$ :

$$\begin{aligned} \frac{dM(t)}{dt} &= \frac{d}{dt} \left[ \sum_g w^g(t) \int_0^{z^g} g\left(\frac{y}{z^g}\right) f^g(y, t) dy \right] \\ &= \frac{d}{dt} \left[ \sum_g w^g(t) \int_0^{\hat{z}^g} g\left(\frac{\hat{y}}{\hat{z}^g}\right) \hat{f}^g(\hat{y}, t) d\hat{y} \right] \\ &= \sum_g \left[ \frac{dw^g(t)}{dt} \int_0^{\hat{z}^g} g\left(\frac{\hat{y}}{\hat{z}^g}\right) \hat{f}^g(\hat{y}, t) d\hat{y} + w^g(t) \int_0^{\hat{z}^g} g\left(\frac{\hat{y}}{\hat{z}^g}\right) \frac{\partial \hat{f}^g(\hat{y}, t)}{\partial t} d\hat{y} \right. \\ &\quad \left. + \frac{d}{d\hat{z}^g} \left[ \int_0^{\hat{z}^g} g\left(\frac{\hat{y}}{\hat{z}^g}\right) \hat{f}^g(\hat{y}, t) d\hat{y} \right] \cdot \left[ \frac{d\hat{z}^g}{dt} \right] \right] \end{aligned}$$

Substituting equation (B7) in the equation above and integrating the equation above over  $t \in [t_0, t_1]$ , we obtain  $\Delta M = \sum_c \Delta_c^* M$ , proving that  $(C, \{\Delta_c^* M(t_0, t_1)\})$  is a poverty decomposition. The time-reversion consistency and subperiod additivity follows immediately from the properties of integration.

### C. Technical Appendix for Footnote 11

In this section, we describe the implementation of numerical integration for  $\gamma < 1$  discussed in Footnote 11. To simplify the presentation, we denote the expression in the square bracket in equation (6) by  $I(t)$ . We approximate this by  $I(t) = I_0 + \tau I_1$  for  $\tau \equiv \frac{t-t_0}{t_1-t_0}$ , where  $I_0 = I(t_0) = I(b_1)$  and  $I_1 = I(t_1)$ . Then, letting  $d \equiv \frac{b_2-t_0}{t_1-t_0}$ , we have:

$$\begin{aligned} \int_{b_1}^{b_2} I(t) \frac{d\tilde{z}}{dt} dt &= \int_0^d (I_0 + \tau I_1) \frac{d\tilde{z}}{d\tau} d\tau \\ &= \int_0^d (I_0 + \tau I_1) \gamma \tau^{\gamma-1} (\tilde{z}_1 - \tilde{z}_0) d\tau \\ &= d^\gamma \left( I_0 - \frac{d\gamma I_1}{\gamma+1} \right) (\tilde{z}_1 - \tilde{z}_0). \end{aligned}$$

We use this formula for the calculation of the integral over the first interval.

### D. Additional Tables

Table D1 provides various types of errors discussed in Footnote 12. In the third column,  $\varepsilon(\delta_*^{GR})$ , we take the absolute difference between the direct calculation of equation (9) and its counterpart obtained by numerical integration using equation (5). The fourth column gives the absolute difference between the observed change in  $P_0$  and the sum of redistribution and growth components obtained from numerical integration. The fifth, sixth, and seventh columns are the corresponding differences for  $P_1$ ,  $P_2$ , and  $W$ , respectively. The last four columns are the absolute difference between the observed change and the sum of six components obtained by numerical integration for various poverty measures.

Table D2 shows the effect of bandwidth on the poverty measures and decomposition results, where ‘‘Direct’’ refers to the direct calculation that does not rely on kernel density estimation. We see that the poverty estimates with our benchmark bandwidth of  $b = 0.01$  are similar to the direct calculation results.

Tables D3, D4, and D5 are the same as Table 4, except that we use  $P_1$ ,  $P_2$ , and  $W$  instead of  $P_0$  as the poverty measure. Similarly, Tables D6, D7, and D8 are the same as Table 5 except for the poverty measure used.

**Table D1: Various Types of Errors**

$t_0$	$t_1$	$\varepsilon(\Delta_*^{GR})$	$\varepsilon(\Delta_* P_0)$	$\varepsilon(\Delta_* P_1)$	$\varepsilon(\Delta_* P_2)$	$\varepsilon(\Delta_* W)$	$\varepsilon(\Delta_{**} P_0)$	$\varepsilon(\Delta_{**} P_1)$	$\varepsilon(\Delta_{**} P_2)$	$\varepsilon(\Delta_{**} W)$
1985	1988	-0.002	-0.002	0.000	0.000	0.000	-0.008	0.006	0.005	0.015
1988	1991	0.003	0.003	0.000	0.000	0.000	0.011	0.005	0.005	0.011
1991	1994	0.001	0.001	0.000	0.000	0.000	-0.021	-0.009	-0.009	-0.024
1994	1997	0.000	0.000	0.000	0.000	0.000	0.015	0.003	0.004	0.009
1997	2000	-0.004	-0.003	0.000	0.000	0.000	-0.009	0.000	0.000	-0.001
2000	2003	0.004	0.003	0.000	0.000	0.000	0.006	0.000	0.000	0.000
2003	2006	-0.004	-0.004	0.000	0.000	0.000	-0.004	-0.002	-0.002	-0.004
2006	2009	0.001	0.001	0.000	0.000	0.000	-0.005	-0.001	-0.001	-0.004
All periods		-0.001	-0.001	-0.001	0.000	-0.001	-0.016	0.002	0.001	0.001

Note: All the errors are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

**Table D2: Linear and Log-Linear Decomposition under Various Bandwidths**

$t_0$	$t_1$	Bandwidth	$P_0(t_0)$	$P_1(t_0)$	$P_2(t_0)$	$W(t_0)$	$\Delta P_0$	eq. (9)	eq. (13)
1985	1988	Direct	35.83	11.12	4.81	15.05	-3.05	-2.82	-2.82
1985	1988	b=0.005	35.82	11.12	4.81	15.05	-3.06	-2.83	-2.83
1985	1988	b=0.010	35.83	11.13	4.82	15.08	-3.07	-2.82	-2.82
1985	1988	b=0.015	35.82	11.14	4.84	15.11	-3.05	-2.78	-2.78
1988	1991	Direct	32.78	9.66	3.96	12.78	0.05	-3.16	-3.16
1988	1991	b=0.005	32.76	9.66	3.96	12.78	0.08	-3.14	-3.14
1988	1991	b=0.010	32.76	9.66	3.97	12.80	0.08	-3.15	-3.15
1988	1991	b=0.015	32.76	9.68	3.98	12.83	0.07	-3.16	-3.16
1991	1994	Direct	32.84	9.89	4.17	13.25	-4.18	-2.42	-2.42
1991	1994	b=0.005	32.85	9.89	4.17	13.26	-4.20	-2.46	-2.46
1991	1994	b=0.010	32.84	9.90	4.18	13.28	-4.21	-2.49	-2.49
1991	1994	b=0.015	32.83	9.92	4.20	13.32	-4.21	-2.49	-2.49
1994	1997	Direct	28.66	8.44	3.51	11.24	-2.67	-8.46	-8.46
1994	1997	b=0.005	28.65	8.44	3.51	11.25	-2.69	-8.48	-8.48
1994	1997	b=0.010	28.63	8.45	3.52	11.27	-2.68	-8.47	-8.47
1994	1997	b=0.015	28.62	8.47	3.54	11.31	-2.69	-8.47	-8.46
1997	2000	Direct	25.99	7.60	3.11	10.04	0.81	0.98	0.98
1997	2000	b=0.005	25.96	7.60	3.12	10.05	0.84	0.98	0.98
1997	2000	b=0.010	25.95	7.61	3.13	10.08	0.85	0.98	0.98
1997	2000	b=0.015	25.94	7.63	3.15	10.13	0.87	0.97	0.97
2000	2003	Direct	26.81	7.82	3.18	10.31	-2.55	-1.02	-1.02
2000	2003	b=0.005	26.80	7.82	3.18	10.32	-2.53	-1.01	-1.01
2000	2003	b=0.010	26.80	7.83	3.20	10.34	-2.51	-1.00	-1.00
2000	2003	b=0.015	26.81	7.85	3.22	10.39	-2.51	-1.01	-1.01
2003	2006	Direct	24.25	7.06	2.90	9.35	2.02	1.73	1.73
2003	2006	b=0.005	24.27	7.07	2.90	9.36	1.99	1.70	1.70
2003	2006	b=0.010	24.29	7.08	2.91	9.39	1.98	1.71	1.71
2003	2006	b=0.015	24.30	7.10	2.93	9.43	1.98	1.73	1.73
2006	2009	Direct	26.27	7.59	3.03	9.93	0.08	1.04	1.04
2006	2009	b=0.005	26.26	7.59	3.04	9.94	0.11	1.07	1.07
2006	2009	b=0.010	26.27	7.60	3.05	9.97	0.09	1.08	1.08
2006	2009	b=0.015	26.28	7.62	3.07	10.01	0.06	1.08	1.08

Note:  $P_0$ ,  $P_1$ ,  $P_2$ , and  $W$  are expressed in percentage and  $\Delta P_0$ , eq. (9), and eq. (13) are in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

**Table D3: Six-Way Decomposition of  $P_1$  by Time Period**

$t_0$	$t_1$	PS	WR	BR	NG	IF	MC	$\Delta P_1$
1985	1988	0.08	0.16	0.43	-7.15	3.75	1.25	-1.47
1988	1991	-0.11	0.67	1.04	-11.07	8.86	0.84	0.23
1991	1994	0.09	-0.42	-0.27	-5.21	5.13	-0.76	-1.44
1994	1997	-0.04	2.05	0.55	-8.03	3.65	0.99	-0.84
1997	2000	-0.11	0.11	0.02	-3.06	3.13	0.13	0.22
2000	2003	-0.07	0.42	-1.00	-1.45	2.12	-0.77	-0.76
2003	2006	0.06	-0.34	-0.10	-2.77	3.26	0.40	0.52
2006	2009	0.02	-0.58	-0.57	-3.68	3.38	1.08	-0.34
All periods		-0.09	2.06	0.11	-42.41	33.28	3.17	-3.87

Note: All the figures for poverty decomposition are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

**Table D4: Six-Way Decomposition of  $P_2$  by Time Period**

$t_0$	$t_1$	PS	WR	BR	NG	IF	MC	$\Delta P_2$
1985	1988	0.04	0.02	0.24	-3.59	1.89	0.54	-0.86
1988	1991	-0.05	0.39	0.54	-5.45	4.36	0.42	0.21
1991	1994	0.04	-0.13	-0.16	-2.57	2.52	-0.35	-0.65
1994	1997	-0.02	1.02	0.26	-3.92	1.79	0.48	-0.39
1997	2000	-0.04	0.01	0.01	-1.50	1.53	0.05	0.07
2000	2003	-0.03	0.30	-0.52	-0.70	1.04	-0.36	-0.28
2003	2006	0.03	-0.26	-0.08	-1.35	1.59	0.20	0.14
2006	2009	0.01	-0.34	-0.27	-1.75	1.62	0.51	-0.22
All periods		-0.03	1.01	0.03	-20.82	16.32	1.50	-2.00

Note: All the figures for poverty decomposition are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

**Table D5: Six-Way Decomposition of  $W$  by Time Period**

$t_0$	$t_1$	PS	WR	BR	NG	IF	MC	$\Delta W$
1985	1988	0.11	0.12	0.66	-10.25	5.39	1.68	-2.29
1988	1991	-0.15	1.05	1.52	-15.71	12.57	1.21	0.47
1991	1994	0.13	-0.51	-0.41	-7.42	7.29	-1.05	-1.98
1994	1997	-0.06	2.90	0.76	-11.37	5.17	1.40	-1.20
1997	2000	-0.14	0.10	0.04	-4.32	4.43	0.16	0.26
2000	2003	-0.10	0.71	-1.46	-2.04	3.00	-1.07	-0.96
2003	2006	0.09	-0.59	-0.18	-3.90	4.60	0.57	0.59
2006	2009	0.02	-0.88	-0.79	-5.12	4.72	1.51	-0.55
All periods		-0.11	2.88	0.14	-60.15	47.17	4.41	-5.66

Note: All the figures for poverty decomposition are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

**Table D6: Six-Way Decomposition of  $P_1$  by Region**

Region	PS	WR	BR	NG	IF	MC	$\Delta P_1$
NCR	-0.01	-0.02	-0.13	-1.41	1.20	0.09	-0.29
CAR	-0.02	0.15	0.01	-0.75	0.58	0.07	0.04
Region I	-0.05	0.16	0.12	-2.73	2.08	0.29	-0.13
Region II	-0.04	0.12	0.00	-1.58	1.21	0.09	-0.20
Region III	0.04	-0.03	0.22	-3.08	2.31	0.40	-0.13
Region IV-A	0.13	0.16	-0.17	-2.93	2.34	0.25	-0.22
Region IV-B	0.04	-0.08	0.10	-1.61	1.21	0.28	-0.06
Region V	-0.08	0.29	-0.09	-4.75	3.81	0.25	-0.57
Region VI	-0.03	0.20	-0.07	-4.53	3.38	0.29	-0.76
Region VII	0.05	0.05	-0.33	-4.06	3.38	0.01	-0.90
Region VIII	-0.06	0.39	-0.31	-2.86	2.23	0.21	-0.40
Region IX	-0.02	0.17	0.09	-1.98	1.53	0.11	-0.12
Region X	-0.03	0.09	0.12	-2.60	2.05	0.20	-0.17
Region XI	-0.02	0.15	0.11	-2.32	1.72	0.25	-0.11
Region XII	0.10	0.22	0.03	-2.07	1.44	0.32	0.04
ARMM	-0.02	-0.19	0.34	-1.45	1.48	-0.05	0.11
Caraga	-0.04	0.24	0.05	-1.71	1.35	0.10	0.00
Philippines	-0.09	2.06	0.11	-42.41	33.28	3.17	-3.87

Note: All the figures for poverty decomposition are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

**Table D7: Six-Way Decomposition of  $P_2$  by Region**

Region	PS	WR	BR	NG	IF	MC	$\Delta P_2$
NCR	0.00	-0.02	-0.05	-0.48	0.41	0.03	-0.11
CAR	-0.01	0.08	0.00	-0.38	0.30	0.03	0.02
Region I	-0.02	0.10	0.06	-1.29	0.98	0.13	-0.05
Region II	-0.02	0.06	0.01	-0.72	0.55	0.04	-0.08
Region III	0.02	0.00	0.10	-1.30	0.97	0.17	-0.05
Region IV-A	0.05	0.07	-0.07	-1.27	1.01	0.11	-0.10
Region IV-B	0.01	-0.05	0.05	-0.79	0.59	0.14	-0.04
Region V	-0.03	0.16	-0.05	-2.59	2.07	0.13	-0.30
Region VI	-0.01	0.08	-0.03	-2.10	1.56	0.13	-0.37
Region VII	0.02	0.02	-0.16	-2.27	1.90	-0.02	-0.51
Region VIII	-0.02	0.18	-0.15	-1.43	1.10	0.11	-0.21
Region IX	-0.01	0.09	0.04	-1.08	0.84	0.05	-0.06
Region X	-0.01	0.04	0.06	-1.40	1.10	0.11	-0.10
Region XI	-0.01	0.09	0.06	-1.18	0.88	0.13	-0.04
Region XII	0.04	0.10	0.02	-1.05	0.73	0.16	0.00
ARMM	-0.01	-0.10	0.13	-0.56	0.58	-0.02	0.02
Caraga	-0.02	0.12	0.04	-0.94	0.74	0.05	0.00
Philippines	-0.03	1.01	0.03	-20.82	16.32	1.50	-2.00

Note: All the figures for poverty decomposition are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.

**Table D8: Six-Way Decomposition of *W* by Region**

<b>Region</b>	<b>PS</b>	<b>WR</b>	<b>BR</b>	<b>NG</b>	<b>IF</b>	<b>MC</b>	<b><math>\Delta W</math></b>
NCR	-0.01	-0.04	-0.17	-1.76	1.50	0.11	-0.37
CAR	-0.02	0.22	0.01	-1.09	0.85	0.10	0.06
Region I	-0.07	0.25	0.17	-3.80	2.89	0.40	-0.17
Region II	-0.05	0.17	0.01	-2.18	1.67	0.12	-0.26
Region III	0.06	-0.03	0.30	-4.11	3.07	0.53	-0.17
Region IV-A	0.17	0.22	-0.23	-3.93	3.13	0.33	-0.31
Region IV-B	0.05	-0.13	0.14	-2.27	1.70	0.41	-0.10
Region V	-0.11	0.44	-0.13	-7.02	5.62	0.36	-0.84
Region VI	-0.04	0.25	-0.09	-6.23	4.64	0.40	-1.08
Region VII	0.06	0.06	-0.47	-6.14	5.12	-0.02	-1.38
Region VIII	-0.08	0.54	-0.43	-4.07	3.15	0.30	-0.59
Region IX	-0.03	0.24	0.11	-2.95	2.30	0.15	-0.18
Region X	-0.04	0.11	0.17	-3.85	3.03	0.31	-0.27
Region XI	-0.03	0.23	0.16	-3.34	2.48	0.35	-0.15
Region XII	0.13	0.29	0.05	-2.99	2.07	0.47	0.02
ARMM	-0.03	-0.28	0.44	-1.88	1.92	-0.06	0.12
Caraga	-0.05	0.34	0.09	-2.54	2.01	0.15	0.00
Philippines	-0.11	2.88	0.14	-60.15	47.17	4.41	-5.66

Note: All the figures for poverty decomposition are expressed in percentage points.

Source: Author's calculation based on Family Income and Expenditure Survey (FIES) data.